b. Find the expected change in the mean conversion for a unit change in temperature x_1 for model 1 when $x_2 = 5$. Does this quantity depend on the specific value of reaction time selected? Why?

c. Find the expected change in the mean conversion for a unit change in temperature x_1 for model 2 when $x_2 = 5$. Repeat this calculation for $x_2 = 2$ and $x_2 = 8$. Does the result depend on the value selected for x_2? Why?

3.22 Show that an equivalent way to perform the test for significance of regression in multiple linear regression is to base the test on R^2 as follows: To test H_0: $\beta_1 = \beta_2 = \ldots = \beta_k$ versus H_1: at least one $\beta_j \neq 0$, calculate

$$F_0 = \frac{R^2(n-p)}{k(1-R^2)}$$

and to reject H_0 if the computed value of F_0 exceeds $F_{\alpha,k,n-p}$, where $p = k + 1$.

3.23 Suppose that a linear regression model with $k = 2$ regressors has been fit to $n = 25$ observations and $R^2 = 0.90$.

$$SS_R = \sum_{i=1} \hat{y}_i^2 - n\bar{y}^2$$

3.25 Consider the multiple linear regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

Using the procedure for testing a general linear hypothesis, show how to test

a. H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta$

b. H_0: $\beta_1 = \beta_2$, $\beta_3 = \beta_4$

c. $H_0 : \beta_1 - 2\beta_2 = 4\beta_3$

$\beta_1 + 2\beta_2 = 0$

Discuss the behavior of these quantities as x_i moves farther from $\bar{x}$,

3.30 Consider the multiple linear regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$. Show that the least-squares estimator can be written as

$$\hat{\boldsymbol{\beta}} = \boldsymbol{\beta} + \mathbf{R}\boldsymbol{\varepsilon} \quad \text{where} \quad \mathbf{R} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$$

3.31 Show that the residuals from a linear regression model can be expressed as $\mathbf{e} = (\mathbf{I} - \mathbf{H})\boldsymbol{\varepsilon}$. [Hint: Refer to Eq. (3.15b).]

3.32 For the multiple linear regression model, show that $SS_R(\boldsymbol{\beta}) = \mathbf{y}'\mathbf{H}\mathbf{y}$.

3.33 Prove that R^2 is the square of the correlation between $\mathbf{y}$ and $\hat{\mathbf{y}}$.

3.34 **Constrained least squares.** Suppose we wish to find the least-squares estimator of $\boldsymbol{\beta}$ in the model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ subject to a set of equality constraints on $\boldsymbol{\beta}$, say $\mathbf{T}\boldsymbol{\beta} = \mathbf{c}$. Show that the estimator is