Causal Mediation Analysis with Multiple Mediators of General Structures

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Motivation

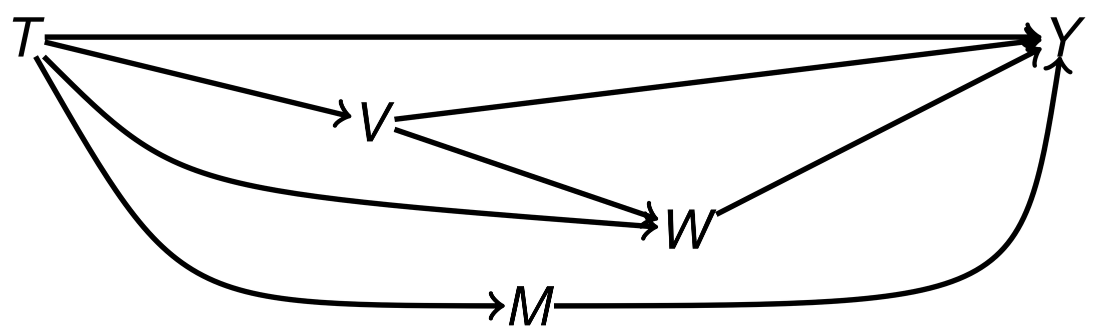
Previous Study

- Counterfactaul definitions of path specific estimands [1]
- Two mediators, causally independent or dependent [2]
- Develop new methods for multiple mediators [3]
- New strategy to assess direct and indirect effect [4]

Limatation

- In general cases, we don't know how they are related

Suggestion



Notation

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$$T = Treatment, X = Confounders,$$
$$M, V, W = Mediators, Y = Outcome$$

Potential Outcome Values

- $M_i(t), V_i(t), W_i(t, V_i(t)), Y_i(t, M_i(t), V_i(t), W_i(t, V_i(t)))$ when $T = t$

Observed Data

- T_i, M_i, V_i, W_i, Y_i for unit i

Consistency Assumption[5]

- For unit i, with observed treatment value $T_i = t$,
  $M_i = M_i(t), V_i = V_i(t), W_i = W_i(t, V_i(t))$,
  $Y_i = Y_i(t, M_i(t), V_i(t), W_i(t, V_i(t)))$ for t=0,1

Effect Decomposition

- Decompose the total effect into direct & indirect effects

First Decomposition : Direct Effect & M Effect

$$\begin{aligned}
\tau &= Y_i(1, V_i(1), W_i(1, V_i(1)), M_i(1)) - Y_i(0, V_i(0), W_i(0, V_i(0)), M_i(0)) \\
&= Y_i(1, V_i(1), W_i(1, V_i(1)), M_i(1)) - Y_i(1, V_i(1), W_i(1, V_i(1)), M_i(0)) \\
&+ Y_i(1, V_i(1), W_i(1, V_i(1)), M_i(0)) - Y_i(0, V_i(1), W_i(1, V_i(1)), M_i(0)) \\
&+ Y_i(0, V_i(1), W_i(1, V_i(1)), M_i(0)) - Y_i(0, V_i(0), W_i(0, V_i(0)), M_i(0))
\end{aligned}$$

Second Decomposition : V Effect & W Effect after V Effect

$$\begin{aligned}
&Y_i(0, V_i(1), W_i(1, V_i(1)), M_i(0)) - Y_i(0, V_i(0), W_i(0, V_i(0)), M_i(0)) \\
=&Y_i(0, V_i(1), W_i(1, V_i(1)), M_i(0)) - Y_i(0, V_i(0), W_i(1, V_i(1)), M_i(0)) \\
+&Y_i(0, V_i(0), W_i(1, V_i(1)), M_i(0)) - Y_i(0, V_i(0), W_i(0, V_i(0)), M_i(0))
\end{aligned}$$

Assumptions

Sequentially Ignorability Assumptions (S.I.A)

$$\{Y_i(t, v, w, m), M_i(t'), V_i(t''), W_i(t''', v')\} \perp\!\!\!\perp T_i \mid X_i = x \quad (1)$$
$$Y_i(t', v, w, m) \perp\!\!\!\perp M_i \mid T_i = t, X_i = x \quad (2)$$
$$\{Y_i(t', v, w, m), W_i(t'', v')\} \perp\!\!\!\perp V_i \mid T_i = t, X_i = x \quad (3)$$
$$Y_i(t', v', w, m) \perp\!\!\!\perp W_i \mid V_i(t) = v, T_i = t, X_i = x \quad (4)$$

for any $t, t', t'', t''', m, v, v', w, x$.

Implication of Assumptions

These assumptions imply,

(a) $Y_i(t, v, w, m) \perp\!\!\!\perp T_i \mid M_i(t') = m', X_i = x$

(b) $Y_i(t, v, w, m) \perp\!\!\!\perp T_i \mid W_i(t'', v'') = w', V_i(t') = v', X_i = x$

(c) $W_i(t, v) \perp\!\!\!\perp T_i \mid V_i(t') = v', X_i = x$

for any $t, t', t'', v, v', v'', w, w', m, m', x$.

Identification

$$\bar{\tau} = \int \{E[Y_i|T_i = 1, X_i = x] - E[Y_i|T_i = 0, X_i = x]\}\, dF_{X_i}(x)$$

$$\bar{\delta}^M(t) = \iint E[Y_i|M_i = m, T_i = t, X_i = x]\,\{dF_{M_i|T_i=1,X_i=x}(m) - dF_{M_i|T_i=0,X_i=x}(m)\}\, dF_{X_i}(x)$$

$$\begin{aligned}
\bar{\delta}^{V,W}(t') = \iiint E[Y_i|W_i = w, V_i = v, T_i = t', X_i = x]\,\{&dF_{V_i|T_i=1,X_i=x}(v)\, dF_{W_i|V_i=v,T_i=1,X_i=x}(w) \\
&- dF_{V_i|T_i=0,X_i=x}(v)\, dF_{W_i|V_i=v,T_i=0,X_i=x}(w)\}\, dF_{X_i}(x)
\end{aligned}$$

$$\begin{aligned}
\bar{\delta}^V(t', t'') = \iiint E[Y_i|W_i = w, V_i = v, T_i = t', X_i = x]\,\{&dF_{V_i|T_i=1,X_i=x}(v) - dF_{V_i|T_i=0,X_i=x}(v)\} \\
&\times dF_{W_i|V_i=v,T_i=t'',X_i=x}(w)\, dF_{X_i}(x)
\end{aligned}$$

Simulation Study

Main data-generating models

$$X_1 \sim N(0, 0.5), \quad X_2 \sim N(-3, 0.5), \quad X_3 \sim N(3, 0.5)$$
$$logit(P(T_i = 1)) = 0.2 + 0.2X_{1i} + 0.7X_{2i} + 0.5X_{3i} \qquad T_i \sim Bernoulli(P(T_i = 1))$$
$$M_i = 1 + T_i + X_{1i} + X_{2i} + X_{3i} + \varepsilon_{iM} \qquad \varepsilon_{iM} \sim N(0, 0.5)$$
$$V_i = 2 + 1.5T_i + 0.7X_{1i} + 0.5X_{2i} + 0.2X_{3i} + \varepsilon_{iV} \qquad \varepsilon_{iV} \sim N(0, 0.4)$$
$$W_i = 3 + 0.8T_i + 1.4V_i + 0.4X_{1i} + 0.4X_{2i} + 0.4X_{3i} + \varepsilon_{iW} \qquad \varepsilon_{iW} \sim N(0, 0.35)$$

Scenario I

$$Y_i = 5 + 1.2T_i + 1.2M_i + 1.4V_i + 0.7W_i + 0.5X_{1i} + 0.4X_{2i} + 0.6X_{3i} + \varepsilon_{iY} \qquad \varepsilon_{iY} \sim N(0, 0.2)$$

Scenario II

$$Y_i = 5 + 1.2T_i + 1.2M_i + 1.4V_i + 0.7W_i + T_iM_i + 0.5X_{1i} + 0.4X_{2i} + 0.6X_{3i} + \varepsilon_{iY} \qquad \varepsilon_{iY} \sim N(0, 0.2)$$

Simulation Result

	Scenario I				Scenario II			
Effects	**Truth**	**n=50**	**n=100**	**n=500**	**Truth**	**n=50**	**n=100**	**n=500**
Total	6.531	0.038(0.164)	0.001(0.063)	0.008(0.014)	8.532	0.120(0.237)	-0.012(0.153)	-0.009(0.024)
M	1.201	-0.019(0.048)	-0.043(0.023)	-0.001(0.005)	2.200	0.038(0.152)	0.021(0.098)	0.002(0.016)
V&W	4.130	0.055(0.033)	0.039(0.009)	0.013(0.002)	4.131	0.087(0.058)	-0.018(0.024)	-0.007(0.005)
Direct	1.200	0.003(0.158)	0.005(0.058)	-0.004(0.013)	2.201	-0.005(0.172)	-0.015(0.060)	-0.003(0.010)
V	2.100	0.021(0.094)	0.016(0.035)	0.017(0.009)	2.101	0.016(0.105)	-0.028(0.048)	0.004(0.007)
W	2.030	0.033(0.109)	0.023(0.040)	-0.004(0.009)	2.030	0.072(0.108)	0.010(0.031)	-0.011(0.007)

Table 1: Biases and MSEs of our estimates

Sensitivity Anls.

Sensitivity parameters

$$\rho_1 = Corr(\varepsilon_{iM}, \varepsilon_{iY})$$
$$\rho_2 = Corr(\varepsilon_{iV}, \varepsilon_{iY})$$
$$\rho_3 = Corr(\varepsilon_{iV}, \varepsilon_{iW})$$

Idea

- *randomized* ⇔ S.I.A (1) holds
- $\rho_1 = 0$ ⇔ S.I.A (2) holds
- $\rho_2 = \rho_3 = 0$ ⇔ S.I.A (3) holds

Sensitivity Analysis Result

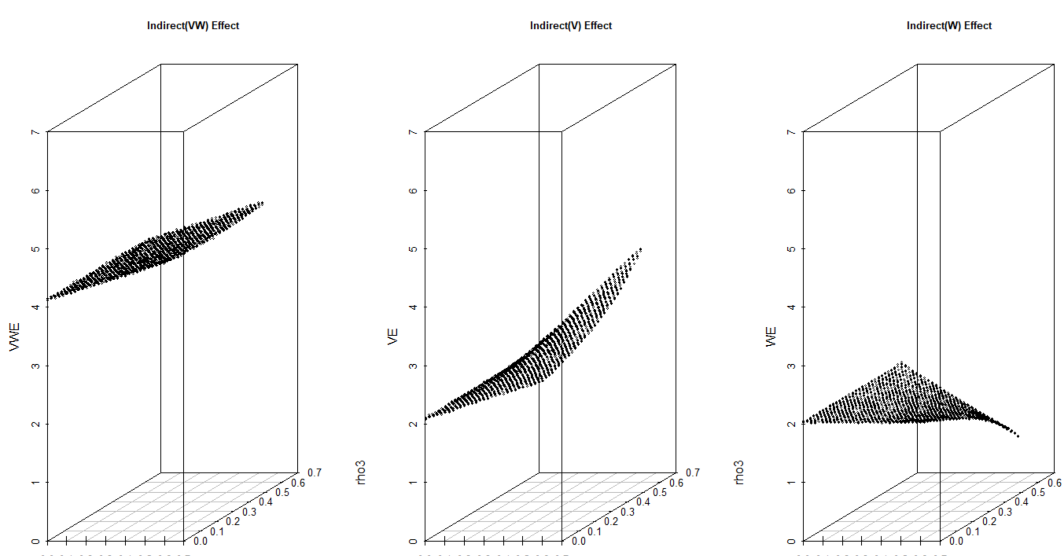

Fig. 1: Changes in estimates according to the sensitivity parameters

Future Study

Sensitivity Analysis

- Sensitivity parameter for the fourth assumption of the sequentially ignorability assumptions
- New sensitivity analysis method with more practical assumptions

Application to real data

- The pollination data : Emission Control Technology

Other relationships between mediators

- Mediators affecting each other
- Mediators that affected by more than one mediator

References

[1] Daniel, R. M., De Stavola, B. L., Cousens, S., and Vansteelandt, S. Causal mediation analysis with multiple mediators. *Biometrics 71*, 1 (2015), 1–14.

[2] Imai, K., and Yamamoto, T. Identification and sensitivity analysis for multiple causal mechanisms: Revisiting evidence from framing experiments. *Political Analysis* (2013), 141–171.

[3] Kim, C., Daniels, M. J., Hogan, J. W., Choirat, C., and Zigler, C. M. Bayesian methods for multiple mediators: Relating principal stratification and causal mediation in the analysis of power plant emission controls. *The annals of applied statistics 13*, 3 (2019), 1927.

[4] VanderWeele, T., and Vansteelandt, S. Mediation analysis with multiple mediators. *Epidemiologic methods 2*, 1 (2014), 95–115.

[5] VanderWeele, T. J. Concerning the consistency assumption in causal inference. *Epidemiology 20*, 6 (2009), 880–883.