

수리통계학 추가숙제 #2

1. Given a random sample of size n from a geometric distribution,
- Find the method of moments estimator of p
 - Find the maximum likelihood estimator of p
 - Use the results from (a) and (b) to compute the method of moments and maximum likelihood estimates from the sample $\{8, 1, 2, 0, 0, 0, 2, 1, 3, 3\}$, which represents the number of Bernoulli trials that resulted in failure before the first success in 10 experiments.

2. Given a random sample of size n from a distribution with a density function given by

$$f(x) = \theta \left(\frac{1}{x}\right)^{\theta+1}, \quad x \geq 1, \theta > 1,$$

- Find the method of moments and the maximum likelihood estimators of θ .
- Find the method of moments and maximum likelihood estimates of θ for the sample $\{3, 4, 2, 1.5, 4, 2, 3, 2, 4, 2\}$.

3. Given the density function

$$f(x) = \frac{3}{\lambda} x^2 e^{-x^{3/\lambda}}, \quad x > 0, \lambda > 0,$$

- Find the maximum likelihood estimator of λ for a random sample of size n .
- Verify that the maximum likelihood estimator is unbiased, consistent, and efficient.
- Find the method of moments estimator of λ for a random sample of size n .

4. Given the density function

$$f(x) = \theta x^{\theta-1} \quad 0 \leq x \leq 1, \theta > 0$$

- What distribution has this density function? Be sure to specify the parameter.
- Find the maximum likelihood estimator of θ for random samples of size n .
- Find the asymptotic variance of the maximum likelihood estimator.
- Find the method of moments estimator of θ for a random sample of size n .
- Calculate the maximum likelihood and method of moments estimates of θ for the sample $\{0.1, 0.7, 0.5, 0.85, 0.9\}$.

5. Given a random sample of size n from an exponential distribution with pdf

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad x \geq 0, \theta > 0,$$

- Find the MLE of θ .
- Given the answer in part (a), what is the MLE of θ^2 .