Nonparametric Statistics

Ch.2 One-sample (location) problem

Estimation associated with the sign statistic

- Point estimation

$$\hat{m} = median\{X_i : 1 \le i \le n\}$$

- Confidence interval

  Let $k_\alpha = b(n, 1/2, \alpha/2)$ (Note that $P(S_n \ge k_\alpha) = \alpha/2$) , then

$$P(n - k_\alpha < S_n < k_\alpha) = 1 - \alpha$$

  And, $\{S_n < k_\alpha\} = \{X_{(n-k_\alpha+1)} < m\}$ and $\{S_n > n - k_\alpha\} = \{X_{(k_\alpha)} > m\}$. Therefore, $100(1-\alpha)\%$ confidence interval for m can be given as

$$(X_{(n-k_\alpha+1)}, X_{(k_\alpha)})$$

  where $X_{(r)}$ denotes r^{th} order statistic.

- When n is large enough,

$$k_\alpha \approx \frac{n}{2} + z_{\alpha/2}\left(\frac{n}{4}\right)^{1/2}.$$

Estimation associated with the signed rank statistic

- Walsh average

$$W_{ij} = \frac{X_i + X_j}{2}, \qquad 1 \le i \le j \le n$$

- We have the following equivalent expression for W_n.

$$W_n = \sum_{i=1}^{n} R_i^+ I(X_i - m_0 > 0) = \sum_{i \le j} I(W_{ij} > m_0).$$

Therefore, the Wilcoxon signed rank statistic can be translated as the sign statistic based on the Walsh average.

- Point estimation

$$\hat{m} = median\{W_{ij} : 1 \le i \le j \le n\}$$

- Confidence interval : Let k_α be the integer satisfying $P(W_n \ge k_\alpha) = \alpha/2$, then $100(1-\alpha)\%$ confidence interval for m is given as

$$(W_{(M-k_\alpha+1)}, W_{(k_\alpha)})$$

where $W_{(r)}$ denotes r^{th} order statistic based on the Walsh average and $M = n(n+1)/2$.

- When n is large enough,

$$k_\alpha \approx \frac{n(n+1)}{4} + z_{\alpha/2}\left(\frac{n(n+1)(2n+1)}{24}\right)^{1/2}.$$

Asymptotic relative efficiency

- In this chapter, ARE shows how efficient the proposed nonparametric estimators (or tests) are compared to t-test or the sample mean.
- If ARE is larger than 1, it means that the nonparametric methods are preferable.
- The following tables represent AREs of the sign test and Wilcoxon signed rank test to the t-test for some selected probability distributions.

	Dist.	Normal	Uniform	Logistic	Double exponential	Cauchy	t(3)	t(5)
sign	ARE	0.637	0.333	0.822	2.000	∞	1.620	0.961

	Dist.	Normal	Uniform	Logistic	Double exponential	Cauchy	t(3)	t(5)
Wilcoxon	ARE	0.955	1.000	1.097	1.500	∞	1.900	1.240