수리통계학 16강 예제 91, 92 풀이

예제 91

확률변수 X와 Y의 결합확률분포가 다음과 같을 때 $g(X,\ Y)=XY$의 기댓값을 구하여라.

Y \ X	0	1	2	3
0	$\frac{1}{8}$	0	0	$\frac{1}{8}$
1	0	$\frac{1}{8}$	$\frac{1}{8}$	0
2	0	$\frac{2}{8}$	$\frac{2}{8}$	0

(풀이)

$$E(XY)=\sum_x\sum_y xyf(x,\ y)=1\cdot1\cdot\frac{1}{8}+1\cdot2\cdot\frac{2}{8}+2\cdot1\cdot\frac{1}{8}+2\cdot2\cdot\frac{2}{8}$$

$$=\frac{1}{8}(1+4+2+8)=\frac{15}{8}$$

예제 92

결합확률밀도함수가 $f(x,\ y)=\begin{cases} x+y, & (0<x<1,\ 0<y<1) \\ 0, & (\text{그 외}) \end{cases}$ 일 때

(1) $E(X)=\int_0^1 xf_1(x)\,dx=\int_0^1 x\left(x+\frac{1}{2}\right)dx=\int_0^1\left(x^2+\frac{1}{2}x\right)dx=\left[\frac{1}{3}x^3+\frac{1}{4}x^2\right]_0^1=\frac{7}{12}$

※$f_1(x)=\int_{-\infty}^{\infty} f(x,\ y)\,dy=\int_0^1 (x+y)\,dy=\left[xy+\frac{1}{2}y^2\right]_0^1=x+\frac{1}{2}$

(2) $E(2X+3Y)=2E(X)+3E(Y)=\frac{14}{12}+\frac{21}{12}=\frac{35}{12}$

※$f_2(y)=\int_0^1 (x+y)\,dx=\left[\frac{1}{2}x^2+xy\right]_0^1=\frac{1}{2}+y$

$\Rightarrow E(Y)=\int_0^1 yf_2(y)\,dy=\int_0^1 y\left(y+\frac{1}{2}\right)dy=\frac{7}{12}$

(3) $E(XY)=\int_0^1\int_0^1 xyf(x,\ y)\,dxdy=\int_0^1\int_0^1 xy(x+y)\,dxdy$

$=\int_0^1 y\int_0^1 (x^2+xy)\,dxdy=\int_0^1 y\left[\frac{1}{3}x^3+\frac{1}{2}x^2y\right]_0^1 dy=\int_0^1\left(\frac{1}{3}y+\frac{1}{2}y^2\right)dy$

$=\left[\frac{1}{6}y^2+\frac{1}{6}y^3\right]_0^1=\frac{2}{6}=\frac{1}{3}$