Additional Homework

Due : 2013. 4. 15

1. Given a random sample of size n from a geometric distribution,
   (a) Find the method of moments estimator of p
   (b) Find the maximum likelihood estimator of p
   (c) Use the results from (a) and (b) to compute the method of moments and maximum likelihood estimates from the sample {8, 1, 2, 0, 0, 0, 2, 1, 3, 3}, which represents the number of Bernoulli trials that resulted in failure before the first success in 10 experiments.

2. Given a random sample of size n from a distribution with a density function given by

$$f(x) = \theta\left(\frac{1}{x}\right)^{\theta+1}, \quad x \geq 1, \theta > 1,$$

   (a) Find the method of moments and the maximum likelihood estimators of θ.
   (b) Find the method of moments and maximum likelihood estimates of θ for the sample {3, 4, 2, 1.5, 4, 2, 3, 2, 4, 2}.

3. Given the density function

$$f(x) = \frac{3}{\lambda}x^2 e^{-x^3}, \quad x > 0, \ \lambda > 0,$$

   (a) Find the maximum likelihood estimator of λ for a random sample of size n.
   (b) Verify that the maximum likelihood estimator is unbiased and consistent.
   (c) Find the method of moments estimator of λ for a random sample of size n.

4. Given the density funcion

$$f(x) = \theta x^{\theta-1} \quad 0 \leq x \leq 1, \ \theta > 0$$

   (a) What distribution has this density function? Be sure to specify the parameter.
   (b) Find the maximum likelihood estimator of θ for random samples of size n.
   (c) Find the asymptotic variance of the maximum likelihood estimator.
   (d) Find the method of moments estimator of θ for a random sample of size n.

5. Given a random sample of size n from an exponential distribution with pdf

$$f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}} \quad x \geq 0, \ \theta > 0,$$

   (a) Find the MLE of θ^2.
   (b) Discuss about the asymptotic distribution of the MLE of θ.