Categorial Data Analysis
Assignment #3

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1. Consider the below 3-way contingency table.

		Response(Y)	
Clinic(Z)	Treatment(X)	Success	Fail
1	A	20	30
	B	30	10
2	A	50	20
	B	20	5

Answer the following questions

(a) Compute the partial odds ratios for Clinic 1 ($\theta_{XY(1)}$) and for Clinic 2 ($\theta_{XY(2)}$).

$$\theta_{XY(1)} = \frac{20 \cdot 10}{30 \cdot 30} = 0.2222.$$

$$\theta_{XY(2)} = \frac{50 \cdot 5}{20 \cdot 20} = 0.625.$$

(b) Construct the marginal table by merging clinic information. Then compute the marginal odds ratio θ_{XY}.

	Success	Failure
A	70	50
B	50	15

$$\theta_{XY} = \frac{70 \cdot 15}{50 \cdot 50} = 0.42.$$

(c) From (a) and (b), can you observe the Simpson's paradox? Explain it. Since all of the conditional odds ratios and the marginal odds ratio are less than 1, partial association and marginal association show the same direction. Therefore we cannot observe the Simpson's paradox in this data.

(d) Suppose the association seems stable across clinic 1 and 2. Compute the Mantel-Haenszel estimator and logit estimator for the common odds ratio. The Mantel-Haenszel estimator becomes

$$\hat{\theta}_{MH} = \frac{\sum_{k=1}^{2} \frac{n_{11k} n_{22k}}{n_{\cdot\cdot k}}}{\sum_{k=1}^{2} \frac{n_{12k} n_{21k}}{n_{\cdot\cdot k}}} = \frac{\frac{20 \cdot 10}{90} + \frac{50 \cdot 5}{95}}{\frac{30 \cdot 30}{90} + \frac{20 \cdot 20}{95}} = 0.3416.$$

And, for the logit estimator, note that

$$w_1 = \left(\frac{1}{n_{111}} + \frac{1}{n_{121}} + \frac{1}{n_{211}} + \frac{1}{n_{221}}\right)^{-1} = \left(\frac{1}{20} + \frac{1}{30} + \frac{1}{30} + \frac{1}{10}\right)^{-1} = 4.6154,$$

$$w_2 \;=\; \left(\frac{1}{n_{112}}+\frac{1}{n_{122}}+\frac{1}{n_{212}}+\frac{1}{n_{222}}\right)^{-1} \;=\; \left(\frac{1}{50}+\frac{1}{20}+\frac{1}{20}+\frac{1}{5}\right)^{-1} \;=\; 3.125.$$

Thus, the logit estimator becomes

$$\hat{\theta}_L \;=\; \exp\left\{\frac{\sum_{k=1}^{2} w_k \log \hat{\theta}_{XY(k)}}{\sum_{k=1}^{2} w_k}\right\} \;=\; \exp\left\{\frac{(4.6154)\log(0.2222)+(3.125)\log(0.625)}{4.6154+3.125}\right\} \;=\; 0.3373.$$