Homework #2

[Probability and Statistical Inference (Hogg, Tanis and Zimmerman), 9th ed. 에서 일부 발췌]

1. 2장 연습문제 중 #2, #3, #5, #8, #11, #12, #13, #15

2. Solve the problems.

2-1) Five cards are selected at random without replacement from a standard, thoroughly shuffled 52-card deck of playing cards. Let X equal the number of face cards (kings, queens, jacks) in the hand. Forty observations of X yielded the following data:

2	1	2	1	0	0	1	0	1	1	0	2	0	2	3	0	1	1	0	3
1	2	0	2	0	2	0	1	0	1	1	2	1	0	1	1	2	1	1	0

(a) Argue that the pmf of X is

$$f(x) = \frac{\binom{12}{x}\binom{40}{5-x}}{\binom{52}{5}}, \quad x = 0,1,2,3,4,5$$

and thus, that $f(0) = \frac{2109}{8330}$, $f(1) = \frac{703}{1666}$, $f(2) = \frac{209}{833}$, $f(3) = \frac{55}{833}$, $f(4) = \frac{165}{21658}$, and $f(5) = \frac{33}{108290}$.

(b) Draw a probability histogram for this distribution.

(c) Determine the relative frequencies of 0, 1, 2, 3, and superimpose the relative frequency histogram on your probability histogram.

2-2) A boiler has four relief valves.
The probability that each opens properly is 0.99.

(a) Find the probability that at least one opens properly.

(b) Find the probability that all four open properly.

2-3) An excellent free-throw shooter attempts several free throws until she misses.

(a) If $p = 0.9$ is her probability of making a free throw, what is the probability of having the first miss on the 13th attempt or later?

(b) If she continues shooting until she misses three, what is the probability that the third miss occurs on the 30th attempt?

2-4) Let X have a Poisson distribution with a mean of 4. Find

(a) $P(2 \le X \le 5)$
(b) $P(X \ge 3)$
(c) $P(X \le 3)$

2-5) A certain type of aluminum screen that is 2 feet wide has, on the average, one flaw in a 100-foot roll. Find the probability that a 50-foot roll has no flaws.

2-6) The weekly demand X for propane gas (in thousands of gallons) has the pdf

$f(x) = 4x^3 e^{-x^4}, \;\; 0 < x < \infty$

If the stockpile consists of two thousand gallons at the beginning of each week (and nothing extra is received during the week), what is the probability of not being able to meet the demand during a given week?

2-7) Nicol (see References) lets the pdf of X be defined by

$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ \dfrac{c}{x^3}, & 1 \le x < \infty \end{cases}$ and $f(x) = 0$, elsewhere.

Find

(a) The value of c so that $f(x)$ is a pdf.

(b) $P(\frac{1}{2} \le X \le 2)$.

2-8) A certain type of aluminum screen 2 feet in width has, on the average, three flaws in a 100-foot roll.

(a) What is the probability that the first 40 feet in a roll contain no flaws?

(b) What assumption did you make to solve part (a)?

2-9) Some dental insurance policies cover the insurer only up to a certain amount, say, M. (This seems to us to be a dumb type of insurance policy because most people should want to protect themselves against large losses.)
Say the dental expense X is a random variable with pdf

$f(x) = (0.001)e^{-\left(\frac{x}{1000}\right)}$, $0 < x < \infty$. Find M so that $P(X < M) = 0.08$.

2-10) The strength X of a certain material is such that its distribution is found by $X = e^{Y}$, where Y is $N(10, 1)$. Find the cdf and pdf of X, and compute $P(10000 < X < 20000)$.

* NOTE : $F(x) = P(X \le x) = P(e^{Y} \le x) = P(Y \le \ln x)$ so that the random variable X is said to have a lognormal distribution.

2-11) The time X to failure of a machine has pdf $f(x) = \left(\frac{x}{4}\right)^3 e^{-\left(\frac{x}{4}\right)^4}$, $0 < x < \infty$.
Compute $P(X > 5 \mid X > 4)$.