Quiz #1 - Solution

2015/09/23

1. Prove the followings.

(i) If $P(A_1) = P(A_2)$ and $P(A_1 \cap A_2^c) = 0$ for two events A_1, A_2, then

$$P(A_1^c \cap A_2) = 0.$$

$P(A_1) = P(A_1 \cap A_2) + P(A_1 \cap A_2^c) = P(A_1 \cap A_2) \quad (i)$
$P(A_2) = P(A_2 \cap A_1) + P(A_2 \cap A_1^c) = P(A_1) + P(A_2 \cap A_1^c) \ by(i) \quad (ii)$
$\therefore P(A_2 \cap A_1^c) = 0 \ by(ii)$

(ii) If A_1, A_2 are disjoint events, then

$$P(A_1 | A_1 \cup A_2) = P(A_1)/[P(A_1) + P(A_2)]$$

$P(A_1 \cup A_2) = P(A_1) + P(A_2),\ P(A_1 \cap (A_1 \cup A_{2)}) = P(A_1)$
$\therefore P(A_1 | A_1 \cup A_2) = P(A_1)/[P(A_1) + P(A_2)]$

2. In a twin engine plane, we are told that the two engines (#1, #2) function independently. We are also told that the plane flies just fine when at least one of the two engines are working. During a flying mission, individually the engine #1 and #2 respectively may fail with probability .001 and .01. Then, during a flying mission, what is the probability that the plane would crash?

A : the plane crash, B_1 : engine #1 fails, B_2 : engine #2 fails
$P(A) = P(B_1 \cap B_2) = P(B_1)P(B_2) = 0.001 \times 0.01 = 0.00001$

3. Answer the following questions.

(i) A boy found a bicycle lock for which the combination was unknown. The correct combination is a four-digit number, $d_1 d_2 d_3 d_4$, where $d_i\ , i = 1,2,3,4$, is selected from 1, 2, 3, 4, 5, 6, 7, and 8. How many different lock combinations are possible with such a lock?

${}_8\Pi_4 = 8^4$

(ii) Three students (S) and six faculty members (F) are on a panel discussing a new college policy. How many lineups are possible, considering only the labels S and F.

$_{9}C_{3}=\frac{9!}{6!3!}=84$

4. At the beginning of a certain study of a group of persons, 15% were classified as heavy smokers, 30% as light smokers, and 55% as nonsmokers. In the five year study, it was determined that the death rates of the heavy and light smokers were five and three times that of the nonsmokers, respectively. A randomly selected participant died over the five-year period; calculate the probability that the participant was a nonsmoker.

A_1 : heavy smoker, A_2 : light smoker, A_3 : nonsmoker

D : died event

$$\begin{aligned}P(A_3|D) &= \frac{P(D|A_3)P(A_3)}{P(D|A_1)P(A_1)+P(D|A_2)P(A_2)+P(D|A_3)P(A_3)} \\ &= \frac{P(D|A_3)P(A_3)}{5\times P(D|A_3)P(A_1)+3\times P(D|A_3)P(A_2)+P(D|A_3)P(A_3)} \\ &= \frac{P(A_3)}{5\times P(A_1)+3\times P(A_2)+P(A_3)} \\ &= \frac{0.55}{5\times 0.15+3\times 0.3+0.55}=0.25\end{aligned}$$

5. (Monty Hall problem) 당신은 어떤 게임쇼에 출연중이다. 세 개의 문 중 하나를 선택할 수 있는데, 그 중 하나의 문 뒤에는 고급 자동차가 있으며, 나머지 두 개의 문 뒤에는 염소가 있다. 당신이 셋 중 하나의 문을 선택한 후, 사회자가 나머지 두 개의 문 중 염소가 있는 문을 열어서 보여준 뒤 이렇게 말하였다. “선택한 문을 바꿀 수 있는 기회를 드리겠습니다. 당신의 결정을 바꾸시겠습니까?” 선택한 문을 바꾸는 것이 자동차를 획득하기 위해서 더 유리한가? 확률적 근거를 들어 답하여라.

A : 선택한 문을 바꾸었을 때 자동차를 획득할 사건 (혹은 남은 문에 자동차가 있는 사건)

B : 처음 선택한 문 뒤에 자동차가 있는 사건

$P(A)=P(A|B)P(B)+P(A|B^c)P(B^c)=0\times\frac{1}{3}+1\times\frac{2}{3}=\frac{2}{3}$

결정을 바꾸는 것이 더 유리하다.