Homework #3

[Probability and Statistical Inference (Hogg, Tanis and Zimmerman), 9th ed. 에서 일부 발췌]

1. 3장 연습문제 중 #2, #4, #5, #10, #12, #14

2. Solve the problems.

2-1) For each of the following functions, determine the constant c so that $f(x,y)$ satisfies the conditions of being a joint pmf for two discrete random variables X and Y :

(a) $f(x,y) = c(x+2y),\ x = 1,2,\ y = 1,2,3.$

(b) $f(x,y) = c(x+y),\ x = 1,2,3,\ y = 1,\ldots,x.$

(c) $f(x,y) = c,$ x and y are integers such that $6 \le x+y \le 8,\ 0 \le y \le 5.$

(d) $f(x,y) = c(\frac{1}{4})^x(\frac{1}{3})^y,\ x = 1,2,\ldots,\ y = 1,2,\ldots.$

2-2) Let the joint pmf of X and Y be defined by

$$f(x,y) = \frac{x+y}{32},\ x = 1,2,\ y = 1,2,3,4$$

(a) Find $f_X(x)$, the marginal pmf of X.

(b) Find $f_Y(y)$, the marginal pmf of Y.

(c) Find $P(X > Y)$.

(d) Find $P(Y = 2X)$.

(e) Find $P(X + Y = 3)$.

(f) Find $P(X \le 3 - Y)$.

(g) Are X and Y independent or dependent? Why or why not?

2-3) Let X and Y have the joint pmf

$$f(x,y) = \frac{x+y}{32}, \ x = 1,2, \ y = 1,2,3,4$$

(a) Display the joint pmf and the marginal pmfs on a graph.

(b) Find $g(x|y)$ and draw a figure, depicting the conditional pmfs for $y = 1,2,3 \text{ and } 4$.

(c) Find $h(y|x)$ and draw a figure, depicting the conditional pmfs for $x = 1$ and 2.

(d) Find $P(1 \le Y \le 3 | X = 1)$, $P(Y \le 2 | X = 2)$ and $P(X = 2 | Y = 3)$.

2-4) Let $f(x,y) = (3/16)xy^2$, $0 \le x \le 2$, $0 \le y \le 2$, be the joint pdf of X and Y.

(a) Find $f_X(x)$ and $f_Y(y)$, the marginal probability density functions.

(b) Are the two random variables independent? Why or why not?

(c) Find $P(X \le Y)$.

2-5) Let T_1 and T_2 be random times for a company to complete two steps in a certain process. Say T_1 and T_2 are measured in days and they have the joint pdf that is uniform over the space $1 < t_1 < 10$, $2 < t_2 < 6$, $t_1 + 2t_2 < 14$. What is $P(T_1 + T_2 > 10)$?

2-6) Let X denote the height in centimeters and Y the weight in kilograms of male college students. Assume that X and Y have a bivariate normal distribution with parameters $\mu_X = 185$, $\sigma_X^2 = 100$, $\mu_Y = 84$, $\sigma_Y^2 = 64$, and $\rho = 3/5$.

(a) Determine the conditional distribution of Y, given that $X = 190$.

(b) Find $P(86.4 < Y < 95.36 | X = 190)$.

2-7) Let X_1, X_2 denote two independent random variables, each with a $\chi^2(2)$ distribution. Find the joint pdf of $Y_1 = X_1$ and $Y_2 = X_2 + X_1$. Note that the support of Y_1, Y_2 is $0 < y_1 < y_2 < \infty$. Also, find the marginal pdf of each of Y_1 and Y_2. Are Y_1 and Y_2 independent?

2-8) Let X_1 and X_2 be independent chi-square random variables with r_1 and r_2 degrees of freedom, respectively. Let $Y_1 = (X_1/r_1)/(X_2/r_2)$ and $Y_2 = X_2$.

(a) Find the joint pdf of Y_1 and Y_2.

(b) Determine the marginal pdf of Y_1 and show that Y_1 has an F distribution. (This is another, but equivalent, way of finding the pdf of F.)

2-9) Let X_1, X_2 be independent random variables representing lifetimes (in hours) of two key components of a device that fails when and only when both components fail. Say each X_i has an exponential distribution with mean 1000. Let $Y_1 = \min(X_1, X_2)$ and $Y_2 = \max(X_1, X_2)$, so that the space of Y_1, Y_2 is $0 < y_1 < y_2 < \infty$.

(a) Find $G(y_1, y_2) = P(Y_1 \le y_1,\ Y_2 \le y_2)$.

(b) Compute the probability that the device fails after 1200 hours; that is, compute $P(Y_2 > 1200)$.