$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \epsilon_i, \qquad i = 1,\ 2,\ \cdots, n$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & x_{11} & x_{21} & \cdots & x_{k1} \\ 1 & x_{12} & x_{22} & \cdots & x_{k2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{1n} & x_{2n} & \cdots & x_{kn} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{pmatrix}$$

$$\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

Find β to Minimize $Q = \sum_{i=1}^{n} \epsilon_i^2 = \boldsymbol{\epsilon}^T \boldsymbol{\epsilon} = (\boldsymbol{y} - X\boldsymbol{\beta})^T (\boldsymbol{y} - X\boldsymbol{\beta})$

$$\frac{\partial Q}{\partial \boldsymbol{\beta}} = \mathbf{0} \longrightarrow \frac{\partial Q}{\partial \boldsymbol{\beta}} = -2X^T(\boldsymbol{y} - X\boldsymbol{\beta}) = \mathbf{0} \longrightarrow \boxed{X^T X\boldsymbol{\beta} = X^T\boldsymbol{y}}$$

Normal Equation

If $X^T X$ is a nonsingular matrix,

$$\boxed{\hat{\boldsymbol{\beta}}_{LSE} = \left(X^T X\right)^T X^T \boldsymbol{y}}$$

Sum of Squares (SS) in Matrix Form

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2 = \boldsymbol{y}^T \left(I_n - \frac{J_n}{n} \right) \boldsymbol{y},$$

where $I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}_{n \times n}$

$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 = \boldsymbol{y}^T \left(P - \frac{J_n}{n} \right) \boldsymbol{y}$$

$J_n = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}_{n \times n}$

where $P = X(X^T X)^{-1} X^T$

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \boldsymbol{y}^T (I_n - P) \boldsymbol{y}$$

ANOVA Table

Source	d.f.	S.S.	M.S.	Fo
Reg.	k	$\boldsymbol{y}^T \left(P - \frac{J_n}{n}\right) \boldsymbol{y}$	SSR/k	$\frac{MSR}{MSE}$
Error	n−k−1	$\boldsymbol{y}^T \left(I_n - P\right) \boldsymbol{y}$	$SSE/(n-k-1)$	
Total	n−1	$\boldsymbol{y}^T \left(I_n - \frac{J_n}{n}\right) \boldsymbol{y}$		

If $F_0 > F_{0.05,\,k,\,n-k-1}$, then Reject $\boxed{H_0 : \beta_1 = \beta_2 = \cdots \ \beta_k = 0}$