Homework #1

[Probability and Statistical Inference (Hogg, Tanis and Zimmerman), 9th ed. 에서 일부 발췌]

1. 1장 연습문제 중 #1, #4, #7, #8, #10, #12
+ H.W (7 page on chap.1 slide in e-class)

2. Show that

(i) $P(A^C|B) = 1 - P(A|B)$

(ii) $P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$

3. Solve the problems.

(3-1) The “eating club” is hosting a make-your-own sundae at which the following are provided:

Ice Cream Flavors	Toppings
Chocolate	Caramel
Cookies ‘n’ cream	Hot fudge
Strawberry	Marshmallow
Vanilla	M&M’s
	Nuts
	Strawberries

(a) How many sundaes are possible using one flavor of ice cream and three different toppings?

(b) How many sundaes are possible using one flavor of ice cream and from zero to six toppings?

(c) How many different combinations of flavors of three scoops of ice cream are possible if it is permissible to make all three scoops the same flavor?

(3-2) A researcher finds that, of 982 men who died in 2002, 221 died from some heart disease. Also, of the 982 men, 334 had at least one parent who had some heart disease. Of the latter 334 men, 111 died from some heart disease. A man is selected from the group of 982. Given that neither of his parents had some heart disease, find the conditional probability that this man died of some heart disease.

(3-3) Consider the birthdays of the students in a class of size r. Assume that the year consists of 365 days.

(i) How many different ordered samples of birthdays are possible (r in sample) allowing repetitions (with replacement)?

(ii) If we can assume that each ordered outcome in part (i) has the same probability, what is the probability that at least two students have the same birthday?

(iii) For what value of r is the probability in part (ii) about equal to 1/2? Is this number surprisingly small? Hint: Use a calculator or computer to find r.

(3-4) Paper is often tested for "burst strength" and "tear strength." Say we classify these strengths as low, middle, and high. Then, after examining 100 pieces of paper, we find the following:

Tear Strength	Burst Strength		
	A_1(low)	A_2(middle)	A_3(high)
B_1(low)	7	11	13
B_2(middle)	11	21	9
B_3(high)	12	9	7

If we select one of the pieces at random, what are the probabilities that it has the following characteristics:

(a) A_1?
(b) $A_3 \cap B_2$?
(c) $A_2 \cup B_3$?
(d) A_1, given that it is B_2?
(e) B_1, given that it is A_3?

(3-5) An urn contains eight red and seven blue balls. A second urn contains an unknown number of red balls and nine blue balls. A ball is drawn from each urn at random, and the probability of getting two balls of the same color is 151/300. How many red balls are in the second urn?

(3-6) If $P(A) = 0.8, P(B) = 0.5$, and $P(A \cup B) = 0.9$, are A and B independent events? Why or why not?

(3-7) Suppose that A, B, and C are mutually independent events and that $P(A) = 0.5, P(B) = 0.8$, and $P(C) = 0.9$. Find the probabilities that (a) all three events occur, (b) exactly two of the three events occur, and (c) none of the events occurs.

(3-8) An urn contains five balls, one marked WIN and four marked LOSE. You and another player take turns selecting a ball at random from the urn, one at a time. The first person to select the WIN ball is the winner. If you draw first, find the probability that you will win if the sampling is done

(a) With replacement.
(b) Without replacement.

(3-9) There is a new diagnostic test for a disease that occurs in about 0.05% of the population. The test is not perfect, but will detect a person with the disease 99% of the time. It will, however, say that a person without the disease has the disease about 3% of the time. A person is selected at random from the population, and the test indicates that this person has the disease. What are the conditional probabilities that

(a) the person has the disease?
(b) the person does not have the disease?

Discuss. Hint: Note that the fraction 0.0005 of diseased persons in the population is much smaller than the error probabilities of 0.01 and 0.03.