$$MSE = \text{Mean Squared Error} = E\left(\frac{SSE}{n-2}\right)$$

$$SSE = \text{Sum of Squares Error} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Prove

$$\boxed{E(MSE) = \sigma^2}$$

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \qquad E(\epsilon_i) = 0, \qquad V(\epsilon_i) = \sigma^2$$

$$\bar{y} = \beta_0 + \beta_1 \bar{x} + \bar{\epsilon}$$

$$y_i - \bar{y} = \underline{\beta_1 (x_i - \bar{x}) + (\epsilon_i - \bar{\epsilon})}$$

$$\hat{e}_i = y_i - \hat{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i = (y_i - \bar{y}) - \hat{\beta}_1 (x_i - \bar{x})$$

$$= (\beta_1 - \hat{\beta}_1)(x_i - \bar{x}) + (\epsilon_i - \bar{\epsilon})$$

$$\sum_i^n \hat{e}_i^2 = \underbrace{(\hat{\beta}_1 - \beta_1)^2 \sum_i^n (x_i - \bar{x})^2}_{\text{HW(1)}} + \underbrace{\sum_i^n (\epsilon_i - \bar{\epsilon})^2}_{\text{HW(2)}} - \underbrace{2(\hat{\beta}_1 - \beta_1) \sum_i^n (x_i - \bar{x})(\epsilon_i - \bar{\epsilon})}_{\text{HW(3)}}$$

$$E\left(\sum_i^n \hat{e}_i^2\right) = \sigma^2 + (n-1)\sigma^2 - 2\sigma^2$$

$$= (n-2)\sigma^2$$

$$\boxed{E(MSE) = \sigma^2}$$

$$E\left\{(\hat{\beta}_1 - \beta_1)^2 \sum_{i}^{n} (x_i - \bar{x})^2\right\} = \sigma^2$$

$$E\left\{\sum_{i}^{n}(\epsilon_i - \bar{\epsilon})^2\right\} = (n-1)\sigma^2$$

$$E\left\{-2(\hat{\beta}_1-\beta_1)\sum_i^n(x_i-\bar{x})(\epsilon_i-\bar{\epsilon})\right\}=-2\sigma^2$$

$$E\left\{-2(\hat{\beta}_1-\beta_1)\sum_i^n(x_i-\bar{x})(\epsilon_i-\bar{\epsilon})\right\}=E\left\{-2(\hat{\beta}_1-\beta_1)\sum_i^n(x_i-\bar{x})\epsilon_i\right\}$$

$$=E\left\{-2\hat{\beta}_1\sum_i^n(x_i-\bar{x})\epsilon_i\right\}+E\left\{2\beta_1\sum_i^n(x_i-\bar{x})\epsilon_i\right\}$$

$$=E\left\{-2\hat{\beta}_1\sum_i^n(x_i-\bar{x})\epsilon_i\right\}+2\beta_1\sum_i^n(x_i-\bar{x})E(\epsilon_i)$$

$$=-2\sum_i^n(x_i-\bar{x})E\left(\hat{\beta}_1\epsilon_i\right)+0$$

$$=-2\sum_i^n(x_i-\bar{x})\mathrm{Cov}\left(\frac{\sum_j^n(x_j-\bar{x})\epsilon_j}{S_{xx}},\epsilon_i\right)$$

$$=-2\sum_i^n(x_i-\bar{x})\frac{(x_i-\bar{x})}{S_{xx}}\sigma^2=-2\sigma^2$$