LECTURE 10-1

ReLU : BETTER NON-LINEARITY

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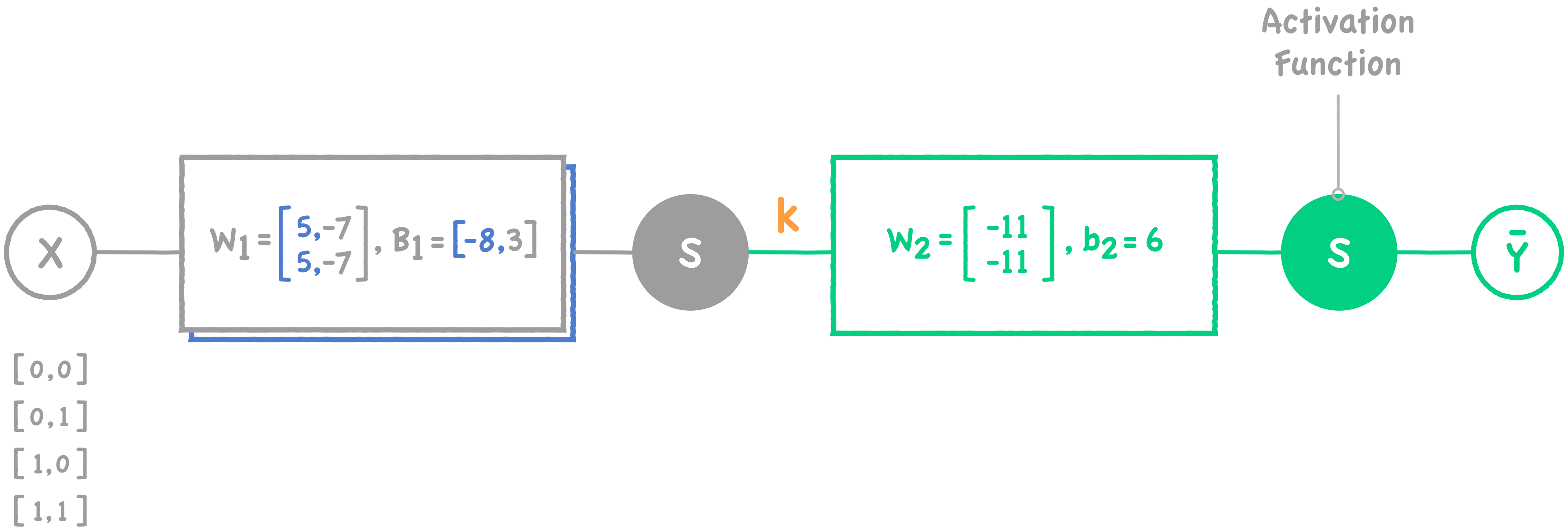
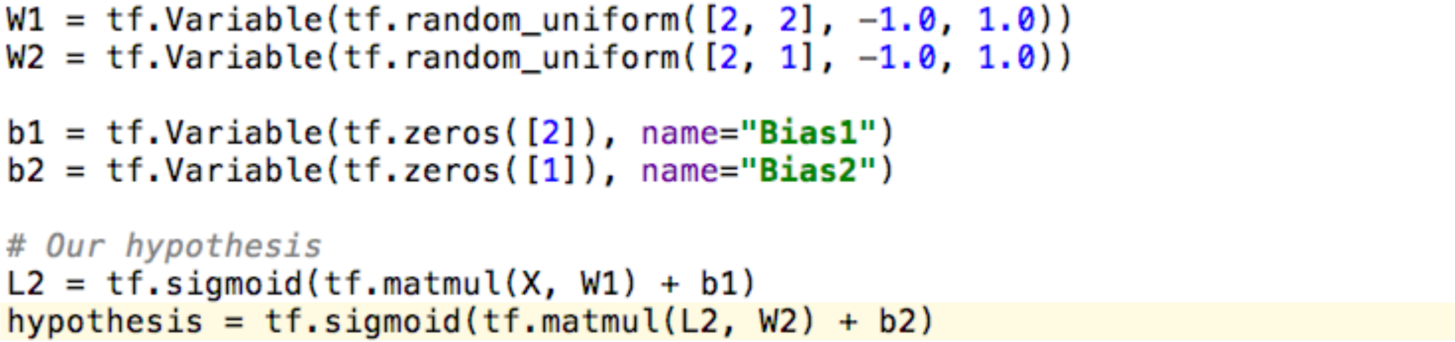
http://hunkim.github.io/ml

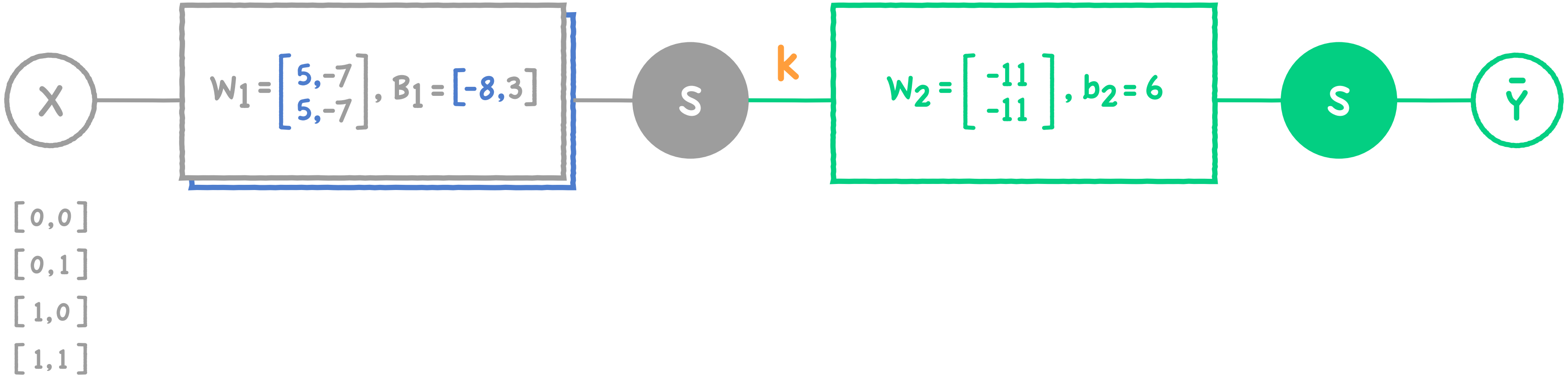
NN for XOR


Activation Function
X
$W_1 = \begin{bmatrix} 5,-7 \\ 5,-7 \end{bmatrix}, B_1 = [-8,3]$
S
k
$W_2 = \begin{bmatrix} -11 \\ -11 \end{bmatrix}, b_2 = 6$
S
$\bar{Y}$
[0,0]
[0,1]
[1,0]
[1,1]

NN for XOR

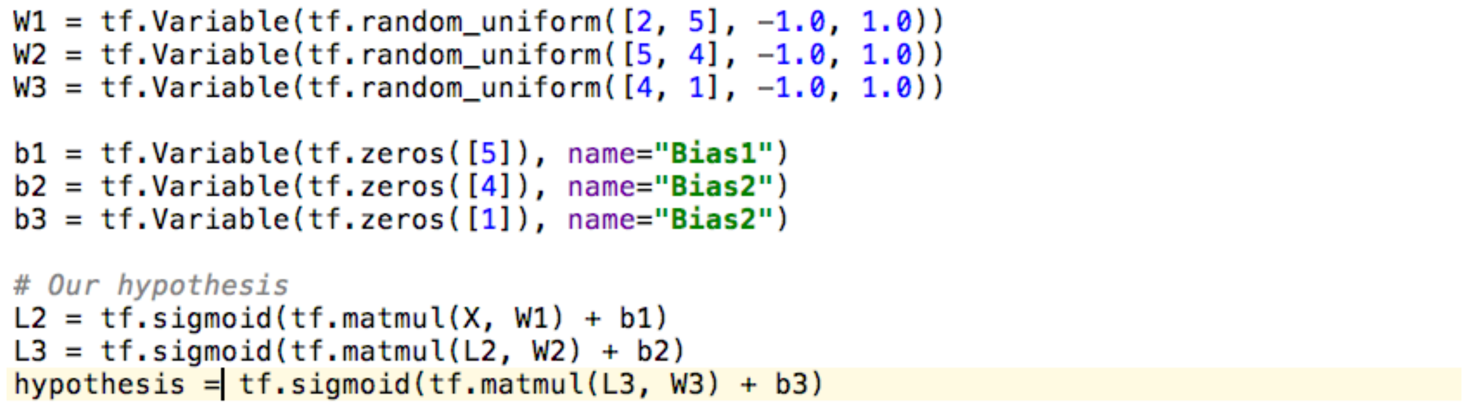


```
W1 = tf.Variable(tf.random_uniform([2, 2], -1.0, 1.0))
W2 = tf.Variable(tf.random_uniform([2, 1], -1.0, 1.0))

b1 = tf.Variable(tf.zeros([2]), name="Bias1")
b2 = tf.Variable(tf.zeros([1]), name="Bias2")

# Our hypothesis
L2 = tf.sigmoid(tf.matmul(X, W1) + b1)
hypothesis = tf.sigmoid(tf.matmul(L2, W2) + b2)
```

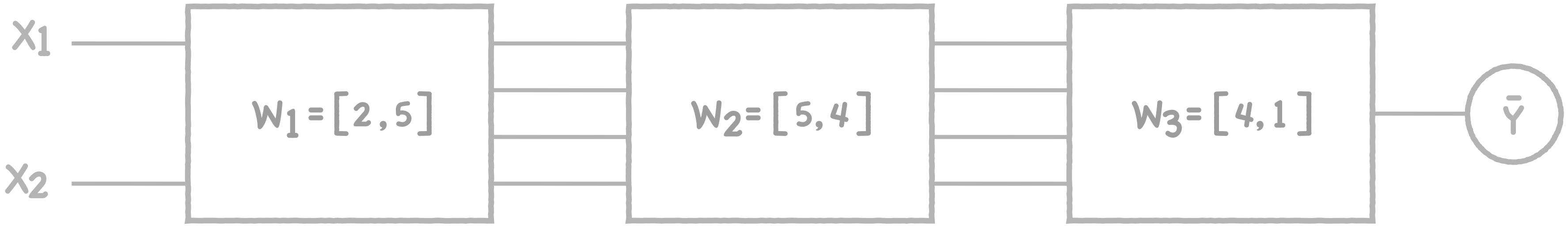


X
$W_1 = \begin{bmatrix} 5,-7 \\ 5,-7 \end{bmatrix}, B_1 = [-8,3]$
S
k
$W_2 = \begin{bmatrix} -11 \\ -11 \end{bmatrix}, b_2 = 6$
S
$\bar{Y}$
[0,0]
[0,1]
[1,0]
[1,1]

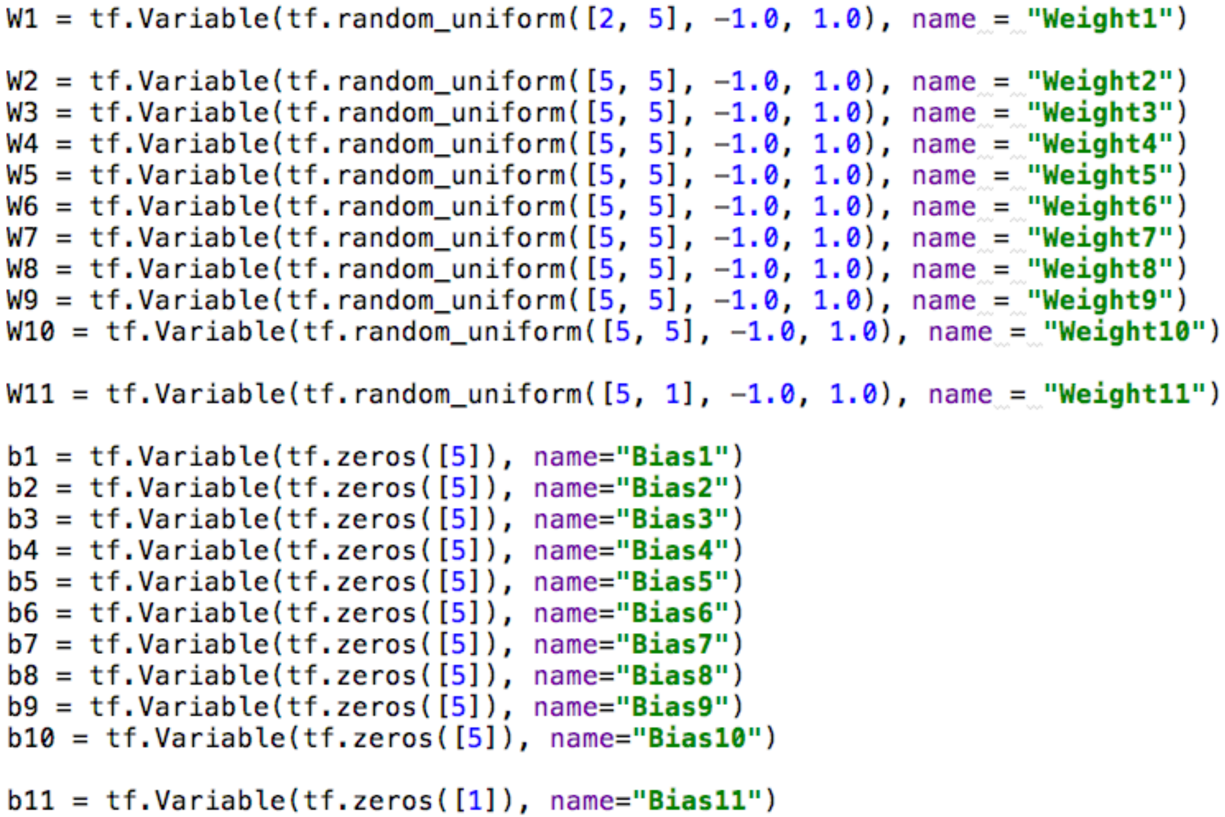
Let's go deep & wide!



```
W1 = tf.Variable(tf.random_uniform([2, 5], -1.0, 1.0))
W2 = tf.Variable(tf.random_uniform([5, 4], -1.0, 1.0))
W3 = tf.Variable(tf.random_uniform([4, 1], -1.0, 1.0))

b1 = tf.Variable(tf.zeros([5]), name="Bias1")
b2 = tf.Variable(tf.zeros([4]), name="Bias2")
b3 = tf.Variable(tf.zeros([1]), name="Bias2")

# Our hypothesis
L2 = tf.sigmoid(tf.matmul(X, W1) + b1)
L3 = tf.sigmoid(tf.matmul(L2, W2) + b2)
hypothesis = tf.sigmoid(tf.matmul(L3, W3) + b3)
```

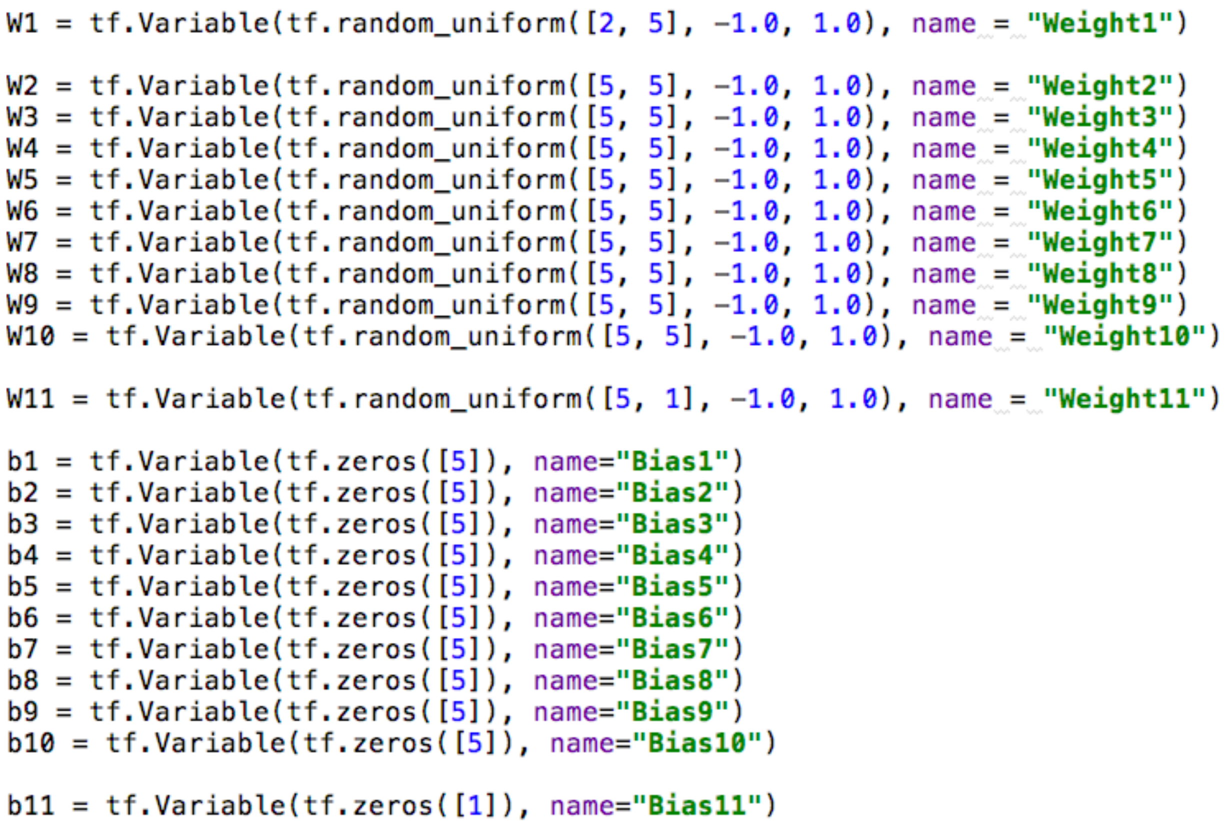


9 Hidden Layers!

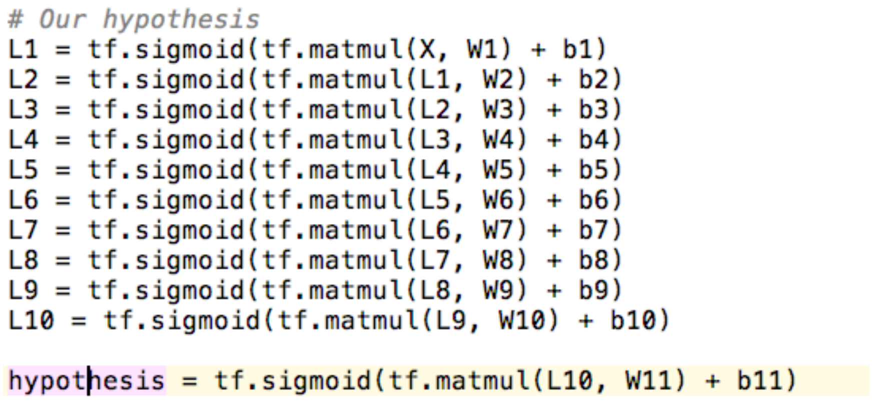


```
W1 = tf.Variable(tf.random_uniform([2, 5], -1.0, 1.0), name = "Weight1")

W2 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight2")
W3 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight3")
W4 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight4")
W5 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight5")
W6 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight6")
W7 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight7")
W8 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight8")
W9 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight9")
W10 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight10")

W11 = tf.Variable(tf.random_uniform([5, 1], -1.0, 1.0), name = "Weight11")

b1 = tf.Variable(tf.zeros([5]), name="Bias1")
b2 = tf.Variable(tf.zeros([5]), name="Bias2")
b3 = tf.Variable(tf.zeros([5]), name="Bias3")
b4 = tf.Variable(tf.zeros([5]), name="Bias4")
b5 = tf.Variable(tf.zeros([5]), name="Bias5")
b6 = tf.Variable(tf.zeros([5]), name="Bias6")
b7 = tf.Variable(tf.zeros([5]), name="Bias7")
b8 = tf.Variable(tf.zeros([5]), name="Bias8")
b9 = tf.Variable(tf.zeros([5]), name="Bias9")
b10 = tf.Variable(tf.zeros([5]), name="Bias10")

b11 = tf.Variable(tf.zeros([1]), name="Bias11")
```

9 Hidden Layers!

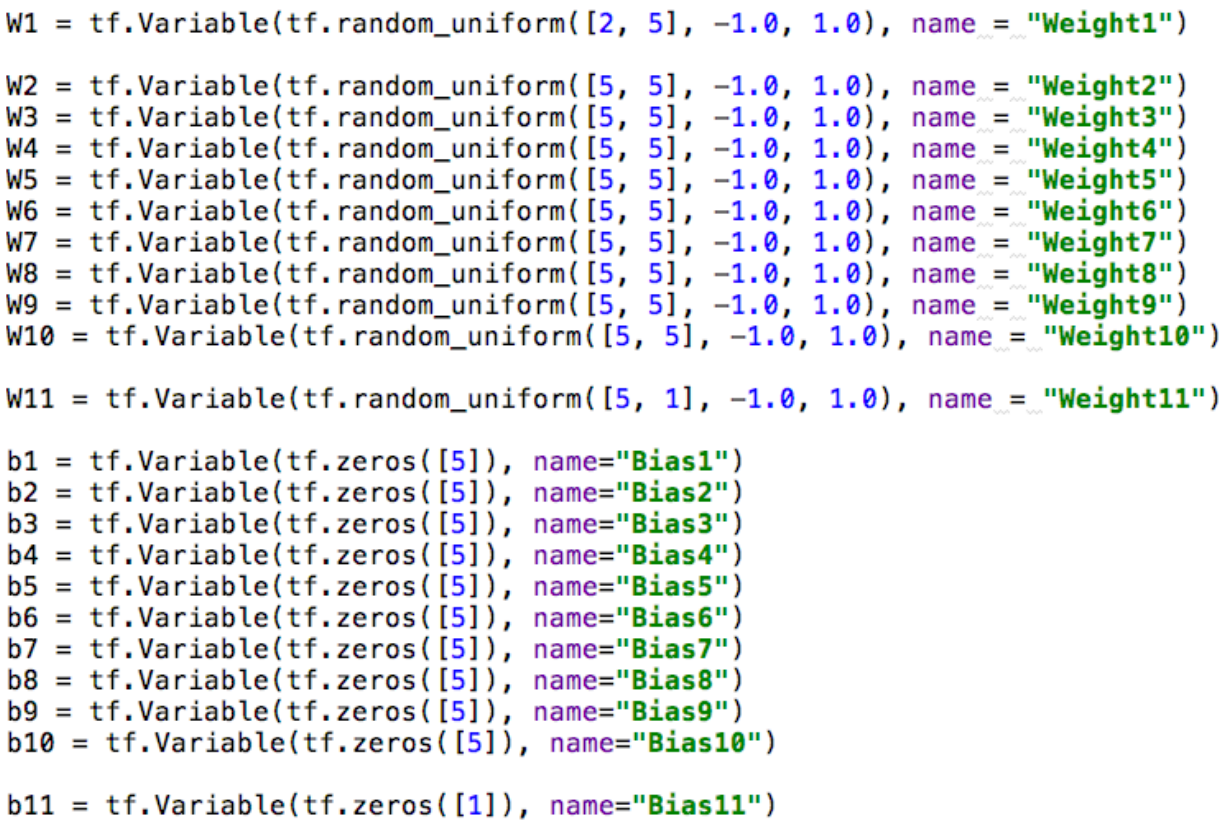


```
W1 = tf.Variable(tf.random_uniform([2, 5], -1.0, 1.0), name = "Weight1")

W2 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight2")
W3 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight3")
W4 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight4")
W5 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight5")
W6 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight6")
W7 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight7")
W8 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight8")
W9 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight9")
W10 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight10")

W11 = tf.Variable(tf.random_uniform([5, 1], -1.0, 1.0), name = "Weight11")

b1 = tf.Variable(tf.zeros([5]), name="Bias1")
b2 = tf.Variable(tf.zeros([5]), name="Bias2")
b3 = tf.Variable(tf.zeros([5]), name="Bias3")
b4 = tf.Variable(tf.zeros([5]), name="Bias4")
b5 = tf.Variable(tf.zeros([5]), name="Bias5")
b6 = tf.Variable(tf.zeros([5]), name="Bias6")
b7 = tf.Variable(tf.zeros([5]), name="Bias7")
b8 = tf.Variable(tf.zeros([5]), name="Bias8")
b9 = tf.Variable(tf.zeros([5]), name="Bias9")
b10 = tf.Variable(tf.zeros([5]), name="Bias10")

b11 = tf.Variable(tf.zeros([1]), name="Bias11")
```

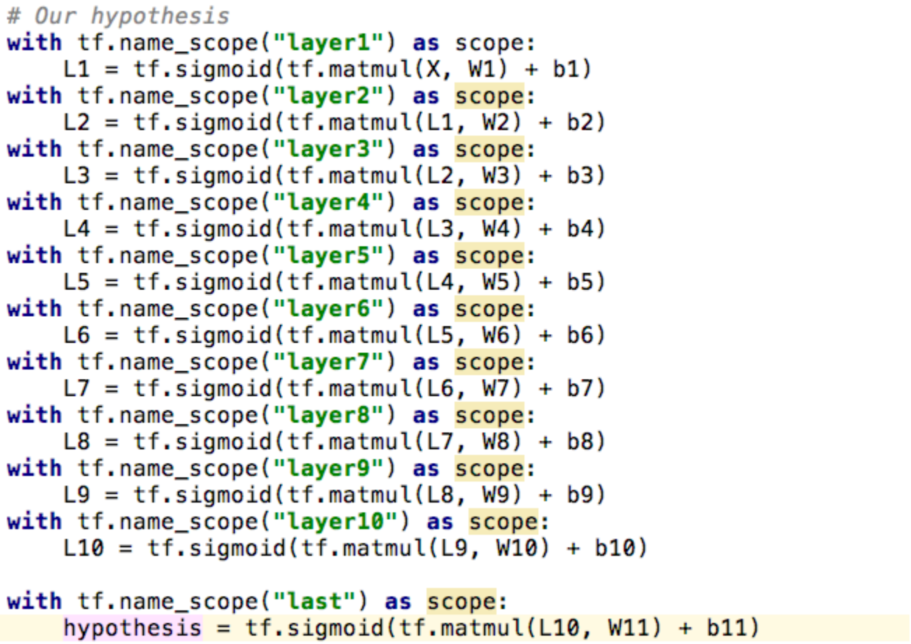


```
# Our hypothesis
L1 = tf.sigmoid(tf.matmul(X, W1) + b1)
L2 = tf.sigmoid(tf.matmul(L1, W2) + b2)
L3 = tf.sigmoid(tf.matmul(L2, W3) + b3)
L4 = tf.sigmoid(tf.matmul(L3, W4) + b4)
L5 = tf.sigmoid(tf.matmul(L4, W5) + b5)
L6 = tf.sigmoid(tf.matmul(L5, W6) + b6)
L7 = tf.sigmoid(tf.matmul(L6, W7) + b7)
L8 = tf.sigmoid(tf.matmul(L7, W8) + b8)
L9 = tf.sigmoid(tf.matmul(L8, W9) + b9)
L10 = tf.sigmoid(tf.matmul(L9, W10) + b10)

hypothesis = tf.sigmoid(tf.matmul(L10, W11) + b11)
```

9 Hidden Layers!

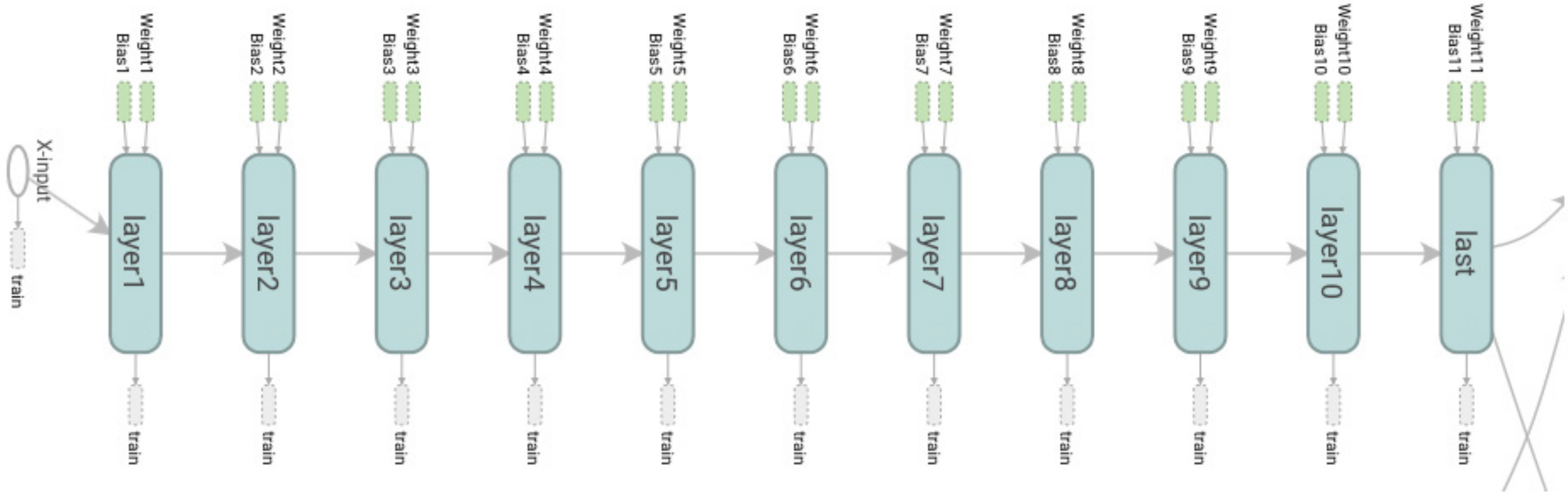


```python
W1 = tf.Variable(tf.random_uniform([2, 5], -1.0, 1.0), name = "Weight1")

W2 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight2")
W3 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight3")
W4 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight4")
W5 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight5")
W6 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight6")
W7 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight7")
W8 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight8")
W9 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight9")
W10 = tf.Variable(tf.random_uniform([5, 5], -1.0, 1.0), name = "Weight10")

W11 = tf.Variable(tf.random_uniform([5, 1], -1.0, 1.0), name = "Weight11")

b1 = tf.Variable(tf.zeros([5]), name="Bias1")
b2 = tf.Variable(tf.zeros([5]), name="Bias2")
b3 = tf.Variable(tf.zeros([5]), name="Bias3")
b4 = tf.Variable(tf.zeros([5]), name="Bias4")
b5 = tf.Variable(tf.zeros([5]), name="Bias5")
b6 = tf.Variable(tf.zeros([5]), name="Bias6")
b7 = tf.Variable(tf.zeros([5]), name="Bias7")
b8 = tf.Variable(tf.zeros([5]), name="Bias8")
b9 = tf.Variable(tf.zeros([5]), name="Bias9")
b10 = tf.Variable(tf.zeros([5]), name="Bias10")

b11 = tf.Variable(tf.zeros([1]), name="Bias11")
```

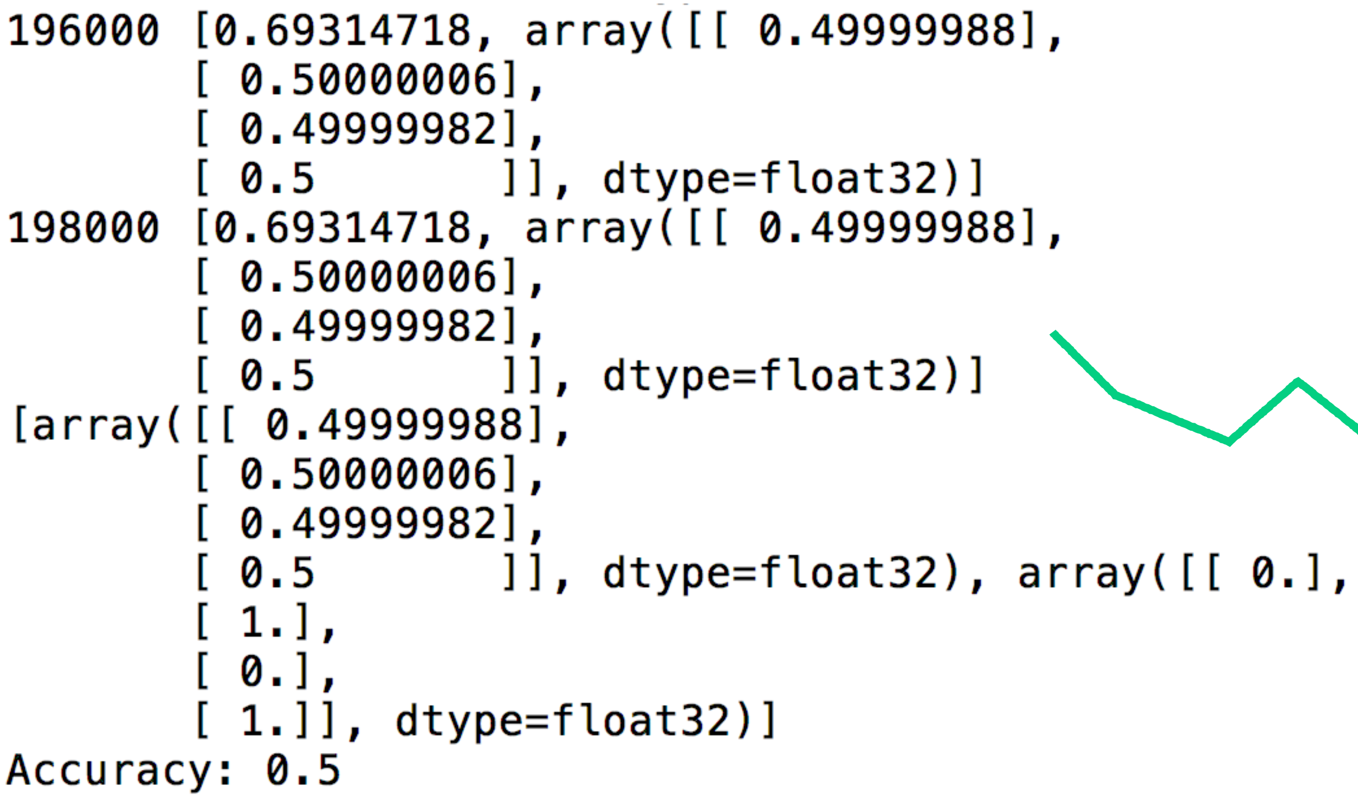


```python
# Our hypothesis
with tf.name_scope("layer1") as scope:
    L1 = tf.sigmoid(tf.matmul(X, W1) + b1)
with tf.name_scope("layer2") as scope:
    L2 = tf.sigmoid(tf.matmul(L1, W2) + b2)
with tf.name_scope("layer3") as scope:
    L3 = tf.sigmoid(tf.matmul(L2, W3) + b3)
with tf.name_scope("layer4") as scope:
    L4 = tf.sigmoid(tf.matmul(L3, W4) + b4)
with tf.name_scope("layer5") as scope:
    L5 = tf.sigmoid(tf.matmul(L4, W5) + b5)
with tf.name_scope("layer6") as scope:
    L6 = tf.sigmoid(tf.matmul(L5, W6) + b6)
with tf.name_scope("layer7") as scope:
    L7 = tf.sigmoid(tf.matmul(L6, W7) + b7)
with tf.name_scope("layer8") as scope:
    L8 = tf.sigmoid(tf.matmul(L7, W8) + b8)
with tf.name_scope("layer9") as scope:
    L9 = tf.sigmoid(tf.matmul(L8, W9) + b9)
with tf.name_scope("layer10") as scope:
    L10 = tf.sigmoid(tf.matmul(L9, W10) + b10)

with tf.name_scope("last") as scope:
    hypothesis = tf.sigmoid(tf.matmul(L10, W11) + b11)
```

Tensorboard Visualization

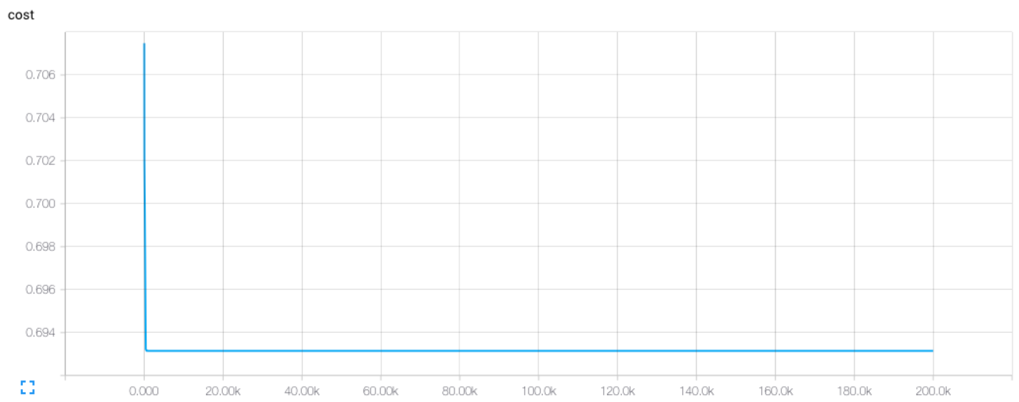


Poor Results?

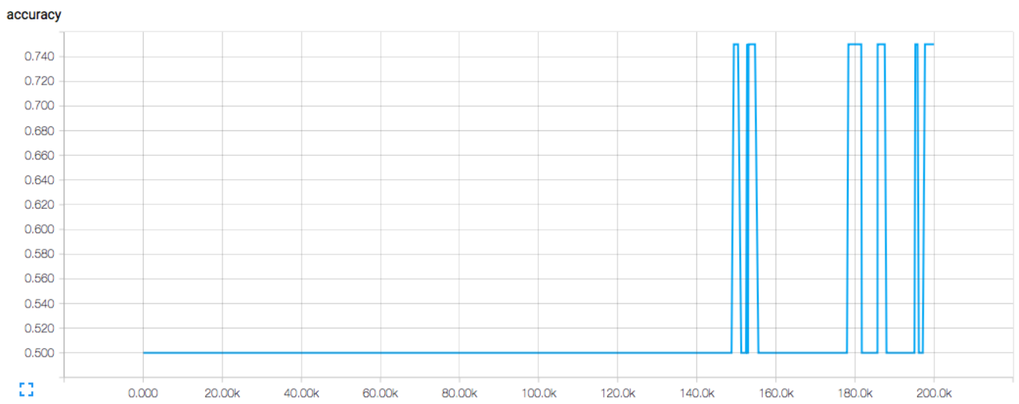


```
196000 [0.69314718, array([[ 0.49999988],
       [ 0.50000006],
       [ 0.49999982],
       [ 0.5       ]], dtype=float32)]
198000 [0.69314718, array([[ 0.49999988],
       [ 0.50000006],
       [ 0.49999982],
       [ 0.5       ]], dtype=float32)]
[array([[ 0.49999988],
       [ 0.50000006],
       [ 0.49999982],
       [ 0.5       ]], dtype=float32), array([[ 0.],
       [ 1.],
       [ 0.],
       [ 1.]], dtype=float32)]
Accuracy: 0.5
```

Tensorboard Cost & Accuracy

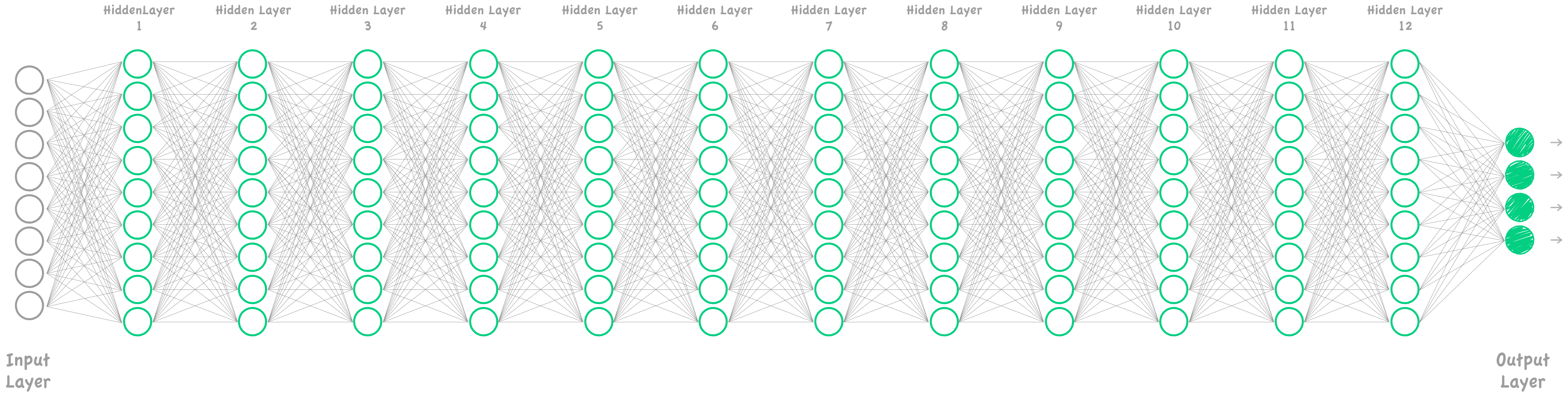


Cost

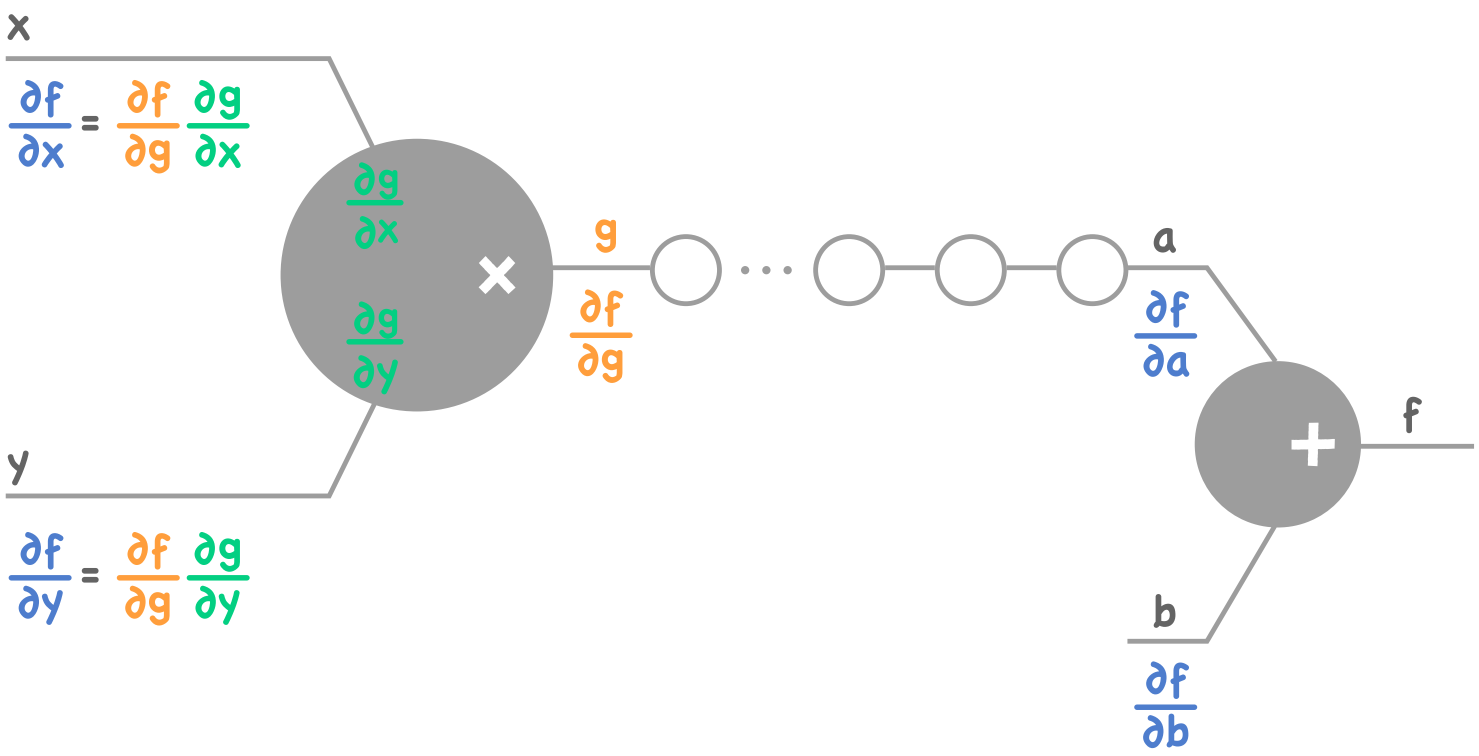


Accuracy

Backpropagation



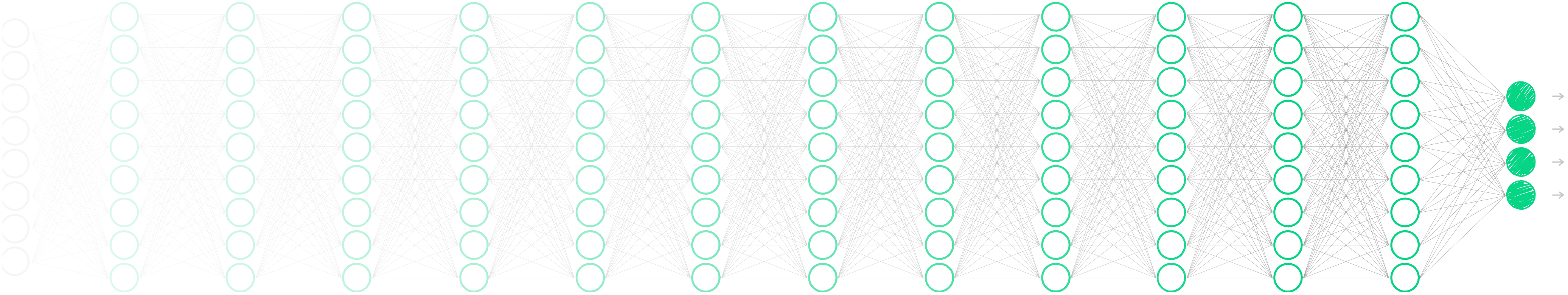
Lec 9-2 : Backpropagation (Chain Rule)



x
$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial x}$
$\frac{\partial g}{\partial x}$
$\frac{\partial g}{\partial y}$
×
g
$\frac{\partial f}{\partial g}$
y
$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial y}$
a
$\frac{\partial f}{\partial a}$
+
f
b
$\frac{\partial f}{\partial b}$

http://cs231n.stanford.edu/

Vanishing Gradient (NN Winter 2 : 1986-2006)

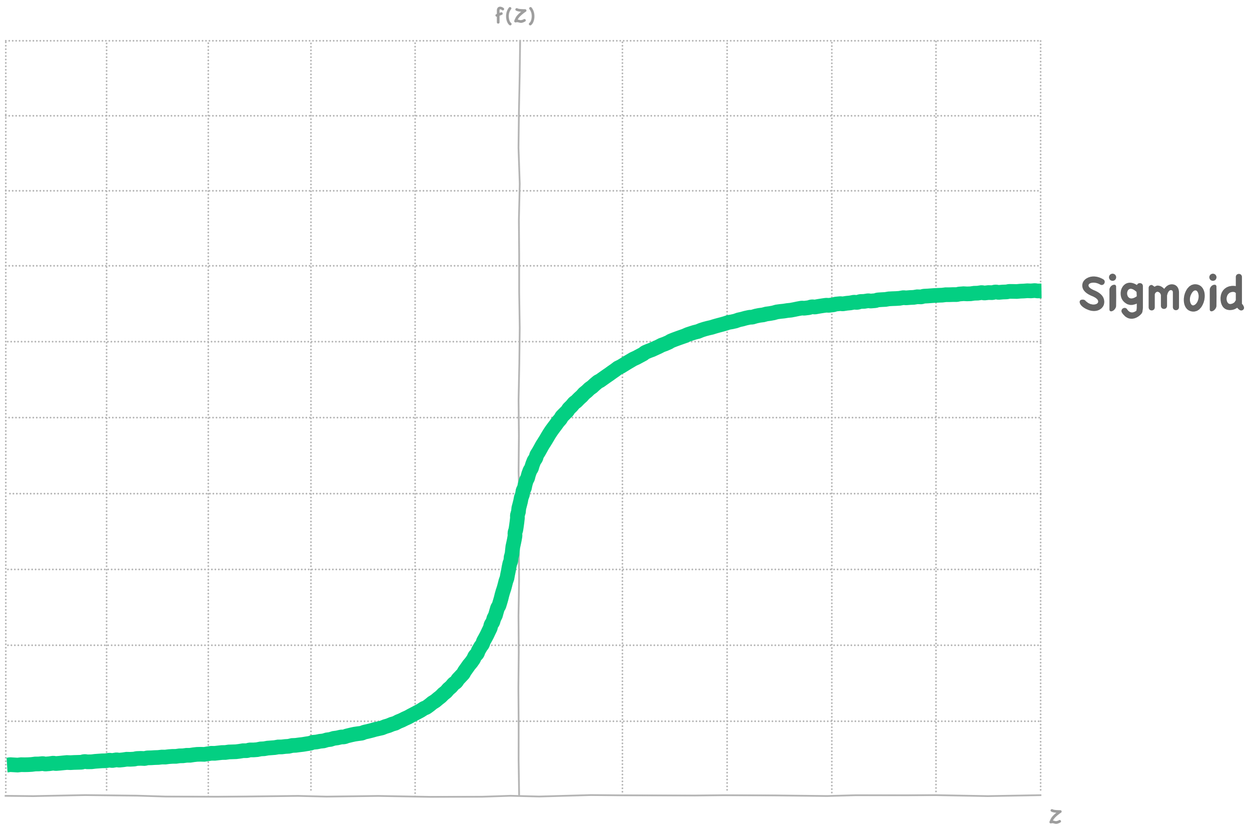


Geoffrey Hinton's Summary of Findings up to Today

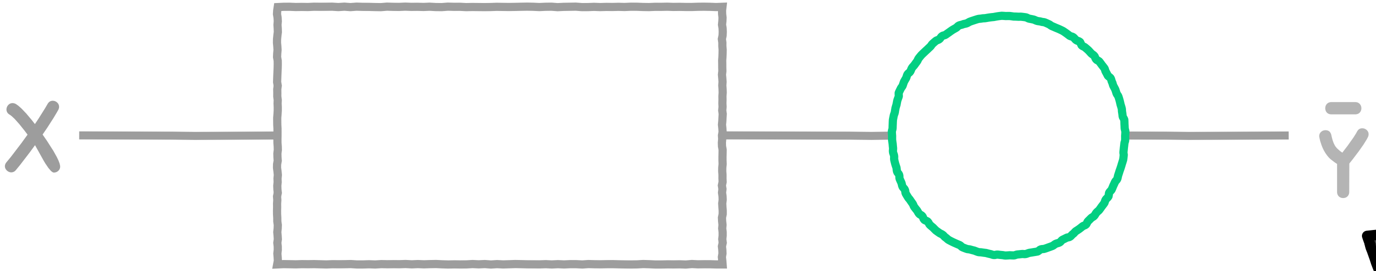
- Our labeled datasets were thousands of times too small
- Our computers were millions of times too slow
- We initialized the weights in a stupid way
- We used the wrong type of non-linearity

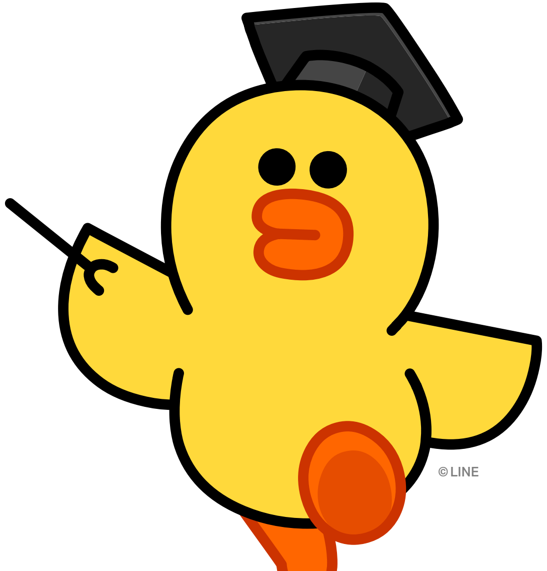
http://www.andreykurenkov.com/writing/a-brief-history-of-neural-nets-and-deep-learning-part-4/

Sigmoid!

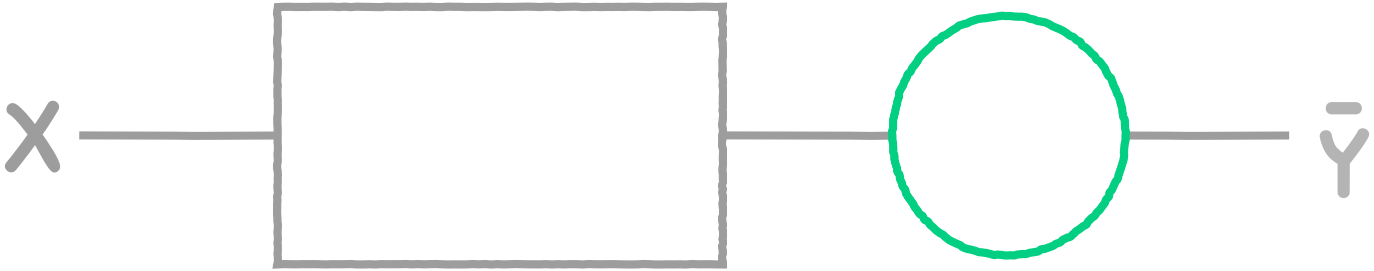


ReLU : Rectified Linear Unit



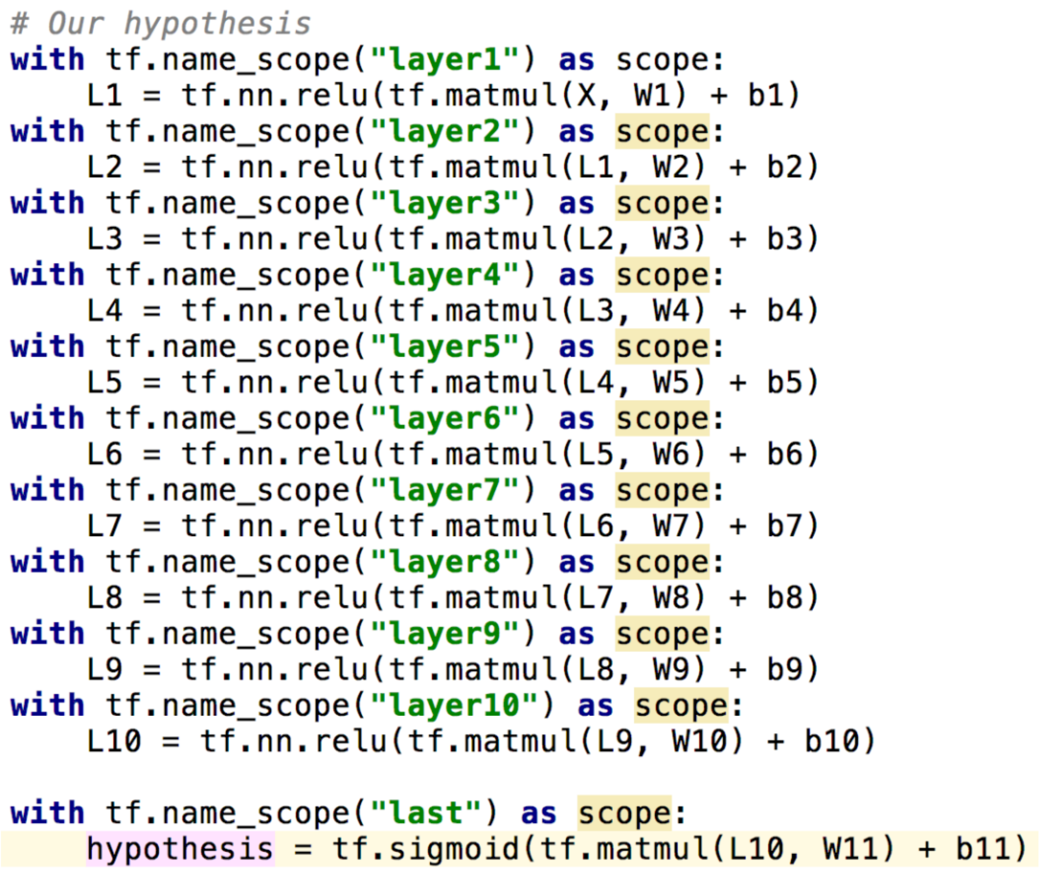

©LINE

ReLU : Rectified Linear Unit



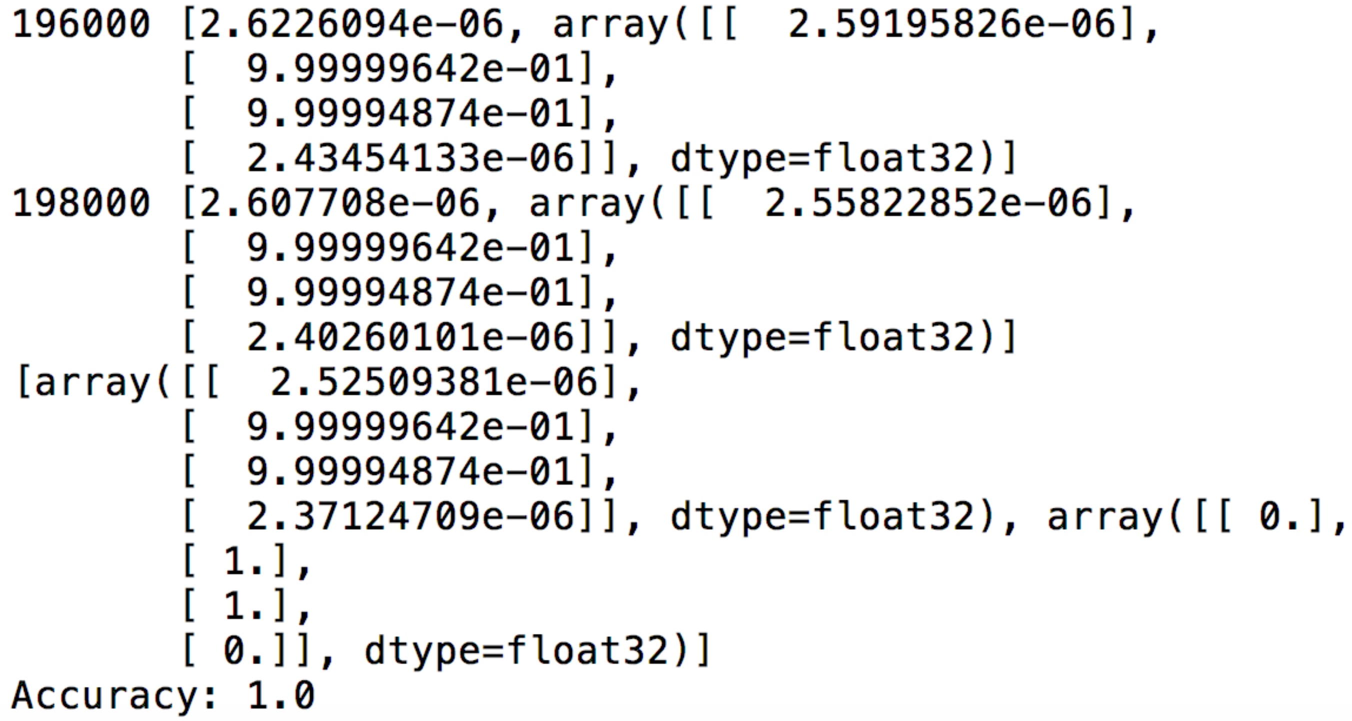
```
L1 = tf.sigmoid(tf.matmul(X,W1) + b1)
L1 = tf.nn.relu(tf.matmul(X,W1) + b1)
```

ReLu



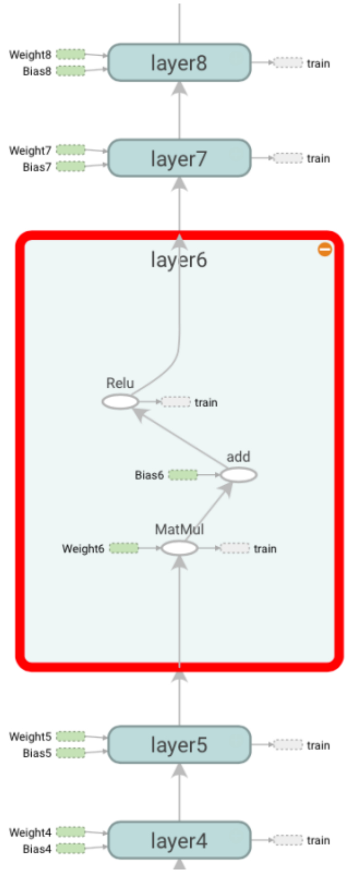
```
# Our hypothesis
with tf.name_scope("layer1") as scope:
    L1 = tf.nn.relu(tf.matmul(X, W1) + b1)
with tf.name_scope("layer2") as scope:
    L2 = tf.nn.relu(tf.matmul(L1, W2) + b2)
with tf.name_scope("layer3") as scope:
    L3 = tf.nn.relu(tf.matmul(L2, W3) + b3)
with tf.name_scope("layer4") as scope:
    L4 = tf.nn.relu(tf.matmul(L3, W4) + b4)
with tf.name_scope("layer5") as scope:
    L5 = tf.nn.relu(tf.matmul(L4, W5) + b5)
with tf.name_scope("layer6") as scope:
    L6 = tf.nn.relu(tf.matmul(L5, W6) + b6)
with tf.name_scope("layer7") as scope:
    L7 = tf.nn.relu(tf.matmul(L6, W7) + b7)
with tf.name_scope("layer8") as scope:
    L8 = tf.nn.relu(tf.matmul(L7, W8) + b8)
with tf.name_scope("layer9") as scope:
    L9 = tf.nn.relu(tf.matmul(L8, W9) + b9)
with tf.name_scope("layer10") as scope:
    L10 = tf.nn.relu(tf.matmul(L9, W10) + b10)

with tf.name_scope("last") as scope:
    hypothesis = tf.sigmoid(tf.matmul(L10, W11) + b11)
```

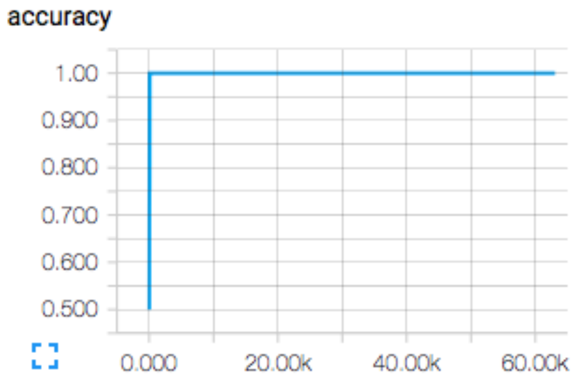
Works very well

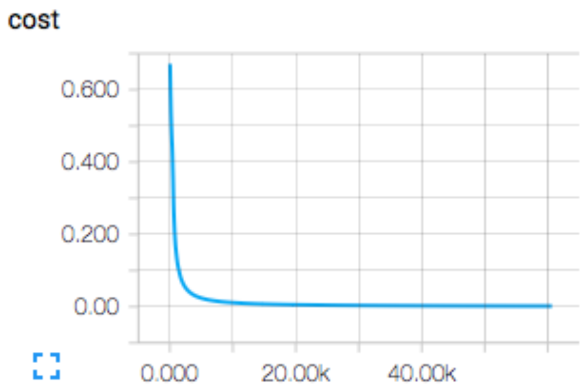


```
196000 [2.6226094e-06, array([[  2.59195826e-06],
       [  9.99999642e-01],
       [  9.99994874e-01],
       [  2.43454133e-06]], dtype=float32)]
198000 [2.607708e-06, array([[  2.55822852e-06],
       [  9.99999642e-01],
       [  9.99994874e-01],
       [  2.40260101e-06]], dtype=float32)]
[array([[  2.52509381e-06],
       [  9.99999642e-01],
       [  9.99994874e-01],
       [  2.37124709e-06]], dtype=float32), array([[ 0.],
       [ 1.],
       [ 1.],
       [ 0.]], dtype=float32)]
Accuracy: 1.0
```

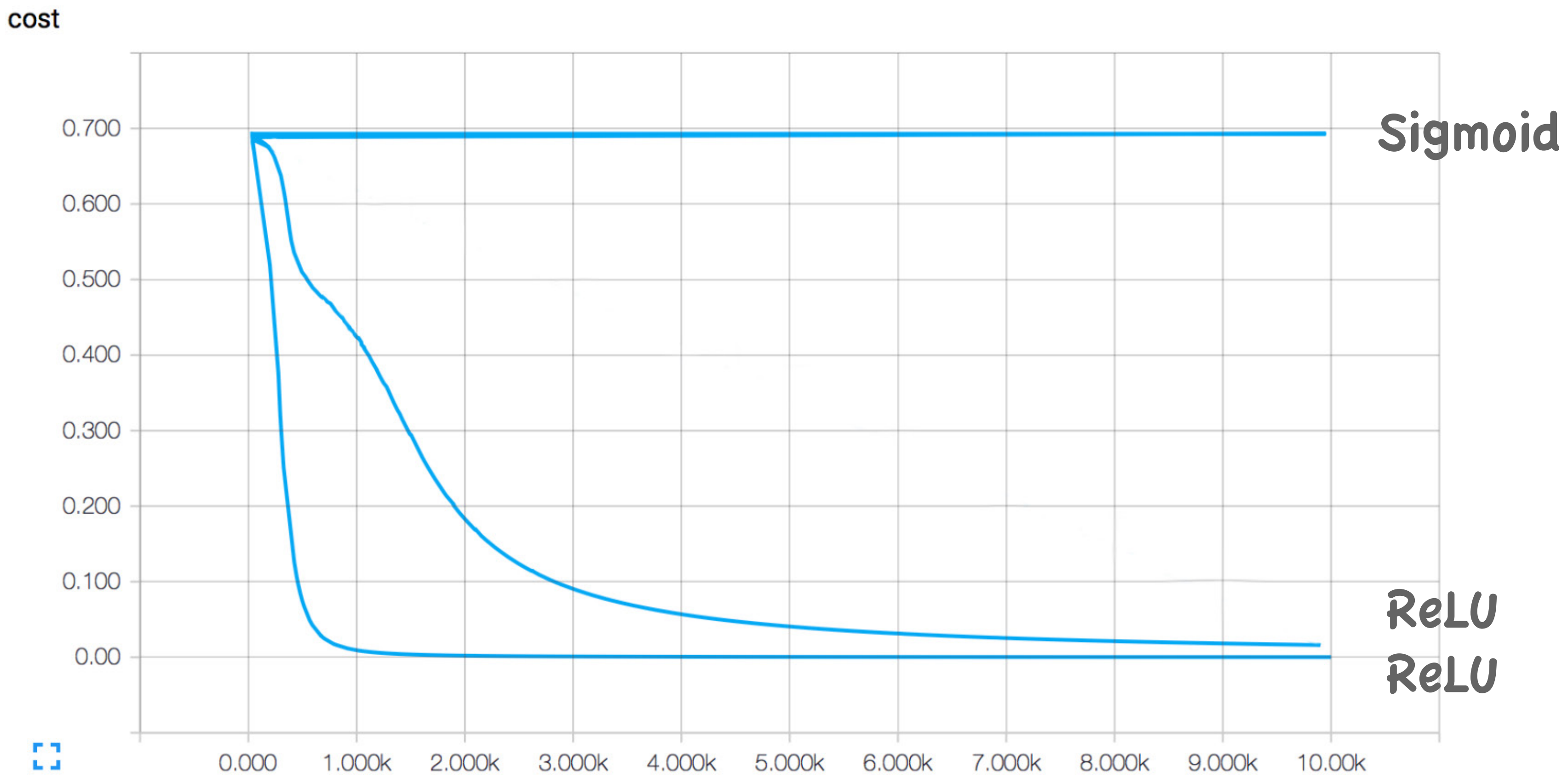
Works very well







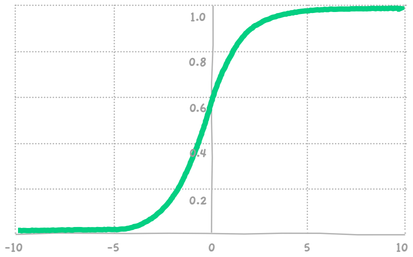
Cost Function



Activation Functions

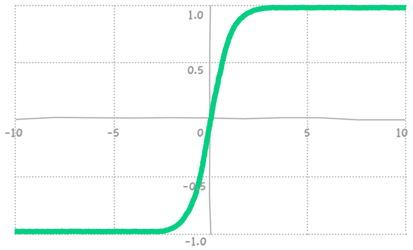
Sigmoid

$\sigma(x) = 1/(1+e^{-x})$



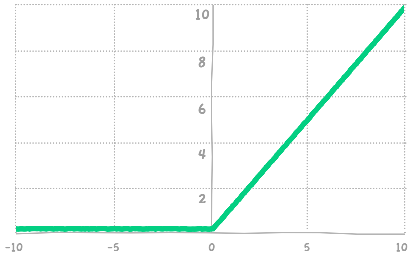
tanh

$\tanh(x)$



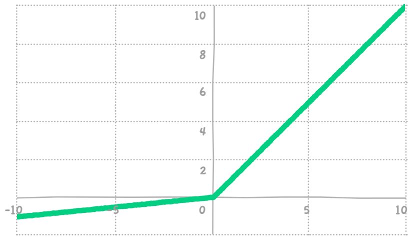
ReLU

$\max(0,x)$



Leaky ReLU

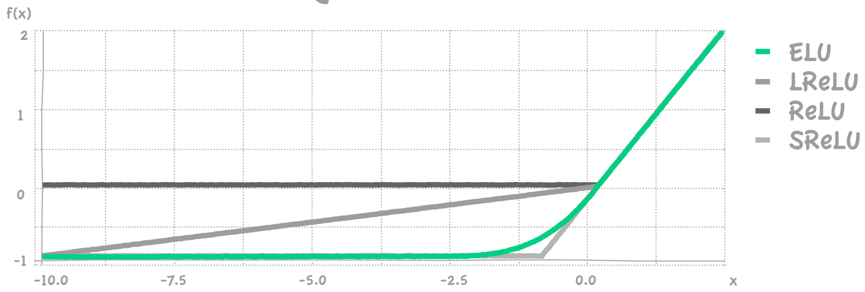
$\max(0.1x, x)$



Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha(\exp(x)-1) & \text{if } x \leq 0 \end{cases}$$



Activation Functions on CIFAR-10

Mishkin et al. 2015

maxout	ReLU	VLReLU	tanh	Sigmoid
93.94	92.11	92.97	89.28	n/c
93.78	91.74	92.40	89.48	n/c
-	91.93	93.09	-	n/c
91.75	90.63	92.27	89.82	n/c
n/c+	90.91	92.43	89.54	n/c

NEXT LECTURE

WEIGHT INITIALIZATION