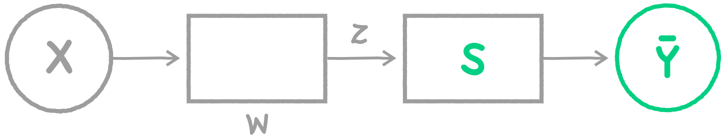
LECTURE 9-1

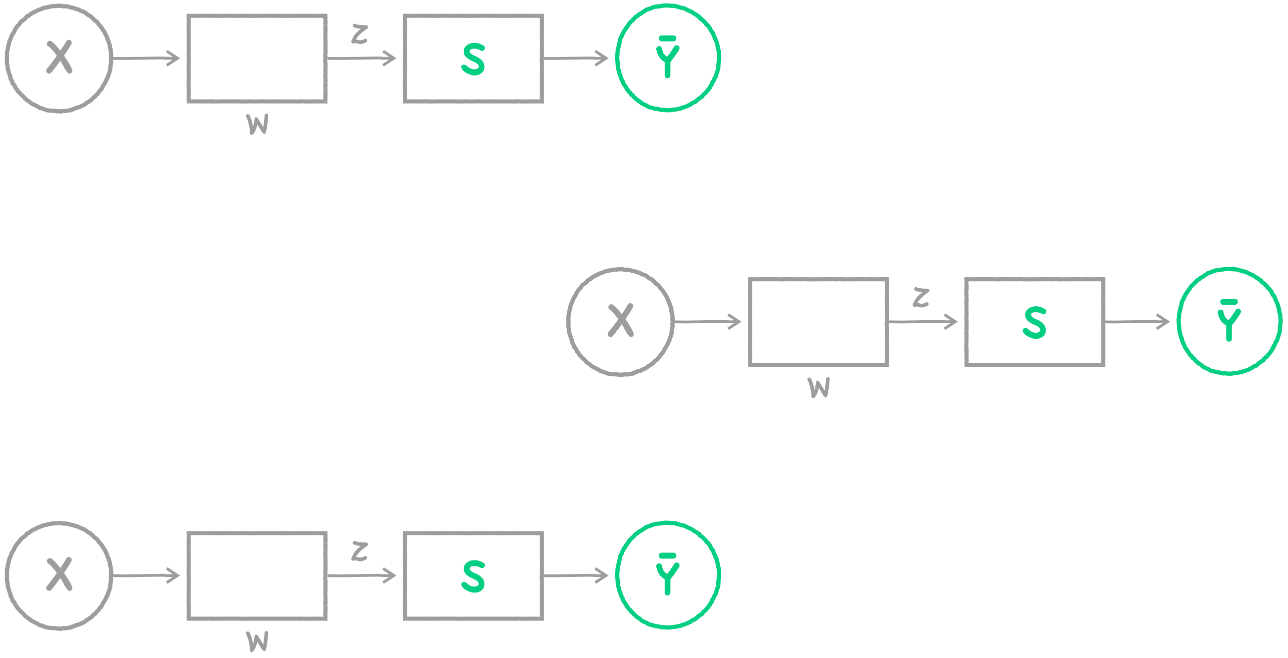
NEURAL NETS(NN) FOR XOR

Sung Kim <hunkim+ml@gmail.com>
http://hunkim.github.io/ml

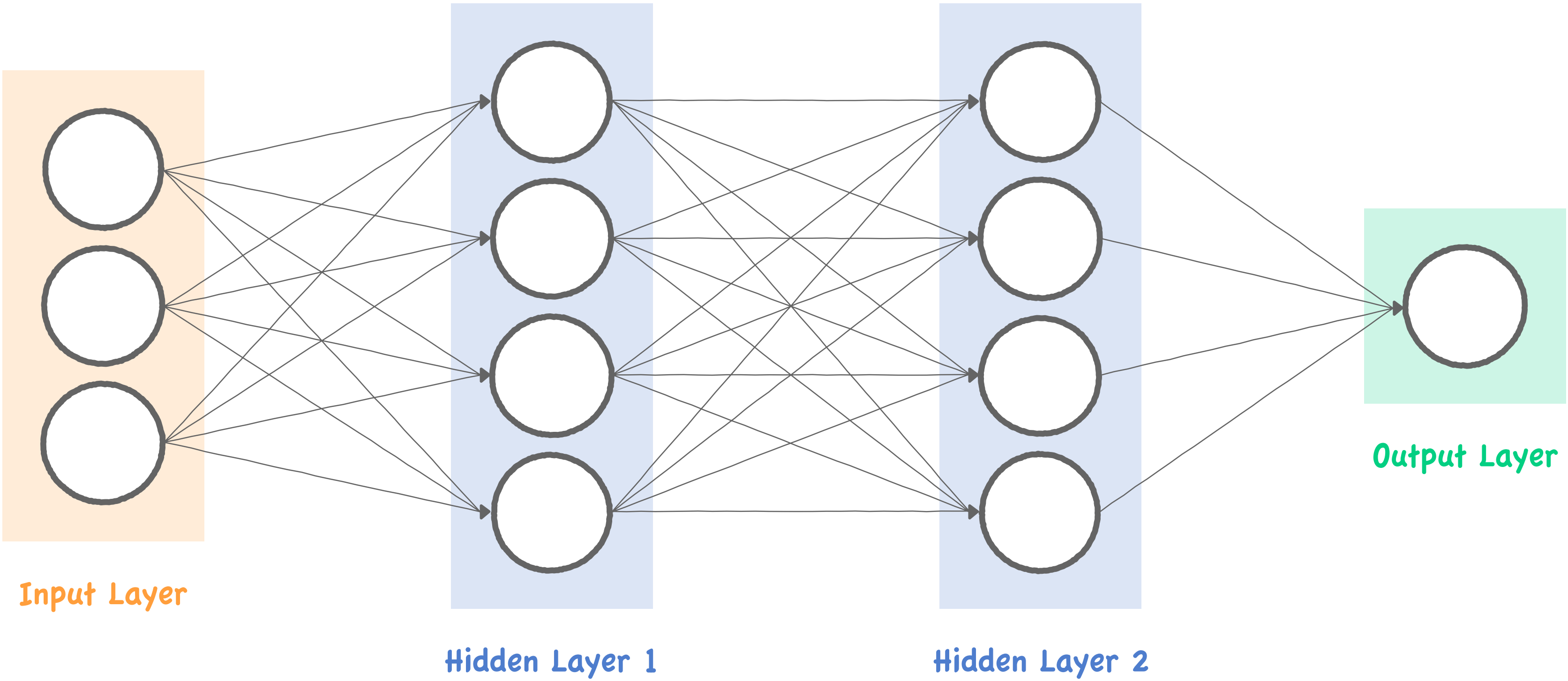
One Logistic Regression Unit Cannot Separate XOR



Multiple Logistic Regression Units



Neural Network(NN)



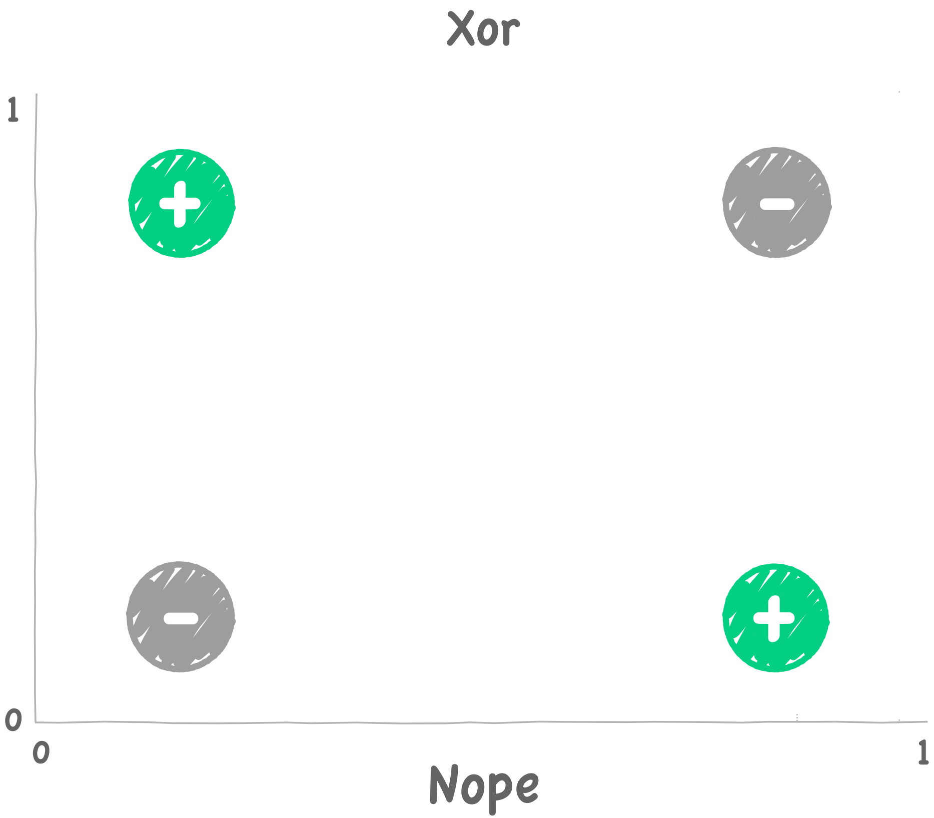
"No one on earth had found a viable way to train"

<Marvin Minsky>

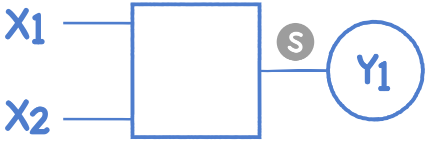
http://cs231n.github.io/convolutional-networks)

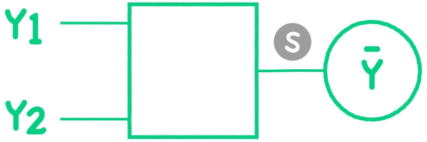
XOR Using NN

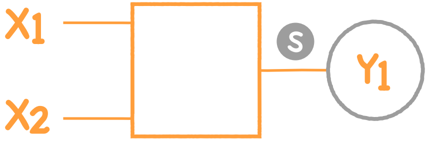
X_1	X_2	XOR
0	0	0(-)
0	1	1(+)
1	0	1(+)
1	1	0(-)



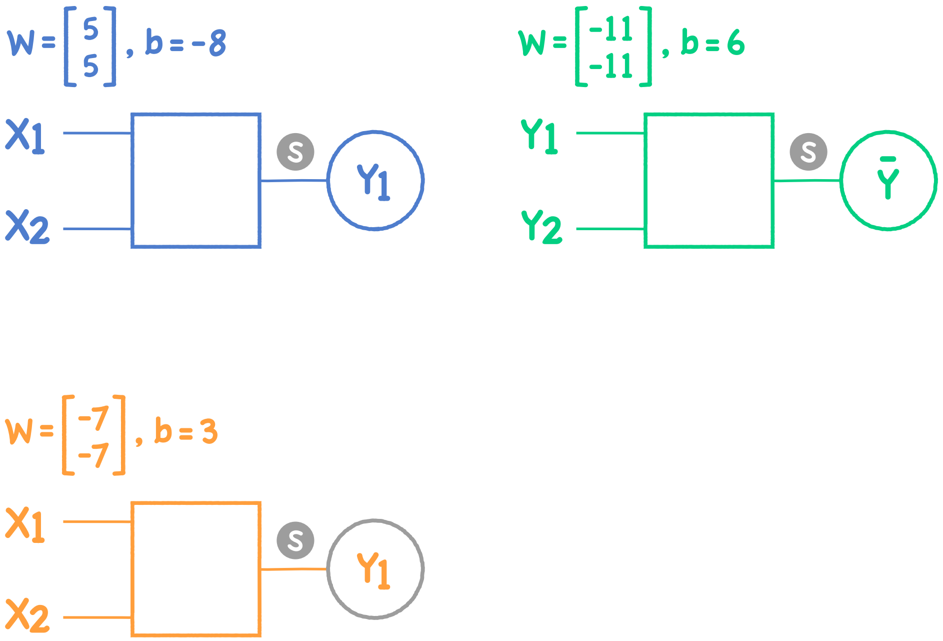
Neural Net







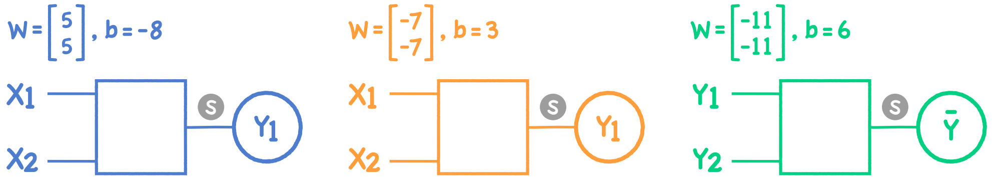
Neural Net



$W = \begin{bmatrix} 5 \\ 5 \end{bmatrix}, b = -8$
X_1
X_2
S
Y_1
$W = \begin{bmatrix} -11 \\ -11 \end{bmatrix}, b = 6$
Y_1
Y_2
S
$\bar{Y}$
$W = \begin{bmatrix} -7 \\ -7 \end{bmatrix}, b = 3$
X_1
X_2
S
Y_1

X_1	X_2	Y_1	Y_2	$\bar{Y}$	XOR
0	0				0
0	1				1
1	0				1
1	1				0

Neural Net



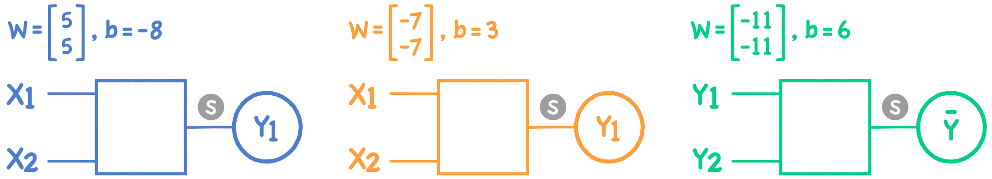
$W = \begin{bmatrix}5\\5\end{bmatrix}$, $b = -8$
X_1
X_2
S
Y_1
$W = \begin{bmatrix}-7\\-7\end{bmatrix}$, $b = 3$
X_1
X_2
S
Y_1
$W = \begin{bmatrix}-11\\-11\end{bmatrix}$, $b = 6$
Y_1
Y_2
S
$\bar{Y}$

$$[0\ 0]\begin{bmatrix}5\\5\end{bmatrix} - 8 =$$

$$[0\ 0]\begin{bmatrix}-7\\-7\end{bmatrix} + 3 =$$

X_1	X_2	Y_1	Y_2	$\bar{Y}$	XOR
0	0				0
0	1				1
1	0				1
1	1				0

Neural Net



$W = \begin{bmatrix}5\\5\end{bmatrix}$, b = -8
X_1
X_2
S
Y_1
$W = \begin{bmatrix}-7\\-7\end{bmatrix}$, b = 3
X_1
X_2
S
Y_1
$W = \begin{bmatrix}-11\\-11\end{bmatrix}$, b = 6
Y_1
Y_2
S
$\bar{Y}$

$[0\ \ 0]\begin{bmatrix}5\\5\end{bmatrix} - 8 = 0 + 0 - 8 = -8$, Sigmoid(-8) = 0

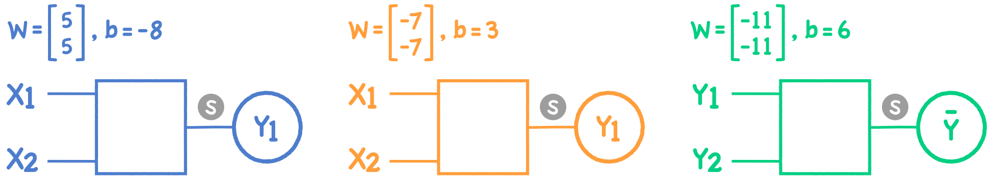
$[0\ \ 0]\begin{bmatrix}-7\\-7\end{bmatrix} + 3 = 0 + 0 + 3 = 3$, Sigmoid(3) = 1

$[0\ \ 1]\begin{bmatrix}-11\\-11\end{bmatrix} + 6 = 0 + (-11) + 6 = -5$

Sigmoid(-5) = 0

X_1	X_2	Y_1	Y_2	$\bar{Y}$	XOR
0	0	0	1	0	0
0	1				1
1	0				1
1	1				0

Neural Net



$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} - 8 = 0 + 5 - 8 = -3, \text{Sigmoid}(-3) = 0$$

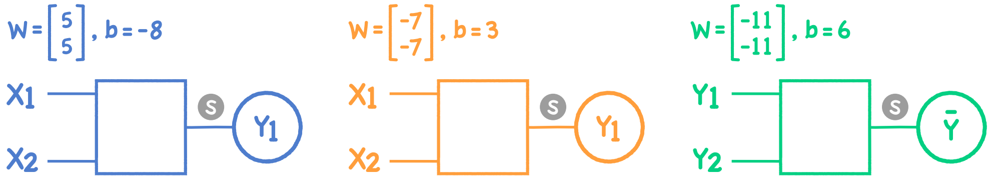
$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} -7 \\ -7 \end{bmatrix} + 3 = 0 + (-7) + 3 = -4, \text{Sigmoid}(-4) = 0$$

$$\begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} -11 \\ -11 \end{bmatrix} + 6 = 0 + 0 + 6 = 6$$

$$\text{Sigmoid}(6) = 1$$

X_1	X_2	Y_1	Y_2	$\bar{Y}$	XOR
0	0	0	1	0	0
0	1	0	0	1	1
1	0				1
1	1				0

Neural Net


W = $\begin{bmatrix}5\\5\end{bmatrix}$, b = -8
X1
X2
S
Y1
W = $\begin{bmatrix}-7\\-7\end{bmatrix}$, b = 3
X1
X2
S
Y1
W = $\begin{bmatrix}-11\\-11\end{bmatrix}$, b = 6
Y1
Y2
S
$\bar{Y}$

$\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}5\\5\end{bmatrix} - 8 = 5 + 0 - 8 = -3$, Sigmoid(−3) = 0

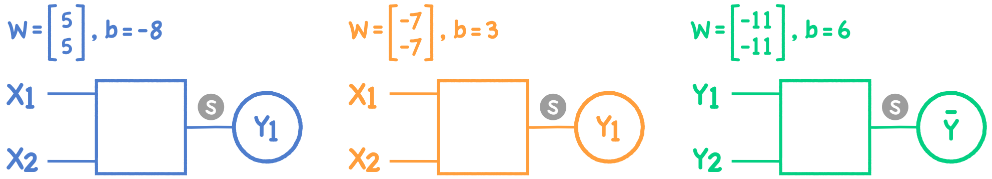
$\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}-7\\-7\end{bmatrix} + 3 = (-7) + 0 + 3 = -4$, Sigmoid(−4) = 0

$\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}-11\\-11\end{bmatrix} + 6 = 0 + 0 + 6 = 6$

Sigmoid(6) = 1

X_1	X_2	Y_1	Y_2	$\bar{Y}$	XOR
0	0	0	1	0	0
0	1	0	0	1	1
1	0	0	0	1	1
1	1				0

Neural Net



$W=\begin{bmatrix}5\\5\end{bmatrix}, b=-8$
X_1
X_2
S
Y_1
$W=\begin{bmatrix}-7\\-7\end{bmatrix}, b=3$
X_1
X_2
S
Y_1
$W=\begin{bmatrix}-11\\-11\end{bmatrix}, b=6$
Y_1
Y_2
S
$\bar{Y}$

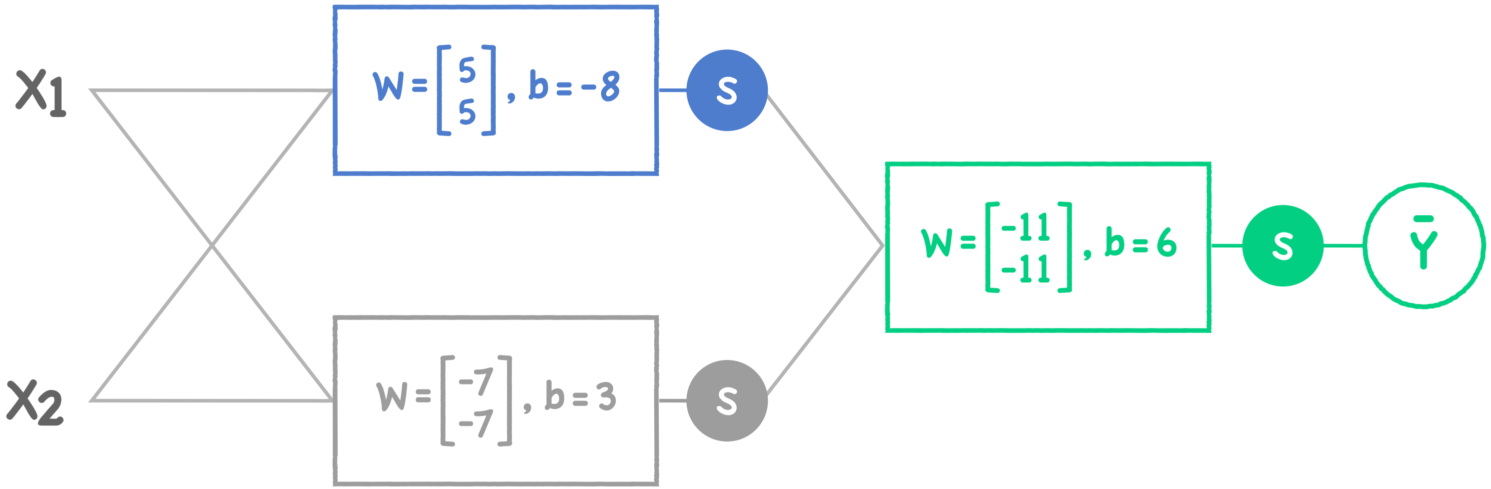
$[1\ \ 1]\begin{bmatrix}5\\5\end{bmatrix} - 8 = 5 + 5 - 8 = 2$, Sigmoid(2) = 1

$[1\ \ 1]\begin{bmatrix}-7\\-7\end{bmatrix} + 3 = (-7) + (-7) + 3 = -11$, Sigmoid(-11) = 0

$[1\ \ 0]\begin{bmatrix}-11\\-11\end{bmatrix} + 6 = (-11) + 0 + 6 = -5$
Sigmoid(-5) = 0

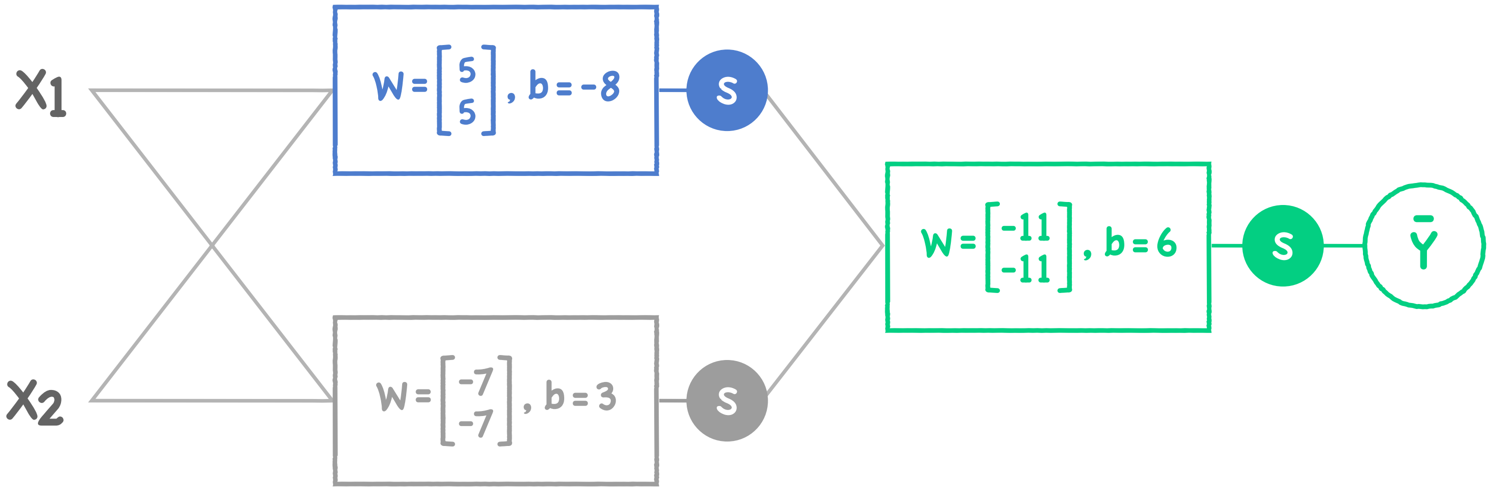
X_1	X_2	Y_1	Y_2	$\bar{Y}$	XOR
0	0	0	1	0	0
0	1	0	0	1	1
1	0	0	0	1	1
1	1	1	0	0	0

Forward Propagation



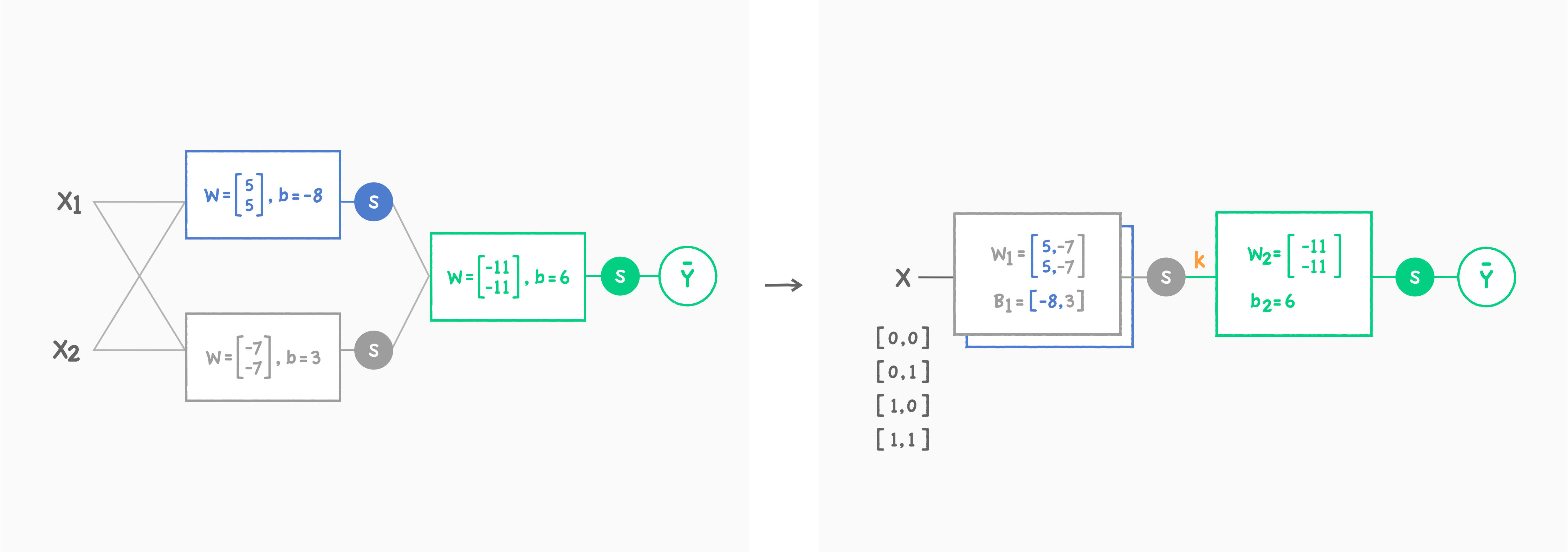
X_1
X_2
$W=\begin{bmatrix}5\\5\end{bmatrix}, b=-8$
$W=\begin{bmatrix}-7\\-7\end{bmatrix}, b=3$
S
S
$W=\begin{bmatrix}-11\\-11\end{bmatrix}, b=6$
S
$\bar{Y}$

Forward Propagation



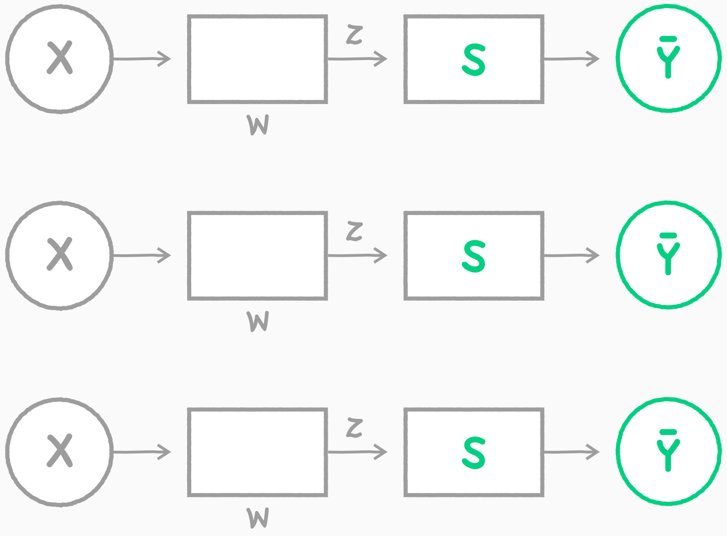
Can you find another W and b for the XOR?

NN

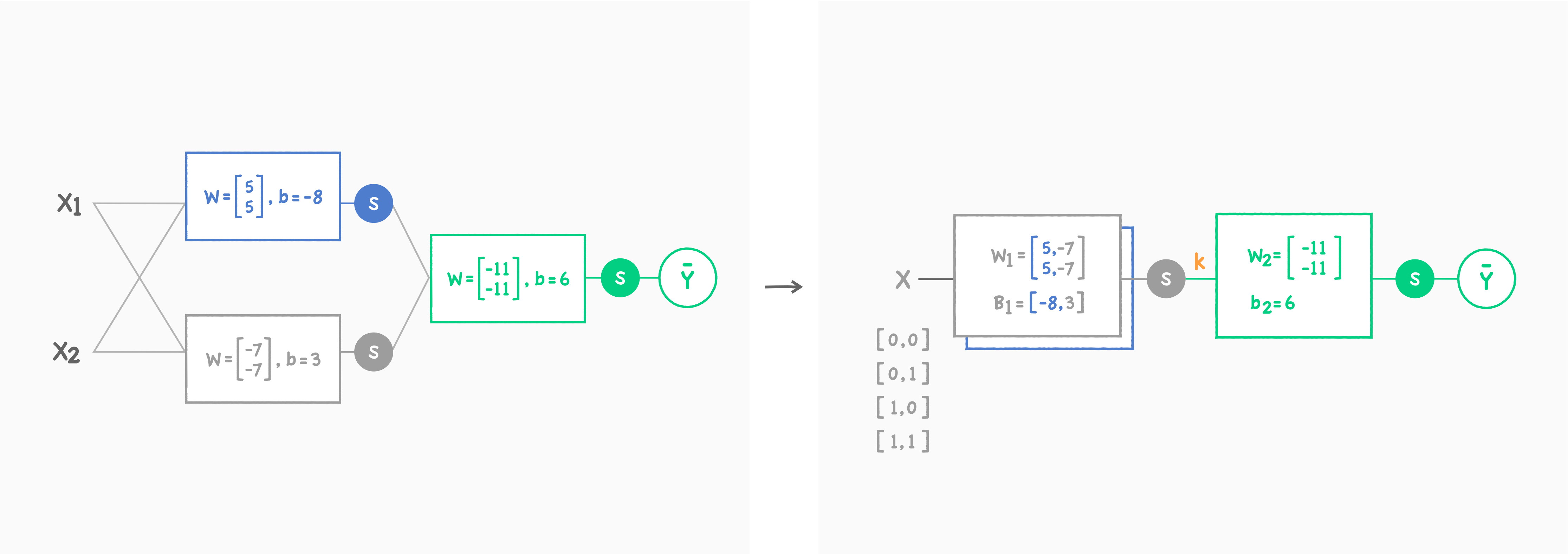

X1
X2
W = [5 5], b = -8
W = [-7 -7], b = 3
S
S
W = [-11 -11], b = 6
S
Ȳ
X
W1 = [5,-7 5,-7]
B1 = [-8,3]
S
k
W2 = [-11 -11]
b2 = 6
S
Ȳ
[0,0]
[0,1]
[1,0]
[1,1]

Recap: Lec 6-1 Multinomial Classification

$$\begin{bmatrix} W_{A1} & W_{A2} & W_{A3} \\ W_{B1} & W_{B2} & W_{B3} \\ W_{C1} & W_{C2} & W_{C3} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} W_{A1}X_1 + W_{A2}X_2 + W_{A3}X_3 \\ W_{B1}X_1 + W_{B2}X_2 + W_{B3}X_3 \\ W_{C1}X_1 + W_{C2}X_2 + W_{C3}X_3 \end{bmatrix} = \begin{bmatrix} \bar{Y}_A \\ \bar{Y}_B \\ \bar{Y}_C \end{bmatrix}$$

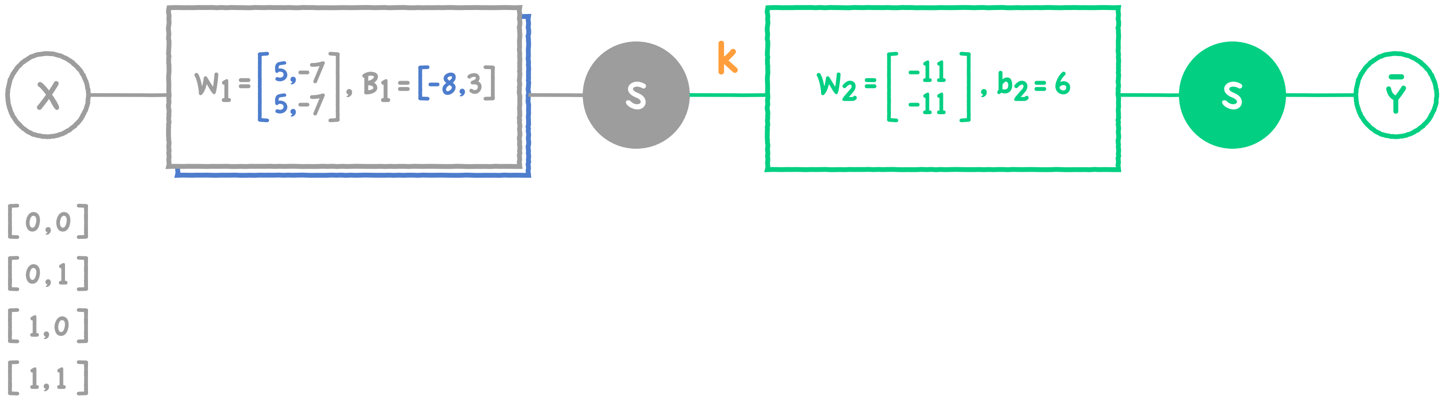


NN



How can we learn W, and b from training data?

NN



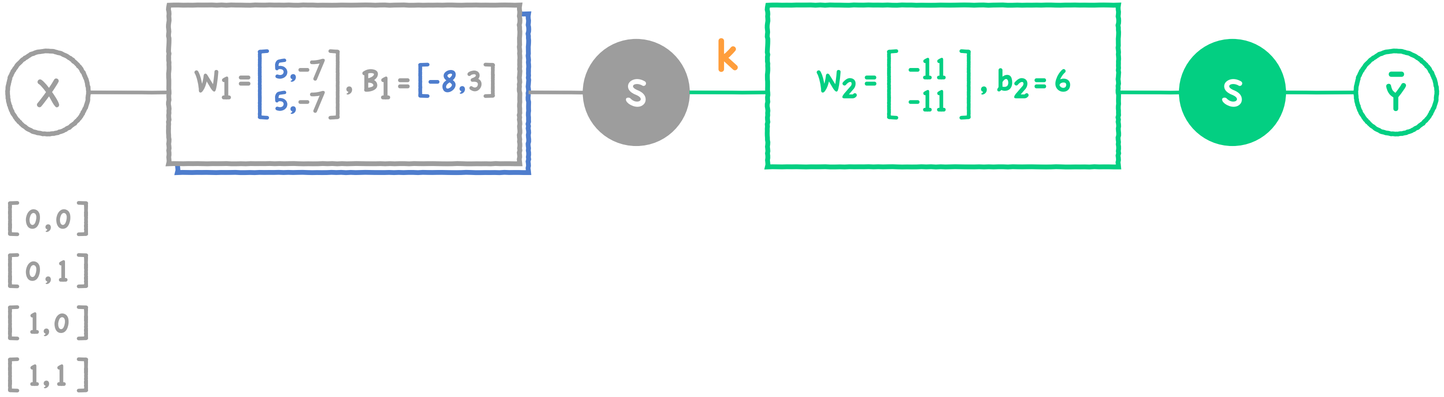
X
$W_1 = \begin{bmatrix} 5,-7 \\ 5,-7 \end{bmatrix}$, $B_1 = [-8,3]$
S
k
$W_2 = \begin{bmatrix} -11 \\ -11 \end{bmatrix}$, $b_2 = 6$
S
$\bar{Y}$
[0,0]
[0,1]
[1,0]
[1,1]

$$K(X) = \text{Sigmoid}(XW_1 + B_1)$$

$$\bar{Y} = H(X) = \text{Sigmoid}(K(X)W_2 + b_2)$$

NN

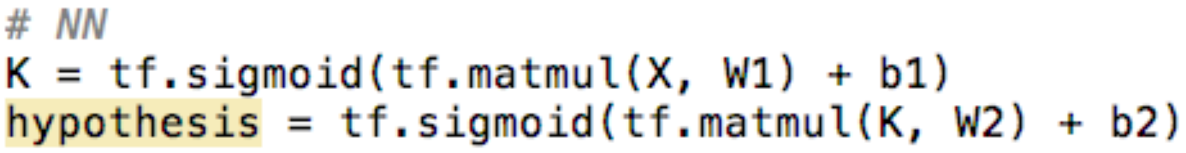
```
# NN
K = tf.sigmoid(tf.matmul(X, W1) + b1)
hypothesis = tf.sigmoid(tf.matmul(K, W2) + b2)
```


X
$W_1 = \begin{bmatrix} 5,-7 \\ 5,-7 \end{bmatrix}, B_1 = [-8,3]$
S
k
$W_2 = \begin{bmatrix} -11 \\ -11 \end{bmatrix}, b_2 = 6$
S
$\bar{Y}$
[0,0]
[0,1]
[1,0]
[1,1]

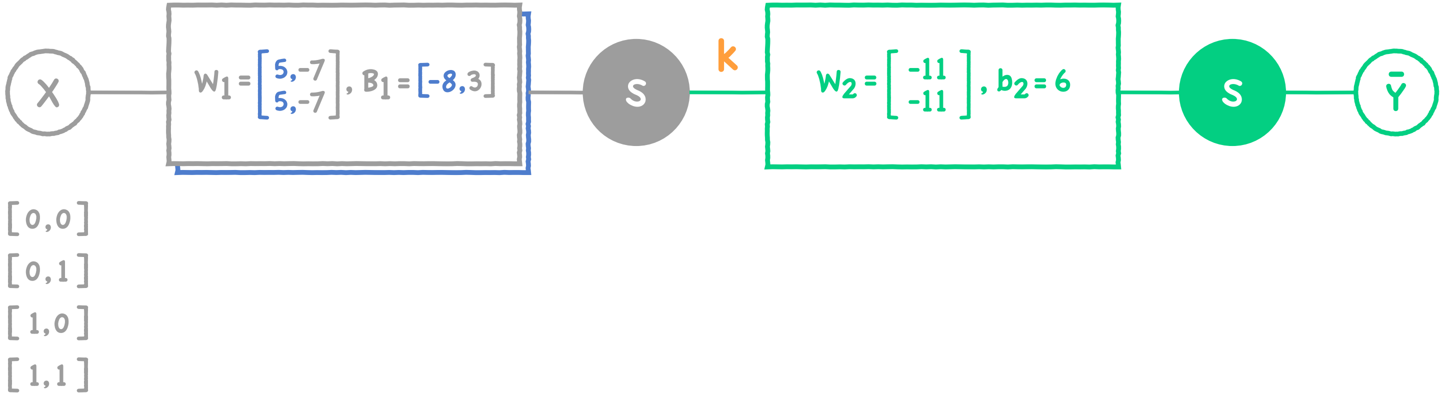
$$K(X) = \mathrm{Sigmoid}(XW_1 + B_1)$$

$$\bar{Y} = H(X) = \mathrm{Sigmoid}(K(X)W_2 + b_2)$$

NN



```
# NN
K = tf.sigmoid(tf.matmul(X, W1) + b1)
hypothesis = tf.sigmoid(tf.matmul(K, W2) + b2)
```



X
$W_1 = \begin{bmatrix} 5,-7 \\ 5,-7 \end{bmatrix}, B_1 = [-8,3]$
S
k
$W_2 = \begin{bmatrix} -11 \\ -11 \end{bmatrix}, b_2 = 6$
S
$\bar{Y}$
[0,0]
[0,1]
[1,0]
[1,1]

$K(X) = \mathrm{Sigmoid}(XW_1 + B_1)$

$\bar{Y} = H(X) = \mathrm{Sigmoid}(K(X)W_2 + b_2)$

How can we learn W1, W2, B1, b2 from training data?

NEXT LECTURE

BACKPROPAGATION