概统 HW7

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No.

Date :

7.12 Since X_1, X_2 independent, $f(x_1, x_2) = e^{-(x_1+x_2)}$ $x_1, x_2 > 0$

Cause $Y_1 = X_1 + X_2$, $Y_2 = X_1 / (X_1 + X_2)$, $X_1 = Y_1 Y_2$

$X_2 = Y_1 - Y_1 Y_2$

$$|J| = \left| \begin{vmatrix} Y_2 & Y_1 \\ 1-Y_2 & -Y_1 \end{vmatrix} \right| = |-Y_1| = Y_1$$

$$g(y_1, y_2) = f(y_1 y_2, y_1 - y_1 y_2)\, y_1 = e^{-(y_1 y_2 + (y_1 - y_1 y_2))} y_1$$

$$= e^{-y_1} y_1 \quad \text{for } y_1 > 0, \; 0 < y_2 < 1$$

$$g(y_1) = \int_0^1 e^{-y_1} y_1 \, dy_2 = e^{-y_1} y_1 \quad y_1 > 0$$

$$g(y_2) = \int_0^\infty e^{-y_1} y_1 \, dy_1 = \Gamma(2) = 1 \quad 0 < y_2 < 1$$

$g(y_1) g(y_2) = g(y_1, y_2)$, so Y_1 Y_2 is independent

7.14 let $x_1 = \sqrt{y}$ $x_2 = -\sqrt{y}$ $x_1 \ge 0$ $x_2 \le 0$

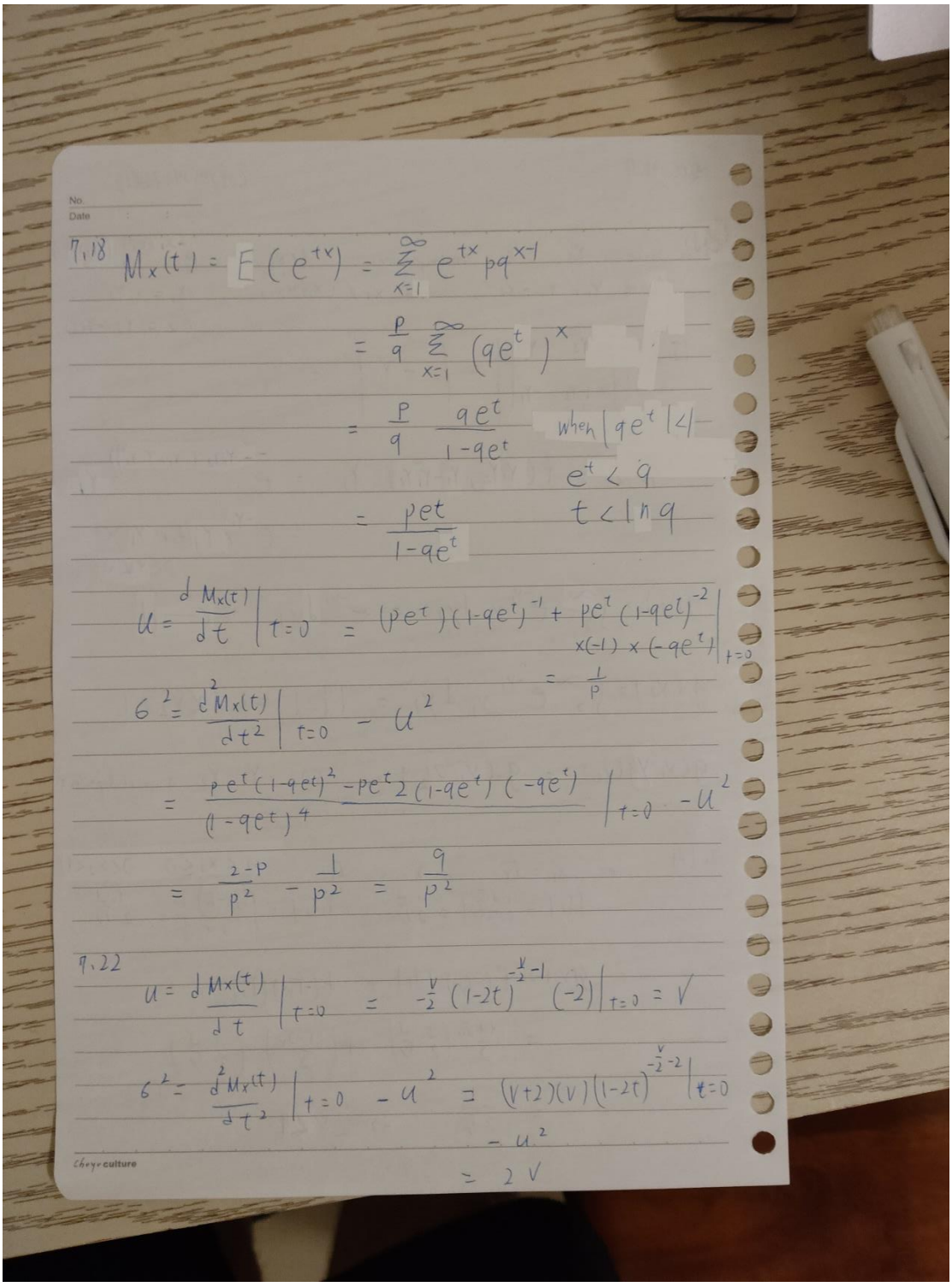
$$|J_1| = \left| \frac{d\sqrt{y}}{dy} \right| = \frac{1}{2}\frac{1}{\sqrt{y}} \qquad |J_2| = \left| \frac{d(-\sqrt{y})}{dy} \right| = \frac{1}{2}\frac{1}{\sqrt{y}}$$

$$g(y) = f(\sqrt{y})|J_1| + f(-\sqrt{y})|J_2|$$

$$= \frac{1+\sqrt{y}}{2}\left(\frac{1}{2}\frac{1}{\sqrt{y}}\right) + \left(\frac{1-\sqrt{y}}{2}\right)\left(\frac{1}{2}\frac{1}{\sqrt{y}}\right)$$

$$= \frac{1}{2\sqrt{y}} \quad \text{for } 0 < y < 1$$

Choyoculture



7.18 $M_x(t) = E(e^{tx}) = \sum_{x=1}^{\infty} e^{tx} pq^{x-1}$

$= \frac{p}{q} \sum_{x=1}^{\infty} (qe^t)^x$

$= \frac{p}{q} \frac{qe^t}{1-qe^t}$ when $|qe^t| < 1$

$e^t < q$

$t < \ln q$

$= \frac{pet}{1-qe^t}$

$u = \frac{d M_x(t)}{dt}\Big|_{t=0} = (pe^t)(1-qe^t)^{-1} + pe^t(1-qe^t)^{-2} \times (-1) \times (-qe^t)\Big|_{t=0}$

$= \frac{1}{p}$

$\sigma^2 = \frac{d^2 M_x(t)}{dt^2}\Big|_{t=0} - u^2$

$= \frac{pe^t(1-qe^t)^2 - pe^t 2(1-qe^t)(-qe^t)}{(1-qe^t)^4}\Big|_{t=0} - u^2$

$= \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{q}{p^2}$

7.22

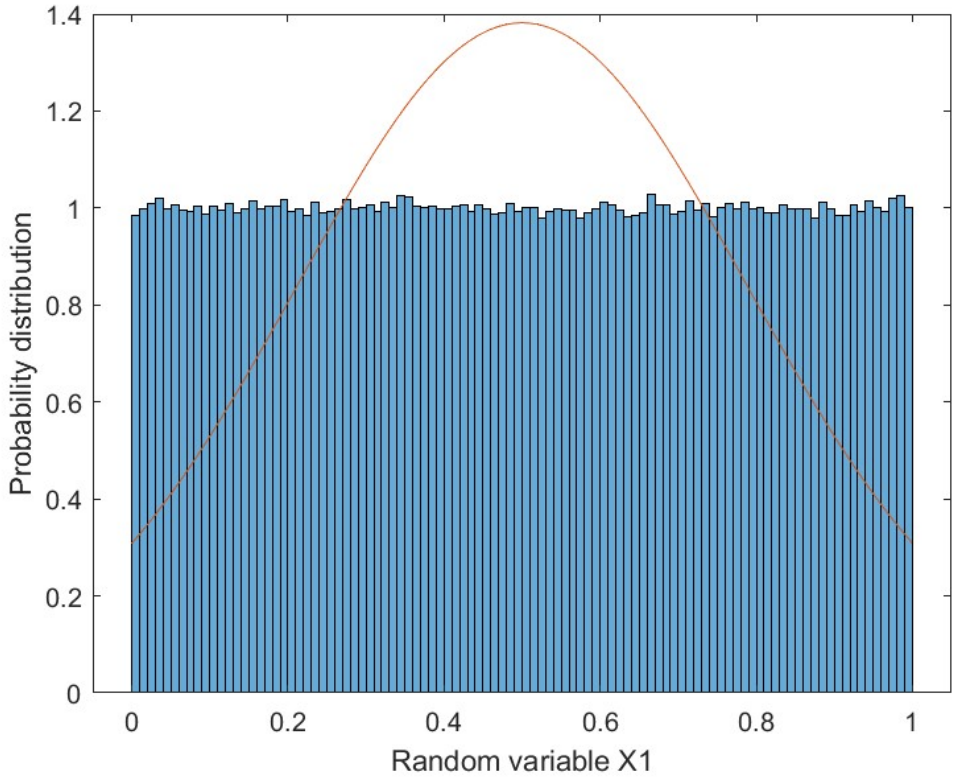
$u = \frac{d M_x(t)}{dt}\Big|_{t=0} = -\frac{v}{2}(1-2t)^{-\frac{v}{2}-1}(-2)\Big|_{t=0} = v$

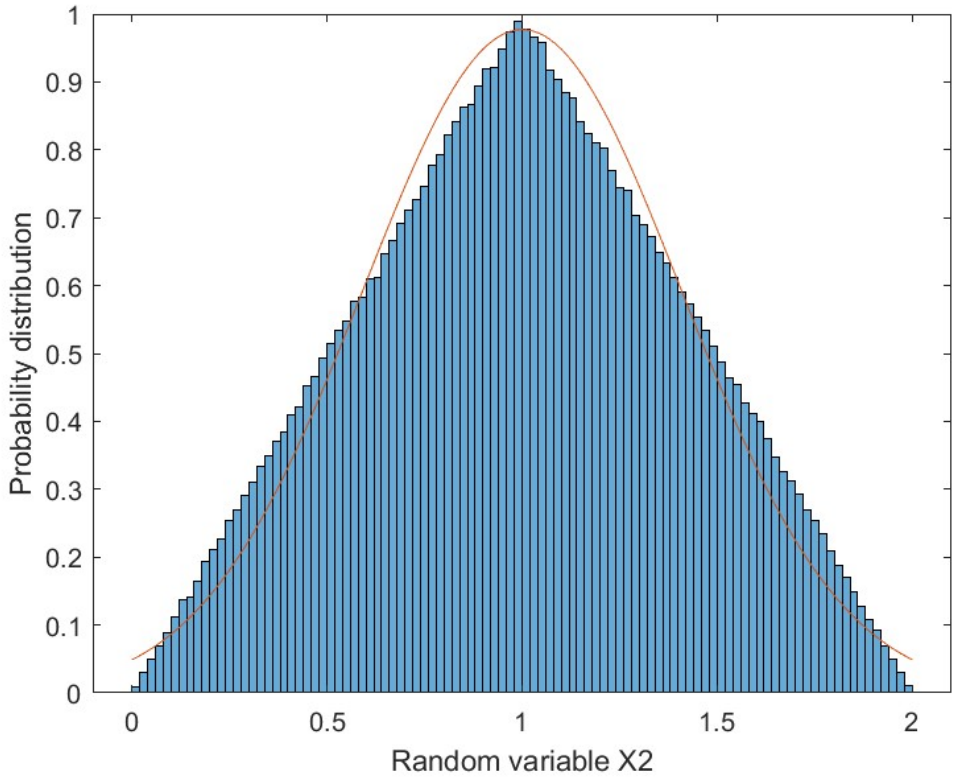
$\sigma^2 = \frac{d^2 M_x(t)}{dt^2}\Big|_{t=0} - u^2 = (v+2)(v)(1-2t)^{-\frac{v}{2}-2}\Big|_{t=0} - u^2$

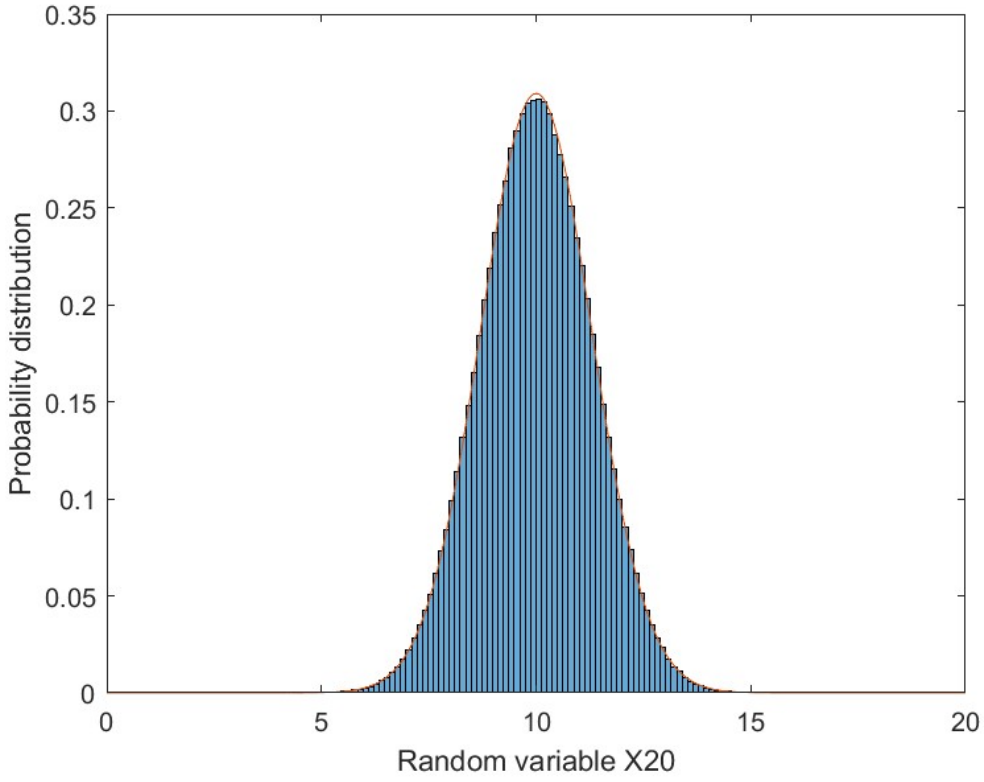
$= 2v$

Matlab.

1. b


Probability distribution
Random variable X1
0
0.2
0.4
0.6
0.8
1
1.2
1.4


Probability distribution
Random variable X2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2



可以看出隨 n 增大，Irwin-Hall distribution 和 normal distribution 誤差越來越小(面積重疊的部分越來越多)