4.24

(a) by 3.39 X is number of orange, Y is number of apple,

$0 \le X, Y \le 4$. and $f(x,y) = \dfrac{C^3_x C^2_y C^3_{4-x-y}}{C^8_4}$

$$E(X^2Y - 2XY) = \sum_{x=0}^{3}\sum_{y=0}^{2} (x^2y - 2xy)\, f(x,y)$$

$$= 0 + 0 + 0 + (-1)\frac{C^3_1 C^2_1 C^3_2}{C^8_4} + (-2)\frac{C^3_1 C^2_2 C^3_1}{C^8_4} + 0$$

$$+ 0 + 0 + 0 + 3\,\frac{C^3_3 C^2_1 C^3_0}{C^8_4} = \frac{-30}{70} = \frac{3}{-7}$$

(b) $g(x) = \dfrac{C^3_x C^5_{4-x}}{C^8_4}$, $h(y) = \dfrac{C^2_y C^6_{4-y}}{C^8_4}$,

$$u_X - u_Y = E(X) - E(Y)$$

$$= \sum_{x=0}^{3} x\,g(x) - \sum_{y=0}^{2} y\,h(y)$$

$$= 0 + 1\times\frac{30}{70} + 2\times\frac{30}{70} + 3\times\frac{5}{70} - 0 - 1\times\frac{40}{70} - 2\times\frac{15}{70}$$

$$= \frac{3}{2} - 1 = \frac{1}{2}$$

4.44 $6xy = E\big((X - u_X)(Y - u_Y)\big)$, and by 4.24-(b)

, $u_X = \frac{3}{2}$, $u_Y = 1$, $6xy = \sum_{x=0}^{3}\sum_{y=0}^{2} (x-1.5)(y-1)\, f(x,y)$

$$= 0 + (-1.5)\times\frac{3}{70} + 0.5\times\frac{3}{70} + 0 + (-0.5)\times\frac{9}{70}$$

$$+ (-0.5)\frac{9}{70} + 0 + (0.5)\frac{3}{70} + (-1.5)\frac{3}{70}$$

$$+ 0 = \frac{3}{-14}$$

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4.60 (a) $E(2X-3Y) = 2E(X) - 3E(Y)$

$$= 2\times\left(2\times(0.15+0.25+0.15) + 4\times(0.1+0.25+0.1)\right)$$

$$- 3\times\left(1\times(0.15+0.1) + 3\times(0.25+0.25)+5\times(0.15+0.1)\right)$$

$$= 2\times 2.9 - 3\times 3 = -3.2$$

(b) $E(XY) = E(X)E(Y)$ cause X, Y are independent,

$E(XY) = E(X)E(Y) = 2.9\times 3 = 8.7$

(by (a) know $E(X)$, $E(Y)$)

4.78 $\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x\, 30x^2(1-x)^2\,dx$

$$= 30\left(\frac{1}{6}x^6 - \frac{2}{5}x^5 + \frac{1}{4}x^4\right)\Big|_0^1$$

$$= \frac{1}{2} - 0 = \frac{1}{2}$$

$\sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2 = 30\left(\frac{1}{7}x^7 - \frac{2}{6}x^6 + \frac{1}{5}x^5\right)\Big|_0^1$

$$- \frac{1}{4}$$

$$= \frac{2}{7} - \frac{1}{4}$$

$$= \frac{1}{28}$$

$\sigma = \sqrt{\frac{1}{28}} \doteq 0.189$

$P(\mu-2\sigma < X < \mu+2\sigma) = P(0.122 < X < 0.878)$

$$= \int_{0.122}^{0.878} 30x^2(1-x^2)\,dx$$

$$= 30\left(\frac{1}{5}x^5 - \frac{2}{4}x^4 + \frac{1}{3}x^3\right)\Big|_{0.122}^{0.878} = 0.970$$

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and by Chebyshev's theorem, $P(\mu-2\sigma < X < \mu+2\sigma) \geq 1-\frac{1}{2^2} = \frac{3}{4}$

so Chebyshev's theorem gives a conservative conclusion.

4.98

(a) $g(x) = 0.12 + 0.04 + 0.04 = 0.2$ when $x=0$

$g(x) = 0.08 + 0.19 + 0.05 = 0.32$ when $x=1$

$g(x) = 0.06 + 0.12 + 0.3 = 0.48$ when $x=2$

$h(y) = 0.12 + 0.08 + 0.06 = 0.26$ when $y=0$

$h(y) = 0.04 + 0.19 + 0.12 = 0.35$ when $y=1$

$h(y) = 0.04 + 0.05 + 0.3 = 0.39$ when $y=2$

$f(X|Y=2) = \frac{f(x,2)}{h(2)} = \frac{0.04}{0.39} = 0.103$ when $x=0, y=2$

$= \frac{0.05}{0.39} = 0.128$ when $x=1, y=2$

$= \frac{0.3}{0.39} = 0.769$ when $x=2, y=2$

(b) $E(x) = \sum_{x=0}^{2} x f(x) = 0 \times 0.2 + 1 \times 0.32 + 2 \times 0.48$

$= 1.28$

$Var(x) = E(x^2) - (E(x))^2 = (0 \times 0.2 + 1 \times 0.32 + 4 \times 0.48)$

$- (1.28)^2$

$= 0.602$

(c) $E(X|Y=2) = \sum_{x=0}^{2} x f(x|Y=2) = 1 \times 0.128 + 2 \times 0.769$

$= 1.666$

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$$Var(X|Y=2) = E(X^2|Y=2) - \left(E(X|Y=2)\right)^2$$

$$= 1 \times 0.128 + 4 \times 0.769 - (1.666)^2$$

$$= 0.428$$