Robotic Navigation and Exploration

HW2 Supplement: Kinematic Model and Path Tracking Control

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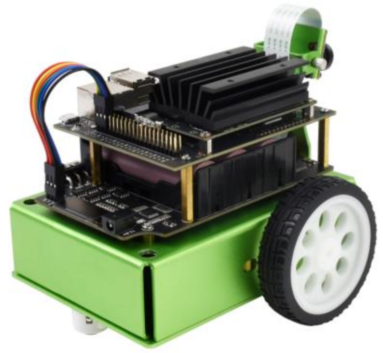
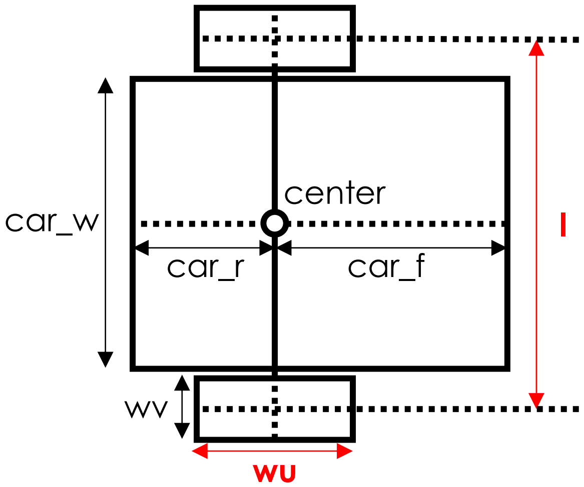
Kinematic Models

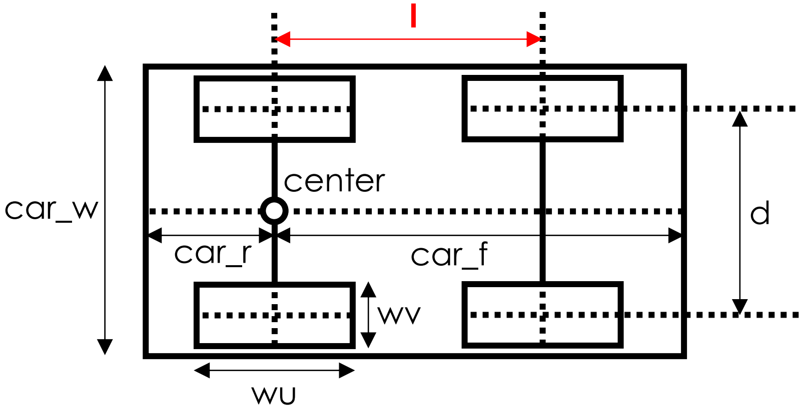
Parameter Settings of Vehicles

Differential Drive





Bicycle



./Simulation/simulator_*.py

States

- Kinematic-Related Physical Parameters
  - x, y, θ, v, ω (position, angle, velocity / angular velocity)
  - (denoted "x", "y", "yaw", "v", "w" in the codes)

```
class State:
    def __init__(self, x=0.0, y=0.0, yaw=0.0, v=0.0, w=0.0):
        self.x = 0.0
        self.y = 0.0
        self.yaw = 0.0
        self.v = 0.0
        self.w = 0.0
        self.update(x, y, yaw, v, w)

    def update(self, x=None, y=None, yaw=None, v=None, w=None):
        if x is not None:
            self.x = x
        if y is not None:
            self.y = y
        if yaw is not None:
            self.yaw = yaw
        if v is not None:
            self.v = v
        if w is not None:
            self.w = w
```

./Simulation/utils.py

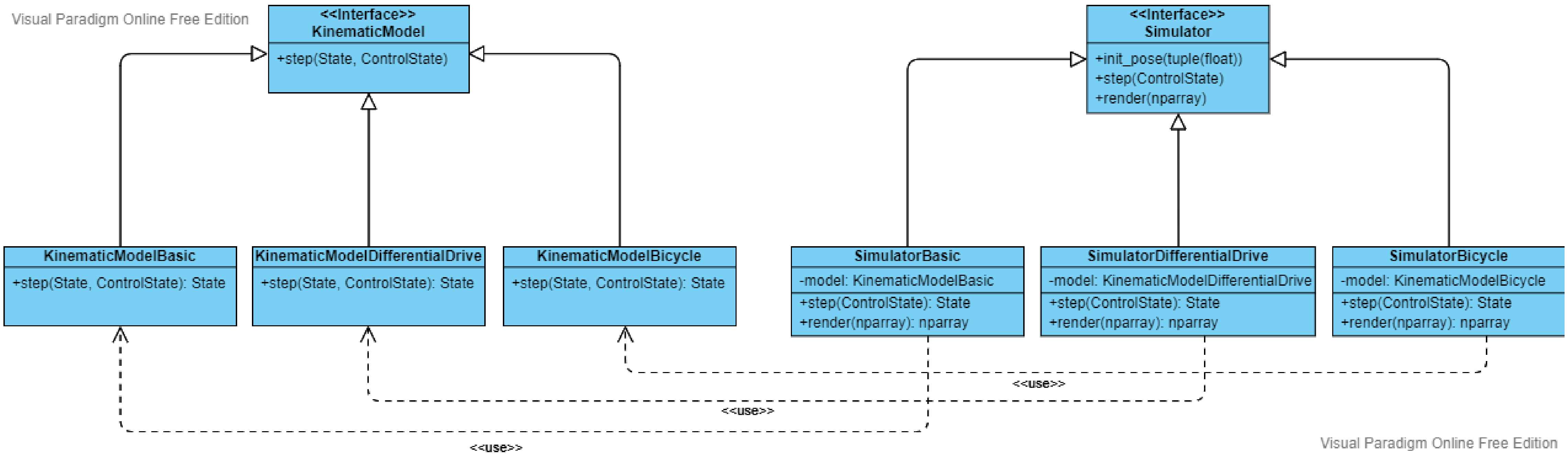
Control States

- Controlling-Related Parameter
  - Basic Model: v, ω (denoted as “v” and “w” in the codes)
  - Differential Drive: $\omega_{left}, \omega_{right}$ (denoted as “lw” and “rw” in the codes)
  - Bicycle Model: a, δ (denoted as “a” and “delta” in the codes)

```python
class ControlState:
    def __init__(self, control_type, *cstate):
        #  Support basic/diff_drive/bicycle
        self.control_type = control_type
        try:
            if control_type == "basic":
                self.v = cstate[0]
                self.w = cstate[1]
            elif control_type == "diff_drive":
                self.lw = cstate[0]
                self.rw = cstate[1]
            elif control_type == "bicycle":
                self.a = cstate[0]
                self.delta = cstate[1]
            else:
                raise NameError("Unknown control type!!")
        except NameError:
            raise
```

./Simulation/utils.py

Class Architecture for Simulation



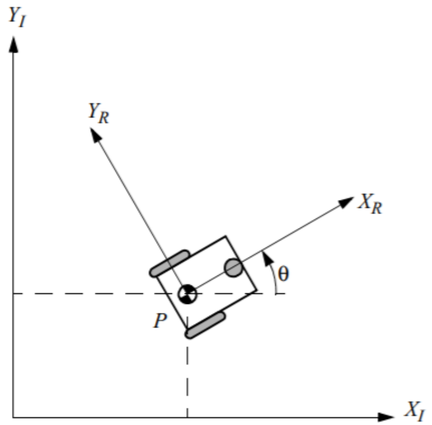
./Simulation/*.py

Basic Kinematic Model

```
class KinematicModelBasic(KinematicModel):
    def __init__(self, dt):
        # Simulation delta time
        self.dt = dt

    def step(self, state:State, cstate:ControlState) -> State:
        v = cstate.v
        w = cstate.w
        x = state.x + v * np.cos(np.deg2rad(state.yaw)) * self.dt
        y = state.y + v * np.sin(np.deg2rad(state.yaw)) * self.dt
        yaw = (state.yaw + state.w * self.dt) % 360
        state_next = State(x, y, yaw, v, w)
        return state_next
```

Beware of the converting between degree and radian.



$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = R(\theta)^{-1}\begin{bmatrix}\dot{x}_R\\ \dot{y}_R\\ \dot{\theta}\end{bmatrix}$$

$$= \begin{bmatrix}cos\theta & -sin\theta & 0\\ sin\theta & cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}v\\ 0\\ \omega\end{bmatrix}$$

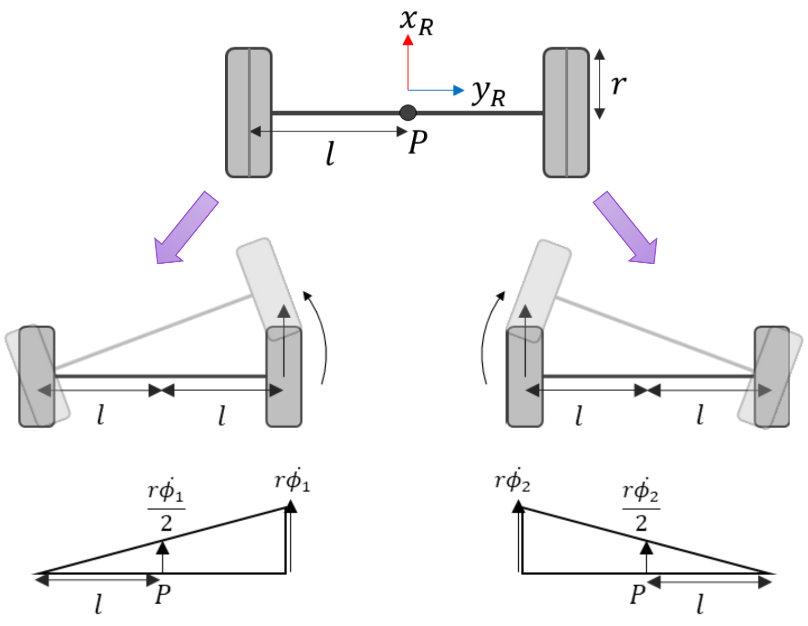
$$= \begin{bmatrix}v\cos(\theta)\\ v\sin(\theta)\\ \omega\end{bmatrix}$$

./Simulation/kinematic_basic.py

Differential Drive Kinematic Model

```
class KinematicModelDifferentialDrive(KinematicModel):
    def __init__(self, r, l, dt):
        # Simulation delta time
        self.r = r
        self.l = l
        self.dt = dt

    def step(self, state:State, cstate:ControlState) -> State:
        x1dot = self.r*np.deg2rad(cstate.rw) / 2
        w1 = np.rad2deg(self.r*np.deg2rad(cstate.rw) / (2*self.l))
        x2dot = self.r*np.deg2rad(cstate.lw) / 2
        w2 = np.rad2deg(self.r*np.deg2rad(cstate.lw) / (2*self.l))
        v = x1dot + x2dot
        w = w1 - w2
        x = state.x + v * np.cos(np.deg2rad(state.yaw)) * self.dt
        y = state.y + v * np.sin(np.deg2rad(state.yaw)) * self.dt
        yaw = (state.yaw + w * self.dt) % 360
        state_next = State(x, y, yaw, v, w)
        return state_next
```



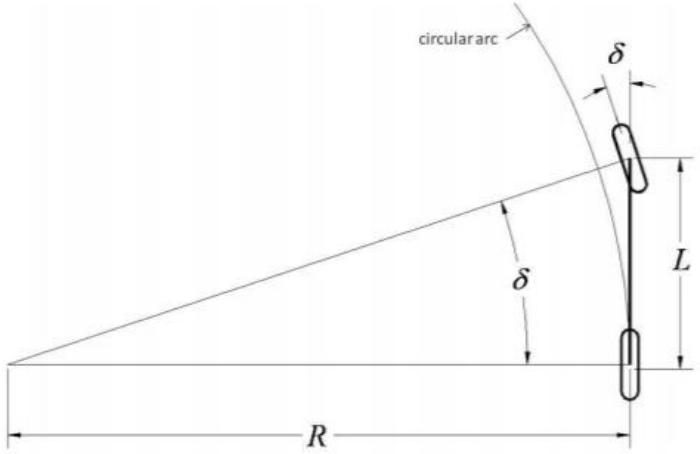
$$\begin{cases} v = \dfrac{r\dot{\phi}_1}{2} + \dfrac{r\dot{\phi}_2}{2} \\ \omega = \dfrac{r\dot{\phi}_1}{2l} - \dfrac{r\dot{\phi}_2}{2l} \end{cases}$$

./Simulation/kinematic_differential_drive.py

Bicycle Kinematic Model

```python
class KinematicModelBicycle(KinematicModel):
    def __init__(self, l, dt):
        # Distance from center to wheel
        self.l = l
        # Simulation delta time
        self.dt = dt

    def step(self, state:State, cstate:ControlState) -> State:
        v = state.v + cstate.a*self.dt
        w = np.rad2deg(state.v / self.l * np.tan(np.deg2rad(cstate.delta)))
        x = state.x + v * np.cos(np.deg2rad(state.yaw)) * self.dt
        y = state.y + v * np.sin(np.deg2rad(state.yaw)) * self.dt
        yaw = (state.yaw + w * self.dt) % 360
        state_next = State(x, y, yaw, v, w)
        return state_next
```



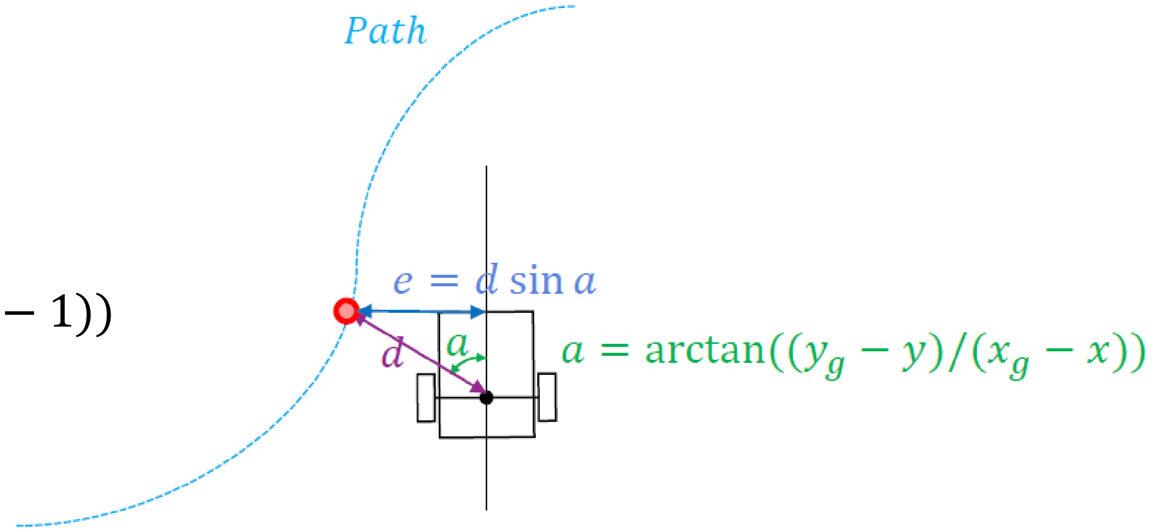
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \frac{\tan(\delta)}{L} \end{bmatrix} v$$

./Simulation/kinematic_bicycle.py

Path Tracking Control

PID Control

$$Output = K_p e(t) + K_i \sum_{0}^{t} e_t + K_d(e(t) - e(t-1))$$



Basic Model

```
min_idx, min_dist = utils.search_nearest(self.path, (x,y))
target = self.path[min_idx]
ang = np.arctan2(self.path[min_idx,1]-y,
self.path[min_idx,0]-x)
ep = min_dist * np.sin(ang)
self.acc_ep += dt*ep
diff_ep = (ep - self.last_ep) / dt
next_w = self.kp*ep + self.ki*self.acc_ep +
self.kd*diff_ep
self.last_ep = ep
return next_w, target
```

Bicycle Model

```
min_idx, min_dist = utils.search_nearest(self.path, (x,y))
target = self.path[min_idx]
ang = np.arctan2(self.path[min_idx,1]-y,
self.path[min_idx,0]-x)
ep = min_dist * np.sin(ang)
self.acc_ep += dt*ep
diff_ep = (ep - self.last_ep) / dt
next_delta = self.kp*ep + self.ki*self.acc_ep +
self.kd*diff_ep
self.last_ep = ep
return next_delta, target
```

./PathTracking/controller_pid_*.py

Pure Pursuit Control for Basic Model

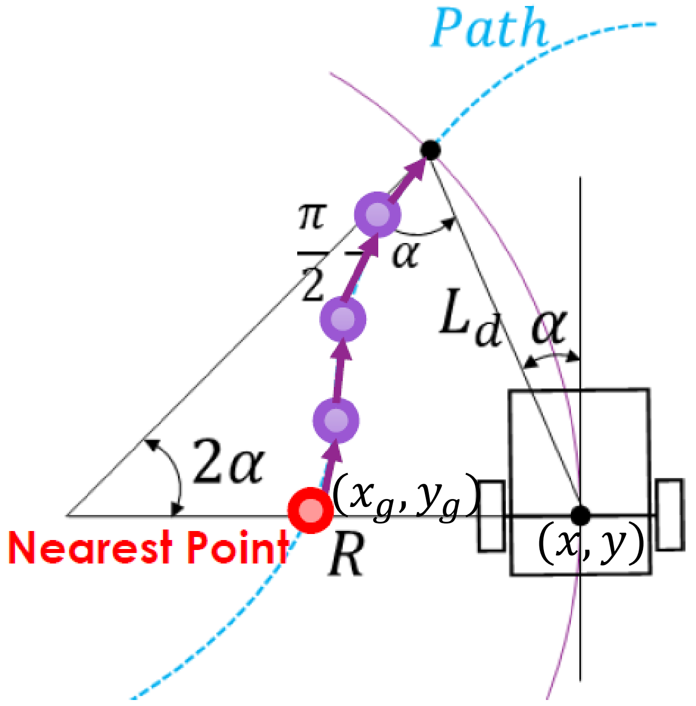
- Concept:
  - Modify the angular velocity to let the center achieve a point on path

$$\alpha = \arctan\left(\frac{y_g - y}{x_g - x}\right) - \theta$$

$$\omega = \frac{2v\sin(\alpha)}{L_d}$$

$L_d = k * v + L_{fc}$, where k, L_{fc} are parameters.

1. Set a distance L_d.
2. Find the nearest point on the path.
3. Search the following point until the distance of the point larger than or equal to L_d.



./PathTracking/controller_pure_pursuit_basic.py

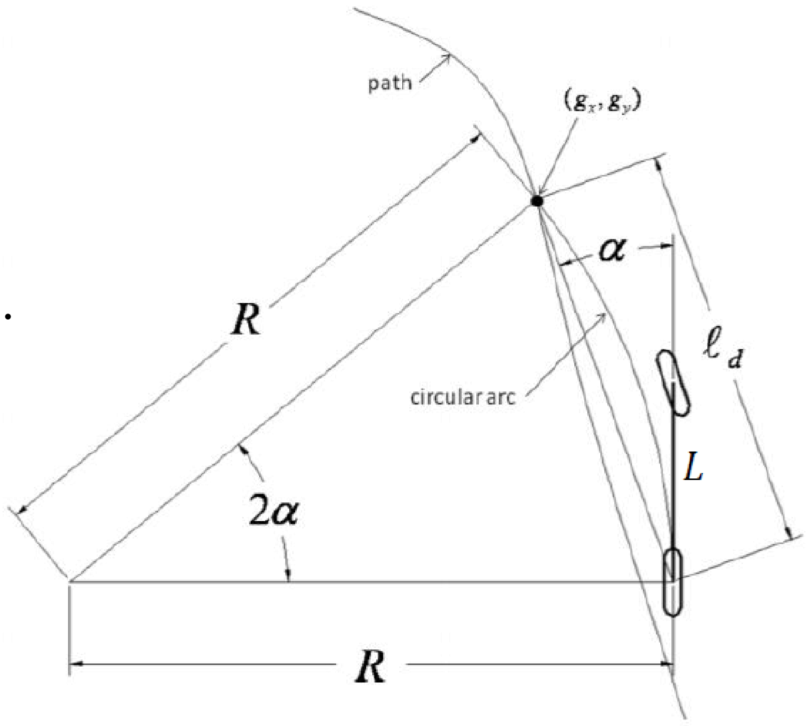
Pure Pursuit Control for Bicycle Model

- Concept:
  - Control the steer to let the rear wheel achieve a point on the path.

$$\alpha = \arctan\left(\frac{y_g - y}{x_g - x}\right) - \theta$$

$$\delta = \arctan\left(\frac{2L\sin(\alpha)}{L_d}\right)$$

$L_d = k * v + L_{fc}$, where k, L_{fc} are parameters.

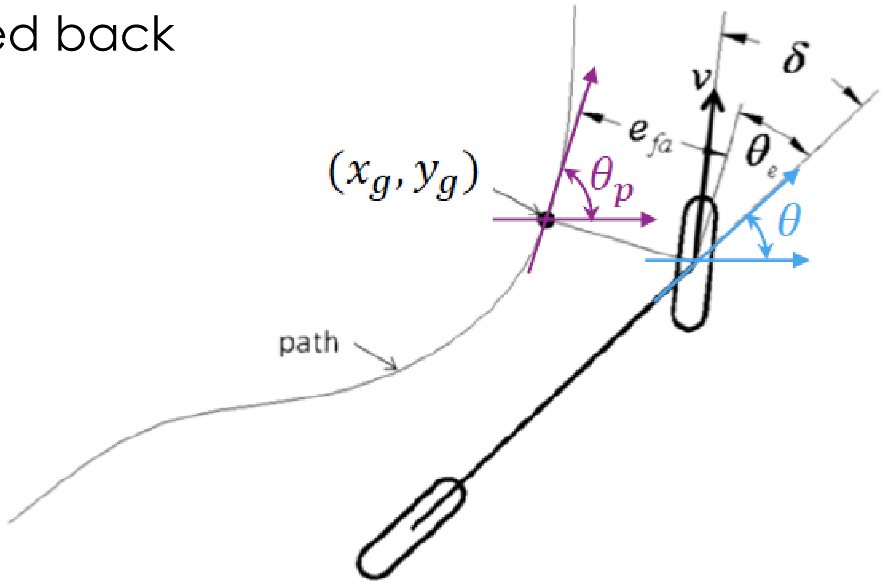


./PathTracking/controller_pure_pursuit_bicycle.py

Stanley Control for Bicycle Model

- Concept:
  - Exponential stability for front wheel feed back
- Some Implementation Details

$$\theta_e = \theta_p - \theta$$
$$\dot{e} = v_f \sin(\delta - \theta_e)$$
$$\delta = \arctan(-\frac{ke}{v_f}) + \theta_e$$
$$e = \begin{bmatrix} x - x_g \\ y - y_g \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \cos(\theta_p + 90) \\ \sin(\theta_p + 90) \end{bmatrix}$$



- Hint: Beware to transform the angle into the boundary of -180~+180

./PathTracking/controller_stanley_bicycle.py

LQR Control

- We have already completed the part of DARE solving.
- Following the steps:
  - Construct the matrix A, B, X of the linear approximation model.

$$Ax + Bu = \underbrace{\begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 0 & v & 0 \\ 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} e \\ \dot{e} \\ \theta \\ \dot{\theta} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{v}{L} \end{bmatrix}}_{\mathbf{B}} \underbrace{\delta}_{\mathbf{u}} \text{ (Bicycle Model)}$$

  - Solve DARE and get the matrix P of the value function.

$$P = Q + A^T PA - A^T PB\left(R + B^T PB\right)^{-1} B^T PA$$

  - Compute the optimal control.

$$u_t^* = -\left(R + B^T P_{t+1} B\right)^{-1} B^T P_{t+1} A x_t$$

./PathTracking/controller_lqr_*.py

Control of Differential Drive

- We only implement the controller for basic and bicycle kinematic model. The control of the differential drive can be simply modified by the controller of basic model.

$$\dot{\phi}_2 = \left(v - \frac{r\dot{\phi}_1}{2}\right)\frac{2}{r} = \frac{2v}{r} - \dot{\phi}_1$$

$$\omega = \frac{r\dot{\phi}_1}{2l} - \frac{r\left(\frac{2v}{r} - \dot{\phi}_1\right)}{2l} = \frac{r\dot{\phi}_1 - v}{l}$$

$$\dot{\phi}_1 = \frac{v}{r} + \frac{\omega l}{r}$$

$$\dot{\phi}_2 = \frac{v}{r} - \frac{\omega l}{r}$$

./navigation.py

Remind

- Check if you complete the “TODO” in the following files:
  - ./PathTracking/controller_pid_basic.py
  - ./PathTracking/controller_pid_bicycle.py
  - ./PathTracking/controller_pure_pursuit_basic.py
  - ./PathTracking/controller_pure_pursuit_bicycle.py
  - ./PathTracking/controller_stanley_bicycle.py
  - ./PathTracking/controller_lqr_basic.py
  - ./PathTracking/controller_lqr_bicycle.py
  - ./PathPlanning/planner_a_star.py
  - ./PathPlanning/planner_rrt_star.py
  - ./navigation.py