We derive the n^{th} derivative of the delta function from the sifting property

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a) \tag{1}$$

with $a = 0$, we integrate by parts the LHS to get

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = [f(x)H(x)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f'(x)H(x)dx \tag{2}$$

It is assumed $f(x)$ vanishes at infinty and therefore becomes

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = -\int_{-\infty}^{\infty} f'(x)\left(\int_{-\infty}^{x} \delta(y)\, dy\right) dx \tag{3}$$

For the integral of the n^{th} derivative of the delta function we can write

$$\int_{-\infty}^{\infty} f(x)\delta^{(n)}(x)dx = -\int_{-\infty}^{\infty} \frac{df}{dx}\delta^{(n-1)}(x)dx \tag{4}$$

Using this we let $f(x) = xg(x)$. Inserting this into the above equation for the first derivative we find

$$\int_{-\infty}^{\infty} xg(x)\delta'(x)dx = -\int_{-\infty}^{\infty}\left(\frac{d}{dx}xg(x)\right)\delta(x)dx \tag{5}$$

$$= -\int_{-\infty}^{\infty} \left(g(x) + xg'(x)\right)\delta(x)dx \tag{6}$$

$$= -\int_{-\infty}^{\infty} g(x)\delta(x)dx \tag{7}$$

This equation implies

$$x\delta'(x) = -\delta(x) \tag{8}$$

Repeating this for the n^th derivative, in general we can write

$$\int_{-\infty}^{\infty} x^n g(x)\delta^{(n)}(x)dx = -\int_{-\infty}^{\infty} \frac{d^n}{dx^n}\left[x^n g(x)\right]\delta^{n-1}(x)dx \tag{9}$$

$$= (-1)^n n! \int_{-\infty}^{\infty} g(x)\delta(x)dx \tag{10}$$

This gives the relation

$$x^n\delta^{(n)}(x) = (-1)^n n!\delta(x) \tag{11}$$