Original Solution to 1D EFJC Partition Function

The extra line of working for Eq. 5.15. From

$$\sin^N(x) = \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^N = \frac{e^{iNx}\left(1 - e^{-2ix}\right)^N}{(2i)^N} \tag{1}$$

using

$$(p+q)^N = \sum_{k=0}^{N} \binom{N}{k} p^k q^{N-k} \tag{2}$$

let $q = 1$ and $p = -e^{-2ix}$ to get

$$\sin^N(x) = \frac{e^{iNx}}{(2i)^N} \sum_{k=0}^{N} \binom{N}{k} (-1)^k e^{-2ikx} \tag{3}$$

$$= \frac{1}{(2i)^N} \sum_{k=0}^{N} \binom{N}{k} (-1)^k e^{ix(N-2k)} \tag{4}$$

We have

$$\sin^N(x) = \frac{1}{(2i)^N} \sum_{k=0}^{N} \binom{N}{k} (-1)^k e^{ix(N-2k)} \tag{5}$$

$$\cos^N(x) = \frac{1}{(2)^N} \sum_{k=0}^{N} \binom{N}{k} e^{ix(N-2k)} \tag{6}$$

Referring back to the partition function eq (5.54) we begin from

$$Z_q(R) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega R} \left[\left(\frac{2A}{\omega p}\right) \cos\omega a \, \sin\frac{\omega p}{2}\right]^N \tag{7}$$

$$= \frac{1}{2\pi} \left(\frac{2A}{p}\right)^N \int_{-\infty}^{\infty} d\omega\, \frac{e^{-i\omega R}}{\omega^N} \cos^N \omega a \sin^N \frac{\omega p}{2} \tag{8}$$

Expanding the cos term we get

$$Z_q(R) = \frac{1}{2\pi}\left(\frac{A}{p}\right)^N \sum_{k=0}^{N} \binom{N}{k} \int_{-\infty}^{\infty} d\omega \, \frac{e^{-i\omega R}}{\omega^N} e^{i\omega a(N-2k)} \sin^N \frac{\omega p}{2} \tag{9}$$

Next, we expand the sin term in the partition function to give

$$Z_q(R) = \frac{1}{2\pi}\left(\frac{A}{2ip}\right)^N \sum_{k=0}^{N}\sum_{k'=0}^{N} \binom{N}{k}\binom{N}{k'}(-1)^{k'} \int_{-\infty}^{\infty} d\omega \, \frac{e^{-i\omega R}}{\omega^N} e^{i\omega a(N-2k)} e^{i\frac{\omega p}{2}(N-2k')} \tag{10}$$

$$= \frac{1}{2\pi}\left(\frac{A}{2ip}\right)^N \sum_{k=0}^{N}\sum_{k'=0}^{N} \binom{N}{k}\binom{N}{k'}(-1)^{k'} \int_{-\infty}^{\infty} d\omega \, \frac{e^{i\omega\alpha}}{\omega^N} \tag{11}$$

where $\alpha = a(N-2k) + \frac{p}{2}(N-2k') - R$. Taking the Fourier transform of $1/\omega^N$ we know that

$$\int_{-\infty}^{\infty} \frac{e^{i\omega\alpha}}{\omega^N} d\omega = \frac{i^N \pi \alpha^{N-1}}{(N-1)!} \operatorname{sgn}(\alpha) \tag{12}$$

The partition function becomes

$$Z_q(R) = \frac{1}{2}\left(\frac{A}{2p}\right)^N \sum_{k=0}^{N}\sum_{k'=0}^{N} \binom{N}{k}\binom{N}{k'}(-1)^{k'} \frac{\alpha^{N-1}}{(N-1)!} \operatorname{sgn}(\alpha) \tag{13}$$

Prof Harker's Method

From Medhurst and Roberts (Math. Comp.,19 (1965),113-117) there is a closed form expression of the nth power integral such that

$$\frac{2}{\pi}\int_0^{\infty} \left(\frac{\sin x}{x}\right)^N \cos bx \, dx = \frac{N}{2^{n-1}} \sum_{0 \leqq r < (b+N)/2} \frac{(-1)^r (b+N-2r)^{N-1}}{r!\,(n-r)!} \tag{14}$$

for $0 \leqq b < N$. We can begin to transform the integral from eq.(9) into this form, but there is a step where Harker seems to drive out a cos term that isn't clear. I do feel this method is more complicated than our original method. I will be in contact with him soon to discuss his method.