HPD and ROPE in the Bernoulli case

Using a flat prior, calculate the HPD with credibility value α = 0.05, that is p = 0.95.Using this make a hypothesis test based on the ROPE range of $\Theta \in [0.45, 0.55]$. What is your conclusion?
HPD region is between, 0.3062 and 0.4943 and % 12.7 of samples inside the ROPE range of $\Theta \in [0.45, 0.55]$

Data is not yet conclusive to make a decision

Bayes factor in the Bernoulli case

$$BF = \frac{p(x|\vartheta = 0.5)}{p(x|\vartheta = 0.4)} = \frac{\vartheta_0^k (1-\vartheta)^{n-k}}{\vartheta_1^k (1-\vartheta)^{n-k}}$$

If $\vartheta_0^k = 0.5, \vartheta_1^k = 0.4, k = 40, n = 100$, BF is equal to

$$BF = \frac{p(x|\vartheta = 0.5)}{p(x|\vartheta = 0.4)} = 0.13$$

Which suggests there is a substantial support for H_1

As a second example compare the two hypothesis: $H_0 = 0.5, H_1 \neq 0.5$

$$BF = \frac{p(k|\vartheta = 0.5)}{p(k|\vartheta \neq 0.5)} = \frac{p(k|\vartheta = 0.5)}{\int_0^1 p(k|\vartheta)\, d\vartheta} = 1.095231$$

Results suggests there is "barely worth mentioning" difference between the the hypothesis

Model comparison and Bayes factor

$$BF(M1, M2) = \frac{p(x|M1)}{p(X|M2)} = \frac{\int \vartheta e^{-\vartheta t} \alpha_0 e^{-\alpha_0}\, d\vartheta}{\int \frac{\mu}{(1+\mu t)^2} \frac{\alpha_0}{(1+\alpha_0 \mu)^2}}$$

If α_0 is equal to $1/10$

$$BF(M1, M2) = \frac{p(x|M1)}{p(X|M2)} = \frac{\vartheta e^{-\vartheta(t+0.1)}\, d\vartheta}{\frac{\mu}{(1+\mu t)^2 (1+\frac{\mu}{10})^2}}$$

For the sequence of t=(9.57, 0.93, 4.04, 3.31, 1.12, 1.71, 6.65, 4.09, 5.83, 2.91) bayes factor equals to

$$BF(M1, M2) = \frac{p(x|M1)}{p(X|M2)} = 0.84$$

Which means M2 more suitable model than M1