Bayesian inference in the binomial case

What is the posterior distribution ? Plot it as well.

$$p(\vartheta|\vec{x}=1,0,0,1,1,1) = \frac{p(x_p=1|\vartheta)p(x_p=0)|\vartheta)p(x_p=0)|\vartheta)p(x_p=1|\vartheta)p(x_p=1|\vartheta)p(x_p=1|\vartheta)(p(\vartheta)}{p(\vec{x})}$$

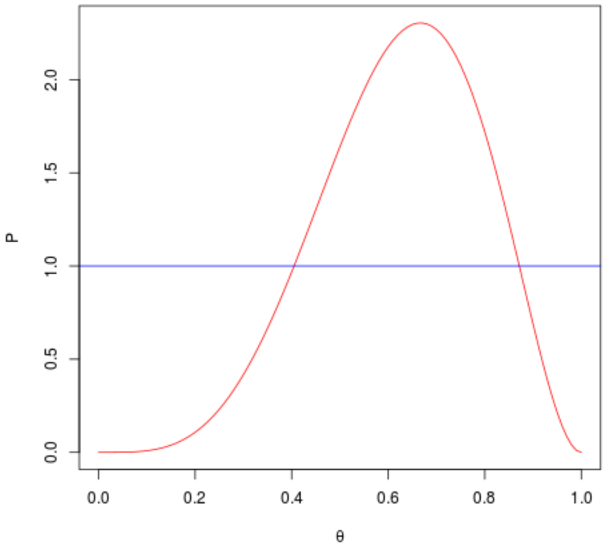

Figure 0.1 – Prior (blue) and posterior (red)

What is the predicted probability p(x p | x) for x p =0, 1?

$$p(x_p = 1|\vec{x}) = \int p(x_p = 1|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.625$$
$$p(x_p = 0|\vec{x}) = \int p(x_p = 0|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.375$$

What is the result, if the experiment would yield x = (0, 1, 1, 0, 1, 1)? Show from the form of the posterior, that the order does not matter.

$$p(\vartheta|\vec{x}=1,0,0,1,1,1) = \frac{p(x_p=1|\vartheta)p(x_p=0)|\vartheta)p(x_p=0)|\vartheta)p(x_p=1|\vartheta)p(x_p=1|\vartheta)p(x_p=1|\vartheta)(p(\vartheta)}{p(\vec{x})}$$

$$p(\vartheta|\vec{x}=0,1,1,0,1,1) = \frac{p(x_p=0|\vartheta)p(x_p=1)|\vartheta)p(x_p=1)|\vartheta)p(x_p=0|\vartheta)p(x_p=1|\vartheta)p(x_p=1|\vartheta)(p(\vartheta)}{p(\vec{x})}$$

Since its a multiplication, order of the likelyhood do not affect the posterior

Bayesian inference for the exponential distribution

Calculate the posterior distribution $p(\vartheta|t)$ after observation of one decay time t. Normalize it and plot it for the experimental result t = 2.3

$$p(\vartheta|t) = \frac{p(t=2.3|\vartheta)(p(\vartheta)}{p(t)}$$

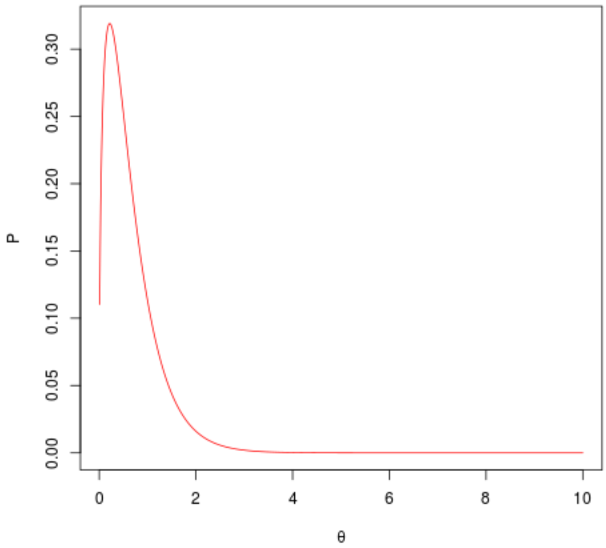

Figure 0.2 – Prior (blue) and posterior (red)

What is the posterior for the observations t = (2.3, 3.0, 2.7)?

$p(\vartheta|\vec{t} = 2.3, 3.0, 2.7) = \frac{p(t_p=2.3)|\vartheta)p(t_p=3.0)|\vartheta)p(t_p=2.7|\vartheta)(p(\vartheta)}{p(\vec{t})}$

As a second option choose the (proper and informative) prior distribution $p(\vartheta) = \vartheta^3 * exp(-\vartheta)/6$ **Plot both the prior and posterior distribution for the second case.**

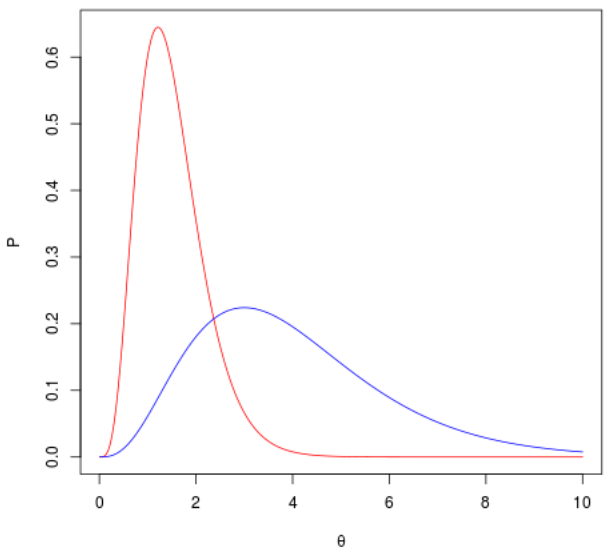

Figure 0.3 – Prior (blue) and posterior (red)

Poisson distribution with an unusual prior

Calculate the posterior distribution $p(\vartheta|\vec{x})$ **numerically for this case**

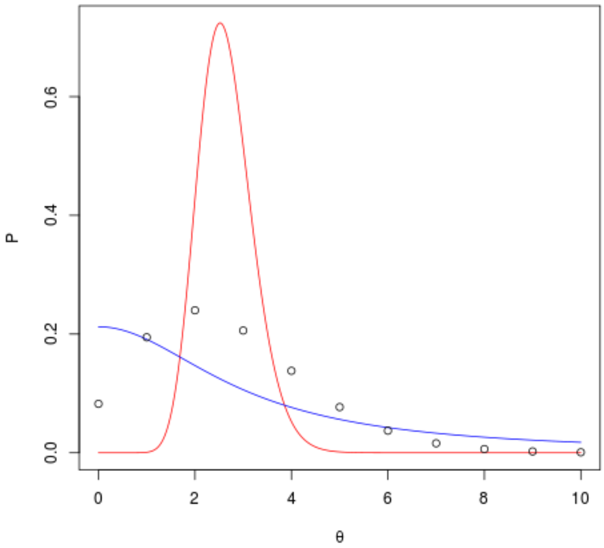

Figure 0.4 – Prior (blue), posterior (red) and $p(\tilde{x}|\vec{x})$ (points)

Use this also to calculate the prediction $p(\tilde{x}|\vec{x})$ **for** $\vec{x}$**= 0, . . . , 10.**

$p(x_p = 0)|\vec{x}) = \int p(x_p = 0|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.082$
$p(x_p = 1)|\vec{x}) = \int p(x_p = 1|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.194$
$p(x_p = 2)|\vec{x}) = \int p(x_p = 2|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.240$
$p(x_p = 3)|\vec{x}) = \int p(x_p = 3|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.206$
$p(x_p = 4)|\vec{x}) = \int p(x_p = 4|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.138$
$p(x_p = 5)|\vec{x}) = \int p(x_p = 5|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.077$
$p(x_p = 6)|\vec{x}) = \int p(x_p = 6|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.037$
$p(x_p = 7)|\vec{x}) = \int p(x_p = 7|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.016$
$p(x_p = 8)|\vec{x}) = \int p(x_p = 8|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.007$
$p(x_p = 9)|\vec{x}) = \int p(x_p = 9|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.002$
$p(x_p = 10)|\vec{x}) = \int p(x_p = 10|\vartheta) * p(\vartheta|\vec{x}) * d\vartheta = 0.001$