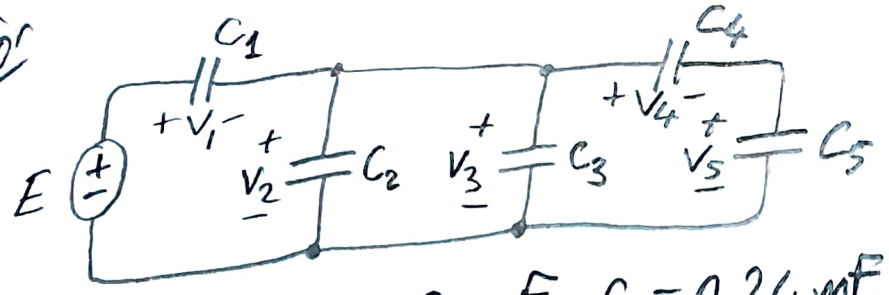
5/



Her bir kapasitör üzerindeki yük ve voltajları bulunuz.

$E = 24V$, $C_1 = 0.3mF$, $C_2 = 0.24mF$,
$C_3 = 0.18mF$, $C_4 = 0.3mF$, $C_5 = 0.45mF$

$C_e = C_1 \$ (C_2 + C_3 + C_4 \$ C_5)$

$= 0.3mF \$ (0.24mF + 0.18mF + 0.3mF \$ 0.45mF)$

$= 0.3mF \$ \left(0.42mF + \frac{0.3 \times 0.45}{0.75} mF\right)$

$= 0.3mF \$ (0.42mF + 0.18mF)$

$= 0.3mF \$ 0.6mF = \frac{0.3 \times 0.6}{0.9} mF = 0.2mF$

$Q = C_e \cdot E = 0.2mF \times 24V = 4.8\,mC$

$Q_1 = Q = 4.8\,mC$, $V_1 = \frac{Q}{C_1} = \frac{4.8mC}{0.3mF} = 16V$

$V_2 = V_3 = E - V_1 = 24V - 16V = 8V$

$Q_2 = C_2 V_2 = 0.24mF \times 8V = 1.92\,mC$

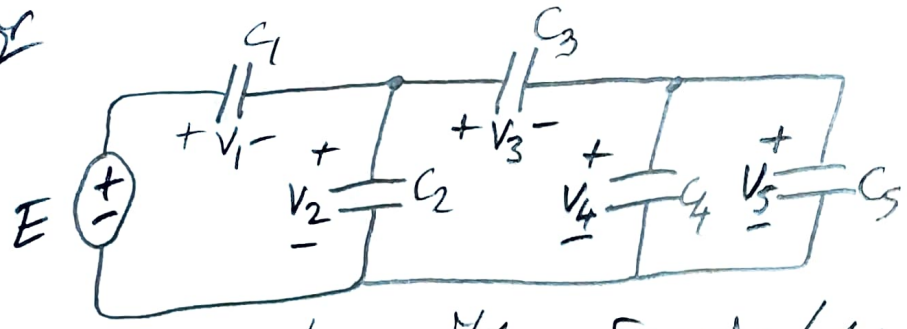
$Q_3 = C_3 V_3 = 0.18mF \times 8V = 1.44\,mC$

$Q_4 = Q_5 = (C_4 \$ C_5) V_3 = (0.3mF \$ 0.45mF) \times 8V$

$= \frac{0.3 \times 0.45}{0.75} mF \times 8V = 0.18mF \times 8V = 1.44\,mC$

$V_4 = \frac{Q_4}{C_4} = \frac{1.44\,mC}{0.3mF} = 4.8\,V$

$V_5 = \frac{Q_5}{C_5} = \frac{1.44\,mC}{0.45\,mF} = 3.2V$



$E = 25V$
$C_1 = 30mF$
$C_2 = 24mF$
$C_3 = 30mF$
$C_4 = 20mF$
$C_5 = 50mF$

Her bir kapasitör üzerindeki yük ve voltajları bulunuz.

$C_e = C_1 \$ (C_2 + C_3 \$ (C_4 + C_5))$

$= 30mF \$ (24mF + 30mF \$ (20mF + 50mF))$

$= 30mF \$ (24mF + 30mF \$ 70mF)$

$= 30mF \$ (24mF + \frac{30 \times 70}{100} mF)$

$= 30mF \$ (24mF + 21mF)$

$= 30mF \$ 45mF = \frac{30 \times 45}{75} mF = 18mF$

$Q = C_e \times E = 18mF \times 25V = 450mC$

$Q_1 = Q = 450mC, \quad V_1 = \frac{Q_1}{C_1} = \frac{450mC}{30mF} = 15V$

$V_2 = E - V_1 = 25V - 15V = 10V$

$Q_2 = C_2 V_2 = 24mF \times 10V = 240mC$

$Q_3 = Q_1 - Q_2 = 450mC - 240mC = 210mC$

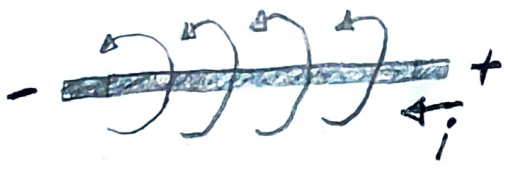
$V_3 = \frac{Q_3}{C_3} = \frac{210mC}{30mF} = 7V$

$V_4 = V_5 = V_2 - V_3 = 10V - 7V = 3V$

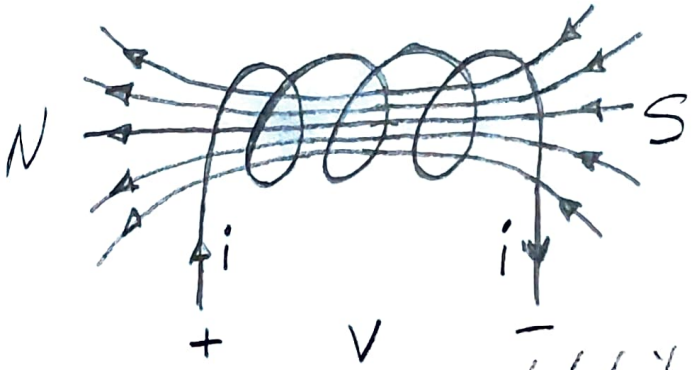
$Q_4 = C_4 V_4 = 20mF \times 3V = 60mC$

$Q_5 = C_5 V_5$
$= 50mF \times 3V$
$= 150mC$

Endüktör (Inductor)

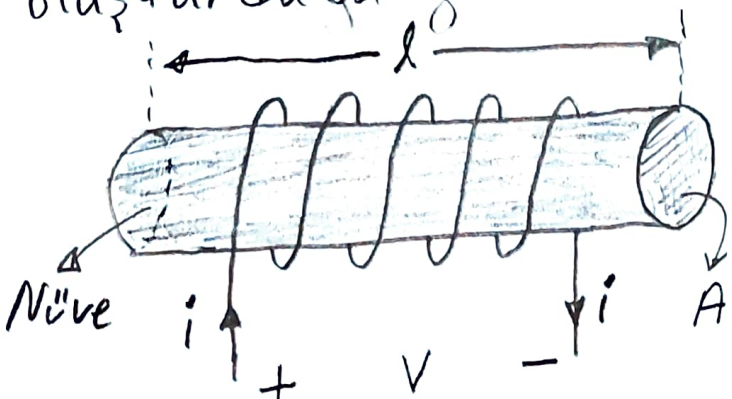


Üzerinden elektrik akımı geçen her tel etrafında manyetik alan oluşturur.



İletken tel bobin haline getirilirse oluşan manyetik alan düz bir mıknasınkine benzer.

Bobinden geçen elektrik akımı manyetik alan oluşturduğu gibi tersi de doğrudur.



Endüktör endükleme işini yapan devre elemanıdır. Pratikte bobin olarak isimlendirilir.

Endüktörün enerjiyi manyetik alan olarak depolama yeteneğine Endüktans denir. Sembolü L, birimi Henry (H)'dir.

n : Sarım Sayısı
A : Kesit Alanı (m^2)
l : Uzunluk (m)

$$L = \mu \frac{n^2 A}{l} = \mu_r \mu_0 \frac{n^2 A}{l} = \mu_r L_0$$

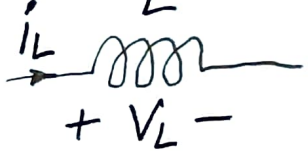
$\mu_0 = 4\pi \times 10^{-7}$ Boşluğun manyetik alan geçirgenliği

μ_r : Bağıl manyetik alan geçirgenlik katsayısı
Boşluk için 1, Hava için ~1, Demir Nüvesi için 400.

Bobin enerjiyi manyetik alanda depolayan devre elemanıdır. Pratikte bobin emaye telden sarılarak elde edilir. Bu telin az da olsa bir direnci vardır. İdeal bobinin iç direnci sıfırdır.

Bobinde manyetik alan oluşturmak için zamana ihtiyaç vardır. Bobinden geçen akım kesildiğinde manyetik alan birden yok olmaz. Manyetik alanın yok olması için yine zamana ihtiyaç vardır.

Bobin, içinden geçen akıma tepki gösterir. Geçen akım DC ise uçları arasındaki gerilim sıfırdır. AC bir akımda frekans artması endüktör gerilimini artırır. Yüksek frekansta açık devre, düşük frekansta kısa devre olur. Bass denilen düşük frekansları geçirir. Endüktör üzerindeki akım artıyorsa tüketici konumunda, azalıyorsa üretici konumundadır.



$\Phi(t) = L\, i_L(t)$ — Manyetik Akı (Weber – Wb)

$$V_L(t) = \frac{d\Phi(t)}{dt} = L\frac{di_L(t)}{dt}$$, L sabit zamandan bağımsız alınırsa

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(z)\,dz = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} V_L(z)\,dz$$

$i_L(0^-) = i_L(0^+)$

$\frac{di_L}{dt} = 0$ ise $V_L = 0$ (Kısa Devre)

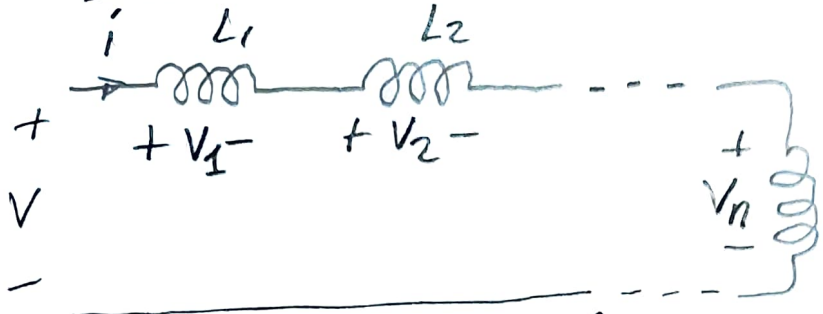
Bobindeki akım değişim hızı 1A/sn ve bu gerilime karşı koyan endüktör gerilimi 1V ise bobinin endüktansı 1H'dir.

$$W_L(t) = \frac{1}{2} L\, i_L^2(t) = \frac{\Phi^2(t)}{2L} = \frac{\Phi(t)\cdot i_L(t)}{2}$$

$$P_L(t) = \frac{dW_L(t)}{dt} = L\, i_L(t)\frac{di_L(t)}{dt} = V_L(t)\cdot i_L(t)$$

$$W_L(t) = \int_{-\infty}^{t} P_L(z)\,dz = L\int_{i_L(-\infty)}^{i_L(t)} i_L\, di_L = \frac{1}{2} L\, i_L^2(t), \quad i_L(-\infty) = 0$$

Endüktörlerin Seri Bağlanması



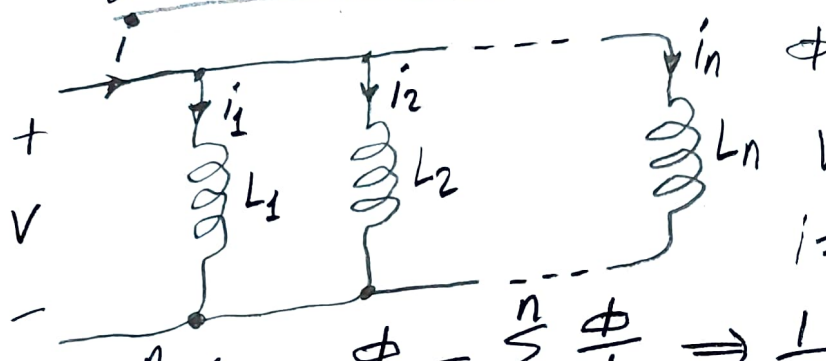
$\Phi = \Phi_1 + \Phi_2 + \cdots + \Phi_n$

$V = V_1 + V_2 + \cdots + V_n$

$i = i_1 = i_2 = \cdots = i_n$

$$\Phi = \sum_{k=1}^{n} \Phi_n \Rightarrow L_e i = \sum_{k=1}^{n} L_k i \Rightarrow L_e = \sum_{k=1}^{n} L_k$$

Endüktörlerin Paralel Bağlanması



$\Phi = \Phi_1 = \Phi_2 = \cdots = \Phi_n$

$V = V_1 = V_2 = \cdots = V_n$

$i = i_1 + i_2 + \cdots + i_n$

$$i = \sum_{k=1}^{n} i_n \Rightarrow \frac{\Phi}{L_e} = \sum_{k=1}^{n} \frac{\Phi}{L_n} \Rightarrow \frac{1}{L_e} = \sum_{k=1}^{n} \frac{1}{L_k}$$

Ör 500 sarımlı bir bobinin uzunluğu 6 cm, kullanılan telin kesit alanı $12\ cm^2$'dir.
a) Bobin nüvesiz ise endüktansı
b) Bobin nüveli ise endüktansı ($\mu_r = 250$ olsun)

$n = 500$, $\ell = 6\ cm = 6 \times 10^{-2}\ m$, $A = 12\ cm^2 = 12 \times 10^{-4}\ m$

a) $L_0 = \mu_0 \frac{n^2 A}{\ell} = 4\pi \times 10^{-7} \frac{500^2 \times 12 \times 10^{-4}}{6 \times 10^{-2}}\ F = 6.28\ mF$

b) $L = \mu_r L_0 = 500 \times 6.28\ mF = 1.57\ F$

Ör Bir bobinden geçen akım 0.25 sn'de 2A'den 5A'e çıktığında üzerinde 6V'luk endüktör gerilimi oluşuyor. Bobinin endüktansını bulunuz.

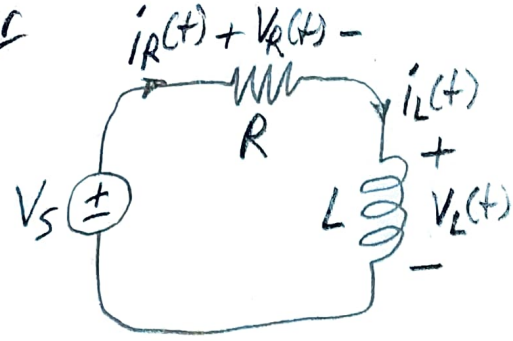
$\Delta t = 0.25\ sn$

$\Delta I = 5A - 2A = 3A$

$$V = L \frac{\Delta I}{\Delta t} \Rightarrow L = V \frac{\Delta t}{\Delta I}$$

$$L = 6V \frac{0.25\ sn}{3A} = 0.5\ H$$

Ör



$$i_R(t) = i_L(t)$$

$$V_L(t) = L\frac{di_L(t)}{dt}$$

$$V_R(t) + V_L(t) = V_S$$

$$R\,i_R(t) + L\frac{di_L(t)}{dt} = V_S$$

$$R\,i_L(t) + L\frac{di_L(t)}{dt} = V_S \Rightarrow \frac{di_L}{dt} + \frac{R}{L}i_L = \frac{V_S}{L}$$

$t = 0^+$ zamanından itibaren devreye V_S sabit gerilimi veriliyor.

$$i_L(t) = \left(A\,e^{-\frac{Rt}{L}} + B\right)u(t) \qquad \tau = L/R$$

yada $i_L(t) = A\,e^{-Rt/L} + B$, $t \geq 0$

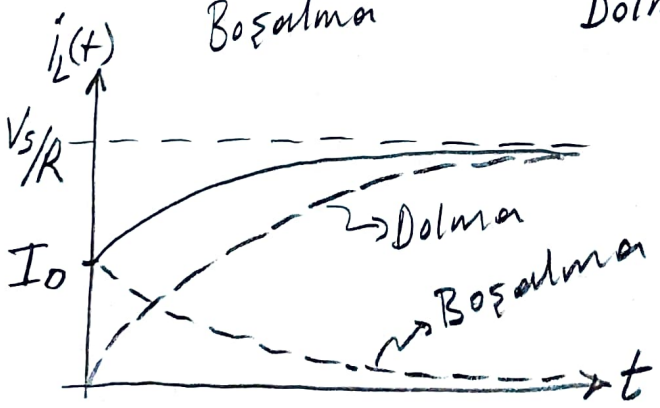
$i_L(0^+) = I_0$, $i_L(\infty) = \frac{V_S}{R}$ (çünkü V_L kısa devre olur)

$t = \infty$ için $B = V_S/R$

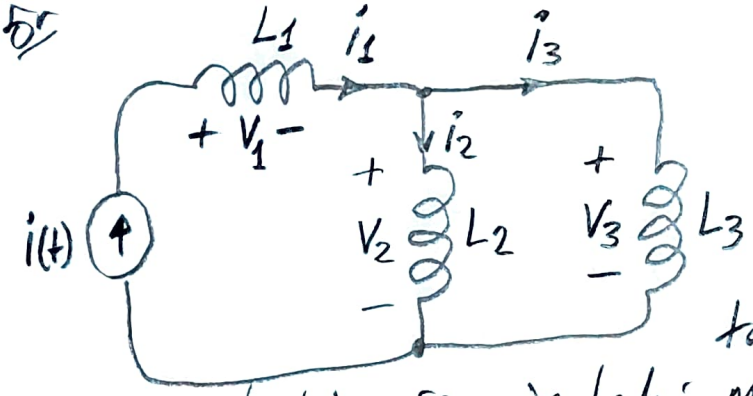
$t = 0$ için $A + B = I_0 \Rightarrow A = I_0 - V_S/R$

$$i_L(t) = (I_0 - V_S/R)\,e^{-Rt/L} + V_S/R$$

$$= \underbrace{I_0\,e^{-Rt/L}}_{\text{Boşalma}} + \underbrace{\frac{V_S}{R}\left(1 - e^{-Rt/L}\right)}_{\text{Dolma}}, \quad t \geq 0$$



5)



$L_1 = 12H$, $L_2 = 10H$, $L_3 = 15H$

$i(t) = (5 - 3e^{-2t})\,u(t)$

Eşdeğer endüktansı ve toplam manyetik akıyı bulunuz. Herbir bobin üzerindeki manyetik akıyı, akımı ve voltajı bulunuz. $t = 0^+$ ve $t = \infty$ için bobinler üzerindeki voltaj ve akımları bulunuz.

$$L_e = L_1 + L_2 // L_3 = 12H + 10H // 15H$$

$$= 12H + \frac{10H \times 15H}{25H} = 12H + 6H = 18H$$

$$\Phi(t) = L_e\, i(t) = 90 - 54e^{-2t}, \quad t \geq 0$$

$$i_1(t) = i(t) = 5 - 3e^{-2t}, \quad t \geq 0$$

$$\Phi_1(t) = L_1\, i_1(t) = 60 - 36e^{-2t}, \quad t \geq 0$$

$$V_1(t) = \frac{d\Phi_1(t)}{dt} = 72e^{-2t}, \quad t \geq 0$$

$$\Phi_2(t) = \Phi_3(t) = \Phi(t) - \Phi_1(t) = 30 - 18e^{-2t}, \quad t \geq 0$$

$$V_2(t) = V_3(t) = \frac{d\Phi_2(t)}{dt} = \frac{d\Phi_3(t)}{dt} = 36e^{-2t}, \quad t \geq 0$$

$$i_2(t) = \frac{\Phi_2(t)}{L_2} = 3 - 1.8e^{-2t}, \quad t \geq 0$$

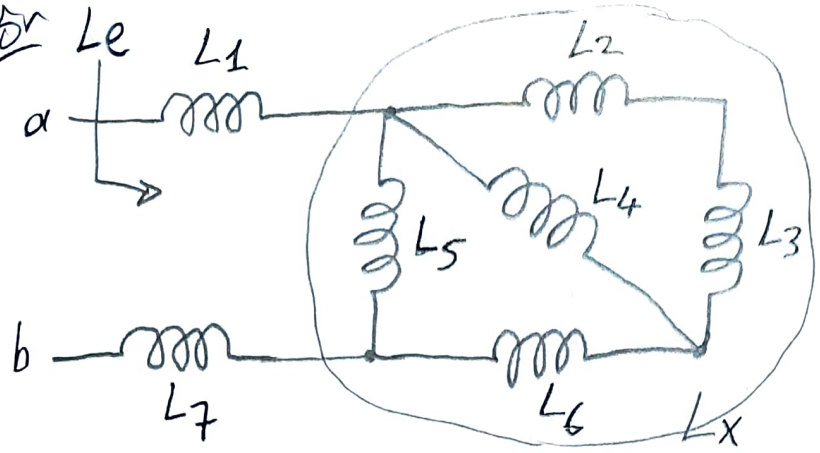
$$i_3(t) = \frac{\Phi_3(t)}{L_3} = 2 - 1.2e^{-2t}, \quad t \geq 0$$

$i_1(0) = 3A$ $\quad V_1(0^+) = 72V$ $\quad i_1(\infty) = 5A$ $\quad V_1(\infty) = 0$

$i_2(0) = 1.2A$ $\quad V_2(0^+) = 36V$ $\quad i_2(\infty) = 3A$ $\quad V_2(\infty) = 0$

$i_3(0) = 0.8A$ $\quad V_3(0^+) = 36V$ $\quad i_3(\infty) = 2A$ $\quad V_3(\infty) = 0$

Ör Le



$L_1 = 5H$, $L_2 = 8H$

$L_3 = 4H$, $L_4 = 24H$

$L_5 = 9H$, $L_6 = 10H$

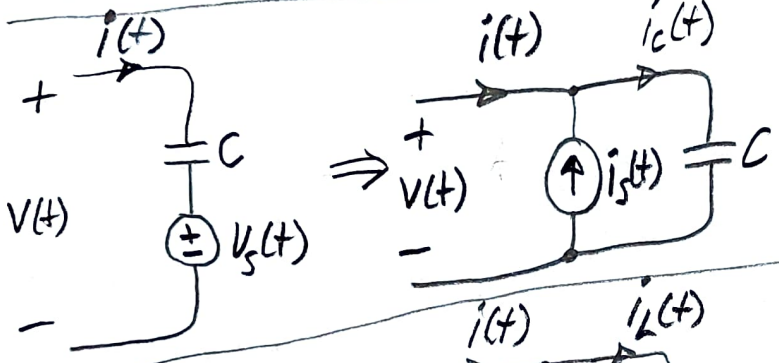
$L_7 = 3H$ ise $L_e = ?$

$$L_x = L_5 // (L_6 + L_4 // (L_2 + L_3)) = 9H // (10H + 24H // (8H + 4H))$$

$$= 9H // (10H + 24H // 12H) = 9H // (10H + 8H)$$

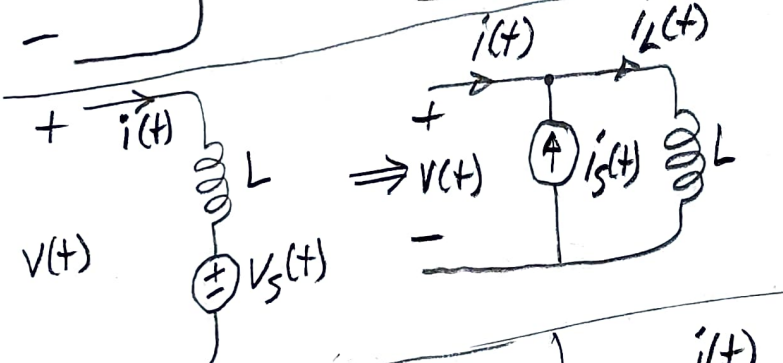
$$= 9H // 18H = 6H$$

$$L_e = L_1 + L_x + L_7 = 5H + 6H + 3H = 14H$$



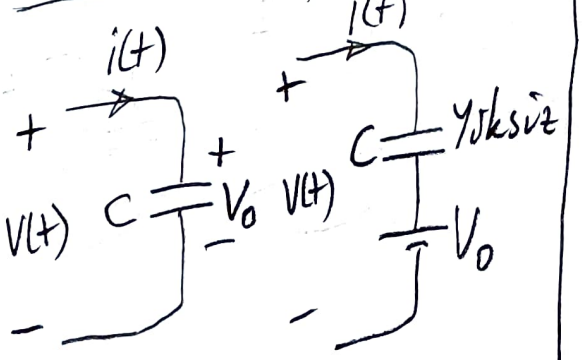
$$i_s(t) = C \frac{dV_s(t)}{dt}$$

$$V_s(t) = \frac{1}{C} \int_{-\infty}^{t} i_s(z)\, dz$$

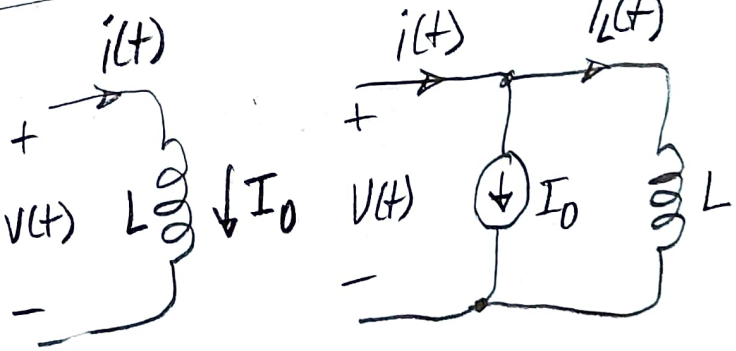


$$V_s(t) = L \frac{di_s(t)}{dt}$$

$$i_s(t) = \frac{1}{L} \int_{-\infty}^{t} V_s(z)\, dz$$

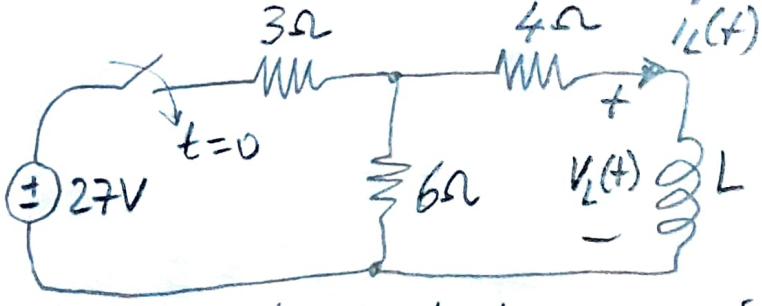


$V_c(0) = V_0$ ise



$i(0) = I_0$ ise $i_L(0) = 0$

Ör



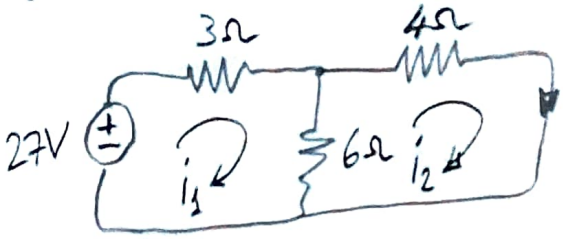
$L = 12H$ ise $\tau = ?$

$i_L(t) = ?$ $V_L(t) = ?$

τ : Zaman Sabiti

$t = 0^+$ için bobin açık devre yani $i_L(0^+) = 0$

$t = \infty$ için bobin kısa devre yani $V_L(\infty) = 0$

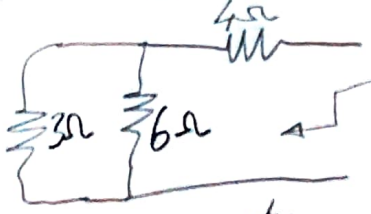


$6i_1 - 10i_2 = 0 \Rightarrow i_2 = 0.6i_1$

$27 - 9i_1 + 6i_2 = 0$

$27 - 9i_1 + 3.6i_1 = 0 \Rightarrow i_1 = 5A$

$i_L(\infty) = i_2 = 0.6i_1 = 3A$



$R_e = 3\Omega // 6\Omega + 4\Omega = 2\Omega + 4\Omega = 6\Omega$

$\tau = L/R_e = \frac{12H}{6\Omega} = 2sn$

$i_L(t) = Ae^{-t/2} + B, \quad t \geq 0$

$i_L(\infty) = B = 3$

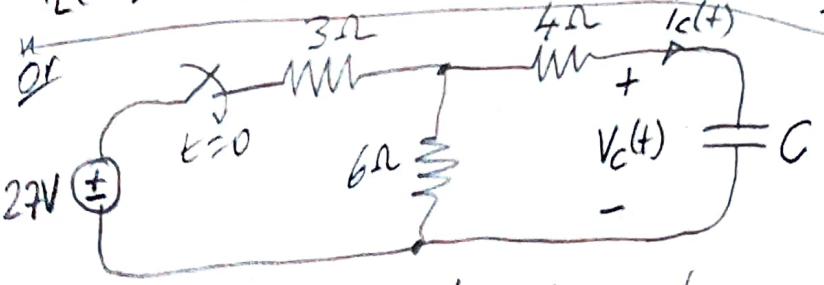
$i_L(0^+) = A + B = 0 \Rightarrow B = -3$

$i_L(t) = 3(1 - e^{-t/2}), \quad t \geq 0$

$V_L(t) = L\frac{di_L(t)}{dt} = 12 \times 3 \times \frac{1}{2}e^{-t/2}$

$= 18e^{-t/2}, \quad t \geq 0$

Ör



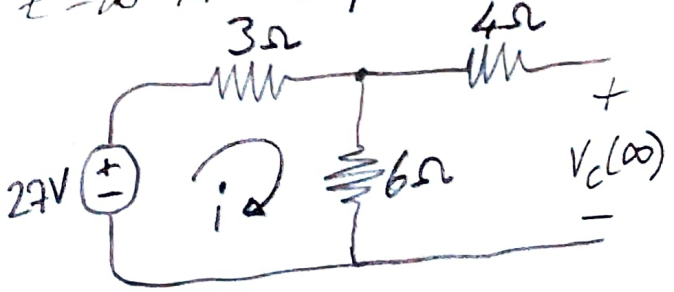
$C = 0.5F$ ise $\tau = ?$

$V_c(t) = ?$ $i_c(t) = ?$

τ : Zaman Sabiti

$t = 0^+$ için kapasitör kısa devre yani $V_c(0^+) = 0$

$t = \infty$ için kapasitör açık devre yani $i_c(\infty) = 0$

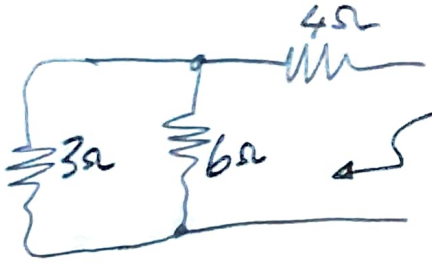


$i = \frac{27V}{3\Omega + 6\Omega} = \frac{27V}{9\Omega} = 6A$

$V_c(\infty) = 6i = 6\Omega \times 3A$

$= 18V$

→ diğer sayfa



$R_e = 3\Omega // 6\Omega + 4\Omega$

$= 2\Omega + 4\Omega = 6\Omega$

$\tau = R_e C = 6\Omega \times 0.5F = 3\,sn$

$v_C(t) = Ae^{-t/3} + B, \quad t \geq 0$

$v_C(\infty) = B = 18$

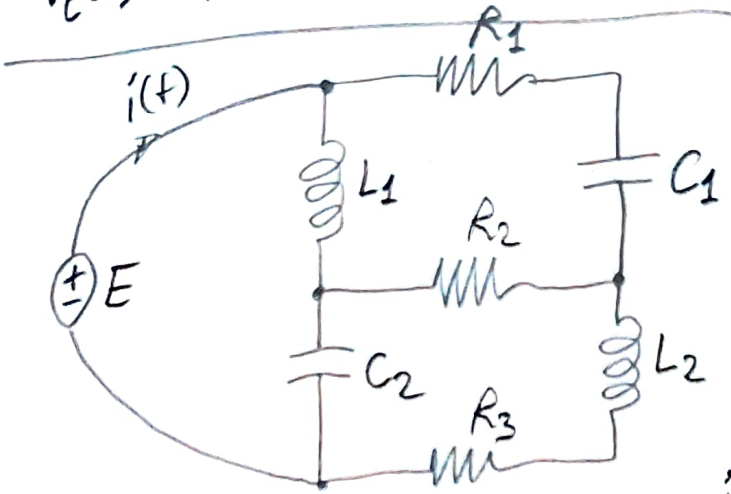
$v_C(0) = A + B = 0 \Rightarrow A = -18$

$v_C(t) = 18\left(1 - e^{-t/3}\right), \quad t \geq 0$

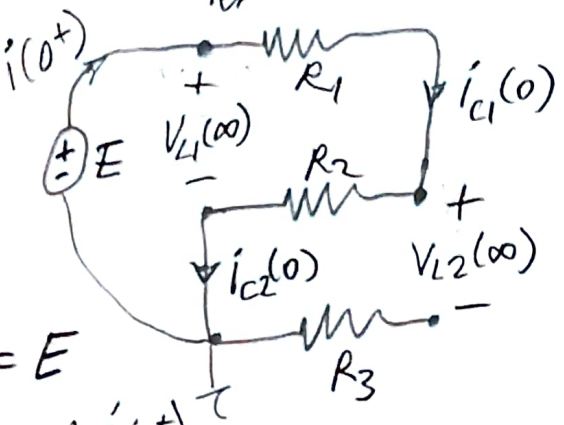
$i_C(t) = C\dfrac{dv_C(t)}{dt}$

$= 0.5 \times 18 \times \frac{1}{3} e^{-t/3}$

$= 3e^{-t/3}, \quad t \geq 0$



$t = 0^+$ anında kapasitörler kısa devre, bobinler açık devredir



$i_{C_1}(0^+) = i_{C_2}(0^+) = i(0^+)$

$= \dfrac{E}{R_1 + R_2}$

$v_{C_1}(0^+) = v_{C_2}(0^+) = 0$

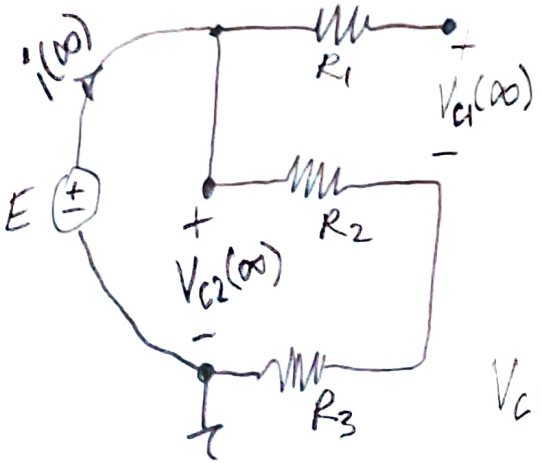
$i_{L_1}(0^+) = i_{L_2}(0^+) = 0$

$v_{L_1}(0^+) = E$

$v_{L_2}(0^+) = R_2\, i(0^+)$

$= \dfrac{R_2 E}{R_1 + R_2}$

$t = \infty$ anında kapasitörler açık devre, bobinler kısa devredir



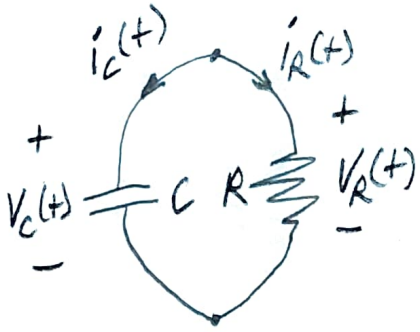
$i_{L_1}(\infty) = i_{L_2}(\infty) = i(\infty) = \dfrac{E}{R_2 + R_3}$

$v_{L_1}(\infty) = v_{L_2}(\infty) = 0$

$i_{C_1}(\infty) = i_{C_2}(\infty) = 0$

$v_{C_1}(\infty) = R_2\, i(\infty) = \dfrac{R_2 E}{R_2 + R_3}, \quad v_{C_2}(\infty) = E$

Birinci Mertebeden Devreler



$V_C(0^+) = V_0$, $\tau = RC$ Zaman Sabiti

5τ Durulma Zamanı

5τ'lik bir sürede kapasitörün boşaldığı veya sabit bir voltaj olduğu kabul edilir.

$V_C(t) = V_R(t)$

$i_C(t) + i_R(t) = 0$

$C\frac{dV_C(t)}{dt} + \frac{V_C(t)}{R} = 0$

$\frac{dV_C}{dt} + \frac{1}{RC}V_C = 0$

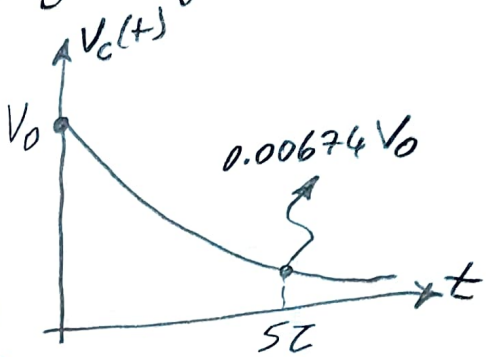
$V_C(t) = A e^{-t/RC} u(t)$

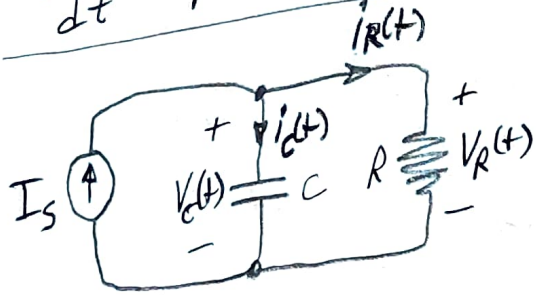
$V_C(0^+) = A = V_0$

$V_C(t) = V_0 e^{-t/RC} u(t)$

$i_C(t) = C\frac{dV_C(t)}{dt} = -i_R(t)$

$= -\frac{V_R(t)}{R} = -\frac{V_0}{R} e^{-t/RC} u(t)$





$V_C(0^+) = V_0$, $t = 0^+$ 'dan itibaren devreye I_S akımı veriliyor.

$\tau = RC$ Zaman Sabiti

5τ Durulama Zamanı

$V_C(t) = V_R(t)$

$i_C(t) + i_R(t) = I_S$

$C\frac{dV_C}{dt} + \frac{V_C}{R} = I_S$

$\frac{dV_C}{dt} + \frac{1}{RC}V_C = \frac{I_S}{C}$

$V_C(t) = \left(A e^{-t/RC} + B\right) u(t)$

$V_C(\infty) = B = R I_S$

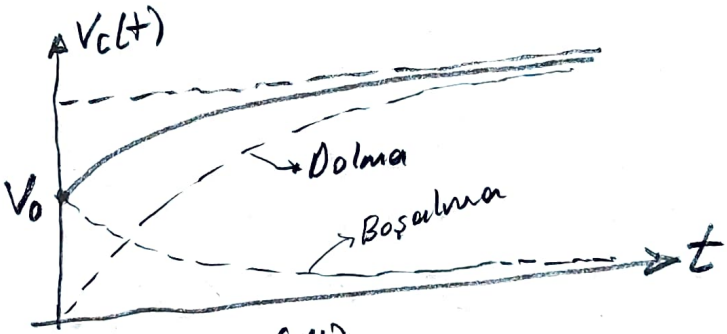
$V_C(0^+) = A + B = V_0$

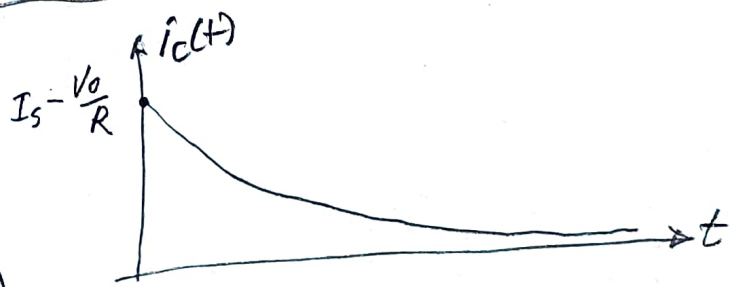
$A = V_0 - B = V_0 - R I_S$

$V_C(t) = \left((V_0 - R I_S) e^{-t/RC} + R I_S\right) u(t)$

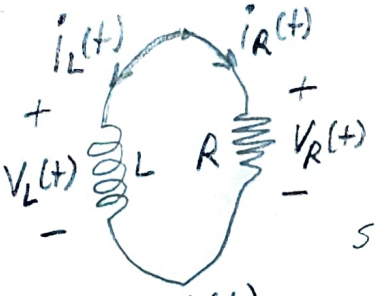
$i_C(t) = C\frac{dV_C}{dt} = I_S - \frac{V_C(t)}{R}$

$= \left(I_S - \frac{V_0}{R}\right) e^{-t/RC} u(t)$





$V_C(t) = \underbrace{V_0 e^{-t/RC} u(t)}_{\text{Boşalma}} + \underbrace{R I_S \left(1 - e^{-t/RC}\right) u(t)}_{\text{Dolma}}$



$i_L(0^+) = I_0$, $\tau = L/R$ Zaman Sabiti

5τ Durulama Zamanı

5τ'lik bir sürede bobinden geçen akımın sıfır veya sabit bir değer olduğu kabul edilir.

$V_L(t) = V_R(t)$

$i_L(t) + i_R(t) = 0$

$i_L(t) + \frac{V_L(t)}{R} = 0$

$i_L(t) + \frac{L}{R}\frac{di_L(t)}{dt} = 0$

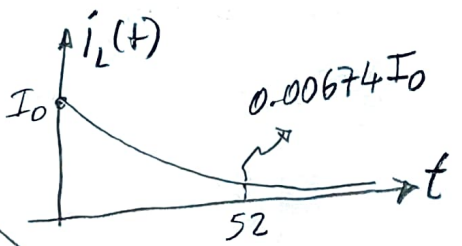
$\frac{di_L}{dt} + \frac{R}{L} i_L = 0$

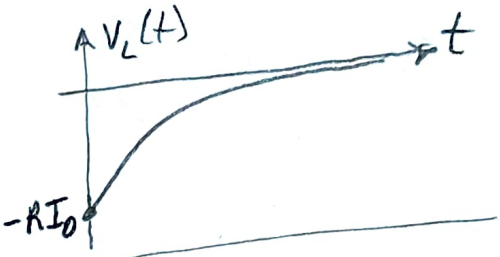
$i_L(t) = A e^{-Rt/L} u(t)$, $i_L(0^+) = A = I_0$

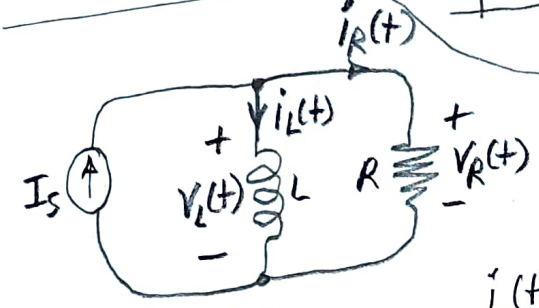
$i_L(t) = I_0 e^{-Rt/L} u(t)$

$V_L(t) = V_R(t) = R i_R(t) = -R i_L(t)$

$= -R I_0 e^{-Rt/L} u(t)$







$i_L(0^+) = I_0$

$t = 0^+$'den itibaren devreye I_S akımı veriliyor.

$V_L(t) = V_R(t)$

$i_L(t) + i_R(t) = I_S$

$i_L(t) + \frac{V_L(t)}{R} = I_S$

$i_L(t) + \frac{L}{R}\frac{di_L(t)}{dt} = I_S$

$\frac{di_L}{dt} + \frac{R}{L} i_L = \frac{R I_S}{L}$

$i_L(t) = \left(A e^{-Rt/L} + B\right) u(t)$

$i_L(\infty) = B = I_S$

$i_L(0^+) = A + B = I_0$

$A = I_0 - B$

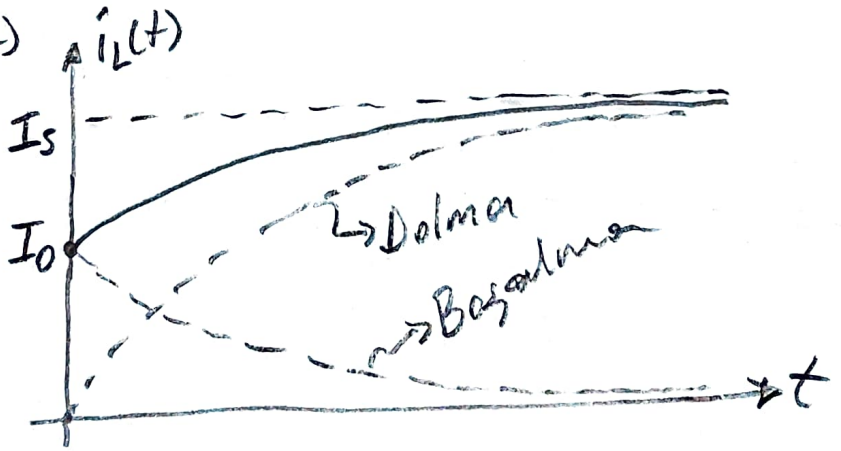
$= I_0 - I_S$

$i_L(t) = \left((I_0 - I_S) e^{-Rt/L} + I_S\right) u(t)$

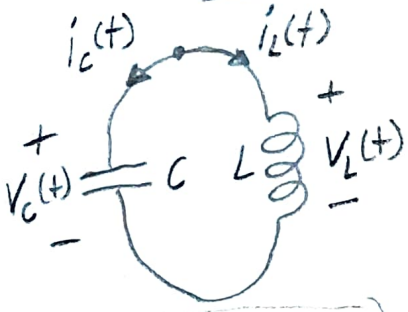
$= \underbrace{I_0 e^{-Rt/L} u(t)}_{\text{Boşalma}} + \underbrace{I_S\left(1 - e^{-Rt/L}\right) u(t)}_{\text{Dolma}}$

$V_L(t) = L\frac{di_L}{dt} = V_R(t) = R i_R(t)$

$= R(I_S - i_L(t)) = R(I_S - I_0) e^{-Rt/L} u(t)$



İkinci Mertebeden Devreler



$V_C(0^+) = V_0$, $i_L(0^+) = I_0$ olsun

$V_C(t) = V_L(t) = L\frac{di_L(t)}{dt}$, $i_C(t) = C\frac{dV_C(t)}{dt}$

$i_C(t) + i_L(t) = 0$

$C\frac{dV_C}{dt} + i_L(t) = 0 \Rightarrow LC\frac{d^2V_C}{dt^2} + L\frac{di_L}{dt} = 0$

$$\frac{d^2V_C}{dt^2} + \frac{1}{LC}V_C = 0$$

$s^2 + \frac{1}{LC} = 0$

$s_{1,2} = \mp j\frac{1}{\sqrt{LC}}$

$V_C(0^+) = A = V_0$

$i_L(0^+) = -B\sqrt{\frac{C}{L}} = I_0$

$B = -I_0\sqrt{\frac{L}{C}}$

$V_C(t) = \left(A\cos\frac{t}{\sqrt{LC}} + B\sin\frac{t}{\sqrt{LC}}\right)u(t)$

$i_L(t) = -i_C(t) = -C\frac{dV_C(t)}{dt}$

$= \sqrt{\frac{C}{L}}\left(A\sin\frac{t}{\sqrt{LC}} - B\cos\frac{t}{\sqrt{LC}}\right)u(t)$

$V_C(t) = \left(V_0\cos\frac{t}{\sqrt{LC}} - \sqrt{\frac{L}{C}}I_0\sin\frac{t}{\sqrt{LC}}\right)u(t)$

$i_L(t) = \left(I_0\cos\frac{t}{\sqrt{LC}} + \sqrt{\frac{C}{L}}V_0\sin\frac{t}{\sqrt{LC}}\right)u(t)$

Yukarıdaki örnek için

$V_C(0^+) = 15V$, $i_L(0^+) = 3A$, $L = 1H$, $C = 40mF$ olursa

$\frac{d^2V_C}{dt^2} + \frac{1}{LC}V_C = 0 \Rightarrow \frac{d^2V_C}{dt^2} + 25V_C = 0 \Rightarrow s^2 + 25 = 0$

$s_{1,2} = \mp 5j$

$V_C(t) = \left(A\cos\frac{t}{\sqrt{LC}} + B\sin\frac{t}{\sqrt{LC}}\right)u(t) = (A\cos 5t + B\sin 5t)u(t)$

$i_L(t) = -i_C(t) = -C\frac{dV_C(t)}{dt} = -40\times10^{-3}(-5A\sin 5t + 5B\cos 5t)u(t)$

$= (0.2A\sin 5t - 0.2B\cos 5t)u(t)$

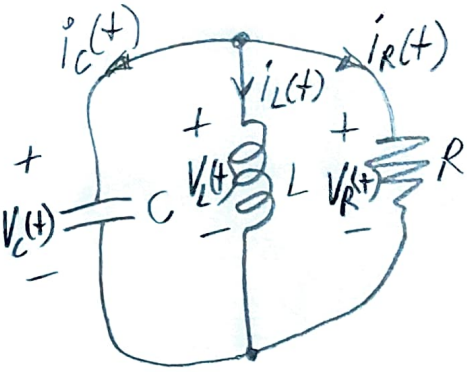
$V_C(0^+) = A = 15$

$i_L(0^+) = -0.2B = 3$

$B = -15$

$V_C(t) = (15\cos 5t - 15\sin 5t)u(t)$

$i_L(t) = (3\cos 5t + 3\sin 5t)u(t)$



$V_C(t) = V_L(t) = V_R(t)$

$i_C(t) + i_L(t) + i_R(t) = 0$

$C\frac{dV_c}{dt} + i_L + \frac{V_c}{R} = 0$

$LC\frac{d^2V_c}{dt} + L\frac{di_L}{dt} + \frac{L}{R}\frac{dV_c}{dt} = 0$

$$\frac{d^2V_c}{dt^2} + \frac{1}{RC}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0$$

$\alpha = \frac{1}{2RC}$ Sönümleme Frekansı

$w = \frac{1}{\sqrt{LC}}$ Salınım Frekansı

$$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + w^2V_c = 0$$

$s^2 + 2\alpha s + w^2 = 0$

$(s+\alpha)^2 + w^2 - \alpha^2 = 0$

$s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - w^2}$

$w_d = \sqrt{w^2 - \alpha^2}$

$\alpha > w$ ise $V_c(t) = (Ae^{s_1 t} + Be^{s_2 t})u(t)$

$\alpha = w$ ise $V_c(t) = (At+B)e^{-\alpha t}u(t)$

$\alpha < w$ ise

$V_c(t) = (A\cos w_d t + B\sin w_d t)e^{-\alpha t}u(t)$

5/ $V_c(0^+) = 7V$, $i_L(0^+) = 2A$, $R = 8\Omega$, $L = 1H$, $C = 25mF$ alınırsa

$$\frac{d^2V_c}{dt^2} + \frac{1}{RC}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0 \Rightarrow \frac{d^2V_c}{dt^2} + 5\frac{dV_c}{dt} + 4V_c = 0$$

$s^2 + 5s + 4 = 0$

$(s+1)(s+4) = 0$

$s_1 = -1$, $s_2 = -4$

$V_c(t) = (Ae^{-t} + Be^{-4t})u(t)$

$i_L(t) = -i_R(t) - i_C(t) = -\frac{V_c(t)}{R} - C\frac{dV_c(t)}{dt}$

$= -\frac{1}{40}(4Ae^{-t} + Be^{-4t})u(t)$

$V_c(0^+) = A + B = 7$

$i_L(0^+) = -\frac{1}{40}(4A+B) = 2$

$A + B = 7$

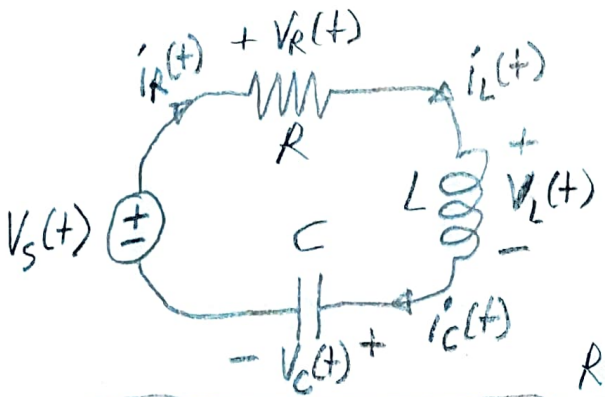
$4A + B = -80$

$-3A = 87$

$A = -29$, $B = 36$

$V_c(t) = (36e^{-4t} - 29e^{-t})u(t)$

$i_L(t) = (2.9e^{-t} - 0.9e^{-4t})u(t)$



$$i_R(t) = i_L(t) = i_C(t) = C\frac{dV_C(t)}{dt}$$

$$V_R(t) + V_L(t) + V_C(t) = V_S(t)$$

$$R\,i_L(t) + L\frac{di_L(t)}{dt} + V_C(t) = V_S(t)$$

$$RC\,i_L(t) + LC\frac{di_L(t)}{dt} + C\,V_C(t) = C\,V_S(t)$$

$$\frac{d^2i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{1}{L}\frac{dV_S}{dt}$$

$\alpha = \frac{R}{2L}$ Sönümleme Frekansı

$w = \frac{1}{\sqrt{LC}}$ Salınım Frekansı

$$\frac{d^2i_L}{dt^2} + 2\alpha\frac{di_L}{dt} + w^2 i_L = \frac{1}{L}\frac{dV_S}{dt}$$

$\frac{dV_S}{dt} = 0$ ise Homojen Çözüm $i_L(t) = i_{L1}(t)$

$\frac{dV_S}{dt} \neq 0$ ise Homojen Olmayan Çözüm $i_L(t) = i_{L1}(t) + i_{L2}(t)$

Ör Yukarıdaki örnek için $R = 24\,\Omega$, $L = 4H$, $C = 10mF$ ve $V_S = 0V$ alınırsa

$V_C(0^+) = -4V$, $i_L(0^+) = 3A$ olsun.

$$\frac{d^2i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC}i_L = 0 \Rightarrow \frac{d^2i_L}{dt^2} + 6\frac{di_L}{dt} + 25\,i_L = 0$$

$$s^2 + 6s + 25 = 0 \Rightarrow (s+3)^2 + 4^2 = 0 \Rightarrow s_{1,2} = -3 \mp 4i$$

$$i_L(t) = (A\cos 4t + B\sin 4t)\,e^{-3t}u(t)$$

$$V_C(t) = -V_R(t) - V_L(t) = -R\,i_L(t) - L\frac{di_L}{dt}$$

$$= \left[(16A - 12B)\sin 4t - (12A + 16B)\cos 4t\right]e^{-3t}u(t)$$

$i_L(0^+) = A = 3$

$V_C(0^+) = -12A - 16B = -4$

$B = \frac{12A - 4}{-16} = -2$

$$i_L(t) = (3\cos 4t - 2\sin 4t)\,e^{-3t}u(t)$$

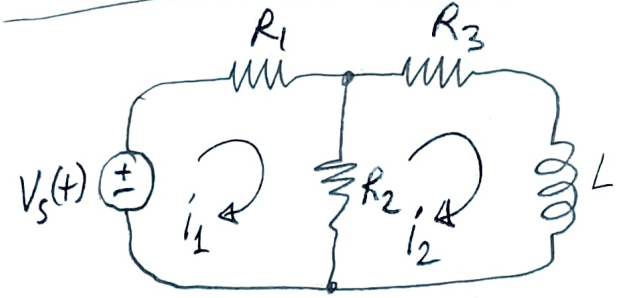
$$V_C(t) = (72\sin 4t - 4\cos 4t)\,e^{-3t}u(t)$$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$

$x(t) = \delta(t)$ ise $y(t) = h(t)$

$x(t) = u(t)$ ise $y(t) = s(t)$

$h(t) = \frac{ds(t)}{dt}$, $s(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$



$V_s(t) = 30\,u(t)$, $i_L(0^+) = 1A$

$R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 3\Omega$

$L = 2H$, $i_L(t) = ?$, $V_L(t) = ?$

$V_s - (R_1+R_2)i_1 + R_2 i_2 = 0$

$30 - 9i_1 + 6i_2 = 0$

$3i_1 - 2i_2 = 10$ 1.denklem

$R_2 i_1 - (R_2+R_3)i_2 - L\frac{di_2}{dt} = 0$

$6i_1 - 9i_2 - 2\frac{di_2}{dt} = 0$ 2.denklem

1. denklemin 2 katından 2. denklemi çıkar.

$2\frac{di_2}{dt} + 5i_2 = 20$

$i_L = i_2$ alınırsa

$\frac{di_L}{dt} + 2.5 i_L = 10$ dif. denklemi

$\frac{di_L}{dt}(\infty) = 0$ olduğundan

$\frac{di_L(\infty)}{dt} + 2.5 i_L(\infty) = 10 \Rightarrow i_L(\infty) = \frac{10}{2.5} A = 4A$

$i_L(t) = (Ae^{-2.5t} + B)\,u(t)$

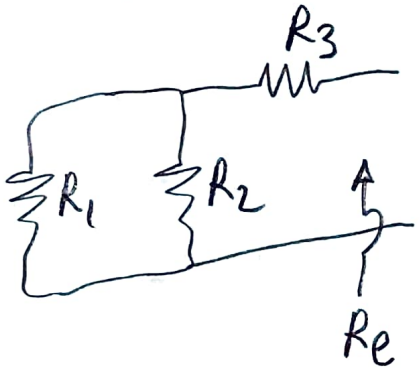
$i_L(t) = (4 - 3e^{-2.5t})\,u(t)$

$i_L(\infty) = B = 4$,

$i_L(0^+) = A + B = 1$

$A = 1 - B = -3$

$V_L(t) = L\frac{di_L}{dt} = 15e^{-2.5t}\,u(t)$



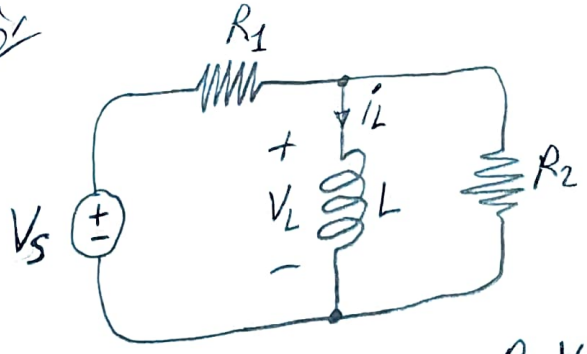
$R_e = R_1 // R_2 + R_3$

$= 3\Omega // 6\Omega + 3\Omega = 2\Omega + 3\Omega = 5\Omega$

$\tau = \frac{L}{R} = \frac{2H}{5\Omega} = 0.4\,sn$

$5\tau = 2sn$ Durulma Zamanı

Ör



$V_s(t) = 5\delta(t)$, $i_L(0^-) = 2A$

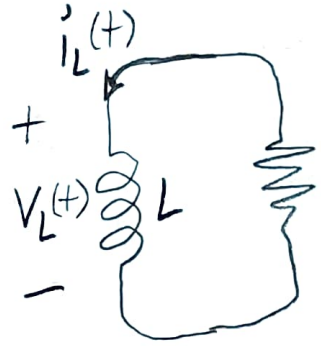
$R_1 = 10\Omega$, $R_2 = 15\Omega$, $L = 3H$

$\tau = ?$ Durulma zamanı $= ?$

$i_L(t) = ?$ $V_L(t) = ?$

$t=0$ için $V_L(t) = \dfrac{R_2 V_s(t)}{R_1 + R_2} = \dfrac{15}{10+15} 5\delta(t) = 3\delta(t)$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} V_L(\xi)\,d\xi = 2A + \frac{1}{3}\int_{0^-}^{0^+} 3\delta(\xi)\,d\xi = 2A + 1A = 3A$$



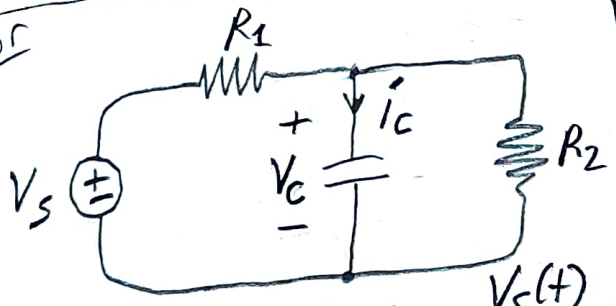
$R_e = R_1 // R_2 = 10\Omega // 15\Omega = 6\Omega$

$\tau = \dfrac{L}{R_e} = \dfrac{3H}{6\Omega} = 0.5\,sn$

Durulma zamanı $= 5\tau = 5 \times 0.5\,sn = 2.5\,sn$

$i_L(t) = 3e^{-2t}u(t) \Rightarrow V_L(t) = L\dfrac{di_L(t)}{dt} = -18e^{-2t}u(t)$

Ör



$V_s(t) = 10\delta(t)$, $V_c(0^-) = 4V$

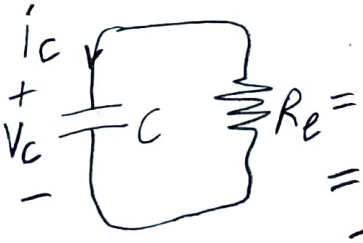
$R_1 = 10\Omega$, $R_2 = 15\Omega$, $C = 0.5F$

$\tau = ?$ Durulma zamanı $= ?$

$V_c(t) = ?$ $i_c(t) = ?$

$t=0$ için $i_c(t) = \dfrac{V_s(t)}{R_1} = \dfrac{10\delta(t)}{10} = \delta(t)$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_c(\xi)\,d\xi = 4V + \frac{1}{0.5}\int_{0^-}^{0^+} \delta(\xi)\,d\xi = 4V + 2V = 6V$$



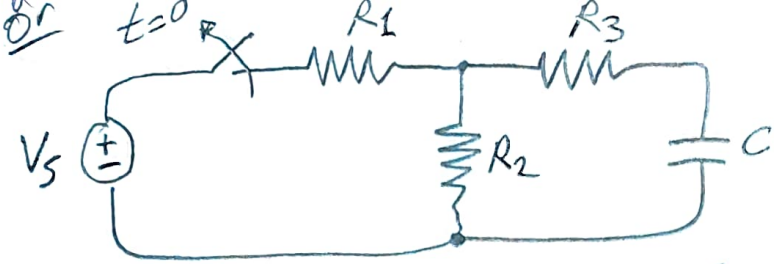
$R_e = R_1 // R_2 = 10\Omega // 15\Omega = 6\Omega$

$\tau = R_e C = 6\Omega \times 0.5F = 3\,sn$

Durulma zamanı $= 5\tau = 5 \times 3\,sn = 15\,sn$

$V_c(t) = 6e^{-t/3}u(t) \Rightarrow i_c(t) = C\dfrac{dV_c(t)}{dt} = -e^{-t/3}u(t)$

Ör

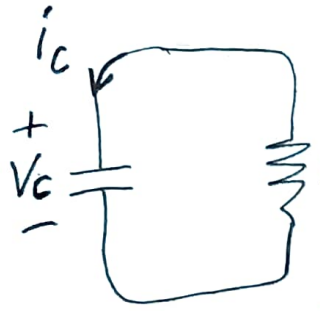


$V_s = 25V$, $R_1 = 60\Omega$
$R_2 = 40\Omega$, $R_3 = 10\Omega$
$C = 0.1F$ $\quad \tau = ?$
Dur. Zamanı $= ?$

$V_c(0^+) = ?$ $\quad W_c(0^+) = ?$ $\quad V_c(t) = ?$ $\quad i_c(t) = ?$

$$V_c(0^+) = V_c(0^-) = \frac{R_2 V_s}{R_1 + R_2} = \frac{40}{60+40}\,25V = 10V$$

$$W_c(0^+) = \frac{1}{2} C V_c^2(0^+) = \frac{1}{2} \times 0.1F \times (10V)^2 = 5J$$



$R_e = R_2 + R_3 = 40\Omega + 10\Omega = 50\Omega$

$\tau = R_e C = 50\Omega \times 0.1F = 5sn$

Durulma Zamanı $= 5\tau = 5 \times 5sn = 25sn$

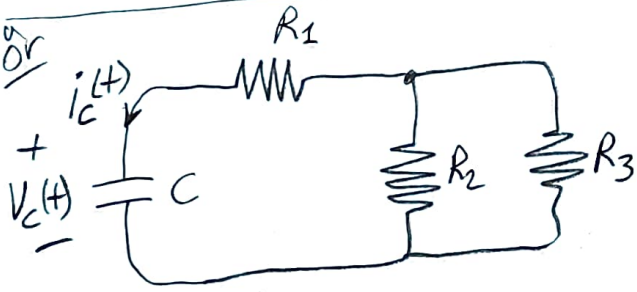
$V_c(t) = V_c(0^+)\, e^{-t/\tau}\, u(t)$

$= 10\, e^{-t/5}\, u(t)$

$$i_c(t) = C \frac{dV_c(t)}{dt}$$

$$= 0.1 \times 10 \times \left(-\frac{1}{5}\right) e^{-t/5}\, u(t) = -\frac{1}{5} e^{-t/5}\, u(t)$$

Ör



$V_c(0^+) = 30V$, $C = 40mF$
$R_1 = 30\Omega$, $R_2 = 60\Omega$, $R_3 = 30\Omega$
Durulma Zamanı $= ?$
$V_c(t) = ?$ $\quad i_c(t) = ?$

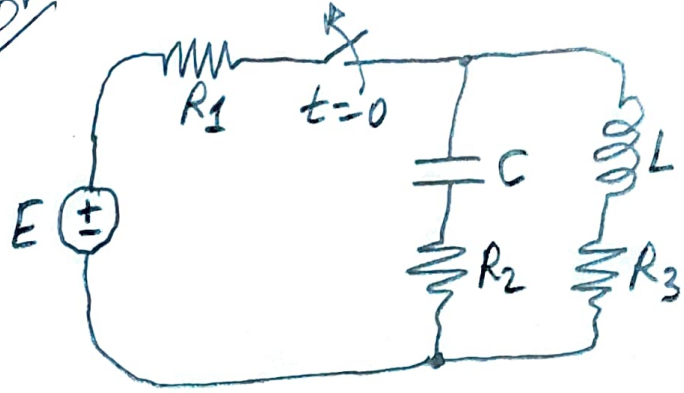
$$R_e = R_1 + R_2 // R_3 = 30\Omega + 60\Omega // 30\Omega = 30\Omega + 20\Omega = 50\Omega$$

$$\tau = R_e C = 50\Omega \times 40mF = 2sn$$

Durulma Zamanı $= 5\tau = 5 \times 2sn = 10sn$

$$V_c(t) = V_c(0^+)\, e^{-t/2}\, u(t) = 30\, e^{-t/2}\, u(t)$$

$$i_c(t) = C \frac{dV_c(t)}{dt} = 40 \times 10^{-3} \times 30 \times \left(-\frac{1}{2}\right) e^{-t/2}\, u(t) = -0.6\, e^{-t/2}\, u(t)$$

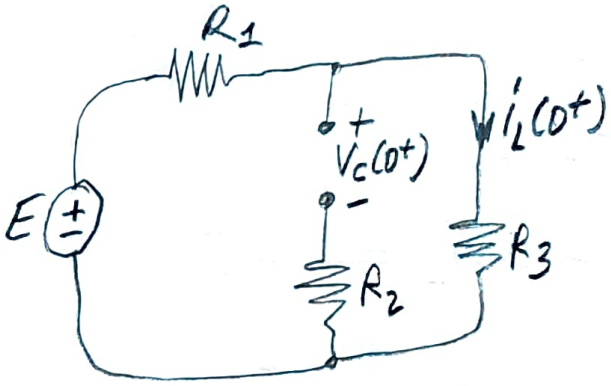


$E=12V,\ C=8mF,\ L=5H$

$R_1=15\Omega,\ R_2=21\Omega,\ R_3=9\Omega$

$\alpha=?\quad w=?$

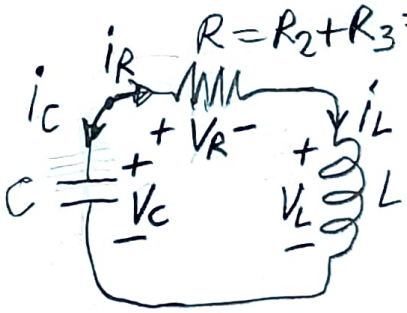
$i_L(t)=?\quad V_c(t)=?$



$$i_L(0^+)=\frac{E}{R_1+R_3}=\frac{12V}{15\Omega+9\Omega}$$

$$=\frac{12V}{24\Omega}=0.5A$$

$$V_c(0^+)=R_3\,i_L(0^+)=9\Omega\times 0.5A=4.5A$$



$$R=R_2+R_3=30\Omega$$

$$\alpha=\frac{R}{2L}=\frac{30\Omega}{2\times 5H}=3$$

$$w=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{5H\times 8mF}}=\frac{1}{\sqrt{0.04}}=5$$

$w>\alpha$ i.e. in $w_d=\sqrt{w^2-\alpha^2}=\sqrt{25-9}=\sqrt{16}=4$

$$i_L(t)=(A\cos 4t+B\sin 4t)\,e^{-3t}u(t)$$

$$\frac{di_L(t)}{dt}=\left[(4B-3A)\cos 4t-(4A+3B)\sin 4t\right]e^{-3t}u(t)$$

$$V_c(t)=V_R(t)+V_L(t)=R\,i_L(t)+L\frac{di_L}{dt}=30\,i_L(t)+5\frac{di_L(t)}{dt}$$

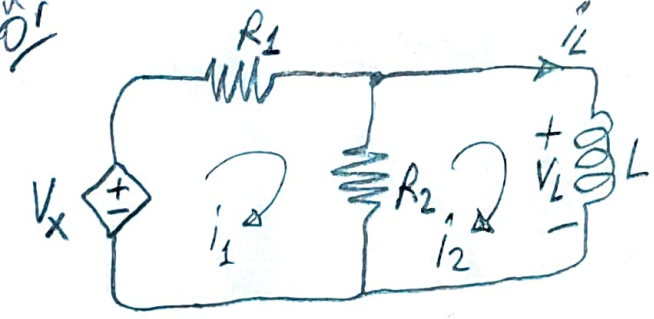
$$=\left[(15A+20B)\cos 4t+(15B-20A)\sin 4t\right]e^{-3t}u(t)$$

$$i_L(0^+)=A=0.5$$

$$V_c(0^+)=15A+20B=4.5\Rightarrow B=\frac{4.5-15A}{20}=-\frac{3}{20}=-0.15$$

$$i_L(t)=(0.5\cos 4t-0.15\sin 4t)\,e^{-3t}u(t)$$

$$V_c(t)=(4.5\cos 4t-12.25\sin 4t)\,e^{-3t}u(t)$$



$V_x(t) = 15 - 7 i_L(t)$

$i_L(0^+) = 5A$

$R_1 = 3\Omega, R_2 = 2\Omega, L = 8H$

$i_L(t) = ?$ $V_L(t) = ?$

$i_2 = i_L$

$V_x - (R_1+R_2) i_1 + R_2 i_2 = 0$

$15 - 7i_L - 5i_1 + 2i_L = 0$

$i_1 = 3 - i_L$

$R_2(i_1 - i_2) - L\frac{di_L}{dt} = 0$

$2(3 - i_L - i_L) - 8\frac{di_L}{dt} = 0$

$\frac{di_L}{dt} + 0.5 i_L = 0.75$

$t = \infty$ için $\frac{di_L}{dt} = 0$ olur.

$\frac{di_L(\infty)}{dt} + 0.5 i_L(\infty) = 0.75 \Rightarrow i_L(\infty) = 1.5A$

$i_L(t) = (Ae^{-0.5t} + B)\,u(t)$

$i_L(\infty) = B = 1.5$

$i_L(0^+) = A + B = 5 \Rightarrow A = 5 - A = 5A - 1.5A = 3.5A$

$i_L(t) = (3.5\,e^{-0.5t} + 1.5)\,u(t)$

$V_L(t) = L\frac{di_L}{dt} = 8 \times 3.5 \times (-0.5)\,e^{-0.5t}\,u(t)$

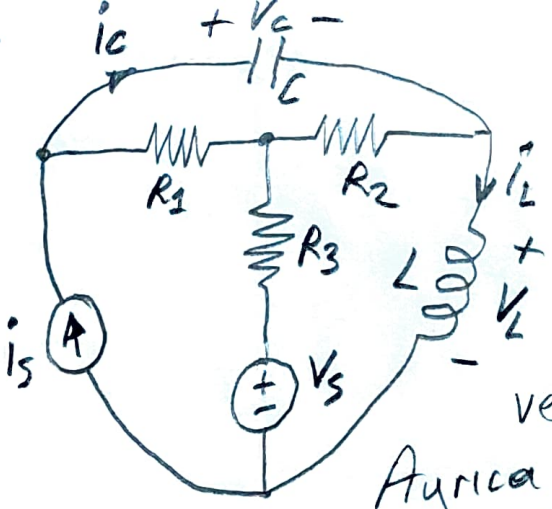
$= -14\,e^{-0.5t}\,u(t)$

$V_x(t) = 15 - 7 i_L(t)$

$= 15 - 7(3.5e^{-0.5t} + 1.5)\,u(t)$

$= (4.5 - 24.5e^{-0.5t})\,u(t)$

6/

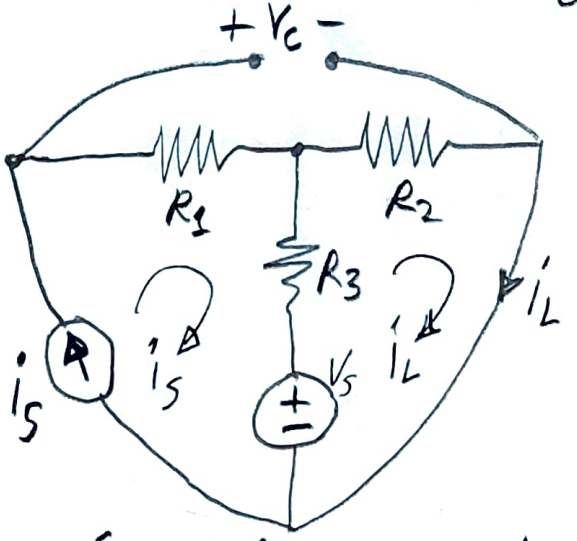


$V_S = 13V$, $i_S = 2A$

$C = 16mF$, $L = 4H$

$R_1 = 8\Omega$, $R_2 = 3\Omega$, $R_3 = 4\Omega$

DC şartlar oluştuğunda üretilen ve tüketilen güçleri bulunuz. Ayrıca kapasitör ve bobinde depolanan enerjileri bulunuz.



$i_S = 2A$

$V_S - (R_2 + R_3) i_L + R_3 i_S = 0$

$13 - 7 i_L + 4 \times 2 = 0$

$i_L = \frac{21}{7} A = 3A$

$V_C = R_1 i_S + R_2 i_L = 8\Omega \times 2A + 3\Omega \times 3A = 16V + 9V = 25V$

$P_{iS} = V_C i_S = 25V \times 2A = 50W$

$P_{VS} = V_S (i_L - i_S) = 13V \times (3A - 2A) = 13W$

$P_{üret} = P_{iS} + P_{VS}$
$= 50W + 13W$
$= 63W$

$P_1 = R_1 i_S^2 = 8\Omega \times (2A)^2 = 32W$

$P_2 = R_2 i_L^2 = 3\Omega \times (3A)^2 = 27W$

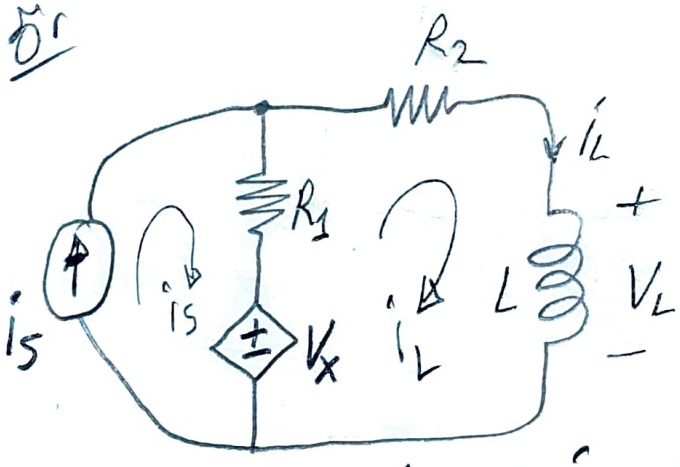
$P_3 = R_3 (i_L - i_S)^2 = 4\Omega \times (3A - 2A)^2 = 4W$

$P_{tük} = P_1 + P_2 + P_3$
$= 32W + 27W + 4W$
$= 63W$

$W_C = \frac{1}{2} C V_C^2 = \frac{1}{2} \times 16mF \times (25V)^2 = 5J$

$W_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \times 4H \times (3A)^2 = 18J$

Ör



$V_x = 5i_L$, $i_L(0^+) = 2A$

$R_1 = 3\Omega$, $R_2 = 5\Omega$, $L = 6H$

$i_s = 5A$ $\quad i_L(t) = ?$

$V_L(t) = ?$

$$V_x - (R_1+R_2)\,i_L + R_1 i_s - L\frac{di_L}{dt} = 0$$

$$5i_L - 8i_L + 3i_s - 6\frac{di_L}{dt} = 0$$

$$3i_s - 3i_L - 6\frac{di_L}{dt} = 0 \Rightarrow \frac{di_L}{dt} + 0.5i_L = 2.5$$

$t=\infty$ için $\frac{di_L}{dt} = 0$ olduğundan

$$\frac{di_L(\infty)}{dt} + 0.5\,i_L(\infty) = 2.5 \Rightarrow i_L(\infty) = 5A$$

$$i_L(t) = \left(Ae^{-0.5t} + B\right)u(t)$$

$$i_L(\infty) = B = 5$$

$$i_L(0^+) = A + B = 2 \Rightarrow A = 2 - B = -3$$

$$i_L(t) = \left(5 - 3e^{-0.5t}\right)u(t)$$

$$V_L(t) = L\frac{di_L}{dt} = 6\times(-3)(-0.5)\,e^{-0.5t}\,u(t)$$

$$= 9e^{-0.5t}\,u(t)$$

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