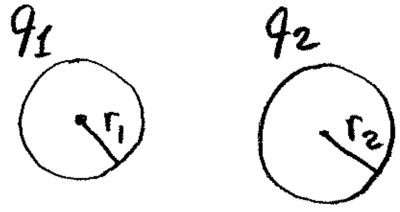
Devre Teorileri



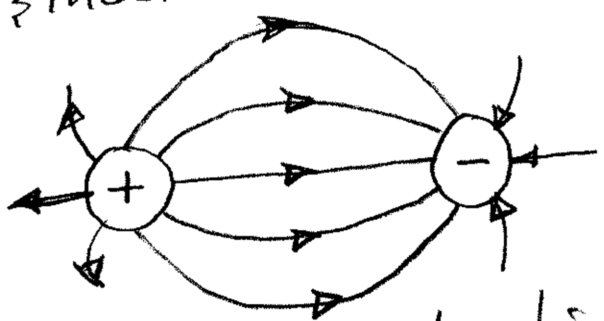
Küreler birbirlerine dokundurulursa üzerlerindeki yeni yükler

$$\bar{q}_1 = q_T \frac{r_1}{r_1+r_2} \ , \quad \bar{q}_2 = q_T \frac{r_2}{r_1+r_2}$$

$$q_T = q_1 + q_2$$
$$= \bar{q}_1 + \bar{q}_2$$

$$F = k\,\frac{q_1 q_2}{d^2}$$ Yitme veya çekme kuvveti.

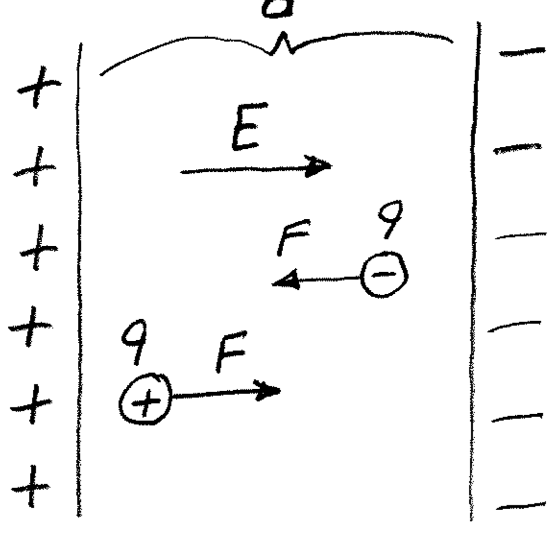
Küreler ikiside pozitif veya ikiside negatif ise birbirlerini iter. Biri pozitif diğeri negatif ise birbirlerini çekerler. İki küre birbirlerine bir tel ile bağlansaydı yükler dengeye gelene kadar telden elektrik akımı geçerdi. Kürelerin içinde elektrik alan sıfırdır. Yüzeyinde veya dışında ise $E = k \cdot q/d^2$.



Kürenin yükü + ise elektrik alan yönü dışarı, − ise içeri doğrudur.

Voltaj ise yönlü bir büyüklük olmayıp formülü

$$V = E.d = k\, q/d$$



Farklı yüklü iki levha arasındaki elektrik alan şiddeti sabittir.

$$V = E.d \Rightarrow E = V/d$$

$$F = q.E = q\frac{V}{d}$$

$$i(t) = \frac{dq(t)}{dt}, \quad q(t) = \int_{-\infty}^{t} i(z)dz = q(t_0) + \int_{t_0}^{t} i(z)dz$$

Birim zamanda geçen yük miktarına akım denir.

$$1\,Amper = \frac{1\,Coulomb}{1\,Saniye}$$ 1 sn'de 1C'luk yük geçmesi 1A'lik akım manasına gelir.

$$w(t) = \int_{-\infty}^{t} p(z).dz = w(t_0) + \int_{t_0}^{t} p(z).dz$$ İş birimi Joule (J)'dür.

$$p(t) = \frac{dw(t)}{dt} = \frac{dw(t)}{dq(t)} \frac{dq(t)}{dt} = v(t).i(t)$$

Güç birimi Watt (W)'dır. $1W = 1V \times 1A = 1VA$

$$1\,Volt = \frac{1\,Joule}{1\,Coulomb}$$ 1C'luk yük 1J'lük iş yaparsa 1V'lük voltaj manasına gelir.

$1\,wh = 3600\,J$ 1 WattSaat 3600 Joule'dür.

Bir lamba üzerinden 1 dakika boyunca 2A'lik akım geçmektedir. Lamba 15 kJ'lük enerjiyi ısı ve ışık olarak dışarı veriyor. Lamba üzerindeki voltajı bul.

$$q = i.t = 2A \times 60\,sn = 120\,C$$

$$v = \frac{w}{q} = \frac{15kJ}{120\,C} = \frac{15000\,J}{120\,C} = 125\,V$$

---

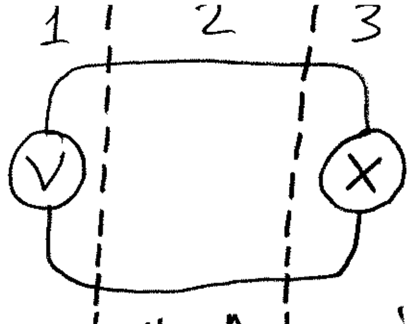
25 W'lık bir lamba 8 saatte kaç kJ'lük enerji tüketir.

$$w = p.t = 25W \times 8h = 200\,wh$$

$$= 200 \times 3600\,J$$

$$= 720000\,J$$

$$= 720\,kJ$$



1- Kaynak (Üreteç)
2- İletim Hattı (Tel)
3- Yük (Tüketeç)

Elektrik Devresi

Bir elektrik devresinde üretilen enerji ile tüketilen enerji birbirlerine eşittir. Yani, devredeki toplam enerji sıfırdır. Elektronların hareket yönü ise tanımlanan akım yönüne zıttır. $I = -I_e$.

Kaynaklar


$v(t)$ Bağımsız Voltaj Kaynağı


Voltaj Kontrollü Voltaj Kaynağı $V_1(t) = K V_2(t)$

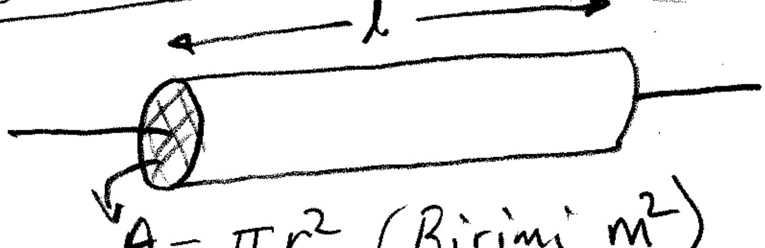

Akım Kontrollü Voltaj Kaynağı $V_1(t) = K i_2(t)$


$i(t)$ Bağımsız Akım Kaynağı


Voltaj Kontrollü Akım Kaynağı $i_1(t) = K V_2(t)$


Akım Kontrollü Akım Kaynağı $i_1(t) = K i_2(t)$

Direnç (R)



$A = \pi r^2$ (Birimi m^2)

$$R = \rho \frac{l}{A} = \frac{l}{\sigma \cdot A}$$

ρ: Özdirenç, $\Omega \cdot m$

σ: İletkenlik, S/m

$$G = \frac{1}{R}$$

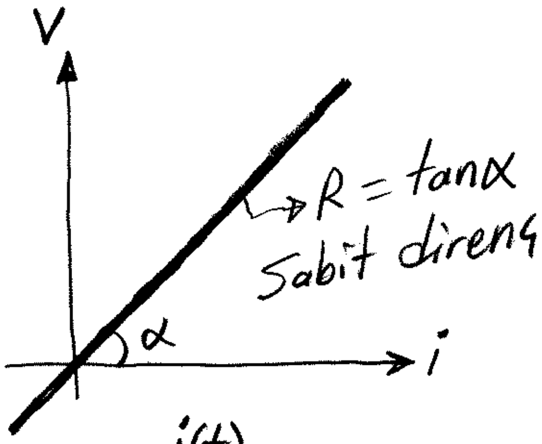
G: Geçirgenlik, mho, ℧, Simens, S

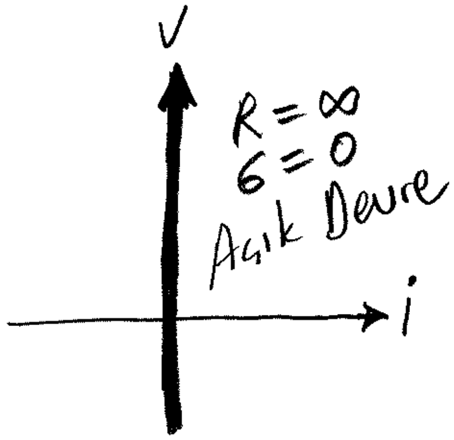
R: Direnç, Ohm, Ω

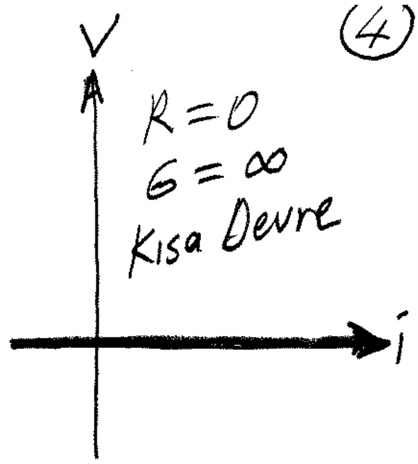
$l = 1km$, $r = 1mm$ olan bakır telin direncini bulunuz.

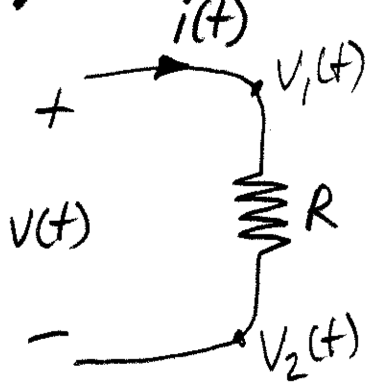
$$\sigma_{cu} = 5.8 \times 10^7 \ S/m$$

$$R_{cu} = \frac{l}{\sigma_{cu} \cdot A} = \frac{10^3}{5.8 \times 10^7 \times \pi \times 10^{-6}} \approx 5.49\,\Omega$$





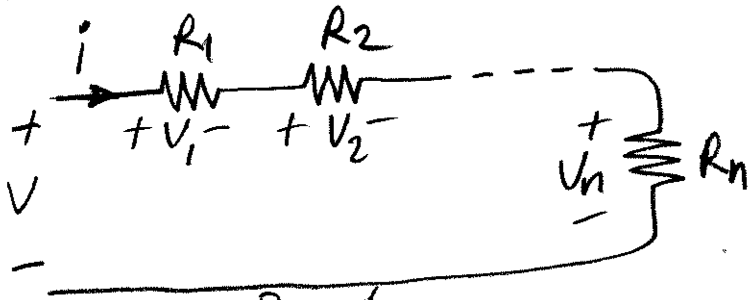




$$v(t) = V_1(t) - V_2(t) > 0$$

$$v(t) = R\, i(t) \qquad p(t) = v(t) \cdot i(t)$$

$$i(t) = G\, v(t) \qquad = R\, i^2(t) = \frac{v^2(t)}{R}$$

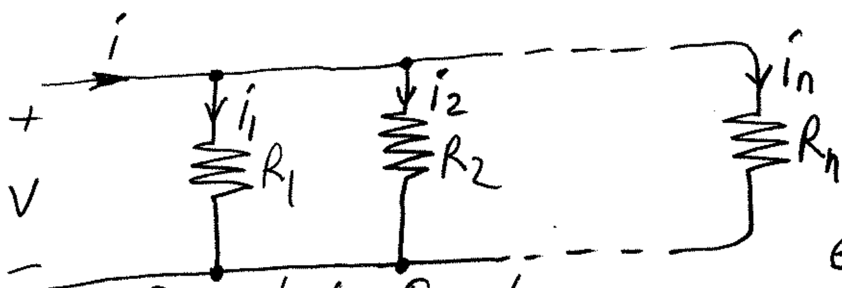


Seri Bağlama

$$i = i_1 = i_2 = \cdots = i_n$$

$$V = V_1 + V_2 + \cdots + V_n$$

$$R_T = R_1 + R_2 + \cdots + R_n$$



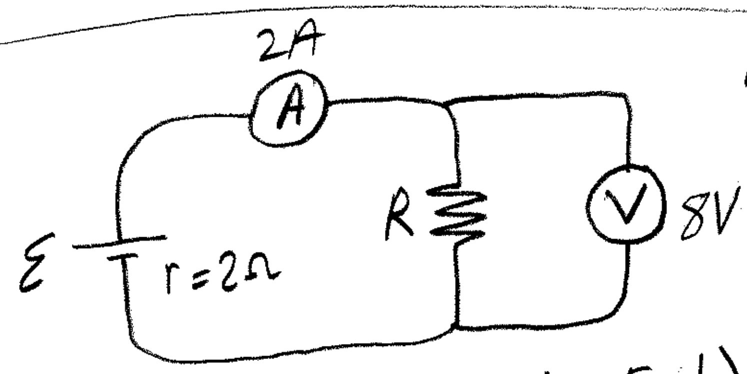
Paralel Bağlama

$$V = V_1 = V_2 = \cdots = V_n$$

$$i = i_1 + i_2 + \cdots + i_n$$

$$G_T = G_1 + G_2 + \cdots + G_n$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$



Ampermetre 2A, Voltmetre 8V ölçüyor.

ε (Elektromotor Kuvvet - Emk) bulunuz.

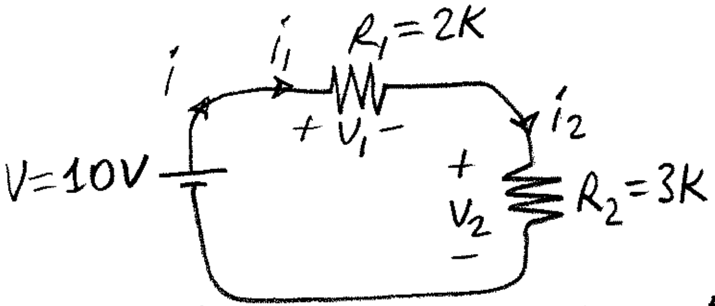
$$R = \frac{V}{I} = \frac{8V}{2A} = 4\Omega$$

$$\varepsilon = (R + r)\, I$$

$$= 6\Omega \times 2A$$

$$= 12V$$

Seri Bağlama



$R_T = R_1 + R_2 = 2K + 3K = 5K$

$i = i_1 = i_2 = V/R_T = \frac{10V}{5K} = 2mA$

$V_1 = R_1 i_1 = 2K \times 2mA = 4V$

$V_2 = R_2 i_2 = 3K \times 2mA = 6V$

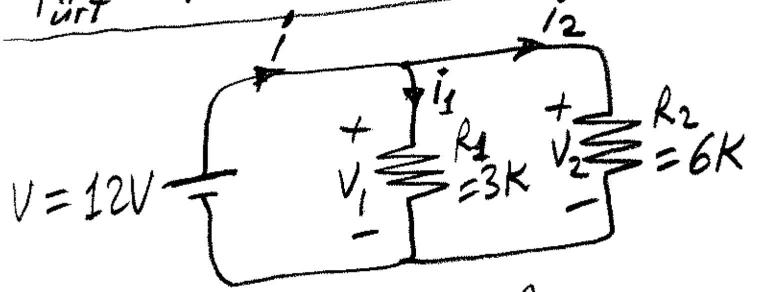
$P_1 = V_1 i_1 = 4V \times 2mA = 8mW$

$P_2 = V_2 i_2 = 6V \times 2mA = 12mW$

$P_{ürt} = V i = 10V \times 2mA = 20mW$

$P_{tük} = P_1 + P_2 = 8mW + 12mW = 20mW$

Paralel Bağlama



$V = V_1 = V_2 = 12V$

$i_1 = V_1/R_1 = 12V/3K = 4mA$

$i_2 = V_2/R_2 = 12V/6K = 2mA$

$i = i_1 + i_2 = 6mA$

$R_{eq} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{3K \times 6K}{3K + 6K} = 2K$

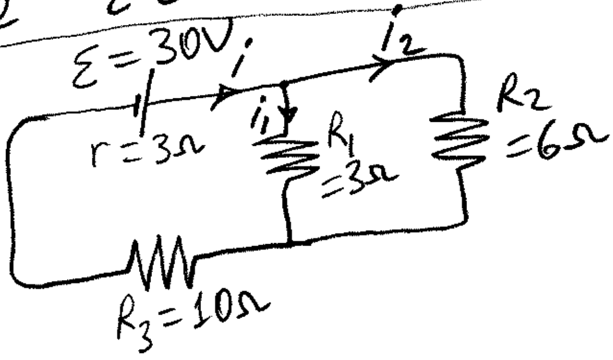
$i = V/R_{eq} = 12V/2K = 6mA$

$P_{ürt} = V i = 12V \times 6mA = 72mW$

$P_1 = V_1 i_1 = 12V \times 4mA = 48mW$

$P_2 = V_2 i_2 = 12V \times 2mA = 24mW$

$P_{yük} = P_1 + P_2 = 48mW + 24mW = 72mW$



Üretecin uçları arasındaki potansiyel farkı $V = ?$

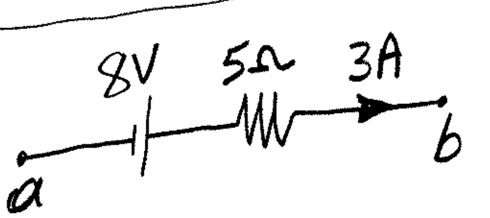
$R_T = r + R_3 + R_1 // R_2$

$= 3\Omega + 10\Omega + 3\Omega // 6\Omega$

$= 15\Omega$

$i = \frac{\mathcal{E}}{R_T} = \frac{30V}{15\Omega} = 2A$

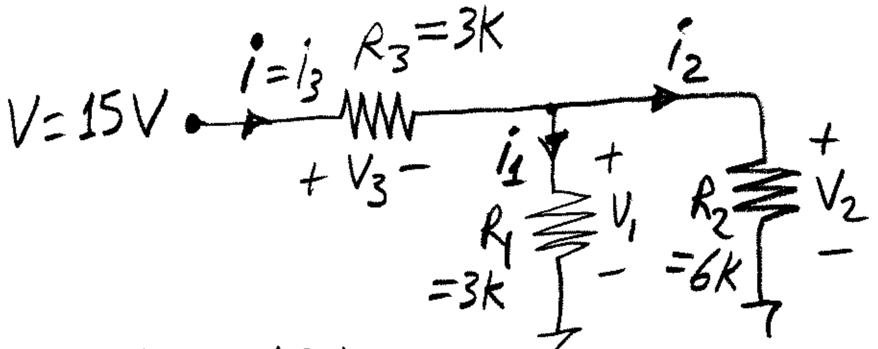
$V = \mathcal{E} - r i = 30V - 3\Omega \times 2A = 24V$



$V_a + 8V - 5\Omega \times 3A = V_b$

$V_{ab} = V_a - V_b = 15V - 8V$

$= 7V$



$$R_{eq} = R_1 // R_2 + R_3$$
$$= \frac{R_1 R_2}{R_1+R_2} + R_3$$
$$= \frac{3K \times 6K}{9K} + 3K = 5K$$

$$i = \frac{V}{R_{eq}} = \frac{15V}{5K} = 3mA = i_3$$

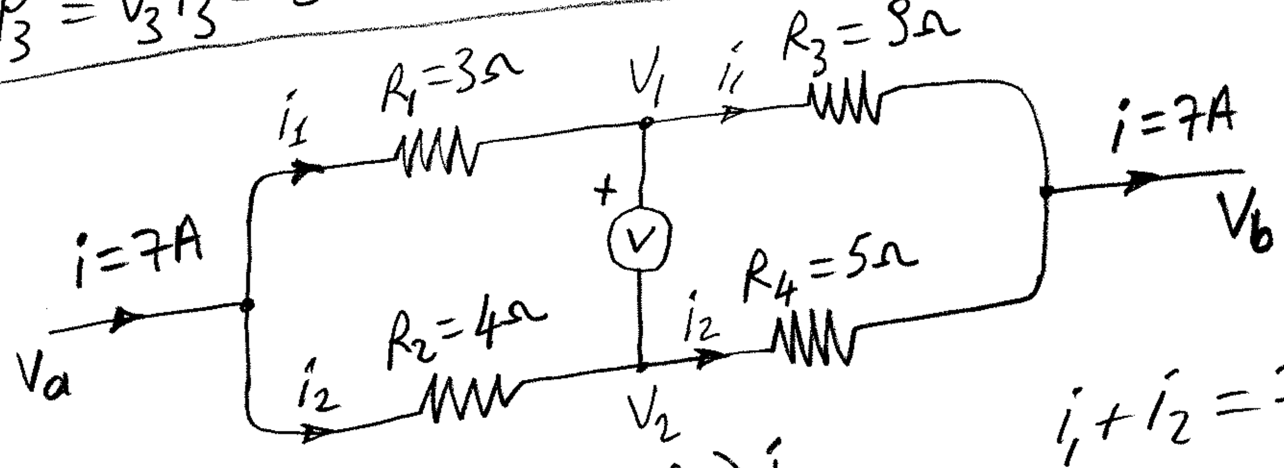
$$V_1 = V_2 = V - R_3 i_3 = 15V - 3K \times 3mA = 6V$$

$$\left.\begin{aligned} i_1 &= V_1/R_1 = 6V/3K = 2mA \\ i_2 &= V_2/R_2 = \frac{6V}{6K} = 1mA \end{aligned}\right\} \quad i = i_1 + i_2 = 3mA$$

$$V_3 = R_3 i_3 = 3K \times 3mA = 9V$$

$$P_1 = V_1 i_1 = 6V \times 2mA = 12mW$$
$$P_2 = V_2 i_2 = 6V \times 1mA = 6mW$$
$$P_3 = V_3 i_3 = 9V \times 3mA = 27mW$$

$$P_{ort} = Vi = 15V \times 3mA = 45mW$$
$$P_{tot} = P_1 + P_2 + P_3 = 45mW$$



$$i_1 + i_2 = 7$$

$$V_{ab} = (R_1 + R_3) i_1 = (R_2 + R_4) i_2$$
$$12 i_1 = 9 i_2 \Rightarrow 4i_1 - 3i_2 = 0$$

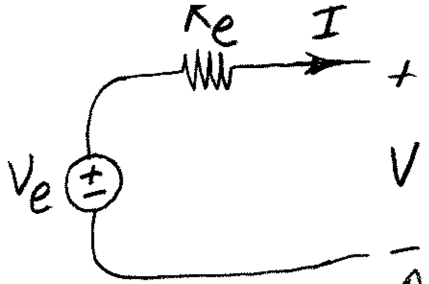
$$V_a = V_1 + R_1 i_1 = V_2 + R_2 i_2$$

$$\begin{aligned} 4i_1 - 3i_2 &= 0 \\ 3i_1 + 3i_2 &= 21 \\ \hline 7i_1 &= 21 \end{aligned}$$

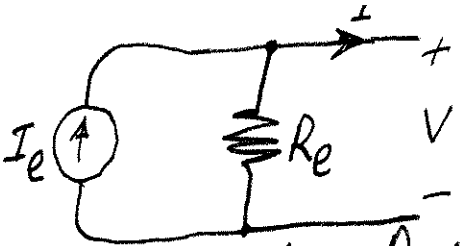
$$i_1 = 3A \qquad i_2 = 7 - i_1 = 4A$$

$$V = V_1 - V_2 = R_2 i_2 - R_1 i_1$$

$$V = 4\Omega \times 4A - 3\Omega \times 3A$$
$$= 16V - 9V$$
$$= 7V$$

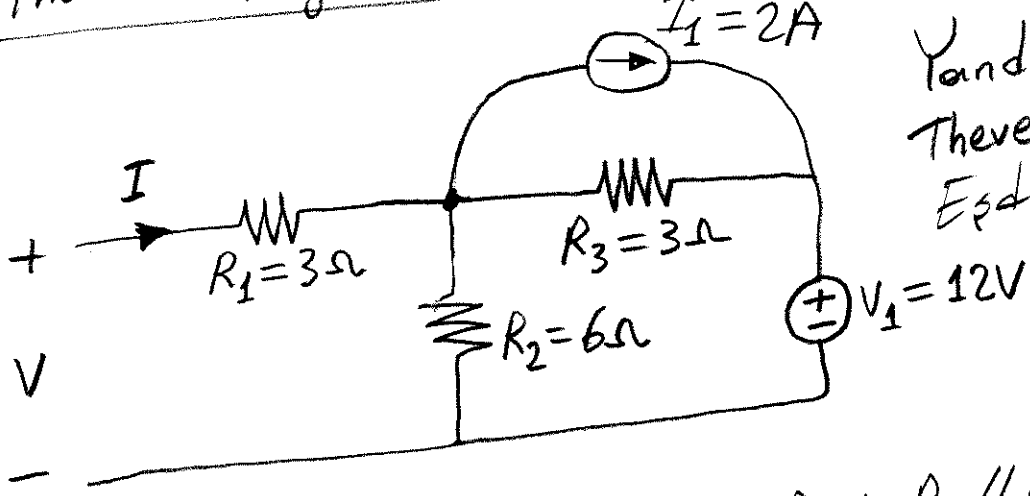


Thevenin Eşdeğer Devre

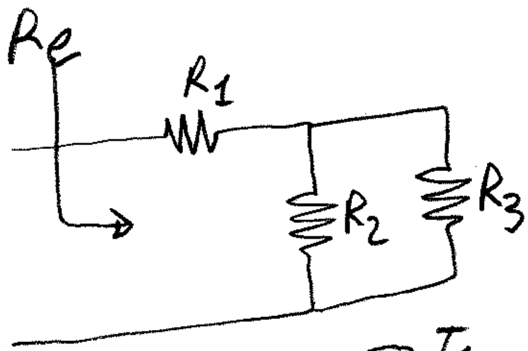


Norton Eşdeğer Devre

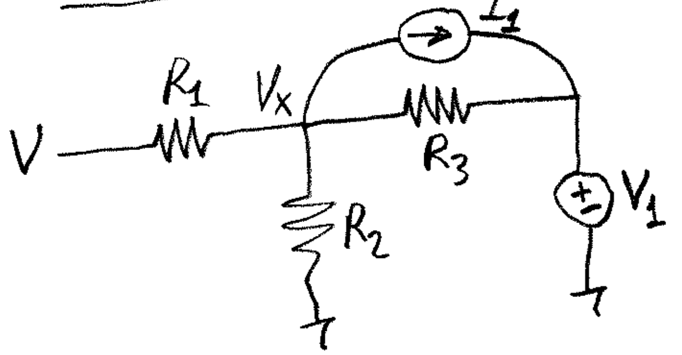
$$V = V_e - R_e I$$
$$= R_e (I_e - I)$$
$$V_e = R_e I_e$$



Yanda verilen devrenin Thevenin ve Norton Eşdeğerini bulunuz.

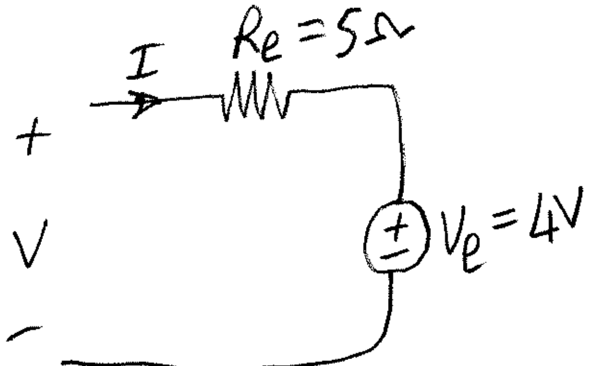


$$R_e = R_1 + R_2 // R_3$$
$$= R_1 + \frac{R_2 R_3}{R_2 + R_3}$$
$$= 3\Omega + \frac{6\Omega \times 3\Omega}{9\Omega} = 5\Omega$$

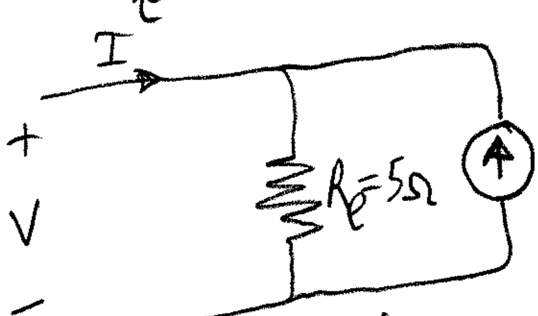


$$I = \frac{V - V_x}{R_1} = \frac{V_x}{R_2} + \frac{V_x - V_1}{R_3} + I_1$$
$$\frac{V - V_x}{3} = \frac{V_x}{6} + \frac{V_x - 12}{3} + 2$$
$$2V - 2V_x = V_x + 2V_x - 24 + 12$$
$$V_x = \frac{2V + 12}{5}$$

$$I = \frac{V - V_x}{R_1} = \frac{V - \frac{2V + 12}{5}}{3} = \frac{V - \overset{V_e}{4}}{\underset{R_e}{5}}$$

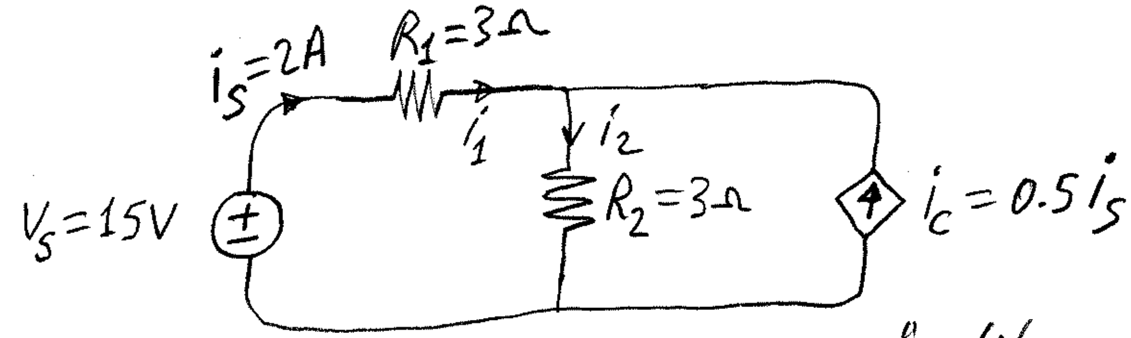


Thevenin Eşd. Devre



$$I_e = \frac{V_e}{R_e} = 0.8A$$

Norton Eşd. Devre



Devre elemanları üzerindeki güçleri bul.

$$P_s = V_s i_s = 15V \times 2A = 30W \text{ Üretilen}$$

$$i_1 = i_s = 2A \quad i_c = 0.5 i_s = 1A \quad i_2 = i_1 + i_c = 3A$$

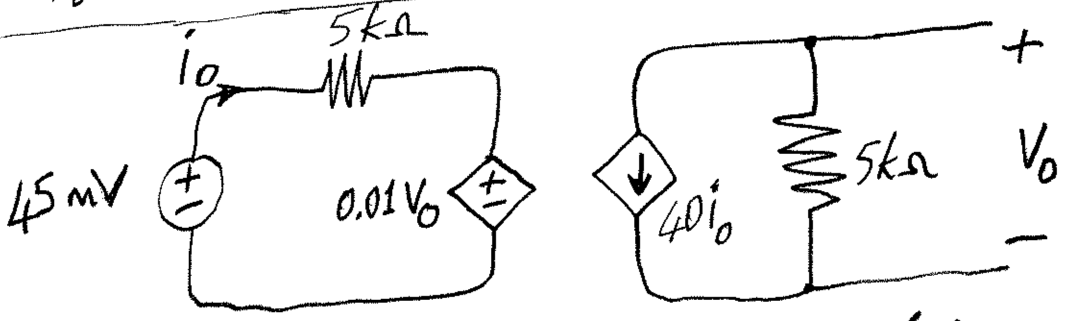
$$P_1 = R_1 i_1^2 = 3\Omega \times (2A)^2 = 12W \text{ Tüketilen}$$

$$P_2 = R_2 i_2^2 = 3\Omega \times (3A)^2 = 27W \text{ Tüketilen}$$

$$V_c = V_2 = R_2 i_2 = 3\Omega \times 3A = 9V$$

$$P_c = V_c \cdot i_c = 9V \times 1A = 9W \text{ Üretilen}$$

$$P_{ürt} = P_s + P_c = 39W \quad P_{tük} = P_1 + P_2 = 39W$$



$i_o = ?$

$V_o = ?$

$$V_o = -5k\Omega \times 40 i_o = -0.2 \times 10^6 \times i_o$$

$$45mV - 5k\Omega \times i_o - 0.01 V_o = 0$$

$$45 \times 10^{-3} - 5 \times 10^3 i_o - 10^{-2} V_o = 0$$

$$45 - 5 \times 10^6 i_o - 10 V_o = 0$$

$$45 - 5 \times 10^6 \times i_o + 2 \times 10^6 i_o = 0$$

$$3 \times 10^6 i_o = 45$$

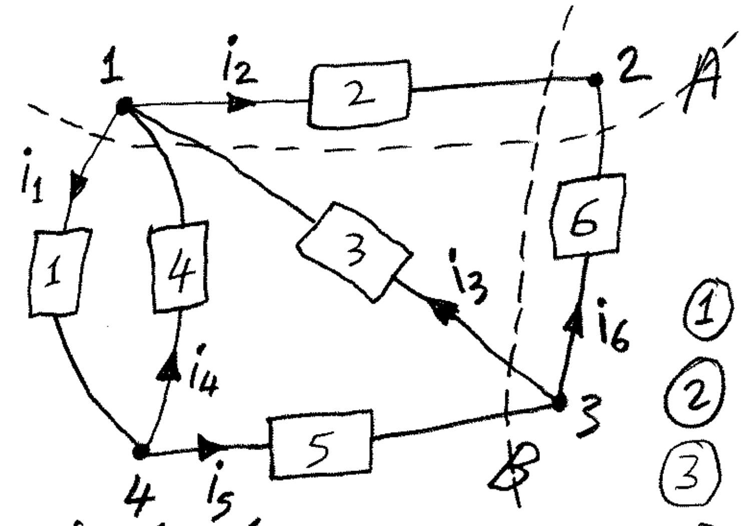
$$i_o = 15 \mu A$$

$$V_o = -0.2 \times 10^6 \times i_o$$

$$= -0.2 \times 10^6 \times 15 \times 10^{-6} V$$

$$= -3V$$

Kirchhoff'un Akım Yasası (KCL) - Nokta Analiz



Çıkan akım pozitif, giren akım negatif alınır. Her nokta için akımların cebirsel toplamı sıfırdır.

(1) $i_1+i_2-i_3-i_4=0$

(2) $-i_2-i_6=0$

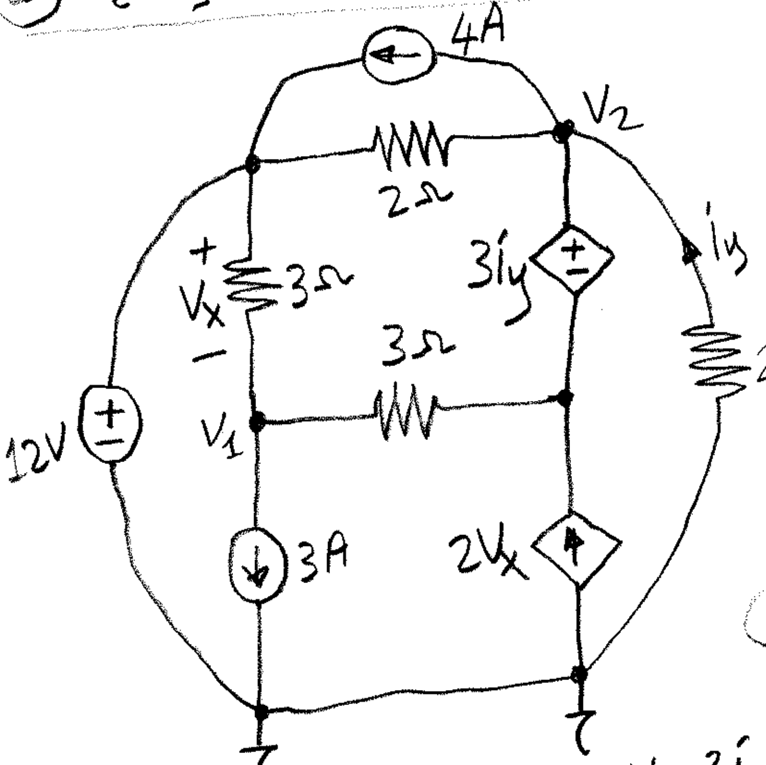
(3) $i_3-i_5+i_6=0$

(4) $-i_1+i_4+i_5=0$

(A) $i_1-i_3-i_4-i_6=0$

1 ile 2.denklem birleşir

(3) $i_2-i_3+i_5=0$, 2 ile 3.denklem birleşir. Eksi ile çarpılır.



$V_x=?$ $\quad i_y=?$

6 noktadan sadece 2'si alınır.

$$V_1=12-V_x\ ,\quad V_2=-2i_y$$

$$3-\frac{V_x}{3}+\frac{V_1-(V_2-3i_y)}{3}=0$$

$$9-V_x+V_1-V_2+3i_y=0$$

$$9-V_x+12-V_x+2i_y+3i_y=0$$

$$2V_x-5i_y=21$$ 1.denklem

$$4+\frac{V_2-12}{2}-i_y-2V_x+\frac{V_2-3i_y-V_1}{3}=0$$

$$4+\frac{-2i_y-12}{2}-i_y-2V_x+\frac{-2i_y-3i_y-12+V_x}{3}=0$$ 2.denklem

$$\cancel{4}-i_y-6-i_y-2V_x-\frac{5}{3}i_y-\cancel{4}+\frac{V_x}{3}=0 \Rightarrow 5V_x+11i_y=-18$$

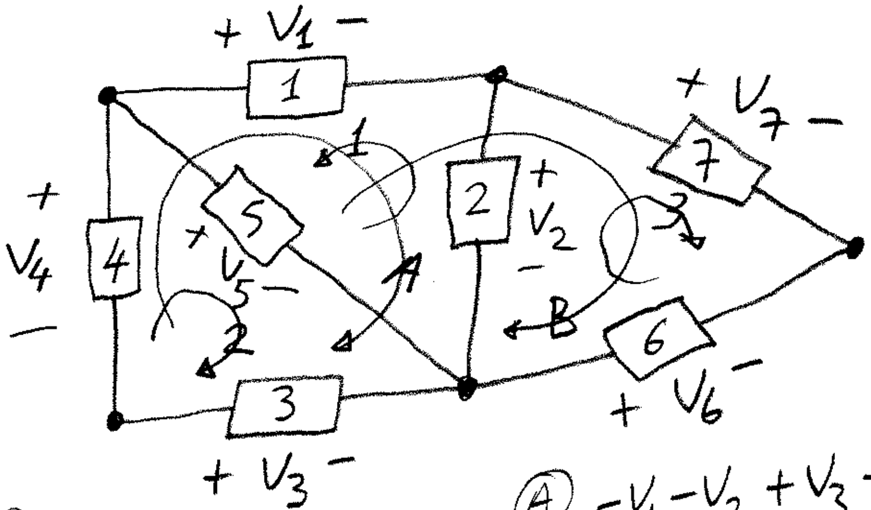
$$\begin{bmatrix}2 & -5 & 21\\ 5 & 11 & -18\end{bmatrix}\sim\begin{bmatrix}2 & -5 & 21\\ 9 & 1 & 24\end{bmatrix}\sim\begin{bmatrix}47 & 0 & 141\\ 9 & 1 & 24\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 0 & 3\\ 9 & 1 & 24\end{bmatrix}\sim\begin{bmatrix}1 & 0 & 3\\ 0 & 1 & -3\end{bmatrix}$$

$V_x=3V$

$i_y=-3A$

Kirchhoff'un Gerilim Yasası (KVL) - Mesh Analiz



Bir göz içinde belirli bir yönde alınan tüm gerilimlerin toplamı sıfırdır.

(1) $V_1 + V_2 - V_5 = 0$

(2) $V_3 + V_4 - V_5 = 0$

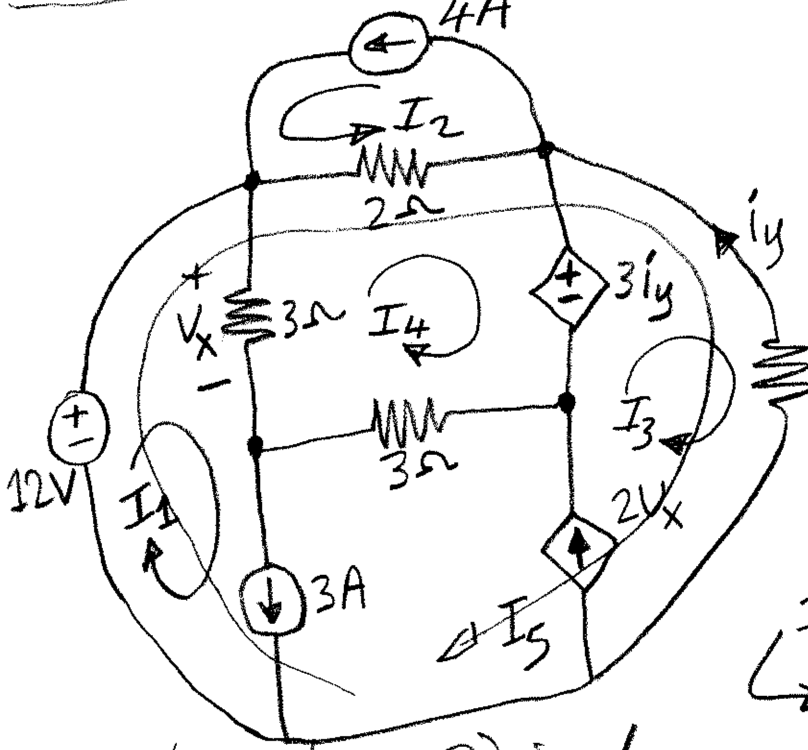
(3) $V_2 + V_6 - V_7 = 0$

(A) $-V_1 - V_2 + V_3 + V_4 = 0$

1 ile 2. denklem birleşir.

(B) $-V_1 + V_5 + V_6 - V_7 = 0$

1 ile 3. denklem birleşir.



$V_x = ?$ $\quad i_y = ?$

$I_1 = 3A \quad I_2 = 4A$

$I_3 = 2V_x$

$I_3 + I_5 = -i_y$

$\to I_5 = -2V_x - i_y$

$I_1 - I_4 = V_x/3$

$\to I_4 = 3 - V_x/3$

5 gözden sadece 2'si alınır. (Akım kaynağı olmayanlar)

$12 - 2(I_2 + I_4 + I_5) + 2i_y = 0$

$12 - 2\left(4 + 3 - \frac{V_x}{3} - 2V_x - i_y\right) + 2i_y = 0 \Rightarrow 7V_x + 6i_y = 3$ (1. denklem)

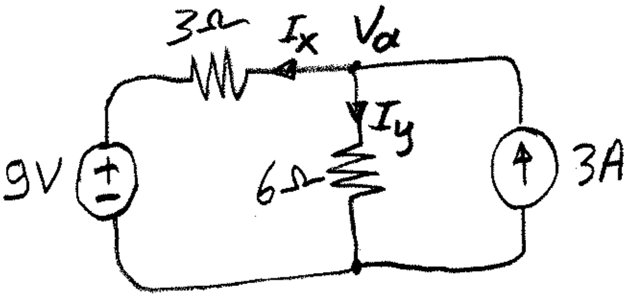
$V_x - 2(I_2 + I_4 + I_5) - 3i_y - 3I_4 = 0 \Rightarrow 20V_x - 3i_y = 69$

$$\begin{bmatrix} 7 & 6 & 3 \\ 20 & -3 & 69 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -3 \end{bmatrix}$$

$V_x = 3V$

$i_y = -3A$

Süper Pozisyon



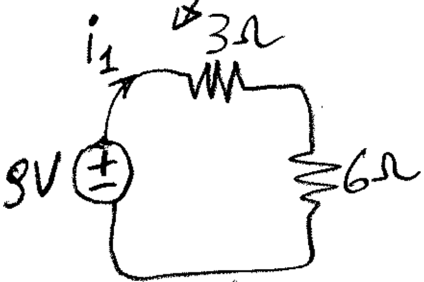
$$\frac{V_a-9}{3}+\frac{V_a}{6}=3 \quad \text{6 ile çarp}$$

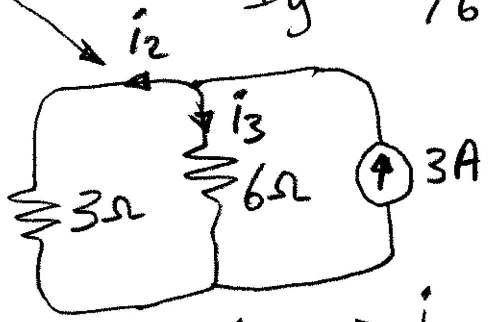
$$2V_a-18+V_a=18$$

$$3V_a=36 \Rightarrow V_a=12V$$

$$I_x=\frac{V_a-9}{3}=1A$$

$$I_y=V_a/6=2A$$





$$i_1=\frac{9V}{3\Omega+6\Omega}=1A$$

$$3i_2=6i_3 \Longrightarrow i_2-2i_3=0$$

$$i_2+i_3=3$$

$$i_2=2A \quad i_3=1A$$

$$I_x=i_2-i_1=2A-1A=1A$$

$$I_y=i_1+i_3=1A+1A=2A$$

Direncin Sıcaklıkla Değişimi

Madde	$\alpha\ (1/°C)$
Bakır	0.0039
Altın	0.0034
Gümüş	0.0038
Silikon	−0.0075
Karbon	−0.0005

$$R_2=R_1\left(1+\alpha(t_2-t_1)\right)$$

α : Direnç Sıcaklık Katsayısı

Cu, Au, Ag elementleri ısıtıldıkça dirençleri artar, Si ve C ise azalır.

Bir telin 20 °C'deki direnci 5Ω, 60 °C'deki direnci ise 6Ω olsun. $\alpha=?$

$t_1=20\ °C \quad t_2=60°C$

$R_1=5\Omega \quad R_2=6\Omega$

$$R_2=R_1\left(1+\alpha(t_2-t_1)\right)$$

$$6=5(1+40\alpha)$$

$$\alpha=\frac{1}{200}=0.005\ 1/°C$$

Bir lambanın direnci 24°C'de 250Ω'dur. Lambanın akkor haline geldiğindeki (yani 1524 °C) direncini bulunuz. Lambadaki telin direnç sıcaklık katsayısı 0.004 1/°C olsun.

$t_1 = 24\,°C \quad t_2 = 1524\,°C \quad \alpha = 0.004\ 1/°C$

$R_1 = 250\,\Omega \quad R_2 = ?$

$$R_2 = R_1\left(1+\alpha(t_2-t_1)\right) = 250\,\Omega \cdot (1+0.004\times 1500) = 1750\,\Omega$$

Direnci 20Ω olan bir ısıtıcı 220V'luk bir gerilime bağlanmıştır. Isıtıcıdan geçen akımı, ısıtıcının gücünü, ısıtıcının 5 dakikada vereceği ısı enerjisini Joule ve Cal cinsinden bulunuz.

$$I = \frac{V}{R} = \frac{220V}{20\Omega} = 11A$$

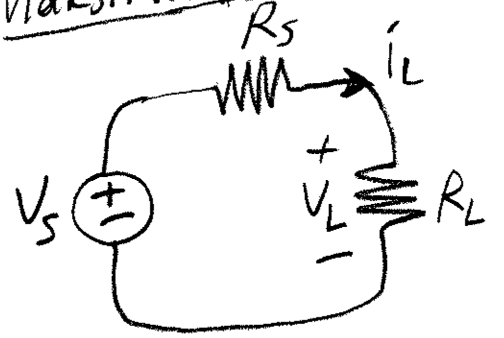
$$P = VI = 220V \times 11A = 2420\,W = 2.42\,kW$$

$$W = P.t = 2420W \times 5 \times 60\,sn = 726000\,J = 726\,kJ$$

1 Joule ⟹ 0.24 Cal

$$W = 726000\,J \times 0.24\,Cal/J = 174240\,Cal = 174.24\,KCal$$

Maksimum Güç Aktarımı



$$i_L = \frac{V_S}{R_S+R_L}$$

$$P_L = R_L i_L^2 = \frac{R_L V_S^2}{(R_S+R_L)^2}$$

$\dfrac{dP_L}{dR_L} = 0$ olunca max. güç aktarımı olur.

$$\frac{dP_L}{dR_L} = \frac{V_S^2(R_S+R_L)^2 - R_L V_S^2 \cdot 2(R_S+R_L)}{(R_S+R_L)^4}$$

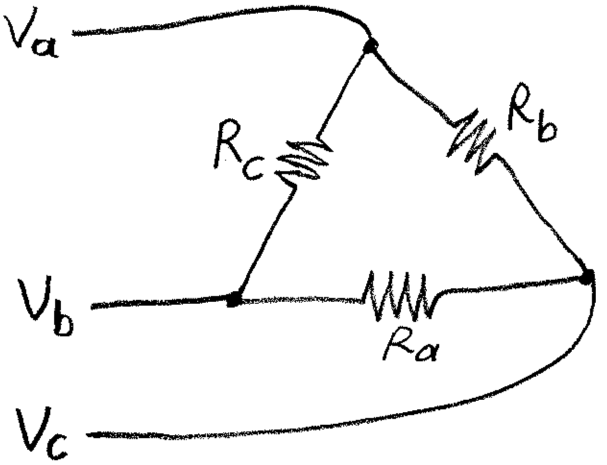
$$= \frac{V_S^2(R_S+R_L) - 2R_L V_S^2}{(R_S+R_L)^3} = \frac{(R_S-R_L)V_S^2}{(R_S+R_L)^3} = 0$$

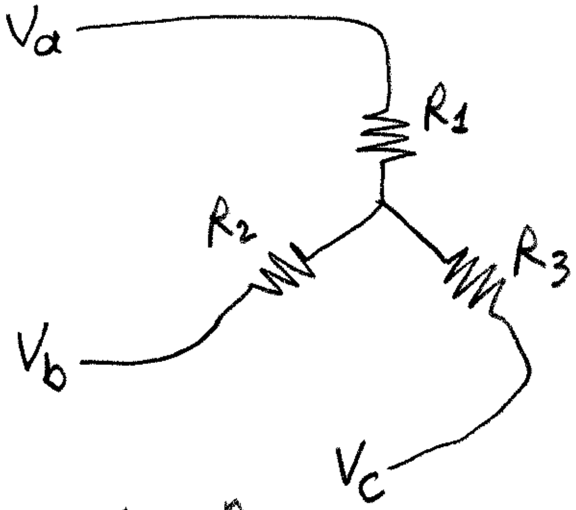
$R_S = R_L$ olunca max. güç aktarımı olur.

$$P_L = R_L i_L^2 \qquad P_T = R_T i_L^2 = 2R_L i_L^2 = 2P_L$$

$$R_T = R_S + R_L = 2R_L \qquad P_L = P_T/2$$

1kWh → 864 kCal





$\Delta \longrightarrow Y$

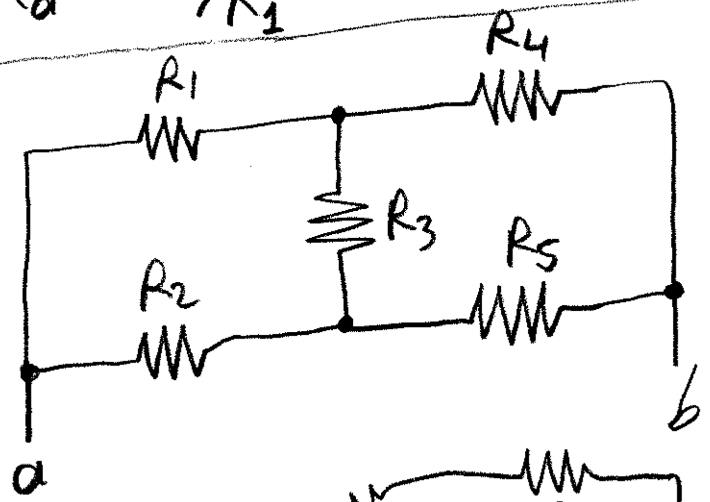
$$R_\Delta = R_a + R_b + R_c$$

$$R_1 = \frac{R_b R_c}{R_\Delta} \qquad R_2 = \frac{R_c R_a}{R_\Delta} \qquad R_3 = \frac{R_a R_b}{R_\Delta}$$

$Y \longrightarrow \Delta$

$$R_Y = R_1R_2 + R_1R_3 + R_2R_3$$

$$R_a = R_Y/R_1 \qquad R_b = R_Y/R_2 \qquad R_c = R_Y/R_3$$

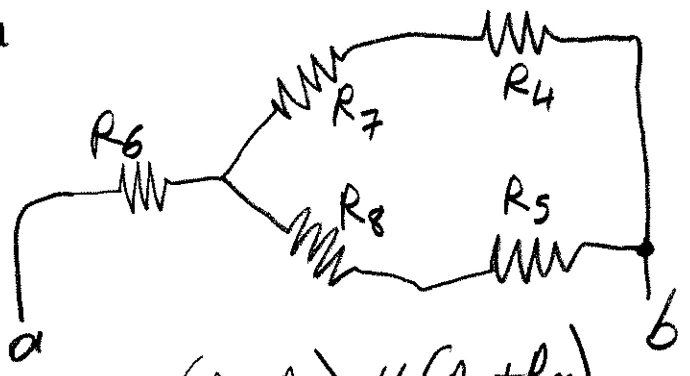


$R_1 = 36\,\Omega$ $\quad R_4 = 9\,\Omega$

$R_2 = 60\,\Omega$ $\quad R_5 = 22\,\Omega$

$R_3 = 48\,\Omega$

$R_{ab} = ?$



$$R_\Delta = R_1 + R_2 + R_3 = 36\,\Omega + 60\,\Omega + 48\,\Omega = 144\,\Omega$$

$$R_6 = \frac{R_1R_2}{R_\Delta} = \frac{36\,\Omega \times 60\,\Omega}{144\,\Omega} = 15\,\Omega$$

$$R_7 = \frac{R_1R_3}{R_\Delta} = \frac{36\,\Omega \times 48\,\Omega}{144\,\Omega} = 12\,\Omega$$

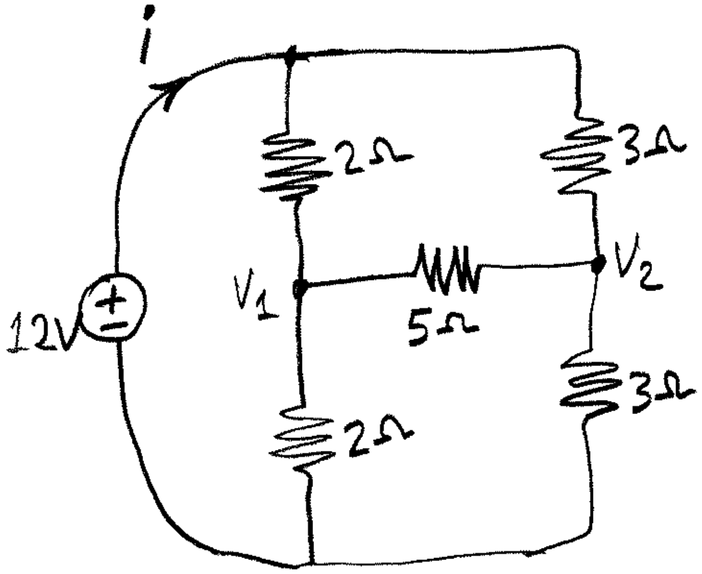
$$R_8 = \frac{R_2R_3}{R_\Delta} = \frac{60\,\Omega \times 48\,\Omega}{144\,\Omega} = 20\,\Omega$$

$$R_{ab} = R_6 + (R_4 + R_7) \,//\, (R_5 + R_8)$$

$$= 15\,\Omega + 21\,\Omega \,//\, 42\,\Omega$$

$$= 15\,\Omega + 14\,\Omega = 29\,\Omega$$

Simetrik Devreler



$$\frac{V_1-V_2}{5}+\frac{V_1}{2}+\frac{V_1-12}{2}=0$$

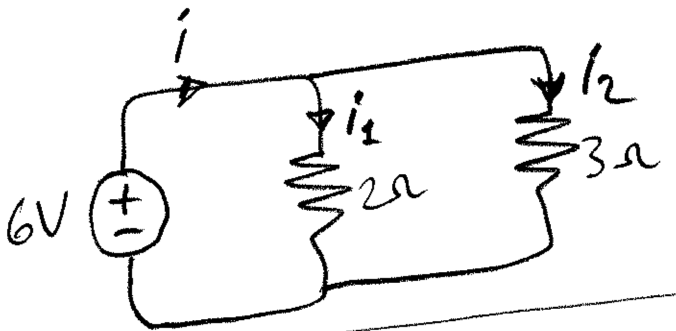
$6V_1-V_2=30$ 1.denklem

$$\frac{V_2-V_1}{5}+\frac{V_2}{3}+\frac{V_2-12}{3}=0$$

$-3V_1+13V_2=60$ 2.denklem

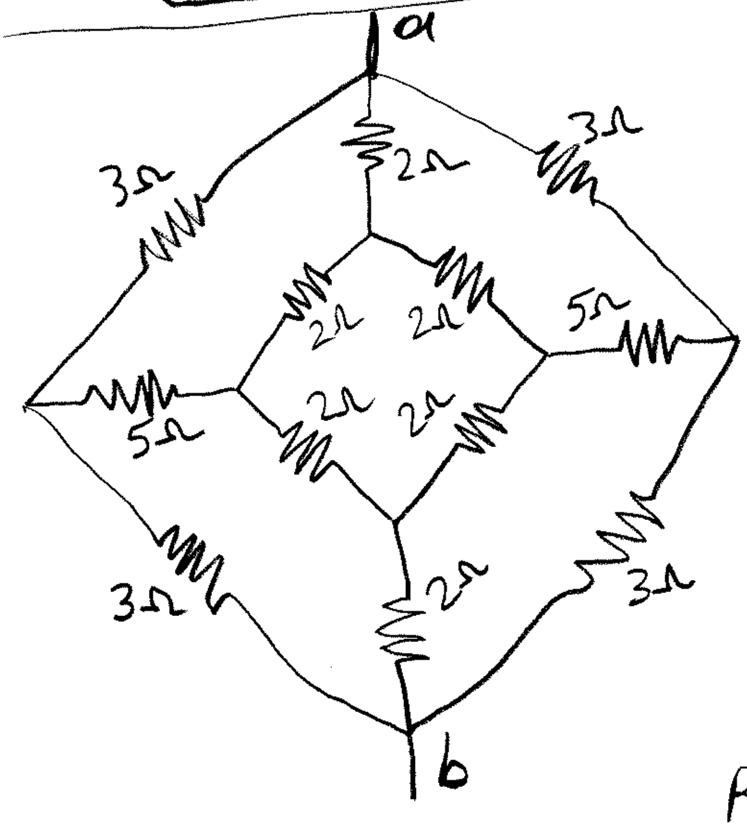
$V_1=V_2=6V$

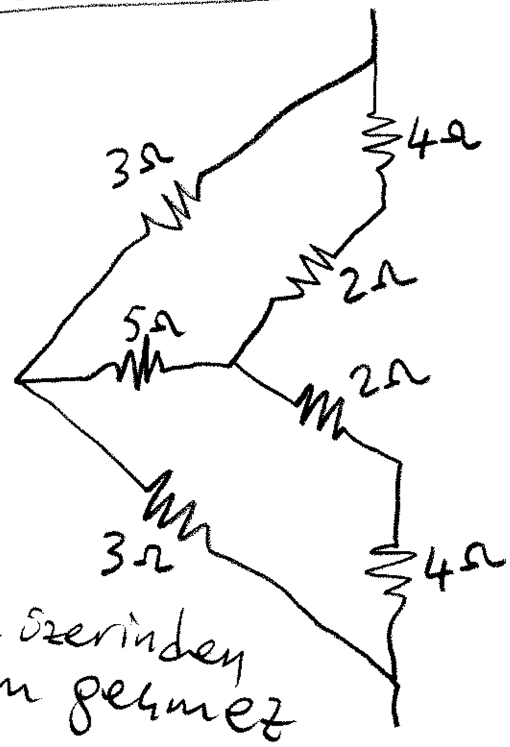
5Ω üzerinden akım geçmez.



$i_1=\frac{6V}{2\Omega}=3A$ $\quad i=i_1+i_2$

$i_2=\frac{6V}{3\Omega}=2A$ $\quad =3A+2A=5A$

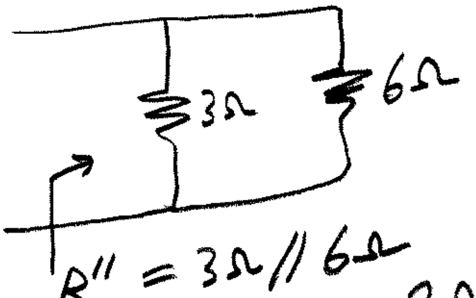




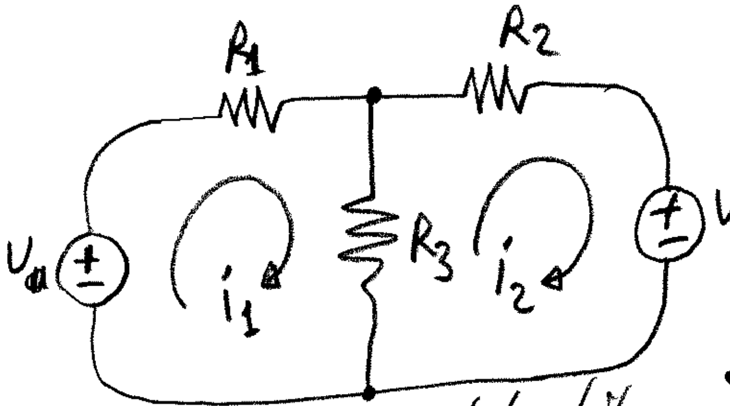
5Ω üzerinden akım geçmez

$R'=2R''=4\Omega$

$R_T=R'/2=2\Omega$



$R''=3\Omega // 6\Omega$
$=2\Omega$



$R_1=2\Omega$ $R_2=3\Omega$ $R_3=5\Omega$ (15)

$V_1=25V$ $V_2=9V$

Devre elemanları üzerlerindeki akımları, voltajları ve güçleri bulunuz.

Üretilen ve tüketilen güçleri bulunuz.

$$(R_1+R_3)i_1-R_3 i_2 = V_a \Rightarrow 7i_1-5i_2=25$$

$$R_3 i_1-(R_2+R_3)i_2 = V_b \Rightarrow 5i_1-8i_2=9$$

$$\begin{bmatrix} 7 & -5 & 25 \\ 5 & -8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \end{bmatrix} \quad \begin{matrix} i_1=5A \\ i_2=2A \end{matrix}$$

$$i_3=i_1-i_2 = 5A-2A = 3A$$

$$V_1=R_1 i_1 = 2\Omega \times 5A = 10V \qquad P_1=V_1 i_1 = 10V\times 5A = 50W$$

$$V_2=R_2 i_2 = 3\Omega \times 2A = 6V \qquad P_2=V_2 i_2 = 6V\times 2A = 12W$$

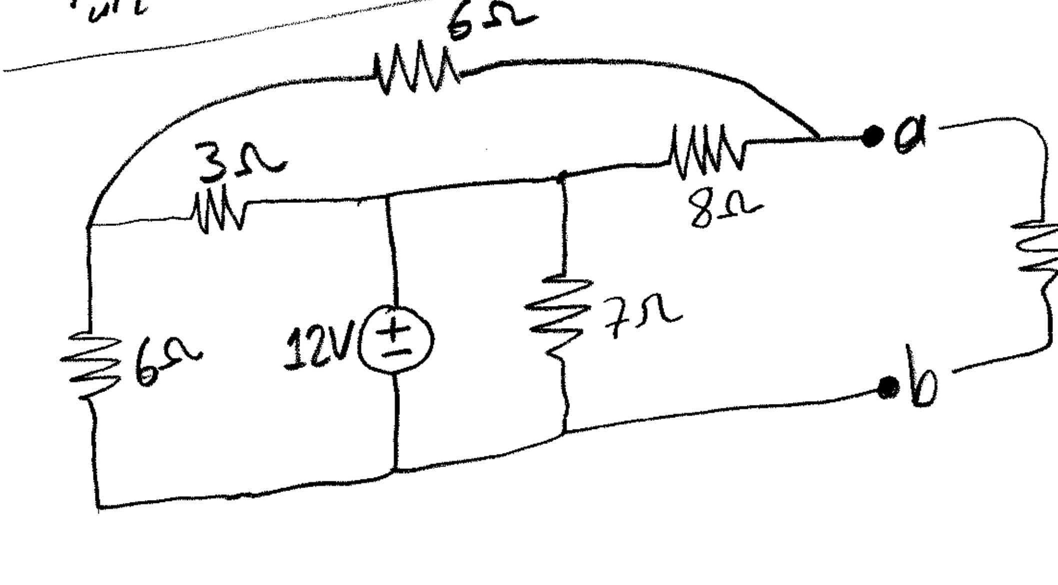
$$V_3=R_3 i_3 = 5\Omega \times 3A = 15V \qquad P_3=V_3 i_3 = 15V\times 3A = 45W$$

$$P_{tük}=P_1+P_2+P_3 = 50W+12W+45W = 107W$$

$P_a = V_a i_1 = 25V\times 5A = 125W$ Üretici konumunda

$P_b = V_b i_2 = 9V\times 2A = 18W$ Tüketici konumunda

$$P_{ürt}=P_a-P_b = 125W-18W = 107W$$

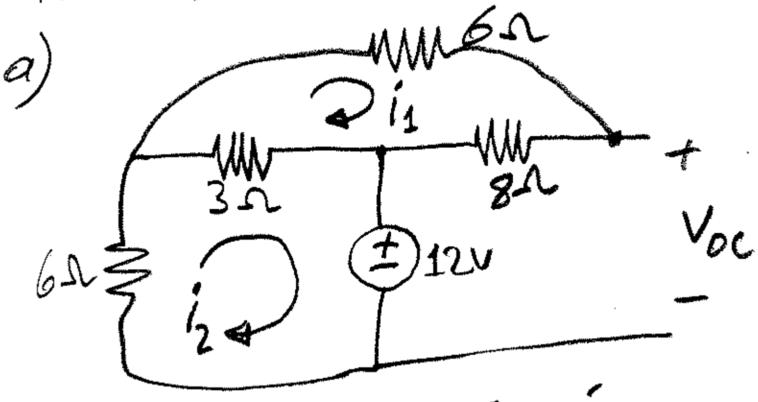


a) R_L direncinin bağlandığı devrenin thevenin ve norton eşdeğer devresini bulunuz.

b) $R_L = 16\Omega$ ise V_L ve i_L bulunuz.

c) R_L lineer değilse yani $i_L = \begin{cases} V_L^2/2 & V_L \geq 0 \\ 0 & V_L < 0 \end{cases}$ ise V_L ve i_L bulunuz.

a)

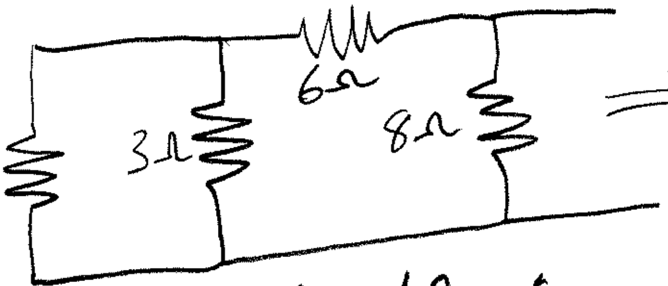


$$17i_1 - 3i_2 = 0$$
$$-3i_1 + 9i_2 = -12$$

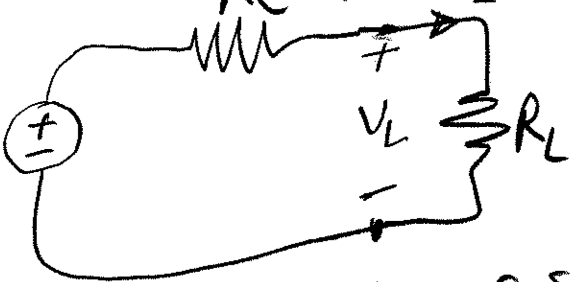
1. denklemin 3 katını 2. denkleme ekle

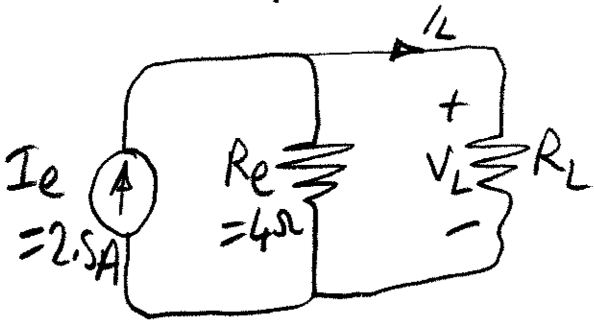
$$48i_1 = -12 \Rightarrow i_1 = -0.25A$$

$$V_{oc} = 12V + 8\Omega \times i_1 = 12V + 8\Omega \times (-0.25A) = 10V = V_e$$



$$\Rightarrow R_e = (6\Omega // 3\Omega + 6\Omega) // 8\Omega$$
$$= 8\Omega // 8\Omega = 4\Omega$$





$$I_e = V_e/R_e = 10V/4\Omega = 2.5A$$

b) R_L lineer

$$i_L = \frac{V_e}{R_e + R_L} = \frac{10V}{4\Omega + 16\Omega} = \frac{10V}{20\Omega} = 0.5A$$

$$V_L = R_L i_L = 16\Omega \times 0.5A = 8V$$

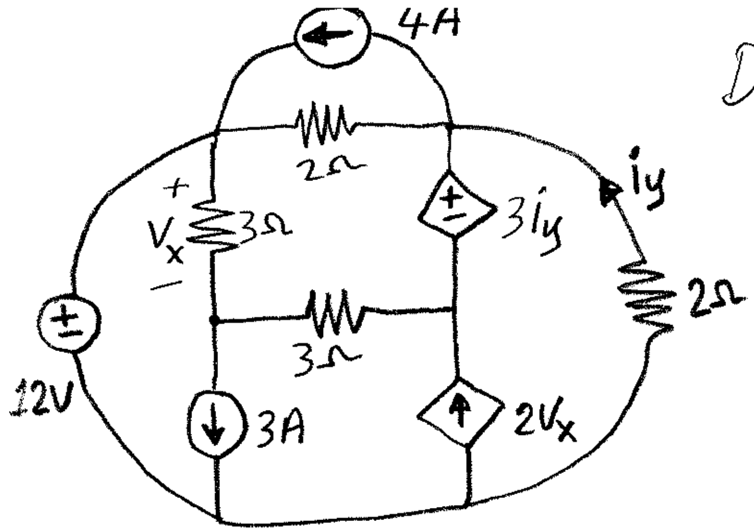
c)

$$V_e = R_e i_L + V_L$$
$$10V = 4\Omega \times 0.5V_L^2 + V_L$$
$$2V_L^2 + V_L - 10 = 0$$
$$(V_L - 2)(2V_L + 5) = 0$$

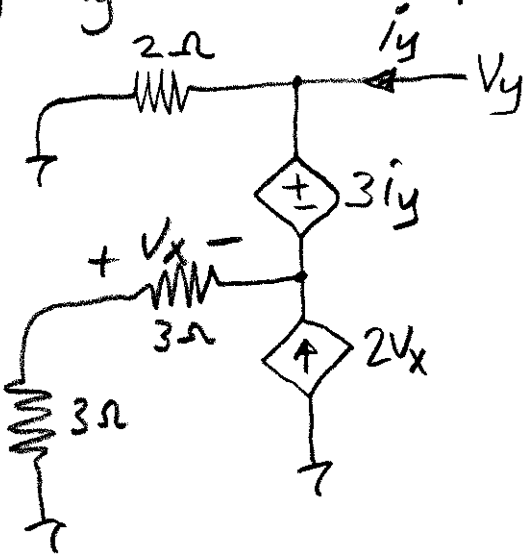
$V_L = 2V$ $\quad \cancel{V_L = -5/2\,V}$

$$i_L = 0.5V_L^2 = 2A$$

$$R_L = V_L/i_L = \frac{2V}{2A} = 1A$$



Devrenin Thevenin eşdeğerini bulup i_y akımını hesaplayınız.

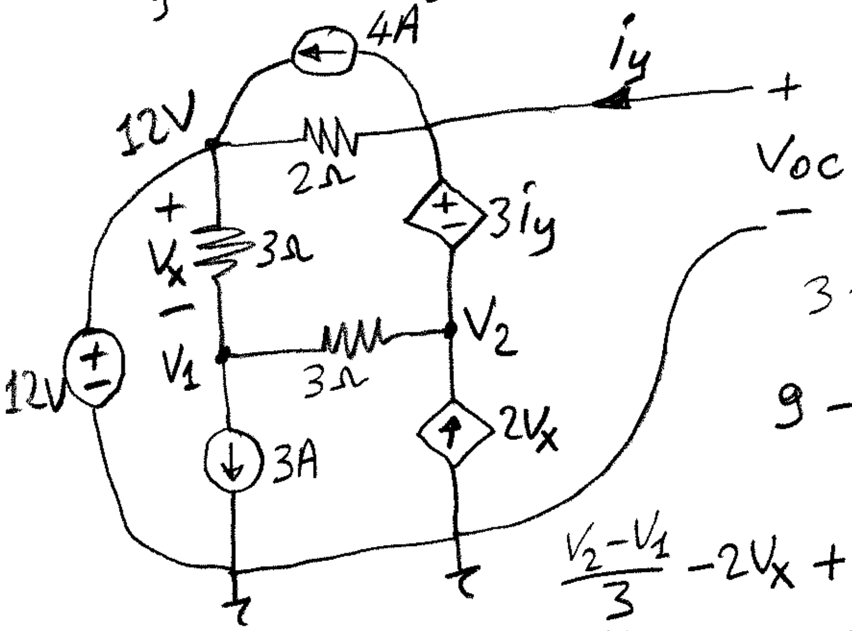


$$\frac{V_y}{2} - i_y - 2V_x - \frac{V_x}{3} = 0$$

$$3V_y - 6i_y - 14V_x = 0 \quad \text{1.denklem}$$

$$V_y - 3i_y + 2V_x = 0 \quad \text{2.denklem}$$

$$V_y = 18V_x\ ,\ i_y = \frac{20}{3}V_x \Rightarrow R_e = \frac{V_y}{i_y} = \frac{18V_x}{\frac{20}{3}V_x} = 2.7\ \Omega$$



$$V_1 = 12 - V_x$$

$$i_y = 0 \Rightarrow V_2 = V_{oc}$$

$$3 - \frac{V_x}{3} + \frac{V_1 - V_2}{3} = 0$$

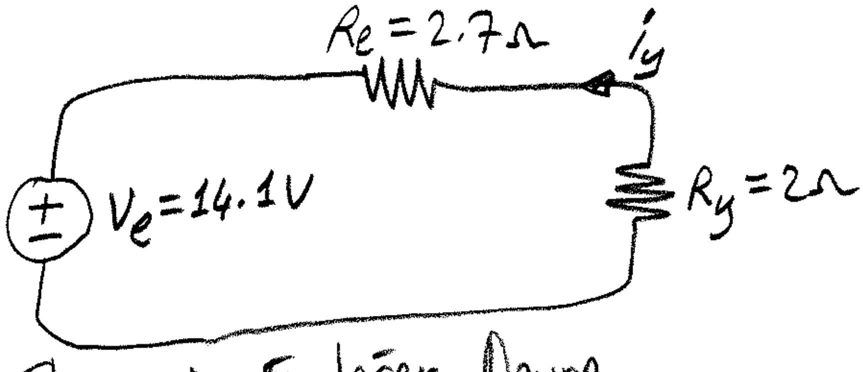
$$9 - V_x + 12 - V_x - V_{oc} = 0$$

$$2V_x + V_{oc} = 21 \quad \text{1.denklem}$$

$$\frac{V_2 - V_1}{3} - 2V_x + 4 + \frac{V_{oc} - 12}{2} = 0$$

$$\frac{V_{oc} - 12 + V_x}{3} - 2V_x + 4 + \frac{V_{oc}}{2} - 6 = 0 \Rightarrow 5V_{oc} - 10V_x = 36 \quad \text{2.denklem}$$

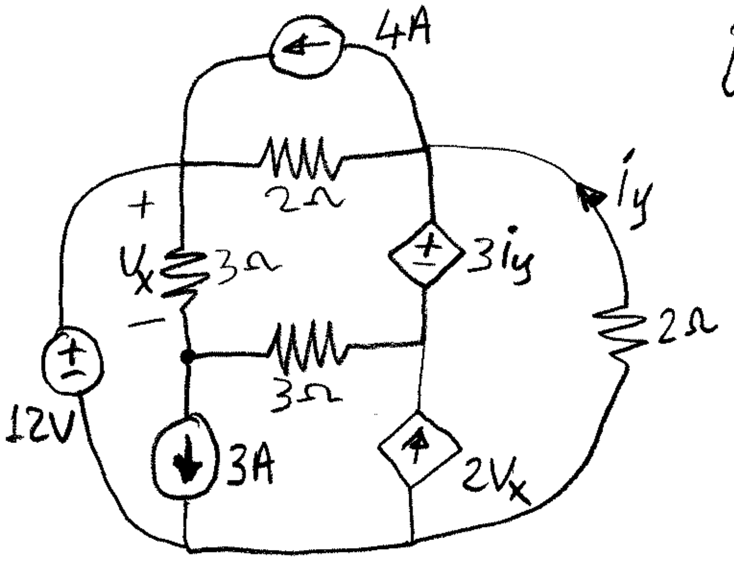
Denklemler çözülünce $V_{oc} = 14.1V = V_e$

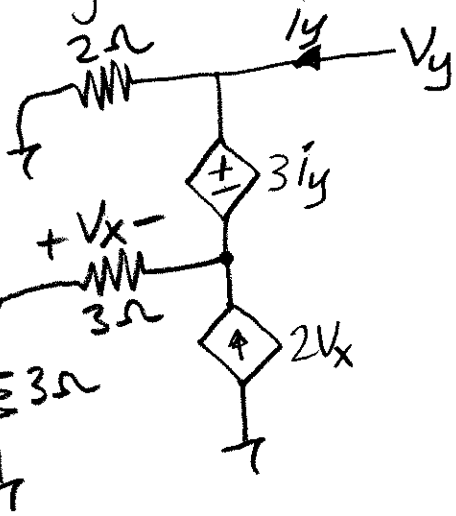


Thevenin Eşdeğer Devre

$$i_y = -\frac{V_e}{R_e + R_y} = -\frac{14.1V}{2.7\Omega + 2\Omega} = -\frac{14.1V}{4.7\Omega} = -3A$$

Devrenin Norton eşdeğerini bulup i_y akımını hesaplayınız.



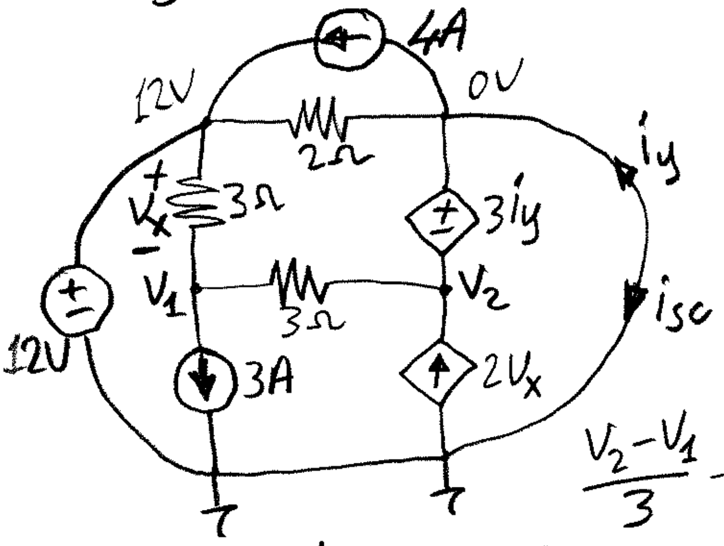


$$\frac{V_y}{2} - i_y - 2V_x - \frac{V_x}{3} = 0$$

$3V_y - 6i_y - 14V_x = 0$ 1. denklem

$V_y - 3i_y + 2V_x = 0$ 2. denklem

$$V_y = 18V_x,\ i_y = \frac{20}{3}V_x \Rightarrow R_e = \frac{V_y}{i_y} = \frac{18V_x}{\frac{20}{3}V_x} = 2.7\,\Omega$$



$V_1 = 12 - V_x$, $V_2 = -3i_y$

$$3 - \frac{V_x}{3} + \frac{V_1 - V_2}{3} = 0$$

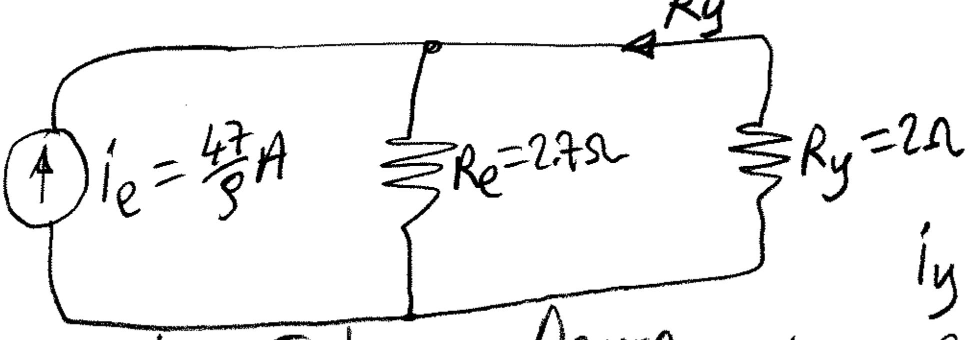
$9 - V_x + 12 - V_x + 3i_y = 0$

$2V_x - 3i_y = 21$ 1. denklem

$$\frac{V_2 - V_1}{3} - 2V_x - i_y + 4 - \frac{12}{2} = 0$$

$\frac{-3i_y - 12 + V_x}{3} - 2V_x - i_y - 2 = 0 \Rightarrow 5V_x + 6i_y = -18$ 2. denklem

Denklemler çözülürse $V_x = \frac{8}{3}V$, $i_y = -\frac{47}{9}A \Rightarrow i_{sc} = -i_y = \frac{47}{9}A$



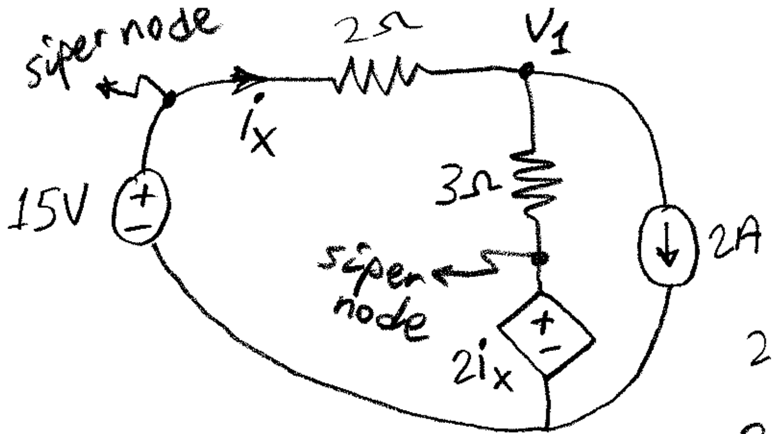
Norton Eşdeğer Devre

$R_e(i_e + i_y) + R_y i_y = 0$

$R_e i_e = -(R_e + R_y) i_y$

$$i_y = -\frac{R_e i_e}{R_e + R_y}$$

$$i_y = -\frac{2.7}{4.7}\frac{47}{9}A = -3A$$

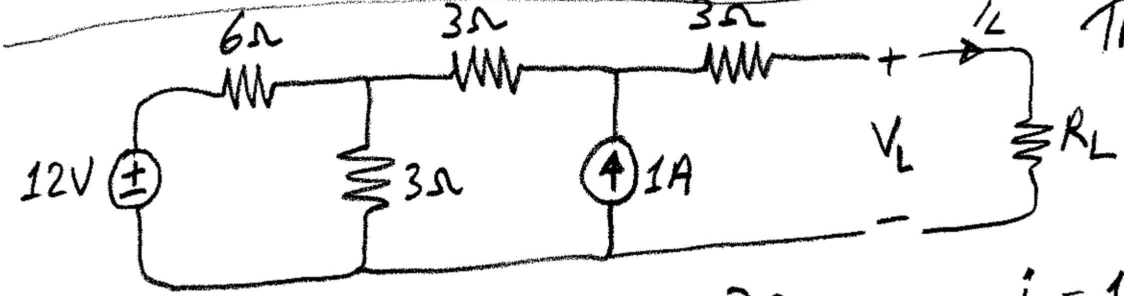


$V_1 = 15 - 2i_x$

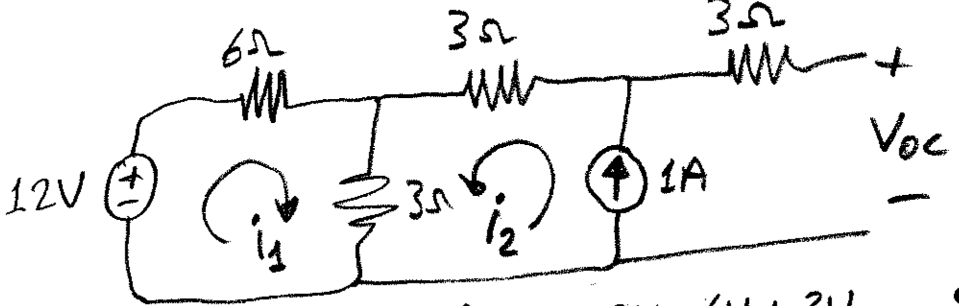
$2 - i_x + \frac{V_1 - 2i_x}{3} = 0$

$2 - i_x + \frac{15 - 2i_x - 2i_x}{3} = 0$

$21 - 7i_x = 0 \Rightarrow i_x = 3A$



Thevenin ve Norton eşdeğerini bulup $R_L = 10$ için $V_L = ?$

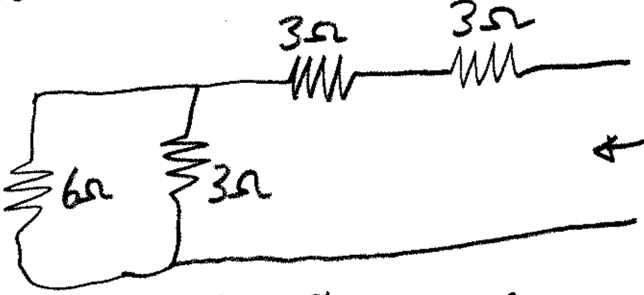


$i_2 = 1A$

$9i_1 + 3i_2 = 12$

$i_2 = \frac{12 - 3i_2}{9} = 1A$

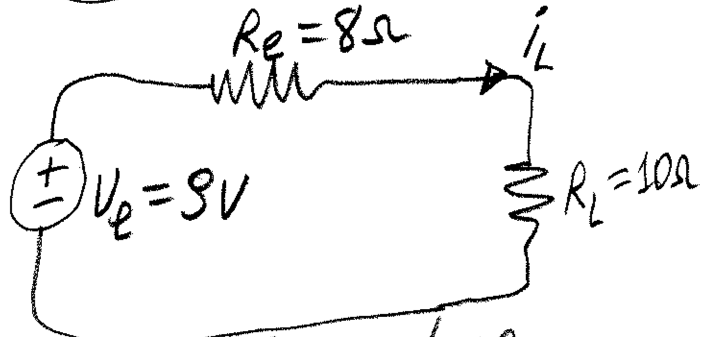
$V_{oc} = 12 - 6i_1 + 3i_2 = 12V - 6V + 3V = 9V = V_e$



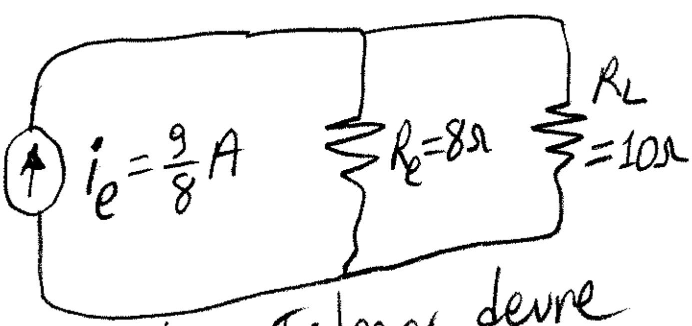
$R_e = 6\Omega // 3\Omega + 3\Omega + 3\Omega$

$= 2\Omega + 3\Omega + 3\Omega$

$= 8\Omega$



Thevenin Eşdeğer devre

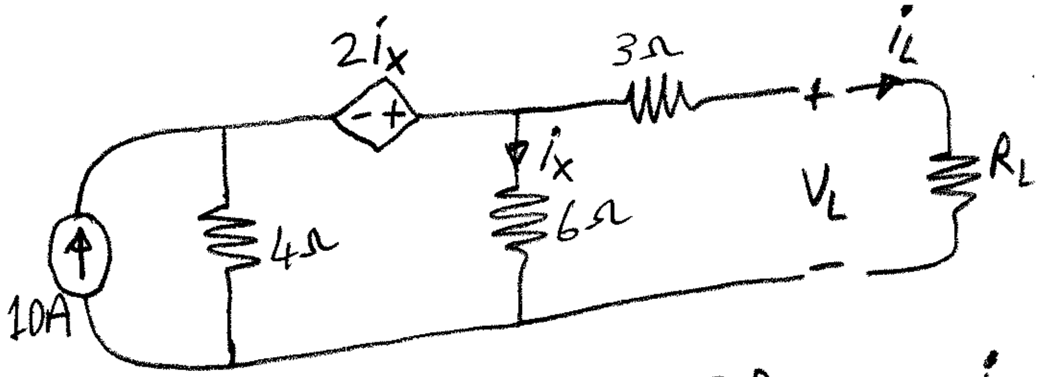


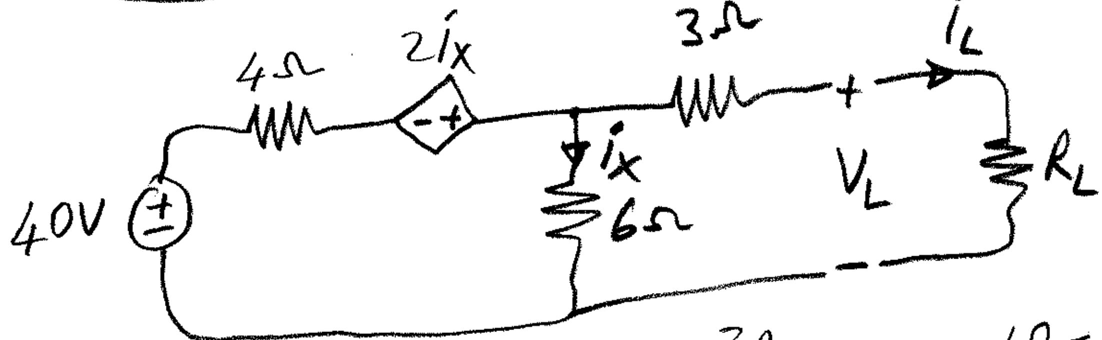
Norton Eşdeğer devre

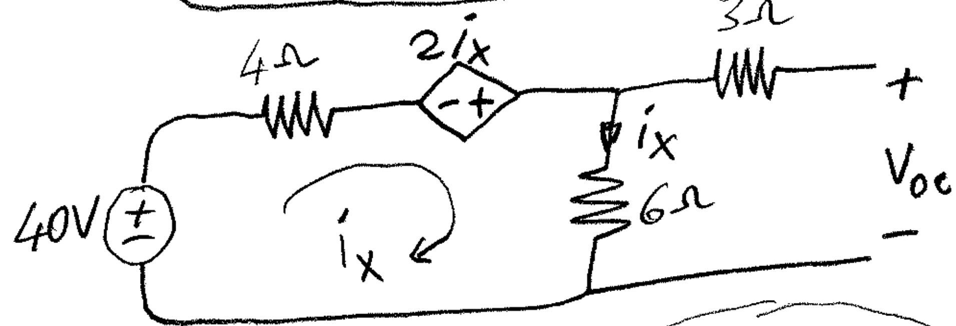
$i_L = \frac{V_e}{R_e + R_L} = \frac{9V}{8\Omega + 10\Omega} = \frac{9V}{18\Omega} = 0.5A$

$V_L = R_L i_L = 10\Omega \times 0.5A = 5V$

Thevenin ve Norton Eşdeğerini bulup $R_L = 4\Omega$ için $V_L = ?$



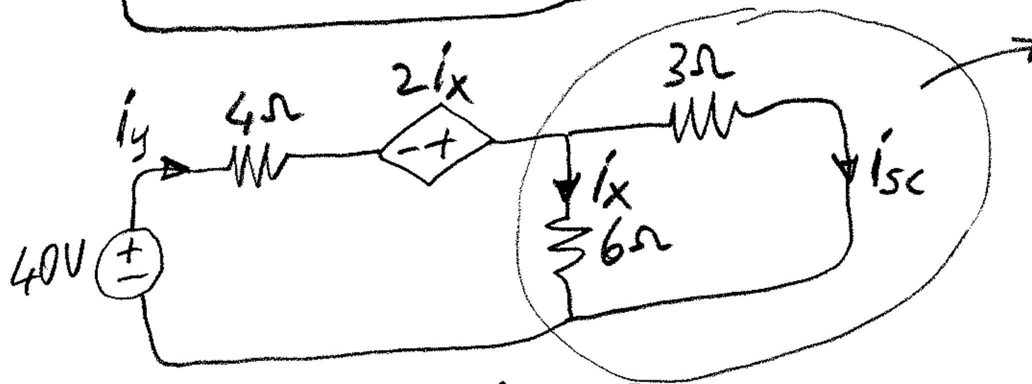




$40 - 10i_x + 2i_x = 0$

$8i_x = 40 \Rightarrow i_x = 5A$

$V_{oc} = 6i_x = 30V$



$\rightarrow 3\Omega // 6\Omega = 2\Omega$

$6i_x = 3i_{sc}$

$i_{sc} = 2i_x$

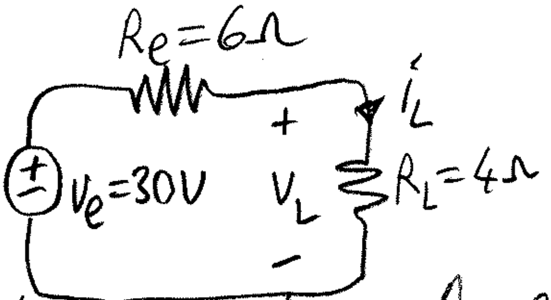
$i_y = i_x + i_{sc}$

$= 3i_x$

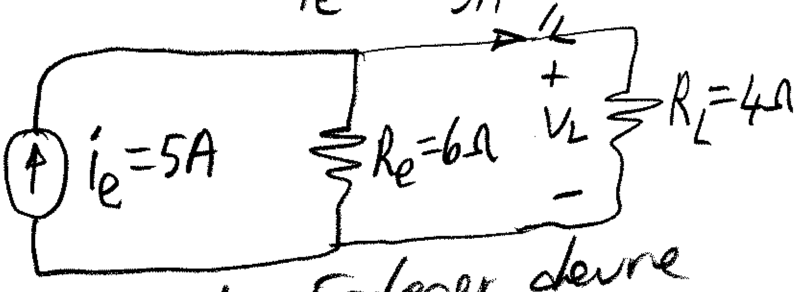
$40 - 4i_y + 2i_x - 6i_x = 0$

$40 - 12i_x + 2i_x - 6i_x = 0 \Rightarrow i_x = 2.5A \Rightarrow i_{sc} = 2i_x = 5A$

$V_e = V_{oc} = 30V, \quad i_e = i_{sc} = 5A, \quad R_e = \frac{V_e}{i_e} = \frac{30V}{5A} = 6\Omega$

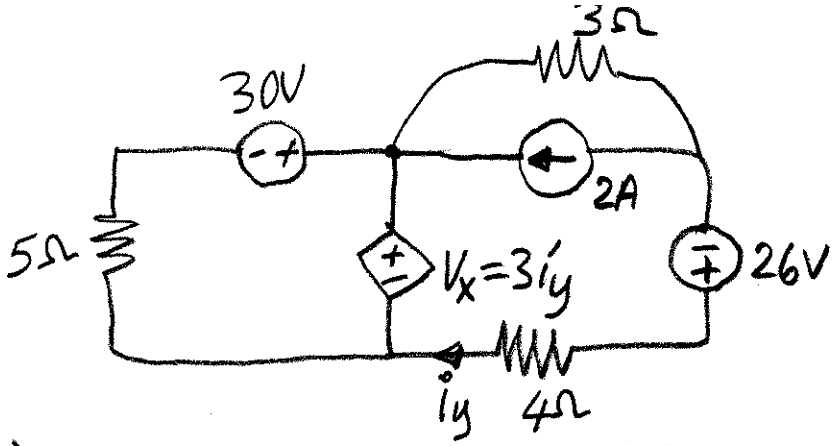


Thevenin Eşdeğer devre



Norton Eşdeğer devre

$i_L = \frac{V_e}{R_e + R_L} = \frac{30V}{6\Omega + 4\Omega} = \frac{30V}{10\Omega} = 3A, \quad V_L = R_L i_L = 12V$

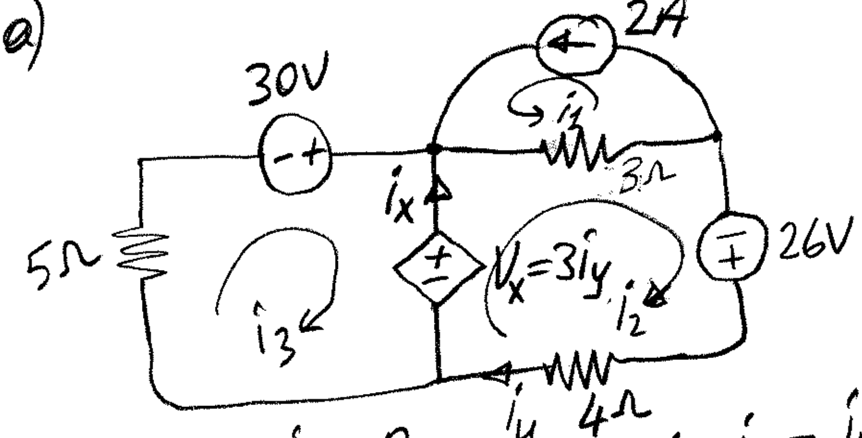


$i_y = ?\ ,\ V_x = ?\ ,\ P_x = ?$

a) Mesh Analiz (KVL) ile

b) Node Analiz (KCL) ile

a)



$i_1 = 2A\ ,\quad i_2 = i_y$

$V_x - 3i_1 - 7i_2 + 26 = 0$

$3i_y - 6 - 7i_y + 26 = 0$

$i_y = 5A \Rightarrow V_x = 15V$

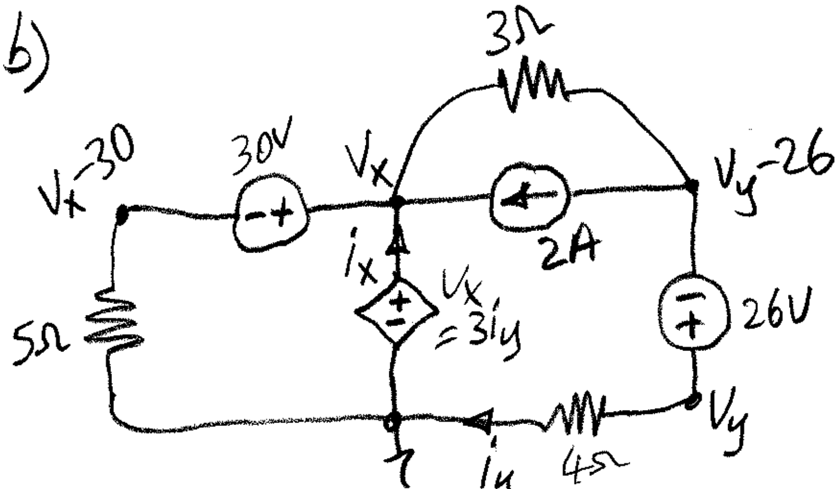
$i_x = i_2 - i_3 = i_y - i_3 = 5A - 3A = 2A$

$30 - V_x - 5i_3 = 0$

$i_3 = \dfrac{30 - V_x}{5} = 3A$

$P_x = V_x i_x = 15V \times 2A = 30W$ Üretici konumunda

b)



$V_x = 3i_y\ ,\quad V_y = 4i_y$

$i_x = i_y + \dfrac{V_x - 30}{5}$

$= 1.6\, i_y - 6$

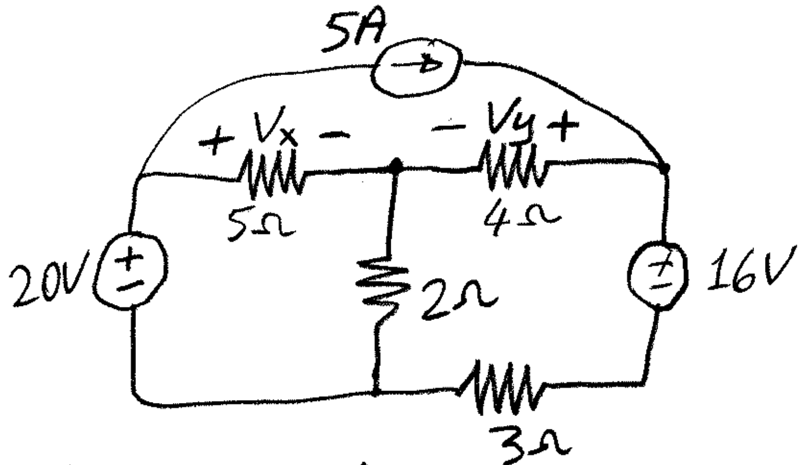
$$\frac{V_x - 30}{5} - i_x - 2 + \frac{V_x - V_y + 26}{3} = 0$$

$$\frac{3i_y - 30}{5} - 1.6 i_y + 6 - 2 + \frac{3i_y - 4i_y + 26}{3} = 0 \Rightarrow i_y = 5A$$

$V_x = 3i_y = 3\Omega \times 5A = 15V$

$i_x = 1.6 i_y - 6 = 1.6 \times 5A - 6A = 8A - 6A = 2A$

$P_x = V_x i_x = 15V \times 2A = 30W$ Üretici konumunda

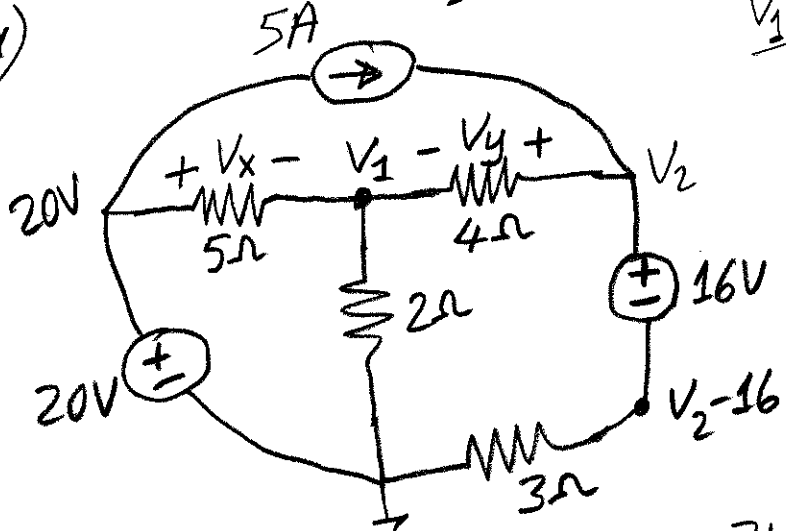


$V_x = ?$ $V_y = ?$

a) Nodal Analiz (KCL) ile

b) Mesh Analiz (KVL) ile

a)



$$\frac{V_1-20}{5}+\frac{V_1}{2}+\frac{V_1-V_2}{4}=0$$

20 ile çarp.

$$4V_1-80+10V_1+5V_1-5V_2=0$$

$$19V_1-5V_2=80 \quad \text{1.denklem}$$

$$\frac{V_2-V_1}{4}-5+\frac{V_2-16}{3}=0$$

$$3V_2-3V_1-60+4V_2-64=0$$

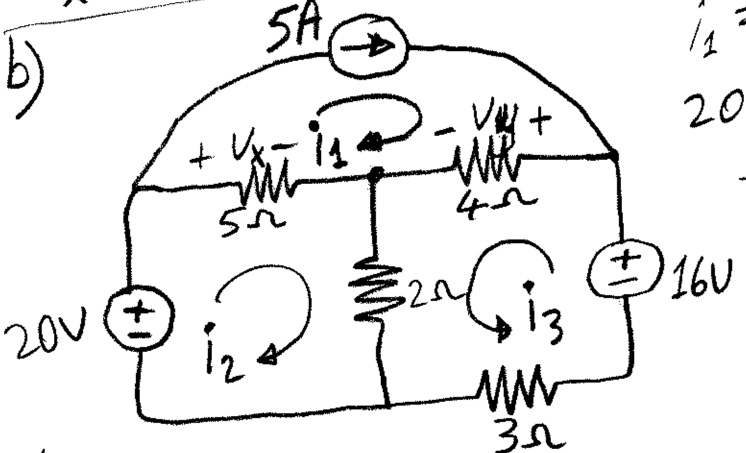
$$7V_2-3V_1=124 \quad \text{2.denklem}$$

denklemler çözülünce

$V_1 = 10V$, $V_2 = 22V$

$V_x = 20 - V_1 = 10V$ $\quad V_y = V_2 - V_1 = 12V$

b)



$i_1 = 5A$

$$20+5i_1-7i_2-2i_3=0$$

$$7i_2+2i_3=45 \quad \text{1.denklem}$$

$$16-4i_1-2i_2-9i_3=0$$

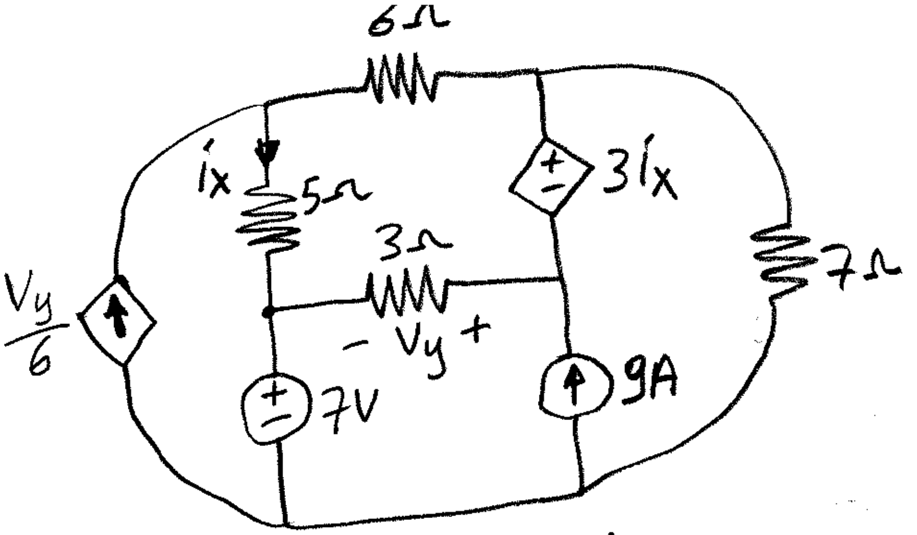
$$2i_2+9i_3=-4 \quad \text{2.denklem}$$

denklemler çözülünce

$i_2 = 7A$, $i_3 = -2A$

$$V_x = 5\Omega(i_2-i_1) = 5\Omega(7A-5A) = 5\Omega \times 2A = 10V$$

$$V_y = 4\Omega(i_1+i_3) = 4\Omega(5A-2A) = 4\Omega \times 3\Omega = 12V$$

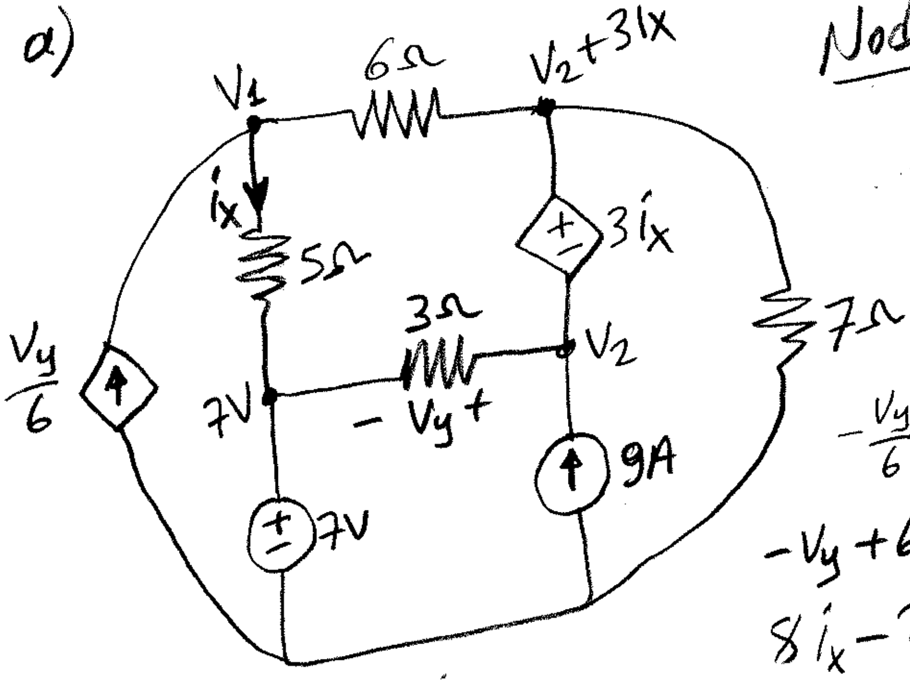


$i_x = ?$ $V_y = ?$

a) Nodal Analiz ile

b) Mesh Analiz ile

a)



Nodal Analiz

$$V_1 = 7 + 5i_x$$

$$V_2 = 7 + V_y$$

$$-\frac{V_y}{6} + i_x + \frac{V_1 - V_2 - 3i_x}{6} = 0$$

$$-V_y + 6i_x + 7 + 5i_x - 7 - V_y - 3i_x = 0$$

$$8i_x - 2V_y = 0 \Rightarrow V_y = 4i_x$$

$$\frac{V_y}{3} - 9 + \frac{V_2 + 3i_x}{7} + \frac{V_2 + 3i_x - V_1}{6} = 0$$

$$\frac{V_y}{3} - 9 + \frac{7 + V_y + 3i_x}{7} + \frac{\cancel{7} + V_y + 3i_x - \cancel{7} - 5i_x}{6} = 0$$

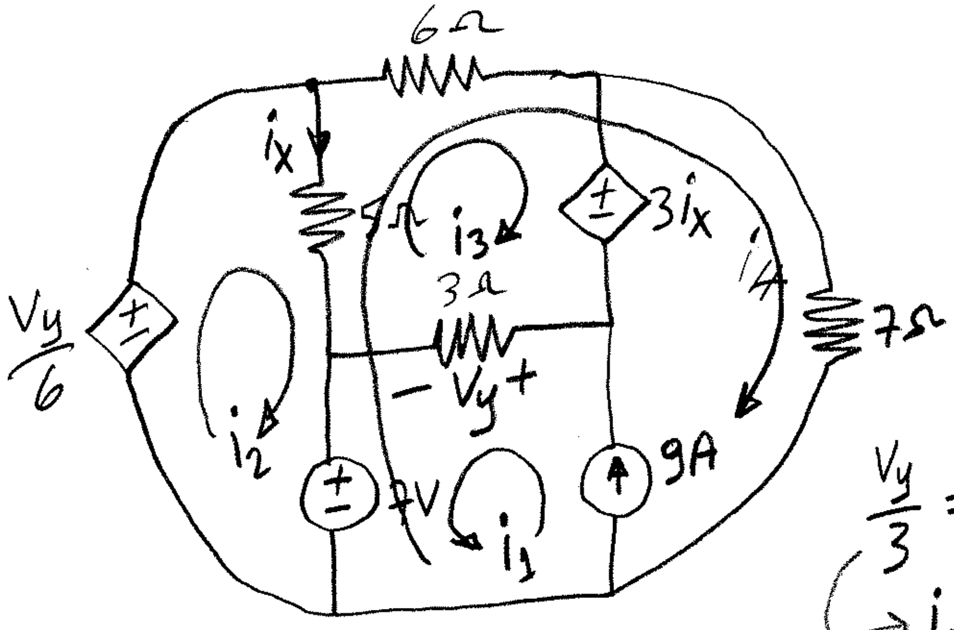
$$\frac{V_y}{3} + \frac{V_y + 3i_x}{7} + \frac{V_y - 2i_x}{6} = 8$$

$$\frac{4i_x}{3} + \frac{4i_x + 3i_x}{7} + \frac{4i_x - 2i_x}{6} = 8$$

$$\frac{4}{3}i_x + i_x + \frac{i_x}{3} = 8 \Rightarrow \frac{8i_x}{3} = 8 \Rightarrow i_x = 3A$$

$$V_y = 4i_x = 4\Omega \times 3A = 12V$$

b)



Mesh Analiz

$$i_1 = 9A$$

$$i_2 = V_y/6$$

$$\frac{V_y}{3} = i_1 + i_3$$

$$\hookrightarrow i_3 = \frac{V_y}{3} - 9$$

$$i_x = i_2 - i_3 - i_4$$

$$\hookrightarrow i_4 = i_2 - i_3 - i_x = \frac{V_y}{6} - \left(\frac{V_y}{3} - 9\right) - i_x = 9 - i_x - V_y/6$$

$$3i_x + 6(i_3 + i_4) - 5i_x + V_y = 0$$

$$3i_x + 6\left(\frac{V_y}{3} \cancel{-9} + \cancel{9} - i_x - \frac{V_y}{6}\right) - 5i_x + V_y = 0$$

$$3i_x + 2V_y - 6i_x - V_y - 5i_x + V_y = 0$$

$$2V_y - 8i_x = 0 \Rightarrow V_y = 4i_x$$

$$7 + 5i_x - 6i_3 - 13i_4 = 0$$

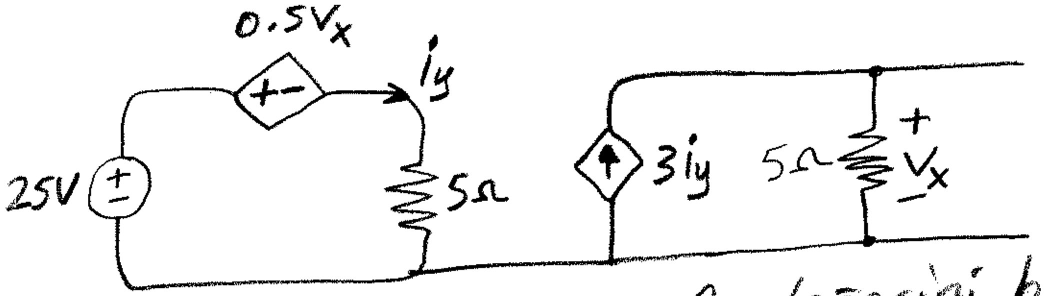
$$7 + 5i_x - 6\left(\frac{V_y}{3} - 9\right) - 13\left(9 - i_x - \frac{V_y}{6}\right) = 0$$

$$7 + 5i_x - 2V_y + 54 - 117 + 13i_x + \frac{13}{6}V_y = 0$$

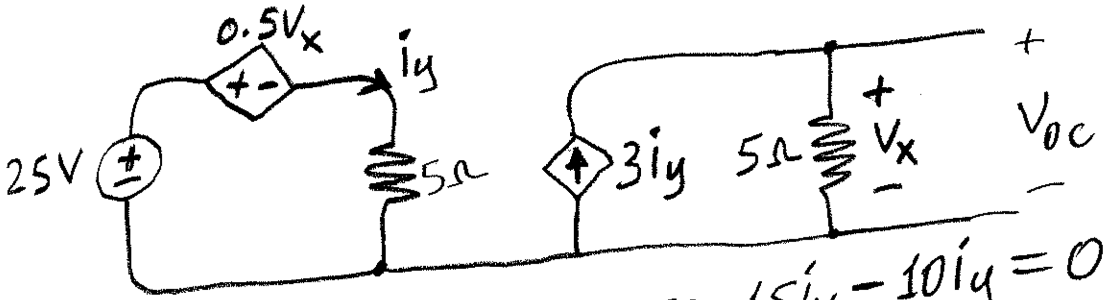
$$18i_x + V_y/6 = 56$$

$$18i_x + \frac{4i_x}{6} = 56 \Rightarrow \frac{56}{3}i_x = 56 \Rightarrow i_x = 3A$$

$$V_x = 4i_x = 4\Omega \times 3A = 12V$$



Devrenin Thevenin eşdeğerini bulup uçlarına R_L direnci bağlayınız. $V_L = i_L^2 + 4i_L + 3$ için R_L değerini bulunuz.



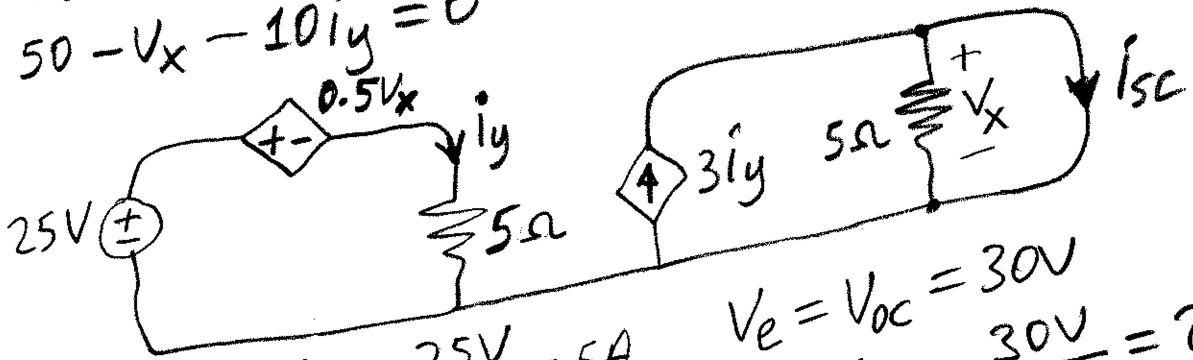
$V_x = 15 i_y$

$25 - 0.5V_x - 5i_y = 0$

$50 - V_x - 10i_y = 0$

$\rightarrow 50 - 15i_y - 10i_y = 0$

$i_y = 2A, \quad V_{oc} = V_x = 15i_y = 30V$

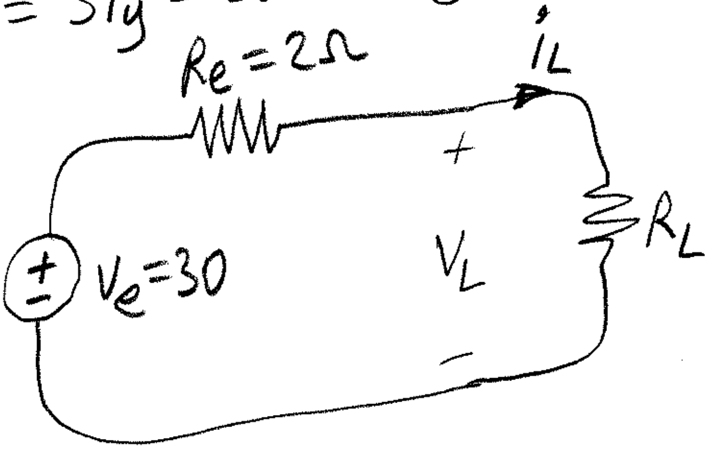


$V_x = 0 \Rightarrow i_y = \dfrac{25V}{5\Omega} = 5A$

$i_{sc} = 3i_y = 15A = i_e$

$V_e = V_{oc} = 30V$

$R_e = \dfrac{V_e}{i_e} = \dfrac{30V}{15A} = 2\Omega$



$V_e - R_e i_L - V_L = 0$

$30 - 2i_L - i_L^2 - 4i_L - 3 = 0$

$i_L^2 + 6i_L - 27 = 0$

$(i_L - 3)(i_L + 9) = 0$

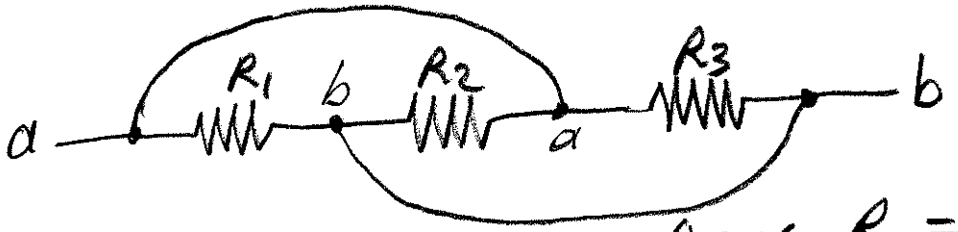
$i_L = 3A \qquad \cancel{i_L = -9A}$

$i_L > 0$ olmalı

$V_L = i_L^2 + 4i_L + 3$

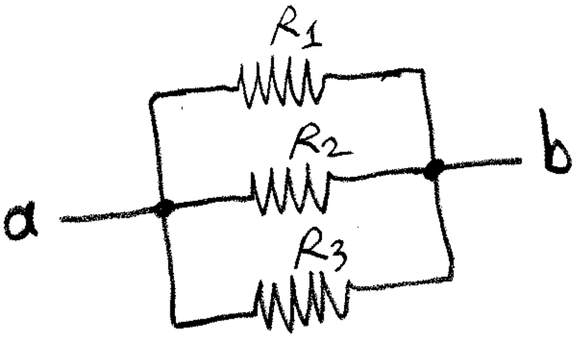
$= 9 + 12 + 3 = 24V$

$R_L = \dfrac{V_L}{i_L} = \dfrac{24V}{3A} = 8\Omega$



$R_1 = 20\Omega$
$R_2 = 12\Omega$
$R_3 = 15\Omega$

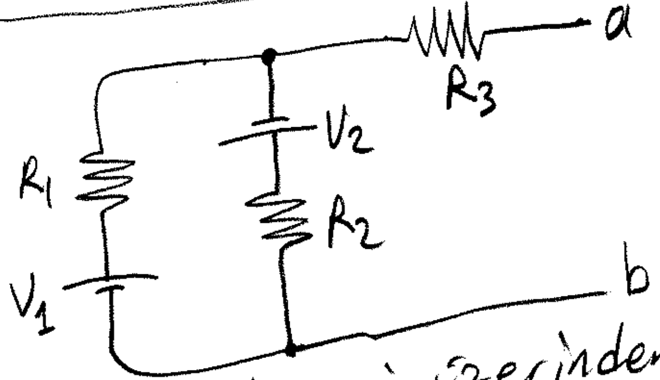
ab uçları arasındaki direnç $R_T = ?$



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

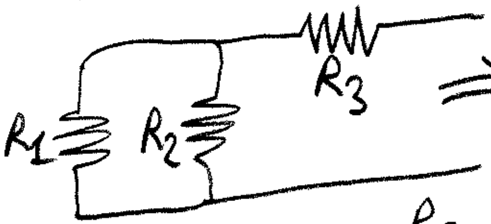
$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$R_T = \frac{20\times 12\times 15}{20\times 12 + 20\times 15 + 12\times 15}\,\Omega = \frac{360\cancel{0}}{24\cancel{0}+30\cancel{0}+18\cancel{0}}\,\Omega = 5\Omega$$

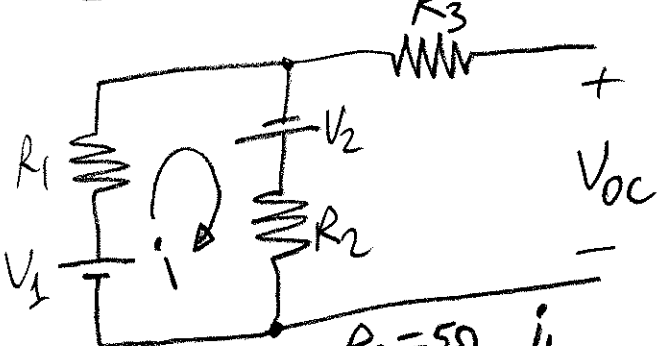


$R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 3\Omega$
$V_1 = 15V$, $V_2 = 12V$

Devrenin Thevenin eşdeğerini bulup uçlarına R_L direnci bağla. R_L direnci üzerinden geçen akım $i_L = 0.5A$ ise R_L ve V_L değerlerini hesapla.

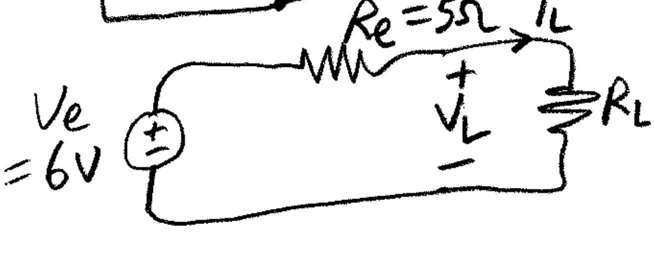


$$\Rightarrow R_e = R_1 // R_2 + R_3 = 3\Omega // 6\Omega + 3\Omega$$
$$= 2\Omega + 3\Omega = 5\Omega$$



$$i = \frac{V_1+V_2}{R_1+R_2} = \frac{15V+12V}{3\Omega+6\Omega}$$
$$= \frac{27V}{9\Omega} = 3A$$

$$V_{oc} = V_1 - R_1 i = 15V - 3\Omega\times 3A = \overset{\overset{V_e}{\parallel}}{6V}$$

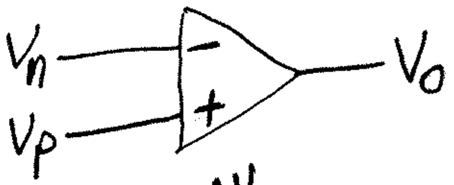


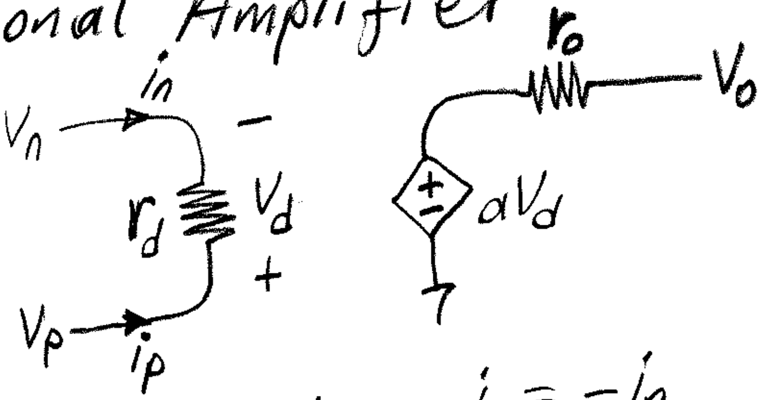
$$i_L = \frac{V_e}{R_e+R_L} = \frac{6V}{5\Omega+R_L} = 0.5A$$

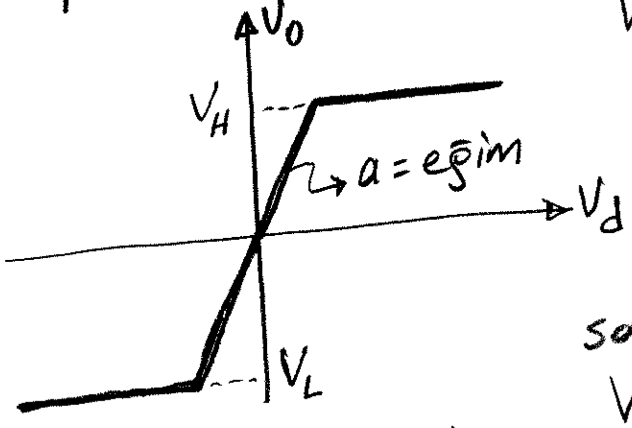
$R_L = 7\Omega$ $V_L = R_L i_L = 3.5V$

İşlemsel Kuvvetlendiriciler

Opamp : Operational Amplifier







$V_d = V_p - V_n$, $i_p = -i_n$

Opamp $\mp 15V$ ile beslenirse saturasyon voltajları

$V_H \approx 15V - 2V = 13V$

$V_L \approx -15V + 2V = -13V$

$a = 200\,000$ için

$\frac{13V}{a} \approx 65\,\mu V$

Artı uç çıkışa verilmez. Verilirse osilasyon yapar. Yani; kararsız durum oluşur.

ideal Opamp

$r_d \to \infty$

$r_o \to 0$

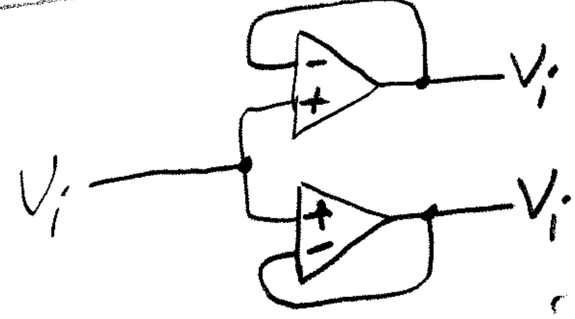
$a \to \infty$

r_d sonsuz ise üzerinden akım geçmez. $i_p = -i_n = 0$ olur.

Yani $V_p = V_n$ olur.

a sonsuz ve r_o sıfır ise $V_d = \frac{V_o}{a} = 0$

Voltaj Takip Edici



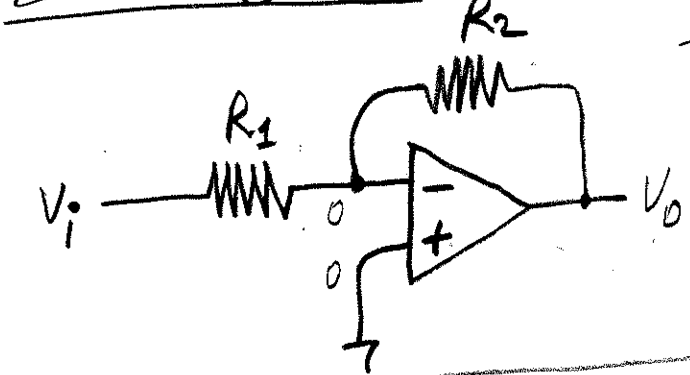
$R_i = \infty$ Giriş direnci

$R_o = 0$ Çıkış direnci

$A = 1$ Yükseltme Oranı

Kaynağın iç direnci varsa iç direncini sıfıra çekerek ideal kaynak oluşturur.

Eviren Devre



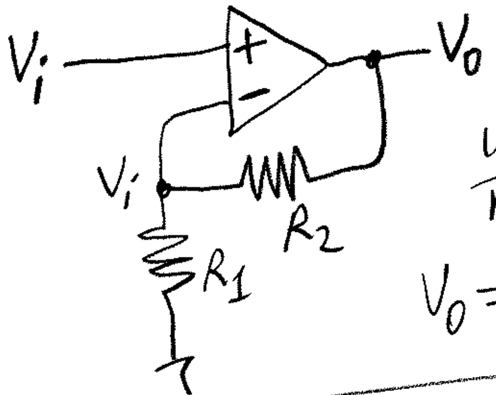
$$\frac{V_i}{R_1}+\frac{V_0}{R_2}=0 \Rightarrow V_0=-\frac{R_2}{R_1}V_i$$

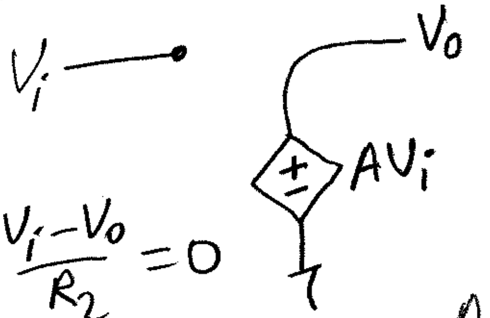
$R_i=R_1$

$R_0=0$

$A=-R_2/R_1$

Evirmeyen Opamp





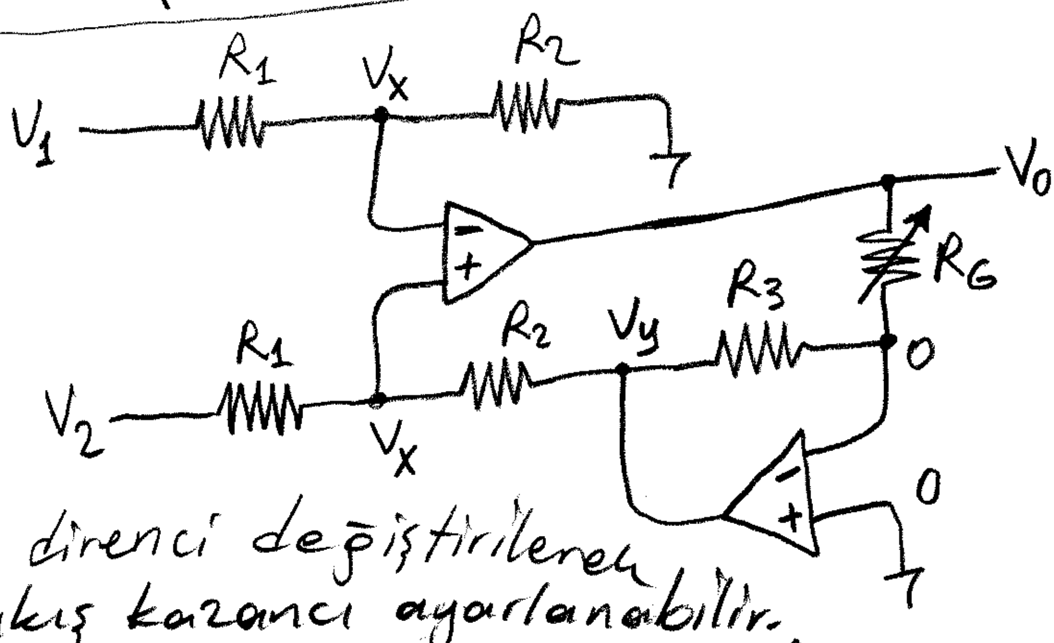
$$\frac{V_i}{R_1}+\frac{V_i-V_0}{R_2}=0$$

$$V_0=\left(1+\frac{R_2}{R_1}\right)V_i$$

$R_i=\infty$

$R_0=0$

$A=1+R_2/R_1$



Çözüm için 3 nokta alınır.

R_6 direnci değiştirilerek çıkış kazancı ayarlanabilir.

$$\frac{V_x-V_1}{R_1}+\frac{V_x}{R_2}=0 \Rightarrow V_x=\frac{R_2}{R_1+R_2}V_1$$

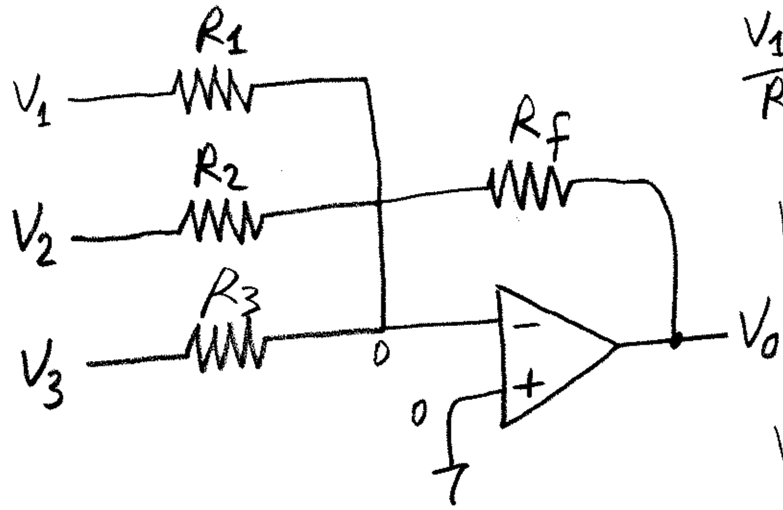
$$\frac{V_x-V_2}{R_1}+\frac{V_x-V_y}{R_2}=0 \Rightarrow V_y=\left(1+\frac{R_2}{R_1}\right)V_x-\frac{R_2}{R_1}V_2=\frac{R_2}{R_1}(V_1-V_2)$$

$$\frac{V_y}{R_3}+\frac{V_0}{R_6}=0 \Rightarrow V_0=-\frac{R_6}{R_3}V_y=\frac{R_2R_6}{R_1R_3}(V_2-V_1)$$

→ Kazanç

Aradaki farkı yükseltir.

Eviren Toplayıcı Devre



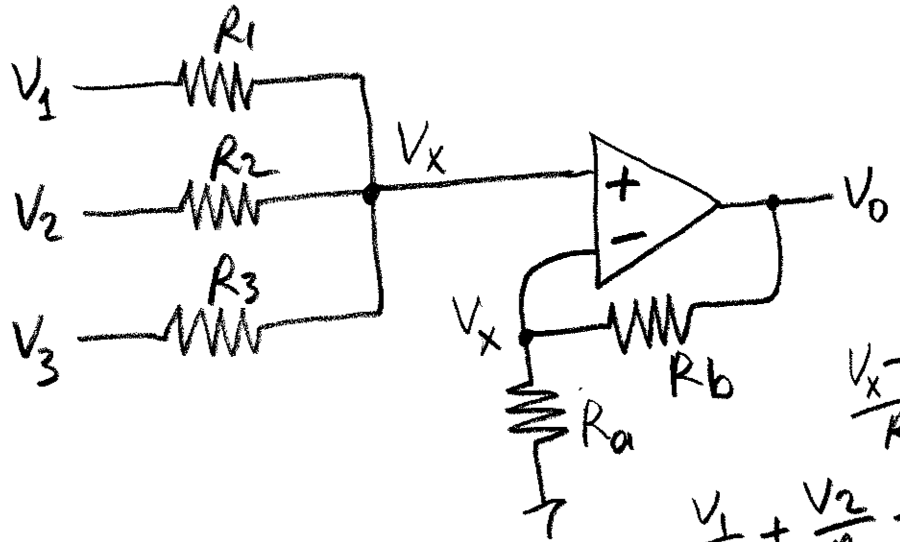
$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_0}{R_f} = 0$$

$$V_0 = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

$R_1 = R_2 = R_3 = R$ alınırsa

$$V_0 = -\frac{R_f}{R}\left(V_1 + V_2 + V_3\right)$$

Evirmeyen Toplayıcı Devre



$$\frac{V_x}{R_a} + \frac{V_x - V_0}{R_b} = 0$$

$$V_0 = \left(1 + \frac{R_b}{R_a}\right) V_x$$

$$\frac{V_x - V_1}{R_1} + \frac{V_x - V_2}{R_2} + \frac{V_x - V_3}{R_3} = 0$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = V_x\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$R_2R_3 \quad R_1R_3 \quad R_1R_2$

$$V_x = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

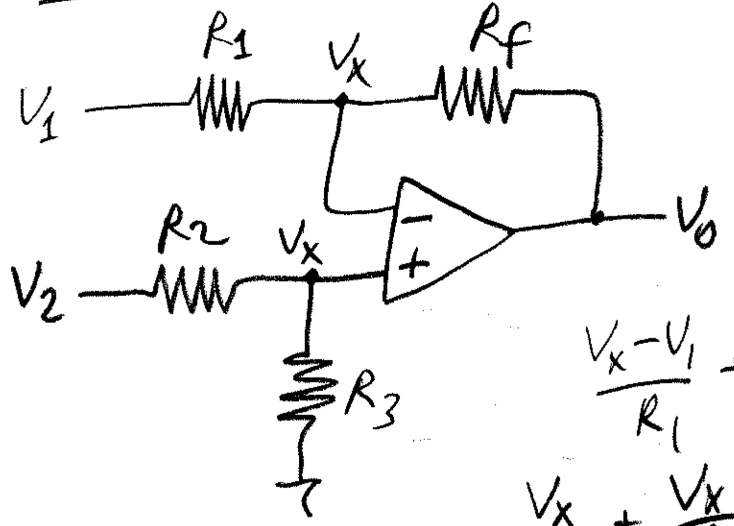
$$V_0 = \left(1 + \frac{R_b}{R_a}\right)V_x = \left(1 + \frac{R_b}{R_a}\right)\frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

$R_1 = R_2 = R_3 = R$ alınırsa

$$V_0 = \left(1 + \frac{R_b}{R_a}\right)\frac{R^3}{3R^2}\,\frac{1}{R}\left(V_1 + V_2 + V_3\right)$$

$$V_0 = \left(1 + \frac{R_b}{R_a}\right)\frac{V_1 + V_2 + V_3}{3}$$

Çıkarıcı Devre



$$\frac{V_x - V_2}{R_2} + \frac{V_x}{R_3} = 0$$

$$V_x = \frac{R_3}{R_2 + R_3} V_2$$

$$\frac{V_x - V_1}{R_1} + \frac{V_x - V_0}{R_f} = 0$$

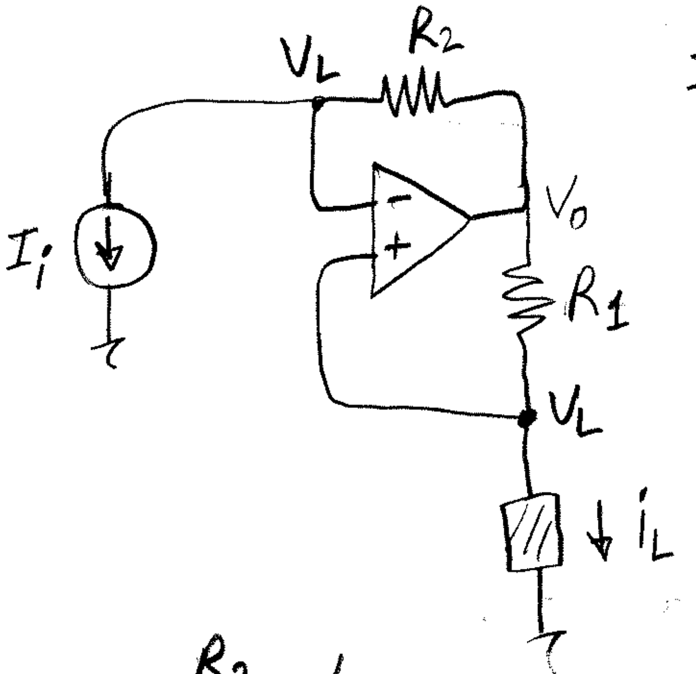
$$\frac{V_x}{R_1} + \frac{V_x}{R_f} = \frac{V_1}{R_1} + \frac{V_0}{R_f}$$

$$\frac{V_1}{R_1} + \frac{V_0}{R_f} = \left(\frac{1}{R_1} + \frac{1}{R_f}\right) V_x = \left(\frac{1}{R_1} + \frac{1}{R_f}\right) \frac{R_3}{R_2 + R_3} V_2$$

$$V_0 = \left(1 + \frac{R_f}{R_1}\right) \frac{R_3}{R_2 + R_3} V_2 - \frac{R_f}{R_1} V_1$$

$\frac{R_f}{R_1} = \frac{R_3}{R_1} = K$ olursa $V_0 = \frac{R_f}{R_1}(V_2 - V_1)$

Akım Yükselticiler



$$I_i + \frac{V_L - V_0}{R_2} = 0$$

$$V_L = V_0 - R_2 I_i$$

$$i_L + \frac{V_L - V_0}{R_1} = 0$$

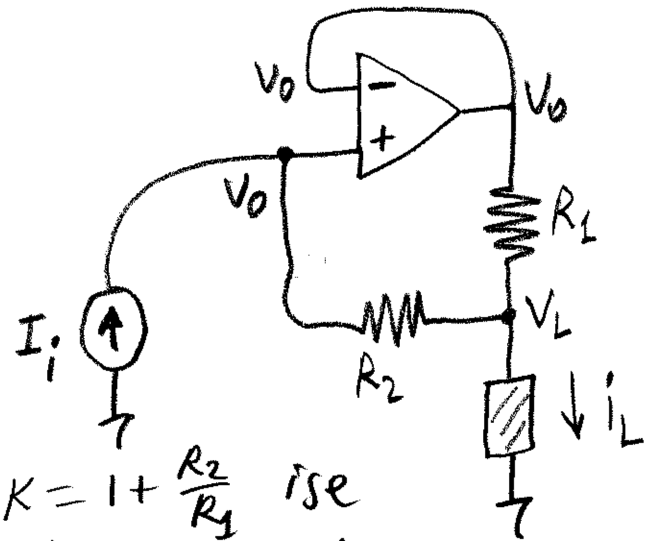
$$i_L = \frac{V_0 - V_L}{R_1}$$

$$i_L = \frac{V_0 - (V_0 - R_2 I_i)}{R_1}$$

$$i_L = \frac{R_2}{R_1} I_i$$

$K = \frac{R_2}{R_1}$ karany ise

$i_L = K I_i$ olur.



$$I_i = \frac{V_0 - V_L}{R_2} \Rightarrow V_L = V_0 - R_2 I_i$$

$$i_L = I_i + \frac{V_0 - V_L}{R_1}$$

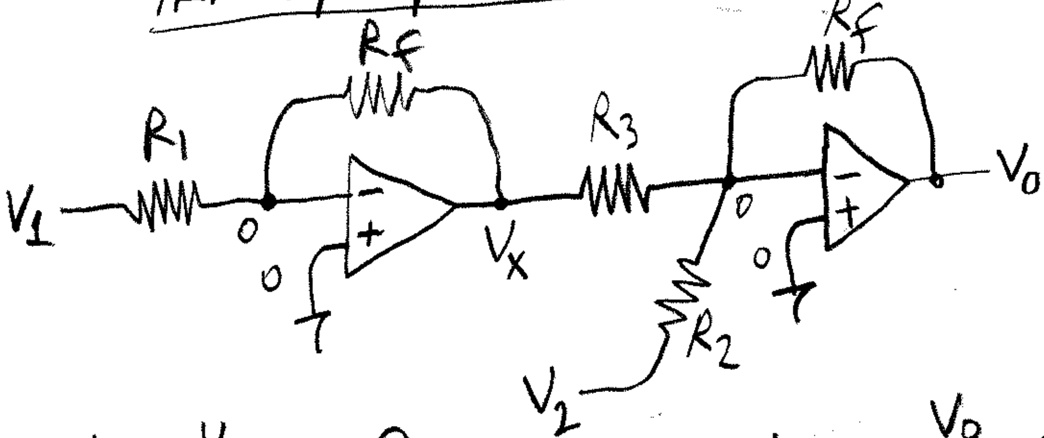
$$i_L = I_i + \frac{V_0 - (V_0 - R_2 I_i)}{R_1}$$

$$i_L = \left(1 + \frac{R_2}{R_1}\right) I_i$$

$K = 1 + \frac{R_2}{R_1}$ ise

$i_L = K I_i$ olur.

İki Opamplı Çıkarıcı Devre



2 nokta seçilir.

$$\frac{V_1}{R_1} + \frac{V_x}{R_f} = 0 \qquad \frac{V_x}{R_3} + \frac{V_2}{R_2} + \frac{V_0}{R_f} = 0$$

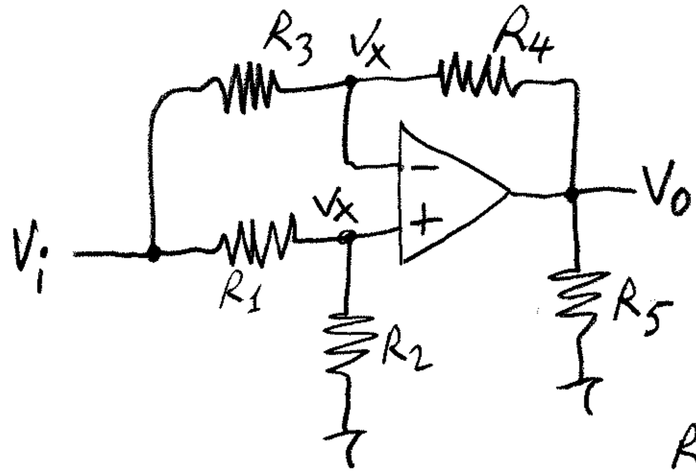
$$V_x = -\frac{R_f}{R_1} V_1 \qquad V_0 = -\frac{R_f}{R_3} V_x - \frac{R_f}{R_2} V_2$$

$$V_0 = -\frac{R_f}{R_3}\left(-\frac{R_f}{R_1} V_1\right) - \frac{R_f}{R_2} V_2$$

$$= \frac{R_f^2}{R_1 R_3} V_1 - \frac{R_f}{R_2} V_2$$

$R_1 = R_2 = R_3 = R_f = R$ alınırsa

$V_0 = V_1 - V_2$ olur.



Yandaki ideal opamp'lı devre için
$A=?$ $R_i=?$ $R_0=?$
en sade halde bulunuz.

$$\frac{V_x-V_i}{R_1}+\frac{V_x}{R_2}=0 \Rightarrow V_x=\frac{R_2}{R_1+R_2}V_i$$

$$\frac{V_x-V_i}{R_3}+\frac{V_x-V_0}{R_4}=0 \Rightarrow V_x=\frac{R_3}{R_3+R_4}V_0+\frac{R_4}{R_3+R_4}V_i$$

$$V_x=V_x \Rightarrow \frac{R_2}{R_1+R_2}V_i=\frac{R_3}{R_3+R_4}V_0+\frac{R_4}{R_3+R_4}V_i$$

$$\frac{R_3}{R_3+R_4}V_0=\left(\frac{R_2}{R_1+R_2}-\frac{R_4}{R_3+R_4}\right)V_i$$

$$V_0=\left(\frac{R_3+R_4}{R_3}\frac{R_2}{R_1+R_2}-\frac{R_4}{R_3}\right)V_i=\frac{R_2R_3-R_1R_4}{(R_1+R_2)R_3}V_i$$

$$A=\frac{R_2R_3-R_1R_4}{(R_1+R_2)R_3} \qquad V_0=AV_i$$

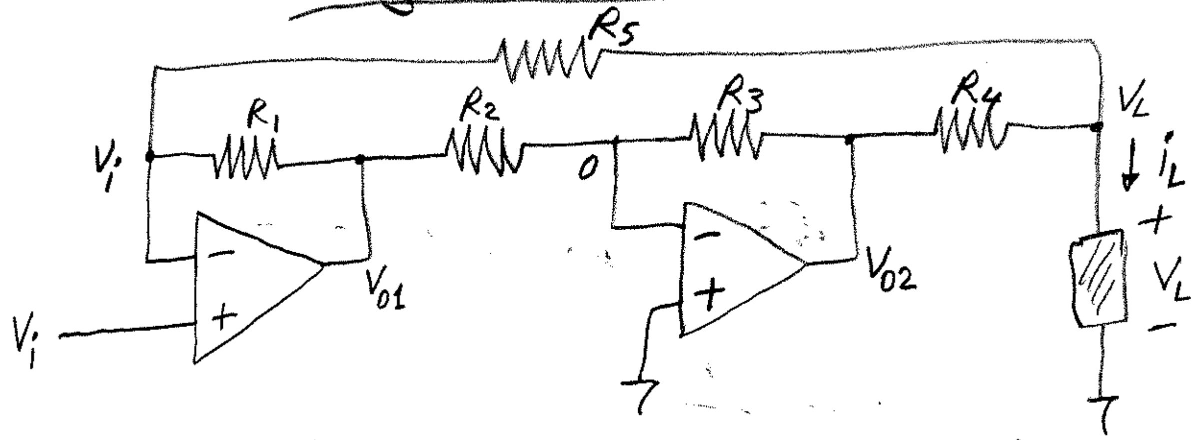
$$I_i=\frac{V_i}{R_1+R_2}+\frac{V_i-V_x}{R_3}=\frac{V_i}{R_1+R_2}+\frac{V_i-\frac{R_2}{R_1+R_2}V_i}{R_3}$$

$$=\left(\frac{1}{R_1+R_2}+\frac{1}{R_3}-\frac{R_2}{(R_1+R_2)R_3}\right)V_i$$

$$R_i=\frac{V_i}{I_i}=\frac{1}{\frac{1}{R_1+R_2}+\frac{1}{R_3}-\frac{R_2}{(R_1+R_2)R_3}}=\frac{(R_1+R_2)R_3}{R_1+R_3}$$

$$R_0=0$$

Voltaj-Akım Dönüştürücü



$$\frac{V_L - V_i}{R_5} + \frac{V_{01} - V_i}{R_1} = 0 \longrightarrow V_{01} = \left(1 + \frac{R_1}{R_5}\right)V_i - \frac{R_1}{R_5}V_L$$

$$\frac{V_{01}}{R_2} + \frac{V_{02}}{R_3} = 0 \rightarrow V_{02} = -\frac{R_3}{R_2}V_{01} = \frac{R_1R_3}{R_2R_5}V_L - \left(1 + \frac{R_1}{R_5}\right)\frac{R_3}{R_2}V_i$$

$$i_L = \frac{V_{02} - V_L}{R_4} + \frac{V_i - V_L}{R_5} = \frac{V_i}{R_5} + \frac{V_{02}}{R_4} - \frac{V_L}{R_4} - \frac{V_L}{R_5}$$

$$= \frac{V_i}{R_5} + \frac{R_1R_3}{R_2R_4R_5}V_L - \left(1 + \frac{R_1}{R_5}\right)\frac{R_3}{R_2R_4}V_i - \frac{V_L}{R_4} - \frac{V_L}{R_5}$$

$$= \left(\frac{1}{R_5} - \left(1 + \frac{R_1}{R_5}\right)\frac{R_3}{R_2R_4}\right)V_i + \left(\frac{R_1R_3}{R_2R_4R_5} - \frac{1}{R_4} - \frac{1}{R_5}\right)V_L$$

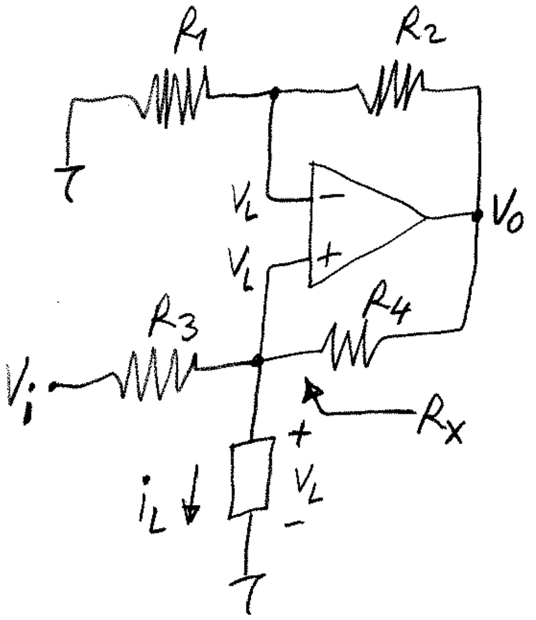
$$\left(\frac{R_2R_4 - R_3R_5 - R_1R_3}{R_2R_4R_5}\right)V_i - \left(\frac{R_1R_3 - R_2R_5 - R_2R_4}{R_2R_4R_5}\right)V_L$$

$R_1 = R_2 = R_3 = R_4 + R_5 = R$ olsun

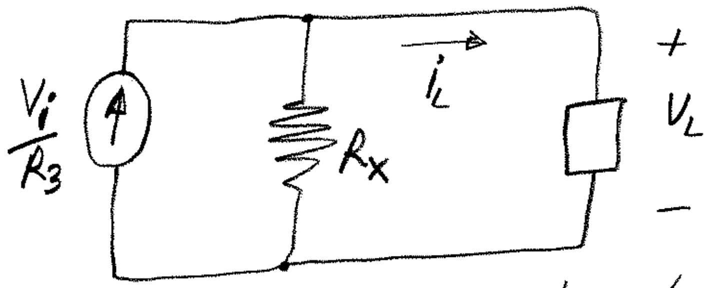
$$i_L = \left(\frac{R.R_4 - R.R_5 - R.R}{R R_4 R_5}\right)V_i - \left(\frac{R.R - RR_5 - RR_4}{R R_4 R_5}\right)V_L$$

$$= \frac{R_4 - R_5 - R}{R_4R_5}V_i - \frac{R - (R_4 + R_5)}{R_4R_5}V_L \rightarrow 0$$

$$= \frac{\cancel{R} - R_5 - R_5 - \cancel{R}}{R_4R_5}V_i = -\frac{2}{R_4}V_i \quad \text{(sabit akım)}$$



Norton eşdeğer devresi



$$i_L = \frac{V_i}{R_3} - \frac{V_L}{R_x}$$

Aşağıdaki denklem bu şekle getirilip R_x bulunur.

$$\frac{V_L}{R_1} + \frac{V_L - V_0}{R_2} = 0 \longrightarrow V_0 = \left(1 + \frac{R_2}{R_1}\right) \cdot V_L$$

$$i_L = \frac{V_i - V_L}{R_3} + \frac{V_0 - V_L}{R_4} = \frac{V_i - V_L}{R_3} + \frac{\left(1 + \frac{R_2}{R_1}\right) V_L - V_L}{R_4}$$

$$= \frac{V_i}{R_3} - \frac{V_L}{R_3} + \frac{R_2}{R_1 R_4} V_L = \frac{V_i}{R_3} - \left(\underset{(R_1 R_4)}{\frac{1}{R_3}} - \underset{(R_3)}{\frac{R_2}{R_1 R_4}}\right) V_L$$

$$= \frac{V_i}{R_3} - \left(\frac{R_1 R_4 - R_2 R_3}{R_1 R_3 R_4}\right) V_L = \frac{V_i}{R_3} - \frac{V_L}{\frac{R_1 R_3 R_4}{R_1 R_4 - R_2 R_3}}$$

$$R_x = \frac{R_1 R_3 R_4}{R_1 R_4 - R_2 R_3} = \frac{R_4}{\frac{R_4}{R_3} - \frac{R_2}{R_1}}$$

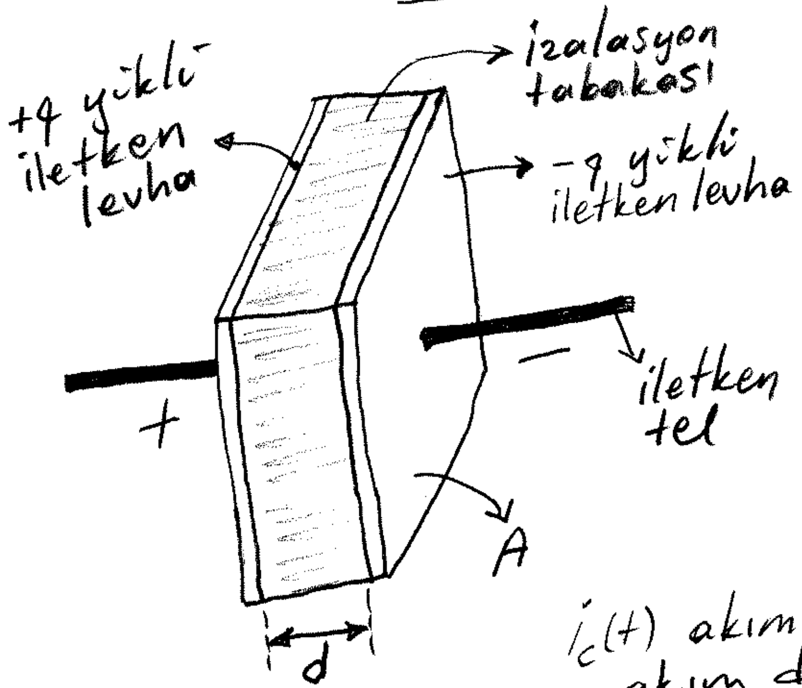
$$\frac{R_4}{R_3} = \frac{R_2}{R_1} = k \text{ ise } R_x = \frac{R_4}{k - k} = \frac{R_4}{0} = \infty$$

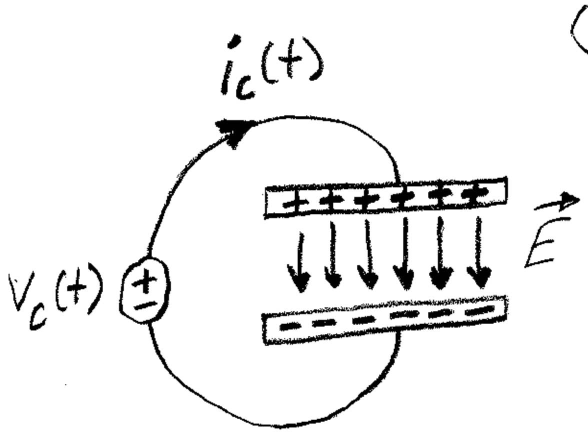
Bu durumda

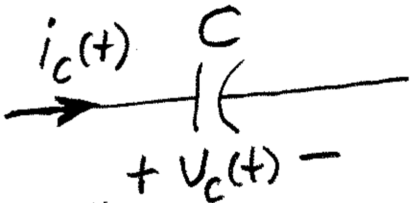
$$i_L = \frac{V_i}{R_3} - \frac{V_L}{R_x} \Longrightarrow i_L = \frac{V_i}{R_3}$$

0 olur

Kapasitör (C)







$i_c(t)$ akımı kapasitörden geçen akım değildir.

d : levhalar arasındaki uzaklık veya izalasyon tabakasının kalınlığı

İzalasyon tabakası ince plastik bir yaprak olabildiği gibi arada boşlukta olabilir.

A : Levhalardan birinin yüzey alanı

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

Boşluğun veya havanın dielektrik katsayısı

ε_r : Plakalar arasındaki yalıtkanın bağıl dielektrik katsayısı. Boşluk için 1, Hava için ~1, Silikon için 12 alınır.

$$C = \varepsilon \cdot \frac{A}{d} = \varepsilon_r \varepsilon_0 \frac{A}{d} = \varepsilon_r C_0$$

Kapasitörün (veya Kondansatörün) şarj depolama kapasitesi kapasidans olarak adlandırılır. Sembolü C'dir. Birimi ise Farad (F)'dir.

Kapasitör, enerjiyi elektrik alanda depolayan devre elemanıdır. Elektrik alan varsa kapasitif etki vardır. Potansiyel farkının değişmesi yüklerin değişmesiyle olur.

Kapasitörler, yüksek frekansta kısa devre özelliği gösterirler. Dolayısıyla tiz denilen yüksek frekansları geçirirler. Üzerlerindeki voltaj artıyorsa tüketici, azalıyorsa üretici konumundadırlar.

Kapasitörler; radyo alıcılarında frekans ayarında, güç kaynaklarında filtre olarak, ateşleme sistemlerinde kıvılcımı yok etmede, elektronik flaş üniteleri gibi pek çok yerde kullanılır.

$$q(t) = C\,V(t) \Rightarrow C = \frac{q(t)}{V(t)}, \quad 1\,\text{Farad} = \frac{1\ \text{Coulomb}}{1\ \text{Volt}}$$

$$i_c(t) = \frac{dq(t)}{dt} = c\,\frac{dV_c(t)}{dt}$$

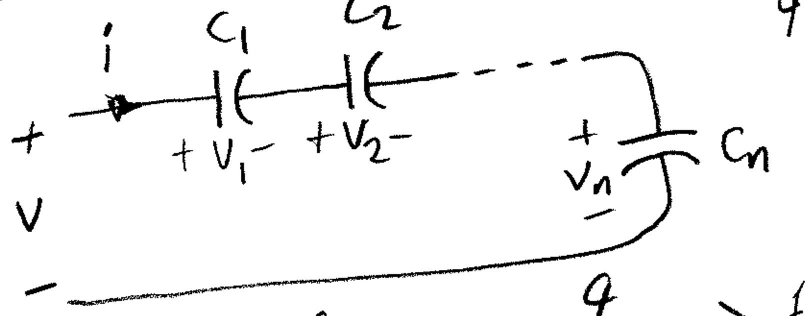
$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(z)\,dz = V_c(t_0) + \frac{1}{C}\int_{t_0}^{t} i_c(z)\,dz$$

$$P_c(t) = \frac{dw_c(t)}{dt} = \frac{dw_c(t)}{dq(t)}\,\frac{dq(t)}{dt} = V_c(t)\cdot i_c(t)$$

$$= V_c(t)\cdot c\,\frac{dV_c(t)}{dt} = \frac{d}{dt}\left(\frac{1}{2}\,C\,V_c^2(t)\right)$$

$$W_c(t) = \frac{1}{2}\,C\,V_c^2(t) = \frac{q^2(t)}{2C} = \frac{q(t)\,V_c(t)}{2}$$

Kapasitörlerin Seri Bağlanması



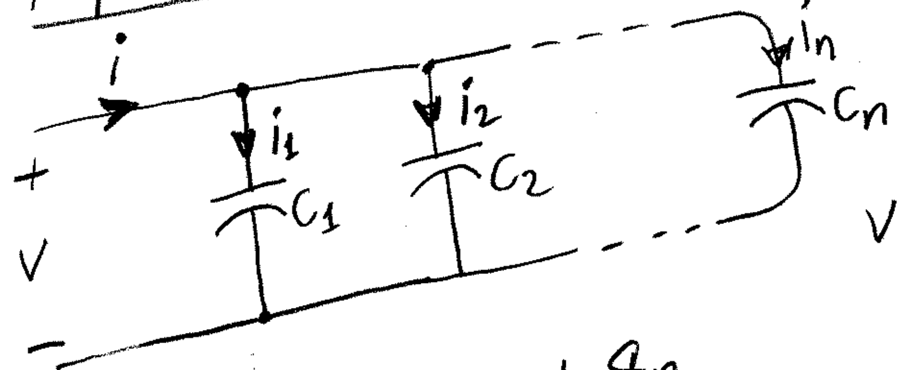
$$q = q_1 = q_2 = \cdots = q_n$$

$$i = i_1 = i_2 = \cdots = i_n$$

$$V = V_1 + V_2 + \cdots + V_n$$

$$\frac{q}{C_e} = \frac{q}{C_1} + \frac{q}{C_2} + \cdots + \frac{q}{C_n} \Rightarrow \frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

Kapasitörlerin Paralel Bağlanması



$$q = q_1 + q_2 + \cdots + q_n$$

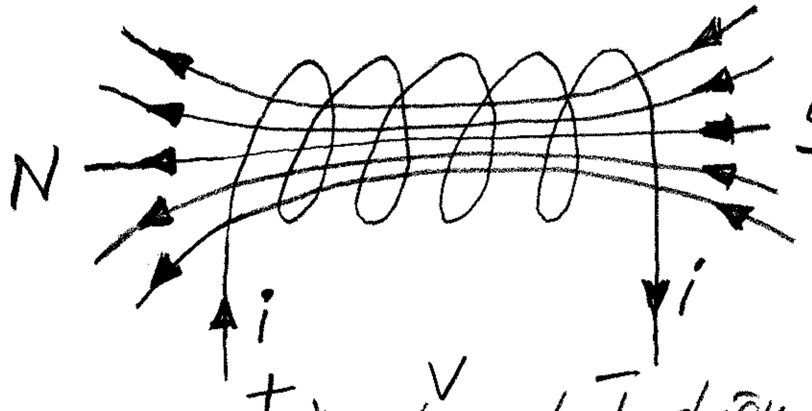
$$i = i_1 + i_2 + \cdots + i_n$$

$$V = V_1 = V_2 = \cdots = V_n$$

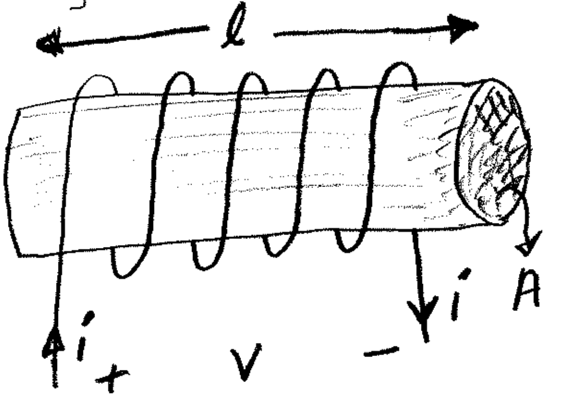
$$q = q_1 + q_2 + \cdots + q_n$$

$$C_e V = C_1 V + C_2 V + \cdots + C_n V \Rightarrow C_e = C_1 + C_2 + \cdots + C_n$$

İndüktör (L)



İletken bir telden akım geçince etrafında manyetik alan oluşur. İletken tel bobin haline getirilirse oluşan manyetik alan düz bir mıknatısınkine benzer. Bobinden geçen akım manyetik alan oluşturduğu gibi tersi de doğrudur.



İndüktör(Endüktans, Bobin)'in Sembolü L, Birimi Henry(H)'dir.

n : Sarım sayısı
A : Yüzey alanı
l : Uzunluk

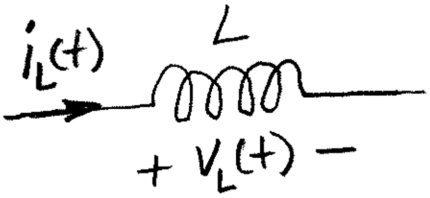
$$L = \mu \frac{n^2 A}{l} = \mu_r \mu_0 \frac{n^2 A}{l} = \mu_r L_0$$

$\mu_0 = 4\pi \times 10^{-7}$ Boşluğun manyetik alan geçirgenliği

μ_r : Bağıl manyetik alan geçirgenlik katsayısı

Boşluk için 1, Hava için ~1, Demir nüvesi için 400 alınır.

Bobin enerjiyi manyetik alanda depolayan devre elemanıdır. Pratikte bobin emaye telden sarılarak elde edilir. Bu telin azda olsa bir direnci vardır. İdeal bobinin iç direnci sıfır alınır. Bobinde manyetik alan oluşturmak için zamana ihtiyaç vardır. Bobinden geçen akım kesildiğinde manyetik alan birden yok olmaz. Manyetik alanın yok olması için yine zamana ihtiyaç vardır. Bobin içinden geçen akıma tepki gösterir. Geçen akım DC ise uçları arasındaki gerilim sıfırdır. AC bir akımda frekans artması endüktans gerilimini artırır. Yüksek frekansta açık devre, düşük frekansta kısa devre olur. Bass devreleri düşük frekansları geçirir. Endüktans enerji çekiyorsa tüketici konumunda, enerji veriyorsa üretici konumundadır.



$$\Phi(t) = L\, i_L(t)$$

$\downarrow$ manyetik akı (weber - Wb)

$$V_L(t) = \frac{d\Phi(t)}{dt} = L\,\frac{di_L(t)}{dt}$$

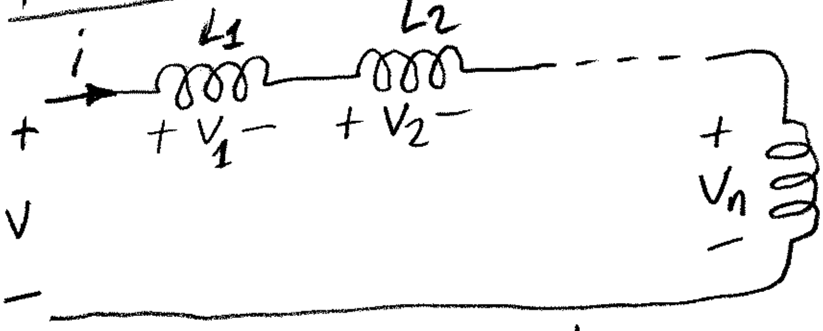
$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(z)\,dz = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} V_L(z)\,dz$$

$$P_L(t) = \frac{dW_L}{dt} = V_L(t)\, i_L(t) = L\,\frac{di_L}{dt}\cdot i_L(t) = \frac{d}{dt}\left(\frac{1}{2} L\, i_L^2(t)\right)$$

$$W_L(t) = \frac{1}{2} L\, i_L^2(t) = \frac{\Phi^2(t)}{2L} = \frac{\Phi(t)\cdot i_L(t)}{2}$$

Bir bobindeki akım değişme hızı 1A/sn ve bu değişime karşı koyan endüksiyon gerilimi 1V ise bobinin endüktansı 1H 'dir.

İndüktörlerin Seri Bağlanması



$$\Phi = \Phi_1 + \Phi_2 + \cdots + \Phi_n$$

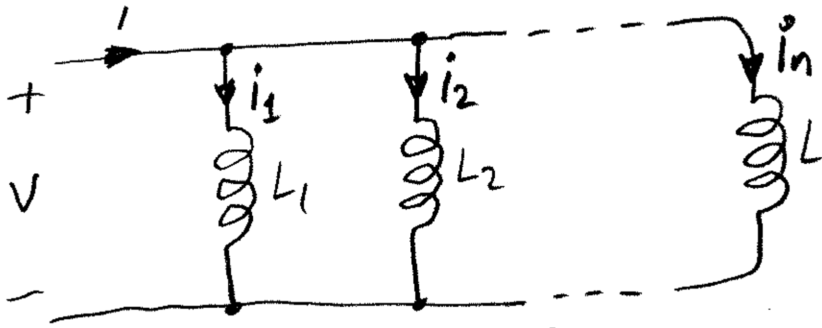
$$V = V_1 + V_2 + \cdots + V_n$$

$$i = i_1 = i_2 = \cdots = i_n$$

$$\Phi = \Phi_1 + \Phi_2 + \cdots + \Phi_n$$

$$L_e i = L_1 i + L_2 i + \cdots + L_n i \Rightarrow L_e = L_1 + L_2 + \cdots + L_n$$

İndüktörlerin Paralel Bağlanması



$$\Phi = \Phi_1 = \Phi_2 = \cdots = \Phi_n$$

$$V = V_1 = V_2 = \cdots = V_n$$

$$i = i_1 + i_2 + \cdots + i_n$$

$$i = i_1 + i_2 + \cdots + i_n$$

$$\frac{\Phi}{L_e} = \frac{\Phi}{L_1} + \frac{\Phi}{L_2} + \cdots + \frac{\Phi}{L_n} \Rightarrow \frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$

5 μF'lık bir kapasitörün uçlarına 12V'luk bir gerilim uygu- (39)
lanıyor. Plakalarda biriken yükü bulunuz. Kapasitörde depolanan enerjiyi bulunuz.

$C = 5\,\mu F \qquad q = CV_c = 5\,\mu F \times 12V = 60\,\mu C$

$V_c = 12V \qquad W = \frac{1}{2} C V_c^2 = \frac{1}{2} \times 5\,\mu F \times (12V)^2 = 0.36\,mJ$

Bir kapasitörde plakalar arası 3mm, plakaların yüzey alanı $15\,cm^2$ olsun. Plakalar arasındaki yalıtkan malzemenin bağıl dielektrik katsayısı 4 olsun.

a) Kapasitörün kapasitesi nedir?

b) Yalıtkan malzemenin dayanabileceği max. elektrik alanı 10^5 ise kapasitörün taşıyabileceği max. voltaj ve yük nedir?

a) $\varepsilon_r = 4$, $d = 3mm = 3 \times 10^{-3} m$, $A = 15 cm^2 = 15 \times 10^{-4} m^2$

$$C = \varepsilon \frac{A}{d} = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

$$= 4 \times 8.85 \times 10^{-12} \frac{15 \times 10^{-4}}{3 \times 10^{-3}} F = 17.7\,pF$$

b) $V = E.d = 10^5 \frac{V}{m} \times 3 \times 10^{-3} m = 300\,V$

$Q = CV = 17.7\,pF \times 300V = 5.31\,nC$

500 sarımlı bir bobinin uzunluğu 6cm, kullanılan telin kesit alanı $12\,cm^2$'dir.

a) Bobin nüvesiz ise Endüktansı

b) Bobin nüveli ise Endüktansı. ($\mu_r = 250$ olsun)

$n = 500$, $l = 6\,cm = 6 \times 10^{-2} m$, $A = 12\,cm^2 = 12 \times 10^{-4} m$

a) $$L_0 = \mu_0 \frac{n^2 A}{l} = 4\pi \times 10^{-7} \frac{500^2 \times 12 \times 10^{-4}}{6 \times 10^{-2}} F$$

$$= 4\pi \times 10^{-7} \frac{25 \times \overset{2}{\cancel{12}} \times 100}{\cancel{6}} F = 6.28\,mF$$

b) $L = \mu_r L_0 = 250 \times 6.28\,mF = 1.57\,F$

Bir bobinden geçen akım 0.25 sn'de 2A'den 5A'e çıktığında üzerinde 6V'luk endüksiyon gerilimi oluşuyor. Bobinin endüktansını bulunuz. (40)

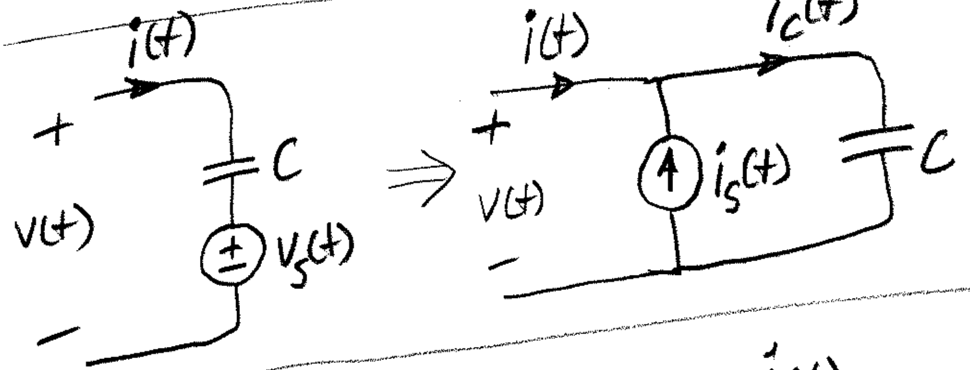
$\Delta t = 0.25\ sn$

$\Delta I = 5A - 2A$

$= 3A$

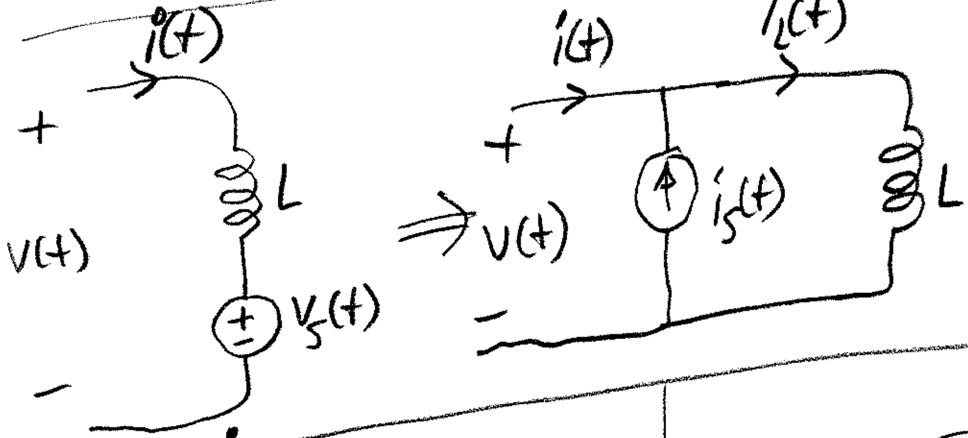
$$V = L\frac{\Delta I}{\Delta t} \Rightarrow L = V \cdot \frac{\Delta t}{\Delta I}$$

$$L = 6V\ \frac{0.25\ sn}{3A} = 0.5\ H$$



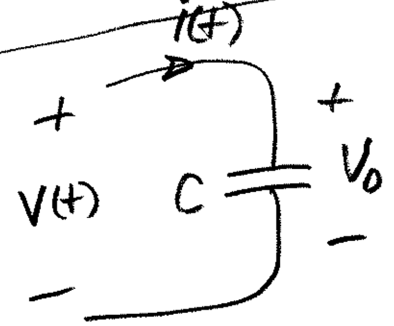
$$i_s(t) = C\frac{dV_s(t)}{dt}$$

$$V_s(t) = \frac{1}{C}\int_{-\infty}^{t} i_s(z)\,dz$$

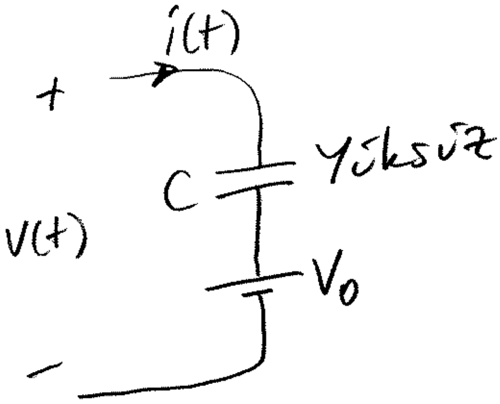


$$V_s(t) = L\frac{di_s(t)}{dt}$$

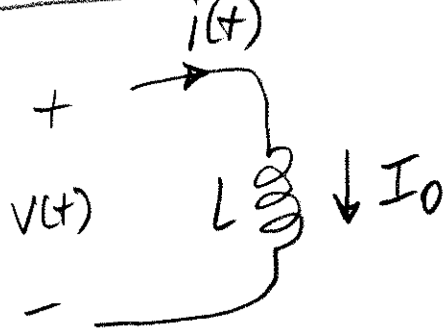
$$i_s(t) = \frac{1}{L}\int_{-\infty}^{t} V_s(z)\,dz$$



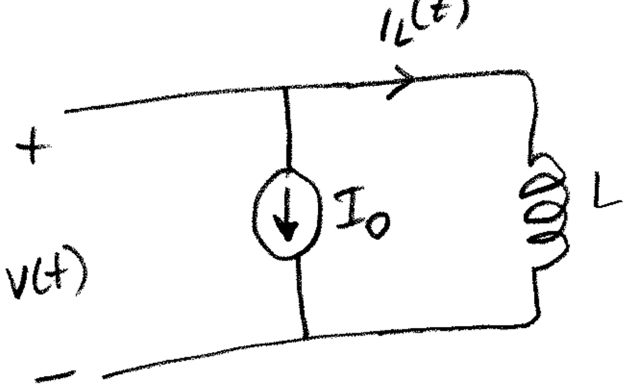
$V_c(0) = V_0$ ise



$V_c(0) = 0$



$i_L(0) = I_0$ ise



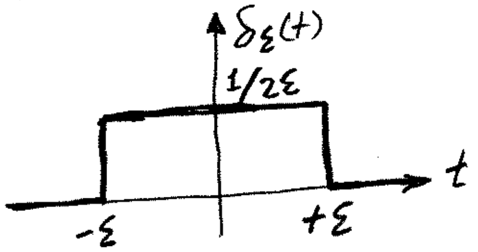
$i_L(0) = 0$

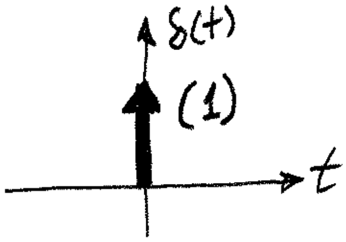
Dalga Formları

① Sabit Fonksiyon $f(t) = A$

② Sinüzoidal Fonksiyon $f(t) = A.\sin(wt+\alpha)$, $w = 2\pi f = 2\pi/T$

③ Birim Dürtü Fonksiyonu





$$\int_{-\infty}^{\infty} \delta_\varepsilon(t)\,dt = \int_{-\varepsilon}^{\varepsilon} \delta_\varepsilon(t)\,dt = 1$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

$\varepsilon \to 0$ için $\delta_\varepsilon(t) \longrightarrow \delta(t)$ olur.

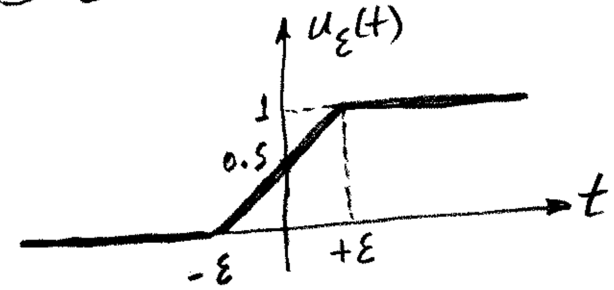
$\delta(-t) = \delta(t)$ çift fonksiyon $\qquad \delta(at) = \delta(t)/|a|$

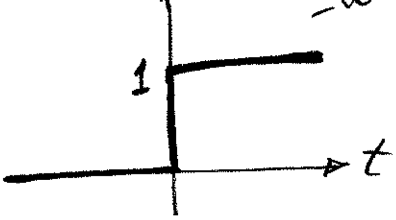
$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = \int_{-\infty}^{\infty} x(0).\delta(t)\,dt = x(0)\int_{-\infty}^{\infty} \delta(t)\,dt = x(0)$$

$$x(t) = \int_{-\infty}^{\infty} x(z)\,\delta(t-z)\,dz = x(t) * \delta(t)$$

④ Birim Basamak Fonksiyonu

$$u(t) = \int_{-\infty}^{t} \delta(z)\,dz = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \\ 1/2 & t = 0 \end{cases}$$



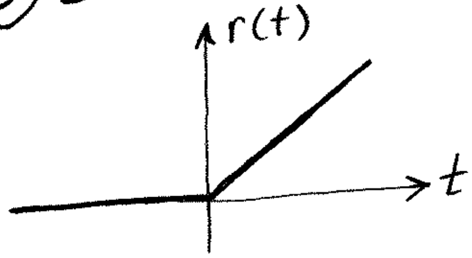


$\varepsilon \to 0$ için

$u_\varepsilon(t) \to u(t)$ olur.

$u(0^-) = 0$, $u(0^+) = 1 \qquad \delta(t) = \dfrac{du(t)}{dt} = u'(t)$

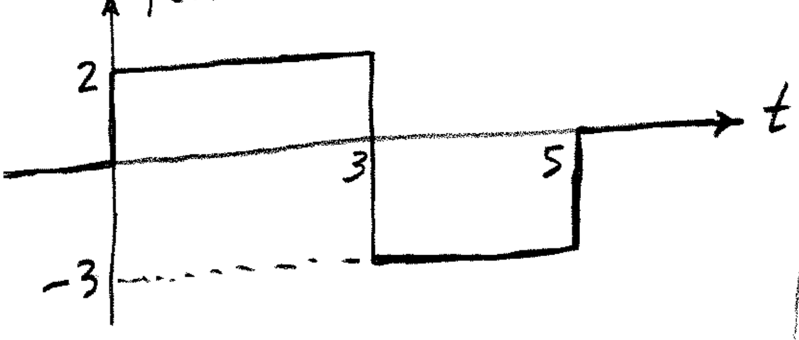
⑤ Birim Rampa Fonksiyonu



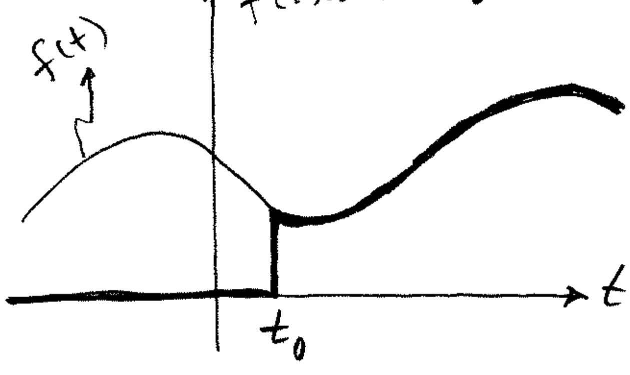
$$r(t) = \int_{-\infty}^{t} u(z)\,dz = t\,u(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

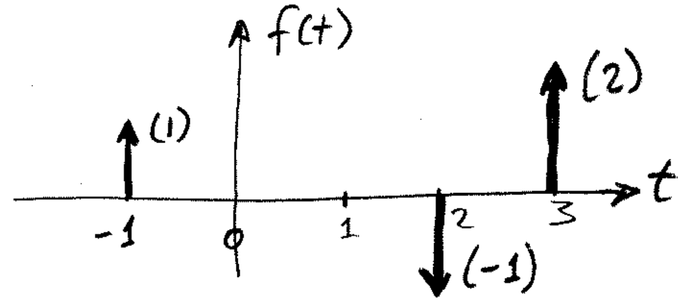
$$u(t) = r'(t) = \frac{dr(t)}{dt}$$

$f(t) = 2u(t) - 5u(t-3) + 3u(t-5)$

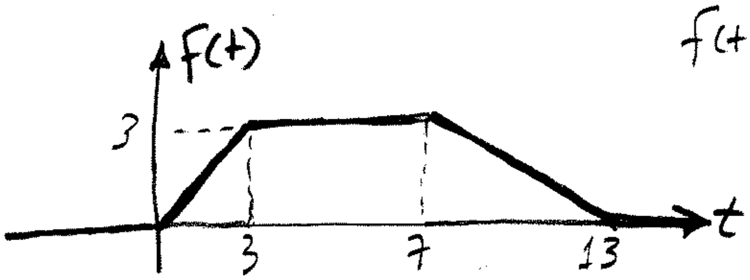


$f(t).u(t-t_0)$

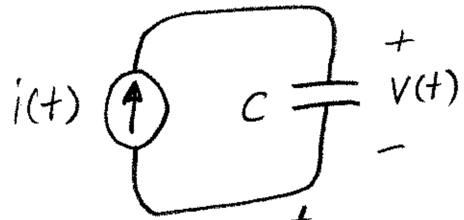




$$f(t) = \delta(t+1) - \delta(t-2) + 2\delta(t-3)$$



$$f(t) = r(t) - r(t-3)$$
$$-0.5\,r(t-7) + 0.5\,r(t-13)$$



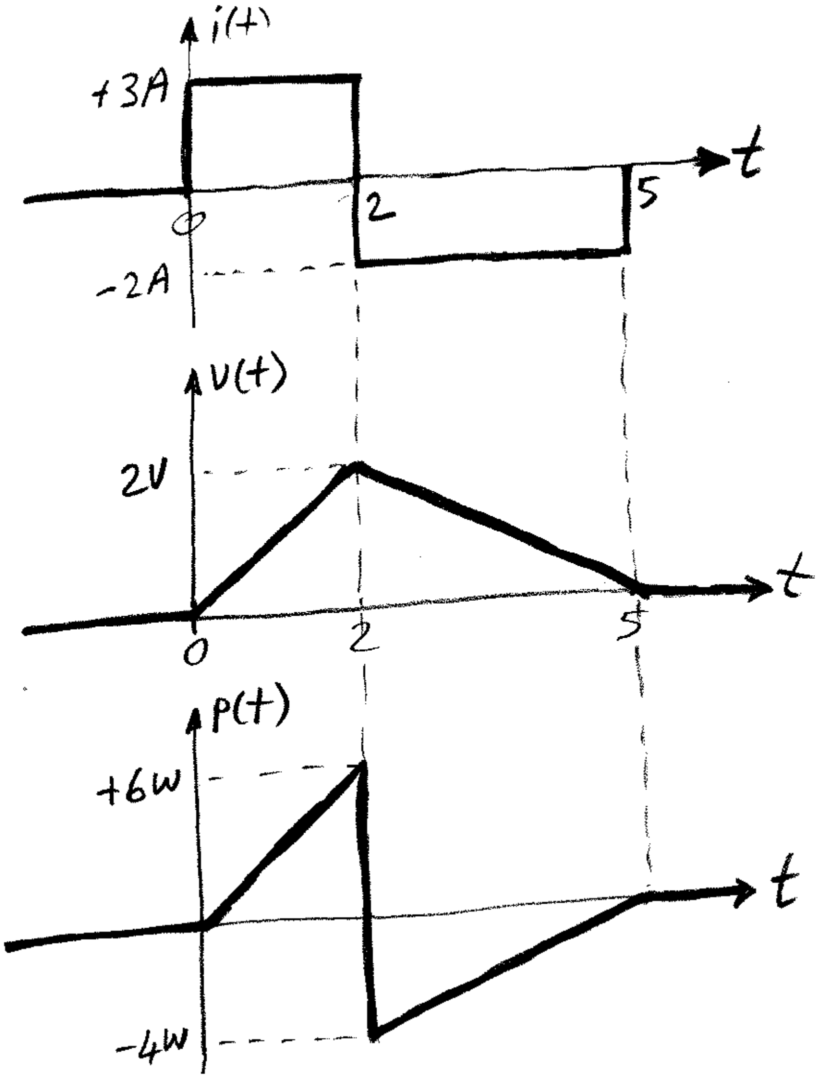
$i(t) = 3u(t) - 5u(t-2) + 2u(t-5)$

$C = 3F$ ise $v(t)$, $p(t)$ bulunuz.

Ve grafiklerini çiziniz.

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(z)\,dz = \int_{-\infty}^{t} u(\tau)\,d\tau - \frac{5}{3}\int_{-\infty}^{t} u(\tau-2)\,d\tau + \frac{2}{3}\int_{-\infty}^{t} u(\tau-5)\,d\tau$$

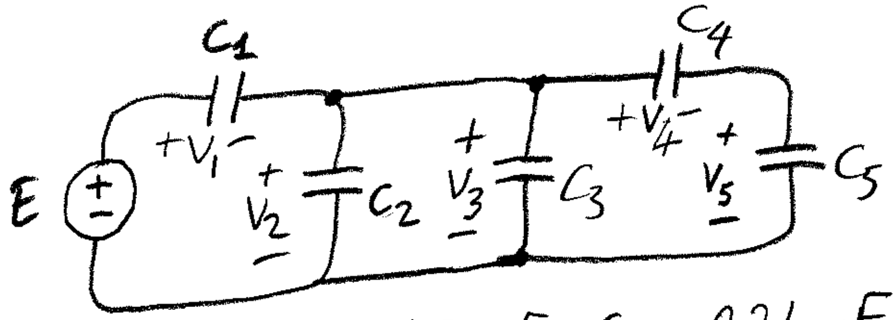
$$= r(t) - \frac{5}{3}r(t-2) + \frac{2}{3}r(t-5)$$



$$p(t) = v(t)\cdot i(t)$$

$$= 3r(t) - 3r(t-2)$$
$$-10u(t-2)$$
$$+\frac{4}{3}r(t-2)$$
$$-\frac{4}{3}r(t-5)$$

$$r(t) = 3r(t) - \frac{5}{3}r(t-2)$$
$$-10u(t-2) - \frac{4}{3}r(t-5)$$



Herbir kapasitör üzerindeki yük ve voltajları bulunuz. (43)

$E = 24V$, $C_1 = 0.3mF$, $C_2 = 0.24mF$, $C_3 = 0.18mF$

$C_4 = 0.3mF$, $C_5 = 0.45mF$

$$C_e = C_1 \$ (C_2 + C_3 + C_4 \$ C_5)$$

$$= 0.3mF \$ (0.24mF + 0.18mF + 0.3mF \$ 0.45mF)$$

$$= 0.3mF \$ \left(0.42mF + \frac{0.3 \times 0.45}{0.75} mF\right)$$

$$= 0.3mF \$ (0.42mF + 0.18mF)$$

$$= 0.3mF \$ 0.6mF = \frac{0.3 \times 0.6}{0.9} mF = 0.2mF$$

$$Q = C_e E = 0.2mF \times 24V = 4.8mC$$

$$Q_1 = Q = 4.8mC \qquad V_1 = \frac{Q_1}{C_1} = \frac{4.8mC}{0.3mF} = 16V$$

$$V_2 = V_3 = E - V_1 = 24V - 16V = 8V$$

$$Q_2 = C_2 V_2 = 0.24mF \times 8V = 1.92mC$$

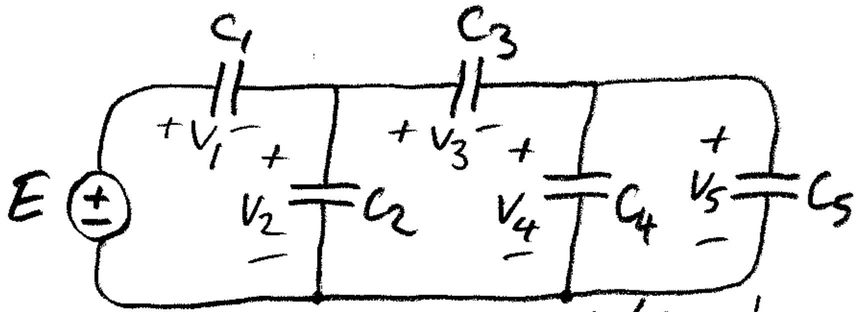
$$Q_3 = C_3 V_3 = 0.18mF \times 8V = 1.44mC$$

$$Q_4 = Q_5 = (C_4 \$ C_5) V_3 = (0.3mF \$ 0.45mF) \times 8V$$

$$= 0.18mF \times 8V = 1.44mC$$

$$V_4 = \frac{Q_4}{C_4} = \frac{1.44mC}{0.3mF} = 4.8V$$

$$V_5 = \frac{Q_5}{C_5} = \frac{1.44mC}{0.45mF} = 3.2V$$



$E = 25V$; $C_1 = 30mF$

$C_2 = 24mF$, $C_3 = 30mF$

$C_4 = 20mF$, $C_5 = 50mF$

Herbir kapasitör üzerindeki yük ve voltajları bulunuz.

$$C_e = C_1 \$ \left(C_2 + C_3 \$ (C_4 + C_5) \right)$$

$$= 30mF \$ \left(24mF + 30mF \$ (20mF + 50mF) \right)$$

$$= 30mF \$ \left(24mF + 30mF \$ 70mF \right)$$

$$= 30mF \$ \left(24mF + 21mF \right)$$

$$= 30mF \$ 45mF = 18\,mF$$

$$Q = C_e E = 18mF \times 25V = 450\,mC$$

$$Q_1 = Q = 450\,mC \quad V_1 = \frac{Q_1}{C_1} = \frac{450mC}{30mF} = 15V$$

$$V_2 = E - V_1 = 25V - 15V = 10V$$

$$Q_2 = C_2 V_2 = 24mF \times 10V = 240\,mC$$

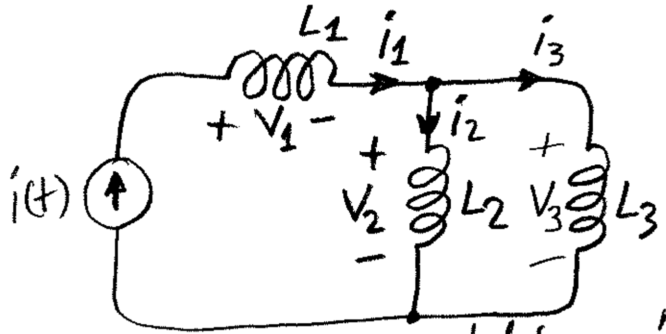
$$Q_3 = Q_1 - Q_2 = 450mC - 240mC = 210\,mC$$

$$V_3 = \frac{Q_3}{C_3} = \frac{210mC}{30mF} = 7V$$

$$V_4 = V_5 = V_2 - V_3 = 10V - 7V = 3V$$

$$Q_4 = C_4 V_4 = 20mF \times 3V = 60\,mC$$

$$Q_5 = C_5 V_5 = 50mF \times 3V = 150\,mC$$



$L_1 = 12H$, $L_2 = 10H$, $L_3 = 15H$

$i(t) = (5 - 3e^{-2t})u(t)$

Eşdeğer endüktansı, toplam manyetik akıyı bulunuz.

Herbir bobin üzerindeki manyetik akıyı, akımı ve voltajı bulunuz.
$t=0^+$ ve $t=\infty$ anlarındaki bobinler üzerindeki voltaj ve akımları bulunuz.

$$L_e = 12H + 10H // 15H = 12H + 6H = 18H$$

$$\Phi(t) = L_e\, i(t) = (90 - 54e^{-2t})u(t)$$

$$i_1(t) = i(t) = (5 - 3e^{-2t})u(t)$$

$$\Phi_1(t) = L_1 i_1(t) = (60 - 36e^{-2t})u(t)$$

$$V_1(t) = \frac{d\Phi_1(t)}{dt} = 72e^{-2t}u(t) + \cancel{24\delta(t)}$$

$$\Phi_2(t) = \Phi_3(t) = \Phi(t) - \Phi_1(t) = (30 - 18e^{-2t})u(t)$$

$$V_2(t) = V_3(t) = \frac{d\Phi_2}{dt} = 36 e^{-2t}u(t) + \cancel{12\delta(t)}$$

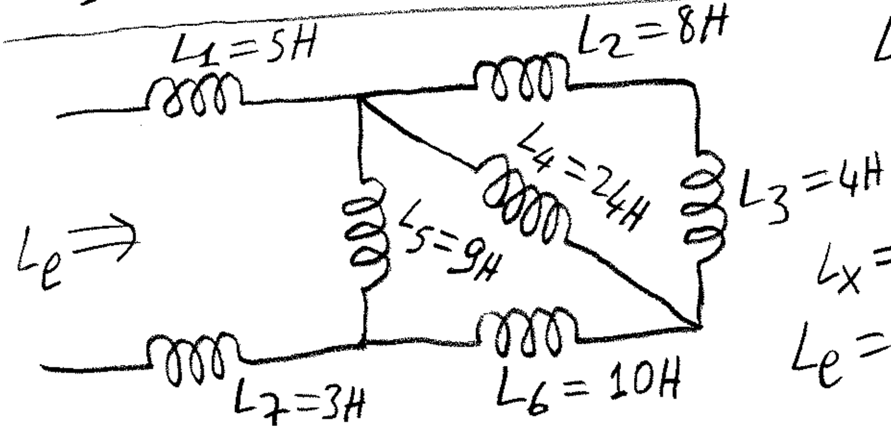
$$i_2(t) = \frac{\Phi_2(t)}{L_2} = (3 - 1.8e^{-2t})u(t)$$

$$i_3(t) = \frac{\Phi_3(t)}{L_3} = (2 - 1.2e^{-2t})u(t)$$

$i(\infty)$
$V(\infty)$

$$i_1(0^+) = 2A, \quad i_2(0^+) = 1.2A, \quad i_3(0^+) = 0.8A$$

$$V_1(0^+) = 72V, \quad V_2(0^+) = V_3(0^+) = 36V$$

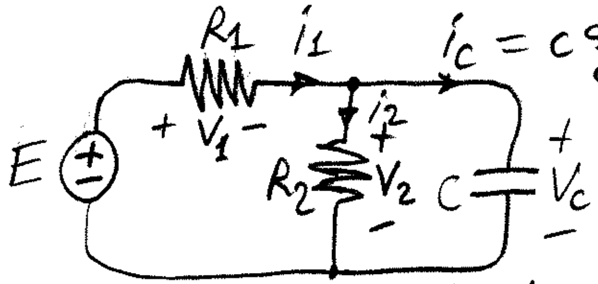


$$L_e = L_1 + L_x + L_7$$

$$L_x = L_5 // (L_6 + L_4 // (L_2 + L_3))$$

$$L_x = 9 // (10 + 24 // 12)H = 6H$$

$$L_e = 5H + 6H + 3H = 14H$$



Kapasitörler $t=0^+$ anında kısa devredir.

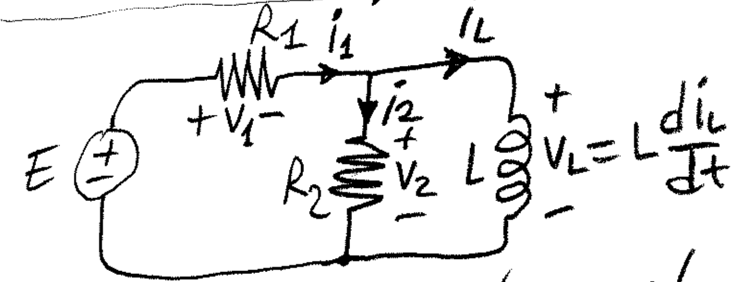
$$i_c(0^+) = \frac{E}{R_1}, \quad V_c(0^+) = 0$$

$t \to \infty$ iken açık devre olur.

$$i_c(\infty) = 0, \quad V_c(\infty) = V_2(\infty) = \frac{R_2 E}{R_1+R_2}$$

Kapasitörün yaklaşık dolma süresi 5τ alınır.

$\tau = (R_1 // R_2) \cdot C$ Zaman sabitesi



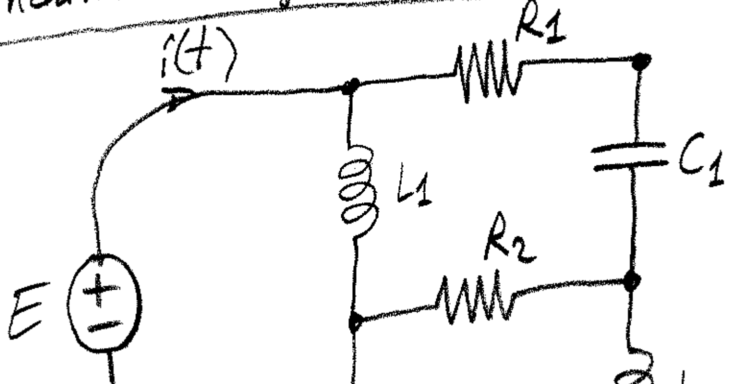
İndüktör $t=0^+$ anında açık devredir.

$$i_L(0^+) = 0 \quad V_L(0^+) = \frac{R_2 E}{R_1+R_2}$$

$t \to \infty$ iken kısa devre olur.

$$i_L(\infty) = E/R_1, \quad V_L(\infty) = 0 \qquad \tau = \frac{L}{R_1 // R_2} \text{ Zaman sabitesi}$$

İndüktörün yaklaşık kısa devre olma süresi 5τ alınır.



$t=0^+$ anında kapasitörler kısa devre, bobinler açık devredir.

$$i(0^+) = i_{C1}(0^+) = i_{C2}(0^+) = \frac{E}{R_1+R_2}$$

$i_{L1}(0^+) = i_{L2}(0^+) = 0$

$$V_{C1}(0^+) = V_{C2}(0^+) = 0$$

$$V_{L1}(0^+) = E, \quad V_{L2}(0^+) = R_2\, i(0^+) = \frac{R_2 E}{R_1+R_2}$$

$t \to \infty$ iken kapasitörler açık devre, bobinler kısa devre olur.

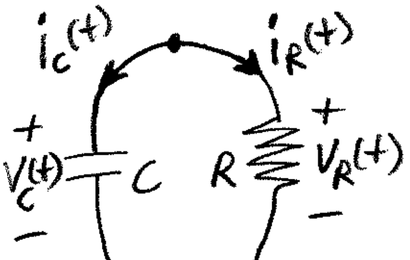
$$i(\infty) = i_{L1}(\infty) = i_{L2}(\infty) = \frac{E}{R_2+R_3} \qquad i_{C1}(\infty) = i_{C2}(\infty) = 0$$

$$V_{L1}(\infty) = V_{L2}(\infty) = 0$$

$$V_{C1}(\infty) = R_2\, i(\infty) = \frac{R_2 E}{R_2+R_3}$$

$$V_{C2}(\infty) = E$$

Birinci Mertebeden Devreler



$$V_C(t) = V_R(t)$$

$$i_C(t) + i_R(t) = 0$$

$$C\frac{dV_C}{dt} + \frac{V_C}{R} = 0$$

$$\frac{dV_C}{dt} + \frac{1}{RC}V_C = 0$$

$V_C(0^+) = V_0$ olsun

$\tau = RC$ Zaman Sabitesi

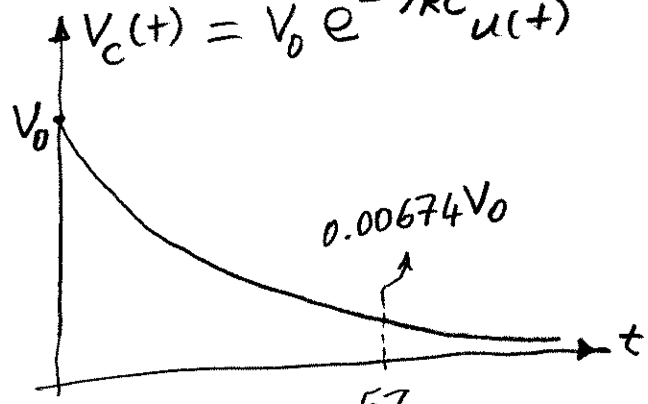
5τ ise Durulma Zamanıdır.

5τ'lık sürede kapasitörün boşaldığı (veya sabit bir voltaj) olduğu kabul edilir.

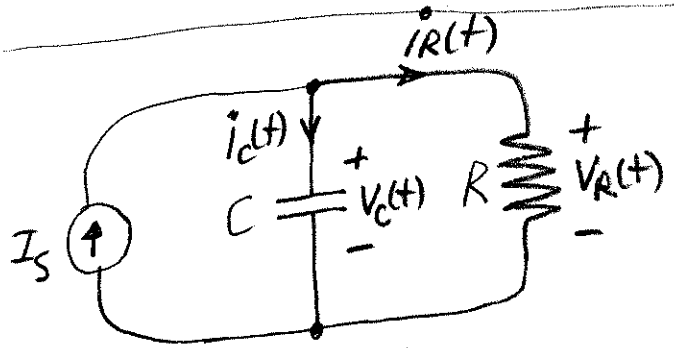
$$V_C(t) = A\,e^{-t/RC}u(t)$$

$$V_C(0^+) = A = V_0$$

$$V_C(t) = V_0\,e^{-t/RC}u(t)$$



$$i_C(t) = C\frac{dV_C(t)}{dt} = -i_R(t) = -\frac{V_R(t)}{R} = -\frac{V_0}{R}e^{-t/RC}u(t)$$



$V_C(0^+) = V_0$ olsun

$t = 0^+$'den itibaren devreye I_S sabit akımı veriliyor.

$$V_C(t) = V_R(t)$$

$$i_C(t) + i_R(t) = I_S$$

$$C\frac{dV_C}{dt} + \frac{V_C}{R} = I_S$$

$$\frac{dV_C}{dt} + \frac{1}{RC}V_C = \frac{I_S}{C}$$

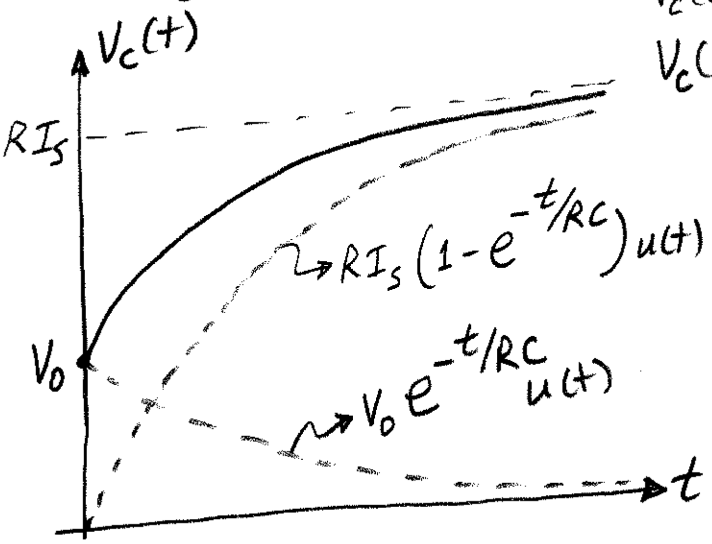
$\tau = RC$ Zaman Sabitesi

5τ Durulma Zamanı

5τ'lık zamanda kapasitör sabit voltaj olur.

$$V_C(t) = \left(A\,e^{-\frac{t}{RC}} + B\right)u(t)$$

$$\left.\begin{array}{l} V_C(0^+) = A + B = V_0 \\ V_C(\infty) = B = RI_S \end{array}\right\} \quad \begin{array}{l} A = V_0 - B \\ \quad = V_0 - RI_S \end{array}$$



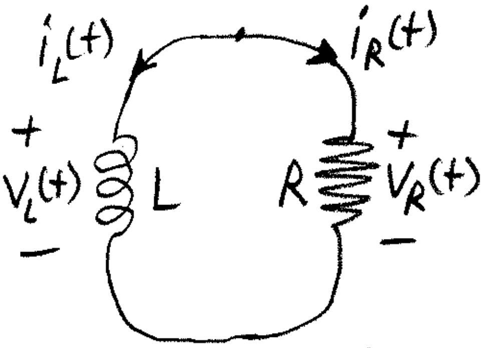
$$V_C(t) = \left((V_0 - RI_S)e^{-t/RC} + RI_S\right)u(t)$$

$$= \underbrace{V_0 e^{-t/RC}u(t)}_{\text{Boşalma}} + \underbrace{RI_S\left(1 - e^{-t/RC}\right)u(t)}_{\text{Dolma}}$$

$$i_C(t) = C\frac{dV_C}{dt} = I_S - \frac{V_C(t)}{R}$$

$$= \left(I_S - \frac{V_0}{R}\right)e^{-t/RC}u(t)$$

$$i_C(0^+) = I_S - \frac{V_0}{R}$$



$V_L(t) = V_R(t)$

$i_L(t) + i_R(t) = 0$

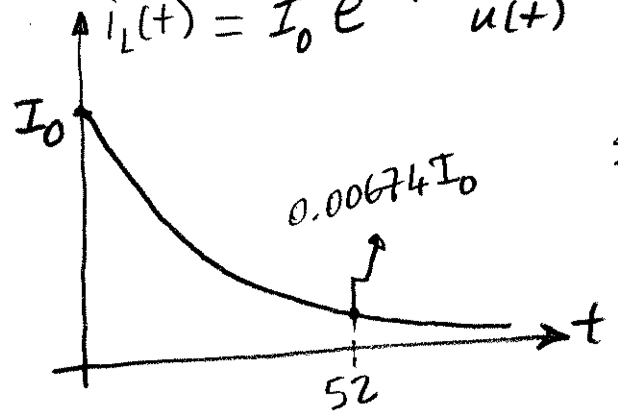
$i_L(t) + \dfrac{V_L(t)}{R} = 0 \rightarrow i_L(t) + \dfrac{L}{R}\dfrac{di_L(t)}{dt} = 0$

$\dfrac{di_L}{dt} + \dfrac{R}{L} i_L = 0$

$i_L(t) = A e^{-t/RC} u(t)$ $\quad i_L(0^+) = A = I_0$

$i_L(0^+) = I_0$ olsun

$i_L(t) = I_0 e^{-t/RC} u(t)$



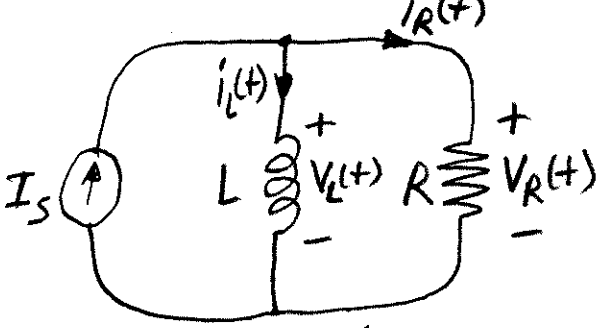
$\tau = L/R$ Zaman Sabitesi

5τ ise Durulma Zamanıdır.

5τ'lık sürede bobinden geçen akımın sıfır (veya sabit bir değer) olduğu kabul edilir.

$V_L(t) = V_R(t) = R i_R(t) = -R i_L(t)$

$= -R I_0 e^{-Rt/L} u(t)$



$V_L(t) = V_R(t)$

$i_L(t) + i_R(t) = I_S$

$i_L(t) + \dfrac{V_L(t)}{R} = I_S \rightarrow i_L(t) + \dfrac{L}{R}\dfrac{di_L}{dt} = I_S$

$\dfrac{di_L}{dt} + \dfrac{R}{L} i_L = \dfrac{R I_S}{L}$

$i_L(0^+) = I_0$ olsun.

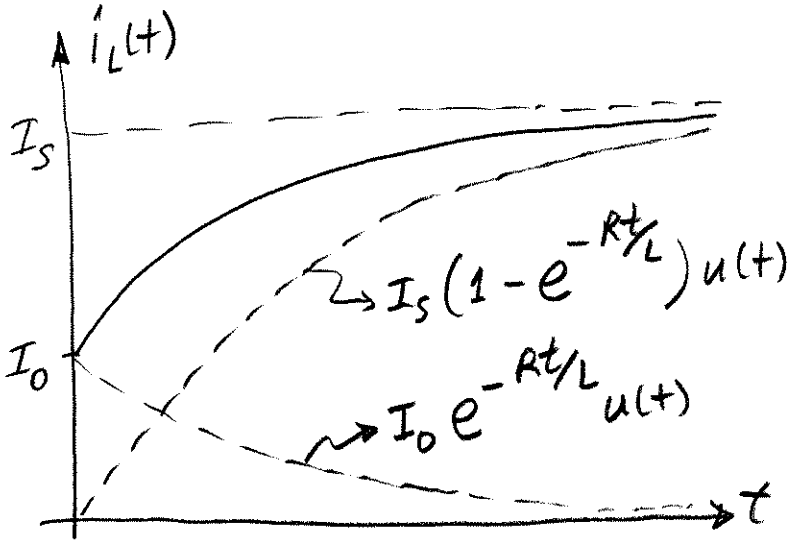
$t=0^+$'den itibaren devreye I_S sabit akımı veriliyor.

$i_L(t) = \left(A e^{-\frac{Rt}{L}} u(t) + B\right) u(t)$

$\left.\begin{array}{l} i_L(0^+) = A + B = I_0 \\ i_L(\infty) = B = I_S \end{array}\right\} \quad \begin{array}{l} A = I_0 - B \\ \quad = I_0 - I_S \end{array}$

$i_L(t) = \left((I_0 - I_S) e^{-\frac{Rt}{L}} u(t) + I_S\right) u(t)$

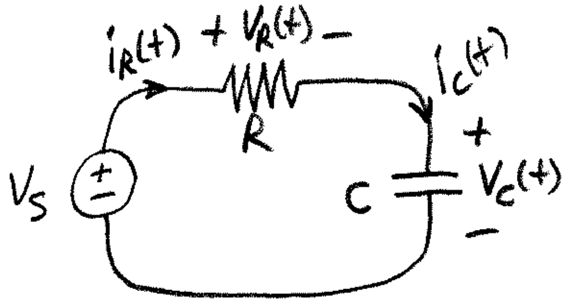
$= \underbrace{I_0 e^{-\frac{Rt}{L}} u(t)}_{\text{Boşalma}} + \underbrace{I_S\left(1 - e^{-\frac{Rt}{L}}\right) u(t)}_{\text{Dolma}}$



$V_L(t) = L\dfrac{di_L}{dt} = V_R(t)$

$= R i_R(t) = R(I_S - i_L(t))$

$= R(I_S - I_0) e^{-\frac{Rt}{L}} u(t)$



$$i_R(t) = i_C(t) = C\frac{dV_C(t)}{dt}$$

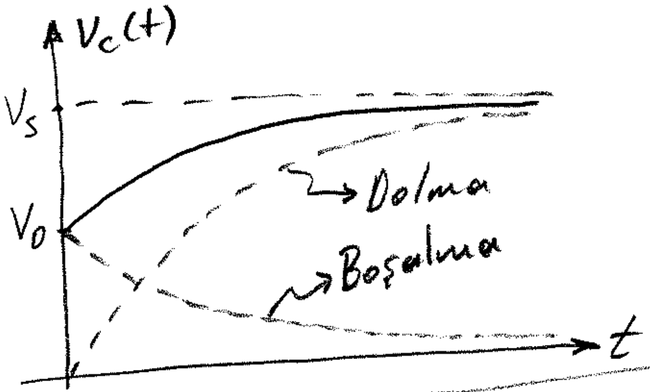
$V_C(t) + V_R(t) = V_S$ çözülünce

$$\frac{dV_C}{dt} + \frac{1}{RC}V_C = \frac{V_S}{RC}$$ elde edilir.

$V_C(0^+) = V_0$ olsun. $t=0^+$'den itibaren devreye V_S sabit voltajı veriliyor.

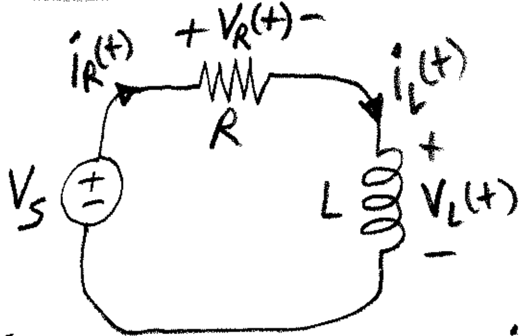
$$V_C(t) = \left((V_0 - V_S)e^{-t/RC} + V_S\right)u(t)$$

$$= \underbrace{V_0 e^{-t/RC}u(t)}_{\text{Boşalma}} + \underbrace{V_S\left(1 - e^{-t/RC}\right)u(t)}_{\text{Dolma}}$$



$$i_C(t) = C\frac{dV_C}{dt} = \frac{V_S - V_C}{R}$$

$$= \frac{V_S - V_0}{R}e^{-t/RC}u(t)$$



$$i_R(t) = i_L(t)\,, \quad V_R(t) + V_L(t) = V_S$$

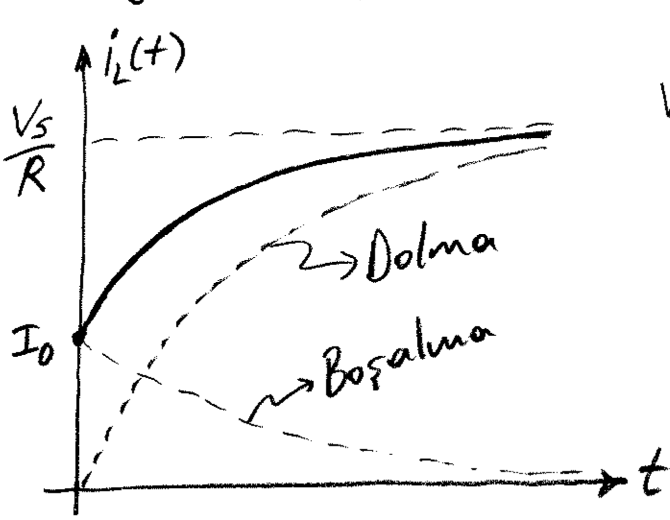
$$R i_L(t) + L\frac{di_L}{dt} = V_S \Rightarrow \frac{di_L}{dt} + \frac{R}{L}i_L = \frac{V_S}{L}$$

Dif. denklemi çözülünce

$i_L(0^+) = I_0$ olsun. $t=0^+$'den itibaren devreye V_S sabit voltajı veriliyor.

$$i_L(t) = \left(\left(I_0 - \frac{V_S}{R}\right)e^{-\frac{Rt}{L}} + \frac{V_S}{R}\right)u(t)$$

$$= \underbrace{I_0 e^{-\frac{Rt}{L}}u(t)}_{\text{Boşalma}} + \underbrace{\frac{V_S}{R}\left(1 - e^{-\frac{Rt}{L}}\right)u(t)}_{\text{Dolma}}$$

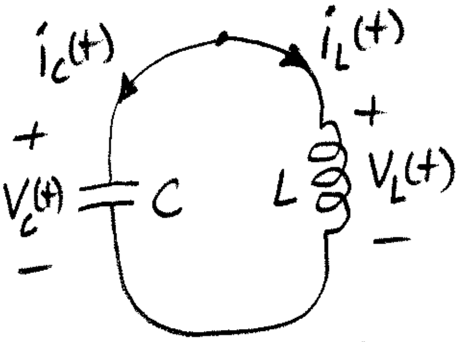


$$V_L(t) = L\frac{di_L(t)}{dt} = V_S - V_R(t)$$

$$= V_S - R\,i_L(t)$$

$$= (V_S - R I_0)e^{-Rt/L}u(t)$$

İkinci Mertebeden Devreler



$$V_c(t) = V_L(t) = L\frac{di_L(t)}{dt}, \quad i_c(t) = C\frac{dV_c(t)}{dt}$$

$$i_c(t) + i_L(t) = 0 \Rightarrow C\frac{dV_c(t)}{dt} + i_L(t) = 0$$

$$C\frac{d^2V_c}{dt} + \frac{di_L}{dt} = 0 \Rightarrow \frac{d^2V_c}{dt^2} + \frac{1}{LC}V_c = 0$$

$V_c(0^+) = V_0$ olsun

$i_L(0^+) = I_0$ olsun

$$s^2 + \frac{1}{LC} = 0 \Rightarrow s = \mp i\frac{1}{\sqrt{LC}}$$

$$V_c(t) = \left(A\cos\frac{t}{\sqrt{LC}} + B\sin\frac{t}{\sqrt{LC}}\right)u(t)$$

$$i_L(t) = -i_c(t) = -C\frac{dV_c(t)}{dt} = \sqrt{\frac{C}{L}}\left(A\sin\frac{t}{\sqrt{LC}} - B\cos\frac{t}{\sqrt{LC}}\right)u(t)$$

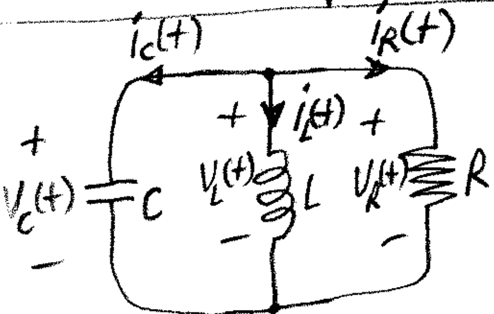
$$V_c(0^+) = A = V_0$$

$$i_L(0^+) = -B\sqrt{\frac{C}{L}} = I_0$$

$$B = -I_0\sqrt{L/C}$$

$$V_c(t) = \left(V_0\cos\frac{t}{\sqrt{LC}} - \sqrt{\frac{L}{C}}I_0\sin\frac{t}{\sqrt{LC}}\right)u(t)$$

$$i_L(t) = \left(I_0\cos\frac{t}{\sqrt{LC}} + \sqrt{\frac{C}{L}}V_0\sin\frac{t}{\sqrt{LC}}\right)u(t)$$



$$V_c(t) = V_L(t) = V_R(t)$$

$$i_c(t) + i_L(t) + i_R(t) = 0$$

$$C\frac{dV_c}{dt} + i_L + \frac{V_c}{R} = 0$$

$$C\frac{d^2V_c}{dt^2} + \frac{di_L}{dt} + \frac{1}{R}\frac{dV_c}{dt} = 0$$

$$LC\frac{d^2V_c}{dt} + L\frac{di_L}{dt} + \frac{L}{R}\frac{dV_c}{dt} = 0$$

$$\frac{d^2V_c}{dt^2} + \frac{1}{RC}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0$$

$\alpha = \frac{1}{2RC}$ Sönümleme frekansı

$$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + w^2V_c = 0$$

$w = \frac{1}{\sqrt{LC}}$ Salınım frekansı

$$s^2 + 2\alpha s + w^2 = 0$$

$$(s+\alpha)^2 + w^2 - \alpha^2 = 0$$

$$s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - w^2}$$

$$w_d = \sqrt{w^2 - \alpha^2}$$

$\alpha > w$ ise

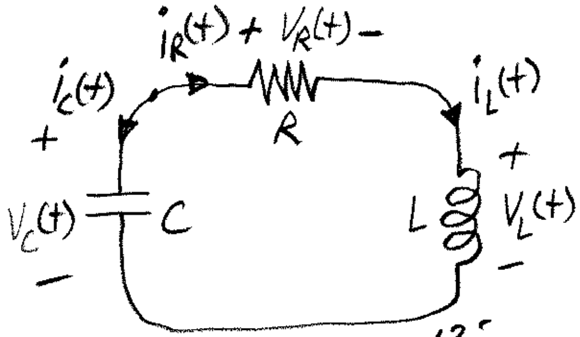
$$V_c(t) = \left(Ae^{s_1t} + Be^{s_2t}\right)u(t)$$

$\alpha = w$ ise

$$V_c(t) = (At + B)e^{-\alpha t}u(t)$$

$\alpha < w$ ise

$$V_c(t) = \left(A\cos w_dt + B\sin w_dt\right)e^{-\alpha t}u(t)$$



$$i_L(t) = i_R(t) = -i_C(t) = -C\frac{dV_C(t)}{dt}$$

$$V_C(t) - V_R(t) - V_L(t) = 0$$

$$V_C(t) - R i_L(t) - L\frac{di_L(t)}{dt} = 0$$

$$\frac{dV_C}{dt} - R\frac{di_L}{dt} - L\frac{d^2 i_L}{dt^2} = 0 \qquad \alpha = \frac{R}{2L} \text{ Sönümleme frekansı}$$

$$-C\frac{dV_C}{dt} + RC\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = 0 \qquad w = \frac{1}{\sqrt{LC}} \text{ Salınım frekansı}$$

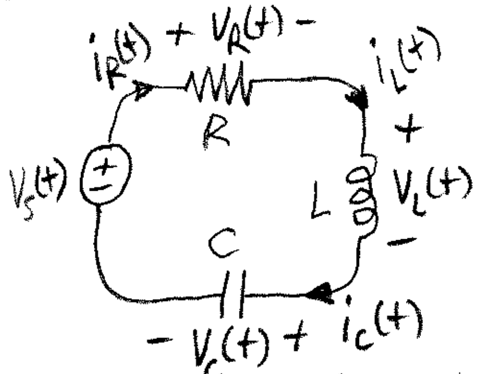
$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0 \Rightarrow \frac{d^2 i_L}{dt^2} + 2\alpha\frac{di_L}{dt} + w^2 i_L = 0$$

$$s^2 + 2\alpha s + w^2 = 0 \Rightarrow s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - w^2}, \quad w_d = \sqrt{w^2 - \alpha^2}$$

$\alpha > w$ ise $i_L(t) = \left(Ae^{s_1 t} + Be^{s_2 t}\right)u(t)$

$\alpha = w$ ise $i_L(t) = (At + B)e^{-\alpha t}u(t)$

$\alpha < w$ ise $i_L(t) = \left(A\cos w_d t + B\sin w_d t\right)e^{-\alpha t}u(t)$



$$i_R(t) = i_L(t) = i_C(t) = C\frac{dV_C(t)}{dt}$$

$$V_R(t) + V_L(t) + V_C(t) = V_S(t)$$

$$R i_L(t) + L\frac{di_L(t)}{dt} + V_C(t) = V_S(t)$$

$$R\frac{di_L}{dt} + L\frac{d^2 i_L}{dt^2} + \frac{dV_C}{dt} = \frac{dV_S}{dt} \Rightarrow \frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = \frac{1}{L}\frac{dV_S}{dt}$$

$\alpha = \frac{R}{2L}$ Sönümleme frekansı

$$\frac{d^2 i_L}{dt^2} + 2\alpha\frac{di_L}{dt} + w^2 i_L = \frac{1}{L}\frac{dV_S}{dt}$$

$w = \frac{1}{\sqrt{LC}}$ salınım frekansı

2. mertebeden homojen olmayan lineer dif. denklemi

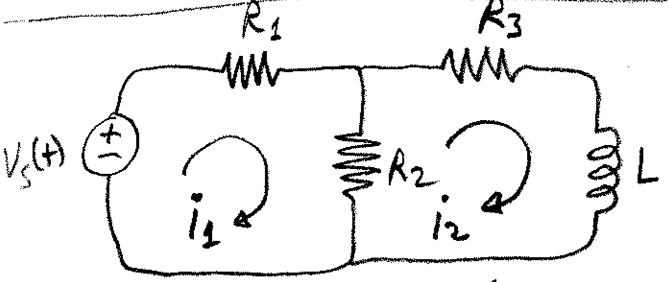
$\frac{dV_S}{dt} = 0$ için i_{L1} homojen çözüm

$\frac{dV_S}{dt} \neq 0$ için i_{L2} hom. olmayan çözüm

$$i_L(t) = i_{L1}(t) + i_{L2}(t)$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$\left.\begin{array}{l} x(t) = \delta(t) \text{ ise } y(t) = h(t) \\ x(t) = u(t) \text{ ise } y(t) = s(t) \end{array}\right\} \quad h(t) = \frac{ds(t)}{dt}, \quad s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$



$V_s(t) = 30u(t), \quad i_L(0^+) = 1A$

$R_1 = 3\Omega, \ R_2 = 6\Omega, \ R_3 = 3\Omega$

$L = 2H, \quad i_L(t) = ?, \quad V_L(t) = ?$

$V_s - (R_1+R_2)i_1 + R_2 i_2 = 0 \Longrightarrow 10 - 3i_1 + 2i_2 = 0$ 1. denklem

$R_2 i_1 - (R_2+R_3)i_2 - L\frac{di_2}{dt} = 0 \Rightarrow 6i_1 - 9i_2 - 2\frac{di_2}{dt} = 0$ 2. denklem

1. denklemin 2 katıyla 2. denklemi topla. $\frac{di_2}{dt} + 2.5 i_2 = 10$

$i_2 = i_L$ olduğundan $\frac{di_L}{dt} + 2.5 i_L = 10$

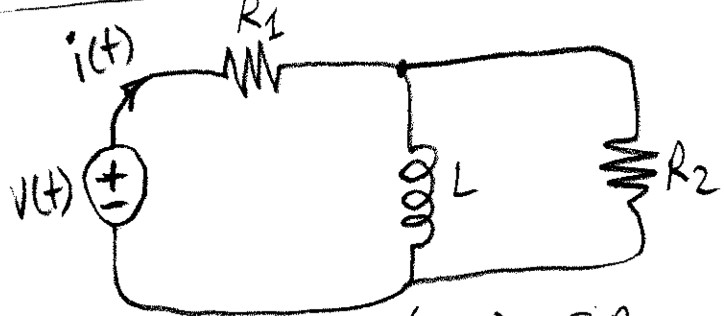
$\frac{di_L(\infty)}{dt} = 0$ olduğundan $i_L(\infty) = 4A$

$i_L(t) = \left(A e^{-2.5t} + B\right) u(t) \qquad i_L(t) = \left(4 - 3e^{-2.5t}\right) u(t)$

$$\left.\begin{array}{l} i_L(0^+) = A + B = 1 \\ i_L(\infty) = B = 4 \end{array}\right\} \begin{array}{l} A = 1 - B \\ \quad = -3 \end{array} \qquad V_L(t) = L\frac{di_L}{dt} = 15 e^{-2.5t} u(t)$$

$R_L = (R_1 \parallel R_2) + R_3 = 3\Omega \parallel 6\Omega + 3\Omega = 5\Omega$

$\tau = \frac{L}{R_L} = 0.4\,sn$ Zaman Sabitesi $\qquad 5\tau = 2sn$ Durulma zamanı



$t = 0$ için $V_L(t) = \frac{R_2 V(t)}{R_1 + R_2} = 12\delta(t)$

$V(t) = 18\delta(t) \quad i_L(0^-) = 3A$

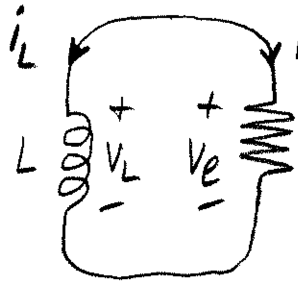
$R_1 = 3\Omega \quad R_2 = 6\Omega \quad L = 4H$

$i(t) = ?$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} V_L(\tau)\, d\tau$$

$$= 3A + \frac{1}{4}\int_{0^-}^{0^+} 12\delta(\tau)\, d\tau$$

$$= 3A + 3\int_{0^-}^{0^+} \delta(\tau)\, d\tau = 6A$$



$R_e = R_1 // R_2 = 2\Omega$

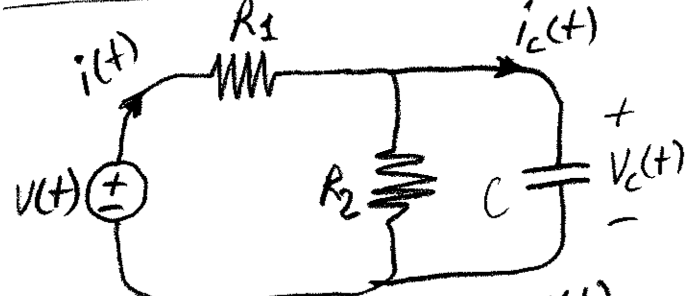
$$i_L(t) + \frac{V_L(t)}{R_e} = 0$$

$$i_L(t) + \frac{L}{R_e}\frac{di_L(t)}{dt} = 0$$

$$\frac{di_L}{dt} + \frac{R_e}{L} i_L = 0 \Rightarrow \frac{di_L}{dt} + 0.5 i_L = 0 \Rightarrow i_L(t) = 6e^{-0.5t}u(t)$$

$$V_L(t) = L\frac{di_L}{dt} = -12e^{-0.5t}u(t) \qquad i(t) = i_L(t) + i_{R2}(t)$$

$$i_{R2}(t) = \frac{V_L(t)}{R_2} = -2e^{-0.5t}u(t) \qquad = 4e^{-0.5t}u(t)$$



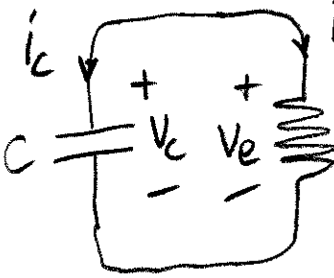
$v(t) = 12\delta(t)$, $V_c(0^-) = 4V$

$R_1 = 3\Omega$, $R_2 = 6\Omega$, $C = 2F$

$i(t) = ?$ $\quad \tau = ?$ $\quad$ Dur. Zam. = ?

$$t=0 \text{ için } \quad i_c(t) = \frac{v(t)}{R_1} = 4\delta(t) + \frac{4}{6}$$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_c(\tau)d\tau = 4V + \frac{1}{2}\int_{0^-}^{0^+} 4\delta(\tau)d\tau = 6V$$



$R_e = R_1 // R_2 = 2\Omega$

$$C\frac{dV_c}{dt} + \frac{V_c}{R_e} = 0$$

$$\frac{dV_c}{dt} + \frac{1}{R_e C}V_c = 0$$

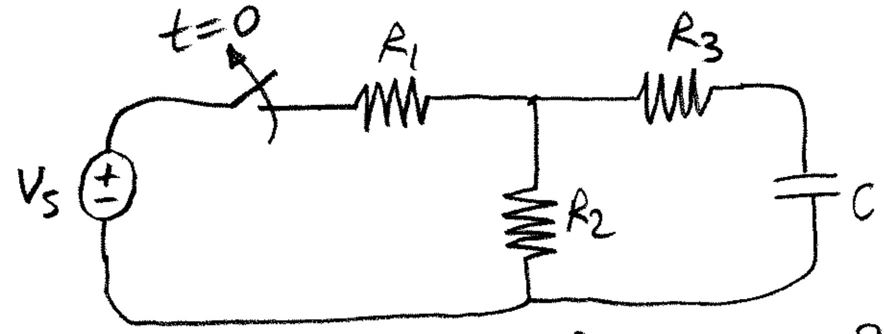
$$\frac{dV_c}{dt} + 0.25V_c = 0 \Rightarrow V_c(t) = 6e^{-0.25t}u(t)$$

$$i_c(t) = C\frac{dV_c}{dt} = -3e^{-0.25t}u(t) \qquad i(t) = i_c(t) + i_R(t)$$

$$i_{R2}(t) = \frac{V_c(t)}{R_2} = e^{-0.25t}u(t) \qquad = -2e^{-0.25t}u(t)$$

$$\tau = R_e C = 2\Omega \times 2F = 4sn$$

Durulma Zamanı $= 5\tau = 20 sn$



$V_s = 25V$, $R_1 = 60\Omega$

$R_2 = 40\Omega$, $R_3 = 10\Omega$

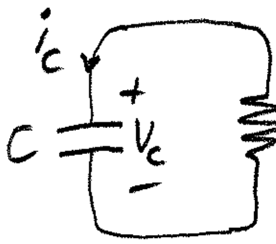
$C = 0.1F$

$V_c(0^+) = ?$, $W_c(0^+) = ?$, $V_c(t) = ?$, $i_c(t) = ?$

$\tau = ?$ Durulma zamanı $= ?$

$$V_c(0^+) = V_c(0^-) = \frac{R_2 V_s}{R_1 + R_2} = \frac{40\Omega \times 25V}{100\Omega} = 10V$$

$$W_c(0^+) = \frac{1}{2} C\, V_c^2(0^+) = \frac{1}{2} \times 0.1F \times (10V)^2 = 5J$$



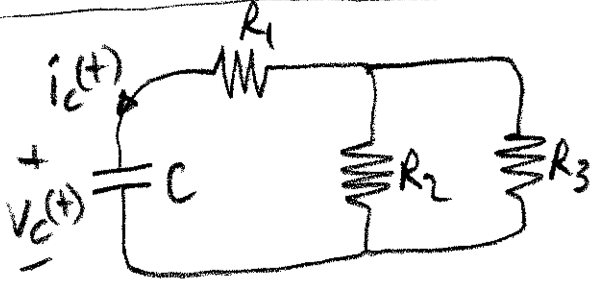
$R_e = R_2 + R_3 = 50\Omega$

$\tau = R_e C = 50\Omega \times 0.1F = 5\,sn$

Durulma zamanı $= 5\tau = 25\,sn$

$$V_c(t) = V_c(0^+)\, e^{-t/\tau} u(t) = 10 e^{-0.2t} u(t)$$

$$i_c(t) = C \frac{dV_c(t)}{dt} = -0.2 e^{-0.2t} u(t)$$



$V_c(0^+) = 30V$, $R_1 = 30\Omega$, $R_2 = 60\Omega$

$C = 40mF$, $R_3 = 30\Omega$

$V_c(t) = ?$ $i_c(t) = ?$ Dur. Zam. $= ?$

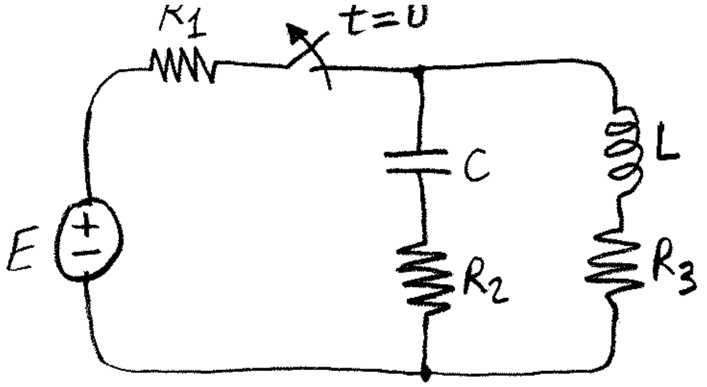
$$R_e = R_1 + R_2 \| R_3 = 30\Omega + 60\Omega \| 30\Omega = 50\Omega$$

$$\tau = R_e C = 50\Omega \times 40mF = 2\,sn$$

Durulma zamanı $= 5\tau = 10\,sn$

$$V_c(t) = V_c(0^+)\, e^{-t/\tau} u(t) = 30 e^{-0.5t} u(t)$$

$$i_c(t) = C \frac{dV_c(t)}{dt} = -0.6 e^{-0.5t} u(t)$$

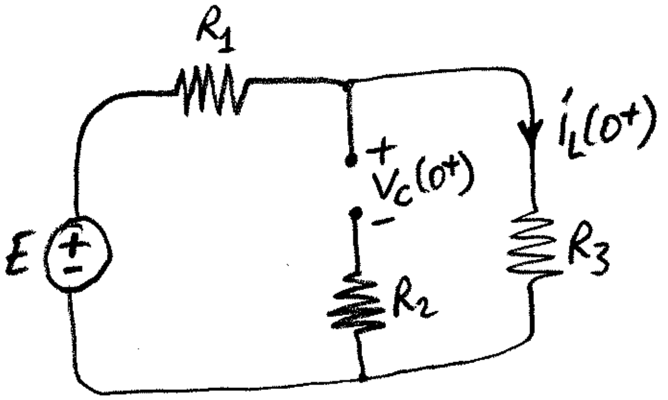


$E = 12V,\ C = 8mF,\ L = 5H$

$R_1 = 15\Omega,\ R_2 = 21\Omega,\ R_3 = 9\Omega$

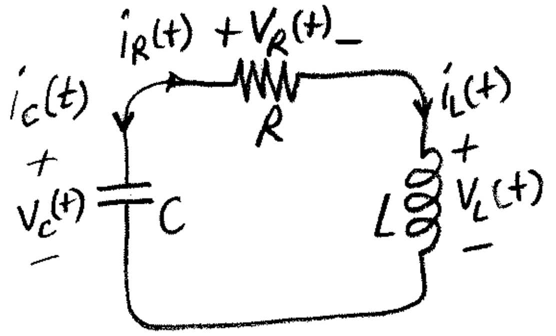
$\alpha = ?\quad w = ?$

$i_L(t) = ?\quad V_c(t) = ?$



$$i_L(0^+) = \frac{E}{R_1 + R_3} = \frac{12V}{24\Omega} = 0.5A$$

$$V_c(0^+) = R_3 i_L(0^+) = 9\Omega \times 0.5A = 4.5V$$



$$R = R_2 + R_3 = 21\Omega + 9\Omega = 30\Omega$$

$$\alpha = \frac{R}{2L} = \frac{30\Omega}{2 \times 5H} = 3$$

$$w = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5H \times 8mF}} = \frac{1}{\sqrt{0.04}} = 5$$

$w > \alpha$ olduğundan

$$w_d = \sqrt{w^2 - \alpha^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

$$i_L(t) = (A\cos 4t + B\sin 4t)e^{-3t}u(t)$$

$$\frac{di_L}{dt} = ((4B - 3A)\cos 4t - (4A + 3B)\sin 4t)e^{-3t}u(t)$$

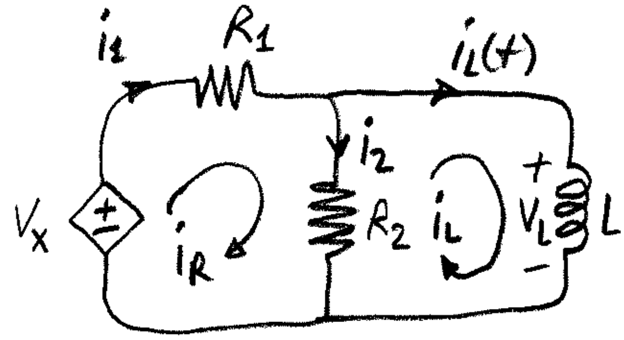
$$V_c(t) = V_R(t) + V_L(t) = R i_L(t) + L\frac{di_L}{dt} = 30 i_L(t) + 5\frac{di_L}{dt}$$

$$= ((15A + 20B)\cos 4t + (15B - 20A)\sin 4t) \cdot e^{-3t} \cdot u(t)$$

$$\left.\begin{aligned} i_L(0^+) &= A = 0.5 \\ V_c(0^+) &= 15A + 20B = 4.5 \end{aligned}\right\} B = \frac{4.5 - 15A}{20} = -0.15$$

$$i_L(t) = (0.5\cos 4t - 0.15\sin 4t)e^{-3t}u(t)$$

$$V_c(t) = (4.5\cos 4t - 12.25\sin 4t)e^{-3t}u(t)$$



$R_1 = 3\Omega$, $R_2 = 2\Omega$

$L = 8H$, $V_x(t) = 15 - 7\, i_L(t)$

$i_L(0^+) = 5A$ $\quad i_L(t) = ?$

$V_L(t) = ?$ $\quad i_1(t) = ?$ $\quad i_2(t) = ?$

$V_x - (R_1 + R_2)\, i_R + R_2\, i_L = 0$

$15 - 7 i_L - 5 i_R + 2 i_L = 0$

$i_R = 3 - i_L$

$R_2(i_R - i_L) - L\dfrac{di_L}{dt} = 0$

$2(3 - i_L - i_L) - 8\dfrac{di_L}{dt} = 0$

$\dfrac{di_L}{dt} + 0.5\, i_L = 0.75$

$t = \infty$ için $\dfrac{di_L}{dt} = 0$ olur.

$i_L(\infty) = \dfrac{0.75}{0.5} A = 1.5 A$

$i_L(t) = \left(A e^{-0.5t} + B\right) u(t)$

$\left.\begin{array}{l} i_L(0^+) = A + B = 5 \\ i_L(\infty) = B = 1.5 \end{array}\right\} A = 3.5$

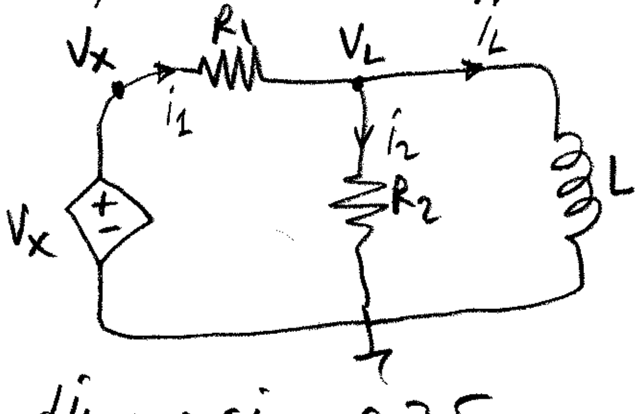
$i_L(t) = \left(3.5 e^{-0.5t} + 1.5\right) u(t)$

$V_L(t) = L\dfrac{di_L}{dt} = -14 e^{-0.5t} u(t)$

$i_1(t) = i_R(t) = 3 - i_L = \left(1.5 - 3.5 e^{-0.5t}\right) u(t)$

$i_2(t) = i_R(t) - i_L(t) = 3 - 2 i_L = -7 e^{-0.5t} u(t)$

Node Analizle yapılsa idi



$\dfrac{V_L - V_x}{R_1} + \dfrac{V_L}{R_2} + i_L = 0$

$(R_1 + R_2) V_L - R_2 V_x + R_1 R_2\, i_L = 0$

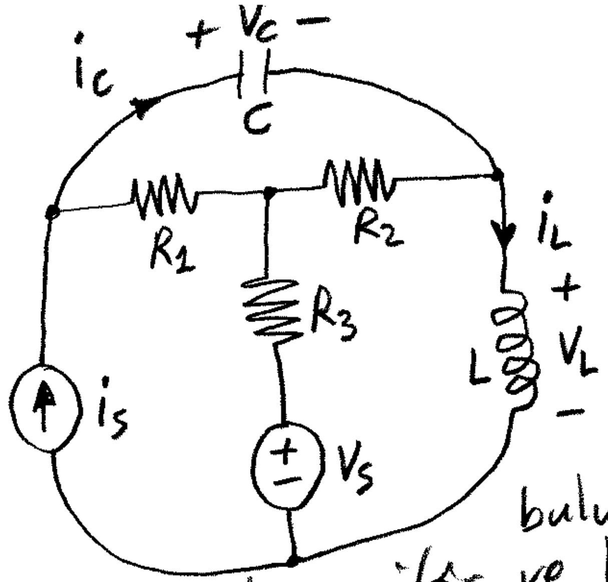
$(R_1 + R_2) L\dfrac{di_L}{dt} - R_2(15 - 7 i_L) + R_1 R_2\, i_L = 0$

$\dfrac{di_L}{dt} + 0.5\, i_L = 0.75$

$t = \infty$ için $\dfrac{di_L}{dt} = 0$ olur.

$i_L(\infty) = 1.5 A$

Diğer kısımlar yukarıdaki gibi yapılır.

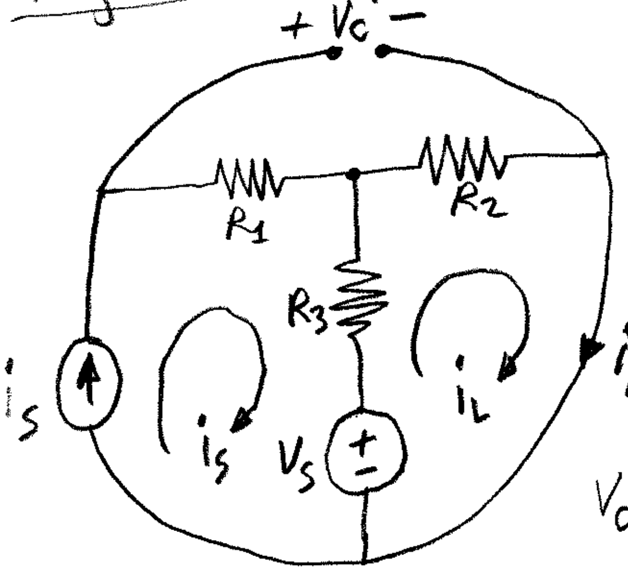


$V_s = 13V$, $i_s = 2A$

$C = 16\,mF$, $L = 4H$

$R_1 = 8\Omega$, $R_2 = 3\Omega$, $R_3 = 4\Omega$

DC şartlar oluştuğunda üretilen ve tüketilen güçleri bulunuz. Ayrıca, dc şartlar oluştuğunda kapasitör ve bobinde depolanan enerjiyi bulunuz.



$$V_s - (R_2 + R_3)\, i_L + R_3 i_s = 0$$

$$i_L = \frac{V_s + R_3 i_s}{R_2 + R_3} = \frac{13V + 4\Omega \times 2A}{3\Omega + 4\Omega}$$

$$= \frac{21V}{7\Omega} = 3A$$

$$V_c = R_1 i_s + R_2 i_L = 8\Omega \times 2A + 3\Omega \times 3A$$

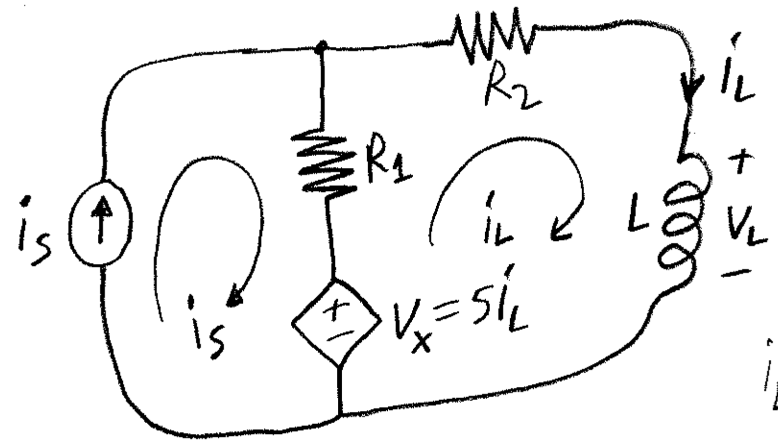
$$= 16V + 9V = 25V$$

$$\left.\begin{aligned} P_i &= V_c i_s = 25V \times 2A = 50W \\ P_V &= V_s (i_L - i_s) = 13V \times (3A - 2A) = 13W \end{aligned}\right\} \begin{aligned} P_{ürt} &= P_i + P_V \\ &= 50W + 13W = 63W \end{aligned}$$

$$\left.\begin{aligned} P_1 &= R_1 i_s^2 = 8\Omega \times (2A)^2 = 32W \\ P_2 &= R_2 i_L^2 = 3\Omega \times (3A)^2 = 27W \\ P_3 &= R_3 (i_L - i_s)^2 = 4\Omega \times (3A - 2A)^2 = 4W \end{aligned}\right\} \begin{aligned} P_{tük} &= P_1 + P_2 + P_3 \\ &= 32W + 27W + 4W \\ &= 63W \end{aligned}$$

$$W_C = \frac{1}{2} C V_c^2 = \frac{1}{2} \times 16\,mF \times (25V)^2 = 5\,J$$

$$W_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \times 4H \times (3A)^2 = 18\,J$$



$R_1 = 3\Omega$, $R_2 = 5\Omega$

$L = 6H$, $i_s = 5A$

$i_L(0^+) = 2A$

$i_L(t) = ?$ $\quad V_L(t) = ?$

$$V_x - (R_1+R_2)\,i_L + R_1 i_s - L\frac{di_L}{dt} = 0$$

$$5i_L - 8i_L + 15 - 6\frac{di_L}{dt} = 0$$

$$15 - 3i_L - 6\frac{di_L}{dt} = 0 \Rightarrow \frac{di_L}{dt} + 0.5\,i_L = 2.5$$

$t = \infty$ için $\dfrac{di_L}{dt} = 0$ olduğundan

$$\frac{di_L(\infty)}{dt} + 0.5\,i_L(\infty) = 2.5 \Rightarrow i_L(\infty) = 5A$$

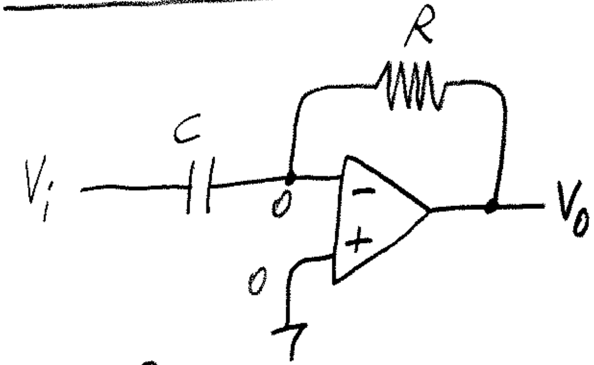
$$i_L(t) = \left(Ae^{-0.5t} + B\right)u(t)$$

$$\left.\begin{aligned} i_L(0^+) &= A + B = 2 \\ i_L(\infty) &= B = 5 \end{aligned}\right\} A = 2 - B = -3$$

$$i_L(t) = \left(5 - 3e^{-0.5t}\right)u(t)$$

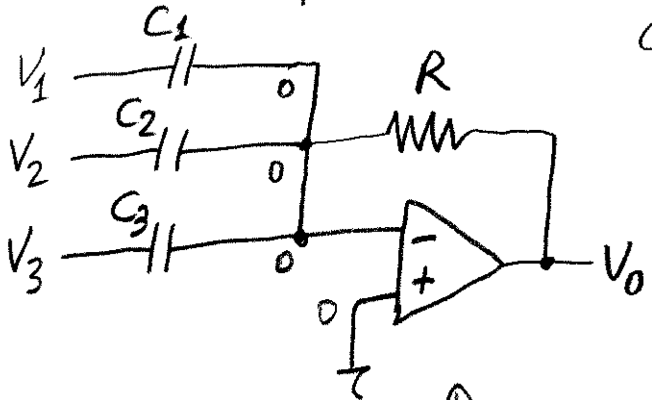
$$V_L(t) = L\frac{di_L(t)}{dt} = 9e^{-0.5t}\,u(t)$$

Türev Alıcı Devre



$$C\frac{dV_i}{dt} + \frac{V_0}{R} = 0$$

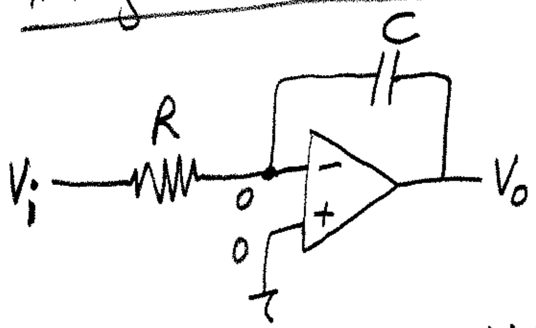
$$V_0(t) = -RC\frac{dV_i(t)}{dt}$$



$$C_1\frac{dV_1}{dt} + C_2\frac{dV_2}{dt} + C_3\frac{dV_3}{dt} + \frac{V_0}{R} = 0$$

$$V_0(t) = -RC_1\frac{dV_1}{dt} - RC_2\frac{dV_2}{dt} - RC_3\frac{dV_3}{dt}$$

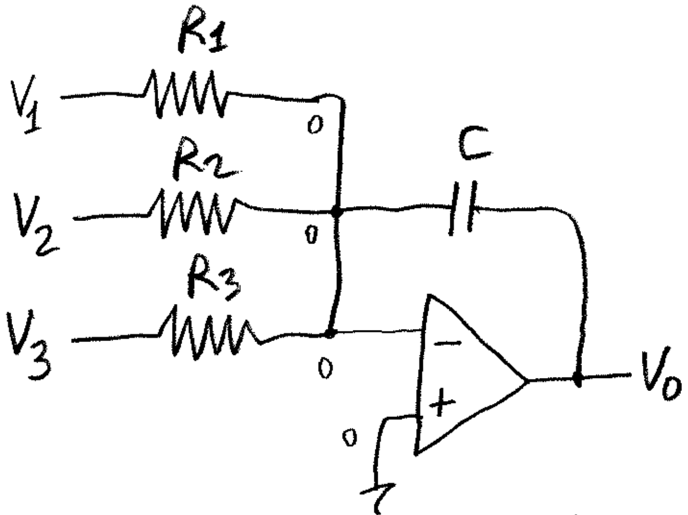
integral Alıcı Devre



$$\frac{V_i}{R} + C\frac{dV_0}{dt} = 0$$

$$\frac{dV_0}{dt} = -\frac{1}{RC}V_i$$

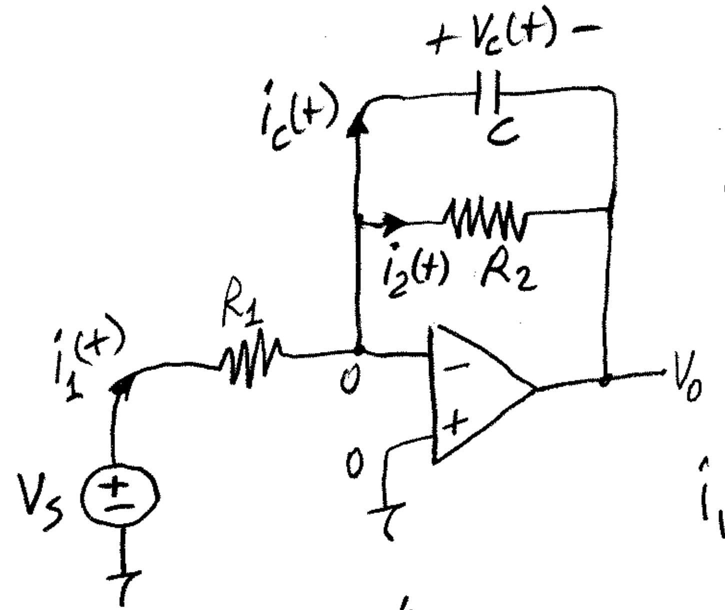
$$V_0(t) = V_0(0) - \frac{1}{RC}\int_0^t V_i(z)\,dz$$



$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + C\frac{dV_0}{dt} = 0$$

$$\frac{dV_0}{dt} = -\frac{V_1}{R_1C} - \frac{V_2}{R_2C} - \frac{V_3}{R_3C}$$

$$V_0(t) = V_0(0) - \frac{1}{R_1C}\int_0^t V_1(z)\,dz - \frac{1}{R_2C}\int_0^t V_2(z)\,dz - \frac{1}{R_3C}\int_0^t V_3(z)\,dz$$



$V_s = 15V$, $R_1 = 12K$, $R_2 = 8K$

$C = 25\mu F$, $V_c(0^+) = 4V$

$\tau = ?$ Dur. Zam. = ?

$V_o(t) = ?$ $i_c(t) = ?$

$i_1(t) = ?$ $i_2(t) = ?$

$$\frac{V_s}{R_1} + \frac{V_o}{R_2} + C\frac{dV_o}{dt} = 0$$

$$\frac{dV_o}{dt} + \frac{1}{R_2 C}V_o = -\frac{V_s}{R_1 C}$$

$$\frac{dV_o}{dt} + 5V_o = -50$$

$\tau = R_2 C = 8K \times 25\mu F = 0.2\,sn$

Durulma Zamanı $= 5\tau = 1\,sn$

$V_o(0^+) = -V_c(0^+) = -4V$

$t = \infty$ için $\frac{dV_o}{dt} = 0$ olduğundan

$V_o(\infty) = -10V$

$$V_o(t) = \left(A e^{-5t} + B\right) u(t)$$

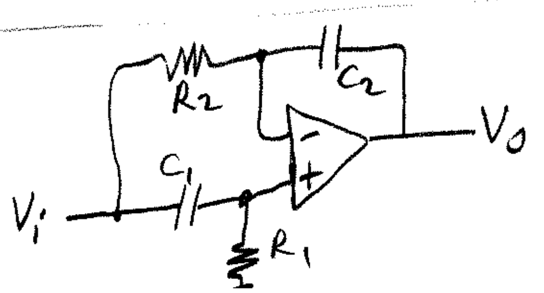
$$\left.\begin{array}{l} V_o(0^+) = A + B = -4 \\ V_o(\infty) = B = -10 \end{array}\right\} A = -4 - B = 6$$

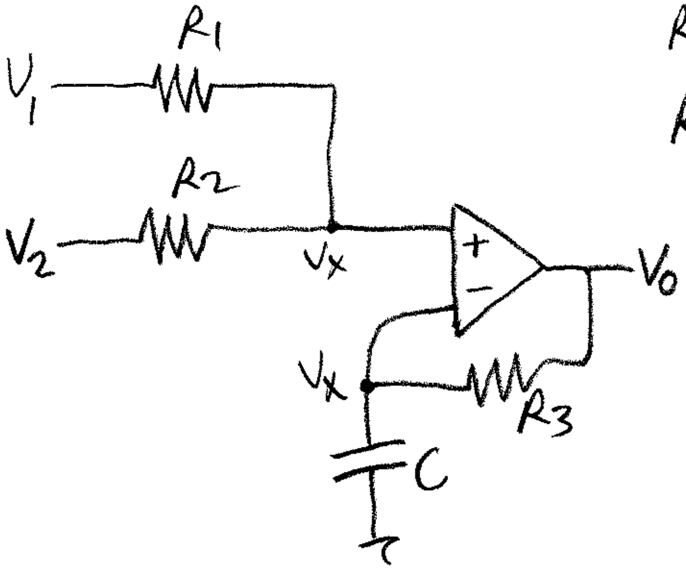
$$V_o(t) = \left(6e^{-5t} - 10\right) u(t)$$

$$i_c(t) = C\frac{dV_c}{dt} = -C\frac{dV_o}{dt} = 0.75 \times 10^{-3} \times e^{-5t} u(t)$$

$$i_2(t) = -\frac{V_o(t)}{R_2} = \left(1.25 - 0.75e^{-5t}\right) \times 10^{-3} \times u(t)$$

$$i_1(t) = i_c(t) + i_2(t) = 1.25 \times 10^{-3} \times u(t)$$





$R_1 = 2k\Omega \quad R_2 = 3k\Omega$

$R_3 = 5k\Omega \quad C = 6\mu F$

$V_1(t) = 9\cos 40t$

$V_2(t) = 7\sin 40t$

$V_o(t) = ?$

$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} = 0$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V_x}{R_1} + \frac{V_x}{R_2}$$

$$V_x = \frac{R_2}{R_1+R_2} V_1 + \frac{R_1}{R_1+R_2} V_2$$

$$V_x = 0.6 V_1 + 0.4 V_2$$

$$V_x = 5.4\cos 40t + 2.8\sin 40t$$

$$\frac{dV_x}{dt} = 112\cos 40t - 216\sin 40t$$

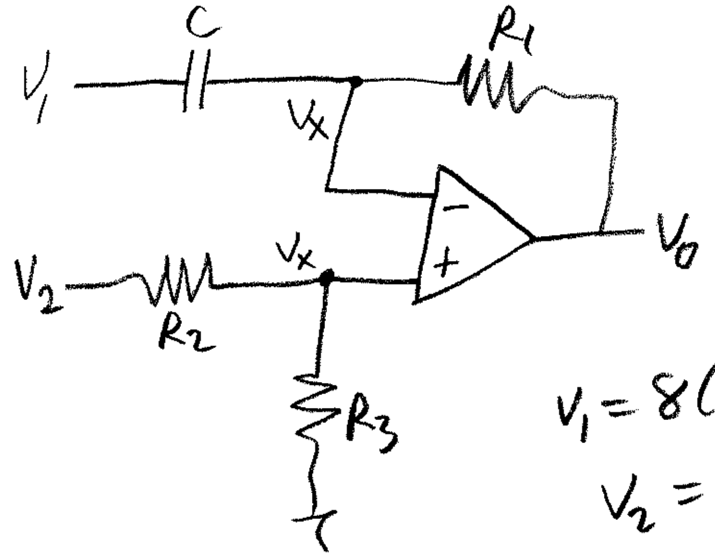
$$C\frac{dV_x}{dt} + \frac{V_x - V_o}{R_3} = 0$$

$$V_o = V_x + R_3 C \frac{dV_x}{dt}$$

$$= V_x + 0.03\frac{dV_x}{dt}$$

$$V_o(t) = (5.4\cos 40t + 2.8\sin 40t) + 0.03(112\cos 40t - 216\sin 40t)$$

$$= 8.76\cos 40t - 3.68\sin 40t$$



$R_1 = 5k\Omega$

$R_2 = 4k\Omega$

$R_3 = 6k\Omega$

$C = 10\,\mu F$

$V_1 = 8\cos 15t$

$V_2 = 6\sin 15t \qquad V_0(t) = ?$

$$\frac{V_x - V_2}{R_2} + \frac{V_x}{R_3} = 0$$

$$\frac{V_x}{R_2} + \frac{V_x}{R_3} = \frac{V_2}{R_2}$$

$$V_x = \frac{R_3}{R_2 + R_3} V_2$$

$$= 0.6 V_2$$

$$C\frac{d}{dt}(V_x - V_1) + \frac{V_x - V_0}{R_1} = 0$$

$$V_0 = V_x + R_1 C \frac{dV_x}{dt} - R_1 C \frac{dV_1}{dt}$$

$$= V_x + 0.05\frac{dV_x}{dt} - 0.05\frac{dV_1}{dt}$$

$$= 0.6V_2 + 0.03\frac{dV_2}{dt} - 0.05\frac{dV_1}{dt}$$

$$V_0(t) = 0.6\,(6\sin 15t) + 0.03\,(6\times 15\cos 15t) - 0.05\,(-8\times 15\sin 15t)$$

$$= 3.6\sin 15t + 2.7\cos 15t + 6\sin 15t$$

$$= 2.7\cos 15t + 9.6\sin 15t$$

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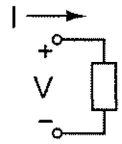
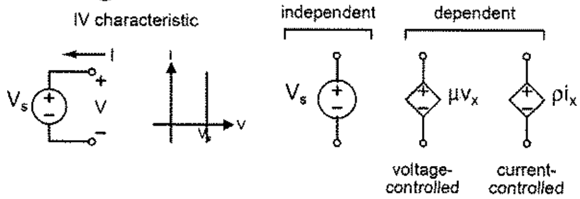
I. CIRCUIT BASICS

- Electrical quantities
  - **Current**: $I = \frac{dq}{dt}$ [Units: C/s = Amps (A)] ◦ **Voltage**: $V = \frac{dw}{dq}$ [Units: J/C = Volts (V)]
  - **Power**: $P = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = VI$
    [Units: J/s = Watts (W)]
    avg power: $\langle P \rangle = \frac{1}{T}\int_0^T I(t)V(t)dt$

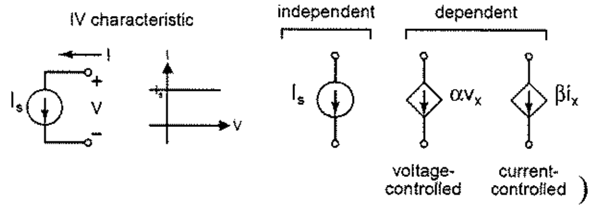
    P = IV > 0: power delivered
    P = IV < 0: power extracted



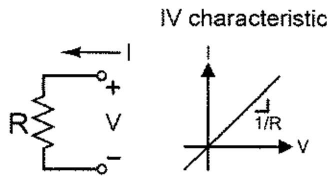
- Primitive circuit elements
  - **Voltage Source**



  - **Current Source**



  - **Resistor** – follows **Ohm's Law**: $V = IR$ (<u>note polarity</u>)
    R = resistance [Units: V/A = Ohms (Ω)]
    G = 1/R = conductance [Units: Siemens (S)]
    Resistor power dissipation: $P = IV = I^2R = \frac{V^2}{R}$

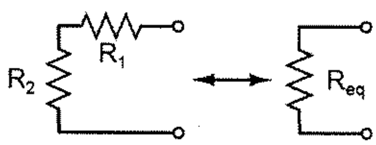


- Circuit definitions
  - **Node** – point where 2 or more circuit elements are connected
  - **Series elements** – same current flows through all elements
  - **Parallel elements** – same voltage across all elements

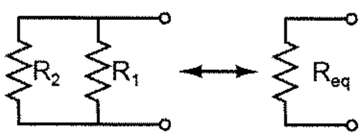
II. CIRCUIT ANALYSIS BASICS

- **KCL** (Kirchhoff's Current Law)
  - Sum of all currents entering a node = 0
  - Sum of all currents leaving a node = 0
  - Σ(currents in) = Σ(currents out)
- **KVL** (Kirchhoff's Voltage Law)
  - Sum of voltage drops around a loop = 0
  - Sum of voltage rises around a loop = 0
  - Σ(voltage drops) = Σ(voltage rises)

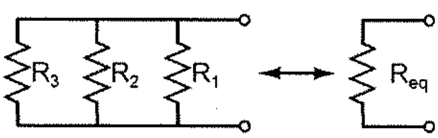
- Series resistors: $R_{eq} = \sum_{k=1}^{n} R_k$
- Parallel resistors: $\frac{1}{R_{eq}} = \sum_{k=1}^{n} \frac{1}{R_k}$



$$R_{eq} = R_1 + R_2$$

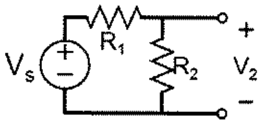


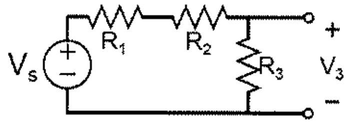
$$R_{eq} = R_1 \parallel R_2 = \frac{R_1R_2}{R_1 + R_2}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

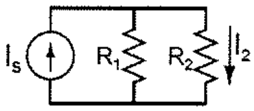
- Voltage divider

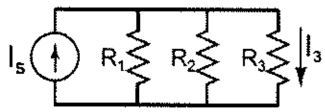




$$V_2 = \frac{R_2}{R_1 + R_2} V_S \qquad V_3 = \frac{R_3}{R_1 + R_2 + R_3} V_S$$

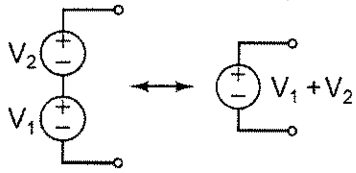
- Current divider

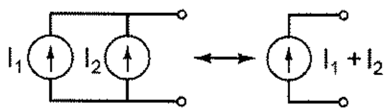




$$I_2 = \frac{R_1}{R_1 + R_2} I_S \qquad I_3 = \frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3} I_S$$

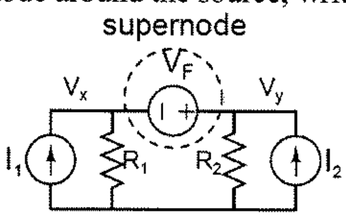
- Source combinations (series voltage sources and parallel current sources)





III. Circuit Analysis Methods

- **Nodal Analysis** – finds unknown node voltages in a circuit; once all node voltages are known, currents can be found through IV relationships of circuit elements (e.g., Ohm's Law)
  1. Choose a reference node ("ground")
  2. Define unknown voltages (those not fixed by voltage sources)
  3. Write KCL at each unknown node, expressing current in terms of node voltages
     - use IV relationships of the circuit elements (e.g., I=V/R for resistors)
  4. Solve the set of independent equations (N eqn's for N unknown node voltages)
- **Supernode** – for a floating voltage source (where both terminals are unknown voltages), define a supernode around the source, write KCL at supernode, and use the voltage source equation



$$I_1 + I_2 = V_x/R_1 + V_y/R_2$$

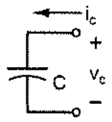
$$V_F = V_y - V_x$$

- **Superposition** – In any linear circuit containing multiple independent sources, any I or V in the circuit can be calculated as the sum of the individual contributions of each source acting alone
  - **Linear circuit** – circuit with only independent sources and linear elements (linear RLC, linear dependent sources). Linear elements have linear IV characteristics.
  1. Leave one source on and turn off all other sources
     → replace voltage source with short circuit (V=0)
     → replace current source with open circuit (I=0)
  2. Find the contribution from the "on" source
  3. Repeat for each independent source.
  4. Sum the individual contributions from each source to obtain the final result

Note: Superposition doesn't work for power, since power is nonlinear ($P=I^2R=V^2/R$)

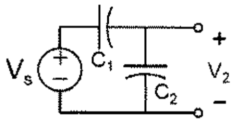
IV. CAPACITORS AND INDUCTORS

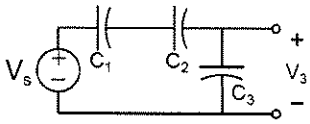
- **Capacitor** – passive circuit element that stores electric energy
  ◦ Capacitance: C = Q/V [Units: Coulombs/Volt = Farads (F)]
  ◦ IV relationship: $i_C = C\dfrac{dv_c}{dt}$



note polarity!

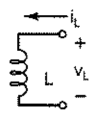
  ◦ Energy stored: $E_c = ½CV^2$
  ◦ voltage across capacitor v_c cannot change instantaneously: $v_c(0^-)=v_c(0^+)$
  ◦ in steady-state, capacitor is an open circuit ($dv_c/dt=0 \rightarrow i_c=0$)
  ◦ low freq: open circuit; high freq: short-circuit
- Parallel capacitors: $C_{eq} = \sum_{k=1}^{n} C_k$
- Series capacitors: $\dfrac{1}{C_{eq}} = \sum_{k=1}^{n}\dfrac{1}{C_k}$
- Capacitive voltage divider





$$V_2 = \frac{C_1}{C_1 + C_2} V_S \qquad V_3 = \frac{1/C_3}{1/C_1 + 1/C_2 + 1/C_3} V_S$$

- **Inductor** – passive circuit element that stores magnetic energy
  ◦ Inductance: L = Φ/I [Units: Webers/Amps = Henrys (H)]
  ◦ IV relationship: $v_L = L\dfrac{di_L}{dt}$



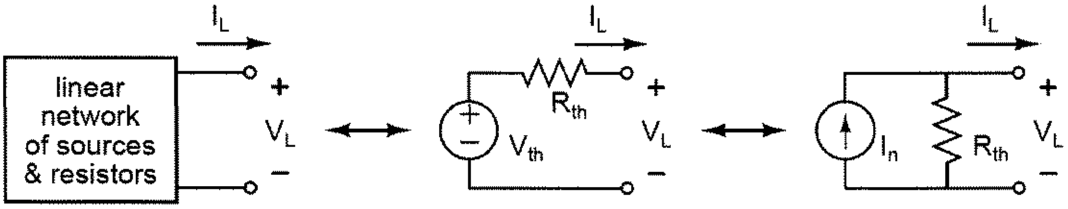
note polarity!

  ◦ Energy stored: $E_L = ½LI^2$
  ◦ current through inductor i_L cannot change instantaneously: $i_L(0^-)=i_L(0^+)$
  ◦ in steady-state, inductor is a short circuit ($di_L/dt=0 \rightarrow v_L=0$)
  ◦ low freq: short circuit; high freq: open-circuit
- Series inductors: $L_{eq} = \sum_{k=1}^{n} L_k$
- Parallel inductors: $\dfrac{1}{L_{eq}} = \sum_{k=1}^{n}\dfrac{1}{L_k}$

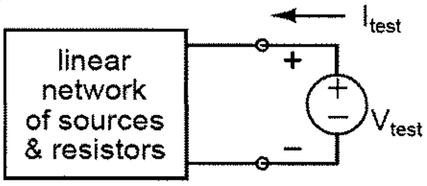
Capacitor and Inductor Summary :

	Capacitor	**Inductor**
IV relationship	$i = C\dfrac{dv}{dt}$	$v = L\dfrac{di}{dt}$
Energy storage	$E_c = ½CV^2$	$E_L = ½LI^2$
Continuity	Voltage: $v_c(0^-)=v_c(0^+)$	Current: $i_L(0^-)=i_L(0^+)$
Steady-state	Open circuit (I=0)	Short circuit (V=0)
Series	$\dfrac{1}{C_{eq}} = \sum_{k=1}^{n}\dfrac{1}{C_k}$	$L_{eq} = \sum_{k=1}^{n} L_k$
Parallel	$C_{eq} = \sum_{k=1}^{n} C_k$	$\dfrac{1}{L_{eq}} = \sum_{k=1}^{n}\dfrac{1}{L_k}$

- **Thevenin/Norton Equivalent Circuit Models** – Any linear 2-terminal network of independent sources and linear resistors can be replaced by an equivalent circuit consisting of 1 independent voltage source in series with 1 resistor (Thevenin) or 1 independent current source in parallel with 1 resistor (Norton). The circuit models have the same IV characteristics.



  ◦ Three variables: $V_{th}=V_{oc}$, $R_{th}=R_N$, $I_N=I_{sc}$.
  ◦ Thevenin/Norton relationship: $V_{th}=I_N R_{th}$ → only 2 of the 3 variables are required
  ◦ $V_{th} = V_{oc}$: open-circuit voltage – Leave the port open (I_L=0) and solve for V_{oc}.
  ◦ $I_N = I_{sc}$: short-circuit current – Short the port (V_L=0) and solve for I_N.
  ◦ R_{thc}: Thevenin/Norton resistance – Turn off all independent sources (leave the dependent sources alone). If there are no dependent sources, simplify the resistive network using series and parallel reductions to find the equivalent resistance. If dependent sources are present, attach I_{test} or V_{test} and use KCL/KVL to find $R_{th}=V_{test}/I_{test}$.

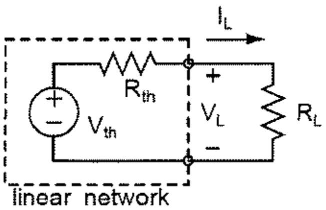


note the direction of I_{test} and the polarity of V_{test}

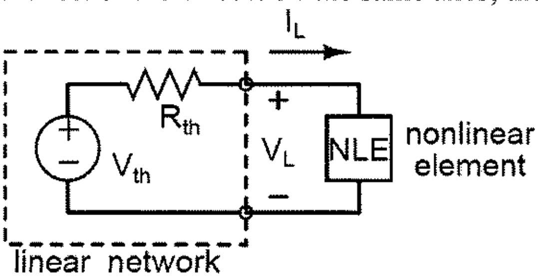
$$R_{th} = \frac{V_{test}}{I_{test}}$$

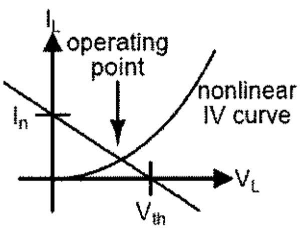
- **Source Transformations** – conversion between Thevenin and Norton equivalent circuits

- **Maximum Power Transfer Theorem**
  → power transferred to load resistor R_L is maximized when $R_L=R_{th}$



- **Load-line Analysis** – graphical method solving circuits with 1 nonlinear circuit element
  → graph the IV curves for the nonlinear circuit element and the Thevenin/Norton equivalent of the rest of the circuit on the same axes; the operating point is where the two curves intersect

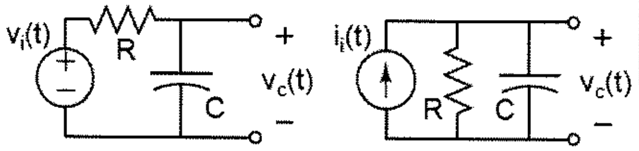




V. First-order Circuits

- **RC circuit** – contains only sources, resistors, and 1 capacitor
- **RL circuit** – contains only sources, resistors, and 1 inductor
  → voltages and currents are described by 1st-order ODE (ordinary differential equation)

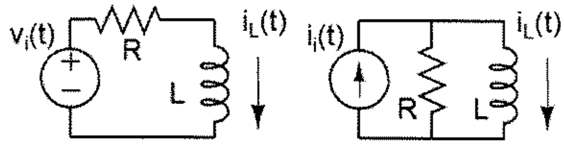
RC Circuits



$RC\frac{dv_C(t)}{dt}+v_C(t)=v_i(t)$ $\quad$ $RC\frac{dv_C(t)}{dt}+v_C(t)=i_i(t)\cdot R$

Time constant: $\tau = RC$

RL Circuits



$\frac{L}{R}\frac{di_L(t)}{dt}+i_L(t)=\frac{v_i(t)}{R}$ $\quad$ $\frac{L}{R}\frac{di_L(t)}{dt}+i_L(t)=i_i(t)$

Time constant: $\tau = L/R$

- **Time-domain Analysis for 1st-order Circuits**
  1. Write the ODE in terms of the variable of interest X(t), using KCL/KVL and IV relationships for R, L, C.
  2. Find the **homogeneous solution** $X_h(t)$ by setting input to 0 and substituting $X_h(t)=Ke^{-t/\tau}$ as the solution to find the time constant τ (τ=RC for RC circuit and τ=L/R for RL circuit) . (Note: The value of K cannot be found until the complete solution is found in Step 4.)
  3. Find the **particular solution** $X_p(t)$. Remember the output follows the form of the input:

input function	constant	exponential	sinusoid
particular solution	A	$Ae^{-\alpha t} + B\cdot te^{-\alpha t}$	Acos(wt)+Bsin(wt)

     Guess the form of the solution and solve the ODE to find any arbitrary constants.
     (Note: For sinusoidal inputs, the particular solution can be found more easily using complex impedance.)
  4. Combine the homogeneous and particular solutions to get the complete solution: $X(t) = X_h(t)+X_p(t)$. Use the initial conditions to find the missing variables (i.e., the K in $X_h(t)$).

Example: Find $v_c(t>0)$ for RC circuit w/ $v_i(t)=V_{DD}$, $v_c(0^-)=0V$.

1) $RC\frac{dv_C(t)}{dt}+v_C(t)=v_i(t)$ $\quad$ 2) $v_{c,h}(t) = Ke^{-t/\tau}$ → $RC\left(-\frac{K}{\tau}\right)e^{-t/\tau}+Ke^{-t/\tau}=0$ → $\tau = RC$
3) Since $v_i(t)$ is a constant, guess $v_{c,p}(t)=A$. Plugging into the ODE, $A=V_{DD}=v_{c,p}(t)$.
4) $v_c(t) = v_{c,h}(t) + v_{c,p}(t) = Ke^{-t/\tau} + V_{DD}$. $v_c(0^-)=v_c(0^+)$ by capacitor voltage continuity. $v_c(0)=0=K+V_{DD}$ → $K=-V_{DD}$. So, $v_c(t) = V_{DD}-V_{DD}e^{-t/\tau}$.

Note: $X_h(t)$ represents the **transient response** of the circuit and should decay to 0 as time passes. $X_p(t)$ represents the **steady-state response** of the circuit which persists after the transients have died away and which takes the form of the input.

- **Time constant τ** – amount of time for the transient exponential response $e^{-t/\tau}$ to decay by 63% (e^{-1} = 0.63). In 5 time constants, the response decays by 99%. Faster circuits have smaller τ.

- **General 1st-order Transient Response for Voltage/Current Step**

$$X(t) = X_f + \left[X(t_o^+) - X_f\right]e^{-(t-t_o^+)/\tau}$$ (X is any voltage or current in the circuit)

X_f = final value, t_o = time voltage/current step occurred
(1) Find initial value $X(t_o^+)$ and final value X_f. Use continuity ($x(0^-)=x(0^+)$) and steady-state rules (open/short) for cap/ind. (2) Calculate τ (τ=RC for RC circuit, τ=L/R for LR circuit). R is the Thevenin equivalent resistance "seen" by the cap/ind.

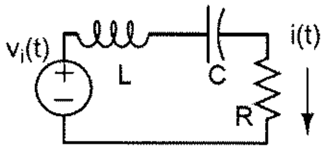
VI. SECOND-ORDER CIRCUITS

- **RLC circuit** – contains only sources, resistors, 1 capacitor, and 1 inductor
  → voltages and currents are described by 2^{nd}-order ODE (ordinary differential equation)

  General 2^{nd}-order ODE: $\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_o^2 x(t) = f(t)$

  α = damping coefficient, ω_o = undamped natural freq (AKA resonant freq)
  $\zeta = \alpha/\omega_o$ = damping ratio, f(t) = forcing function (related to the input)

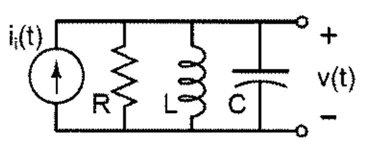
Series RLC Circuit



$\alpha = \frac{1}{2}\frac{R}{L}$ $\omega_o = \frac{1}{\sqrt{LC}}$

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = \frac{1}{L}\frac{dv_i(t)}{dt}$$

Parallel RLC Circuit



$\alpha = \frac{1}{2RC}$ $\omega_o = \frac{1}{\sqrt{LC}}$

$$\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = \frac{1}{C}\frac{di_i(t)}{dt}$$

- **Time-domain Analysis for 2^{st}-order Circuits**
  1. Write the ODE in terms of the variable of interest X(t), using KCL/KVL and IV relationships for R, L, C.
  2. Obtain the **characteristic equation** by setting the input to 0 and substituting $X(t)=Ke^{st}$ into the ODE: $s^2+2\alpha s+\omega_o^2=0$. Find α and ω_o. The roots of the characteristic equation are $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$; the form of the solution depends on the damping ratio $\zeta = \alpha/\omega_o$.
  3. Find the **homogeneous solution** $X_h(t)$ depending on ζ:

     overdamped: $\alpha > \omega_o, \zeta > 1$ $X_h(t) = K_1 e^{(-\alpha+\sqrt{\alpha^2-\omega_o^2})t} + K_2 e^{(-\alpha-\sqrt{\alpha^2-\omega_o^2})t}$

     critically damped: $\alpha = \omega_o, \zeta = 1$ $X_h(t) = K_1 e^{-\alpha t} + K_2 t e^{-\alpha t}$

     underdamped: $\alpha < \omega_o, \zeta < 1$ $X_h(t) = K_1 e^{-\alpha t}\cos(\omega_n t) + K_2 e^{-\alpha t}\sin(\omega_n t)$

     $\omega_n = \sqrt{\omega_o^2 - \alpha^2}$ = damped natural frequency

     (Note: The value of K_1 and K_2 cannot be found until the complete solution is found.)
  4. Find the **particular solution** $X_p(t)$. Remember the output follows the form of the input:

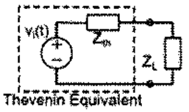
input function	constant	exponential	Sinusoid
particular solution	A	$Ae^{-\alpha t} + B\cdot te^{-\alpha t}$	Acos(wt)+Bsin(wt)

     Guess the form of the solution and solve the ODE to find any arbitrary constants.
     (Note: For sinusoidal inputs, the particular solution can be found more easily using complex impedance.)
  5. Combine the homogeneous and particular solutions to get the complete solution: $X(t) = X_h(t)+X_p(t)$. Use the initial conditions to find the missing variables (i.e., K_1, K_2 in $X_h(t)$).

VII. SINUSOIDAL STEADY-STATE ANALYSIS

Any steady-state (SS) voltage or current in a linear time-invariant (LTI) circuit with a sinusoidal input source is sinusoidal with the same frequency. Only the magnitude and phase (relative to the source) may be different.

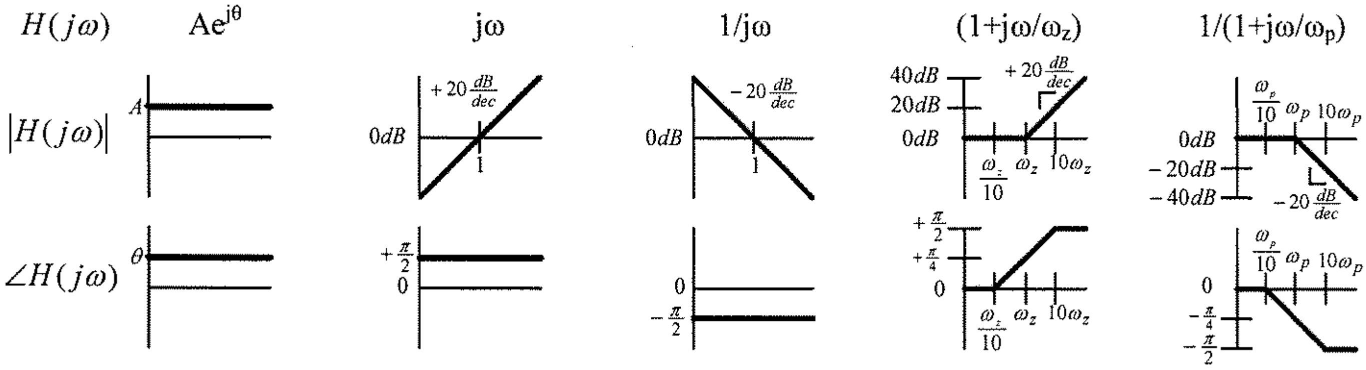
- **Phasors** – vectors (i.e., complex numbers) that represent sinusoids. Since all V,I in the circuit are sinusoids with the same frequency, only magnitude & phase are needed to describe any V,I.
  sinusoids: $v(t) = V\cos(\omega t+\theta) = \mathrm{Re}[Ve^{j(\omega t+\theta)}] = \mathrm{Re}[Ve^{j\theta}e^{\omega t}]$ → phasor: $Ve^{j\theta} = V\angle\theta$
  $v(t) = V\sin(\omega t+\theta) = V\cos(\omega t+\theta-\pi/2)$ → phasor: $V\angle(\theta-\pi/2)$
  → For convenience, define phasors in terms of <u>cosine</u> (i.e., the real part of a complex exponential)
- Euler's Identity: $e^{jx} = \cos(x) + j\sin(x)$, $\cos(x) = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$, $\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$
- Differentiation/integration become algebraic operations w/ phasors (i.e., complex exponentials)
  $\frac{d}{dt} \Leftrightarrow j\omega$ $\quad \int dt \Leftrightarrow \frac{1}{j\omega}$ $\quad$ Ex: $\frac{d}{dt}\left(e^{j(\omega t+\theta)}\right) = j\omega e^{j(\omega t+\theta)}$
- **Capacitor Impedance**: $Z_C = \frac{1}{j\omega C}$ → **ICE** – Current (I) LEADS Voltage (EMF) by 90°
- **Inductor Impedance**: $Z_L = j\omega L$ → **ELI** – Voltage (EMF) LEADS Current (I) by 90°
- **Complex Impedance/Generalized Ohm's Law**: $Z = \frac{V}{I}$
  → allows for easy nodal analysis (no differential equations); series/parallel resistor laws apply
- **Maximum Average Power Transfer Theorem**
  → power transferred to load impedance Z_L is maximized when $Z_L = Z_{th}^*$



- **Decibel (dB)** – unit of measure for <u>ratios</u> of power, voltage, and current levels (often used to express gain). Power: $1\text{dB}=10\log_{10}(P_1/P_2)$; V,I: $1\text{dB}=20\log_{10}(V_1/V_2)=20\log_{10}(I_1/I_2)$
- **Frequency Response** – system's input→output transfer function vs. frequency (given sinusoidal input). Both magnitude and phase plots are needed (output freq = input freq)
  - General transfer function – can be written as a product of poles and zeroes

$$H(\omega) = Ae^{j\theta}\cdot(j\omega)^n\cdot\frac{\left(1+\frac{j\omega}{\omega_{z1}}\right)\left(1+\frac{j\omega}{\omega_{z2}}\right)\cdots}{\left(1+\frac{j\omega}{\omega_{p1}}\right)\left(1+\frac{j\omega}{\omega_{p2}}\right)\cdots}$$

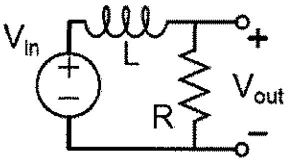
  - **zeroes** – roots of the numerator
  - **poles** – roots of the denominator
  - **Break point frequency** ω_{BP} – poles and zeros are break point freq's
    → at a zero frequency, the magnitude is +3dB ($=\sqrt{2}$) and the phase is +45°
    → at a pole frequency, the magnitude is -3dB ($=1/\sqrt{2}$) and the phase is -45°
- **Bode Plot** – logarithmic plots for frequency response



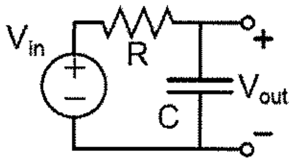
  - to draw Bode plot for general transfer function, add individual pole and zero plots

- **Filters**

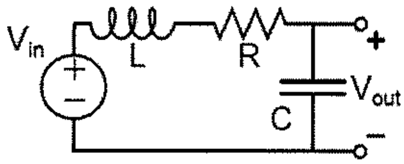
  ◦ Lowpass Filter (LPF) – V_C in RC circuit / V_R in RL circuit / V_C and RLC circuit
  (for current output, switch from series to parallel and switch L and C)



$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1+j\omega L/R}$

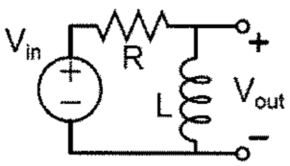


$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1+j\omega RC}$

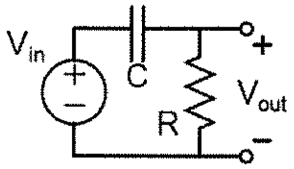


$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1+j\omega RC+(j\omega)^2 LC}$

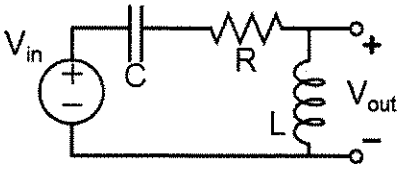
  ◦ Highpass Filter (HPF) – V_L in RL circuit / V_R in RC circuit / V_L in RLC circuit
  (for current output, switch from series to parallel and switch L and C)



$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega L/R}{1+j\omega L/R}$

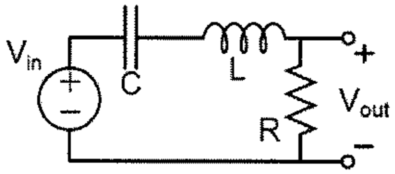


$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega RC}{1+j\omega RC}$

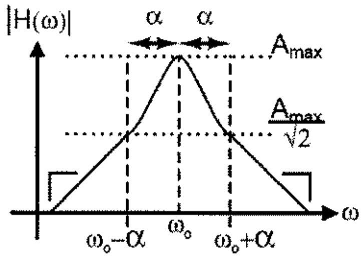


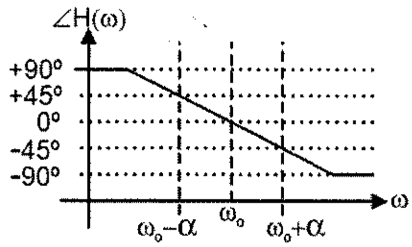
$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{(j\omega)^2 LC}{1+j\omega RC+(j\omega)^2 LC}$

  ◦ Bandpass Filter (BPF) – V_R, I_R in RLC circuit



$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega RC}{1+j\omega RC+(j\omega)^2 LC}$





  → at low freq, cap. impedance $Z_C = \frac{1}{j\omega C}$ dominates → $I = \frac{V_{in}}{Z_{tot}} \approx \frac{V_{in}}{Z_C} = j\omega C V_{in}, V_{out} = IR \approx j\omega RC V_{in}$

  → at high freq, ind. impedance $Z_L = j\omega L$ dominates → $I = \frac{V_{in}}{Z_{tot}} \approx \frac{V_{in}}{Z_L} = \frac{V_{in}}{j\omega L}, V_{out} = IR \approx \frac{V_{in}}{j\omega L/R}$

  ◦ **Resonant Frequency** $\omega_o = \frac{1}{\sqrt{LC}}$

  → At ω_o, $Z_C = \frac{1}{j\omega_o C} = -j\sqrt{\frac{L}{C}} = -jZ_o$, $Z_L = j\omega_o L = +j\sqrt{\frac{L}{C}} = +jZ_o$ → $V_{out} = V_{in}$

  (capacitor and inductor impedances are equal in magnitude, opposite in sign)

  → **Characteristic Impedance**: $Z_o = \sqrt{L/C}$

  ◦ BPF **Bandwidth Δω** = 2α = difference between half-power frequencies

  ◦ **Quality Factor Q** – (1) measure of "peakiness" or filter selectivity (high Q → low bandwidth)
  (2) measure of energy stored vs. energy dissipated (high Q → low loss)

  $Q = \frac{\omega_o}{\Delta\omega} = \frac{\omega_o}{2\alpha} = \frac{1}{2\zeta}$ series RLC: $Q = \frac{Z_o}{R} = \frac{\sqrt{L/C}}{R}$ parallel RLC: $Q = \frac{R}{Z_o} = \frac{R}{\sqrt{L/C}}$

  <u>Tradeoffs</u>: Bandwidth/selectivity/speed/energy loss
  (e.g., high Q → low Δω (high selectivity) → low α → slow transients $e^{-\alpha t}$)