Kapılar (Gates)

Ve (And) Kapısı

n girişli Ve kapısı için tüm girişler 1 ise $f=1$, diğer durumlarda $f=0$ olur.

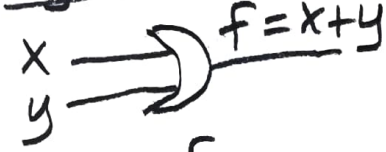
x	y	f
0	0	0
0	1	0
1	0	0
1	1	1

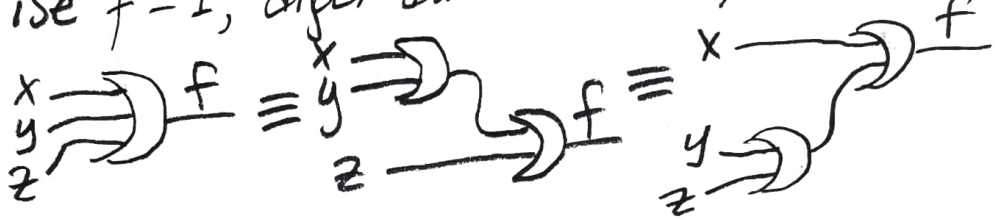
Doğruluk Tablosu

$xyz = (xy)z = x(yz)$ Birleşme Kuralı

Veya (Or) Kapısı

n girişli Veya kapısı için en az bir giriş 1 ise $f=1$, diğer durumlarda $f=0$ olur.





x	y	f
0	0	0
0	1	1
1	0	1
1	1	1

Doğruluk Tablosu

$x+y+z = (x+y)+z = x+(y+z)$ Birleşme Kuralı

Değil (Not) Kapısı

x	f
0	1
1	0

Doğruluk Tablosu

Giriş 0 ise $f=1$, diğer türlü $f=0$

Nand (Ve Değil) Kapısı

Nor (Veya Değil) Kapısı

Evrensel Kapılar

Düşük maliyetinden dolayı mantık devreleri genellikle Nand ve Nor kapıları kullanılarak üretilir.

4 girişli and kapısı

$f = ((ab)' + (cd)')'$

$= (ab)''(cd)'' = abcd$

4 girişli or kapısı

$f = ((a+b)'(c+d)')'$

$= (a+b)'' + (c+d)''$

$= a+b+c+d$

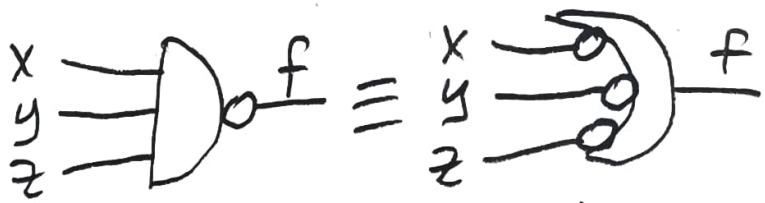
Ve Değil (Nand) Kapısı

$f=(xy)'=x'+y'$

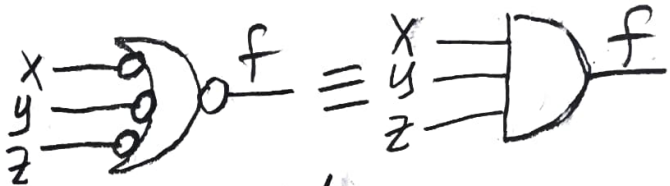
x	y	f
0	0	1
0	1	1
1	0	1
1	1	0

Doğruluk Tablosu

n girişli Nand kapısı için tüm girişler 1 ise f=0, diğer durumlarda f=1



$f=(xyz)'=x'+y'+z'$



$f=(x'+y'+z')'=xyz$

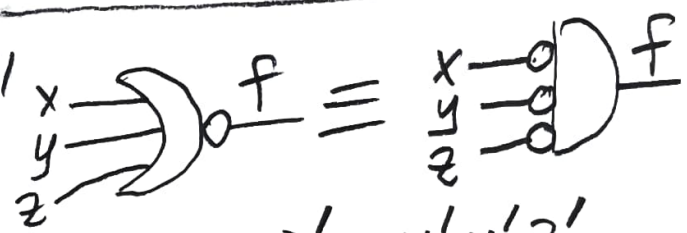
Veya Değil (Nor) Kapısı

$f=(x+y)'=x'y'$

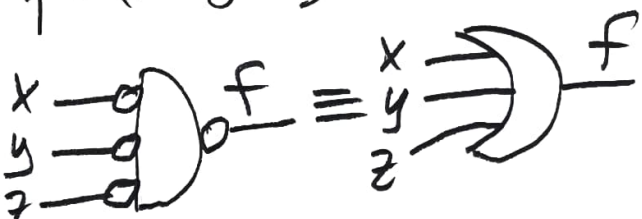
x	y	f
0	0	1
0	1	0
1	0	0
1	1	0

Doğruluk Tablosu

n girişli Nor kapısı için tüm girişler 0 ise f=1, diğer durumlarda f=0



$f=(x+y+z)'=x'y'z'$



$f=(x'y'z')'=x+y+z$

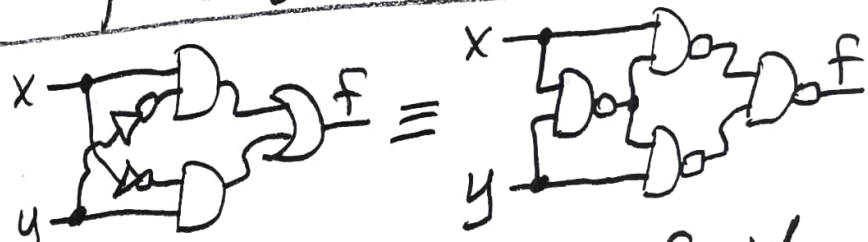
Ya Da (Exor) Kapısı

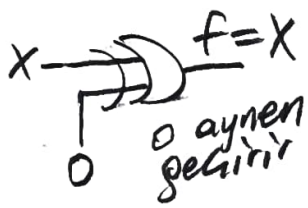
$f=x\oplus y$

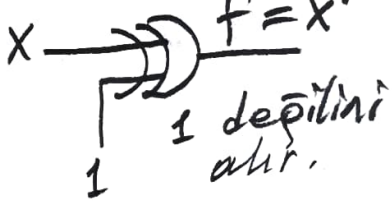
$=x'y+xy'$

x	y	f
0	0	0
0	1	1
1	0	1
1	1	0

Doğruluk Tablosu







$x\oplus y=y\oplus x$

$f=\begin{cases}0, & x=y\\ 1, & x\neq y\end{cases}$

$x\oplus x=0$

$x\oplus x'=1$

$x\oplus 0=x$

$x\oplus 1=x'$

$x\oplus y\oplus z=(x\oplus y)\oplus z$

$=x\oplus(y\oplus z)$

n girişli Exor Kapısı için 1'lerin sayısı tek ise f=1, çift ise f=0 olur.

Eşdeğer (Exnor) Kapısı

$f = x \odot y = xy + x'y'$

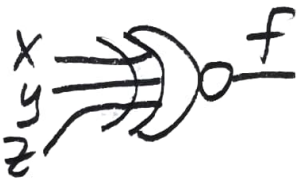
x	y	f
0	0	1
0	1	0
1	0	0
1	1	1

$x \odot y = y \odot x$

$f = \begin{cases} 0, & x \neq y \\ 1, & x = y \end{cases}$

$x \odot y \odot z = (x \odot y) \odot z$

$= x \odot (y \odot z)$



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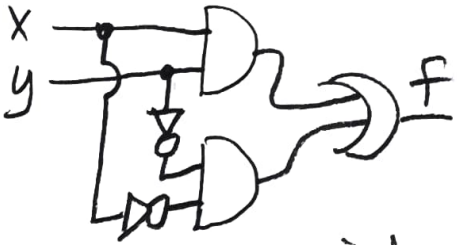

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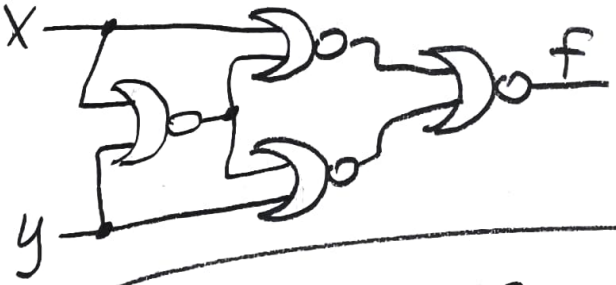


n girişli Exnor Kapısı için 0'ların sayısı çift ise $f=1$, tek ise $f=0$ olur

$x \oplus y \oplus z = x \odot y \odot z$ Değişken Sayısı Tek ise

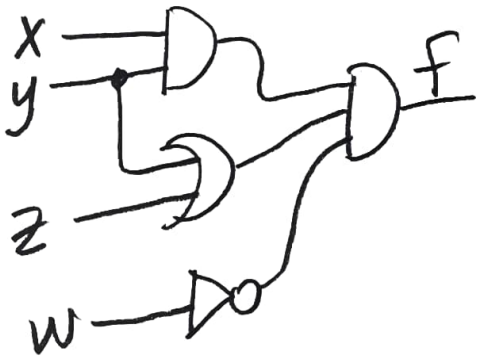
$(x \oplus y)' = x \odot y$

$(x \odot y)' = x \oplus y$

Değişken sayısı çift ise $x \oplus y \oplus z \oplus w = (x \odot y \odot z \odot w)'$

$(x \oplus y \oplus z)' = x \oplus y \odot z = x \odot y \oplus z$

$f = xy(y+z)w' = ((xy)(y+z)w')'' = ((xy)' + (y+z)' + w)'$



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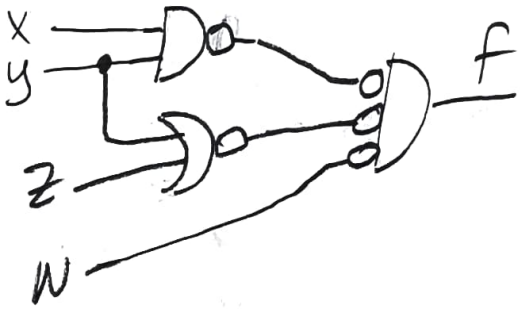

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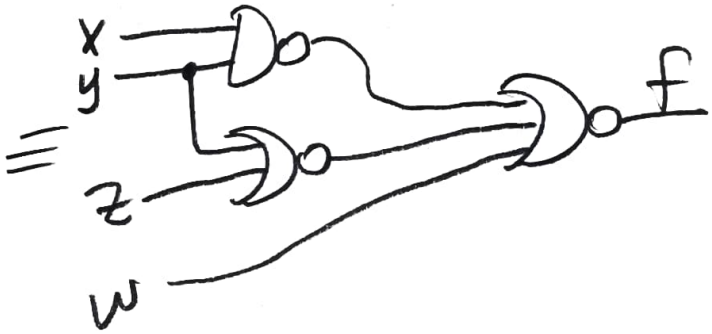
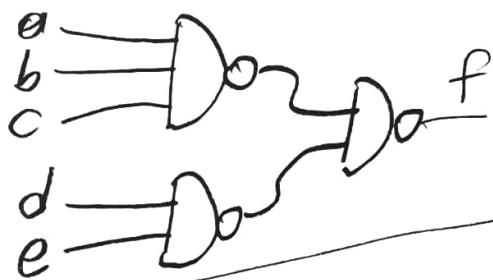
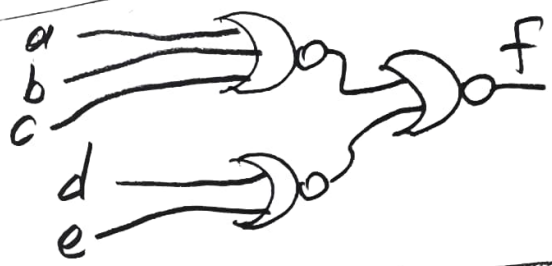




$f = ((abc)'(de)')' = abc + de$

Çarpımlar Toplamı



$f = ((a+b+c)' + (d+e)')'$

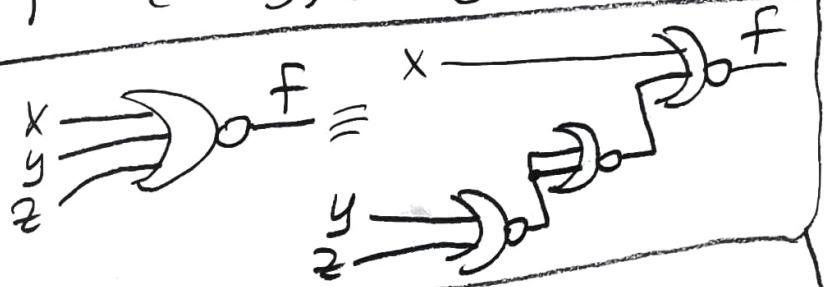
$= (a+b+c)(d+e)$

Toplamlar Çarpımı

$f = x + y'z + x'yz'$ Çarpımlar Toplamı } Standart

$f = y(x+z')(x'+y'+z)$ Toplamlar Çarpımı } form

$f = (x+y)(x'+y'z)$ Standart form değil



$f = x \oplus y \oplus z$

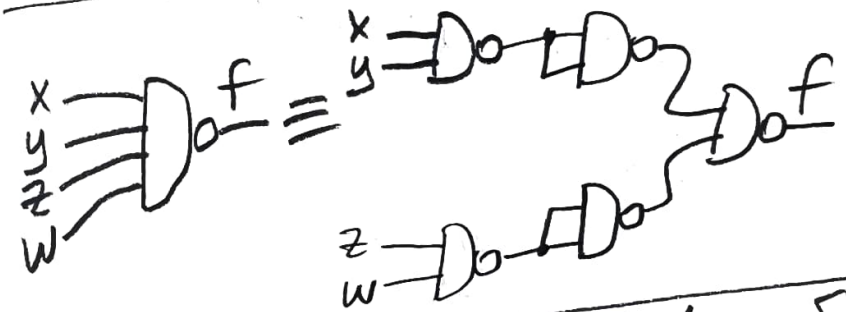
$= (x \oplus y) \oplus z$

$= (x \odot y) z + (x \oplus y) z'$

$= (xy + x'y') z + (x'y + xy') z'$

$= xyz + x'y'z + x'yz' + xy'z'$

$= \Sigma(1,2,4,7)$



$f_1 = x \odot y \odot z \odot w$ Sıfırların Sayısı Çift

$= x'y'z'w' + x'y'zw + x'yz'w + x'yzw'$

$+ xy'z'w + xy'zw' + xyz'w' + xyzw$

$= \Sigma(0,3,5,6,9,10,12,15)$

$f_2 = x \oplus y \oplus z \oplus w$ Birlerin Sayısı Tek

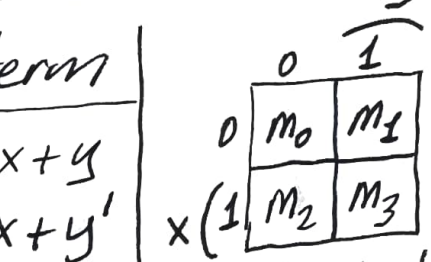
$= \Sigma(1,2,4,8,7,11,13,14)$

$f = f_1 + f_2 = 1$

$f_1 = f_2'$

Karno Harita Yöntemi

x y	Minterm	Maxterm
0 0	$m_0 = x'y'$	$M_0 = x+y$
0 1	$m_1 = x'y$	$M_1 = x+y'$
1 0	$m_2 = xy'$	$M_2 = x'+y$
1 1	$m_3 = xy$	$M_3 = x'+y'$



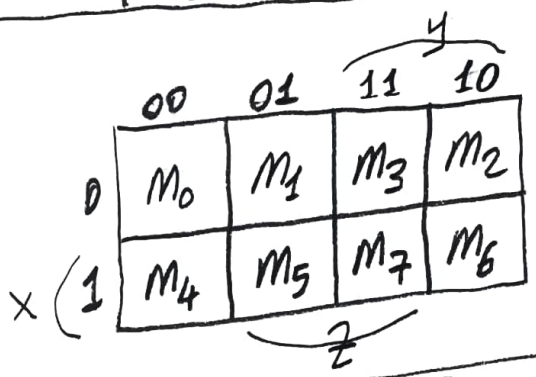
İki değişkenli Karno harita

m_i : Minterm

M_i : Maxterm

$m_i' = M_i$

m_i için 1 yapan, M_i için 0 yapan olacak



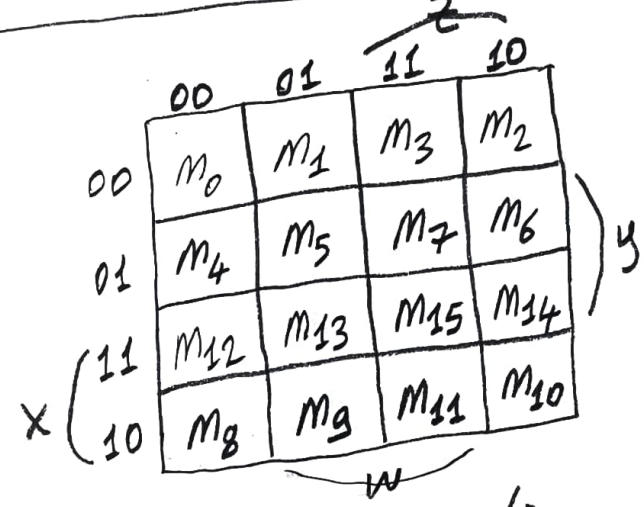
$(5)_{10} = (1\,0\,1)_2$

$m_5 = xy'z$

$M_5 = x'+y+z'$

Haritada m_5 haricinde kalan bütün bölge M_5'e aittir

$M_5 = m_5'$



$(9)_{10} = (1\,0\,0\,1)_2$

$m_9 = xy'z'w$

$M_9 = m_5' = x'+y+z+w'$

Dört değişkenden daha fazlası için harita yöntemi yerine tablo yöntemi kullanılır.

Dört değişkenli Karno haritası

Tablo yönteminde hesaplama çok olduğundan bilgisayar yazılımı kullanmak gerekir.

$f(x,y,z) = \Sigma(0,1,4,5,7) = m_0 + m_1 + m_4 + m_5 + m_7$ Birinci Kanonik Açılım

$= x'y'z' + x'y'z + xy'z' + xy'z + xyz$

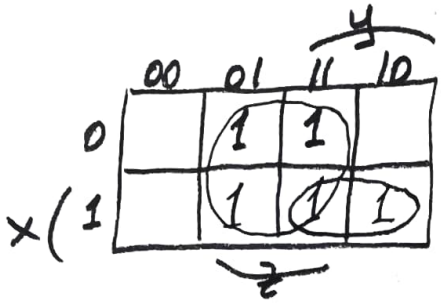
$= \Pi(2,3,6) = M_2 M_3 M_6$ İkinci Kanonik Açılım

$= (x+y'+z)(x+y'+z')(x'+y'+z)$

$f' = \Sigma(2,3,6) = \Pi(0,1,4,5,7)$

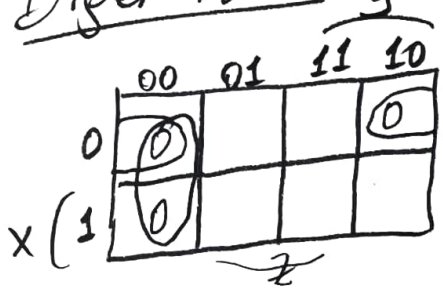
$f = (f')' = f''$

$f(x,y,z) = \Sigma(1,3,5,6,7)$ ifadesini Karno Haritası kullanarak sadeleştir.



$f = xy + z$

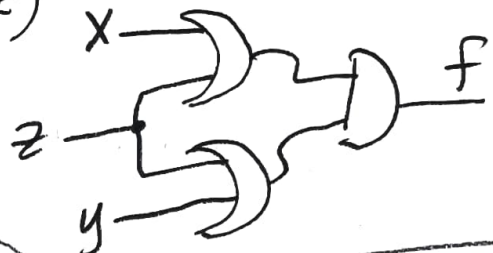
Diğer Türlü



$f' = y'z' + x'z'$

$f = (f')' = (y'z' + x'+z') = (y+z)(x+z)$

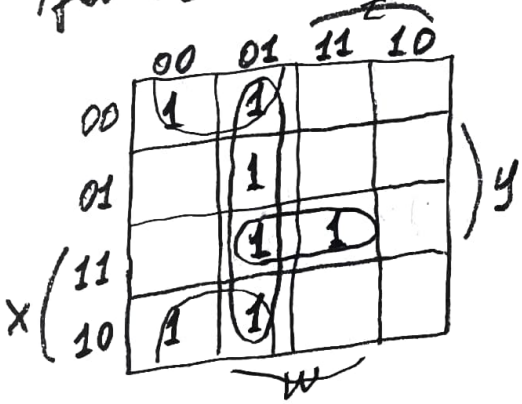
$= (x+z)(y+z)$



$f(x,y,z,w) = \Sigma(0,1,5,8,9,13,15)$

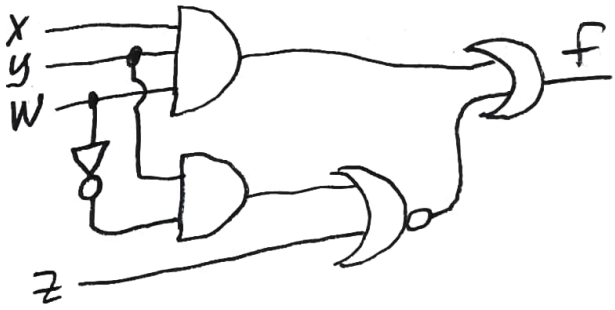
$= \Pi(2,3,4,6,7,10,11,12,14)$

ifadesini karno haritası kullanarak sadeleştir.

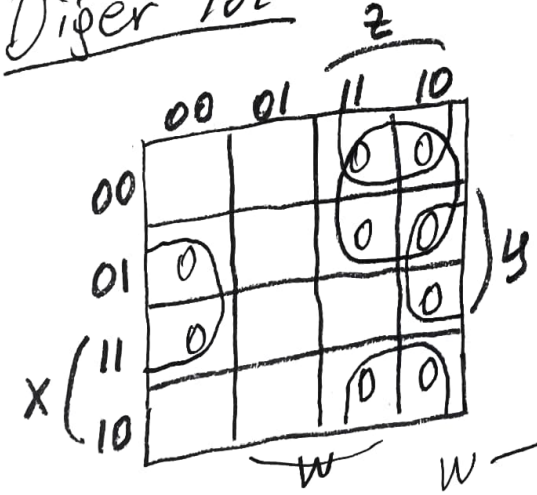


$f = y'z' + z'w + xyw = xyw + z'(y'+w)$

$= xyw + (z + yw')'$



Diğer Yol

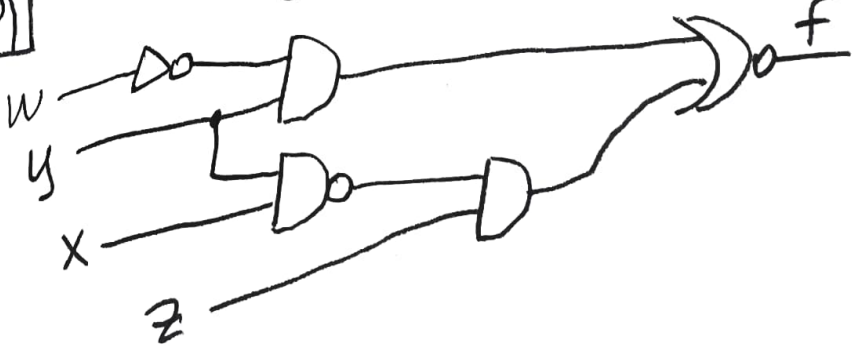


$f' = yw' + x'z + y'z$

$f = (f')' = (yw' + x'z + y'z)'$

$= (y'+w)(x+z')(y+z')$

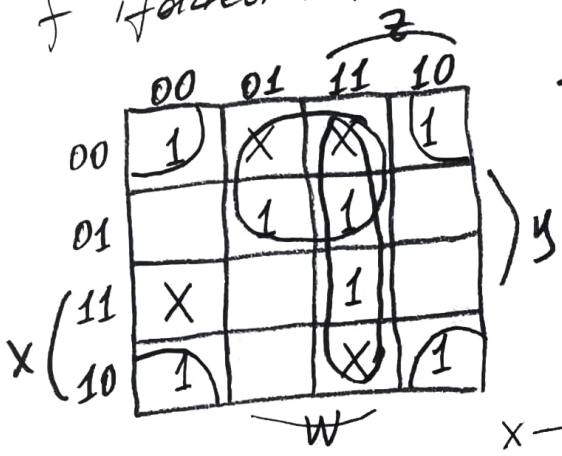
$= (y'+w)(xy+z') = (yw' + (xy)'z)'$



$f(x,y,z,w) = \Sigma(0,2,5,7,8,10,15)$

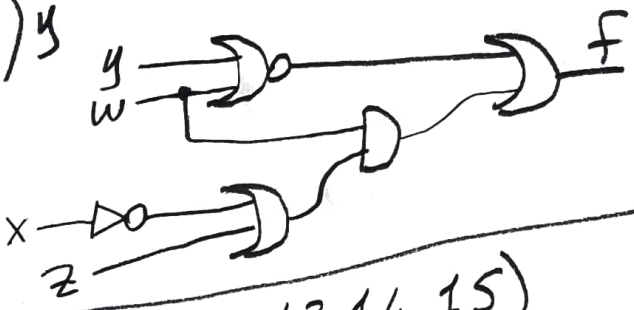
$d(x,y,z,w) = \Sigma(1,3,11,12)$ → Gerekirse

f ifadesini karno haritası kullanarak sadeleştir.



$f = y'w' + x'w + zw$

$= (y+w)' + (x'+z)w$

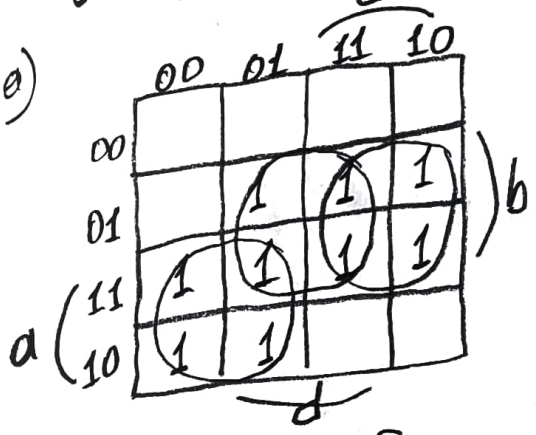


$f(a,b,c,d) = \Sigma(5,6,7,8,9,12,13,14,15)$

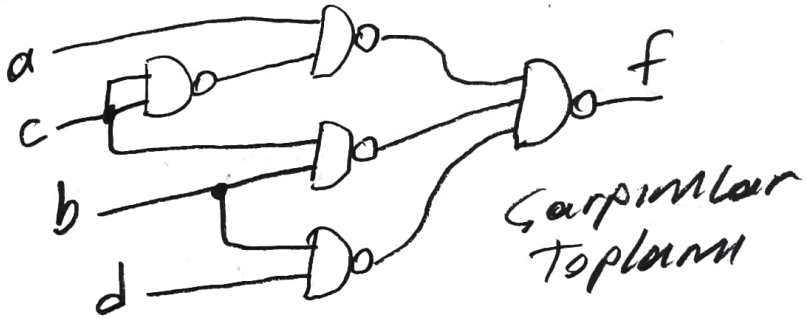
a) Çarpımlar Toplamı

b) Toplamlar Çarpımı

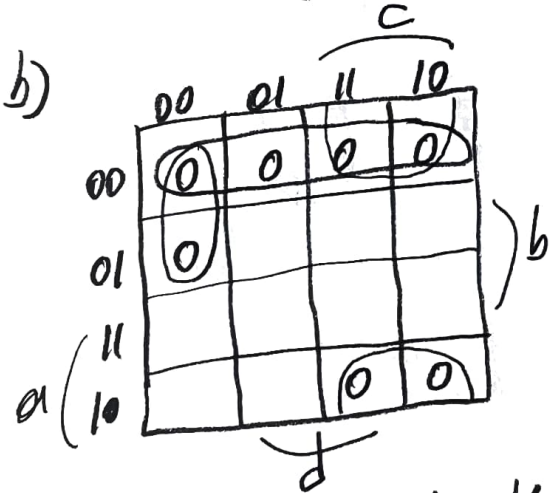
a)



$f = ac' + bc + bd$



b)

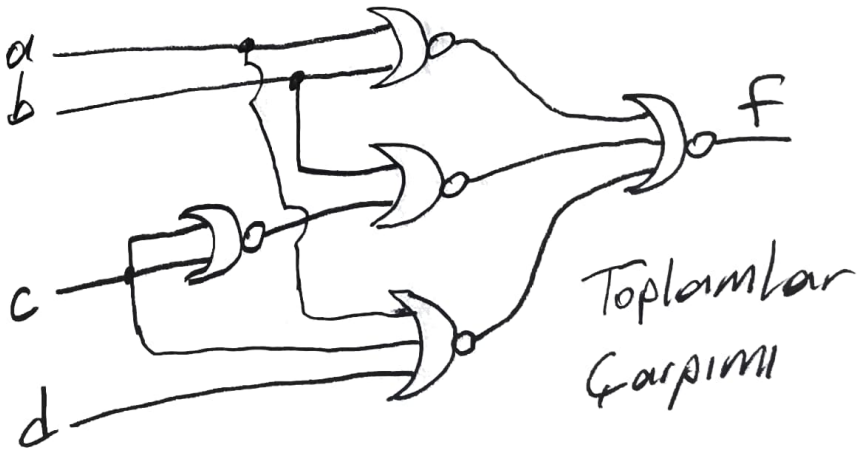


$f' = a'b' + b'c + a'c'd'$

$f = (f')'$

$f = (a'b' + b'c + a'c'd')'$

$= (a+b)(b+c')(a+c+d)$

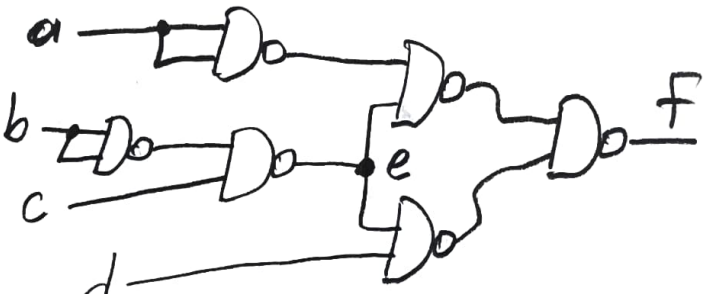


$f = a'b + a'c' + bd + c'd$ ifadesi ile verilen boole fonksiyonunu iki girişli nand kapıları ile gerçekle.

$f = a'b + a'c' + bd + c'd$

$= a'(b+c') + (b+c')d$

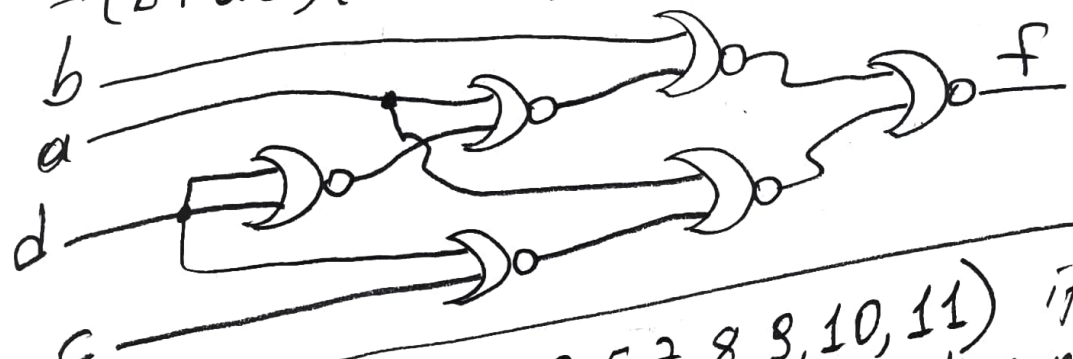
$= a'\underbrace{(b'c)'}_{e} + \underbrace{(b'c)'}_{e}d$



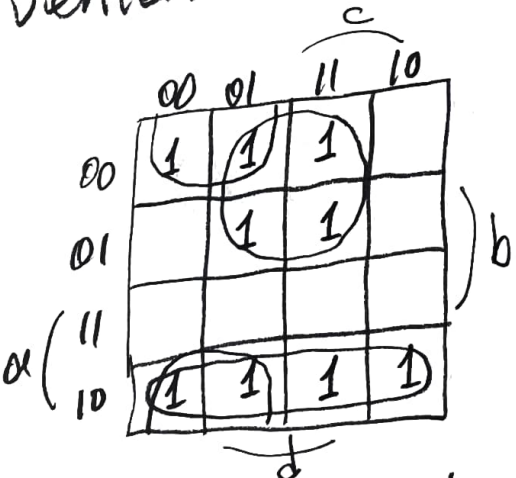
$f = (a'+b)(b+d)(a+c'd')$ ifadesi ile verilen boole fonksiyonunu iki girişli nor kapıları ile gerçekle.

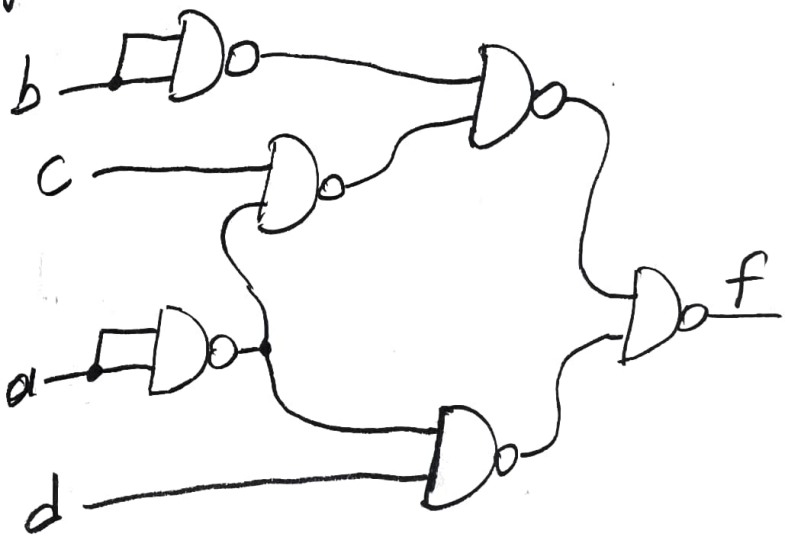
$f = (a'+b)(b+d)(a+c'd')$

$= (b+a'd)(a+c'd') = (b+(a+d')')(a+(c+d)')$



$f(a,b,c,d) = \Sigma(0,1,3,5,7,8,9,10,11)$ ifadesi ile verilen boole fonksiyonunu iki girişli nand kapılarıyla gerçekle.



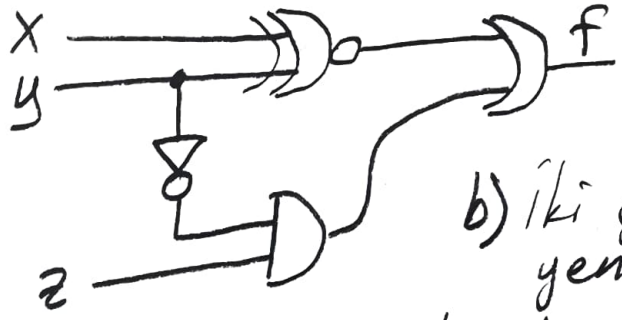


$f = ab' + b'c' + a'd$

$= b'(a+c') + a'd$

$= b'(a'c)' + a'd$

Şekilde verilen mantık devresini,



a) $f = \Sigma(?)$ formunda ifade et.

b) İki girişli nand kapılarıyla yeniden gerçekle

c) İki girişli nor kapılarıyla yeniden gerçekle.

a) $f = x \odot y + y'z$

$= xy + x'y' + y'z$

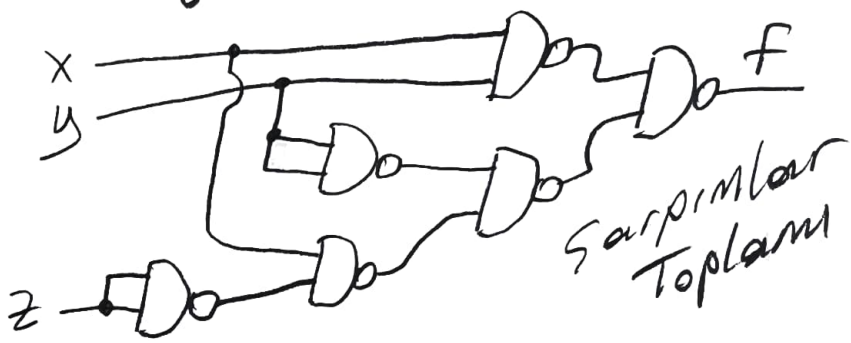
x \ yz	00	01	11	10
0	1	1		
1		1	1	1

$f(x,y,z) = \Sigma(0,1,5,6,7)$

b) $f = xy + x'y' + y'z$

$= xy + y'(x'+z)$

$= xy + y'(xz')'$



c)

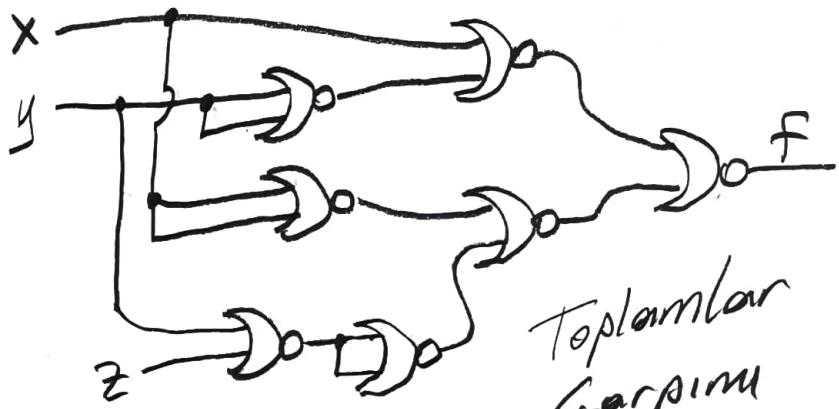
x \ yz	00	01	11	10
0			0	0
1	0			

$f' = x'y + xy'z'$

$f = (f')' = (x'y + xy'z')'$

$= (x+y')(x'+y+z)$

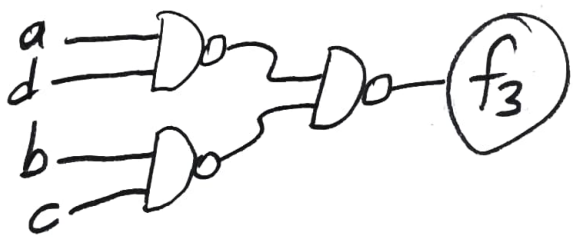
$= (x+y')(x' + (y+z)'')$



bd	f_1
00	0
01	0
10	1
11	0

d \ ab	00	01	11	10
0	0	1	1	0
1	1	1	0	0

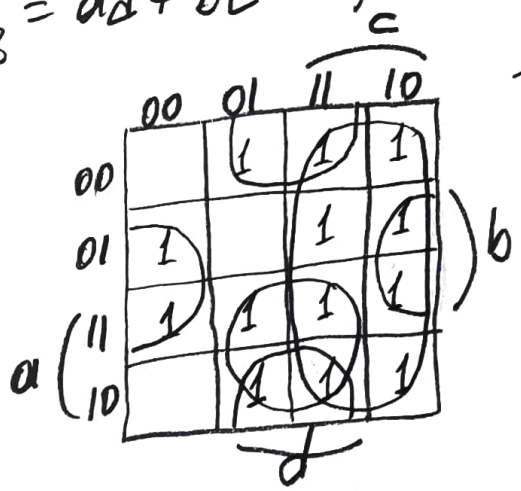
(f_2, c)



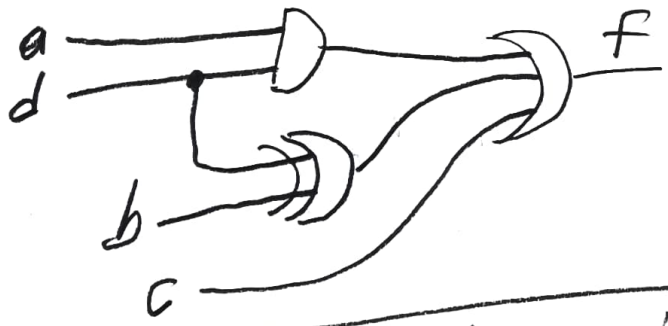
$f = f_1 + f_2 + f_3$ fonksiyonunu en az kapı elemanı kullanarak tasarla

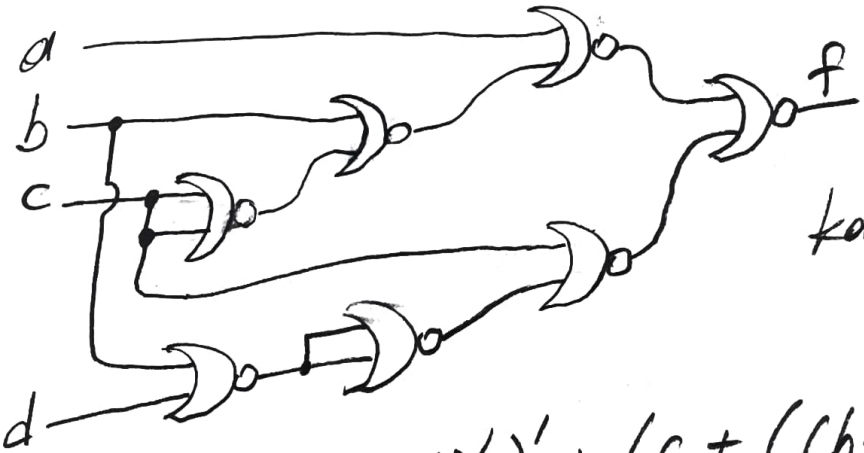
$f_1 = bd'$
$f_2 = b'd + cd'$
$f_3 = ad + bc$

$f = f_1 + f_2 + f_3$
$= bd' + b'd + cd' + ad + bc$



$f = c + ad + b'd + bd'$
$= c + ad + b \oplus d$



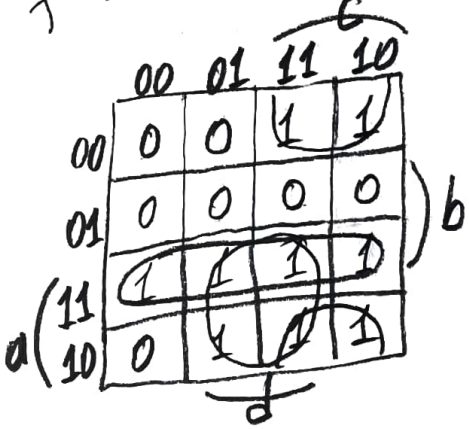


Yanda verilen mantık devresini iki girişli NAND kapılarıyla yeniden tasarla.

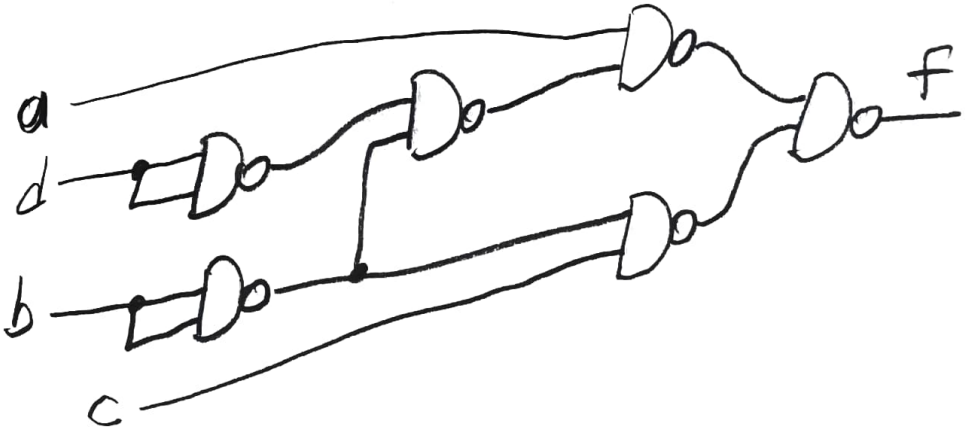
$f = ((a + (b+c')')' + (c + ((b+d')')')')'$

$= (a+b'c)(c+b+d) = (a+b')(a+c)(b+c+d)$

$f' = a'b + a'c' + b'c'd'$



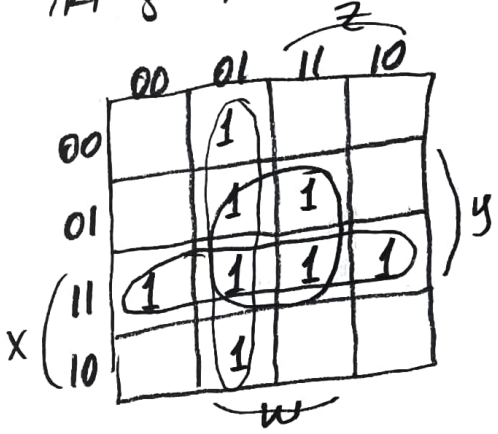
$f = ab + ad + b'c$
$= b'c + a(b+d)$
$= b'c + a(b'd')'$



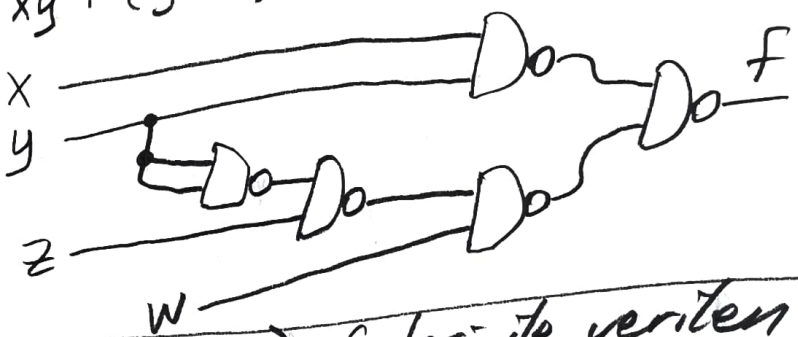
$f(a,b,c,d) = ad + ac'd' + a'cd + bcd'$ denklemi ile verilen boole fonksiyonunu boole kurallarını kullanarak sadeleştir.

$$f = ad + ac'd' + a'cd + bcd'$$
$$= ad(c+c') + ac'd' + a'cd + bcd'$$
$$= \underline{acd} + \underline{ac'd} + \underline{ac'd'} + \underline{a'cd} + bcd'$$
$$= \underbrace{(a+a')}_{1} cd + ac'\underbrace{(d+d')}_{1} + bcd'$$
$$= cd + ac' + bcd' = ac' + c(d+bd')$$
$$= ac' + c(d+b)\underbrace{(d+d')}_{1} = ac' + c(b+d)$$

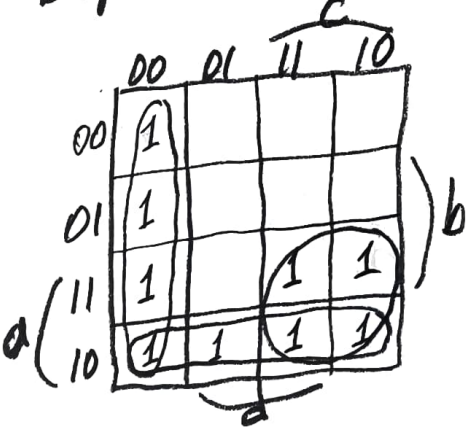
$f(x,y,z,w) = \Sigma(1,5,7,9,12,13,14,15)$ boole fonksiyonunu iki girişli Nand kapılarıyla tasarla.



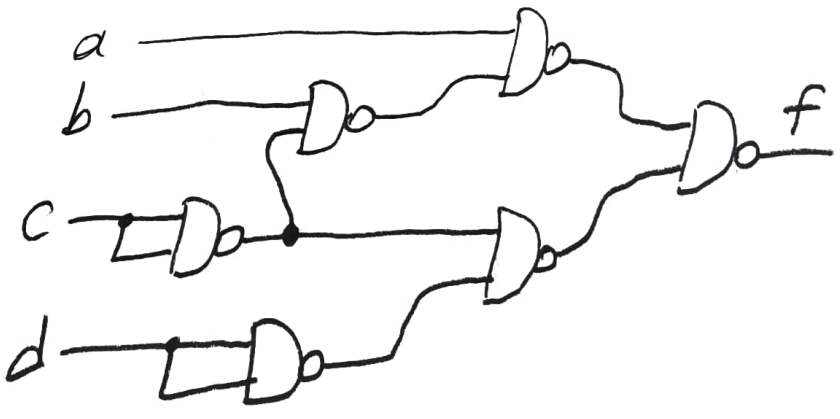
$$f = xy + yw + z'w$$
$$= xy + (y+z')w = xy + (y'z)'w$$

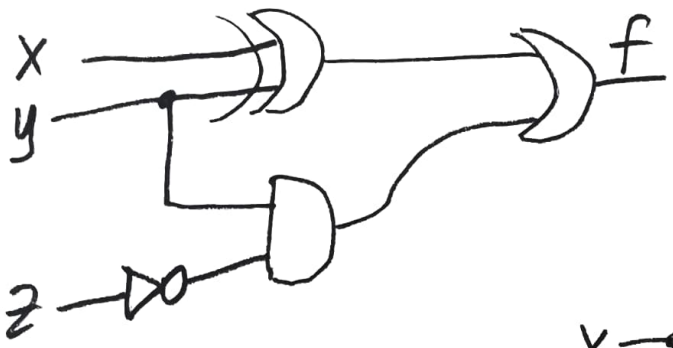


$f(a,b,c,d) = \Sigma(0,4,8,9,10,11,12,14,15)$ ifadesi ile verilen boole fonksiyonunu en az sayıda iki girişli Nand kapısı kullanarak tasarla.



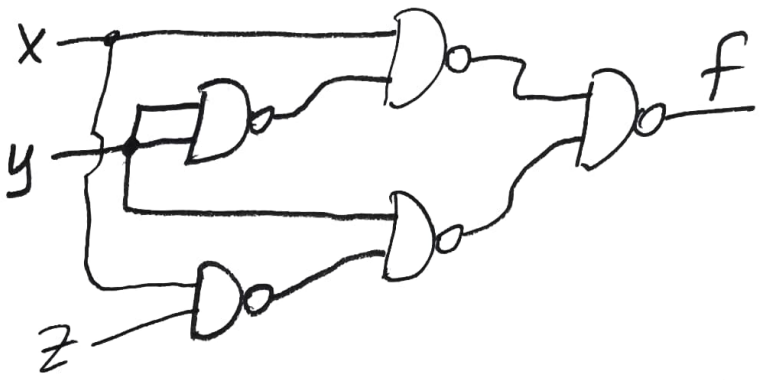
$$f = ab' + ac + c'd'$$
$$= a(b'+c) + c'd' = a(bc')' + c'd'$$

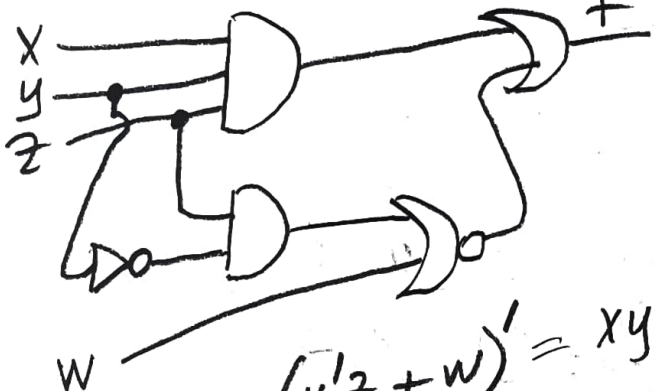




Yanda verilen mantık devresini iki girişli Nand kapılarıyla yeniden tasarlar.

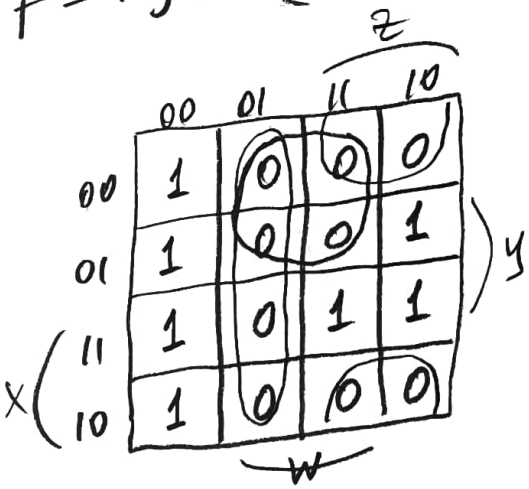
$$f = x \oplus y + yz'$$
$$= x'y + xy' + yz'$$
$$= xy' + y(x' + z')$$
$$= xy' + y(xz)'$$



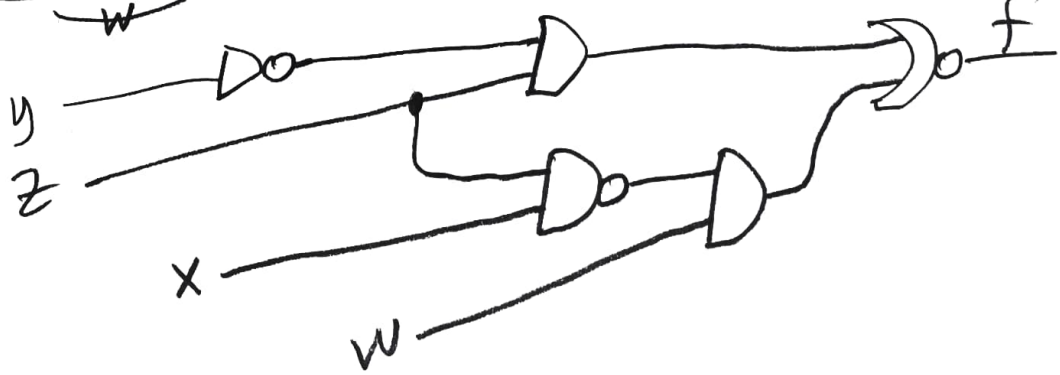


Yanda verilen sayısal mantık devresini $f = f''$ mantığı ile en az kapı elemanı kullanarak tasarla.

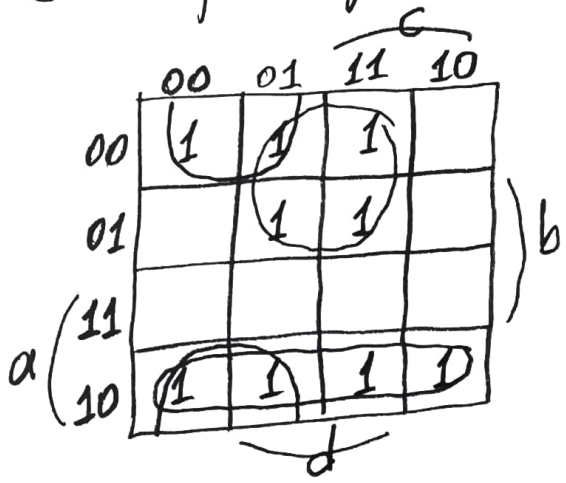
$$f = xyz + (y'z + w)' = xyz + (y + z')w'$$
$$= xyz + yw' + z'w'$$



$$f' = z'w + x'w + y'z$$
$$= (z' + x')w + y'z$$
$$= y'z + (xz)'w$$
$$f = (y'z + (xz)'w)'$$



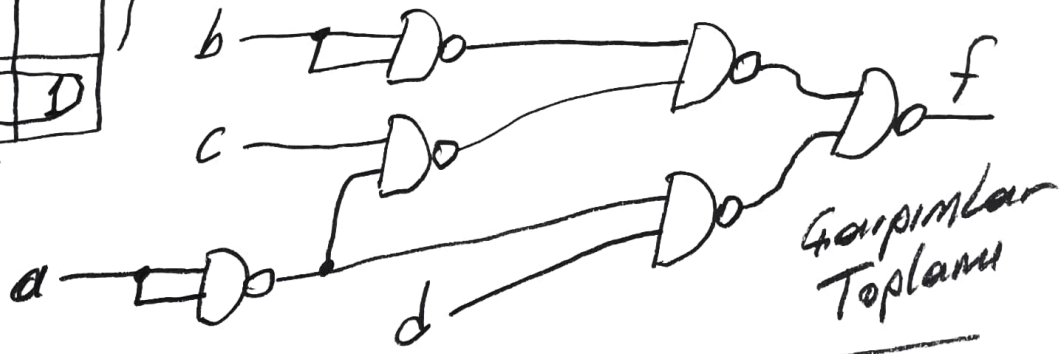
$f(a,b,c,d) = \Sigma(0,1,3,5,7,8,9,10,11)$ ifadesi ile verilen boole fonksiyonunu iki girişli Nand kapılarıyla gerçekle



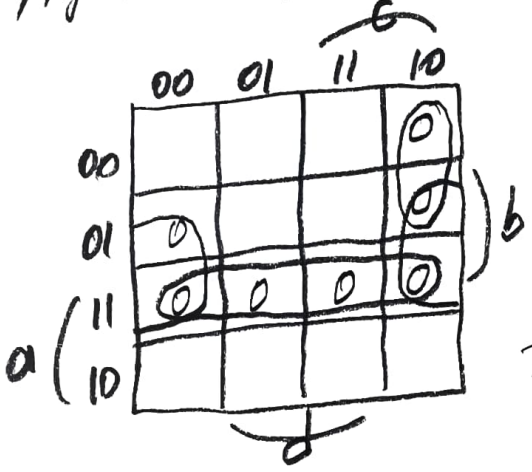
$f = ab' + b'c' + a'd$

$= b'(a+c') + a'd$

$= b'(a'c)' + a'd$



Aynı soruyu Nor kapılarıyla yapsaydık.



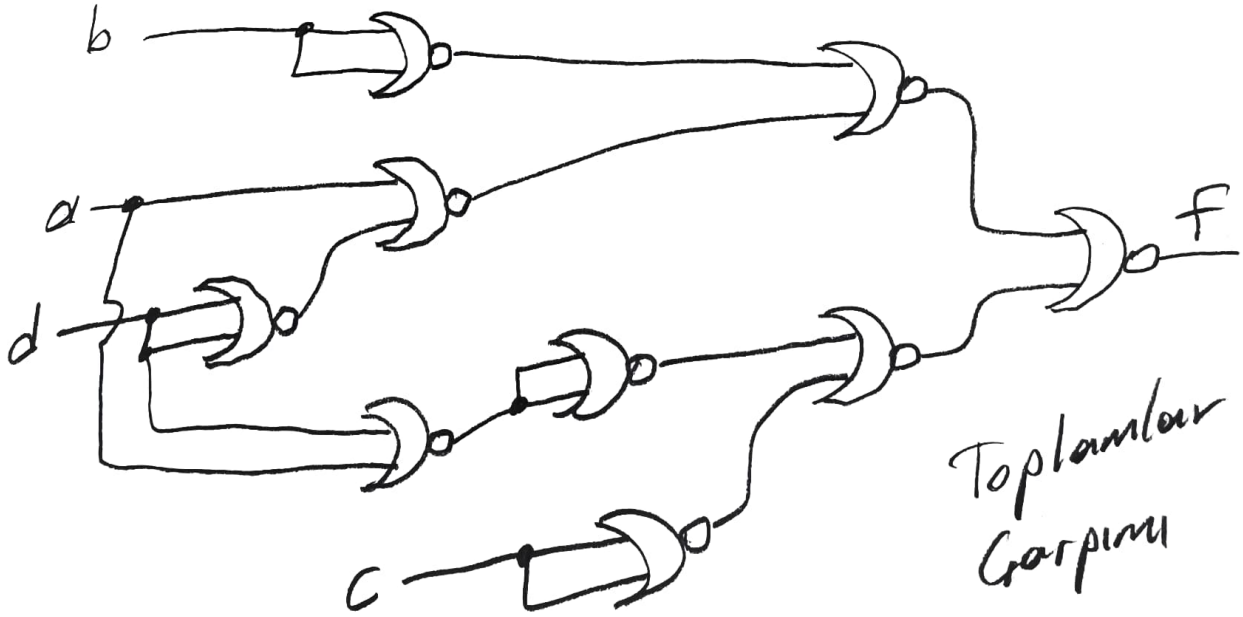
$f' = ab + bd' + a'cd'$

$f = (f')' = (ab + bd' + a'cd')'$

$f = (a'+b')(b'+d)(a+c'+d)$

$= (b' + a'd)(c' + (a+d))$

$= (b' + (a+d')')(c' + (a+d)'')$



Tablo Yöntemi (Quine McClustey Yöntemi)

Değişken sayısı 6 veya daha fazla olduğunda harita yöntemini kullanmak zorlaşır. Aşırı ölçüde artan kare sayısı makul bir bitişik kareler seçimini zorlaştırır. Bu durumda tablo yöntemi kullanılır. Fakat, tablo yönteminde matematiksel işlemler çok olduğundan bir bilgisayar yazılımı yazmak gerekir.

$f(a,b,c,d) = \Sigma(1,2,3,4,5,9,10,11,13)$

Tablo yöntemi kullanarak boole fonksiyonunu sadeleştir.

	a	b	c	d	
1	0	0	0	1	✓
2	0	0	1	0	✓
4	0	1	0	0	✓
3	0	0	1	1	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
11	1	0	1	1	✓
13	1	1	0	1	✓

	a	b	c	d	
(1,3)	0	0	X	1	✓
(1,5)	0	X	0	1	✓
(1,9)	X	0	0	1	✓
(2,3)	0	0	1	X	✓
(2,10)	X	0	1	0	✓
(4,5)	0	1	0	X	X
(3,11)	X	0	1	1	✓
(5,13)	X	1	0	1	✓
(9,11)	1	0	X	1	✓
(9,13)	1	X	0	1	✓
(10,11)	1	0	1	X	✓

	a	b	c	d
(1,3) (9,11)	X	0	X	1
(1,5) (9,13)	X	X	0	1
~~(1,9) (3,11)~~	~~X~~	~~0~~	~~X~~	~~1~~
~~(1,9) (5,13)~~	~~X~~	~~X~~	~~0~~	~~1~~
(2,3) (10,11)	X	0	1	X
~~(2,10) (3,11)~~	~~X~~	~~0~~	~~1~~	~~X~~

		1	2	3	4	5	9	10	11	13
b'd	(1,3) (9,11)	X		X			X		X	
c'd	(1,5) (9,13)	X		X		X	X			X
b'c	(2,3) (10,11)		X	X				X	X	
a'bc'	(4,5)				X	X				

Farklılık

$$f = c'd + b'c + a'bc'$$
$$= b'c + c'(a'b + d)$$