Bilgisayar
Mat III
(Tağer Bd. öl.)

Diferansiyel Denklemler
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Bir veya daha fazla bağımlı değişkenin, bir veya daha fazla bağımsız değişkene göre türevlerini içeren denkleme <u>diferansiyel denklem</u> adı verilir.

Dif. Denklem → Adi Dif. Denklem (Bağımsız değişken bir tane)
Dif. Denklem → Kısmi Dif. Denklem (Bağımsız değişken birden fazla)

Bir diferansiyel denklemde bulunan en yüksek mertebeden türevinin mertebesine <u>dif. denklemin mertebesi</u> denir.

Bir diferansiyel denklemde bulunan en yüksek mertebeden türevinin kuvvetine <u>dif. denklemin derecesi</u> denir.

$$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0$$ 2. mertebeden 1. dereceden adi dif. denklem

$$\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t$$ 4. mertebeden 1. dereceden adi dif. denklem

$$(s'')^3 + t^2 s' + s = 0$$ 2. mertebeden 3. dereceden adi dif. denklem (t bağımsız, s bağımlı değişken)

$$\left.\begin{array}{l} \dfrac{\partial^2 U}{\partial x^2} + \dfrac{\partial^2 U}{\partial y^2} + \dfrac{\partial^2 U}{\partial z^2} = 0 \\ (U_{xx} + U_{yy} + U_{zz} = 0) \end{array}\right\}$$ x, y, z bağımsız değişkenler, u bağımlı değişken
2. mertebeden 1. dereceden kısmi dif. denklem

$$U_t = U_{xx} + 2.U.U_x$$ 2. mertebeden 1. dereceden kısmi dif. denklem

$$v + m\frac{dv}{dm} = v^2$$ Roket uçuşuna ait denklem

$$\frac{d^2y}{dx^2} = \frac{w}{H}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$ Asılı Teller denklemi

$$\frac{d^2x}{dt^2} + kx = 0$$ Sarkacın küçük salınımlı hareketlerinde ortaya çıkar

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = f(t)$$ Devre teorisinde karşımıza çıkar

$$\tau(t) = D(q)\cdot q_{tt} + M(q, q_t) + G(q)$$ Robot kolunun dinamik denkleminde ortaya çıkar

n. mertebeden adi diferansiyel denklem

$$g(x, y, y', y'', \ldots, y^{(n)}) = 0$$

g fonksiyonu $y, y', y'', \ldots, y^{(n)}$ değişkenleri cinsinden lineer bir fonksiyon ise dif. denklemi lineer, değilse dif. denklemi lineer değildir.

$$a_0(x)\, y^{(n)} + \ldots + a_{n-1}(x) \cdot y' + a_n(x) \cdot y = b(x)$$ $a_0(x) \neq 0$ n. mertebeden lineer adi dif. denklem

$$\frac{d^4y}{dx^4} + x^2\frac{d^3y}{dx^3} + x^3\frac{dy}{dx} = xe^x$$ 4. mertebeden lineer adi dif. denklem

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$$ 2. mertebeden lineer adi dif. denklem

$$\left.\begin{aligned} &\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y^2 = 0 \\ &\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^2 + 6y = 0 \\ &\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0 \end{aligned}\right\}$$ 2. mertebeden lineer olmayan adi diferansiyel denklemler.

Açık Çözüm, Kapalı Çözüm (Her ikisi de basit çözümdir)

ör $\frac{d^2y}{dx^2} + y = 0$ dif. denk. açık çözümünün $y = 2\sin x + 3\cos x$ olduğunu gösteriniz.

$\frac{d^2y}{dx^2} + y = 0$ olmalı

$$\left.\begin{aligned} &y = 2\sin x + 3\cos x \\ &y' = 2\cos x - 3\sin x \\ &y'' = -2\sin x - 3\cos x \end{aligned}\right\}$$

$\frac{d^2y}{dx^2} + y = 0$ ✓

$y'' + y = -\cancel{2\sin x} - \cancel{3\cos x} + \cancel{2\sin x} + \cancel{3\cos x} = 0$

ör $x + y\frac{dy}{dx} = 0$ dif. denk. $|x| < 5$ için kapalı çözümünün $x^2 + y^2 = 25$ olduğunu gösteriniz

$x^2 + y^2 = 25 \rightarrow 2x + 2yy' = 0 \rightarrow y' = \frac{dy}{dx} = -x/y$

$x + y\frac{dy}{dx} = 0$ olmalı $\quad x + y \cdot \left(-\frac{x}{y}\right) = x - x = 0$ ✓

Başlangıç Değer Problemi

$g(x, y, y', y'', y''') = 0$ dif. denk. veriliyor.

$y(x_0)$, $y'(x_0)$, $y''(x_0)$ verilirse sabitler bulunabilir.

Ör $y' - 2x + 3 = 0$, $y(2) = 3$ ise $y = ?$

$$\frac{dy}{dx} - 2x + 3 = 0 \rightarrow dy = (2x-3)dx \rightarrow y = \int (2x-3)dx = x^2 - 3x + c$$

$x = 2$ için $y = 3$ olur.

$$y = x^2 - 3x + c \longrightarrow 3 = 4 - 6 + c \longrightarrow c = 5$$

$$y = x^2 - 3x + 5$$

Ör $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$, $y(0) = 6$, $y'(0) = 2$ ise $y = ?$

$$m^2 + m - 6 = 0 \qquad y = c_1 e^{2x} + c_2 e^{-3x}$$

$$(m-2)(m+3) = 0 \qquad y' = 2c_1 e^{2x} - 3c_2 e^{-3x}$$

$$m = 2, m = -3$$

$$\left.\begin{aligned} y(0) &= c_1 + c_2 = 6 \\ y'(0) &= 2c_1 - 3c_2 = 2 \end{aligned}\right\} \begin{aligned} c_1 &= 4 \\ c_2 &= 2 \end{aligned} \qquad y = 4e^{2x} + 2e^{-3x}$$

Diferansiyel Denklemin Oluşturulması

Ör $y = c_1 x^2 + c_2 e^{-2x}$ fonk. kullanarak dif. denk. oluştur.

$$\left.\begin{aligned} y' &= 2c_1 x - 2c_2 e^{-2x} \\ y'' &= 2c_1 + 4c_2 e^{-2x} \end{aligned}\right\} \begin{aligned} y'' + 2y' &= 2c_1 + 4c_1 x \\ y''' + 2y'' &= 4c_1 \rightarrow y^{(4)} + 2y''' = 0 \end{aligned}$$

iki bilinmeyen olduğunda 2. mertebeden dif. denk. oluşturulabilir.

$$\left.\begin{aligned} y' + 2y &= 2c_1 x(x+1) \\ y'' + 2y' &= 2c_1(2x+1) \end{aligned}\right\} c_1 \text{ çekilir ise} \quad x(x+1)y'' + (2x^2-1)y' - 2(2x+1)y = 0$$

Ör $y = a.\cos(3x) + b.\sin(3x)$ fonk. kullanarak dif. denk. oluşur. (4)

$y' = -3a\sin(3x) + 3b\cos(3x)$

$y'' = -9a\cos(3x) - 9b\sin(3x) = -9\left(a\cos(3x + b\sin(3x))\right) = -9y$

$y'' + 9y = 0$ bulunur.

Ör Merkezi orijin olan çember ailesinin oluşturduğu dif. denk.

$x^2 + y^2 = r^2 \xrightarrow{\text{türev}} 2x + 2yy' = 0 \xrightarrow[\text{dx ile çarp}]{\text{2'ye böl.}} x\,dx + y\,dy = 0$

Ör Merkezi orijin olan elips ailesinin oluşturduğu dif. denk.

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \xrightarrow{\text{türev}} 2\left(\frac{x}{a}\right)\cdot\left(\frac{1}{a}\right) + 2\left(\frac{y}{b}\right)\left(\frac{y'}{b}\right) = 0$

$a^2yy' + b^2x = 0 \longrightarrow \frac{a^2}{b^2} = -\frac{x}{y.y'}$

$a^2\left((y')^2 + y.y''\right) + b^2 = 0 \longrightarrow \frac{a^2}{b^2} = \frac{-1}{y.y'' + (y')^2}$

$y'' + \frac{1}{y}(y')^2 - \frac{1}{x}y' = 0$ dif. denk.

Ör Düzlemde bütün doğruların oluşturduğu dif. denklem.

$y = mx + b \longrightarrow y' = m \longrightarrow y'' = 0$

Ör Düzlemde orijinden geçen doğruların oluşturduğu dif. denk.

$y = mx \longrightarrow y' = m$

$y = mx = xy' \longrightarrow x\frac{dy}{dx} - y = 0$

$x\,dy - y\,dx = 0$

Ör Simetri ekseni $x=2$ doğrusu üzerinde olan parabollerin oluş. dif. denk.

$y = ax^2 + bx + c \longrightarrow y' = 2ax + b = 0 \longrightarrow x = -\frac{b}{2a} = 2$

$b = -4a$

$y = ax^2 - 4ax + c \longrightarrow \left.\begin{array}{l} y' = 2ax - 4a \\ y'' = 2a \end{array}\right\} (x-2)y'' - y' = 0$

veya direk $y = a(x-2)^2 + h$

Ör Düzlemde r yarıçaplı, (a,b) merkezli çemberlerin oluşturduğu dif. denklemi bul.

$(x-a)^2+(y-b)^2=r^2$

$2(x-a)+2(y-b)y'=0$ sadeleştir.

$x-a+(y-b)y'=0$

$1+(y')^2+(y-b)y''=0$

$3y'y''+(y-b)y'''=0$

$y-b=\dfrac{1+(y')^2}{-y''}$

$3y'y''+\dfrac{1+(y')^2}{-y''}\cdot y'''=0$ $-y''$ ile çarp

$(1+(y')^2)y'''-3y'(y'')^2=0$

Ör Düzlemde $x^2+cy^2=4$ eğri ailesinin dik yörüngelerinin oluşturduğu dif. denklemi bul.

$x^2+cy^2=4 \longrightarrow c=\dfrac{4-x^2}{y^2}$

türevi $\longrightarrow 2x+2cyy'=0 \xrightarrow{\text{sadeleştir}} x+cyy'=0$

$x+\dfrac{4-x^2}{y^2}\cdot y\cdot y'=2 \Longrightarrow xy+(4-x^2)y'=0$

Bu dif. denk. $y'=-1/y'$ konursa eğri ailesinin dik yörüngelerinin oluşturduğu dif. denk. bulunur.

$xy+(4-x^2)\cdot\left(-\dfrac{1}{y'}\right)=0 \Longrightarrow x.y.y'+x^2-4=0$

Ör m kütleli bir cisim yüksekten serbest düşmeye bırakılıyor. Cisme etki eden hava direnci cismin hızı ile doğru orantılı. Yerçekimi g ise. Yol ve türevlerine bağlı dif. denk. oluştur.

$f=ma=m.v'=mg-kv$ Cisme etki eden kuvvet

$v'=x''$ yazarsak $mx''=mg-kx' \Rightarrow x''+\dfrac{k}{m}x'=g$

Ör 250 gr tuz suda eritiliyor. Erime hızı henüz erimemiş tuz miktarıyla doğru orantılı ise erime hızını zamana bağlı veren dif. denk bul.

y: tuz miktarı

t: zaman $\quad k$: orantı sabiti

$y'=k(250-y)$

Birinci Mertebeden Diferansiyel Denklemler

$$F(x,y)=C \longrightarrow dF(x,y)=\frac{\partial F(x,y)}{\partial x}dx+\frac{\partial F(x,y)}{\partial y}dy=0$$

$$M(x,y)=\frac{\partial F(x,y)}{\partial x} \qquad N(x,y)=\frac{\partial F(x,y)}{\partial y}$$ alınırsa

$$M(x,y)\,dx+N(x,y)\,dy=0$$

$$\frac{\partial^2 F(x,y)}{\partial y\,\partial x}=\frac{\partial^2 F(x,y)}{\partial x\,\partial y} \xrightarrow{\text{yani}} \frac{\partial}{\partial y}\left(\frac{\partial F(x,y)}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial F(x,y)}{\partial y}\right)$$

$$\frac{\partial M(x,y)}{\partial y}=\frac{\partial N(x,y)}{\partial x}$$ Normalde birbirlerine eşit olmalıdır. Fakat, dif. denk. sadeleştirme yapılırsa eşit olmaz.

$$\frac{\partial M(x,y)}{\partial y}=\frac{\partial N(x,y)}{\partial x}$$ ise Tam Dif. Denklem

$$\frac{\partial M(x,y)}{\partial y}\neq\frac{\partial N(x,y)}{\partial x}$$ ise Tam olmayan Dif. Denklem

Ör $xy^2+2x^3y+3x+y^3=5$ kapalı fonk. dif. denklemine dönüştür.

$$F(x,y)=xy^2+2x^3y+3x+y^3$$

$$\frac{\partial F}{\partial x}dx+\frac{\partial F}{\partial y}dy=0 \Rightarrow \underbrace{(y^2+6x^2y+3)}_{M}dx+(\underbrace{2xy+2x^3+3y^2}_{N})dy=0$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=2y+6x^2$$ Tam. dif. denklem

Ör $x^4y^3+0.5x^2y^4=2.5$ kapalı fonk. dif. denk. dönüştür.

$$F(x,y)=x^4y^3+0.5x^2y^4=2.5$$

$$\frac{\partial F}{\partial x}dx+\frac{\partial F}{\partial y}dy=0 \rightarrow (4x^3y^3+xy^4)dx+(3x^4y^2+2x^2y^3)dy=0$$

xy^2 ile sadeleştirirsek

$$\underbrace{(4x^2y+y^2)}_{M}dx+(\underbrace{3x^3+2xy}_{N})dy=0$$ olur.

$$\frac{\partial M}{\partial y}=4x^2+2y\neq\frac{\partial N}{\partial x}=9x^2+2y$$

Tam olmayan dif. denklem.

Tam Diferansiyel Denklemler

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ ise tam dif. denklem

1. Yol

$$M(x,y) = \frac{\partial F(x,y)}{\partial x} \longrightarrow \partial F(x,y) = M(x,y).\partial x$$

$$F(x,y) = \int M(x,y).\partial x + \phi(y)$$

$$N(x,y) = \frac{\partial F(x,y)}{\partial y} = \frac{\partial}{\partial y}\int M(x,y).\partial x + \phi'(y)$$

$$\phi'(y) = N(x,y) - \int \frac{\partial M(x,y)}{\partial y}.\partial x$$

→ $\phi(y)$ bulunup $F(x,y)$ fonk. yerine konursa dif. denk çözülmüş olur.

2. Yol

$$N(x,y) = \frac{\partial F(x,y)}{\partial y} \longrightarrow \partial F(x,y) = N(x,y).\partial y$$

$$F(x,y) = \int N(x,y).\partial y + \phi(x)$$

$$M(x,y) = \frac{\partial F(x,y)}{\partial x} = \frac{\partial}{\partial x}\int N(x,y).\partial y + \phi'(x)$$

$$\phi'(x) = N(x,y) - \int \frac{\partial N(x,y)}{\partial x}.\partial y$$

→ $\phi(x)$ bulunup $F(x,y)$ fonk yerine konursa dif. denk. çözülmüş olur.

Tam olmayan Diferansiyel Denklemler

$\mu(x,y)$ çarpanı ile çarpılarak tam hale getirilir.

$$\mu(x,y)M(x,y)\,dx + \mu(x,y)N(x,y)\,dy = 0$$ Tam dif. oldu.

$$\frac{\partial}{\partial y}\big(\mu(x,y)M(x,y)\big) = \frac{\partial}{\partial x}\big(\mu(x,y)N(x,y)\big)$$

$$\frac{\partial \mu(x,y)}{\partial y}M(x,y) + \mu(x,y)\frac{\partial M(x,y)}{\partial y} = \frac{\partial \mu(x,y)}{\partial x}N(x,y) + \mu(x,y)\frac{\partial N(x,y)}{\partial x}$$

$$N(x,y)\frac{\partial \mu(x,y)}{\partial x} - M(x,y)\frac{\partial \mu(x,y)}{\partial y} = \underbrace{\left[\frac{\partial M(x,y)}{\partial y} - \frac{\partial N(x,y)}{\partial x}\right]}_{\text{dif. denk. tam olsa idi burası sıfır olurdu}}.\mu(x,y)$$

μ x'e bağlı bir fonksiyon ise

$$N(x,y)\frac{d\mu(x)}{dx}=\left[\frac{\partial M(x,y)}{\partial y}-\frac{\partial N(x,y)}{\partial x}\right]\cdot\mu(x)$$

$$\frac{d\mu(x)}{\mu(x)}=\frac{1}{N(x,y)}\left[\frac{\partial M(x,y)}{\partial y}-\frac{\partial N(x,y)}{\partial x}\right]dx=g(x)\,dx$$

$$\int\frac{d\mu(x)}{\mu(x)}=\ln\mu(x)=\int g(x)\,dx\longrightarrow\mu(x)=e^{\int g(x)dx}$$

μ y'ye bağlı bir fonksiyon ise

$$-M(x,y)\frac{d\mu(y)}{dy}=\left[\frac{\partial M(x,y)}{\partial y}-\frac{\partial N(x,y)}{\partial x}\right]\cdot\mu(y)$$

$$\frac{d\mu(y)}{\mu(y)}=\frac{1}{M(x,y)}\left[\frac{\partial N(x,y)}{\partial x}-\frac{\partial M(x,y)}{\partial y}\right]\cdot dy=g(y)\cdot dy$$

$$\int\frac{d\mu(y)}{\mu(y)}=\ln\mu(y)=\int g(y)\,dy\longrightarrow\mu(y)=e^{\int g(y)dy}$$

$v=x$ ise $\frac{\partial v}{\partial x}=1$ $\frac{\partial v}{\partial y}=0$

$v=y$ ise $\frac{\partial v}{\partial x}=0$ $\frac{\partial v}{\partial y}=1$

Farklı bir bakış

$v=v(x,y)$ ve $\mu=\mu(v)$ olsun

$$N\frac{\partial\mu}{\partial x}-M\frac{\partial\mu}{\partial y}=\left[\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right]\cdot\mu$$

$$\left.\begin{aligned}\frac{\partial\mu}{\partial x}&=\frac{\partial\mu}{\partial v}\frac{\partial v}{\partial x}\\ \frac{\partial\mu}{\partial y}&=\frac{\partial\mu}{\partial v}\frac{\partial v}{\partial y}\end{aligned}\right\}\quad N\frac{\partial\mu}{\partial v}\frac{\partial v}{\partial x}-M\frac{\partial\mu}{\partial v}\frac{\partial v}{\partial y}=\left[\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right]\cdot\mu$$

$$\frac{\partial\mu}{\partial v}\left(N\frac{\partial v}{\partial x}-M\frac{\partial v}{\partial y}\right)=\left[\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right]\cdot\mu$$

$$\frac{\mu'(v)}{\mu(v)}=\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N\frac{\partial v}{\partial x}-M\frac{\partial v}{\partial y}}$$

$v=x+y\to\frac{\partial v}{\partial x}=1$ $\frac{\partial v}{\partial y}=1$

$v=x-y\to\frac{\partial v}{\partial x}=1$ $\frac{\partial v}{\partial y}=-1$

$v=x^2+y^2\to\frac{\partial v}{\partial x}=2x$ $\frac{\partial v}{\partial y}=2y$

$v=x^2y\to\frac{\partial v}{\partial x}=2xy$ $\frac{\partial v}{\partial y}=x^2$

Ör $(2xy^2 - y\sin x + 2x - 1)\,dx + (2x^2y + \cos x + \frac{1}{y})\,dy = 0$ çöz.

$M = 2xy^2 - y\sin x + 2x - 1 \longrightarrow \frac{\partial M}{\partial y} = 4xy - \sin x$

$N = 2x^2y + \cos x + \frac{1}{y} \longrightarrow \frac{\partial N}{\partial x} = 4xy - \sin x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Tam dif. denk.

1.yol

$F(x,y) = \int M(x,y)\,\partial x + \phi(y) = \int (2xy^2 - y\sin x + 2x - 1)\,\partial x + \phi(y)$

$= x^2y^2 + y\cos x + x^2 - x + \phi(y)$

$N(x,y) = \frac{\partial F(x,y)}{\partial y} = 2x^2y + \cos x + \phi'(y)$

$\phi'(y) = 1/y$

$\phi(y) = \int \frac{dy}{y} = \ln y$

$F(x,y) = x^2y^2 + y\cos x + x^2 - x + \ln y = C$

2.yol

$F(x,y) = \int N(x,y)\,\partial y + \phi(x) = \int (2x^2y + \cos x + \frac{1}{y})\,\partial y + \phi(x)$

$= x^2y^2 + y\cos x + \ln y + \phi(x)$

$M(x,y) = \frac{\partial F(x,y)}{\partial x} = 2xy^2 - y\sin x + \phi'(x)$

$\phi'(x) = 2x - 1$

$\phi(x) = x^2 - x$

$F(x,y) = x^2y^2 + y\cos x + \ln y + x^2 - x = C$

Ör $(3x^2 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0$ dif. denk. çöz.

$M(x,y) = 3x^2 + 4xy \longrightarrow \frac{\partial M}{\partial y} = 4x$

$N(x,y) = 2x^2 + 2y \longrightarrow \frac{\partial N}{\partial x} = 4x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Tam dif. denk.

1.yol

$F(x,y) = \int M(x,y)\,\partial x + \phi(y) = \int (3x^2 + 4xy)\,\partial x + \phi(y)$

$= x^3 + 2x^2y + \phi(y)$

$N(x,y) = \frac{\partial F(x,y)}{\partial y} = 2x^2 + \phi'(y)$

$\phi'(y) = 2y$

$\phi(y) = y^2$

$F(x,y) = x^3 + 2x^2y + y^2 = C$

2.yol

$F(x,y) = \int N(x,y)\,\partial y + \phi(x) = \int (2x^2 + 2y)\,\partial y + \phi(x)$

$= 2x^2y + y^2 + \phi(x)$

$M(x,y) = \frac{\partial F(x,y)}{\partial x} = 4xy + \phi'(x)$

$\phi'(x) = 3x^2 \rightarrow \phi(x) = x^3$

$F(x,y) = 2x^2y + y^2 + x^3 = C$

Gruplama Yöntemi

Ör $(3x^2+4xy)dx+(2x^2+2y)dy=0\ ,\ y(0)=2$

$$\underbrace{3x^2dx}_{x^3}+\underbrace{(4xy\,dx+2x^2dy)}_{2x^2y}+\underbrace{2y\,dy}_{y^2}=\underbrace{0}_{c}$$

$$x^3+2x^2y+y^2=c$$ (Bu yöntemi kullanmak için dif. denk. Tam olmalı)

Ör $(2x\cos y+3x^2y)dx+(x^3-x^2\sin y-y)dy=0$

$$M=2x\cos y+3x^2y\rightarrow\frac{\partial M}{\partial y}=-2x\sin y+3x^2$$
$$N=x^3-x^2\sin y-y\rightarrow\frac{\partial N}{\partial x}=3x^2-2x\sin y$$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Tam dif. denk. grup. yön. kul.

$$2x\cos y\,dx+3x^2y\,dx+x^3dy-x^2\sin y\,dy-y\,dy=0$$
$$(2x\cos y\,dx-x^2\sin y\,dy)+(3x^2y\,dx+x^3dy)-y\,dy=0$$
$$x^2\cos y+x^3y-\frac{y^2}{2}=c\rightarrow 0+0-2=c\Rightarrow c=-2$$

dif. denk. çöz.

Ör $(3y+4xy^2)dx+(2x+3x^2y)dy=0$

$$M(x,y)=3y+4xy^2\rightarrow\frac{\partial M}{\partial y}=3+8xy$$
$$N(x,y)=2x+3x^2y\rightarrow\frac{\partial N}{\partial x}=2+6xy$$

$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$ Dif. Denk. Tam değil

$\mu(x,y)=x^2y$ şeklinde verilmiş olsun

$$x^2y(3y+4xy^2)dx+x^2y(2x+3x^2y)dy=0$$
$$3x^2y^2dx+4x^3y^3dx+2x^3y\,dy+3x^4y^2dy=0$$
$$\underbrace{(3x^2y^2dx+2x^3y\,dy)}_{x^3y^2}+\underbrace{(4x^3y^3+3x^4y^2dy)}_{x^4y^3}=\underbrace{0}_{c}$$
$$x^3y^2+x^4y^3=c$$

(11)

Ör $(2xye^y + y^2\cos x - xy^2\sin x)dx + (x^2e^y + x^2ye^y + 2xy\cos x)dy = 0$
dif. denk. gruplama yöntemiyle çöz.

$$\underbrace{2xye^y dx + (x^2e^y + x^2ye^y)dy}_{x^2ye^y} + \underbrace{(y^2\cos x - xy^2\sin x)dx + 2xy\cos x\, dy}_{xy^2\cos x} = \underbrace{0}_{C}$$

$$x^2ye^y + xy^2\cos x = C$$

Ör $(y+\sin x)dx + \sin x.\cos x.dy = 0$ dif. denk. tam olmayıp
μ'nün x'e bağlı olduğu biliniyor. $\mu(x) = ?$

$$\left.\begin{array}{l} M = y + \sin x \longrightarrow \dfrac{\partial M}{\partial y} = 1 \\ N = \sin x\cos x \longrightarrow \dfrac{\partial N}{\partial x} = \cos^2 x - \sin^2 x = \cos 2x \end{array}\right\} \begin{array}{l} \dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x} \\ \text{Tam değil.} \end{array}$$

$$g(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{1 - \cos 2x}{\sin x\cos x} = \frac{\cancel{\cos^2 x} + \sin^2 x - (\cancel{\cos^2 x} - \sin^2 x)}{\sin x\cos x} = 2\tan x$$

$$\mu(x) = e^{\int g(x)dx} = e^{2\int \tan x dx} = e^{2\int \frac{\sin x dx}{\cos x}} = e^{-2\int \frac{du}{u}}$$

$u = \cos x$
$du = -\sin x dx$

$$= e^{-2\ln u} = e^{\ln u^{-2}} = u^{-2} = \frac{1}{u^2} = \sec^2 x$$

Ör $(x - 2x^2y)dy - ydx = 0$ dif. denk. tam olmayıp
x'e bağlı bir integrasyon çarpanı var. $\mu(x) = ?$ ve çöz.

$$\left.\begin{array}{l} M = -y \longrightarrow \dfrac{\partial M}{\partial y} = -1 \\ N = x - 2x^2y \longrightarrow \dfrac{\partial N}{\partial x} = 1 - 4xy \end{array}\right\} \begin{array}{l} \dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x} \\ \text{Tam değil.} \end{array}$$

$$g(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-1 - 1 + 4xy}{x - 2x^2y} = \frac{-2(1-2xy)}{x(1-2xy)} = -2/x$$

$$\mu(x) = e^{\int g(x)dx} = e^{-2\int \frac{dx}{x}} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = 1/x^2$$

$\frac{1}{x^2}$ ile çarp $\Rightarrow \left(\frac{1}{x} - 2y\right)dy - \frac{y}{x^2}dx = 0 \Rightarrow \left(\frac{1}{x}dy - \frac{y}{x^2}dx\right) - 2ydy = 0$

$$\frac{y}{x} - y^2 = C$$

$(2xe^y - \frac{x}{y^2})dx + (x^2e^y + \frac{x^2}{y^3} + 2y)dy = 0$ dif. denk.

tam olduğunu göster ve çöz -

$M = 2xe^y - \frac{x}{y^2} \longrightarrow \frac{\partial M}{\partial y} = 2xe^y + \frac{2x}{y^3}$

$N = x^2e^y + \frac{x^2}{y^3} + 2y \longrightarrow \frac{\partial N}{\partial x} = 2xe^y + \frac{2x}{y^3}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Tam dif. denk-

$F(x,y) = \int M(x,y)\,\partial x + \phi(y) = \int (2xe^y - \frac{x}{y^2})\,\partial x + \phi(y)$

$= x^2e^y - \frac{x^2}{2y^2} + \phi(y)$

$N(x,y) = \frac{\partial F(x,y)}{\partial y} = x^2e^y + \frac{x^2}{y^3} + \phi'(y)$

$\phi'(y) = 2y$

$\phi(y) = y^2$

$F(x,y) = x^2e^y - \frac{x^2}{2y^2} + y^2 = C$

$y' = \frac{3x-y}{x}$ dif. denk. çöz.

$y' = \frac{3x-y}{x} = \frac{dy}{dx} \Longrightarrow (3x-y)dx = x\,dy$

$(y-3x)dx + x\,dy = 0$

$M = y - 3x \longrightarrow \frac{\partial M}{\partial y} = 1$

$N = x \longrightarrow \frac{\partial N}{\partial x} = 1$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Tam dif. denk.

$F(x,y) = \int M(x,y)\,\partial x + \phi(y) = \int (y-3x)\,\partial x + \phi(y)$

$= xy - \frac{3x^2}{2} + \phi(y)$

$N = \frac{\partial F}{\partial y} = x + \phi'(y)$

$\phi'(y) = 0$

$\phi(y) = c_1$

$F(x,y) = xy - \frac{3x^2}{2} = C$

Ör $(\cos x - x\sin x + 2x\sin^3 y)dx + (3x^2\cos y\sin^2 y + 1/y)\,dy = 0$ (13)

dif. denk. tam ise gruplama yön. çöz.

$M = \cos x - x\sin x + 2x\sin^3 y \longrightarrow \frac{\partial M}{\partial y} = 6x\sin^2 y\cos y$

$N = 3x^2\cos y\sin^2 y + 1/y \longrightarrow \frac{\partial N}{\partial x} = 6x\sin^2 y\cos y$

} Tam dif. denk.

$$\underbrace{(\cos x - x\sin x)dx}_{x\cos x} + \underbrace{\frac{dy}{y}}_{\ln y} + \underbrace{(2x\sin^3 y\,dx + 3x^2\sin^2 y\cos y\,dy)}_{x^2\sin^3 y} = \underbrace{0}_{c}$$

$$x\cos x + \ln y + x^2\sin^3 y = C$$

Gruplama kullanmasaydık.

$$F(x,y) = \int N(x,y)\,\partial y + \phi(x) = \int \left(3x^2\sin^2 y\cos y + \frac{1}{y}\right)dy + \phi(x)$$

$$= x^2\sin^3 y + \ln y + \phi(x)$$

$$M = \frac{\partial F}{\partial x} = 2x\sin^3 y + \phi'(x) \longrightarrow \phi'(x) = \cos x - x\sin x$$

$$\phi(x) = x\sin x$$

$$F(x,y) = x^2\sin^3 y + \ln y + x\sin x = C$$

Ör $(1 + e^{2y})dx + (2xe^{2y} + y^2)dy = 0$ dif. denk. çöz.

$M = 1 + e^{2y} \longrightarrow \frac{\partial M}{\partial y} = 2e^{2y}$

$N = 2xe^{2y} + y^2 \longrightarrow \frac{\partial N}{\partial x} = 2e^{2y}$

} Tam dif. denk.

$$F(x,y) = \int M(x,y)\,\partial x + \phi(y) = \int (1 + e^{2y})\,\partial x + \phi(y)$$

$$= x + xe^{2y} + \phi(y)$$

$$N(x,y) = \frac{\partial F}{\partial y} = 2xe^{2y} + \phi'(y) \longrightarrow \phi'(y) = y^2$$

$$\phi(y) = y^3/3$$

$$F(x,y) = x + xe^{2y} + \frac{y^3}{3} = C$$

1) $(e^{3y}-2xye^{-x^2y}+y^2)dx+(3xe^{3y}-x^2e^{-x^2y}+2xy+1)dy=0$ (14)

dif. denk. çöz.

$M(x,y)=e^{3y}-2xye^{-x^2y}+y^2 \longrightarrow \frac{\partial M}{\partial y}=3e^{3y}-2xe^{-x^2y}+2x^3ye^{-x^2y}+2y$

$N(x,y)=3xe^{3y}-x^2e^{-x^2y}+2xy+1 \longrightarrow \frac{\partial N}{\partial x}=3e^{3y}-2xe^{-x^2y}+2x^3ye^{-x^2y}+2y$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ olduğundan dif. denk. tamdır.

$F(x,y)=\int M(x,y)\partial x+\phi(y)=\int(e^{3y}-2xye^{-x^2y}+y^2)\partial x+\phi(y)$

$=xe^{3y}+e^{-x^2y}+xy^2+\phi(y)$

$N(x,y)=\frac{\partial F}{\partial y}=3xe^{3y}-x^2e^{-x^2y}+2xy+\phi'(y) \longrightarrow \phi'(y)=1$, $\phi(y)=y$

$F(x,y)=xe^{3y}+e^{-x^2y}+xy^2+y=C$

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2) $(x\sqrt{x^2+y^2}-y)dx+(y\sqrt{x^2+y^2}-x)dy=0$

dif. denk. çözünüz.

$M(x,y)=x\sqrt{x^2+y^2}-y \longrightarrow \frac{\partial M}{\partial y}=\frac{xy}{\sqrt{x^2+y^2}}-1$

$N(x,y)=y\sqrt{x^2+y^2}-x \longrightarrow \frac{\partial N}{\partial x}=\frac{xy}{\sqrt{x^2+y^2}}-1$

Tam dif. denk.

$F(x,y)=\int M(x,y)\partial x+\phi(y)$

$=\int(x\sqrt{x^2+y^2}-y)\partial x+\phi(y)$

$u=\sqrt{x^2+y^2}$

$u^2=x^2+y^2$

$2u\frac{\partial u}{\partial x}=2x$

$u\,\partial u=x\,\partial x$

$=\int u^2\partial u-xy+\phi(y)$

$=\frac{u^3}{3}-xy+\phi(y)=\frac{1}{3}(x^2+y^2)^{3/2}-xy+\phi(y)$

$N(x,y)=\frac{\partial F}{\partial y}=y\sqrt{x^2+y^2}-x+\phi'(y) \longrightarrow \phi'(y)=0$

$\phi(y)=c_1$

$F(x,y)=\frac{1}{3}(x^2+y^2)^{3/2}-xy+c_1=c_2 \Rightarrow (x^2+y^2)^{3/2}-3xy=C$

8/ $(2xy+y\ln y-y^2\sin x)dx+(x+y\cos x)dy=0$ dif. denk.
y'ye bağlı integrasyon çarpanı var. $\mu(y)=?$ ve çöz.

$$\left.\begin{aligned} M&=2xy+y\ln y-y^2\sin x \rightarrow \frac{\partial M}{\partial y}=2x+\ln y+1-2y\sin x \\ N&=x+y\cos x \longrightarrow \frac{\partial N}{\partial x}=1-y\sin x \end{aligned}\right\}\text{Tam değil.}$$

$$g(y)=\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}=\frac{(1-y\sin x)-(2x+\ln y+1-2y\sin x)}{2xy+y\ln y-y^2\sin x}$$

$$=\frac{+y\sin x-2x-\ln y}{-y(y\sin x-2x-\ln y)}=-1/y \checkmark$$

$$\mu(y)=e^{\int g(y)dy}=e^{-\int dy/y}=e^{-\ln y}=e^{\ln y^{-1}}=1/y$$

denklem $1/y$ ile çarpılırsa

$$(2x+\ln y-y\sin x)dx+(x/y+\cos x)dy=0$$

$$\left.\begin{aligned} M&=2x+\ln y-y\sin x \longrightarrow \frac{\partial M}{\partial y}=\frac{1}{y}-\sin x \\ N&=\frac{x}{y}+\cos x \longrightarrow \frac{\partial N}{\partial x}=\frac{1}{y}-\sin x \end{aligned}\right\}\text{Tam dif. denk.}$$

$$F(x,y)=\int N(x,y)\partial y+\Phi(x)=\int\left(\frac{x}{y}+\cos x\right)\partial y+\Phi(x)$$

$$=x\ln y+y\cos x+\Phi(x)$$

$$M(x,y)=\frac{\partial F}{\partial x}=\ln y-y\sin x+\Phi'(x) \rightarrow \Phi'(x)=2x$$

$$\Phi(x)=x^2$$

$$F(x,y)=x\ln y+y\cos x+x^2=c$$

Gruplama yön. yaparsak

$$(2x+\ln y-y\sin x)dx+\left(\frac{x}{y}+\cos x\right)dy=0$$

$$2xdx+\left(\ln y\,dx+\frac{x}{y}dy\right)+(-y\sin x\,dx+\cos x\,dy)=0$$

$$x^2+x\ln y+y\cos x=c$$

Ayrışabilir Diferansiyel Denklemler

$M(x)\,dx + N(y)\,dy = 0$ şeklindeki dif. denklemleridir.

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 0$ aynı zamanda tamdırlar.

$f(x)g(y)\,dx + p(x)h(y)\,dy = 0$ ise ayrışabilir hale getirilir. $\frac{f(x)}{p(x)}dx + \frac{h(y)}{g(y)}dy = 0$ elde edilir.

$y' + \frac{y^3+1}{3xy^2(x^2+1)} = 0$

Ör $(1+2x)y\,dy + (1+y^2)\,dx = 0$ çöz.

$$\frac{dx}{2x+1} + \frac{y\,dy}{1+y^2} = 0 \longrightarrow \frac{2\,dx}{2x+1} + \frac{2y\,dy}{1+y^2} = 0$$

$$\int \frac{2\,dx}{2x+1} + \int \frac{2y\,dy}{1+y^2} = C_1 \qquad \begin{array}{l} u = 2x+1 \\ du = 2\,dx \end{array} \qquad \begin{array}{l} v = 1+y^2 \\ dv = 2y\,dy \end{array}$$

$$\int \frac{du}{u} + \int \frac{dv}{v} = C_1$$

$$\ln|u| + \ln|v| = C_1 = \ln|c| = \ln|uv|$$

$$uv = c \longrightarrow (2x+1)(1+y^2) = c$$

$ye^{-2x}\,dy + x(y^2+1)\,dx = 0$

Ör $y' = \frac{x(1+y^2)}{y(1+x^2)}$ dif. denk. çöz.

$$\frac{dy}{dx} = \frac{x(1+y^2)}{y(1+x^2)} \longrightarrow \frac{2x\,dx}{1+x^2} - \frac{2y\,dy}{1+y^2} = 0 \qquad \begin{array}{l} u = 1+x^2 \\ du = 2x\,dx \\ v = 1+y^2 \\ dv = 2y\,dy \end{array}$$

$$\int \frac{2x\,dx}{1+x^2} - \int \frac{2y\,dy}{1+y^2} = C_1$$

$$\int \frac{du}{u} - \int \frac{dv}{v} = C_1$$

$$\ln|u| - \ln|v| = C_1 \longrightarrow \ln\left|\frac{u}{v}\right| = C_1 = \ln|c|$$

$$\frac{u}{v} = c \Rightarrow \frac{1+x^2}{1+y^2} = c$$

Ör $x.\cos y.dx - e^x.\sin y.dy = 0$ dif. denk. değişkenlerine ayrılabilir hale getirip çöz.

$e^x \cos y$ ile bölersek

$$x e^{-x} dx - \frac{\sin y}{\cos y} dy = 0 \rightarrow \int x e^{-x} dx - \int \frac{\sin y}{\cos y} dy = c$$

$$\ln|\cos y| - (x+1).e^{-x} = c$$

Ör $y' = \sin(x-y+5)$ dif. denk. çöz.

$$u = x-y+5 \longrightarrow \frac{du}{dx} = 1 - \frac{dy}{dx} = 1 - y' = 1 - \sin u$$

$$\frac{du}{1-\sin u} - dx = 0 \Rightarrow \int \frac{du}{1-\sin u} - x = c \Rightarrow \tan u + \sec u - x = c$$

$$\tan(x-y+5) + \sec(x-y+5) - x = c$$

Ör $y' = (3x+3y+8)^2$ dif. denk. çöz.

$$u = 3x+3y+8 \longrightarrow \frac{du}{dx} = 3 + 3\frac{dy}{dx} = 3 + 3y' = 3 + 3u^2 = 3(1+u^2)$$

$$\frac{du}{1+u^2} - 3dx = 0 \longrightarrow \int \frac{du}{1+u^2} - 3\int dx = 0$$

$$\arctan u - 3x = c \longrightarrow \arctan(3x+3y+8) - 3x = c$$

Ör $y' = \tan^2(x+y)$ dif. denk. çöz.

$$u = x+y \longrightarrow \frac{du}{dx} = 1 + y' = 1 + \tan^2 u = \sec^2 u = \frac{1}{\cos^2 u}$$

$$\cos^2 u\, du - dx = 0 \longrightarrow \int \cos^2 u\, du - \int dx = c_1$$

$$\int \frac{1+\cos 2u}{2} du - x = c_1 \longrightarrow \frac{u}{2} + \frac{\sin 2u}{4} - x = c_1$$

$$\frac{x+y}{2} + \frac{\sin(2x+2y)}{4} - x = c_1 \Rightarrow y - x + \frac{1}{2}\sin(2x+2y) = c$$

Ör $(x+y+1)y' = 1$ dif. denk. çöz.

$$u = x+y+1 \longrightarrow \frac{du}{dx} = 1 + \frac{dy}{dx} = 1 + y' = 1 + \frac{1}{u} = \frac{u+1}{u}$$

$$\frac{u\,du}{u+1} - dx = 0 \rightarrow du - dx - \frac{du}{u+1} = 0 \rightarrow u - x - \ln|u+1| = c_1$$

$$x+y+1 - x - \ln|x+y+1+1| = c_1 \Rightarrow y - \ln|x+y+2| = c$$

$$(3x+8)(y^2+4)dx - 4y(x^2+5x+6)dy = 0$$

$$\longrightarrow (x+3)(x+2)^2 = c\,(y^2+4)^2$$

$y(1-xy)dx - x(1+xy)dy = 0$ dif. denk. çöz.

$$y(1-xy)dx = x(1+xy)dy \longrightarrow \frac{dy}{dx} = \frac{y(1-xy)}{x(1+xy)}$$

$$u = xy \rightarrow \frac{du}{dx} = y + x\frac{dy}{dx} = y + x\cdot\frac{y(1-xy)}{x(1+xy)}$$

$$\frac{du}{dx} = y + y\,\frac{1-xy}{1+xy} = \frac{2y}{1+xy}$$, x ile çarp.

$$x\frac{du}{dx} = \frac{2xy}{1+xy} = \frac{2u}{1+u} \longrightarrow \frac{1+u}{u}du = \frac{2dx}{x}$$

$$du + \frac{du}{u} + 2\frac{dx}{x} = 0 \longrightarrow u + \ln|u| - 2\ln|x| = c$$

$$xy + \ln|xy| - \ln|x|^2 = c \longrightarrow xy + \ln\left|\frac{y}{x}\right| = c$$

$(x-2\sin y+3)dx + (2x-4\sin y-3)\cdot\cos y\,dy = 0$ dif. denk. ayrısabilir ~~hale~~ getirip çöz.

$$u = \sin y \longrightarrow du = \cos y\cdot dy$$

$$(x-2u+3)dx + (2x-4u-3)du = 0$$

$$\frac{du}{dx} = -\frac{x-2u+3}{2x-4u-3}\ ,\quad v = x-2u$$

$$\frac{dv}{dx} = 1 - 2\frac{du}{dx} = 1 + 2\,\frac{x-2u+3}{2x-4u-3} = 1 + 2\,\frac{v+3}{2v-3} = \frac{4v+3}{2v-3}$$

$$\frac{2v-3}{4v+3}dv - dx = 0 \longrightarrow \left(\frac{1}{2} - \frac{9}{2}\,\frac{1}{4v+3}\right)dv - dx = 0$$

$$4dv - 9\,\frac{4dv}{4v+3} - 8\,dv = 0$$

$$4\int dv - 9\int\frac{4dv}{4v+3} - 8\int dv = c_1 \longrightarrow 4v - 9\ln|4v+3| - 8x = c_1$$

$$v = x - 2u = x - 2\sin y$$

$$4(x-2\sin y) - 9\ln|4x-8\sin y+3| = c_1$$

$$4x + 8\sin y + 9\ln|4x-8\sin y+3| = c$$

Ör $(2+2x^2\cdot\sqrt{y})\cdot y\,dx+(x^2\sqrt{y}+2)\,x\,dy=0$ dif. denk. çöz.

$$\frac{dy}{dx}=-\frac{y(2+2x^2\cdot\sqrt{y})}{x(2+x^2\cdot\sqrt{y})}\ ,\quad u=x^2\cdot\sqrt{y}$$

$$\frac{du}{dx}=2x\sqrt{y}+\frac{x^2}{2\sqrt{y}}\frac{dy}{dx}=2x\sqrt{y}-\frac{x^2}{2\sqrt{y}}\frac{y(2+2x^2\cdot\sqrt{y})}{x(2+x^2\cdot\sqrt{y})}$$

$$x\frac{du}{dx}=2x^2\sqrt{y}-x^2\cdot\sqrt{y}\,\frac{1+x^2\cdot\sqrt{y}}{2+x^2\sqrt{y}}=2u-\frac{u(1+u)}{2+u}=\frac{u(u+3)}{u+2}$$

$$\frac{u+2}{u(u+3)}du-\frac{dx}{x}=0\longrightarrow 2\frac{du}{u}+\frac{du}{u+3}-3\frac{dx}{x}=0$$

$$2\ln|u|+\ln|u+3|-3\ln|x|=c_1\longrightarrow\ln\left|\frac{u^2(u+3)}{x^3}\right|=c_1=\ln|c|$$

$$\frac{u^2(u+3)}{x^3}=c\longrightarrow\frac{x^4y(x^2\sqrt{y}+3)}{x^3}=c\longrightarrow xy(x^2\cdot\sqrt{y}+3)=c$$

Homojen Diferansiyel Denklemler

$F(kx,ky)=k^nF(x,y)$ veya $M(kx,ky)=k^n\cdot M(x,y)$

$N(kx,ky)=k^n\cdot N(x,y)$

$u=y/x$ yazılarak homojenlik kontrol edilir.

Ör $(2x-y)\,dy=(2y-x)\,dx$ dif. denk. çöz.

$$\frac{dy}{dx}=\frac{2y-x}{2x-y}=\frac{2\frac{y}{x}-1}{2-y/x}=\frac{2u-1}{2-u}\ ,\quad u=y/x$$

$$y=xu\longrightarrow y'=u+xu'=\frac{2u-1}{2-u}\Rightarrow xu'=\frac{u^2-1}{2-u}=x\frac{du}{dx}$$

$$\frac{u-2}{u^2-1}du+\frac{dx}{x}=0\longrightarrow\frac{u-2}{(u-1)(u+1)}=\frac{a}{u-1}+\frac{b}{u+1}\qquad a=-1/2,\ b=3/2$$

$$-\frac{1}{2}\frac{du}{u-1}+\frac{3}{2}\frac{du}{u+1}+\frac{dx}{x}=0$$

$$3\frac{du}{u+1}-\frac{du}{u-1}+2\frac{dx}{x}=0$$

$$3\ln|u+1|-\ln|u-1|+2\ln|x|=c_1$$

$$\rightarrow\ln\left|\frac{(u+1)^3x^2}{u-1}\right|=\ln|c|$$

$$\frac{(u+1)^3x^2}{u-1}=c\longrightarrow\frac{(x+y)^3}{y-x}=c$$

Ör $(x^2-3y^2)\,dx + 2xy\,dy = 0$ dif. denk. çöz.

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} = \frac{3}{2}\left(\frac{y}{x}\right) - \frac{1}{2}\left(\frac{x}{y}\right) = \frac{3u}{2} - \frac{1}{2u} \quad , \quad u = y/x$$

$$y = xu \longrightarrow y' = u + xu' = \frac{3u}{2} - \frac{1}{2u} \longrightarrow x\frac{du}{dx} = \underset{(u)}{\frac{u}{2}} - \underset{(1)}{\frac{1}{2u}} = \frac{u^2-1}{2u}$$

$$\frac{2u\,du}{u^2-1} - \frac{dx}{x} = 0 \longrightarrow \int \frac{2u\,du}{u^2-1} - \int \frac{dx}{x} = c_1$$

$$\ln|u^2-1| - \ln|x| = c_1 \longrightarrow \ln\left|\frac{u^2-1}{x}\right| = \ln|c| = c_1$$

$$\frac{u^2-1}{x} = c \longrightarrow \frac{\frac{y^2}{x^2}-1}{x} = c \longrightarrow \frac{y^2}{x^2} - 1 = cx \Rightarrow y^2 - x^2 = cx^3$$

Ör $(y+\sqrt{x^2+y^2})\,dx - x\,dy = 0$, $y(1) = 0$ dif. denk. çöz

$$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+(y/x)^2} = u + \sqrt{1+u^2} \quad , \quad u = y/x$$

$$y = xu \longrightarrow y' = \cancel{u} + xu' = \cancel{u} + \sqrt{1+u^2} \longrightarrow x\frac{du}{dx} = \sqrt{1+u^2}$$

$$\frac{du}{\sqrt{1+u^2}} - \frac{dx}{x} = 0 \longrightarrow \int \frac{du}{\sqrt{1+u^2}} - \int \frac{dx}{x} = c_1$$

$u = \tan\theta$ diyerek çöz

$$\ln|u + \sqrt{1+u^2}| - \ln|x| = c_1 \longrightarrow \ln\left|\frac{u+\sqrt{1+u^2}}{x}\right| = c_1 = \ln|c|$$

$$\frac{u+\sqrt{1+u^2}}{x} = c = \frac{y+\sqrt{x^2+y^2}}{x^2} \qquad x=1, y=0 \longrightarrow c = \frac{0+1}{1} = 1$$

$$y + \sqrt{x^2+y^2} - x^2 = 0 \longrightarrow y = \frac{x^2-1}{2}$$

Ör $2x^3y\,dx + (x^4+y^4)\,dy = 0$ dif. denk. çöz.

$$\frac{dy}{dx} = \frac{-2x^3y}{x^4+y^4} \longrightarrow \frac{dx}{dy} = \frac{x^4+y^4}{-2x^3y} = -\frac{x}{2y} - \frac{1}{2}\left(\frac{y}{x}\right)^3 = -\frac{1}{2}u - \frac{1}{2u^3} \quad , \quad u = x/y$$

$$x = yu \longrightarrow \frac{dx}{dy} = u + y\frac{du}{dy} = -\frac{1}{2}u - \frac{1}{2u^3} \Rightarrow y\frac{du}{dy} = -\underset{(u^3)}{\frac{3u}{2}} - \underset{(1)}{\frac{1}{2u^3}} = -\frac{3u^4+1}{2u^3}$$

$$\frac{2u^3\,du}{3u^4+1} + \frac{dy}{y} = 0 \longrightarrow \frac{1}{6}\ln|3u^4+1| + \ln|y| = c_1 \rightarrow \ln|y^6(3u^4+1)| = c_2 = \ln|c$$

$$y^6(3u^4+1) = c \longrightarrow 3x^4y^2 + y^6 = c \longrightarrow y^2(3x^4+y^4) = c$$

Ör $(x^2+y^2)dy + 2y^2dx = 0$ dif. denk. çöz.

$$\frac{dy}{dx} = \frac{-2y^2}{x^2+y^2} = -2\,\frac{(y/x)^2}{1+(y/x)^2} = -\frac{2u^2}{1+u^2}\ , \quad u = y/x$$

$$y = xu \rightarrow y' = u + xu' = -\frac{2u^2}{1+u^2} \rightarrow x\frac{du}{dx} = -\frac{u(u+1)^2}{u^2+1}$$

$$\frac{u^2+1}{u(u+1)^2}du + \frac{dx}{x} = 0 \xrightarrow[\text{ayır}]{\text{çarpanlarına}} \int\frac{du}{u} - 2\int\frac{du}{(u+1)^2} + \int\frac{dx}{x} = C$$

$$\ln|u| + \frac{2}{u+1} + \ln|x| = C \longrightarrow \ln|y| + \frac{2x}{x+y} = C$$

<u>diğer yol</u>

$$\frac{dx}{dy} = \frac{x^2+y^2}{-2y^2} = -\frac{1}{2}\left(\frac{x}{y}\right)^2 - \frac{1}{2} = -\frac{u^2}{2} - \frac{1}{2} = \frac{u^2+1}{-2}\ , \quad u = x/y$$

$$x = yu \longrightarrow \frac{dx}{dy} = u + y\frac{du}{dy} = \frac{u^2+1}{-2} \rightarrow y\frac{du}{dy} = -\frac{(u+1)^2}{2}$$

$$\frac{dy}{y} + 2\frac{du}{(u+1)^2} = 0 \longrightarrow \ln|y| - \frac{2}{u+1} = C \Rightarrow \ln|y| - \frac{2y}{x+y} = C$$

Ör $(y-\sqrt{xy})dx - x\,dy = 0$ dif. denk. çöz.

$$\frac{dy}{dx} = \frac{y-\sqrt{xy}}{x} = \frac{y}{x} - \sqrt{\frac{y}{x}} = u - \sqrt{u}\ , \quad u = y/x$$

$$y = xu \longrightarrow y' = \cancel{u} + xu' = \cancel{u} - \sqrt{u} \longrightarrow x\frac{du}{dx} = -\sqrt{u}$$

$$\frac{du}{\sqrt{u}} + \frac{dx}{x} = 0 \longrightarrow 2\sqrt{u} + \ln|x| = C \longrightarrow 2\sqrt{\frac{y}{x}} + \ln|x| = C$$

Ör $xy' = y - x\cos^2(y/x)$ dif. denk. çöz

$$y' = \frac{y}{x} - \cos^2(y/x) = u - \cos^2 u\ , \quad u = y/x$$

$$y = xu \longrightarrow y' = \cancel{u} + xu' = \cancel{u} - \cos^2 u \longrightarrow x\frac{du}{dx} = -\cos^2 u$$

$$\frac{du}{\cos^2 u} + \frac{dx}{x} = 0 \longrightarrow \sec^2 u\,du + \frac{dx}{x} = 0$$

$$\int \sec^2 u\,du + \int\frac{dx}{x} = C$$

$$\tan u + \ln|x| = C \longrightarrow \tan(y/x) + \ln|x| = C$$

ör $y' = \dfrac{y}{x-\sqrt{xy}}$ dif. denk. çöz.

$$y' = \frac{y/x}{1-\sqrt{y/x}} = \frac{u}{1-\sqrt{u}} \quad , \quad \frac{y}{x} = u$$

$$y = xu \rightarrow y' = u + xu' = \frac{u}{1-\sqrt{u}}$$

$$x\frac{du}{dx} = \frac{u}{1-\sqrt{u}} - u = \frac{u\sqrt{u}}{1-\sqrt{u}} \Rightarrow \frac{1-\sqrt{u}}{u\sqrt{u}}du - \frac{dx}{x} = 0$$

$$\int u^{-3/2}du - \int \frac{du}{u} - \int \frac{dx}{x} = c_1$$

$$-2u^{-1/2} - \ln|u| - \ln|x| = c_1 = -c$$

$$\frac{2}{\sqrt{u}} + \ln|ux| = c \Rightarrow 2\sqrt{\frac{x}{y}} + \ln|y| = c$$

ör $\left(xe^{y/x} + y - \frac{y^2}{x}\right)dx + (y-x)dy = 0$ dif. denk. çöz.

$$\frac{dy}{dx} = \frac{xe^{y/x} + y - y^2/x}{x-y} = \frac{e^{y/x} + y/x - (y/x)^2}{1 - y/x} = \frac{e^u + u - u^2}{1-u} \quad , \quad u = y/x$$

$$y = xu \rightarrow y' = u + xu' = \frac{e^u + u - u^2}{1-u} \Rightarrow x\frac{du}{dx} = -u + \frac{e^u + u - u^2}{1-u}$$

$$= \frac{e^u}{1-u}$$

$$(u-1)e^{-u}du + \frac{dx}{x} = 0$$

$$\int (u-1)e^{-u}du + \int \frac{dx}{x} = c \Rightarrow \ln|x| - ue^{-u} = c \Rightarrow \ln|x| - \frac{y}{x}e^{-y/x} = c$$

ör $y' = \tan(2x+y) - 2$ genel çözüm

$$u = 2x + y \rightarrow u' = 2 + y' = 2 + \tan u - 2 = \tan u \Rightarrow \frac{du}{dx} = \tan u$$

$$\frac{\cos u}{\sin u}du - dx = 0 \rightarrow \ln|\sin u| - x = c_1 \Rightarrow \ln|\sin u| = c_1 + x$$

$$e^{\ln|\sin u|} = e^{c_1 + x} = e^{c_1}e^x = ce^x$$

$$\sin u = ce^x \Rightarrow \sin(2x+y) = ce^x$$

$$y = -2x + \arcsin(ce^x)$$

Ayrılabilir Hale Getirilebilen Dif. Denklemleri

$$(a_1x+b_1y+c_1)\,dx+(a_2x+b_2y+c_2)\,dy=0$$

$\left.\begin{matrix} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{matrix}\right\}$ düzlem iki doğru $\quad \dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=k$ ise doğrular paralel.

$u=a_1x+b_1y+c \rightarrow du=a_1dx+b_1dy$ ile çözeriz.

Ör $(2x+4y+5)dx+(3x+6y-2)dy=0$ dif. denk. çöz.

$u=x+2y \rightarrow du=dx+2dy \rightarrow dy=\dfrac{du-dx}{2}$

$\rightarrow (2u+5)dx+(3u-2)\dfrac{du-dx}{2}=0$

$dx+\dfrac{3u-2}{u+12}du=0 \rightarrow \int dx+3\int du-38\int\dfrac{du}{u+12}=0$

$x+3u-38\ln|u+12|=C_1$

$4x+6y-38\ln|x+2y+12|=C_1 \Rightarrow 2x+3y-19\ln|x+2y+12|=C$

Ör $(2x+3y)dx+(4x+6y-8)dy=0$ dif. denk. çöz.

$u=2x+3y \rightarrow du=2dx-3dy \rightarrow dy=\dfrac{du-2dx}{3}$

$\rightarrow u\,dx+(2u-8)\dfrac{du-2dx}{3}=0$

$3u\,dx+(2u-8)(du-2dx)=0$

$dx+\dfrac{2u-8}{16-u}du=0 \rightarrow dx-2du-24\dfrac{du}{u-16}=0$

$x-2u-24\ln|u-16|=C_1 \rightarrow x-2(2x+3y)-24\ln|2x+3y-16|=C_1$

$x+2y+8\ln|2x+3y-16|=C$

Ör $(2x+6y+5)dx+(3x+9y-6)dy=0$ dif. denk. çöz.

$u=x+3y \rightarrow du=dx+3dy \rightarrow dy=\dfrac{du-dx}{3}$

$\rightarrow (2u+5)dx+(3u-6)\dfrac{du-dx}{3}=0 \rightarrow (2u+5)dx+(u-2)(du-dx)=0$

$(u+7)dx+(u-2)du=0 \rightarrow dx+\dfrac{u-2}{u+7}du=0$

$dx+du-9\dfrac{du}{u+7}=0 \rightarrow x+u-9\ln|u+7|=C$

$2x+3y-9\ln|x+3y+7|=C$

2) $(2x-y+4)dx+(x+2y+7)dy=0$ dif. denk. çöz.

$2x-y+4=0 \rightarrow y=2x+4$
$x+2y+7=0 \rightarrow y=\frac{x+7}{-2}$ } doğruları düzlemde bir noktada birleşir.

$y=y \rightarrow 2x+4=\frac{x+7}{-2} \rightarrow \begin{matrix} x=-3 \\ y=-2 \end{matrix}$ $(-3,-2)$ noktasından iki doğru kesişir.

$x=-3+t \rightarrow dx=dt$
$y=-2+z \rightarrow dy=dz$

$(2x-y+4)dx+(x+2y+7)dy=0$

$(\cancel{-6}+2t+\cancel{2}-z+\cancel{4})dt+(\cancel{-3}+t\cancel{-4}+2z+\cancel{7})dz=0$

$(2t-z)dt+(t+2z)dz=0$

$\frac{dz}{dt}=\frac{z-2t}{2z+t}=\frac{z/t-2}{2z/t+1}=\frac{u-2}{2u+1}$, $u=z/t$

$z=tu \rightarrow \frac{dz}{dt}=u+t\frac{du}{dt}=\frac{u-2}{2u+1} \Rightarrow t\frac{du}{dt}=-2\frac{u^2+1}{2u+1}$

$\frac{2u+1}{u^2+1}du+2\frac{dt}{t}=0 \rightarrow \frac{2u\,du}{u^2+1}+\frac{du}{u^2+1}+2\frac{dt}{t}=0$

$\ln(u^2+1)+\arctan u+2\ln|t|=c$

$\arctan u+\ln(t^2(u^2+1))=c$

$\arctan(z/t)+\ln(z^2+t^2)=c \Rightarrow \arctan\left(\frac{y+2}{x+3}\right)+\ln\left((x+3)^2+(y+2)^2\right)=c$

Ör $(x-y+1)dx+(3-x-y)dy=0$ dif. denk. çöz.

$x-y+1=0 \rightarrow y=x+1$
$3-x-y=0 \rightarrow y=3-x$ } $y=y$, $x+1=3-x \rightarrow \begin{matrix} x=1 \\ y=2 \end{matrix}$ $(1,2)$ noktası

$x=1+t \rightarrow dx=dt$
$y=2+z \rightarrow dy=dz$

$(x-y+1)dx+(3-x-y)dy=0$

$\rightarrow (x-y+1)dx+(\cancel{3}\cancel{-1}-t\cancel{-2}-z)dz=0$

$(\cancel{1}+t\cancel{-2}-z+\cancel{1})dt$

$(t-z)dt-(t+z)dz=0 \Rightarrow \frac{dz}{dt}=\frac{t-z}{t+z}=\frac{1-z/t}{1+z/t}=\frac{1-u}{1+u}$, $u=z/t$

$$z = tu \rightarrow \frac{dz}{dt} = u + t\frac{du}{dt} = \frac{1-u}{1+u}$$

$$\frac{dt}{t} + \frac{u+1}{u^2+2u-1}du = 0 \rightarrow 2\frac{dt}{t} + \frac{2(u+1)du}{u^2+2u-1} = 0$$

$$2\int\frac{dt}{t} + \int\frac{2(u+1)du}{u^2+2u-1} = C_1$$

$$2\ln|t| + \ln(u^2+2u-1) = C_1 \Rightarrow \ln(t^2u^2+2ut^2-t^2) = C_1$$

$$t^2u^2+2ut^2-t^2 = C_2 \Rightarrow z^2+2zt-t^2 = C_2$$

$$(y-2)^2+2(y-2)(x-1)-(x-1)^2 = C_2$$

$$y^2-4y+4+2xy-2y-4x-x^2+2x-1 = C_2$$

$$y^2-6y-x^2-2x+2xy = C$$

Ör $(2x-3y+8)dx+(3x+2y+7)dy = 0$ dif. denk. çöz.

$$2x-3y+8 = 0 \rightarrow y = \frac{2x+8}{3}$$
$$3x+2y+7 = 0 \rightarrow y = \frac{3x+7}{-2}$$

$$y = y \qquad \frac{2x+8}{3} = \frac{3x+7}{-2} \Rightarrow -4x-16 = 9x+21$$
$$13x+37 = 0$$
$$x = -3,\ y = 1$$

$$x = -3+t \rightarrow dx = dt$$
$$y = 1+z \rightarrow dy = dz$$

$$\rightarrow (\cancel{-6}+2t\cancel{-3}-3z\cancel{+8})dt + (\cancel{-9}+3t\cancel{+2}+2z\cancel{+7})dz = 0$$

$$(2t-3z)dt+(3t+2z)dz = 0$$

$$\frac{dz}{dt} = \frac{3z-2t}{2z+3t} = \frac{3\,z/t-2}{2\,z/t+3} = \frac{3u-2}{2u+3}, \quad u = z/t$$

$$z = tu \rightarrow \frac{dz}{dt} = u+t\frac{du}{dt} = \frac{3u-2}{2u+3} \Rightarrow t\frac{du}{dt} = -2\,\frac{u^2+1}{2u+3}$$

$$\frac{2u+3}{u^2+1}du + 2\frac{dt}{t} = 0 \Rightarrow \frac{2u\,du}{u^2+1} + 3\frac{du}{u^2+1} + 2\frac{dt}{t} = 0$$

$$\ln(u^2+1) + 3\arctan u + 2\ln|t| = C$$

$$\ln((u^2+1)t^2) + 3\arctan u = C$$

$$\ln(t^2+z^2) + 3\arctan(z/t) = C$$

$$\ln((x+3)^2+(y-1)^2) + 3\arctan\left(\frac{y-1}{x+3}\right) = C$$

$(4x+2y+1)dx+(5x+3y+1)dy=0$ dif. denk. çöz.

$(x-2y+1)dx+(4x-3y-6)dy=0$ çöz

$(5x+2y+1)dx+(2x+y+1)dy=0$ çöz

Ör $(x-y-1)dx + (4y+x-1)dy = 0$ dif. denk. çöz.

$x-y-1=0 \to y = x-1$
$4y+x-1=0 \to y = \frac{1-x}{4}$ $\Big\}$ $y=y$, $x-1 = \frac{1-x}{4} \to 4x-4 = 1-x$, $x=1,\ y=0$ $(1,0)$ noktası

$x = 1+t \to dx = dt$
$y = z \to dy = dz$

$(1+t-z-1)dt + (4z+1+t-1)dz = 0$

$(t-z)dt + (t+4z)dz = 0$

$$\frac{dz}{dt} = \frac{z-t}{4z+t} = \frac{z/t - 1}{4z/t + 1} = \frac{u-1}{4u+1}, \quad u = z/t$$

$$z = tu \to \frac{dz}{dt} = u + t\frac{du}{dt} = \frac{u-1}{4u+1} \Rightarrow t\frac{du}{dt} = \frac{u-1}{4u+1} - u = -\frac{4u^2+1}{4u+1}$$

$$\frac{4u+1}{4u^2+1}du + \frac{dt}{t} = 0 \to \int \frac{8u\,du}{4u^2+1} + \int\frac{2du}{4u^2+1} + 2\int\frac{dt}{t} = C$$

$\ln(1+4u^2) + \arctan(2u) + 2\ln|t| = C$

$\ln\left(t^2(1+4u^2)\right) + \cancel{\arctan}(2u) = C$

$\ln(t^2+4z^2) + \arctan\left(\frac{2z}{t}\right) = C$

$\ln\left((x-1)^2+y^2\right) + \arctan\left(\frac{2y}{x-1}\right) = C$

Ör $y(2-xy)dx - x(1+xy)dy = 0$ dif. denk. çöz.

$$\frac{dy}{dx} = \frac{y(2-xy)}{x(1+xy)}, \quad u = xy \text{ seçersek}$$

$$\frac{du}{dx} = y + x\frac{dy}{dx} = y + y\frac{2-xy}{1+xy} = y\left(1+\frac{2-xy}{1+xy}\right) = \frac{3y}{1+xy}$$

$$x\frac{du}{dx} = \frac{3xy}{1+xy} = \frac{3u}{1+u} \Rightarrow \frac{u+1}{u}du - 3\frac{dx}{x} = 0$$

$$du + \frac{du}{u} - 3\frac{dx}{x} = 0 \to \int du + \int\frac{du}{u} - 3\int\frac{dx}{x} = C$$

$u + \ln|u| - 3\ln|x| = C \to u + \ln\left|\frac{u}{x^3}\right| = C$

$xy + \ln\frac{|y|}{x^2} = C$

Ör $y' = \frac{e^x}{y}\cos^2(x^2-y^2) + x/y$, $u = x^2-y^2$ kul. dif. denk. çöz

$u = x^2-y^2 \to u' = 2x - 2yy' = 2x - 2y\left(\frac{e^x}{y}\cos^2(x^2-y^2) + x/y\right)$

$= \cancel{2x} - 2e^x\cos^2(x^2-y^2) - \cancel{2x} = -2e^x\cos^2 u = \frac{du}{dx}$

$\frac{du}{dx} + 2e^x\cos^2 u = 0 \Rightarrow \frac{du}{\cos^2 u} + 2e^x dx = 0$

$\int \sec^2 u\, du + \int 2e^x dx = C \longrightarrow \tan u + 2e^x = C \to \tan(x^2-y^2) + 2e^x = C$

Ör $y(xy+1)dx + x(1+xy+x^2y^2)dy = 0$ dif. denk. çöz.

$\frac{dy}{dx} = -\frac{y(xy+1)}{x(1+xy+x^2y^2)} = -\frac{xy(xy+1)}{x^2(x^2y^2+xy+1)} = -\frac{u(u+1)}{x^2(u^2+u+1)}$, $u = xy$

$u = xy \to \frac{du}{dx} = y + x\frac{dy}{dx} = y - x\frac{u(u+1)}{x^2(u^2+u+1)}$

$x\frac{du}{dx} = xy - \frac{u(u+1)}{u^2+u+1} = u - \frac{u(u+1)}{u^2+u+1} = \frac{u^3}{u^2+u+1}$

$\frac{u^2+u+1}{u^3}du - \frac{dx}{x} = 0$ $\qquad \frac{u^2+u+1}{u^3} = \frac{a}{u} + \frac{b}{u^2} + \frac{c}{u^3}$ $\quad a=1,\ b=1,\ c=1$

$\frac{du}{u} + \frac{du}{u^2} + \frac{du}{u^3} - \frac{dx}{x} = 0 \to \ln|u| - \underset{(2u)}{\frac{1}{u}} - \underset{(1)}{\frac{1}{2u^2}} - \ln|x| = C_1$

$\ln\left|\frac{u}{x}\right| - \frac{2u+1}{2u^2} = C_1 \Rightarrow \ln|y| - \frac{xy+1}{2x^2y^2} = C_1$

$\ln|y| = C_1 + \frac{xy+1}{2x^2y^2} \Rightarrow y = c\, e^{\frac{xy+1}{2x^2y^2}}$

Ör $(x^2+9)y' + xy = 0$ dif. denk. çöz.

$(x^2+9)\frac{dy}{dx} + xy = 0 \to (x^2+9)dy + xy\,dx = 0$

$\frac{dy}{y} + \frac{x\,dx}{x^2+9} = 0 \longrightarrow 2\frac{dy}{y} + \frac{2x\,dx}{x^2+9} = 0$

$2\ln|y| + \ln(x^2+9) = C_1 \longrightarrow \ln y^2(x^2+9) = C_1$

$y^2(x^2+9) = C$

1. mertebeden lineer diferansiyel denklemler

$y' + P(x).y = Q(x)$ şeklindeki dif. denk.

$(P(x).y - Q(x))\,dx + dy = 0$

$M(x,y) = P(x).y - Q(x) \longrightarrow \frac{\partial M}{\partial y} = P(x)$

$N(x,y) = 1 \longrightarrow \frac{\partial N}{\partial x} = 0$

Tam değil. Tam olması için $P(x) = 0$ olmalı.

μ integrasyon çarpanı kullanılır.

$g(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = P(x)$ $g(x)$ x'e bağlı olduğundan μ'de x'e bağlıdır.

$$\underbrace{\mu(x)\left(P(x).y - Q(x)\right)}_{M(x,y)} dx + \underbrace{\mu(x)}_{N(x,y)} dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \mu(x)P(x) = \mu'(x) \Rightarrow \frac{\mu'(x)}{\mu(x)} = P(x)$$

$$\frac{d\mu(x)}{\mu(x)} = P(x)\,dx \rightarrow \ln|\mu(x)| = \int P(x)\,dx \Rightarrow \mu(x) = e^{\int P(x)dx}$$

$$F(x,y) = \int N(x,y)\,\partial y + \Phi(x) = \int \mu(x)\,\partial y + \Phi(x) = y\,\mu(x) + \Phi(x)$$

$$M(x,y) = \frac{\partial F(x,y)}{\partial x} = y\,\mu'(x) + \Phi'(x) = \mu(x)P(x)y - \mu(x)Q(x)$$

$\mu'(x) = \mu(x)P(x)$ ✓

$\Phi'(x) = -\mu(x)Q(x)$

$\Phi(x) = -\int \mu(x)Q(x)\,dx$

$$F(x,y) = y\,\mu(x) + \Phi(x) = c$$
$$= y\,\mu(x) - \int \mu(x)Q(x)\,dx = c$$

$$y = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)\,dx\right)$$

$$\mu(x) = e^{\int P(x)dx}$$

Ör $y' + \tan x \cdot y + \cot^2 x = 0$ dif. denk. çöz.

$$y' + \tan x \cdot y = -\cot^2 x \longrightarrow P(x) = \tan x, \quad Q(x) = -\cot^2 x$$

$$\mu(x) = e^{\int P(x)dx} = e^{\int \tan x\, dx} = e^{\int \frac{\sin x\, dx}{\cos x}} = e^{\ln(\sec x)} = \sec x$$

$$y = \frac{1}{\mu(x)}\left(c + \int \mu(x) Q(x)\, dx\right) = \frac{1}{\sec x}\left(c - \int \sec x \cdot \cot^2 x\, dx\right)$$

$$= \cos x\left(c - \int \frac{\cos x\, dx}{\sin^2 x}\right) = \cos x \cdot \left(c + \frac{1}{\sin x}\right) = c \cdot \cos x + \cot x$$

Ör $(x^2+1)y' + 2xy = x^2$ dif. denk. çöz.

$$y' + \frac{2x}{x^2+1} y = \frac{x^2}{x^2+1} \Longrightarrow P(x) = \frac{2x}{x^2+1}, \quad Q(x) = \frac{x^2}{x^2+1}$$

$$\mu(x) = e^{\int P(x)dx} = e^{\int \frac{2x\, dx}{x^2+1}} = e^{\ln(x^2+1)} = x^2+1$$

$$y = \frac{1}{\mu(x)}\left(c + \int \mu(x) Q(x)\, dx\right) = \frac{1}{x^2+1}\left(c + \int x^2 dx\right)$$

$$= \frac{1}{x^2+1}\left(c + \frac{x^3}{3}\right)$$

$(x^2-1)y' + 2y = (x+1)^2$ dif. denk. çöz. $y(2) = 3$

Ör $(x+1)y' - y = e^x (x+1)^2$, $y(0) = 3$ dif. denk. çöz.

$$y' - \frac{1}{x+1} y = (x+1)e^x \longrightarrow P(x) = -\frac{1}{x+1}, \quad Q(x) = (x+1)e^x$$

$$\mu(x) = e^{\int P(x)dx} = e^{-\int \frac{dx}{x+1}} = e^{-\ln(x+1)} = \frac{1}{x+1}$$

$$y = \frac{1}{\mu(x)}\left(c + \int \mu(x) Q(x)\, dx\right)$$

$$= (x+1)\left(c + \int e^x dx\right) = (x+1)(c + e^x)$$

$$y(0) = 3 \rightarrow c = 2 \longrightarrow y = (x+1)(2 + e^x)$$

Ör $x\,dy-(4y+x^6e^x)dx=0$ dif. denk. çöz.

$$\frac{dy}{dx}=\frac{4y+x^6e^x}{x}=\frac{4}{x}y+x^5e^x \rightarrow y'-\frac{4}{x}y=x^5e^x$$

$$P(x)=-\frac{4}{x}\ ,\ Q(x)=x^5e^x$$

$$\mu(x)=e^{\int P(x)dx}=e^{\int -\frac{4dx}{x}}=e^{-4\ln x}=\frac{1}{x^4}$$

$$y=\frac{1}{\mu(x)}\left(c+\int \mu(x)Q(x)dx\right)=x^4\Big(c+\underbrace{\int xe^x dx}_{\text{kısmi integral uygula}}\Big)$$

$$=x^4\left(c+(x-1)e^x\right)$$

Ör $x^2(x-1)y'+2x^2y=x+1$ dif. denk. çöz.

$$y'+\frac{2}{x-1}y'=\frac{x+1}{x^2(x-1)} \qquad P(x)=\frac{2}{x-1}\ ,\ Q(x)=\frac{x+1}{x^2(x-1)}$$

$$\mu(x)=e^{\int P(x)dx}=e^{2\int\frac{dx}{x-1}}=e^{2\ln(x-1)}=(x-1)^2$$

$$y=\frac{1}{\mu(x)}\left(c+\int \mu(x)Q(x)dx\right)=\frac{1}{(x-1)^2}\left(c+\int\frac{x^2-1}{x^2}dx\right)$$

$$=\frac{1}{(x-1)^2}\left(c+\int dx-\int\frac{dx}{x^2}\right)=\frac{1}{(x-1)^2}\left(c+x+\frac{1}{x}\right)=\frac{x^2+cx+1}{x(x-1)^2}$$

$xy'+(3x+1)y=e^{-x}$ dif. denk. çöz.

Ör $y+x(2+y')+3y'=0$ dif. denk. çöz.

$$y+2x+xy'+3y'=0 \rightarrow (x+3)y'+y=-2x$$

$$y'+\frac{1}{x+3}y=-\frac{2x}{x+3} \qquad P(x)=\frac{1}{x+3}\ ,\ Q(x)=-\frac{2x}{x+3}$$

$$\mu(x)=e^{\int P(x)dx}=e^{\int\frac{dx}{x+3}}=e^{\ln(x+3)}=x+3$$

$$y=\frac{1}{\mu(x)}\left(c+\int \mu(x)Q(x)dx\right)=\frac{1}{x+3}\left(c+\int (x+3)\frac{-2x}{x+3}dx\right)$$

$$=\frac{1}{x+3}\left(c-\int 2x\,dx\right)=\frac{c-x^2}{x+3}$$

$x^2y'+x(x+2)y=e^x$ dif. denk. çöz.

Ör $y'+1=4e^{-(x+y)}\sin x$, $y(0)=0$ dif. denk. çöz.

$$e^{y}y'+e^{y}=4e^{-x}\sin x \;,\; u=e^{y}\rightarrow u'=e^{y}\cdot y'$$

$$u'+u=4e^{-x}\sin x \longrightarrow P(x)=1 \;,\; Q(x)=4e^{-x}\sin x$$

$$\mu(x)=e^{\int P(x)dx}=e^{\int dx}=e^{x}$$

$$u=\frac{1}{\mu(x)}\left(c+\int \mu(x)Q(x)dx\right)=e^{-x}\left(c+\int 4\sin x\,dx\right)$$

$$=e^{-x}\left(c-4\cos x\right)=e^{y}$$

$$x=0,\; y=0 \Rightarrow e^{0}(c-4\cos 0)=e^{0}\rightarrow c-4=1\rightarrow c=5$$

$$e^{y}=e^{-x}(5-4\cos x)\rightarrow e^{x+y}=5-\cos x$$

$$x+y=\ln(5-\cos x)\longrightarrow y=-x+\ln(5-\cos x)$$

Ör $y^{2}dx+(3xy-1)dy=0$ dif. denk. çöz.

$$y'=\frac{dy}{dx}=\frac{y^{2}}{1-3xy}\Longrightarrow \frac{dx}{dy}=\frac{1-3xy}{y^{2}}=\frac{1}{y^{2}}-\frac{3}{y}x$$

$$x'+\frac{3}{y}x=y^{-2}\longrightarrow P(y)=\frac{3}{y} \;,\; Q(y)=y^{-2}$$

$$\mu(y)=e^{\int P(y)dy}=e^{\int \frac{3dy}{y}}=e^{3\ln y}=y^{3}$$

$$x=\frac{1}{\mu(y)}\left(c+\int \mu(y)Q(y)dy\right)$$

$$=\frac{1}{y^{3}}\left(c+\int y\,dy\right)=\frac{1}{y^{3}}\left(c+\frac{y^{2}}{2}\right)$$

$$=\frac{c}{y^{3}}+\frac{1}{2y}$$

Ör $\frac{\sin y}{\cos^2 y} y' - \frac{1}{\cos y} = -xe^x$ dif. denk. çöz.

$u = \frac{1}{\cos y} \longrightarrow u' = \frac{\sin y}{\cos^2 y} y'$

$u' - u = -xe^x \longrightarrow P(x) = -1 \; , \; Q(x) = -xe^x$

$\mu(x) = e^{\int P(x)dx} = e^{-\int dx} = e^{-x}$

$u = \frac{1}{\mu(x)}\left(c + \int \mu(x) . Q(x) . dx\right) = e^x\left(c - \int x dx\right)$

$= e^x\left(c - \frac{x^2}{2}\right) = \frac{1}{\cos y} \Rightarrow \cos y = \frac{2e^{-x}}{c - x^2} \Rightarrow y = \arccos\left(\frac{2e^{-x}}{c - x^2}\right)$

Ör $x^2 \cos y . y' - 2x \sin y = -1$ dif. denk. çöz.

$u = \sin y \longrightarrow u' = \cos y . y'$ $\quad P(x) = -2/x$

$x^2 u' - 2xu = -1 \longrightarrow u' - \frac{2}{x} u = -\frac{1}{x^2} \quad Q(x) = -\frac{1}{x^2}$

$\mu(x) = e^{\int P(x)dx} = e^{-2\int \frac{dx}{x}} = e^{-2\ln x} = 1/x^2$

$u = \frac{1}{\mu(x)}\left(c + \int \mu(x) Q(x) dx\right) = x^2\left(c - \int \frac{dx}{x^4}\right) = x^2\left(c + \frac{1}{3x^3}\right)$

$= cx^2 + \frac{1}{3x} = \sin y \Rightarrow y = \arcsin\left(cx^2 + \frac{1}{3x}\right)$

Ör $e^y y' + (1 + e^y)\cot x = 5e^{\cos x}$ dif. denk. çöz.

$u = 1 + e^y \longrightarrow u' = e^y y' \longrightarrow u' + u . \cot x = 5e^{\cos x}$ $\quad P(x) = \cot x$, $Q(x) = 5e^{\cos x}$

$\mu(x) = e^{\int P(x)dx} = e^{\int \cot x dx} = e^{\int \frac{\cos x \, dx}{\sin x}} = e^{\ln(\sin x)} = \sin x$

$u = \frac{1}{\mu(x)}\left(c + \int \mu(x) Q(x) dx\right) = \frac{1}{\sin x}\left(c + 5\int \sin x \, e^{\cos x} dx\right)$

$= \frac{1}{\sin x}\left(c - 5e^{\cos x}\right) = 1 + e^y \Rightarrow (1 + e^y)\sin x + 5e^{\cos x} = c$

Bernoulli Diferansiyel Denklemi (lineer değil)

$$y' + P(x)y = Q(x)\cdot y^n \quad , \quad n \neq 0 \; , \; n \neq 1$$

$$y^{-n}y' + P(x)\,y^{1-n} = Q(x) \qquad u = y^{1-n}$$
$$u' = (1-n)\,y^{-n}y'$$

$$(1-n)y^{-n}y' + (1-n)P(x)\,y^{1-n} = (1-n)\,Q(x)$$

$$u' + \underbrace{(1-n)P(x)}_{P(x)}\,u = \underbrace{(1-n)Q(x)}_{Q(x)} \quad \text{lineer hale geldi}$$

Ör $x\,dy + y(1-x^3y^5)\,dx = 0$ dif. denk. çöz.

$$x\,y' + y(1-x^3y^5) = 0 \qquad u = y^{-5}$$
$$u' = -5y^{-6}y'$$

$$x\,y' + y = x^3y^6 \longrightarrow y^{-6}y' + \frac{1}{x}y^{-5} = x^2$$

$$-5y^{-6}y' - \frac{5}{x}y^{-5} = -5x^2 \Longrightarrow u' - \frac{5}{x}u = -5x^2 \qquad P(x) = -5/x \quad Q(x) = -5x^2$$

$$\mu(x) = e^{\int P(x)dx} = e^{-5\int\frac{dx}{x}} = e^{-5\ln x} = \frac{1}{x^5}$$

$$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)\,dx\right) = x^5\left(c - 5\int x^{-3}dx\right)$$

$$= x^5\left(c + \frac{5}{2x^2}\right) = y^{-5} \Longrightarrow y = \left(cx^5 + \frac{5x^3}{2}\right)^{-1/5}$$

Ör $x^2y - x^3y' = y^4\cos x$ dif. denk. çöz.

$$y' - \frac{1}{x}y = -\frac{\cos x}{x^3}y^4 \quad , \quad -3y^{-4} \text{ ile çarp}$$

$$u = y^{-3} \longrightarrow u' = -3y^{-4}y'$$

$$-3y^{-4}y' + \frac{3}{x}y^{-3} = \frac{3\cos x}{x^3}$$

$$u' + \frac{3}{x}u = \frac{3\cos x}{x^3} \qquad P(x) = \frac{3}{x} \qquad Q(x) = \frac{3\cos x}{x^3}$$

$$\mu(x) = e^{\int P(x)dx} = e^{3\int\frac{dx}{x}} = e^{3\ln x} = x^3$$

$$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)\,dx\right) = \frac{1}{x^3}\left(c + 3\int \cos x\,dx\right) = \frac{c + 3\sin x}{x^3} = y^{-3}$$

$$y = \frac{x}{(c + 3\sin x)^{1/3}}$$

Ör $xy'-(1+y^2)\arctan y - x^2e^x(1+y^2)=0$ dif. denk. çöz.

$y=\tan u \qquad 1+y^2 = 1+\tan^2 u = \sec^2 u$

$y' = \sec^2 u . u' \qquad u = \arctan y$

$x\sec^2 u . u' - u.\sec^2 u - x^2e^x \sec^2 u = 0 \qquad P(x) = -1/x$

$xu' - u - x^2e^x = 0 \longrightarrow u' - \frac{1}{x}u = xe^x \qquad Q(x) = xe^x$

$$\mu(x) = e^{\int P(x)dx} = e^{-\int\frac{dx}{x}} = e^{-\ln x} = \frac{1}{x}$$

$$u = \frac{1}{\mu(x)}\left(c+\int \mu(x)Q(x)dx\right) = x\left(c+\int e^x dx\right)$$

$$= x(c+e^x) \Rightarrow y = \tan u = \tan(cx+xe^x)$$

Ör $(1+e^y)y'+y+e^y = e^{-x}$ dif. denk. çöz.

$u = y+e^y \longrightarrow u' = y'+e^yy' = (1+e^y)y'$

$u'+u = e^{-x} \qquad P(x)=1,\ Q(x) = e^{-x}$

$$\mu(x) = e^{\int P(x)dx} = e^{\int dx} = e^x$$

$$u = \frac{1}{\mu(x)}\left(c+\int\mu(x)Q(x)dx\right) = e^{-x}\left(c+\int dx\right) = e^{-x}(c+x)$$

$$= y+e^y \Longrightarrow ye^x + e^{x+y} - x = c$$

Ör $y'.\cos y - \sin y = e^{-x}\cos x . \sin^2 y$ dif. denk. çöz.

$\frac{\cos y}{\sin^2 y}y' - \frac{1}{\sin y} = e^{-x}\cos x \qquad u = \frac{-1}{\sin y} \Rightarrow u' = \frac{\cos y}{\sin^2 y}y'$

$u'+u = e^{-x}\cos x \qquad P(x) = +1,\ Q(x) = e^{-x}\cos x$

$$\mu(x) = e^{\int P(x)dx} = e^{\int dx} = e^x$$

$$u = \frac{1}{\mu(x)}\left(c+\int\mu(x)Q(x)dx\right) = e^{-x}\left(c+\int\cos x\,dx\right) = e^{-x}(c+\sin x) = -\frac{1}{\sin y}$$

$$\sin y = \frac{-e^x}{c+\sin x} = \frac{e^x}{c_1-\sin x} \Rightarrow y = \arcsin\left(\frac{e^x}{c_1-\sin x}\right)$$

Ör $y' - \frac{1}{3x} y = y^4 \ln x$ dif. denk. çöz.

$u = y^{-3} \longrightarrow u' = -3y^{-4}y'$, $-3y^{-4}y'$ ile dif. denk. çarpılırsa

$-3y^{-4}y' + \frac{1}{x} y^{-3} = -3\ln x$

$u' + \frac{1}{x} u = -3\ln x \longrightarrow P(x) = \frac{1}{x}$, $Q(x) = -3\ln x$

$\mu(x) = e^{\int P(x)dx} = e^{\int \frac{dx}{x}} = e^{\ln x} = x$

$u = \frac{1}{\mu(x)} \left(c + \int \mu(x) Q(x) dx \right) = \frac{1}{x} \left(c - 3 \int x \ln x \, dx \right)$

$= \frac{1}{x} \left(c - \frac{3x^2}{4} (2\ln x - 1) \right) = y^{-3} \Rightarrow y = \left(\frac{c}{x} + \frac{3x}{4} - \frac{3x^2}{2} \ln x \right)^{-1/3}$

Ör $3y' \cos x + y \sin x = 1/y^2$ dif. denk. çöz.

$3y^2y' + y^3 \tan x = \frac{1}{\cos x}$ $u = y^3 \longrightarrow u' = 3y^2y'$

$u' + u.\tan x = \sec x$, $P(x) = \tan x$, $Q(x) = \sec x$

$\mu(x) = e^{\int P(x)dx} = e^{\int \tan x \, dx} = e^{\int \frac{\sin x}{\cos x} dx} = e^{-\ln(\cos x)} = \sec x$

$u = \frac{1}{\mu(x)} \left(c + \int \mu(x) Q(x) dx \right) = \cos x \left(c + \int \sec^2 x \, dx \right)$

$= \cos x \, (c + \tan x) = y^3$

$y^3 = c.\cos x + \sin x \longrightarrow y = (c.\cos x + \sin x)^{1/3}$

$3x^2y^2y' - y^6 - xy^3 = 0$ dif. denk. çöz.

Ör $y' \cos x - \frac{y}{4} \sin x = y^{-3}$ dif. denk. çöz.

$4y^3y' - \tan x . y^4 = 4\sec x$ $u = y^4 \longrightarrow u' = 4y^3y'$

$u' - u.\tan x = 4\sec x$ $P(x) = -\tan x$, $Q(x) = 4\sec x$

$\mu(x) = e^{\int P(x)dx} = e^{-\int \tan x dx} = e^{-\int \frac{\sin x \, dx}{\cos x}} = e^{\ln(\cos x)} = \cos x$

$u = \frac{1}{\mu(x)} \left(c + \int \mu(x) Q(x) dx \right) = \sec x . \left(c + 4\int dx \right) = \sec x . (c + 4x)$

$= y^4 \Rightarrow y = \left(\frac{4x + c}{\cos x} \right)^{1/4}$

Ör $2(x^2-1)y\cdot y' - xy^2 = x(x^2-1)$ dif. denk. çöz.

$$2yy' - \frac{x}{x^2-1}y^2 = x \quad , \quad u = y^2 \rightarrow u' = 2yy'$$

$$u' - \frac{x}{x^2-1}u = x \qquad p(x) = -\frac{x}{x^2-1} \quad , \quad Q(x) = x$$

$$\mu(x) = e^{\int p(x)dx} = e^{-\frac{1}{2}\int \frac{2x\,dx}{x^2-1}} = e^{-\frac{1}{2}\ln(x^2-1)} = \frac{1}{\sqrt{x^2-1}}$$

$$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = \sqrt{x^2-1}\left(c + \int \frac{x\,dx}{\sqrt{x^2-1}}\right)$$

$$= \sqrt{x^2-1}\left(c + \sqrt{x^2-1}\right) = c\sqrt{x^2-1} + x^2 - 1 = y^2 \Rightarrow \frac{y^2-x^2+1}{\sqrt{x^2-1}} = c$$

Ör $y' - xy = xy^3e^{-x^2}$ dif. denk. çöz.

$$-2y^{-3}y' + 2xy^{-2} = -2xe^{-x^2} \qquad u = y^{-2} \rightarrow u' = -2y^{-3}y'$$

$$u' + 2xu = -2xe^{-x^2} \qquad p(x) = 2x \, , \; Q(x) = -2xe^{-x^2}$$

$$\mu(x) = e^{\int p(x)dx} = e^{\int 2x\,dx} = e^{x^2}$$

$$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = e^{-x^2}\left(c - \int 2x\,dx\right)$$

$$= e^{-x^2}(c - x^2) = y^{-2} \longrightarrow y = \mp\sqrt{\frac{e^{x^2}}{c-x^2}}$$

$3y' + y = (1-2x)y^4$ Bernoulli dif. denk. çöz

Ör $y' + y = y^2(\cos x - \sin x)$ dif. denk. çöz.

$$-y^{-2}y' - y^{-1} = \sin x - \cos x \qquad u = y^{-1} \, , \; u' = -y^{-2}y'$$

$$u' - u = \sin x - \cos x \qquad p(x) = -1 \, , \; Q(x) = \sin x - \cos x$$

$$\mu(x) = e^{\int p(x)dx} = e^{-\int dx} = e^{-x}$$

$$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = e^{x}\left(c + \int (\sin x - \cos x)e^{-x}dx\right)$$

$$= e^{x}\left(c - \sin x . e^{-x}\right) = ce^{x} - \sin x = y^{-1}$$

$$y = \frac{1}{ce^{x} - \sin x}$$

Riccati Diferansiyel Denklemi

$$y' = P(x).y^2 + Q(x).y + R(x), \quad P(x) \neq 0 \quad \text{lineer değil.}$$

1 özel çözüm biliniyorsa

y_1 özel çözüm ise $y = y_1 + \frac{1}{u} \longrightarrow y' = y_1' - \frac{u'}{u^2}$

$$y_1' = P(x).y_1^2 + Q(x).y_1 + R(x)$$

$$y' = y_1' - \frac{u'}{u^2} = P(x)\left(y_1 + \frac{1}{u}\right)^2 + Q(x).\left(y_1 + \frac{1}{u}\right) + R(x)$$

$$y' - y_1' = -\frac{u'}{u^2} = P(x)\left(\left(y_1 + \frac{1}{u}\right)^2 - y_1^2\right) + Q(x)\left(y_1 + \frac{1}{u} - y_1\right)$$

$$= P(x)\left(\frac{2y_1}{u} + \frac{1}{u^2}\right) + Q(x).\frac{1}{u}$$

$$u' + \underbrace{\left(2P(x)y_1 + Q(x)\right)}_{P(x)} u = \underbrace{-P(x)}_{Q(x)} \Rightarrow u' + P(x)\,u = Q(x)$$

1. mertebeden lineer dif. denk.

2 özel çözüm biliniyorsa

$$\left.\begin{aligned} y' &= P(x)y^2 + Q(x)y + R(x) \\ y_1' &= P(x)y_1^2 + Q(x)y_1 + R(x) \\ y_2' &= P(x)y_2^2 + Q(x)y_2 + R(x) \end{aligned}\right\} \begin{aligned} y' - y_1' &= P(x)(y^2 - y_1^2) + Q(x)(y - y_1) \\ y' - y_2' &= P(x)(y^2 - y_2^2) + Q(x)(y - y_2) \end{aligned}$$

$$\left.\begin{aligned} \frac{y' - y_1'}{y - y_1} &= P(x)(y + y_1) + Q(x) \\ \frac{y' - y_2'}{y - y_2} &= P(x)(y + y_2) + Q(x) \end{aligned}\right\} \begin{aligned} &\frac{d}{dx}\left(\ln(y - y_1)\right) - \frac{d}{dx}\left(\ln(y - y_2)\right) \\ &= P(x)(y_1 - y_2) \end{aligned}$$

$$\frac{d}{dx}\left(\ln\left(\frac{y - y_1}{y - y_2}\right)\right) = P(x).(y_2 - y_1) \Rightarrow \ln\left(\frac{y - y_1}{y - y_2}\right) = \int (y_1 - y_2).P(x).dx + C_1$$

$$\frac{y - y_1}{y - y_2} = C.\,e^{\int (y_1 - y_2).P(x).dx}$$

3 özel çözüm biliniyorsa

$$\frac{y-y_1}{y-y_2} = C\,\frac{y_3-y_1}{y_3-y_2}$$ eşitliği sağlanır.

4 özel çözüm biliniyorsa

$$\frac{y_4-y_1}{y_4-y_2} = C\,\frac{y_3-y_1}{y_3-y_2}$$ eşitliği sağlanır.

Çözümlerden denklemi oluşturma

$$\left.\begin{aligned} y' &= P(x)y^2 + Q(x)y + R(x) \\ y_1' &= P(x)y_1^2 + Q(x)y_1 + R(x) \\ y_2' &= P(x)y_2^2 + Q(x)y_2 + R(x) \\ y_3' &= P(x)y_3^2 + Q(x)y_3 + R(x) \end{aligned}\right\} \begin{bmatrix} y' & y^2 & y & 1 \\ y_1' & y_1^2 & y_1 & 1 \\ y_2' & y_2^2 & y_2 & 1 \\ y_3' & y_3^2 & y_3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ P(x) \\ Q(x) \\ R(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} y' & y^2 & y & 1 \\ y_1' & y_1^2 & y_1 & 1 \\ y_2' & y_2^2 & y_2 & 1 \\ y_3' & y_3^2 & y_3 & 1 \end{vmatrix} = 0$$ çözülürse Riccati diferansiyel denklemi elde edilir.

Ör: $y' + y^2 + \frac{y}{x} - \frac{4}{x^2} = 0$ Riccati dif. denk. bir özel çözümü $y_1 = 2/x$ ise genel çözüm.

$$y = y_1 + \frac{1}{u} = \frac{2}{x} + \frac{1}{u} \Rightarrow y' = -\frac{2}{x^2} - \frac{u'}{u^2}$$

$$\to -\frac{2}{x^2} - \frac{u'}{u^2} + \left(\frac{2}{x} + \frac{1}{u}\right)^2 + \frac{\frac{2}{x} + \frac{1}{u}}{x} - \frac{4}{x^2} = 0$$

$$-\cancel{\frac{2}{x^2}} - \frac{u'}{u^2} + \cancel{\frac{4}{x^2}} + \frac{4}{xu} + \frac{1}{u^2} + \cancel{\frac{2}{x^2}} + \frac{1}{xu} - \cancel{\frac{4}{x^2}} = 0$$

$$-\frac{u'}{u^2} + \frac{5}{xu} + \frac{1}{u^2} = 0 \Rightarrow u' - \frac{5}{x}u = 1, \quad P(x) = -5/x, \quad Q(x) = 1$$

$$\mu(x) = e^{\int P(x)dx} = e^{-5\int \frac{dx}{x}} = e^{-5\ln x} = \frac{1}{x^5} = x^{-5}$$

$$u = \frac{1}{\mu(x)}\left(C + \int \mu(x)Q(x)dx\right) = x^5\left(C + \int x^{-5}dx\right) = x^5\left(C + \frac{x^{-4}}{-4}\right) = Cx^5 - x/4$$

$$y = y_1 + \frac{1}{u} = \frac{2}{x} + \frac{1}{Cx^5 - x/4} = \frac{2}{x}\left(1 + \frac{2}{4Cx^4 - 1}\right)$$

Ör $y' = -y^2 + 2x^2y + 2x - x^4$ Riccati dif. denkleminin bir özel çözümü $y_1 = x^2$ ise genel çözüm.

$y = y_1 + \frac{1}{u} = x^2 + \frac{1}{u} \longrightarrow y' = 2x - \frac{u'}{u^2}$

$2x - \frac{u'}{u^2} = -\left(x^2 + \frac{1}{u}\right)^2 + 2x^2\left(x^2 + \frac{1}{u}\right) + 2x - x^4$

$\cancel{2x} - \frac{u'}{u^2} = -\cancel{x^4} - \frac{1}{u^2} - \cancel{\frac{2x^2}{u}} + \cancel{2x^4} + \cancel{\frac{2x^2}{u}} + \cancel{2x} - \cancel{x^4}$

$u' = 1 = \frac{du}{dx} \longrightarrow du = dx \longrightarrow u = x + c \longrightarrow y = y_1 + \frac{1}{u} = x^2 + \frac{1}{x+c}$

Ör $xy' = (y-x)^2 + 2y - x$ Riccati dif. denk. üç özel çözümü

$y_1 = x,\ y_2 = x-2,\ y_3 = \frac{x^3 - 2x^2 - x}{x^2 - 1}$ ise genel çözüm.

$\frac{y - y_1}{y - y_2} = c\,\frac{y_3 - y_1}{y_3 - y_2} \Rightarrow \frac{y - x}{y - x + 2} = c\,\frac{\frac{x^3 - 2x^2 - x}{x^2 - 1} - x}{\frac{x^3 - 2x^2 - x}{x^2 - 1} - x + 2}$

$\frac{y - x}{y - x + 2} = c\,\frac{x^3 - 2x^2 - x - x(x^2 - 1)}{x^3 - 2x^2 - x - (x-2)(x^2 - 1)} = c\,\frac{\cancel{x^3} - 2x^2 \cancel{-x} \cancel{-x^3} \cancel{+x}}{\cancel{x^3} \cancel{-2x^2} \cancel{-x} \cancel{-x^3} \cancel{+x} \cancel{+2x^2} - 2} = cx^2$

$\frac{y - x + 2 - 2}{y - x + 2} = 1 - \frac{2}{y - x + 2} = cx^2 \Rightarrow y = x - 2 + \frac{2}{1 - cx^2}$

Ör $y_1 = x,\ y_2 = 1,\ y_3 = -x$ özel çözümleri bilinen Riccati dif. denk. oluştur ve genel çözümü bul.

$$\begin{vmatrix} y' & y^2 & y & 1 \\ 1 & x^2 & x & 1 \\ 0 & 1 & 1 & 1 \\ -1 & x^2 & -x & 1 \end{vmatrix} = 0 \longrightarrow (2x^3 - 2x)y' = 2y^2 + (2x^3 - 2)y - 2x^2$$

$y' = \frac{1}{x(x^2-1)}y^2 + \frac{x^2 + x + 1}{x(x-1)}y - \frac{x}{x^2 - 1}$

$\frac{y - y_1}{y - y_2} = c_1\,\frac{y_3 - y_1}{y_3 - y_2} \Longrightarrow \frac{y - x}{y - 1} = c_1\,\frac{-x - x}{-x - 1} = c_1\,\frac{2x}{x+1}$

$y = \frac{x(x + 1 - 2c_1)}{(1 - 2c_1)x + 1} = \frac{x(x + c)}{cx + 1}$

$i)$ $xy' = (y-x)^2 + 2y - x$ Riccati dif. denk bir özel çözümü ④
$y_1 = x$ ise genel çözüm.

$$y = y_1 + \frac{1}{u} = x + \frac{1}{u} \longrightarrow y' = 1 - \frac{u'}{u^2}$$

$$x\left(1 - \frac{u'}{u^2}\right) = \left(x + \frac{1}{u} - x\right)^2 + 2\left(x + \frac{1}{u}\right) - x$$

$$\cancel{x} - x\frac{u'}{u^2} = \frac{1}{u^2} + \cancel{2x} + \frac{2}{u} - \cancel{x}$$

$$u' + \frac{2}{x}u = -\frac{1}{x} \qquad P(x) = 2/x \,,\ Q(x) = -1/x$$

$$\mu(x) = e^{\int P(x)dx} = e^{2\int \frac{dx}{x}} = e^{2\ln x} = x^2$$

$$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = \frac{1}{x^2}\left(c_1 - \int x\,dx\right) = \frac{c_1 - x^2/2}{x^2}$$

$$y = x + \frac{1}{u} = x + \frac{x^2}{c_1 - x^2/2} = x - 2 + \frac{2c_1}{c_1 - x^2/2} = x - 2 + \frac{2}{1 + cx^2}$$

$ii)$ $y' = \sec^2 x - y\tan x + y^2$ Riccati dif. denk. bir özel çözümü
$y_1 = \tan x$ ise genel çözüm.

$$y = y_1 + \frac{1}{u} = \tan x + \frac{1}{u} \longrightarrow y' = \sec^2 x - \frac{u'}{u^2}$$

$$\sec^2 x - \frac{u'}{u^2} = \sec^2 x - \left(\tan x + \frac{1}{u}\right).\tan x + \left(\tan x + \frac{1}{u}\right)^2$$

$$\cancel{\sec^2 x} - \frac{u'}{u^2} = \cancel{\sec^2 x} - \cancel{\tan^2 x} - \frac{\tan x}{u} + \cancel{\tan^2 x} + \frac{1}{u^2} + \frac{2\tan x}{u}$$

$$u' + u.\tan x = -1 \qquad P(x) = \tan x \,,\ Q(x) = -1$$

$$\mu(x) = e^{\int P(x)dx} = e^{\int \tan x\,dx} = e^{\ln(\sec x)} = \sec x$$

$$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = \frac{1}{\sec x}\left(c - \int \sec x\,dx\right)$$

$$= \cos x.\left(c - \ln|\sec x + \tan x|\right)$$

$$y = \tan x + \frac{1}{u} = \tan x + \frac{\sec x}{c - \ln|\sec x + \tan x|}$$

$x\frac{dy}{dx} = y^2 + y - x^2$ $y_1 = x$ ise çöz

$\frac{dy}{dx} = -8xy^2 + 4x(4x+1)y - 8x^3 - 4x^2 + 1$ $y_1 = x$ ise çöz

Tekil Çözüm : Birinci mertebeden $g(x,y,y')=0$ dif. denklemi ile $\partial g/\partial y' = 0$ denklemi arasında y' çekilerek elde edilen $\phi(x,y)=0$ denkleminin gösterdiği eğriye Diskriminant eğrisi denir. Bu eğri veya bunun bir kolu, dif. denkleminin bir çözümü olabilir. Bu çözüme Tekil Çözüm denir.

Ör $y = xy' + 4(y')^2$ dif. denk. tekil çözümünü bul.

$g(x,y,y') = y - xy' - 4(y')^2 = 0$

$\frac{\partial g}{\partial y'} = -x - 8y' = 0 \rightarrow y' = -x/8$ dif. denk. yerine koy.

$y = x.(-x/8) + 4(-x/8)^2 = -\frac{x^2}{8} + \frac{4x^2}{64} = -x^2/16$ Tekil çözüm

Ör $y^2 = \frac{4}{1+(y')^2}$ dif. denk. tekil çözümünü bul.

$g(x,y,y') = y^2(1+(y')^2) - 4 = 0$

$\frac{\partial g}{\partial y'} = 2y^2 y' = 0 \rightarrow y' = 0$ dif. denk. yerine koy.

$y^2 = \frac{4}{1+(y')^2} = \frac{4}{1+0} = 4 \rightarrow y^2 - 4 = 0$ tekil çözüm

Ör $y = xy' + \frac{1}{y'}$ dif. denk. tekil çözümünü bul.

$g(x,y,y') = y - xy' - \frac{1}{y'}$

$\partial g/\partial y' = -x + \frac{1}{(y')^2} = 0 \rightarrow (y')^2 = \frac{1}{x}$ dif. denk. yerine koy

$y = xy' + \frac{1}{y'} \Rightarrow y^2 = (xy' + \frac{1}{y'})^2 = x^2(y')^2 + \frac{1}{(y')^2} + 2x\cancel{y'}.\frac{1}{\cancel{y'}}$

$y^2 = x^2(y')^2 + \frac{1}{(y')^2} + 2x = x^2\frac{1}{x} + x + 2x = 4x$

Tekil çözüm

Clairaut Diferansiyel Denklemi

$\boxed{y = xy' + f(y')}$ şeklindeki dif. denklemleri

$p = y'$ yazılırsa $y = xp + f(p) \rightarrow y' = \cancel{p} + xp' + f'(p).p' = \cancel{p}$

$$p'(x + f'(p)) = 0$$

$\rightarrow x + f'(p) = 0 \Rightarrow x = -f'(p)$ dif. denk. yerine koy.

$y = -pf'(p) + f(p)$ Tekil çözüm

Genel çözümün gösterdiği doğru ailesinin zarfıdır. p çekilirse zarfın kartezyen koordinattaki denklemi elde edilir

$p' = 0$
$p = c$ dif. denk. yerine koy.

$y = cx + f(c)$
Doğru ailesinin genel çözümü

Lagrange Diferansiyel Denklemi

$\boxed{y = x.g(y') + f(y')}$ şeklindeki dif. denklemleri

$g(y') = y'$ olursa Clairaut dif. denk.

$p = y'$ yazılırsa

$y = x\,g(p) + f(p) \rightarrow y' = g(p) + (x\,g'(p) + f'(p)).p' = p$

$(g(p) - p) + (x\,g'(p) + f'(p)).p' = 0$ x'e göre lineer dif. denk.

$$\mu(p) = e^{\int \frac{dp}{g(p)-p}}$$

$$x = \frac{1}{(g(p)-p).\mu(p)}\left(c - \int \mu(p).f'(p).dp\right) = h(p,c)$$

$$y = h(p,c).g(p) + f(p)$$

Genel çözümün parametrik gösterimi

p çekilirse x ve y değişkenlerine bağımlı bir fonksiyon elde edilir. Diğer türlü fonksiyon parametrik olarak kalır.

Ör $y = xy' + \frac{1}{y'}$ dif. denk. genel ve varsa tekil çözümü.

$p = y' \rightarrow y = xp + \frac{1}{p} \rightarrow y' = \cancel{p} + xp' - \frac{p'}{p^2} = \cancel{p}$

$p'\left(x - \frac{1}{p^2}\right) = 0$

$\hookrightarrow x - \frac{1}{p^2} = 0 \rightarrow x = \frac{1}{p^2}$ dif. denk. yerine koy

$p' = 0$
$p = c$ dif. denk. yerine koy

$y = \frac{1}{p^2} p + \frac{1}{p} = 2/p$, p çekilirse

$y = cx + \frac{1}{c}$ Genel çözüm

$\frac{1}{p^2} = x = y^2/4 \Rightarrow y^2 = 4x$ tekil çözüm
genel çözümün zarfı

Ör $y = xy' + 4(y')^2$ dif. denk. genel ve varsa tekil çözümü.

$p = y' \rightarrow y = xp + 4p^2 \rightarrow y' = \cancel{p} + xp' + 8pp' = \cancel{p}$

$p'(x + 8p) = 0$

$\hookrightarrow x + 8p = 0 \rightarrow x = -8p$ dif. denk. yerine koy.

$p' = 0$
$p = c$ dif. denk. yerine koy

$y = (-8p)\cdot p + 4p^2 = -4p^2$, p çekilirse

$y = cx + 4c^2$
genel çözüm

$y = -x^2/16$ tekil çözüm

Ör $y = xy' - \frac{1}{(y')^2}$ dif. denk. genel ve varsa tekil çözümü.

$p = y' \rightarrow y = xp - \frac{1}{p^2} \rightarrow y' = \cancel{p} + xp' + \frac{2p'}{p^3} = \cancel{p}$

$2(y')^2 - xy' + y = 0$
Genel ve varsa tekil çözüm

$p' \cdot \left(x + \frac{2}{p^3}\right) = 0$

$\hookrightarrow x + \frac{2}{p^3} = 0$

$p' = 0$
$p = c$ dif. denk. yerine koy

$x = -2/p^3$ dif. denk. yerine koy

$y = cx - \frac{1}{c^2}$
Genel çözüm

$y = (-2/p^3)p - \frac{1}{p^2} = -3/p^2$

p çekilirse

$\frac{x^2}{4} + \frac{y^3}{27} = 0$ Tekil çözüm

Ör $y = xy' + \sqrt{4+(y')^2}$ dif. denk. genel ve varsa tekil çözümü

$p = y' \longrightarrow y = xp + \sqrt{4+p^2}$

$\underline{y'} = \cancel{p} + xp' + \frac{1}{2}(4+p^2)^{-1/2} \cdot 2p\,p' = \cancel{p}$

$$p'\left(x + \frac{p}{\sqrt{4+p^2}}\right) = 0$$

$p' = 0$

$p = c$ dif. denk. yerine koy

$y = cx + \sqrt{4+c^2}$

genel çözüm

$\hookrightarrow x + \frac{p}{\sqrt{4+p^2}} = 0 \Rightarrow x = \frac{-p}{\sqrt{4+p^2}}$ dif. denk. yerine koy.

$y = \left(\frac{-p}{\sqrt{4+p^2}}\right) \cdot p + \sqrt{4+p^2} = \frac{4}{\sqrt{4+p^2}}$

p çekilirse $y = 2\sqrt{1-x^2}$ Tekil çözüm

Ö $y = x + (y')^2 - \frac{2}{3}(y')^3$ Lagrange dif. denk. genel ve varsa tekil ~~çözümünü~~ bul.

$p = y' \longrightarrow y = x + p^2 - \frac{2p^3}{3} \longrightarrow y' = 1 + 2p\,p' - 2p^2p' = p$

$2p(1-p)p' = p - 1 \Rightarrow (p-1) + 2p(p-1)\frac{dp}{dx} = 0$

$(p-1)\frac{dx}{dp} + 2p(p-1) = 0$

$$(p-1) \cdot \left(\frac{dx}{dp} + 2p\right) = 0$$

$p = 1$

denklemde yerine koy

$y = x + 1 - 2/3$

$= x + 1/3$

tekil çözüm

$\hookrightarrow \frac{dx}{dp} + 2p = 0 \rightarrow dx + 2p\,dp = 0$

$\int dx + \int 2p\,dp = 0$

$x = c - p^2$ denk. yerine koy.

$y = c - p^2 + p^2 - \frac{2p^3}{3} = c - \frac{2p^3}{3}$

p çekilirse

$4(c-x)^3 - 9(c-y)^2 = 0$ Genel çözüm

Ör: $y=-xy'+(y')^2$ dif. denk. genel çözümü

$p=y' \to y=-xp+p^2 \to y'=-p-xp'+2pp'=p$

$2p=(2p-x)p' \longrightarrow \frac{dp}{dx}=\frac{2p}{2p-x} \longrightarrow \frac{dx}{dp}=\frac{2p-x}{2p}$

$\frac{dx}{dp}+\frac{1}{2p}x=1 \quad P(p)=\frac{1}{2p}\,,\ Q(p)=1$

$\mu(p)=e^{\int P(p)dp}=e^{\int\frac{dp}{2p}}=e^{\frac{1}{2}\ln p}=\sqrt{p}$

$x=\frac{1}{\mu(p)}\left(c+\int\mu(p)Q(p)dp\right)=\frac{1}{\sqrt{p}}\left(c+\int\sqrt{p}\,dp\right)$

$=\frac{c+\frac{2}{3}p^{3/2}}{\sqrt{p}}=\frac{c}{\sqrt{p}}+\frac{2}{3}p$ dif. denk. yerine koy.

$y=-\left(\frac{c}{\sqrt{p}}+\frac{2p}{3}\right)p+p^2=\frac{p^2}{3}-c\sqrt{p}$ p çekilemiyor Genel çözüm parametrik olarak kalır.

$(y')^2\cos^2 y+\sin x.\cos x.\cos y.y'-\sin y.\cos^2 x=0$ dif. denk. genel ve varsa tekil çözümü

Ör: $y=2xy'+(y')^2$ dif. denk. genel çözümünü bul.

$p=y' \longrightarrow y=2xp+p^2 \longrightarrow y'=2p+2xp'+2pp'=p$

$(2x+2p)p'=-p \Rightarrow p'=\frac{-p}{2x+2p}=\frac{dp}{dx}$

$\frac{dx}{dp}=\frac{2x+2p}{-p} \Rightarrow \frac{dx}{dp}+\frac{2}{p}x=-2\,,\ P(p)=2/p\,,\ Q(p)=-2$

$\mu(p)=e^{\int P(p)dp}=e^{2\int\frac{dp}{p}}=e^{2\ln p}=p^2$

$x=\frac{1}{\mu(p)}\left(c+\int\mu(p)Q(p)dp\right)$

$=\frac{1}{p^2}\left(c-2\int p^2dp\right)=\frac{c-2/3\,p^3}{p^2}=\frac{c}{p^2}-\frac{2}{3}p$ dif. denk. yerine koy

$y=2\left(\frac{c}{p^2}-\frac{2}{3}p\right)\cdot p+p^2$

$=\frac{2c}{p}-\frac{p^2}{3}$

p çekilemiyor
Genel çözüm parametrik olarak kalır.

Ör $x(y')^2+(1-y)y'-1=0$ Genel ve varsa tekil çözüm.

$p=y' \rightarrow xp^2+(1-y)p-1=0$

$y=1+xp-1/p \rightarrow y'=\cancel{p}+xp'+\frac{p'}{p^2}=\cancel{p}$

$p'(x+1/p^2)$

$p'=0$
$p=c$ dif-denk. yerine koy

$y=1+cx-1/c$
Genel çözüm

$\hookrightarrow x+1/p^2=0 \rightarrow x=-1/p^2$ dif-denk. yerine koy.

$y=1-\frac{1}{p^2}p-1/p=1-2/p$

p çekilirse $(y-1)^2+4x=0$ tekil çözüm

Ör $x(y')^2+4=yy'$ Genel çözüm ve varsa tekil çözüm

$p=y' \rightarrow xp^2+4=yp \rightarrow y=xp+4/p$

$x=yy'-x(y')^2$ dif. denk. genel çözümü

$y'=\cancel{p}+xp'-4/p^2\, p'=\cancel{p}$

$p'(x-4/p^2)=0$

$p'=0$
$p=c$ dif-denk. yerine koy

$y=cx+4/c$
Genel çözüm

$\hookrightarrow x-4/p^2=0 \rightarrow x=4/p^2$ dif-denk yerine koy

$y=(4/p^2)\cdot p+4/p=8/p$

p çekilirse $y^2=16x$ tekil çözüm

Ör $y=x(1+y')+(y')^2$ dif.denk. genel ve varsa tekil çözümü

$p=y' \rightarrow y=x(1+p)+p^2 \rightarrow y'=1+\cancel{p}+xp'+2pp'=\cancel{p}$

$p'=-\frac{1}{x+2p}=\frac{dp}{dx} \rightarrow \frac{dx}{dp}+x=-2p \quad P(p)=1,\ Q(p)=-2p$

$\mu(p)=e^{\int P(p)dp}=e^{\int dp}=e^p$

$x=\frac{1}{\mu(p)}\left(c+\int \mu(p)Q(p)dp\right)=e^{-p}\left(c-2\int pe^p dp\right)$

$=e^{-p}\left(c-2(p-1)e^p\right)=ce^{-p}-2p+2$ dif. denk. yerine koy.

$y=(ce^{-p}-2p+2)(1+p)+p^2=c(p+1)e^{-p}+2-p^2$

p çekilmiyor. genel çözüm parametrik olarak kalır.

y' değişkenine göre çözülebilen dif. denklemleri

Ör: $(y')^2-(y+\sin x)y'+y\sin x=0$ Genel çözüm

$p=y' \longrightarrow p^2-(y+\sin x)p+y\sin x=0$

$(p-y)(p-\sin x)=0$

$p=y=\frac{dy}{dx}$

$\frac{dy}{y}=dx$

$\ln y=x+c_1$

$y=e^{x+c_1}=c_2e^x$

$y-c_2e^x=0$

$p=\sin x=\frac{dy}{dx}$

$dy=\sin x\,dx$

$y=c_3-\cos x$

$y+\cos x-c_3=0$

1. mertebeden dif. denk. sadece bir keyfi değer içerir. Bu yüzden $c_2=c_3=c$ olmalıdır.

iki denklem çarpılırsa

$(y-ce^x)(y+\cos x-c)=0$ Genel çözüm

Ör: $2y(y')^2+(6x^2y-1)y'-3x^2=0$ Genel çözüm

$p=y' \Rightarrow 2y\,p^2+(6x^2y-1)p-3x^2=0$

$p^2+\frac{6x^2y-1}{2y}p-\frac{3x^2}{2y}=0$

$(p-\frac{1}{2y})(p+3x^2)=0$

$p-\frac{1}{2y}=0$

$\frac{dy}{dx}-\frac{1}{2y}=0$

$2y\,dy-dx=0$

$y^2-x-c_1=0$

$p+3x^2=0$

$\frac{dy}{dx}+3x^2=0$

$dy+3x^2dx=0$

$y+x^3-c_2=0$

1. mertebeden dif. denk. sadece bir keyfi değer içerir. $c_1=c_2=c$ olmalı

iki denklem çarpılırsa

$(y^2-x-c)(y+x^3-c)=0$ Genel çözüm

Ör $x^2(y')^2-4xyy'-5y^2=0$ Genel çözüm.

$p=y' \longrightarrow x^2p^2-4xyp-5y^2=0$

$$p^2-\frac{4y}{x}p-\frac{5y^2}{x^2}=0$$

$$\left(p-\frac{5y}{x}\right)\left(p+\frac{y}{x}\right)=0$$

$p-\frac{5y}{x}=0$

$\frac{dy}{dx}-\frac{5y}{x}=0$

$\frac{dy}{y}-5\frac{dx}{x}=0$

$\ln y-5\ln x=c_1$

$y-c_2x^5=0$

$p+\frac{y}{x}=0$

$\frac{dy}{dx}+\frac{y}{x}=0$

$\frac{dy}{y}+\frac{dx}{x}=0$

$\ln y+\ln x=c_3$

$xy-c_4=0$

1. mertebeden dif. denk. sadece bir keyfi değer içerir

$c_2=c_4=c$

iki denklem çarpılırsa

$(y-cx^5)(xy-c)=0$

Genel çözüm

Ör $(y')^2+y^2=1$ Genel çözüm.

$p=y' \longrightarrow p^2+y^2=1 \longrightarrow p^2-(1-y^2)=0$

$$\left(p+\sqrt{1-y^2}\right)\left(p-\sqrt{1-y^2}\right)=0$$

$p+\sqrt{1-y^2}=0$

$\frac{dy}{dx}+\sqrt{1-y^2}=0$

$\frac{dy}{\sqrt{1-y^2}}+dx=0$

$\arcsin y+x-c_1=0$

$p-\sqrt{1-y^2}=0$

$\frac{dy}{dx}-\sqrt{1-y^2}=0$

$\frac{dy}{\sqrt{1-y^2}}-dx=0$

$\arcsin y-x-c_2=0$

1. mertebeden dif. denklemi sadece bir keyfi değer içerir.

$c_1=c_2=c$ olmalı.

iki denklem çarpılırsa

$$(\arcsin y+x-c)(\arcsin y-x-c)=0$$

Genel çözüm

Yüksek Mertebeli Diferansiyel Denklemler

Bağımlı Değişken (y) İçermeyen dif. denklemleri

Ör $xy'' + 3x(y')^2 - 2y' = 0$ dif. denk. genel çözümü

$u = y'$, $u' = y'' \rightarrow xu' + 3xu^2 - 2u = 0 \rightarrow u' - \frac{2}{x}u = -3u^2$ $\quad -u^{-2}$ ile çarp

$-u^{-2}u' + \frac{2}{x}u^{-1} = 3 \qquad v = u^{-1} \rightarrow v' = -u^{-2}u'$

$v' + \frac{2}{x}v = 3 \qquad P(x) = 2/x, \; Q(x) = 3$

$\mu(x) = e^{\int P(x)dx} = e^{2\int \frac{dx}{x}} = e^{2\ln x} = x^2$

$V = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = \frac{1}{x^2}\left(c_1 + \int 3x^2 dx\right)$

$= \frac{c_1 + x^3}{x^2} = \frac{1}{u} = \frac{1}{y'} = \frac{dx}{dy} \rightarrow dy = \frac{x^2 dx}{x^3 + c_1}$

$y = \int \frac{x^2 dx}{x^3 + c_1} = \frac{1}{3}\ln(x^3 + c_1) + c_2$

Ör $x^3 y'' = 1$ Genel çözüm

$u = y'$, $u' = y'' \rightarrow x^3 u' = 1 \rightarrow u' = \frac{1}{x^3} = \frac{du}{dx} \rightarrow du = \frac{dx}{x^3}$

$u = \int \frac{dx}{x^3} = c_1 - \frac{1}{2x^2} = y' = \frac{dy}{dx} \rightarrow dy = c_1 dx - \frac{dx}{2x^2}$

$y = c_1 x + \frac{1}{2x} + c_2$

Ör $xy''' - 2y'' = 0$ Genel çözüm

$u = y''$, $u' = y''' \rightarrow xu' - 2u = 0 \rightarrow x\frac{du}{dx} - 2u = 0$

$\frac{du}{u} - 2\frac{dx}{x} = 0 \rightarrow \ln u - 2\ln x = c \rightarrow \ln\frac{u}{x^2} = c = \ln c_a$

$u = c_a x^2 = y''$

$y' = \int y'' dx = \int c_a x^2 dx = \frac{c_a}{3}x^3 + c_b$

$y = \int y' dx = \int\left(\frac{c_a}{3}x^3 + c_b\right)dx = \frac{c_a}{12}x^4 + c_b x + c_c \rightarrow y = c_1 x^4 + c_2 x + c_3$

Genel çözüm

$y'' = \frac{y'}{x} + x.\cos x$ genel çözüm

$u = y',\ u' = y'' \rightarrow u' - \frac{1}{x}u = x\cos x \qquad P(x) = -1/x \quad Q(x) = x\cos x$

$\mu(x) = e^{\int P(x)dx} = e^{-\int \frac{dx}{x}} = e^{-\ln x} = 1/x$

$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = x\left(c + \int \cos x\, dx\right)$

$= x(c + \sin x) = cx + x\sin x = y' = \frac{dy}{dx}$

$y = c\int x\,dx + \underbrace{\int x\sin x\, dx}_{\text{kısmi integ.}} = c\frac{x^2}{2} + \sin x - x\cos x + C_2$

$= C_1 x^2 + C_2 + \sin x - x\cos x$

Ör $xy''' + y'' = 12x$ dif. denk. genel çözümü $\qquad P(x) = 1/x \quad Q(x) = 12$

$u = y',\ u' = y'' \rightarrow xu' + u = 12x \rightarrow u' + \frac{1}{x}u = 12$

$\mu(x) = e^{\int P(x)dx} = e^{\int \frac{dx}{x}} = e^{\ln x} = x$

$u = \frac{1}{\mu(x)}\left(C_1 + \int \mu(x)Q(x)dx\right) = \frac{1}{x}\left(C_1 + \int 12x\,dx\right) = \frac{C_1}{x} + 6x = y''$

$y' = \int y''dx = C_1\ln x + 3x^2 + C_2$

$y = \int y'dx = C_1 x(\ln x - 1) + x^3 + C_2 x + C_3 = x^3 + C_1 x\ln x + \underbrace{(C_2 - C_1)}_{C_2}x + C_3$

Ör $xy^{(4)} - 2y''' = x^3$ Genel çözüm $\qquad P(x) = -2/x \quad Q(x) = x^2$

$u = y''',\ u' = y^{(4)} \rightarrow xu' - 2u = x^3 \rightarrow u' - \frac{2}{x}u = x^2$

$\mu(x) = e^{\int P(x)dx} = e^{-2\int \frac{dx}{x}} = e^{-2\ln x} = 1/x^2$

$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = x^2\left(c + \int dx\right) = x^2(c + x) = cx^2 + x^3 = y'''$

$y'' = \int y'''dx = \frac{x^4}{4} + \frac{cx^3}{3} + C_a \rightarrow y' = \int y''dx = \frac{x^5}{20} + \frac{cx^4}{12} + C_a x + C_b$

$y = \int y'dx = \frac{x^6}{120} + \frac{cx^5}{60} + \frac{C_a x^2}{2} + C_b x + C_c$

$= \frac{x^6}{120} + C_1 x^5 + C_2 x^2 + C_3 x + C_4$

Ör $x^2y'' - (y')^2 + 2xy' - 2x^2 = 0$ Genel çözüm

$u = y'$, $u' = y'' \rightarrow x^2u' - u^2 + 2xu - 2x^2 = 0$

$u' = 2 - 2\left(\frac{u}{x}\right) + \left(\frac{u}{x}\right)^2 = 2 - 2v + v^2$, $v = \frac{u}{x}$ homojen

$u = xv \rightarrow u' = v + xv' = 2 - 2v + v^2 \rightarrow x\frac{dv}{dx} = v^2 - 3v + 2$

$\frac{dx}{x} - \frac{dv}{v^2 - 3v + 2} = 0 \rightarrow \frac{dx}{x} + \frac{dv}{v-1} - \frac{dv}{v-2} = 0$

$\ln|x| + \ln|v-1| - \ln|v-2| = c \rightarrow \ln\left|\frac{x(v-1)}{v-2}\right| = c = \ln|c_1|$

$\frac{x(v-1)}{v-2} = c_1 = \frac{x\left(\frac{u}{x} - 1\right)}{\frac{u}{x} - 2} = \frac{x(u-x)}{u-2x} \rightarrow u = \frac{x^2 - 2c_1x}{x - c_1}$

$u = x - c_1 - \frac{c_1^2}{x - c_1} = y' \rightarrow y = \int y'dx = \frac{x^2}{2} - c_1x - c_1^2\ln|x - c_1| + c_2$

Ör $x^3y'' - x^2y' = 3 - x^2$ Genel çözüm

$u = y'$, $u' = y'' \rightarrow x^3u' - x^2u = 3 - x^2 \rightarrow u' - \frac{1}{x}u = \frac{3 - x^2}{x^3}$ $\quad P(x) = -1/x$, $Q(x) = \frac{3 - x^2}{x^3}$

$\mu(x) = e^{\int P(x)dx} = e^{-\int\frac{dx}{x}} = e^{-\ln x} = \frac{1}{x}$

$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = x\left(c + \int\frac{3 - x^2}{x^4}dx\right)$

$= x\left(c + 3\int x^{-4}dx - \int x^{-2}dx\right) = x\left(c - \frac{1}{x^3} + \frac{1}{x}\right) = cx - \frac{1}{x^2} + 1 = y' = \frac{dy}{dx}$

$dy = \left(cx - \frac{1}{x^2} + 1\right)dx \rightarrow y = \frac{cx^2}{2} + \frac{1}{x} + x + c_2 = c_1x^2 + x + \frac{1}{x} + c_2$

Ör $xy'' - y' = x^2e^x$ Genel çözüm

$u = y'$, $u' = y'' \rightarrow xu' - u = x^2e^x \rightarrow u' - \frac{1}{x}u = xe^x$ $\quad P(x) = -1/x$, $Q(x) = xe^x$

$\mu(x) = e^{\int P(x)dx} = e^{-\int\frac{dx}{x}} = e^{-\ln x} = 1/x$

$u = \frac{1}{\mu(x)}\left(c + \int\mu(x)Q(x)dx\right) = x\left(c + \int e^x dx\right)$

$= x(c + e^x) = cx + xe^x = y' = \frac{dy}{dx}$

$dy = (cx + xe^x)dx \rightarrow y = \int y'dx = \int(cx + xe^x)dx$

$= \frac{cx^2}{2} + (x-1)e^x + c_2 = c_1x^2 + (x-1)e^x + c_2$

Bağımsız değişken (x) içermeyen diferansiyel denklemler

Ör $y'' = y(y')^3$ Genel çözüm

$u = y' \rightarrow u' = y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = y'\frac{du}{dy} = u\frac{du}{dy}$

$u\frac{du}{dy} = yu^3 \rightarrow y\,dy - \frac{du}{u^2} = 0 \rightarrow \frac{y^2}{2} + \frac{1}{u} = C_a$

$\frac{1}{u} = C_a - \frac{y^2}{2} = \frac{1}{y'} = \frac{dx}{dy} \rightarrow (C_a - y^2/2)\,dy = dx$

$C_a y - \frac{y^3}{6} = x + C_b \rightarrow 6C_a y - y^3 = 6x + 6C_b \rightarrow y^3 + C_1 y + 6x + C_2 = 0$

Ör $y^2 y'' + (y')^3 = 0$ Genel çözüm

$u = y' \rightarrow u' = y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = y'\frac{du}{dy} = u\frac{du}{dy}$

$y^2 u\frac{du}{dy} + u^3 = 0 \rightarrow \frac{du}{u^2} + \frac{dy}{y^2} = 0 \rightarrow -\frac{1}{u} - \frac{1}{y} = C_1$

$\frac{1}{u} = -C_1 - \frac{1}{y} = \frac{1}{y'} = \frac{dx}{dy} \rightarrow (C_1 + \frac{1}{y})\,dy + dx = 0$

$C_1 y + \ln|y| + x = C_2$

Ör $(y+1)y'' = (y')^2$ Genel çöz. $u = y'$, $u' = y'' = u\frac{du}{dy}$

$(y+1)u\frac{du}{dy} = u^2 \rightarrow \frac{du}{u} = \frac{dy}{y+1} \rightarrow \ln|u| = \ln|y+1| + C$

$u = e^{C + \ln|y-1|} = C_1(y-1) = y' = \frac{dy}{dx}$

$\frac{dy}{y+1} = C_1 dx \longrightarrow \ln|y+1| = C_1 x + C_2$

Ör $yy'' + (1+y)(y')^2 = 0$ Genel çöz. $u = y'$, $u' = y'' = u\frac{du}{dy}$

$yu\frac{du}{dy} + (1+y)u^2 = 0 \rightarrow \frac{du}{u} + \frac{1+y}{y}dy = 0$

$\frac{du}{u} + \frac{dy}{y} + dy = 0 \longrightarrow \ln|u| + \ln|y| + y = C$

$\ln|uy| = C - y \rightarrow uy = e^{C-y} = C_1 e^{-y} \rightarrow u = \frac{C_1}{ye^y} = y' = \frac{dy}{dx}$

$ye^y dy = C_1 dx \rightarrow \int ye^y dy = \int C_1 dx$

$(y-1)e^y = C_1 x + C_2$

Ör. $y(y-1)y'' + (y')^2 = 0$ Genel çöz.

$$u = y',\quad u' = y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = y'\frac{du}{dy} = u\frac{du}{dy}$$

$$y(y-1)u\frac{du}{dy} + u^2 = 0 \rightarrow \frac{du}{u} + \frac{dy}{y(y-1)} = 0$$

$$\frac{du}{u} + \frac{dy}{y-1} - \frac{dy}{y} = 0 \rightarrow \ln|u| + \ln|y-1| - \ln|y| = C$$

$$\ln\left|\frac{u(y-1)}{y}\right| = C = \ln|c_1| \rightarrow \frac{u(y-1)}{y} = c_1 \rightarrow u = \frac{c_1 y}{y-1} = y' = \frac{dy}{dx}$$

$$\frac{y-1}{y}dy = c_1 dx \rightarrow dy - \frac{dy}{y} = c_1 dx \rightarrow y - \ln|y| = c_1 x + c_2$$

Ör. $(y')^2 + 2yy'' = 0$ Genel çöz. $u = y',\ u' = y'' = u\frac{du}{dy}$

$$u^2 + 2yu\frac{du}{dy} = 0 \rightarrow \frac{1}{2}\frac{dy}{y} + \frac{du}{u} = 0 \rightarrow \frac{1}{2}\ln|y| + \ln|u| = C$$

$$\ln|\sqrt{y}\,u| = C = \ln|c_a| \rightarrow \sqrt{y}\,u = c_a \rightarrow u = \frac{c_a}{\sqrt{y}} = y' = \frac{dy}{dx}$$

$$\sqrt{y}\,dy = c_a dx \rightarrow \frac{2}{3}y^{3/2} = c_a x + c_b \rightarrow y^{3/2} = \frac{3c_a}{2}x + \frac{3c_b}{2}$$

$$y = \left(\frac{3c_a}{2}x + \frac{3c_b}{2}\right)^{2/3} = (c_1 x + c_2)^{2/3}$$

Ör. $y'' - (y')^2 + y(y')^3 = 0$ Genel çözüm $\quad u = y'$, $u' = y'' = u\frac{du}{dy}$

$$u\frac{du}{dy} - u^2 + yu^3 = 0$$

$$\frac{du}{dy} - u = -yu^2, \quad -u^{-2} \text{ ile çarp.} \quad v = u^{-1},\ \frac{dv}{dy} = -u^{-2}\frac{du}{dy}$$

$$-u^{-2}\frac{du}{dy} + u^{-1} = y \rightarrow \frac{dv}{dy} + v = y, \quad P(y) = 1,\ Q(y) = y$$

$$\mu(y) = e^{\int P(y)dy} = e^{\int dy} = e^y$$

$$v = \frac{1}{\mu(y)}\left(c_1 + \int \mu(y)Q(y)dy\right) = e^{-y}\left(c_1 + \int ye^y dy\right)$$

$$= e^{-y}\left(c_1 + (y-1)e^y\right) = c_1e^{-y} + y - 1 = \frac{1}{u} = \frac{1}{y'} = \frac{dx}{dy}$$

$$dx - (c_1e^{-y} + y - 1)dy = 0 \rightarrow x + c_1e^{-y} - \frac{y^2}{2} + y = c_2$$

y ve türevlerine göre homojen dif. denklemler

Ör $xyy'' - x(y')^2 + yy' = 0$ Genel çözüm.

$x\frac{y''}{y} - x\left(\frac{y'}{y}\right)^2 + \frac{y'}{y} = 0 \qquad \frac{y'}{y} = u, \quad \frac{y''}{y} = u' + u^2$

$x(u'+u^2) - xu^2 + u = 0 \rightarrow xu' + \cancel{xu^2} - \cancel{xu^2} + u = 0$

$x\frac{du}{dx} + u = 0 \rightarrow \frac{du}{u} + \frac{dx}{x} = 0 \rightarrow \ln|u| + \ln|x| = c$

$\ln|ux| = c = \ln|c_a| \rightarrow ux = c_a \rightarrow u = \frac{c_a}{x} = \frac{y'}{y} = \frac{dy}{y\,dx}$

$\frac{dy}{y} = c_a\frac{dx}{x} \rightarrow \ln|y| = c_a\ln|x| + c_b$

$y = e^{c_a\ln|x| + c_b} = c_2 x^{c_1}$

Ör $y^2 - 2xyy' + x^2(y')^2 - x^2yy'' = 0$ Genel çözüm

$1 - 2x\frac{y'}{y} + x^2\left(\frac{y'}{y}\right)^2 - x^2\frac{y''}{y} = 0 \qquad \frac{y'}{y} = u, \quad \frac{y''}{y} = u' + u^2$

$1 - 2xu + x^2u^2 - x^2(u'+u^2) = 0$

$1 - 2xu + \cancel{x^2u^2} - x^2u' - \cancel{x^2u^2} = 0 \rightarrow u' + \frac{2}{x}u = \frac{1}{x^2} \qquad P(x) = 2/x \quad Q(x) = 1/x^2$

$\mu(x) = e^{\int P(x)dx} = e^{2\int\frac{dx}{x}} = e^{2\ln x} = x^2$

$u = \frac{1}{\mu(x)}\left(c + \int \mu(x)Q(x)dx\right) = \frac{1}{x^2}\left(c + \int dx\right) = \frac{c+x}{x^2}$

$= \frac{c}{x^2} + \frac{1}{x} = \frac{y'}{y} = \frac{dy}{y\,dx} \rightarrow \frac{dy}{y} = \left(\frac{c}{x^2} + \frac{1}{x}\right)dx$

$\ln|y| = -\frac{c}{x} + \ln|x| + c_a \rightarrow y = e^{-c/x}e^{\ln x}e^{c_a} = c_2 x e^{c_1 x}$

Ör $y'' - 3y' = 0$ Genel çözüm

$u = y', \; u' = y'' \rightarrow u' - 3u = 0 \rightarrow \frac{du}{dx} - 3u = 0$

$\frac{du}{u} - 3dx = 0$

$\int\frac{du}{u} - 3\int dx = c \rightarrow \ln|u| = 3x + c \rightarrow u = e^{3x+c}$

$u = e^c e^{3x} = y' = \frac{dy}{dx}$

$y = c_1 e^{3x} + c_2$

$xyy'' - 4x(y')^2 + 4yy' = 0$

$x^2y'' + xy' - 4y = 0$

$(y'')^2 - 4(y')^2 = 0$

n. mertebeden lineer diferansiyel denklemler

$$a_0(x)\cdot y^{(n)}+\dots+a_{n-2}(x)\,y''+a_{n-1}(x)\,y'+a_n(x)\,y=b(x),\quad a_0(x)\neq 0$$

$b(x)=0$ ise dif. denklemi homojen, değilse homojen değildir.

$a_i(x)=k_i$, $i=0,1,2,\dots,n$ Katsayılar sabit ise dif. denk. sabit katsayılıdır.

$y'''-3y''+4y'-12y=xe^x$ sabit katsayılı homojen olmayan 3. mertebeden lineer dif. denklemi.

$(x^2-1)y''+4xy'+2y=0$ 2. mertebeden homojen lineer dif. denklemi

Lineer dif. denkleminin oluşturulması

Ör: $y=(c_1+c_2x)e^x+x^2+3x$ genel çözüm ise lineer dif. denk. oluştur.

$y-x^2-3x,\ e^x,\ xe^x$ 3 tane

$y=y_h+y_p$

$y_h=c_1e^x+c_2xe^x$ Homojen kısmın çözümü

$y_p=x^2+3x$ Homojen olmayan kısmın çözümü

$$\begin{vmatrix} y-x^2-3x & e^x & xe^x \\ y'-2x-3 & e^x & (x+1)e^x \\ y''-2 & e^x & (x+2)e^x \end{vmatrix}=0 \xrightarrow{\text{çöz}} y''-2y'+y=x^2-x-4$$

Ör: $y=c_1e^{2x}+c_2x$ genel çözüm ise lineer dif. denk. oluştur

$$\begin{vmatrix} y & e^{2x} & x \\ y' & 2e^{2x} & 1 \\ y'' & 4e^{2x} & 0 \end{vmatrix}=0 \longrightarrow y''-\frac{4x}{2x-1}y'+\frac{4}{2x-1}y=0$$

Ör: $y=c_1e^{3x}+2x^2-3x+5$ genel çözüm ise lineer dif. denk. oluştur

$$\begin{vmatrix} y-2x^2+3x-5 & e^{3x} \\ y'-4x+3 & 3e^{3x} \end{vmatrix}=0 \longrightarrow y'-3y=-6x^2+13x-18$$

Ör: $y=c_1x^2+c_2/x+3x$ genel çözüm ise lineer dif. denk. oluştur.

$$\begin{vmatrix} y-3x & x^2 & 1/x \\ y'-3 & 2x & -1/x^2 \\ y'' & 2 & 2/x^3 \end{vmatrix}=0 \longrightarrow y''-\frac{2}{x^2}y=-6/x$$

Ör $y = c_1 x^3 + c_2 x + 3x^2 + 5$ genel çözüm ise lineer dif. denk. oluştur. (2t)

$$\begin{vmatrix} y-3x^2-5 & x^3 & x \\ y'-6x & 3x^2 & 1 \\ y''-6 & 6x & 0 \end{vmatrix} = 0 \rightarrow x^2y'' - 3xy' + 3y = -3x^2 + 15$$

Ör $y = c_1 \cos(3x) + c_2 \sin(3x) + 2x + 3$ genel çözüm ise lineer dif. denk. oluştur.

$$\begin{vmatrix} y-2x-3 & \cos(3x) & \sin(3x) \\ y'-2 & -3\sin(3x) & 3\cos(3x) \\ y'' & -9\cos(3x) & -9\sin(3x) \end{vmatrix} = 0 \rightarrow y'' + 9y = 18x + 27$$

Homojen lineer dif. denklemlerinde mertebe indirgeme

Ör $x^2y'' - xy' - 3y = 0$ dif. denk. bir özel çözümü $y_1 = x^3$ ise genel çözüm.

$$y = y_1 u = x^3 u \rightarrow y' = 3x^2 u + x^3 u'$$
$$y'' = 6xu + 6x^2u' + x^3u''$$
$$x^2(6xu + 6x^2u' + x^3u'') - x(3x^2u + x^3u') - 3x^3u = 0$$
$$\cancel{6x^3u} + 6x^4u' + x^5u'' - \cancel{3x^3u} - x^4u' - \cancel{3x^3u} = 0$$
$$u'' + \frac{5}{x}u' = 0, \quad v = u', \quad v' = u'' = \frac{dv}{dx}$$
$$\frac{dv}{dx} + \frac{5}{x}v = 0 \rightarrow \frac{dv}{v} + 5\,dx = 0 \rightarrow \ln|v| + 5\ln|x| = \ln|c|$$
$$vx^5 = c \rightarrow v = c/x^5 = u' = \frac{du}{dx} \rightarrow du = c\frac{dx}{x^5}$$
$$u = -\frac{c}{4x^4} + c_2 = c_1x^{-4} + c_2 \rightarrow y = x^3u = c_2x^3 + c_1/x$$

Ör $(2x+1)x^2y'' + 2(x+1)xy' - 2xy = 0$ dif. denk. bir özel çözümü $y_1 = x+1$ ise genel çözüm

$$y = y_1 u = (x+1)u \rightarrow y' = u + (x+1)u'$$
$$y'' = 2u' + (x+1)u''$$

$$(2x+1)x^2(2u'+(x+1)u'')+2(x+1)x(u+(x+1)u')-2x(x+1)u=0$$ (5)

$$x^2(x+1)(2x+1)u''+2x^2(2x+1)u'+2x(x+1)^2u'+2x\cancel{(x+1)}u-2x\cancel{(x+1)}u=0$$

$$u''+\frac{6x^2+6x+2}{x(x+1)(2x+1)}u'=0 \qquad v=u',\ v'=u''=\frac{dv}{dx}$$

$$\frac{dv}{dx}+\frac{6x^2+6x+2}{x(x+1)(2x+1)}v=0 \rightarrow \frac{dv}{v}+2\frac{dx}{x}+2\frac{dx}{x+1}-2\frac{dx}{2x+1}=0$$

$$\ln|v|+2\ln|x|+2\ln|x+1|-\ln|2x+1|=c=\ln|c_1|$$

$$\ln\left|\frac{vx^2(x+1)^2}{2x+1}\right|=\ln|c_1| \longrightarrow v=c_1\frac{2x+1}{x^2(x+1)^2}=u'=\frac{du}{dx}$$

$$\frac{du}{dx}=c_1\left(\frac{1}{x^2}-\frac{1}{(x+1)^2}\right)\longrightarrow du=c_1\frac{dx}{x^2}-c_1\frac{dx}{(x+1)^2}$$

$$u=\frac{c_1}{x+1}-\frac{c_1}{x}+c_2 \Rightarrow y=(x+1)u=c_1-c_1\frac{x+1}{x}+c_2(x+1)$$
$$=c_2(x+1)-c_1/x$$

Ör $y''+(\tan x-2\cot x)y'+2y\cot^2x=0$ dif denk. bir özel çözümü $y_1=\sin x$ ise genel çözüm.

$$y=y_1u=u.\sin x \longrightarrow y'=u'\sin x+u\cos x$$
$$y''=u''\sin x+2u'\cos x-u\sin x$$

$$(u''\sin x+2u'\cos x-u\sin x)+(\tan x-2\cot x)(u'\sin x+u\cos x)+2u\sin x\cot^2x=0$$

$$u''\sin x+2u'\cos x-\cancel{u\sin x}+\frac{\sin^2x-2\cos^2x}{\cos x}u'+\cancel{\frac{\sin^2x-2\cos^2x}{\sin x}}u+\cancel{\frac{2\cos^2x}{\sin x}}u=0$$

$$u''+\frac{\sin x}{\cos x}u'=0,\quad v=u',\ v'=u''=\frac{dv}{dx}$$

$$\frac{dv}{dx}+\frac{\sin x}{\cos x}v=0 \longrightarrow \frac{dv}{v}+\frac{\sin x}{\cos x}dx=0$$

$$\int\frac{dv}{v}+\int\frac{\sin x}{\cos x}dx=c \Rightarrow \ln|v|-\ln|\cos x|=c=\ln|c_1|$$

$$\ln\left|\frac{v}{\cos x}\right|=\ln|c_1| \longrightarrow v=c_1\cos x=\frac{du}{dx}$$

$$du=c_1\cos x\,dx \rightarrow u=c_1\sin x+c_2$$

$$y=u.\sin x=(c_1\sin x+c_2).\sin x$$

Ör $(x^2-1)y'' + 4xy' + 2y = 0$ dif. denk. bir özel çözümü $y_1 = 1/x+1$ ise genel çözüm (20)

$y = y_1 u = \frac{u}{x+1} \longrightarrow y' = \frac{u'}{x+1} - \frac{u}{(x+1)^2}$

$y'' = \frac{u''}{x+1} - \frac{2u'}{(x+1)^2} + \frac{2u}{(x+1)^3}$

$(x-1)(x+1)\left(\frac{u''}{x+1} - \frac{2u'}{(x+1)^2} + \frac{2u}{(x+1)^3}\right) + 4x\left(\frac{u'}{x+1} - \frac{u}{(x+1)^2}\right) + \frac{2u}{x+1} = 0$

$u'' + \frac{2}{x-1}u' = 0 \quad , \quad v = u' \, , \; v' = u'' = \frac{dv}{dx}$

$\frac{dv}{dx} + \frac{2}{x-1}v = 0 \rightarrow \frac{dv}{v} + 2\frac{dx}{x-1} = 0 \rightarrow \ln|v| + 2\ln|x-1| = c$

$\ln|v(x-1)^2| = \ln|c_1| \rightarrow v = \frac{c_1}{(x-1)^2} = \frac{du}{dx} \rightarrow du = c_1 \frac{dx}{(x-1)^2}$

$u = -\frac{c_1}{x-1} + c_2 \rightarrow y = \frac{u}{x+1} = \frac{c_2}{x+1} - \frac{c_1}{x^2-1}$

Ör $xy'' - (x+1)y' + y = 0$ dif. denk. bir özel çözümü $y_1 = e^x$ ise genel çözüm.

$y = y_1 u = ue^x \longrightarrow y' = u'e^x + ue^x$

$y'' = u''e^x + 2u'e^x + ue^x$

$x(u''e^x + 2u'e^x + ue^x) - (x+1)(u'e^x + ue^x) + ue^x = 0$

$u'' + \frac{x-1}{x}u' = 0 \quad , \quad v = u' \, , \; v' = u'' = \frac{dv}{dx}$

$\frac{dv}{dx} + \frac{x-1}{x}v = 0 \rightarrow \frac{dv}{v} + dx - \frac{dx}{x} = 0 \Rightarrow \ln|v| + x - \ln|x| = c$

$\ln\left|\frac{v}{x}\right| = c - x \rightarrow v = c_1 x e^{-x} = \frac{du}{dx}$

$du = c_1 x e^{-x} dx \rightarrow u = c_2 - c_1(x+1)e^{-x} \rightarrow y = ue^x$
$= c_2 e^x - c_1(x+1)$

Ör $(x^2+1)y'' - 2xy' + 2y = 0$ dif. denk. bir özel çözümü $y_1 = x$ ise genel çözüm

$y = y_1 u = xu \rightarrow y' = u + xu' \, , \; y'' = 2u' + xu''$

$(x^2+1)(2u' + xu'') - 2x(u + xu') + 2xu = 0$

$u'' + \frac{2}{x(x^2+1)}u' = 0 \longrightarrow \frac{dv}{v} + \frac{2}{x(x^2+1)}dx = 0$

$v = u' \, , \; v' = u'' = \frac{dv}{dx}$

$\xrightarrow{\text{sonuç olarak}} y = c_1(x^2-1) + c_2 x$

Homojen olmayan lineer dif. denk. mertebe indirgeme

$y_h = c_1 y_1 + c_2 y_2 + \cdots + c_n y_n$ Homojen kısmın çözümü

$y = c_{1x} y_1 + c_{2x} y_2 + \cdots + c_{nx} y_n$ Genel çözüm

$$\left.\begin{aligned}
c_{1x}' y_1 + c_{2x}' y_2 + \cdots + c_{nx}' y_n &= 0 \\
c_{1x}' y_1' + c_{2x}' y_2' + \cdots + c_{nx}' y_n' &= 0 \\
&\vdots \\
c_{1x}' y_1^{(n-2)} + c_{2x}' y_2^{(n-2)} + \cdots + c_{nx}' y_n^{(n-2)} &= 0 \\
c_{1x}' y_1^{(n-1)} + c_{2x}' y_2^{(n-1)} + \cdots + c_{nx}' y_n^{(n-1)} &= \frac{b(x)}{a_0(x)}
\end{aligned}\right\}$$

Parametre Değişim Yöntemi (Sabitler Değişimi Yöntemi)

$$\begin{bmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & & y_n' \\ \vdots & \vdots & & \vdots \\ y_1^{(n-2)} & y_2^{(n-2)} & \cdots & y_n^{(n-2)} \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{bmatrix} \begin{bmatrix} c_{1x}' \\ c_{2x}' \\ \vdots \\ \\ c_{nx}' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{b(x)}{a_0(x)} \end{bmatrix}$$

2. mertebeden dif. denk. için

$a_0(x) y'' + a_1(x) y' + a_2(x) y'' = b(x) \rightarrow y_h = c_1 y_1 + c_2 y_2$ ise

Genel çözüm

$y = c_{1x} y_1 + c_{2x} y_2$

$$\begin{aligned} c_{1x}' y_1 + c_{2x}' y_2 &= 0 \\ c_{1x}' y_1' + c_{2x}' y_2' &= b(x)/a_0(x) \end{aligned}$$

Homojen kısmın özel çözümü $y_1 = x$ ise genel çözüm

Ör $y'' - \frac{3}{x} y' + \frac{3}{x^2} y = 2x - 1$

$y_h = y_1 u = xu \rightarrow y_h' = u + xu'$, $y_h'' = 2u' + xu''$

$y_h'' - \frac{3}{x} y_h' + \frac{3}{x^2} y_h = 0$

$(2u' + xu'') - \frac{3}{x}(u + xu') + \frac{3}{x^2}(xu) = 0$

$2u' + xu'' - \cancel{\frac{3u}{x}} - 3u' + \cancel{\frac{3u}{x}} = 0$

$u'' - \frac{1}{x} u' = 0$, $v = u'$, $v' = u'' = \frac{dv}{dx}$

$\frac{dv}{dx} - \frac{1}{x} v = 0$

$\frac{dv}{v} - \frac{dx}{x} = 0$

$\ln|v| - \ln|x| = \ln|c|$

$\ln\left|\frac{v}{x}\right| = \ln|c|$

$\frac{v}{x} = c$

$$v = cx = \frac{du}{dx} \longrightarrow du = cx\,dx$$

$$u = \frac{cx^2}{2} + c_2 = c_1x^2 + c_2 \longrightarrow y_h = xu = c_1x^3 + c_2x$$

$$\left.\begin{aligned} c_{1x}'x^3 + c_{2x}'x &= 0 \\ c_{1x}'3x^2 + c_{2x}' &= 2x-1 \end{aligned}\right\} \begin{bmatrix} x^3 & x & 0 \\ 3x^2 & 1 & 2x-1 \end{bmatrix} \sim \begin{bmatrix} x^2 & 1 & 0 \\ 3x^2 & 1 & 2x-1 \end{bmatrix}$$

$$\sim \begin{bmatrix} x^2 & 1 & 0 \\ 0 & -2 & 2x-1 \end{bmatrix} \sim \begin{bmatrix} x^2 & 1 & 0 \\ 0 & 1 & \frac{1}{2}-x \end{bmatrix} \sim \begin{bmatrix} x^2 & 0 & x-\frac{1}{2} \\ 0 & 1 & \frac{1}{2}-x \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{1}{x}-\frac{1}{2x^2} \\ 0 & 1 & \frac{1}{2}-x \end{bmatrix}$$

$$c_{1x}' = \frac{1}{x} - \frac{1}{2x^2} \longrightarrow c_{1x} = \ln|x| + \frac{1}{2x} + c_1$$

$$c_{2x}' = \frac{1}{2} - x \longrightarrow c_{2x} = \frac{x}{2} - \frac{x^2}{2} + c_2$$

$$y = c_{1x}x^3 + c_{2x}x = \left(\ln|x| + \frac{1}{2x} + c_1\right)x^3 + \left(\frac{x}{2} - \frac{x^2}{2} + c_2\right)x$$

$$= \underbrace{c_1x^3 + c_2x}_{y_h \text{ (hom. çözüm)}} + \underbrace{x^2 + x^3\left(\ln|x| - \tfrac{1}{2}\right)}_{y_p \text{ (özel çözüm)}}$$

Ör. $x^2y'' + 4xy' + 2y = e^x$ ~~homojen~~ kısmın özel çözümü $y_1 = 1/x$ ise genel çözü

$$y_h = y_1u = \frac{u}{x} \longrightarrow y_h' = \frac{u'}{x} - \frac{u}{x^2}, \quad y_h'' = \frac{u''}{x} - \frac{2u'}{x^2} + \frac{2u}{x^3}$$

$$x^2y_h'' + 4xy_h' + 2y_h = 0$$

$$x^2\left(\frac{u''}{x} - \frac{2u'}{x^2} + \frac{2u}{x^3}\right) + 4x\left(\frac{u'}{x} - \frac{u}{x^2}\right) + 2\frac{u}{x} = 0$$

$$xu'' - 2u' + \cancel{\frac{2u}{x}} + 4u' - \cancel{\frac{4u}{x}} + \cancel{\frac{2u}{x}} = 0$$

$$u'' + \frac{2}{x}u' = 0, \quad v = u', \quad v' = u'' = \frac{dv}{dx}$$

$$\frac{dv}{dx} + \frac{2}{x}v = 0 \longrightarrow \frac{dv}{v} + 2\frac{dx}{x} = 0 \longrightarrow \ln|v| + 2\ln|x| = \ln|c|$$

$$\ln|x^2v| = \ln|c| \longrightarrow x^2v = c \longrightarrow v = \frac{c}{x^2} = \frac{du}{dx}$$

$$du = c\frac{dx}{x^2} \longrightarrow u = -\frac{c}{x} + c_2 = \frac{c_1}{x} + c_2$$

$$y_h = \frac{u}{x} = \frac{c_1}{x^2} + \frac{c_2}{x}$$

$$\left.\begin{aligned} c'_{1x}\frac{1}{x^2}+c'_{2x}\frac{1}{x}&=0\\ c'_{1x}\left(-\frac{2}{x^3}\right)+c'_{2x}\left(-\frac{1}{x^2}\right)&=e^x/x^2\end{aligned}\right\}\begin{bmatrix}\frac{1}{x^2} & \frac{1}{x} & 0\\ -\frac{2}{x^3} & -\frac{1}{x^2} & \frac{e^x}{x^2}\end{bmatrix}\sim\begin{bmatrix}1 & x & 0\\ 2 & x & -xe^x\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & x & 0\\ 1 & 0 & -xe^x\end{bmatrix}\sim\begin{bmatrix}0 & x & xe^x\\ 1 & 0 & -xe^x\end{bmatrix}\sim\begin{bmatrix}1 & 0 & -xe^x\\ 0 & 1 & e^x\end{bmatrix}$$

$$c'_{1x}=-xe^x\longrightarrow c_{1x}=(1-x)e^x+c_1$$

$$c'_{2x}=e^x\longrightarrow c_{2x}=e^x+c_2$$

$$y=\frac{c_{1x}}{x^2}+\frac{c_{2x}}{x}=\frac{(1-x)e^x+c_1}{x^2}+\frac{e^x+c_2}{x}=\underbrace{\frac{c_1}{x^2}+\frac{c_2}{x}}_{y_h}+\underbrace{\frac{e^x}{x^2}}_{y_p}$$

Ör $x^2y''-4xy'+6y=4x-6$ dif. denk. $y_h=c_1x^2+c_2x^3$ ise genel çözüm

$$\left.\begin{aligned} c'_{1x}x^2+c'_{2x}x^3&=0\\ c'_{1x}2x+c'_{2x}3x^2&=\frac{4x-6}{x^2}\end{aligned}\right\}\begin{bmatrix}x^2 & x^3 & 0\\ 2x & 3x^2 & \frac{4x-6}{x^2}\end{bmatrix}\sim\begin{bmatrix}1 & x & 0\\ 2 & 3x & \frac{4x-6}{x^3}\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & x & 0\\ 0 & x & \frac{4x-6}{x^3}\end{bmatrix}\sim\begin{bmatrix}1 & 0 & \frac{6-4x}{x^3}\\ 0 & x & \frac{4x-6}{x^3}\end{bmatrix}\sim\begin{bmatrix}1 & 0 & \frac{6-4x}{x^3}\\ 0 & 1 & \frac{4x-6}{x^4}\end{bmatrix}$$

$$c'_{1x}=\frac{6-4x}{x^3}=\frac{6}{x^3}-\frac{4}{x^2}\longrightarrow c_{1x}=\frac{4}{x}-\frac{3}{x^2}+c_1$$

$$c'_{2x}=\frac{4x-6}{x^4}=\frac{4}{x^3}-\frac{6}{x^4}\longrightarrow c_{2x}=\frac{2}{x^3}-\frac{2}{x^2}+c_2$$

$$y=c_{1x}x^2+c_{2x}x^3=\left(\frac{4}{x}-\frac{3}{x^2}+c_1\right)x^2+\left(\frac{2}{x^3}-\frac{2}{x^2}+c_2\right)x^3$$

$$=4x-3+c_1x^2+2-2x+c_2x^3$$

$$=\underbrace{c_1x^2+c_2x^3}_{y_h}+\underbrace{2x-1}_{y_p}$$

Ör $x^2y'' + 10xy' + 8y = x^2$ dif. denk. $y_h = \frac{c_1}{x} + \frac{c_2}{x^8}$ ise genel çözüm

$$\left.\begin{aligned} c_{1x}' \frac{1}{x} + c_{2x}' \frac{1}{x^8} &= 0 \\ c_{1x}' \frac{-1}{x^2} + c_{2x}' \frac{-8}{x^9} &= 1 \end{aligned}\right\} \begin{bmatrix} \frac{1}{x} & \frac{1}{x^8} & 0 \\ -\frac{1}{x^2} & -\frac{8}{x^9} & 1 \end{bmatrix} \sim \begin{bmatrix} x^7 & 1 & 0 \\ x^7 & 8 & -x^9 \end{bmatrix}$$

$$\sim \begin{bmatrix} x^7 & 1 & 0 \\ 0 & 7 & -x^9 \end{bmatrix} \sim \begin{bmatrix} x^7 & 1 & 0 \\ 0 & 1 & -x^9/7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & x^2/7 \\ 0 & 1 & -x^9/7 \end{bmatrix}$$

$$c_{1x}' = \frac{x^2}{7} \rightarrow c_{1x} = \frac{x^3}{21} + c_1 \qquad y = \frac{c_{1x}}{x} + \frac{c_{2x}}{x^8}$$

$$c_{2x}' = -\frac{x^9}{7} \rightarrow c_{2x} = -\frac{x^{10}}{70} + c_2$$

$$y = \left(\frac{x^3}{21} + c_1\right)\frac{1}{x} + \left(-\frac{x^{10}}{70} + c_2\right)\frac{1}{x^8} = \frac{c_1}{x} + \frac{c_2}{x^8} + \frac{x^2}{30}$$

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n. mertebeden sabit katsayılı lineer dif. denklemleri

$$a_0 y^{(n)} + \dots + a_{n-2} y'' + a_{n-1} y' + a_n y = b(x), \quad a_0 \neq 0$$

$b(x) = 0$ olursa dif. denk. homojendir.

Sabit Katsayılı Lineer Dif. Denklemleri

Ör $y' + 2y = 0$ Genel çözüm

$r + 2 = 0 \rightarrow r = -2 \rightarrow y = c_1 e^{-2x}$

Ör $y' - 3y = 0$ Genel çözüm

$r - 3 = 0 \rightarrow r = 3 \rightarrow y = c_1 e^{3x}$

Ör $y' = 0$ Genel çözüm

$r = 0 \rightarrow y = c_1$

Ör $y'' + 2y' - 3y = 0$ Genel çözüm

$r^2 + 2r - 3 = 0$

$(r-1)(r+3) = 0$

$r = 1, \; r = -3$

$y = c_1 e^{x} + c_2 e^{-3x}$

Ör $y''' - 2y'' - y' + 2y = 0$ Genel çözüm

$r^3 - 2r^2 - r + 2 = 0$

$(r-1)(r+1)(r-2) = 0$

$r = 1, \; r = -1, \; r = 2$

$y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x}$

Ör $y^{(4)}-2y'''+11y''-2y'+10y=0$ Genel çözüm

$r^4-2r^3+11r^2-2r+10=0$

$(r^2-2r+10)(r^2+1)=0$ $\longrightarrow r_{1,2}=\mp i$

$((r-1)^2+9)(r^2+1)=0$ $\quad r_{3,4}=1\mp 3i$

$y=c_1\cos x+c_2\sin x+e^x(c_3\cos 3x+c_4\sin 3x)$

Ör $y''-6y'+9y=0$ Genel çözüm

$r^2-6r+9=0 \longrightarrow (r-3)^2=0 \longrightarrow r_1=r_2=3$

$y=(c_1+c_2x)e^{3x}$

Ör $y^{(5)}-2y^{(4)}+y'''=0$ Genel çözüm

$r^5-2r^4+r^3=0 \longrightarrow r^3(r-1)^2=0 \longrightarrow r_1=r_2=r_3=0,\ r_4=r_5=1$

$y=c_1+c_2x+c_3x^2+(c_4+c_5x)e^x$

Ör $y^{(4)}+6y'''+12y''+8y'=0$ Genel çözüm

$r^4+6r^3+12r^2+8r=0$

$r(r^3+6r^2+12r+8)=0 \longrightarrow r(r+2)^3=0$

$r_1=0,\ r_2=r_3=r_4=-2 \longrightarrow y=c_1+(c_2+c_3x+c_4x^2)e^{-2x}$

Ör $y^{(4)}+8y''+16y=0$ Genel çözüm

$r^4+8r^2+16=0$

$(r^2+4)^2=0$ $\quad y=(c_1+c_2x)\cos 2x+(c_3+c_4x)\sin 2x$

$r_1=r_2=2i$

$r_3=r_4=-2i$

Ör $y''-4y=0$ Genel çözüm

$r^2-4=0 \rightarrow (r-2)(r+2)=0 \rightarrow y=c_1e^{2x}+c_2e^{-2x}$

$r_1=2,\ r_2=-2$

Ör $y''+y'=0$ Genel çözüm

$r^2+r=0 \rightarrow r(r+1)=0 \rightarrow y=c_1+c_2e^{-x}$

$r_1=0,\ r_2=-1$

Ör $y'''-9y'=0$ Genel çözüm

$r^3-9r=0 \rightarrow r(r^2-9)=0 \rightarrow r(r-3)(r+3)=0 \rightarrow r_1=0,\ r_2=3,\ r_3=-3$

$y=c_1+c_2e^{3x}+c_3e^{-3x}$

Ör $y^{(4)}-2y'''-y''+2y'=0$ Genel çözüm

$r^4-2r^3-r^2+2r=0$

$r(r-1)(r+1)(r-2)=0 \rightarrow r_1=0,\ r_2=1,\ r_3=-1,\ r_4=2$

$y=c_1+c_2e^{x}+c_3e^{-x}+c_4e^{2x}$

Ör $y''+9y=0$ Genel çözüm

$r^2+9=0 \rightarrow r^2=-9=9i^2 \rightarrow r_{1,2}=\mp 3i \rightarrow y=c_1\cos 3x+c_2\sin 3x$

Ör $y'''+4y'=0$ Genel çözüm

$r^3+4r=0 \rightarrow r(r^2+4)=0 \rightarrow r(r-2i)(r+2i)=0$

$r_1=0,\ r_2=2i,\ r_3=-2i \rightarrow y=c_1+c_2\cos 2x+c_3\sin 2x$

Ör $y''-6y'+13y=0$ Genel çözüm

$r^2-6r+13=0 \rightarrow (r-3)^2+4=0 \rightarrow r_{1,2}=3\mp 2i$

$y=e^{3x}\left(c_1\cos 2x+c_2\sin 2x\right)$

Ör $y'''-3y''+4y'-12y=0$ Genel çözüm

$r^3-3r^2+4r-12=0$

$(r-3)(r^2+4)=0 \rightarrow r_1=3,\ r_2=2i,\ r_3=-2i$

$y=c_1e^{3x}+c_2\cos 2x+c_3\sin 2x$

Ör $y'''-4y''+5y'=0$ Genel çözüm

$r^3-4r^2+5r=0$

$r(r^2-4r+5)=0 \rightarrow r\left((r-2)^2+1\right)=0 \rightarrow y=c_1+e^{2x}(c_2\cos x+c_3\sin x)$

$r_1=0,\ r_2=2+i,\ r_3=2-i$

$$((r-2)^2+1)^3 \cdot r^2 \cdot (r-2)^3 \cdot (r^2+9)(r^2-9)$$

$r_1=r_2=r_3=2+i$, $r_4=r_5=r_6=2-i$ $\quad$ $r_7=r_8=0$ $\quad$ $r_9=r_{10}=r_{11}=2$ $\quad$ $r_{12}=3i$, $r_{13}=-3i$ $\quad$ $r_{14}=3$, $r_{15}=-3$

$$y=(c_1+c_2x+c_3x_2)e^{2x}\cos x+(c_4+c_5x+c_6x^2)e^{2x}\sin x+c_7+c_8x$$
$$(c_9+c_{10}x+c_{11}x^2)e^{2x}+c_{12}\cos 3x+c_{13}\sin 3x+c_{14}e^{3x}+c_{15}e^{-3x}$$

$$(r^2+4)^3((r-3)^2+4)^2 \cdot (r+2) \cdot r^4 \cdot (r^2-9)^2$$

$r_1=r_2=r_3=2i$, $r_4=r_5=r_6=-2i$ $\quad$ $r_7=r_8=3+2i$, $r_9=r_{10}=3-2i$ $\quad$ $r_{11}=-2$ $\quad$ $r_{12}=r_{13}=r_{14}=r_{15}=0$ $\quad$ $r_{16}=r_{17}=3$, $r_{18}=r_{19}=-3$

$$y=(c_1+c_2x+c_3x^2)\cos 2x+(c_4+c_5x+c_6x^2)\sin 2x$$
$$+(c_7+c_8x)e^{3x}\cos 2x+(c_9+c_{10}x)e^{3x}\sin 2x+c_{11}e^{-2x}$$
$$+c_{12}+c_{13}x+c_{14}x^2+c_{15}x^3+(c_{16}+c_{17}x)e^{3x}+(c_{18}+c_{19}x)e^{-3x}$$

Sabit Katsayılı Homojen olmayan lineer dif. denklemleri

$$a_0y^{(n)}+\dots+a_{n-2}y''+a_{n-1}y'+a_ny=b(x)\neq 0$$

$y=y_h+y_p$ $\quad$ y_h: Homojen çözüm $\quad$ y_p: özel çözüm

Homojen Kısımda Katlı Kök Yoksa

$b(x)=b \rightarrow y_p=c$

$b(x)=b e^{mx} \rightarrow y_p=ce^{mx}$

$b(x)=b_1\cos\beta x+b_2\sin\beta x \rightarrow y_p=c_1\cos\beta x+c_2\sin\beta x$ (b_1, b_2 'den bir tanesi 0 olabilir)

$b(x)=e^{mx}(b_1\cos\beta x+b_2\sin\beta x) \rightarrow y_p=e^{mx}(c_1\cos\beta x+c_2\sin\beta x)$

$b(x)=b_0x^n+\dots+b_{n-1}x+b_n \rightarrow y_p=c_0x^n+\dots+c_{n-1}x+c_n$

$b(x)=(b_0x^n+\dots+b_{n-1}x+b_n)e^{mx} \rightarrow y_p=(c_0x^n+\dots+c_{n-1}x+c_n)e^{mx}$

$b(x)=b_1(x)\cos\beta x+b_2(x)\sin\beta x \rightarrow y_p=c_1(x)\cos\beta x+c_2(x)\sin\beta x$

$b(x)=(b_1(x)\cos\beta x+b_2(x)\sin\beta x)e^{mx} \rightarrow y_p=(c_1(x)\cos\beta x+c_2(x)\sin\beta x)e^{mx}$

$$\left.\begin{array}{l} b_1(x) = x^3+3x \\ b_2(x) = x^2+4 \end{array}\right\} n=3 \to c_1(x),\ c_2(x)$$ 3 mertebeden iki polinom

<u>Homojen kısımda katlı kök varsa</u>

K tane katlı kök olsun bu durumda çözüm x^k ile çarpılır.

$$y_h(x) = \underbrace{c_1 + c_2 x}_{k_1=2} + \underbrace{c_3 e^{3x}}_{k_2=1}, \quad b(x) = b_1 + b_2 e^{3x} \text{ olsun}$$

$$y_p(x) = c_4 x^2 + c_5 x e^{3x} \text{ olur.}$$

b) $y_h = (c_1+c_2x)e^{2x}\cos 3x + (c_3+c_4x)e^{2x}\sin 3x + c_5\cos 2x + c_6 \sin 2x$

$b(x) = xe^{2x}\sin 3x + x^2\cos 2x$ ise $y_p = ?$

$r_1 = r_2 = 2+3i$, $r_3 = r_4 = 2-3i$ → iki tane x^2 ile çarp

$r_5 = 2i$, $r_6 = -2i$ → 1 tane x ile çarp

$$y_p = x^2\left((b_0x+b_1)e^{2x}\cos 3x + (b_2x+b_3)e^{2x}\sin 3x\right)$$
$$+\, x\left((b_4x^2+b_5x+b_6)\cos 2x + (b_7x^2+b_8x+b_9)\sin 2x\right)$$

$y_h = c_1 + c_2x + c_3x^2 + (c_4+c_5x)\cos 2x + (c_6+c_7x)\sin 2x$

$b(x) = x+2+\cos 3x + e^x \to y_p = ?$

c) $y_h = c_1e^{2x} + c_2x^2 + c_3x + c_4 + c_5\cos 3x + c_6\sin 3x$

$b(x) = e^{2x}\cos 3x + x + \cos 3x$ ise $y_p = ?$

$r_1 = 2$ — 1 tane fakat b(x)'te yok

$r_2 = r_3 = r_4 = 0$ → 3 tane x^3 ile çarp

$r_5 = 3i$, $r_6 = -3i$ → 1 tane x ile çarp

$$y_p = e^{2x}(b_0\cos 3x + b_1\sin 3x) + x^3(b_2x+b_3) + x(b_4\cos 3x + b_5\sin 3x)$$

d) $y_h = (c_1+c_2x+c_3x^2)e^{-2x} + c_4e^x + c_5x + c_6$

$b(x) = xe^{-2x} + x^2 + e^{2x}$ ise $y_p = ?$

$r_1 = r_2 = r_3 = -2 \quad r_4 = 1 \quad r_5 = r_6 = 0$

$$y_p = x^3(b_0x+b_1)e^{-2x} + x^2(b_2x^2+b_3x+b_4) + b_5e^{2x}$$

Belirli Katsayılar Metodu

Ör $y'' - 3y' + 2y = 2x^2 + 3$ Genel çözüm

$r^2 - 3r + 2 = 0 \rightarrow (r-1)(r-2) = 0 \rightarrow r_1 = 1,\ r_2 = 2$

$y_h = c_1 e^x + c_2 e^{2x}$

$$\left.\begin{aligned} y_p &= b_0 x^2 + b_1 x + b_2 \\ y_p' &= 2b_0 x + b_1 \\ y_p'' &= 2b_0 \end{aligned}\right\} \quad \begin{aligned} & y_p'' - 3y_p' + 2y_p = 2x^2 + 3 \\ & 2b_0 - 3(2b_0 x + b_1) + 2(b_0 x^2 + b_1 x + b_2) = 2x^2 + 3 \\ & 2b_0 x^2 + (2b_1 - 6b_0)x + (2b_2 - 3b_1 + 2b_0) = 2x^2 + 3 \end{aligned}$$

$$\left.\begin{aligned} 2b_0 &= 2 \longrightarrow b_0 = 1 \\ 2b_1 - 6b_0 &= 0 \longrightarrow b_1 = 3 \\ 2b_2 - 3b_1 + 2b_0 &= 3 \longrightarrow b_2 = 5 \end{aligned}\right\} \quad \begin{aligned} & y_p = x^2 + 3x + 5 \\ & y = y_h + y_p = c_1 e^x + c_2 e^{2x} + x^2 + 3x + 5 \end{aligned}$$

Ör $y''' - y'' = x - e^x + 3\cos x + \sin x$ Genel çözüm

$r^3 - r^2 = 0 \rightarrow r^2(r-1) = 0 \rightarrow r_1 = r_2 = 0,\ r_3 = 1$

$y_h = c_1 + c_2 x + c_3 e^x$

$y_p = x^2(b_0 x + b_1) + x(b_2 e^x) + b_3 \cos x + b_4 \sin x$

$\quad = b_0 x^3 + b_1 x^2 + b_2 x e^x + b_3 \cos x + b_4 \sin x$

$y_p' = 3b_0 x^2 + 2b_1 x + b_2 e^x + b_2 x e^x - b_3 \sin x + b_4 \cos x$

$y_p'' = 6b_0 x + 2b_1 + 2b_2 e^x + b_2 x e^x - b_3 \cos x - b_4 \sin x$

$y_p''' = 6b_0 + 3b_2 e^x + b_2 x e^x + b_3 \sin x - b_4 \cos x$

$y_p''' - y_p'' = x - e^x + 3\cos x + \sin x$

$= (6b_0 - 2b_1) - 6b_0 x + b_2 e^x + (b_3 - b_4)\cos x + (b_3 + b_4)\sin x$

$-6b_0 = 1 \rightarrow b_0 = -1/6 \qquad b_2 = -1 \qquad b_3 - b_4 = 3$

$6b_0 - 2b_1 = 0 \rightarrow b_1 = -1/2 \qquad\qquad b_3 + b_4 = 1$

$\qquad\qquad b_3 = 2,\ b_4 = -1$

$$y_p = -\frac{x^3}{6} - \frac{x^2}{2} - xe^x + 2\cos x - \sin x$$

$$y = y_h + y_p = c_1 + c_2 x + c_3 e^x - \frac{x^3}{6} - \frac{x^2}{2} - xe^x + 2\cos x - \sin x$$

$$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 3x^2 + 4\sin x - 2\cos x$$

25 $y'' - 2y' + y = x + e^{x} + e^{-x}$ Genel çözüm.

$r^2 - 2r + 1 = 0 \longrightarrow (r-1)^2 = 0 \longrightarrow r_1 = r_2 = 1 \longrightarrow y_h = c_1 e^x + c_2 x e^x$

$$\left.\begin{aligned} y_p &= b_0 x + b_1 + b_2 x^2 e^x + b_3 e^{-x} \\ y_p' &= b_0 + 2b_2 x e^x + b_2 x^2 e^x - b_3 e^{-x} \\ y_p'' &= 2b_2 e^x + 4b_2 x e^x + b_2 x^2 e^x + b_3 e^{-x} \end{aligned}\right\} \; y_p'' - 2y_p' + y_p = x + e^x + e^{-x}$$

$$(2b_2 e^x + 4b_2 x e^x + b_2 x^2 e^x + b_3 e^{-x}) - 2(b_0 + 2b_2 x e^x + b_2 x^2 e^x - b_3 e^{-x})$$
$$+ (b_0 x + b_1 + b_2 x^2 e^x + b_3 e^{-x}) = x + e^x + e^{-x}$$

$$b_0 x + b_1 - 2b_0 + 2b_2 e^x + 4b_3 e^{-x} = x + e^x + e^{-x}$$

$b_0 = 1$ $\qquad 2b_2 = 1 \longrightarrow b_2 = 1/2$

$b_1 - 2b_0 = 0 \longrightarrow b_1 = 2$ $\qquad 4b_3 = 1 \longrightarrow b_3 = 1/4$

$$y = y_h + y_p = c_1 e^x + c_2 x e^x + \frac{x^2}{2} e^x + \frac{e^{-x}}{4} + x + 2$$

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26 $y'' + 4y = e^x + \sin 2x + x^2 + 7x$ Genel çözüm

$r^2 + 4 = 0 \longrightarrow r_1 = 2i,\ r_2 = -2i \longrightarrow y_h = c_1 \cos 2x + c_2 \sin 2x$

$$y_p = b_0 e^x + x(b_1 \cos 2x + b_2 \sin 2x) + b_3 x^2 + b_4 x + b_5$$
$$y_p' = b_0 e^x + b_1 \cos 2x + b_2 \sin 2x + x(-2b_1 \sin 2x + 2b_2 \cos 2x) + 2b_3 x + b_4$$
$$y_p'' = b_0 e^x - 4b_1 \sin 2x + 4b_2 \cos 2x + x(-4b_1 \cos 2x - 4b_2 \sin 2x) + 2b_3$$

$$y_p'' + 4y_p = e^x + \sin 2x + x^2 + 7x$$
$$= 5b_0 e^x - 4b_1 \sin 2x + 4b_2 \cos 2x + 4b_3 x^2 + 4b_4 x + 2b_3 + 4b_5$$

$5b_0 = 1 \longrightarrow b_0 = 1/5 \qquad -4b_1 = 1 \longrightarrow b_1 = -1/4 \qquad 4b_2 = 0 \longrightarrow b_2 = 0$

$4b_3 = 1 \longrightarrow b_3 = 1/4 \qquad 4b_4 = 7 \longrightarrow b_4 = 7/4 \qquad 2b_3 + 4b_5 = 0 \longrightarrow b_5 = -1/8$

$$y_p = \frac{e^x}{5} + x\left(-\frac{1}{4}\cos 2x\right) + \frac{1}{4}x^2 + \frac{7x}{4} - 1/8$$

$$y = y_h + y_p = c_1 \cos 2x + c_2 \sin 2x + \frac{e^x}{5} - \frac{x}{4}\cos 2x + \frac{x^2}{4} + \frac{7x}{4} - 1/8$$

Ör $y''-y=e^x\sin x$ dif. denk. veriliyor

a) Parametre Değişim Yöntemiyle çöz.

b) Belirsiz Katsayılar Yöntemiyle çöz.

a) $r^2-1=0 \rightarrow (r-1)(r+1)=0 \rightarrow r_1=1,\ r_2=-1 \rightarrow y_h=c_1e^x+c_2e^{-x}$

$$\left.\begin{aligned} c_{1x}'e^x+c_{2x}'e^{-x}&=0\\ c_{1x}'e^x-c_{2x}'e^{-x}&=e^x\sin x \end{aligned}\right\}\ \begin{bmatrix} e^x & e^{-x}\\ e^x & -e^{-x}\end{bmatrix}\begin{bmatrix} c_{1x}'\\ c_{2x}'\end{bmatrix}=\begin{bmatrix}0\\ e^x\sin x\end{bmatrix}$$

$$\begin{bmatrix} e^x & e^{-x} & 0\\ e^x & -e^{-x} & e^x\sin x\end{bmatrix}\sim\begin{bmatrix} e^x & e^{-x} & 0\\ 2e^x & 0 & e^x\sin x\end{bmatrix}\sim\begin{bmatrix} 1 & e^{-2x} & 0\\ 2 & 0 & \sin x\end{bmatrix}$$

$$\sim\begin{bmatrix} 0 & e^{-2x} & -\frac{1}{2}\sin x\\ 1 & 0 & \frac{1}{2}\sin x\end{bmatrix}\sim\begin{bmatrix} 1 & 0 & \frac{1}{2}\sin x\\ 0 & 1 & -\frac{1}{2}e^{2x}\sin x\end{bmatrix}$$

$$c_{1x}'=\frac{1}{2}\sin x \longrightarrow c_{1x}=c_1-\frac{1}{2}\cos x$$

$$c_{2x}'=-\frac{1}{2}e^{2x}\sin x \longrightarrow c_{2x}=c_2+\frac{e^{2x}}{10}(\cos x-2\sin x)$$

$$y=c_{1x}e^x+c_{2x}e^{-x}=c_1e^x+c_2e^{-x}-\frac{1}{5}e^x(2\cos x+\sin x)$$

b) $r^2-1=0 \rightarrow (r-1)(r+1)=0 \rightarrow r_1=1,\ r_2=-1 \rightarrow y_h=c_1e^x+c_2e^{-x}$

$$\left.\begin{aligned} y_p&=e^x(b_1\cos x+b_2\sin x)\\ y_p'&=e^x((b_1+b_2)\cos x+(b_2-b_1)\sin x)\\ y_p''&=e^x(2b_2\cos x-2b_1\sin x) \end{aligned}\right\}\ y_p''-y_p=e^x\sin x$$

$$y_p''-y_p=e^x(2b_2\cos x-2b_1\sin x)-e^x(b_1\cos x+b_2\sin x)$$

$$=e^x((2b_2-b_1)\cos x-(2b_1+b_2)\sin x)=e^x\sin x$$

$$\left.\begin{aligned} 2b_2-b_1&=0\\ b_2+2b_1&=-1 \end{aligned}\right\}\ \begin{bmatrix} -1 & 2 & 0\\ 2 & 1 & -1\end{bmatrix}\sim\begin{bmatrix} 1 & -2 & 0\\ 0 & 5 & -1\end{bmatrix}\sim\begin{bmatrix} 1 & 0 & -2/5\\ 0 & 1 & -1/5\end{bmatrix}\begin{matrix}\rightarrow b_1\\ \rightarrow b_2\end{matrix}$$

$$y_p=e^x\left(-\tfrac{2}{5}\cos x-\tfrac{1}{5}\sin x\right)=-\frac{e^x}{5}(2\cos x+\sin x)$$

$$y=y_h+y_p=c_1e^x+c_2e^{-x}-\frac{e^x}{5}(2\cos x+\sin x)$$

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=2e^x-10\sin x \qquad y(0)=2 \quad y'(0)=4$$

Ör $y'' + y = \sec x$ Parametre Kestirim Yön. Genel çözüm

$r^2+1=0 \to r_1=i,\ r_2=-i \to y_h = c_1\cos x + c_2\sin x$

$$\left.\begin{aligned} c_{1x}'\cos x + c_{2x}'\sin x &= 0 \\ -c_{1x}'\sin x + c_{2x}'\cos x &= \sec x \end{aligned}\right\} \begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}\begin{bmatrix}c_{1x}'\\ c_{2x}'\end{bmatrix}=\begin{bmatrix}0\\ \sec x\end{bmatrix}$$

$$\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & \sec x\end{bmatrix} \sim \begin{bmatrix}\cos x\sin x & \sin^2 x & 0\\ -\cos x\sin x & \cos^2 x & 1\end{bmatrix} \sim \begin{bmatrix}\cos x\sin x & \sin^2 x & 0\\ 0 & 1 & 1\end{bmatrix}$$

$$\sim \begin{bmatrix}1 & \tan x & 0\\ 0 & 1 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 0 & -\tan x\\ 0 & 1 & 1\end{bmatrix}$$

$c_{1x}' = -\tan x \to c_{1x} = c_1 + \ln|\cos x|$

$c_{2x}' = 1 \to c_{2x} = c_2 + x$

$y = c_{1x}\cos x + c_{2x}\sin x$
$= c_1\cos x + c_2\sin x$
$+ \cos x.\ln|\cos x|$
$+ x\sin x$

Ör $y'' + 4y = 4\tan 2x$ P.K.Y. ile çöz.

$r^2+4=0 \to r_1=2i,\ r_2=-2i \to y_h = c_1\cos 2x + c_2\sin 2x$

$$\left.\begin{aligned} c_{1x}'\cos 2x + c_{2x}'\sin 2x &= 0 \\ -2c_{1x}'\sin 2x + 2c_{2x}'\cos 2x &= 4\tan 2x \end{aligned}\right\} \begin{bmatrix}\cos 2x & \sin 2x\\ -\sin 2x & \cos 2x\end{bmatrix}\begin{bmatrix}c_{1x}'\\ c_{2x}'\end{bmatrix}=\begin{bmatrix}0\\ 2\tan 2x\end{bmatrix}$$

$$\begin{bmatrix}\cos 2x & \sin 2x & 0\\ -\sin 2x & \cos 2x & 2\tan 2x\end{bmatrix} \sim \begin{bmatrix}\cos 2x\sin 2x & \sin^2 2x & 0\\ -\cos 2x\sin 2x & \cos^2 2x & 2\sin 2x\end{bmatrix}$$

$$\sim \begin{bmatrix}\cos 2x\sin 2x & \sin^2 2x & 0\\ 0 & 1 & 2\sin 2x\end{bmatrix} \sim \begin{bmatrix}\cos 2x\sin 2x & 0 & -2\sin^3 2x\\ 0 & 1 & 2\sin 2x\end{bmatrix}$$

$$\sim \begin{bmatrix}1 & 0 & -\dfrac{2\sin^2 2x}{\cos 2x}\\ 0 & 1 & 2\sin 2x\end{bmatrix}$$

$c_{1x}' = -\dfrac{2\sin^2 2x}{\cos 2x} \to c_{1x} = c_1 + \sin 2x - \ln|\sec 2x + \tan 2x|$

$c_{2x}' = 2\sin 2x \to c_{2x} = c_2 - \cos 2x$

$y = c_{1x}\cos 2x + c_{2x}\sin 2x$
$= c_1\cos 2x + c_2\sin 2x + \cancel{\cos 2x\sin 2x} - \cos 2x.\ln|\sec 2x + \tan 2x| - \cancel{\cos 2x\sin 2x}$
$= c_1\cos 2x + c_2\sin 2x - \cos 2x.\ln|\sec 2x + \tan 2x|$

Cauchy-Euler Dif. Denklemi

$$a_0 x^n y^{(n)} + \cdots + a_{n-2} x^2 y'' + a_{n-1} x y' + a_n y = b(x)$$

$$x = e^t \rightarrow \frac{dx}{dt} = e^t = x \rightarrow \frac{dt}{dx} = e^{-t} = \frac{1}{x} \qquad D = \frac{d}{dt}$$

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = e^{-t}\frac{dy}{dt} = \frac{1}{x}\frac{dy}{dt} \qquad xy' \longrightarrow Dy$$

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) \qquad x^2y'' \longrightarrow D(D-1)y$$

$$y''' = \frac{d^3y}{dx^3} = \frac{d}{dx}(y'') = \frac{1}{x^3}\left(\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right) \qquad x^3y''' \longrightarrow D(D-1)(D-2)y$$

x yerine $ax+b$ olsa idi

$$ax+b = e^t \longrightarrow \frac{dx}{dt} = \frac{1}{a}e^t = \frac{ax+b}{a} \longrightarrow \frac{dt}{dx} = ae^{-t} = \frac{a}{ax+b}$$

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{a}{ax+b}\frac{dy}{dt} \qquad (ax+b)y' \longrightarrow aDy$$

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{a^2}{(ax+b)^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) \qquad (ax+b)^2y'' \longrightarrow a^2D(D-1)y$$

$$y''' = \frac{d^3y}{dx^3} = \frac{d}{dx}(y'') = \frac{a^3}{(ax+b)^3}\left(\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right) \qquad (ax+b)^3y''' \longrightarrow a^3D(D-1)(D-2)y$$

Ör $x^3y''' + 3x^2y'' = \ln x + x$ Genel çözüm

$x = e^t$ ise $\ln x = t$, $\frac{1}{x} = e^{-t}$ olur.

$$D(D-1)(D-2)y + 3D(D-1)y = t + e^t$$

$$(D^3 - D)y = t + e^t \longrightarrow \frac{d^3y}{dt^3} - \frac{dy}{dt} = t + e^t$$

$r^3 - r = 0$

$r(r^2-1) = 0$

$r(r-1)(r+1) = 0$

$r_1 = 0,\ r_2 = 1,\ r_3 = -1$

$y_h = c_1 + c_2e^t + c_3e^{-t}$

$$y_p = t(at+b) + t(ce^t) = at^2 + bt + cte^t$$

$$\frac{dy_p}{dt} = 2at + b + ce^t + cte^t \qquad \frac{d^2y_p}{dt^2} = 2a + 2ce^t + cte^t$$

$$\frac{d^3y_p}{dt^3} = 3ce^t + cte^t$$

$$\frac{d^3y_p}{dt^3} - \frac{dy_p}{dt} = t + e^t = 2ce^t - 2at - b$$

$$a = -\tfrac{1}{2}\ ,\ b = 0\ ,\ c = \tfrac{1}{2}$$

$$y_p = -\frac{t^2}{2} + \frac{t}{2}e^t$$

$$y = y_h + y_p = c_1 + c_2 e^t + c_3 e^{-t} - \frac{t^2}{2} + \frac{t}{2}e^t$$

$$= c_1 + c_2 x + \frac{c_3}{x} + \frac{\ln x}{2}(x - \ln x)$$

$y'' + 4y = 4\sin^2 x$

$y'' + 4y = 4\cos^2 x$

$y^{(4)} + 4y = 0$

$y^{(4)} - y = 0$

$xyy'' = 4x(y')^2 - 4yy'$

Ör $x^2y'' + 4xy' + 2y = x^2$ Genel çözüm

$x = e^t \rightarrow xy' = Dy\ ,\ x^2y'' = D(D-1)y$

$D(D-1)y + 4Dy + 2y = e^{2t}$

$(D^2 + 3D + 2)y = e^{2t}$

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{2x}$$

$r^2 + 3r + 2 = 0$

$(r+1)(r+2) = 0 \rightarrow r_1 = -1,\ r_2 = -2$

$y_h = c_1 e^{-t} + c_2 e^{-2t}$

$y_p = ae^{2t}$ Katlı kök yok

$$\cancel{\frac{dy_p}{dt}} = 2ae^{2t} \qquad \cancel{\frac{d^2y_p}{dt^2}} = 4ae^{2t}$$

$$\frac{d^2y_p}{dt^2} + 3\frac{dy_p}{dt} + 2y_p = e^{2x} = 12ae^{2t} \rightarrow \cancel{a = 1/12}$$

$$y = y_h + y_p = c_1e^{-t} + c_2e^{-2t} + \frac{1}{12}e^{2t} = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{x^2}{12}$$

$y''' - y' = 2x + 1 - 4\cos x + 2e^x$ dif. denk. belirsiz katsayılar yöntemiyle çöz.

$y(0) = 2$ $y'(0) = 1$ $y''(0) = 3$

Ör $(x-1)^2y'' - 4(x-1)y' + 6y = x$ Genel çözüm

$D(D-1)y - 4Dy + 6y = 1 + e^t\ ,\quad x - 1 = e^t$

$(D^2 - 5D + 6)y = 1 + e^t$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 1 + e^t$$

$r^2 - 5r + 6 = 0 \rightarrow (r-2)(r-3) = 0$

$r_1 = 2,\ r_2 = 3$

$y_h = c_1e^{2t} + c_2e^{3t}$

$y_p = a + be^t$ Katlı kök yok.

$$\frac{d^2y_p}{dt^2} - 5\frac{dy_p}{dt} + 6y_p = 1 + e^t = 6a + 2be^t$$

$$\frac{dy_p}{dt} = be^t \qquad \frac{d^2y_p}{dt^2} = be^t$$

$a = 1/6 \quad b = 1/2$

$$y = y_h + y_p = c_1e^{2t} + c_2e^{3t} + \frac{1}{6} + \frac{1}{2}e^t$$

$$= c_1(x-1)^2 + c_2(x-1)^3 + \frac{x-1}{2} + \frac{1}{6}$$

Ör. $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 4y = 4x^2 - 6x^3$, $y(2)=4$, $y'(2)=-1$ (+5)

dif. denk. genel çözümü

$x = e^t \rightarrow \frac{1}{x} = e^{-t}$, $xy' = Dy$, $x^2y'' = D(D-1)y$

$D(D-1)y - 4Dy + 4y = 4e^{2t} - 6e^{3t}$

$(D^2 - 5D + 4)y = 4e^{2t} - 6e^{3t}$ $\qquad r^2 - 5r + 4 = 0$, $(r-1)(r-4) = 0$, $r_1 = 1$, $r_2 = 4$

$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 4e^{2t} - 6e^{3t}$ $\qquad y_h = c_1e^t + c_2e^{4t}$

$y_p = ae^{2t} + be^{3t}$ Katlı kök yok

$\frac{dy_p}{dt} = 2ae^{2t} + 3be^{3t}$, $\frac{d^2y_p}{dt^2} = 4ae^{2t} + 9be^{3t}$

$\frac{d^2y_p}{dt^2} - 5\frac{dy_p}{dt} + 4y_p = 4e^{2t} - 6e^{3t} = -2ae^{2t} - 2be^{3t}$ $\qquad a = -2$, $b = 3$

$y_p = -2e^{2t} + 3e^{3t}$ $\qquad y = y_h + y_p = c_1e^t + c_2e^{4t} - 2e^{2t} + 3e^{3t}$

$= c_1x + c_2x^4 - 2x^2 + 3x^3$

$y(2) = 4 \longrightarrow 2c_1 + 16c_2 - 8 + 24 = 4 \longrightarrow c_1 + 8c_2 = -6$

$y' = c_1 + 4c_2x^3 - 4x + 9x^2$

$y'(2) = -1 \longrightarrow c_1 + 32c_2 - 8 + 36 = -1 \longrightarrow c_1 + 32c_2 = -29$

$c_1 = 5/3 \quad c_2 = -23/24$

$y = \frac{5x}{3} - \frac{23x^4}{24} - 2x^2 + 3x^3$

$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$ Genel çözüm

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$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 4\ln x$ Genel çözüm

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$(3x+2)^2y'' + 3(3x+2)y' - 36y = 3x^2 + 4x + 1$ Genel çözüm

$(3x+2)y' = 3Dy \qquad (3x+2)^2y'' = 9D(D-1)y \qquad 3x+2 = e^t$

$9D(D-1)y + 9Dy' - 36y = 3\left(\frac{e^t-2}{3}\right)^2 + 4\left(\frac{e^t-2}{3}\right) + 1$

bu şekilde çöz.

Dönüşüm Kullanarak Çözüm

Ör $(1-x^2)y''-xy'+4y=2x^2-1$ dif-denk. $x=\cos t$ dönüşümünü kullanarak çöz.

$$x=\cos t \longrightarrow \frac{dx}{dt}=-\sin t \longrightarrow \frac{dt}{dx}=-\frac{1}{\sin t}$$

$$y'=\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=-\frac{1}{\sin t}\frac{dy}{dt}$$

$$y''=\frac{d^2y}{dx^2}=\frac{d}{dx}(y')=\frac{d}{dt}(y')\cdot\frac{dt}{dx}=\frac{1}{\sin^2 t}\left(\frac{d^2y}{dt^2}-\frac{\cos t}{\sin t}\frac{dy}{dt}\right)$$

$$(1-\cos^2 t)\frac{1}{\sin^2 t}\left(\frac{d^2y}{dt^2}-\frac{\cos t}{\sin t}\frac{dy}{dt}\right)+\frac{\cos t}{\sin t}\frac{dy}{dt}+4y=2\cos^2 t-1$$

$$\frac{d^2y}{dt^2}+4y=\cos 2t \qquad r^2+4=0 \longrightarrow r_1=2i,\ r_2=-2i$$

$$y_h=c_1\cos 2t+c_2\sin 2t$$

$y_p=t(a\cos 2t+b\sin 2t)$ Katlı kök var

$$\frac{dy_p}{dt}=(a\cos 2t+b\sin 2t)+t(-2a\sin 2t+2b\cos 2t)$$

$$\frac{d^2y_p}{dt^2}=(-4a\sin 2t+4b\cos 2t)+t(-4a\cos 2t-4b\sin 2t)$$

$$\frac{d^2y_p}{dt^2}+4y_p=\cos 2t=-4a\sin 2t+4b\cos 2t \qquad a=0,\ b=1/4$$

$$y_p=\frac{t}{4}\sin 2t$$

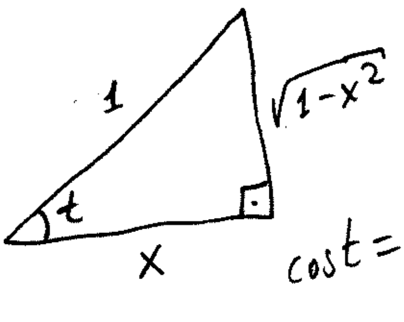
$$y=y_h+y_p=c_1\cos 2t+c_2\sin 2t+\frac{t}{4}\sin 2t$$

$$=c_1(2x-1)+c_2(2x\sqrt{1-x^2})+\frac{1}{4}(\arccos x).(2x\sqrt{1-x^2})$$

$$=c_1(2x-1)+2c_2x\sqrt{1-x^2}+\frac{x}{2}\sqrt{1-x^2}.\arccos x$$

$$\cos 2t=2\cos^2 t-1=2x-1$$

$$\sin 2t=2\cos t\sin t=2x\sqrt{1-x^2}$$



$\cos t=\sqrt{1-x^2}$