Fourier Seri Açılımı

Bütün peryodik fonksiyonlar sinüs veya kosinüs terimlerinin toplamı seklinde ifade edilebilir.

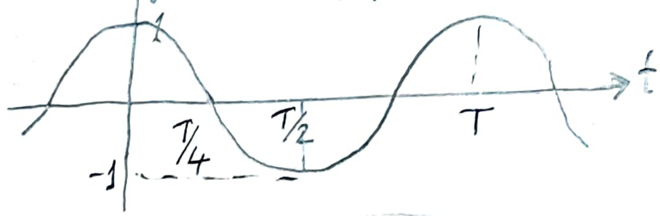
$f(t)$ peryodik bir fonksiyon ise $f(t) = f(t + nT)$, $n \in \mathbb{Z}$

T : peryod (sn)

f : frekans (Hz)

$f = \frac{1}{T}$

$f(t) = \cos\left(\frac{2\pi}{T}t\right)$



$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n w t) + b_n \sin(n w t)\right)$$

$$= \alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos(n w t + \theta_n)$$

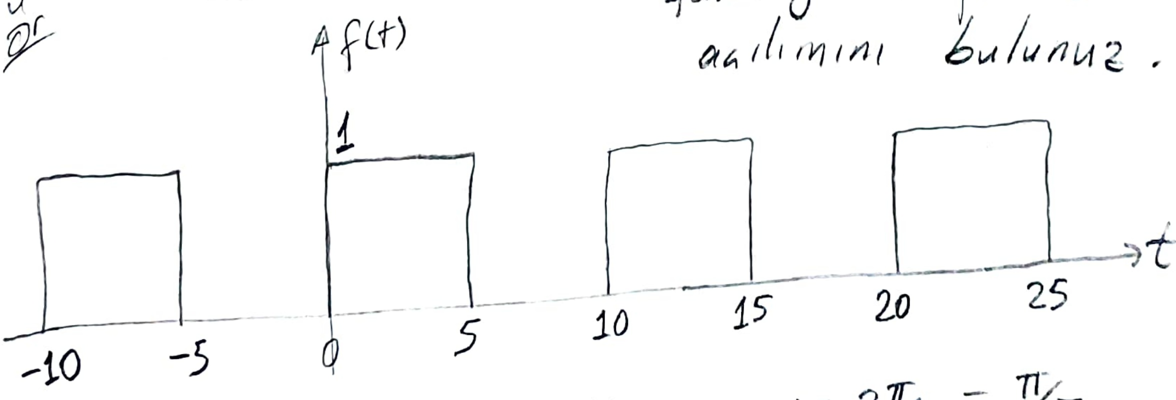
a_n, b_n veya α_n, θ_n değişkenleri fourier katsayılarıdır.

$\alpha_0 = a_0$, $\alpha_n = \sqrt{a_n^2 + b_n^2}$, $\theta_n = -\arctan\left(\frac{b_n}{a_n}\right)$, $w = 2\pi f = \frac{2\pi}{T}$

$$a_0 = \frac{1}{T}\int_T f(t)\,dt, \quad a_n = \frac{2}{T}\int_T f(t)\cos(n w t)\,dt$$

$$b_n = \frac{2}{T}\int_T f(t)\sin(n w t)\,dt$$

Ör: aşağıda verilen peryodik fonksiyonun fourier seri açılımını bulunuz.



$T = 10$ sn $\quad w = 2\pi/T = \pi/5$

$$a_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{10}\int_0^5 dt = \frac{1}{2}$$

$$a_n = \frac{2}{T}\int_T f(t)\cos(nwt)\,dt = \frac{2}{10}\int_0^5 \cos\left(\frac{n\pi}{5}t\right)dt$$

$$= \frac{1}{5}\,\frac{\sin\left(\frac{n\pi}{5}t\right)}{\frac{n\pi}{5}}\Bigg|_0^5 = \frac{1}{n\pi}\left(\sin(n\pi) - \sin(0)\right) = 0$$

$$b_n = \frac{2}{T}\int_T f(t)\sin(nwt)\,dt = \frac{2}{10}\int_0^5 \sin\left(\frac{n\pi}{5}t\right)dt$$

$$= \frac{1}{5}\,\frac{\cos\left(\frac{n\pi}{5}t\right)}{-\frac{n\pi}{5}}\Bigg|_0^5 = -\frac{1}{n\pi}\left(\cos(n\pi) - \cos(0)\right)$$

$$= \frac{1}{n\pi}\left(1 - \cos(n\pi)\right) = \begin{cases} 0 & , n \text{ çift ise} \\ \frac{2}{n\pi} & , n \text{ tek ise} \end{cases}$$

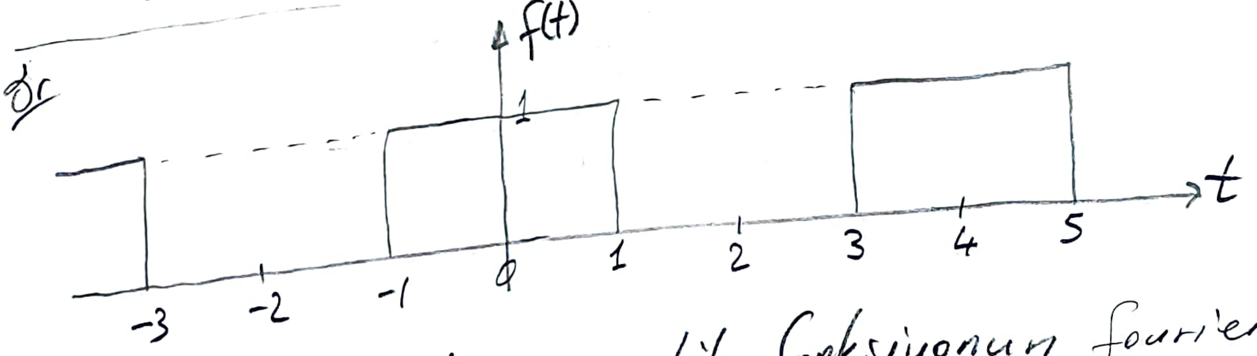
$$a_n = \left\{\tfrac{1}{2}, 0, 0, 0, \ldots\right\}, \quad b_n = \left\{0, \frac{2}{\pi}, 0, \frac{2}{3\pi}, 0, \frac{2}{5\pi}, \ldots\right\}$$

ilk 6 terim istenseydi

$$a_0 = \tfrac{1}{2}, \quad a_1 = 0, \quad a_2 = 0, \quad a_3 = 0, \quad a_4 = 0, \quad a_5 = 0$$

$$b_0 = 0, \quad b_1 = \tfrac{2}{\pi}, \quad b_2 = 0, \quad b_3 = \frac{2}{3\pi}, \quad b_4 = 0, \quad b_5 = \tfrac{2}{5\pi}$$

Ör



Yukarıda verilen peryodik fonksiyonun fourier seri açılımını bulunuz.

$T = 4$

$$w = \frac{2\pi}{T} = \tfrac{\pi}{2}$$

$$a_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{4}\int_{-1}^{+1} dt = \tfrac{1}{2}$$

$$a_n = \frac{2}{T}\int_T f(t)\cos(nwt)\,dt = \frac{2}{4}\int_{-1}^{1}\cos\left(\frac{n\pi}{2}t\right)dt$$

$$= \frac{1}{2}\,\frac{\sin\left(\frac{n\pi}{2}t\right)}{\frac{n\pi}{2}}\Bigg|_{-1}^{+1} = \frac{1}{n\pi}\left(\sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right)\right)$$

$$= \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0, & n \text{ çift ise} \\ 2/n\pi, & n = 1, 5, 9, 13, \ldots \\ -2/n\pi, & n = 3, 7, 11, 15, \ldots \end{cases}$$

$$b_n = \frac{2}{T}\int_T f(t)\sin(nwt)\,dt = \frac{2}{4}\int_{-1}^{+1}\sin\left(\frac{n\pi}{2}t\right)dt$$

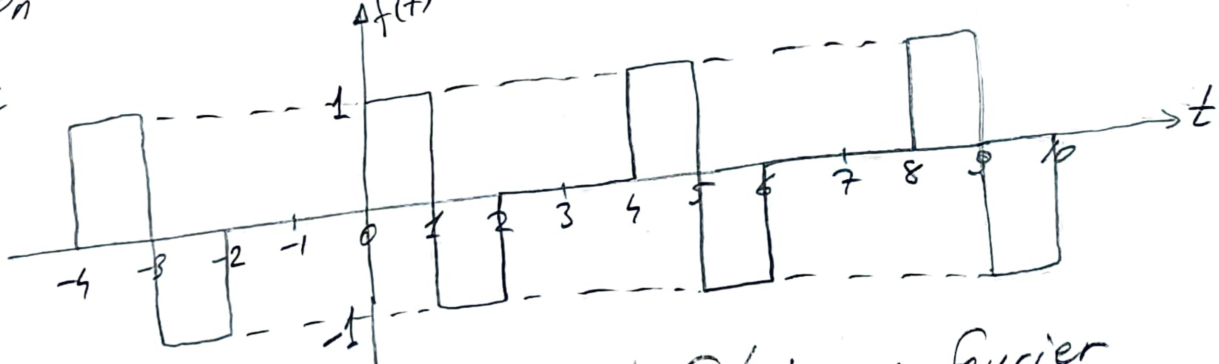
$$= \frac{1}{2}\,\frac{\cos\left(\frac{n\pi}{2}t\right)}{-\frac{n\pi}{2}}\Bigg|_{-1}^{+1} = -\frac{1}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(-\frac{n\pi}{2}\right)\right)$$

$$= -\frac{1}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{2}\right)\right) = 0$$

$$a_n = \left\{\frac{1}{2}, \frac{2}{\pi}, 0, -\frac{2}{3\pi}, 0, \frac{2}{5\pi}, 0, -\frac{2}{7\pi}, 0, \frac{2}{8\pi}, \ldots\right\}$$

$$b_n = 0, \quad n = 1, 2, 3, \ldots$$

Ör



Yukarıda verilen peryodik fonksiyonun fourier seri açılımını bulunuz.

$T = 4$

$w = \frac{2\pi}{T} = \pi/2$

$$a_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{4}\left(\int_0^1 dt - \int_1^2 dt\right)$$

$$= \frac{1}{4}(1-1) = 0$$

$$a_n = \frac{2}{T}\int_T f(t)\cos(n\omega t)\,dt = \frac{2}{4}\left(\int_0^1 \cos\left(\frac{n\pi}{2}t\right)dt - \int_1^2 \cos\left(\frac{n\pi}{2}t\right)dt\right)$$ 10b

$$= \frac{1}{2}\left(\frac{\sin\left(\frac{n\pi}{2}t\right)}{n\pi/2}\Bigg|_0^1 - \frac{\sin\left(\frac{n\pi}{2}t\right)}{n\pi/2}\Bigg|_1^2\right)$$

$$= \frac{1}{n\pi}\left(\sin\left(\frac{n\pi}{2}\right) - \underbrace{\sin(0)}_{0} - \underbrace{\sin(n\pi)}_{0} + \sin\left(\frac{n\pi}{2}\right)\right)$$

$$= \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0, & n \text{ çift ise} \\ 2/n\pi, & n = 1, 5, 9, 13, \dots \\ -2/n\pi, & n = 3, 7, 11, 15, \dots \end{cases}$$

$$b_n = \frac{2}{T}\int_T f(t)\sin(n\omega t)\,dt = \frac{2}{4}\left(\int_0^1 \sin\left(\frac{n\pi}{2}t\right)dt - \int_1^2 \sin\left(\frac{n\pi}{2}t\right)dt\right)$$

$$= \frac{1}{2}\left(\frac{\cos\left(\frac{n\pi}{2}t\right)}{-n\pi/2}\Bigg|_0^1 - \frac{\cos\left(\frac{n\pi}{2}t\right)}{-n\pi/2}\Bigg|_1^2\right)$$

$$= -\frac{1}{n\pi}\left(\underbrace{\cos\left(\frac{n\pi}{2}\right)}_{0} - \cos(0) - \cos(n\pi) + \underbrace{\cos\left(\frac{n\pi}{2}\right)}_{0}\right)$$

$$= \frac{1}{n\pi}\left(1 + \cos(n\pi)\right) = \begin{cases} 0, & n \text{ tek ise} \\ 2/n\pi, & n \text{ çift ise} \end{cases}$$

$$a_n = \left\{0, \frac{2}{\pi}, 0, -\frac{2}{3\pi}, 0, \frac{2}{5\pi}, 0, -\frac{2}{7\pi}, 0, \frac{2}{9\pi}, 0, \dots\right\}$$

$$b_n = \left\{0, 0, \frac{1}{\pi}, 0, \frac{1}{2\pi}, 0, \frac{1}{3\pi}, 0, \frac{1}{4\pi}, 0, \frac{1}{5\pi}, 0, \dots\right\}$$

S: $f(t) = 5 + \cos\left(\frac{\pi}{9}t\right) + \sin\left(\frac{\pi}{6}t + \pi/3\right)$ şeklinde (107)

verilen fonksiyonun fourier seri açılımını bulunuz.

$$f(t) = 5 + \cos\left(\frac{\pi}{9}t\right) + \sin\left(\frac{\pi}{6}t + \pi/3\right)$$

$$= 5 + \cos\left(\frac{\pi}{9}t\right) + \sin\left(\frac{\pi}{6}t\right)\cos(\pi/3) + \cos\left(\frac{\pi}{6}t\right)\sin(\pi/3)$$

$$= 5 + \cos\left(\frac{\pi}{9}t\right) + 0.5 \times \sin\left(\frac{\pi}{6}t\right) + \frac{\sqrt{3}}{2} \times \cos\left(\frac{\pi}{6}t\right)$$

$$= a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(nwt) + b_n \sin(nwt)\right)$$

$$\left.\begin{aligned} w_1 &= \frac{2\pi}{T_1} = \frac{\pi}{9} \rightarrow T_1 = 18 \\ w_2 &= \frac{2\pi}{T_2} = \frac{\pi}{6} \rightarrow T_2 = 12 \end{aligned}\right\}$$ en küçük ortak katları bulunmalı.

$$\begin{array}{r|l} 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$$

$$T = 2 \times 2 \times 3 \times 3 = 36$$

$$w = \frac{2\pi}{T} = \frac{2\pi}{36} = \pi/18$$

$$f(t) = 5 + \cos(2wt) + 0.5\cos(3wt) + \frac{\sqrt{3}}{2}\sin(3wt)$$

$a_0 = 5$, $a_2 = 1$, $a_3 = 0.5$, $b_3 = \sqrt{3}/2$

$$a_n = \begin{cases} 5 & n=0 \\ 0 & n=1 \\ 1 & n=2 \\ 0.5 & n=3 \\ 0 & n \geq 4 \end{cases} \qquad b_n = \begin{cases} \sqrt{3}/2 & n=3 \\ 0 & n \neq 3 \end{cases}$$

Matris türevleri

$$\frac{d}{dt}A(t) = \dot{A}(t) = \left[\frac{d}{dt}a_{ij}(t)\right]$$

$$\frac{d}{dt}\left(A(t).B(t)\right) = \dot{A}(t).B(t) + A(t).\dot{B}(t)$$

Ör $\frac{d}{dt}A^{-1}(t) = ?$

$$A(t)A^{-1}(t) = I \longrightarrow \frac{d}{dt}(A.A^{-1}) = \frac{d}{dt}(I) = 0$$

$$\frac{dA}{dt}.A^{-1} + A\frac{d}{dt}A^{-1} = 0$$

$$A\frac{d}{dt}A^{-1} = -\dot{A}A^{-1} \Longrightarrow \frac{dA^{-1}}{dt} = -A^{-1}.\dot{A}.A^{-1}$$

Ör $\frac{d}{dt}A^3 = ?$

$$\frac{d}{dt}A^2 = \frac{d}{dt}(A.A) = \dot{A}A + A\dot{A}$$

$$\frac{d}{dt}A^3 = \frac{d}{dt}(A.A^2) = \dot{A}A^2 + A\frac{d}{dt}A^2$$

$$= \dot{A}A^2 + A\left(\dot{A}A + A\dot{A}\right) = \dot{A}A^2 + A\dot{A}A + A^2\dot{A}$$

matris fonksiyonlar

$AB \neq BA$ (Eşit olmak zorunda değil)

$A^0 = I$

fakat $A.f(A) = f(A).A$ (eşit olur)

$$f(A) = e^{At} = \sum_{n=0}^{\infty}\frac{(At)^n}{n!} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \cdots$$

$$\frac{d}{dt}e^{At} = Ae^{At} = e^{At}.A$$

$$\int_0^t e^{At}dt = A^{-1}\left(e^{At} - I\right)$$

A'nın tersi varsa

Cayley–Hamilton Teoremi

$$f(A) = a_n A^n + a_{n-1} A^{n-1} + \cdots + a_2 A^2 + a_1 A + a_0 I$$

$$f(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0$$

$$f(s) = d(s).q(s) + r(s)$$

$$d(s) = \det(sI - A) = 0 \Rightarrow \begin{array}{l} f(s) = r(s) \\ f(A) = r(A) \end{array}$$

$d(s) = 0 \Rightarrow s_1, s_2, \ldots, s_m$ deperleri bulunur.

$$r(s) = \alpha_0 + \alpha_1 s + \alpha_2 s^2 + \cdots + \alpha_{m-1} s^{m-1}$$

$$\underbrace{\begin{bmatrix} 1 & s_1 & s_1^2 & \cdots & s_1^{m-1} \\ 1 & s_2 & s_2^2 & \cdots & s_2^{m-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & s_m & s_m^2 & \cdots & s_m^{m-1} \end{bmatrix}}_{\text{Vandermonde matrisi}} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{m-1} \end{bmatrix} = \begin{bmatrix} f(s_1) \\ f(s_2) \\ \vdots \\ f(s_m) \end{bmatrix}$$

ör $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, $A^5 = ?$

$$sI - A = \begin{bmatrix} s & -1 \\ 2 & s-3 \end{bmatrix}$$

$$\det(sI - A) = \begin{vmatrix} s & -1 \\ 2 & s-3 \end{vmatrix} = 0$$

$$d(s) = s^2 - 3s + 2 = 0$$

$$f(s) = s^5$$

$$\begin{array}{l|l} s^5 & s^2 - 3s + 2 \rightarrow d(s) \\ \hline \vdots & s^3 + 3s^2 + 7s + 15 \rightarrow q(s) \\ \hline 31s - 30 \rightarrow r(s) & \end{array}$$

$$f(s) = \underbrace{d(s)}_{0} q(s) + r(s) = r(s)$$

$$A^5 = f(A) = 31A - 30I$$

$$= 31 \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} - 30 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 31 \\ -62 & +93 \end{bmatrix} + \begin{bmatrix} -30 & 0 \\ 0 & -30 \end{bmatrix} = \begin{bmatrix} -30 & 31 \\ -62 & 63 \end{bmatrix}$$

Ör $A=\begin{bmatrix}-2 & -6\\ 1 & 3\end{bmatrix}$, $f(A)=A^5t+3A^2+e^{A^2t}$

$$sI-A=s\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}-2&-6\\1&3\end{bmatrix}=\begin{bmatrix}s+2&6\\-1&s-3\end{bmatrix}$$

$$d(s)=\begin{vmatrix}s+2&6\\-1&s-3\end{vmatrix}=(s+2)(s-3)+6=s^2-s=s(s-1)=0$$

$s_1=0$, $s_2=1$

$$f(s)=r(s)=\alpha_0+\alpha_1 s=s^5t+3s^2+e^{s^2t}$$

$f(0)=\alpha_0=1$

$f(1)=\alpha_0+\alpha_1=t+3+e^t \longrightarrow \alpha_1=e^t+t+2$

$$f(A)=\alpha_0 I+\alpha_1 A=\begin{bmatrix}1&0\\0&1\end{bmatrix}+(e^t+t+2)\begin{bmatrix}-2&-6\\1&3\end{bmatrix}$$

$$=\begin{bmatrix}-2e^t-2t-3 & -6e^t-6t-12\\ e^t+t+2 & 3e^t+3t+7\end{bmatrix}$$

Ör $A=\begin{bmatrix}0&1\\-2&3\end{bmatrix}$ a) $f(A)=A^5+2A^3$

b) $f(A)=e^{At}$

$$sI-A=s\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}0&1\\-2&3\end{bmatrix}=\begin{bmatrix}s&-1\\2&s-3\end{bmatrix}.$$

$$d(s)=\det(sI-A)=\begin{vmatrix}s&-1\\2&s-3\end{vmatrix}=s(s-3)+2=s^2-3s+2=0$$

$s_1=1$, $s_2=2$

a) $f(s)=r(s)=\alpha_0+\alpha_1 s=s^5+2s^3$

$$\left.\begin{aligned}\alpha_0+\alpha_1&=3\\ \alpha_0+2\alpha_1&=48\end{aligned}\right\}\quad \begin{aligned}\alpha_0&=-42\\ \alpha_1&=45\end{aligned}$$

$f(s)=45s-42$

$$f(A)=45A-42I=45\begin{bmatrix}0&1\\-2&3\end{bmatrix}-42\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}-42&45\\-90&93\end{bmatrix}$$

b) $f(s) = r(s) = \alpha_0 + \alpha_1 s = e^{st}$

$$\left.\begin{aligned} \alpha_0 + \alpha_1 &= e^{t} \\ \alpha_0 + 2\alpha_1 &= e^{2t} \end{aligned}\right\} \begin{bmatrix} 1 & 1 & e^{t} \\ 1 & 2 & e^{2t} \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & e^{t} \\ 0 & 1 & e^{2t} - e^{t} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 2e^{t} - e^{2t} \\ 0 & 1 & e^{2t} - e^{t} \end{bmatrix}$$

$\alpha_0 = 2e^{t} - e^{2t}$

$\alpha_1 = e^{2t} - e^{t}$

$f(s) = e^{st} = \alpha_0 + \alpha_1 s = (2e^{t} - e^{2t}) + (e^{2t} - e^{t}) s$

$f(A) = e^{At} = \alpha_0 I + \alpha_1 A$

$= (2e^{t} - e^{2t}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (e^{2t} - e^{t}) \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2e^{t} - e^{2t} & -e^{t} + e^{2t} \\ 2e^{t} - 2e^{2t} & -e^{t} + 2e^{2t} \end{bmatrix}$

Ör $A = \begin{bmatrix} 0 & -1 & 1 \\ -2 & 2 & 0 \\ 0 & 3 & -1 \end{bmatrix}$ veriliyor.

a) $f(A) = A^4 + 2A^3 - 7A^2 + A + 3I$

b) $f(A) = \cos(At)$

fonk. cayley-hamilton teoremini kullanarak çözünüz.

$sI - A = s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 1 \\ -2 & 2 & 0 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} s & 1 & -1 \\ 2 & s-2 & 0 \\ 0 & -3 & s+1 \end{bmatrix}$

$d(s) = \det(sI - A) = \begin{vmatrix} s & 1 & -1 \\ 2 & s-2 & 0 \\ 0 & -3 & s+1 \end{vmatrix} = s^3 - s^2 - 4s + 4 = 0$

$s_1 = 1, \; s_2 = 2, \; s_3 = -2$

$$f(s) = d(s)\,q(s) + r(s)$$

(112)

$$\begin{array}{l|l} f(s) = s^4+2s^3-7s^2+s+3 & s^3-s^2-4s+4 \rightarrow d(s) \\ \underline{-\;s^4-s^3-4s^2+4s} & \overline{s+3} \\ \quad 3s^3-3s^2-3s+3 & \\ \underline{-\;3s^3-3s^2-12s+12} & \\ \quad 9s-9 \rightarrow r(s) & \end{array}$$

$$f(s) = r(s) = \alpha_0 + \alpha_1 s + \alpha_2 s^2$$

$$\alpha_0 \to -9, \quad \alpha_1 \to 9, \quad \alpha_2 \to 0$$

$$f(s) = r(s) = 9(s-1)$$

$$f(A) = 9(A-I) = 9\left(\begin{bmatrix} 0 & -1 & 1 \\ -2 & 2 & 0 \\ 0 & 3 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -9 & -9 & 9 \\ -18 & 9 & 0 \\ 0 & 27 & -18 \end{bmatrix}$$

$$s_1 = 1,\ s_2 = 2,\ s_3 = -2$$

b) $f(s) = \cos(st) = \alpha_0 + \alpha_1 s + \alpha_2 s^2$

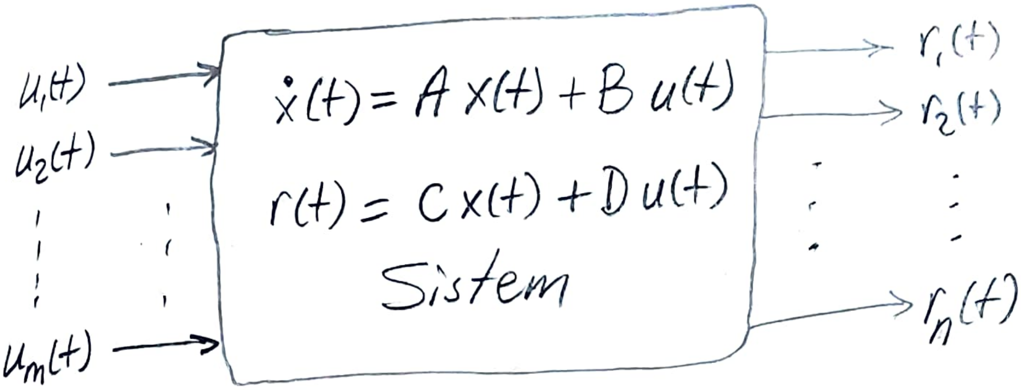
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \cos t \\ \cos 2t \\ \cos 2t \end{bmatrix} \xrightarrow{\text{çöz.}} \begin{array}{l} \alpha_0 = \frac{4}{3}\cos t - \frac{1}{3}\cos 2t \\ \alpha_1 = 0 \\ \alpha_2 = \frac{1}{3}\cos 2t - \frac{1}{3}\cos t \end{array}$$

$$f(A) = \alpha_0 I + \alpha_1 A + \alpha_2 A^2$$

$$= \alpha_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \alpha_1 \begin{bmatrix} 0 & -1 & 1 \\ -2 & 2 & 0 \\ 0 & 3 & -1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 2 & 1 & -1 \\ -4 & 6 & -2 \\ -6 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2\cos t + \cos 2t}{3} & \frac{\cos 2t - \cos t}{3} & \frac{\cos t - \cos 2t}{3} \\ \frac{4\cos t - 4\cos 2t}{3} & \frac{5\cos 2t - 2\cos t}{3} & \frac{2\cos t - 2\cos 2t}{3} \\ 2\cos t - 2\cos 2t & \cos 2t - \cos t & \cos t \end{bmatrix}$$

Sistem



A, B, C, D : sistem matrisleri

$x(t)$: sistemin iç değişkenleri

$$\underbrace{\dot{x}(t)}_{k\times 1} = \underbrace{A}_{k\times k}\ \underbrace{x(t)}_{k\times 1} + \underbrace{B}_{k\times m}\ \underbrace{u(t)}_{m\times 1}$$

$$\underbrace{r(t)}_{n\times 1} = \underbrace{C}_{n\times k}\ \underbrace{x(t)}_{k\times 1} + \underbrace{D}_{n\times m}\ \underbrace{u(t)}_{m\times 1}$$

$$x(t) = e^{At}.x(0) + (e^{At} - I).A^{-1}.B.u(t)$$

kullanılarak $x(t)$ hesaplanır. sonra $r(t)$'de yerine konulursa $r(t)$ hesaplanır.

<u>sistem elektrik devresi olsun</u>

$u(t)$: Devredeki akım ve voltaj kaynaklarıdır.

$x(t)$: Devre elemanları üzerindeki akım/voltaj değerleridir. Diğerleri cinsinden ifade edilebilenler alınmaz. parametre sayısını düşürmek için

$r(t)$: Devrede bulunması istenilen noktalar üzerindeki akım/voltaj değerleri.

S: $A=\begin{bmatrix}0 & 1\\ -2 & 3\end{bmatrix}$ $B=\begin{bmatrix}0 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$, $u(t)=\begin{bmatrix}1\\1\\e^{-t}\end{bmatrix}$, $x(0)=\begin{pmatrix}1\\-1\end{pmatrix}$

$C=\begin{bmatrix}1 & 0\\ 2 & -1\end{bmatrix}$ $D=\begin{bmatrix}1 & 0 & 2\\ 1 & 4 & 3\end{bmatrix}$

Yukarıdaki sistem için $r(t)=?$

$$d(s)=\det(sI-A)=\det\left(s\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}0&1\\-2&3\end{bmatrix}\right)=\begin{vmatrix}s & -1\\ 2 & s-3\end{vmatrix}$$

$$=s(s-3)+2=s^2-3s+2=(s-1)(s-2)=0 \quad s_1=1,\ s_2=2$$

$$f(s)=e^{st}=\alpha_0+\alpha_1 s$$

$$\left.\begin{aligned}\alpha_0+\alpha_1&=e^t\\ \alpha_0+2\alpha_1&=e^{2t}\end{aligned}\right\}\begin{bmatrix}1 & 1 & e^t\\ 1 & 2 & e^{2t}\end{bmatrix}\sim\begin{bmatrix}1 & 0 & 2e^t-e^{2t}\\ 0 & 1 & -e^t+e^{2t}\end{bmatrix}$$

$$\alpha_0=2e^t-e^{2t},\quad \alpha_1=-e^t+e^{2t}$$

$$f(A)=e^{At}=\alpha_0 I+\alpha_1 A=\alpha_0\begin{bmatrix}1&0\\0&1\end{bmatrix}+\alpha_1\begin{bmatrix}0&1\\-2&3\end{bmatrix}$$

$$=\begin{bmatrix}\alpha_0 & \alpha_1\\ -2\alpha_1 & \alpha_0+3\alpha_1\end{bmatrix}=\begin{bmatrix}2e^t-e^{2t} & -e^t+e^{2t}\\ -2e^t+2e^{2t} & -e^t+2e^{2t}\end{bmatrix}$$

$$[A\ I]\sim[I\ A]\Longrightarrow A^{-1}=\begin{bmatrix}3/2 & -1/2\\ 1 & 0\end{bmatrix}$$

$$x(t)=e^{At}x(0)+(e^{At}-I)A^{-1}Bu(t)$$

$$=\begin{bmatrix}2e^t-e^{2t} & -e^t+e^{2t}\\ -2e^t+2e^{2t} & -e^t+2e^{2t}\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$+\begin{bmatrix}2e^t-e^{2t}-1 & -e^t+e^{2t}\\ -2e^t+2e^{2t} & -e^t+2e^{2t}-1\end{bmatrix}\begin{bmatrix}3/2 & -1/2\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\e^{-t}\end{bmatrix}$$

$$x(t) = \begin{bmatrix} 3e^{t}-2e^{2t} \\ -e^{t} \end{bmatrix} + \begin{bmatrix} 2e^{t}-\frac{1}{2}e^{2t}-3/2 & -e^{t}+\frac{1}{2}e^{2t}+\frac{1}{2} \\ -4e^{t}+5e^{2t}-1 & e^{t}-e^{2t} \end{bmatrix} \begin{bmatrix} e^{-t} \\ 0 \end{bmatrix}$$ 115

$$= \begin{bmatrix} 3e^{t}-2e^{2t} \\ -e^{t} \end{bmatrix} + \begin{bmatrix} 2-\frac{1}{2}e^{t}-\frac{3}{2}e^{-t} \\ -4+5e^{t}-e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} 2+\frac{5}{2}e^{t}-2e^{2t}-\frac{3}{2}e^{-t} \\ -4+4e^{t}-e^{-t} \end{bmatrix}$$

$$r(t) = Cx(t) + Du(t)$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2+5/2\,e^{t}-2e^{2t}-3/2\,e^{-t} \\ -4+4e^{t}-e^{-t} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} 2+5/2\,e^{t}-2e^{2t}-3/2\,e^{-t} \\ 8+e^{t}-4e^{2t}-2e^{-t} \end{bmatrix} + \begin{bmatrix} 1+2e^{-t} \\ 5+3e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} 3+5/2\,e^{t}-2e^{2t}+1/2\,e^{-t} \\ 13+e^{t}-4e^{2t}+e^{-t} \end{bmatrix}$$

Ör. $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $u = 1$, $x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\dot{x} = Ax + bu$

$t \geq 0$ için $x(t) = ?$

$$[A\ I] \sim [I\ A^{-1}] \Longrightarrow A^{-1} = \begin{bmatrix} 3/2 & -1/2 \\ 1 & 0 \end{bmatrix}$$

A önceki örneğin aynısı olduğundan
Cayley-Hamilton yöntemi kullanılarak

$$f(A) = e^{At} = \begin{pmatrix} 2e^{t}-e^{2t} & e^{2t}-e^{t} \\ -2e^{t}+2e^{2t} & 2e^{2t}-e^{t} \end{pmatrix}$$

$$x(t) = e^{At} x(0) + (e^{At} - I) A^{-1} B u(t)$$

$$= \begin{pmatrix} 2e^t - e^{2t} & -e^t + e^{2t} \\ -2e^t + 2e^{2t} & -e^t + 2e^{2t} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} 2e^t - e^{2t} - 1 & -e^t + e^{2t} \\ -2e^t + 2e^{2t} & -e^t + 2e^{2t} - 1 \end{pmatrix} \cdot \begin{pmatrix} 3/2 & -1/2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot 1$$

$$= \begin{pmatrix} 2e^t - e^{2t} \\ -2e^t + 2e^{2t} \end{pmatrix} + \begin{pmatrix} 2e^t - e^{2t} - 1 & -e^t + e^{2t} \\ -2e^t + 2e^{2t} & -e^t + 2e^{2t} - 1 \end{pmatrix} \begin{pmatrix} -1/2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2e^t - e^{2t} \\ -2e^t + 2e^{2t} \end{pmatrix} + \begin{pmatrix} -e^t + \frac{1}{2}e^{2t} + 1/2 \\ e^t - e^{2t} \end{pmatrix} = \begin{pmatrix} e^t - \frac{1}{2}e^{2t} + 1/2 \\ -e^t + e^{2t} \end{pmatrix}$$

Ör $f(t) = 5 + 4\sin\left(\frac{\pi t}{2}\right) + 6\cos\left(\frac{\pi t}{3} + \pi/6\right)$ ile verilen fonksiyonun fourier seri katsayılarını bulunuz. (117)

$$w_1 = \frac{2\pi}{T_1} = \frac{\pi}{2} \Rightarrow T_1 = 4$$
$$w_2 = \frac{2\pi}{T_2} = \frac{\pi}{3} \Rightarrow T_2 = 6$$

En küçük ortak katlar $T = 12$, $w = \frac{2\pi}{T} = \frac{\pi}{6}$

$$f(t) = 5 + 4\sin(3wt) + 6\cos(2wt + \pi/6)$$
$$= 5 + 4\sin(3wt) + 6\left(\cos(2wt)\cos(\pi/6) - \sin(2wt)\sin(\pi/6)\right)$$
$$= 5 + 4\sin(3wt) + 3\sqrt{3}\cos(2wt) - 3\sin(2wt)$$

$$a_n = \begin{cases} 5 & n=0 \\ 3\sqrt{3} & n=2 \\ 0 & \text{else} \end{cases} \qquad b_n = \begin{cases} -3 & n=2 \\ 4 & n=3 \\ 0 & \text{else} \end{cases}$$

Ör $A = \begin{bmatrix} 1.5 & 0.75 \\ 1 & 2.5 \end{bmatrix}$ $f(A) = 2^A + 3A$ fonksiyonunu Cayley-Hamilton teoremini kullanarak bulunuz.

$$d(s) = \det(sI - A)$$
$$= \begin{vmatrix} s-1.5 & -0.75 \\ -1 & s-2.5 \end{vmatrix} = (s-1.5)(s-2.5) - 0.75$$
$$= s^2 - 4s + 3 = (s-1)(s-3) = 0$$
$$s_1 = 1,\ s_2 = 3$$

s_1 ve s_2 için

$$f(s) = 2^s + 3s = r(s) = \alpha_0 + \alpha_1 s$$

$$\left.\begin{aligned} \alpha_1 + \alpha_2 &= 5 \\ \alpha_1 + 3\alpha_2 &= 17 \end{aligned}\right\} \begin{bmatrix} 1 & 1 & 5 \\ 1 & 3 & 17 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 6 \end{bmatrix} \quad \begin{aligned} \alpha_0 &= -1 \\ \alpha_1 &= 6 \end{aligned}$$

$$f(s) = 6s - 1 \Rightarrow f(A) = 6A - I$$

$$f(A) = 6\begin{bmatrix} 1.5 & 0.75 \\ 1 & 2.5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 4.5 \\ 6 & 14 \end{bmatrix}$$

Lineer Olmayan Denklem Sistemleri

$$\left.\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= 0 \\ f_2(x_1, x_2, \ldots, x_n) &= 0 \\ &\vdots \\ f_n(x_1, x_2, \ldots, x_n) &= 0 \end{aligned}\right\}$$

En az bir tanesi lineer olmayan terim içerirse n değişkenli lineer olmayan denklem sistemi olur. Değilse lineer denklem sistemidir.

$x = [x_1, x_2, \ldots, x_n]^T$ Çözüm vektörü ise tüm denklemleri sağlar.

Lineer olmayan denklem sistemleri, mühendislik problemlerinde çok sık kullanılır. Tek değişkenli ise tek denklemden, n değişkenli ise n denklemden oluşur. Çok değişkenli olanları çözmek için bilgisayar yazılımı zorunludur.

Tek değişkenli ise $f(x)=0$ kapalı formunda gösterilir ve tek denklemden oluşur. Örneğin $x^3+3x^2+5x-7=0$ denklemi polinom formunda lineer olmayan bir denklemdir. $(5x-7)$ lineer kısım, (x^3+3x^2) ise lineer olmayan kısımdır.

$e^x + x\sin x = 0$ denklemi de lineer değildir. Denklemin birden fazla kökü olabilir. Kökler x eksenini kestiği noktalardır. Kökleri bulmak denklemi çözmek demektir. Kökleri bulmak için çoğu zaman analitik yöntem işe yaramaz.

Tek değişkenli ise grafik çizimi ile kökler bulunabilir. Fakat çözüm hassas olmayabilir. Hassas bir çözüm yani hata yüzdesi çok düşük bir çözüm için sayısal analiz yöntemlerinden birine ihtiyaç vardır.