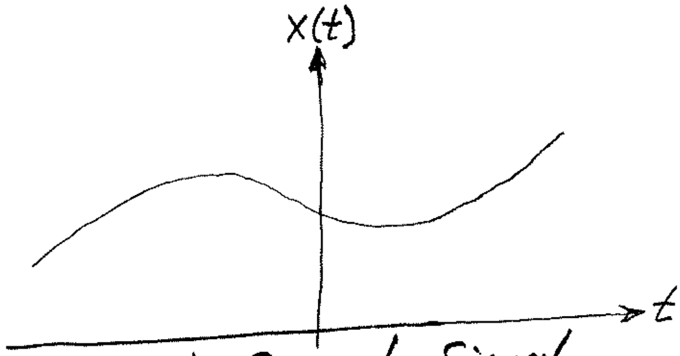
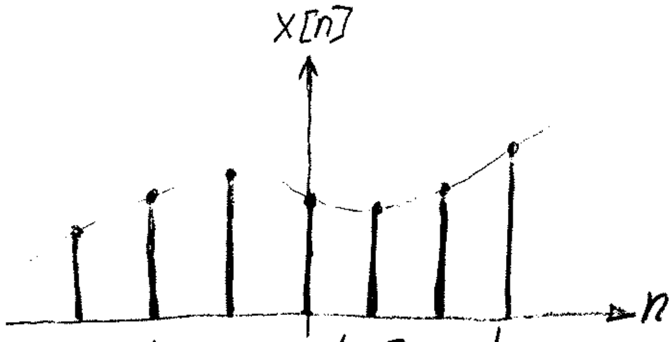
Bilgisayar.

Sinyaller ve Sistemler



Sürekli Zamanlı Sinyal



Ayrık Zamanlı Sinyal

T_s : Örnekleme peryodu

$f_s = \frac{1}{T_s}$: Örnekleme peryodu

$x[n] = x(nT_s)\,,\ n \in \mathbb{Z}$

Sürekli zamanlı sistemler zamanın her anında tanımlıdır. Kesikli zamanlı sistemler ise sadece seçilen zaman aralıklarında tanımlıdır. Sinyal uygun örnekleme hızında örneklenirse bilgi kaybı olmaz. Örneklenen bir sinyalin tekrar elde edilebilmesi için $f_s \geq 2f_{max}$ olmalıdır.

$$\left.\begin{array}{l} x(t) = x(t+kT) \\ x[n] = x[n+kN] \end{array}\right\} \begin{array}{l}\text{Peryodik Sinyal} \\ n,k,N \in \mathbb{Z}\end{array}$$

T : Sürekli zamanlı sinyalin peryodu

N : Kesikli zamanlı sinyalin peryodu

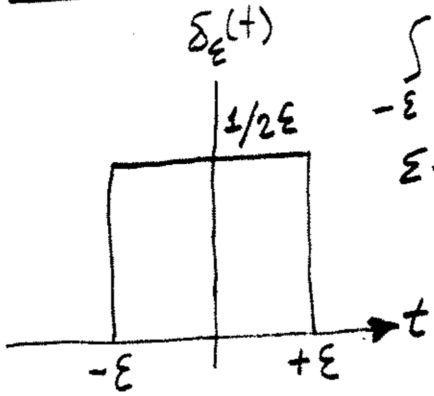
$$\left.\begin{array}{l} x(-t) = x(t) \\ x[-n] = x[n] \end{array}\right\} \text{çift sinyal}$$

$$\left.\begin{array}{l} x(-t) = -x(t) \\ x[-n] = -x[n] \end{array}\right\} \text{tek sinyal}$$

$$x(t) = x_e(t) + x_o(t)$$
$$x_e(t) = \frac{x(t)+x(-t)}{2}$$
$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

$$x[n] = x_e[n] + x_o[n]$$
$$x_e[n] = \frac{x[n]+x[-n]}{2}$$
$$x_o[n] = \frac{x[n]-x[-n]}{2}$$

Birim Dürtü Fonksiyonu



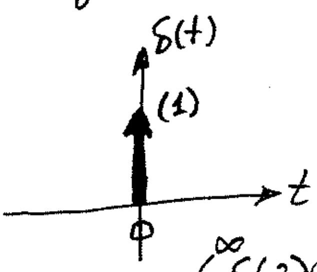
$$\int_{-\varepsilon}^{\varepsilon} \delta_\varepsilon(t)\,dt = 1$$

$\varepsilon \to 0$ için

$$\delta_\varepsilon(t) \to \delta(t)$$

$$\int_{0^-}^{0^+} \delta(t)\,dt = 1$$

$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & t\neq 0 \end{cases}$$



$$\delta(-t) = \delta(t)$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\int_{-\infty}^{\infty} \delta(z)\,dz = 1$$

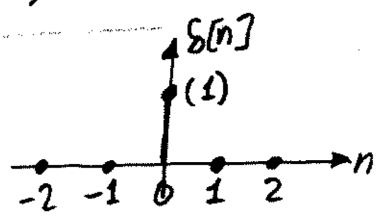
$$x(t)\delta(t) = x(0)\delta(t)$$

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} x(z)\delta(z)\,dz = x(0)$$

$$x(t) = \int_{-\infty}^{\infty} x(z)\delta(t-z)\,dz$$

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$$



$$\delta[k-n] = \begin{cases} 1 & n=k \\ 0 & n\neq k \end{cases}$$

$$\delta[-n] = \delta[n] \qquad \delta[k-n] = \delta[n-k]$$

$$x[n]\delta[n] = x[0]\delta[n]$$

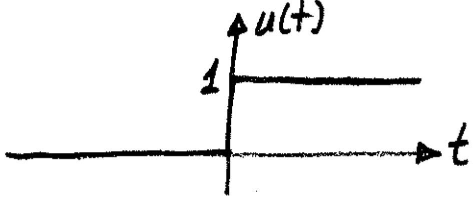
$$x[n]\delta[n-k] = x[k]\delta[n-k]$$

$$\sum_{k=-\infty}^{\infty} \delta[k] = 1$$

$$\sum_{k=-\infty}^{\infty} x[k]\delta[k] = x[0]$$

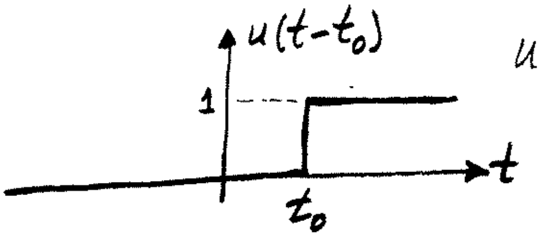
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Birim Basamak Fonksiyonu



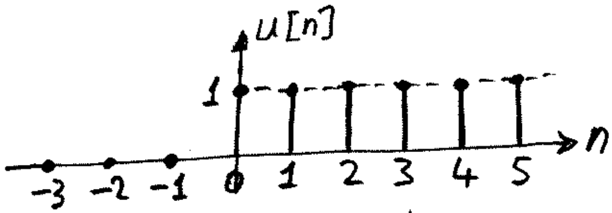
$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

$$\delta(t) = u'(t) = \frac{du(t)}{dt}$$



$$u(t-t_0) = \begin{cases} 1 & t>t_0 \\ 0 & t<t_0 \end{cases}$$ ötelenmiş birim basamak sinyali

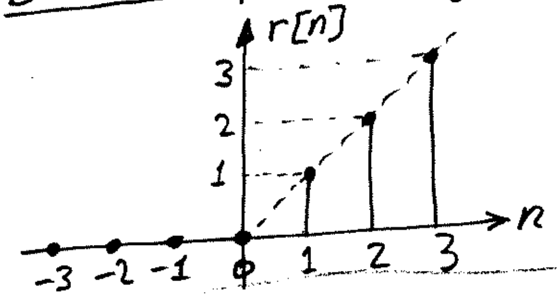
$$u[n] = \sum_{k=-\infty}^{n} \delta[k] = \begin{cases} 1 & n \geq 0 \\ 0 & n<0 \end{cases}$$



$$\delta[n] = u[n] - u[n-1]$$

$$u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & n<k \end{cases}$$

Birim Rampa Fonksiyonu



$$r[n] = n\,u[n] = \begin{cases} n & n \geq 0 \\ 0 & n<0 \end{cases}$$

$$r[n] - r[n-1] = u[n]$$

$$x[n] = \{4, \underset{\uparrow}{2}, 1, -3, 6\}$$

$$x[n] = 4\delta[n+1] + 2\delta[n] + \delta[n-1] - 3\delta[n-2] + 6\delta[n-3]$$

$$x[n] = r[n+1] - r[n-2] - 3u[n-3]$$

$$= (n+1)u[n+1] - (n-2)u[n-2] - 3u[n-3]$$

$n < -1 \longrightarrow x[n] = 0$

$x[-1] = 0$

$x[0] = 1$

$x[1] = 2$

$x[2] = 3$

$n \geq 3 \longrightarrow x[n] = (n+1) - (n-2) - 3 = 0$

$$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$$

$$x[n] = 3r[n+1] - 4r[n-2] + r[n-4] - 2u[n] - 5u[n-5]$$

$$= 3(n+1)u[n+1] - 4(n-2)u[n-2] + (n-4)u[n-4] - 2u[n] - 5u[n-5]$$

$n \leq -2 \longrightarrow x[n] = 0$

$x[-1] = 0$

$x[0] = 3-0+0-2-0 = 1$

$x[1] = 6-0+0-2-0 = 4$

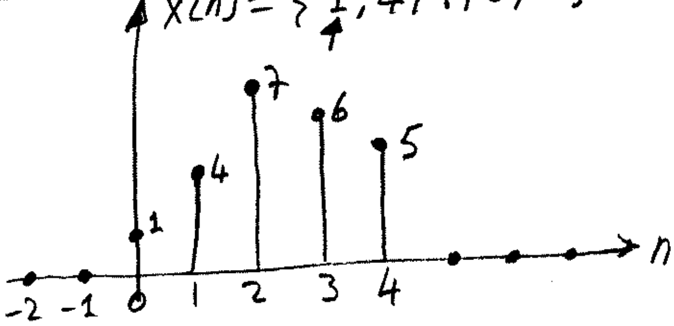
$x[2] = 9-0+0-2-0 = 7$

$x[3] = 12-4+0-2-0 = 6$

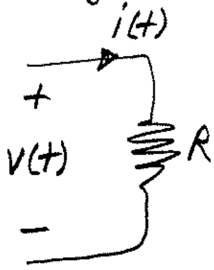
$x[4] = 15-8+0-2-0 = 5$

$x \geq 5 \rightarrow x[n] = 3(n+1) - 4(n-2) + (n-4) - 2 - 5 = 0$

$$x[n] = \{\underset{\uparrow}{1}, 4, 7, 6, 5\}$$



Enerji ve Güç Sinyalleri



Direnç üzerinde harcanan güç.

$p(t) = v(t) \cdot i(t) = R\, i^2(t)$

Ohm başına olsa idi $p(t) = \dfrac{v(t)\, i(t)}{R} = i^2(t)$ olurdu.

E : ohm başına düşen enerji

P : ohm başına düşen ortalama güç.

$$E = \lim_{T\to\infty} \int_{-T}^{T} i^2(t)\,dt \quad \text{Joule} \qquad P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} i^2(t)\,dt \quad \text{Watt}$$

x(t) sürekli zaman sinyali	x[n] ayrık zaman sinyali
$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \lim_{T\to\infty}\int_{-T}^{T} x^2(t)\,dt$	$E = \sum_{n=-\infty}^{\infty} x^2[n]$
$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt$	$P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} x^2[n]$

1. $0 < E < \infty$ ise $x(t)$ (veya $x[n]$) enerji sinyalidir.
2. $0 < P < \infty$ ise $x(t)$ (veya $x[n]$) güç sinyalidir.
3. Bu koşullar sağlanmıyorsa sinyal ne enerji ne de güç sinyalidir.

$x(t) = A e^{-\alpha t} u(t), \quad \alpha > 0$

$$E = \lim_{T\to\infty}\int_{-T}^{T} x^2(t)\,dt = \lim_{T\to\infty}\int_{0}^{T} A^2 \cdot e^{-2\alpha t}\,dt = \lim_{T\to\infty} \frac{A^2 \cdot e^{-2\alpha t}}{-2\alpha}\Bigg|_0^T$$

$$= \lim_{T\to\infty} \frac{A^2}{2\alpha}\left(1 - e^{-2\alpha T}\right) = \frac{A^2}{2\alpha}, \quad x(t) \text{ enerji sinyalidir.}$$

$\alpha = 0$ olsa idi $x(t) = A u(t)$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \lim_{T\to\infty}\frac{1}{2T}\int_{0}^{T} A^2\,dt = \frac{A^2}{2}, \quad x(t) \text{ güç sinyalidir.}$$

$x(t) = A\cos(wt + \alpha)$

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} A^2\cos^2(wt+\alpha)\,dt$$

$$= \lim_{T\to\infty}\frac{A^2}{2T}\int_{-T}^{T}\frac{1+\cos(2wt+2\alpha)}{2}\,dt = \lim_{T\to\infty}\frac{1}{2T}\left(\frac{t}{2} + \frac{\sin(2wt+2\alpha)}{4w}\right)\Bigg|_{-T}^{T}$$

$$= \lim_{T\to\infty}\frac{A^2}{2T}\left(T + \frac{\sin(\cancel{2wT}+2\alpha) - \sin(\cancel{-2wT}+2\alpha)}{4w}\right) = \frac{A^2}{2} \quad x(t) \text{ güç sinyalidir.}$$

$x[n] = a^n$ iki taraflı kesikli zaman sinyali

$$x[n] = a^n u[n] \begin{cases} \rightarrow |a|>1 & \text{artar} \\ \rightarrow |a|<1 & \text{azalır} \end{cases}$$

$$x(t) = \text{rect}(t) = \begin{cases} 1 & |t| \le 1/2 \\ 0 & |t| > 1/2 \end{cases}$$

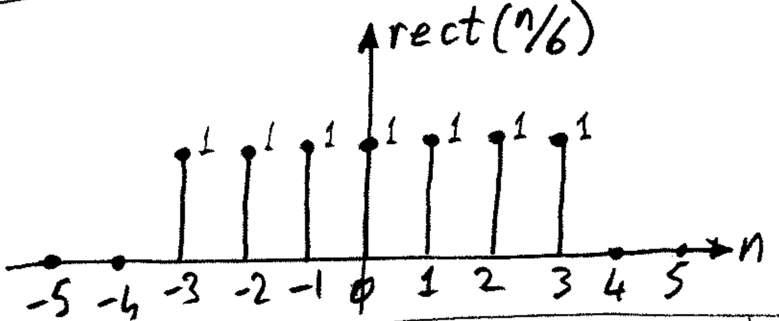
$$x(t) = \text{rect}\left(\frac{t}{2T}\right) = \begin{cases} 1 & |t| \le T \\ 0 & |t| > T \end{cases}$$

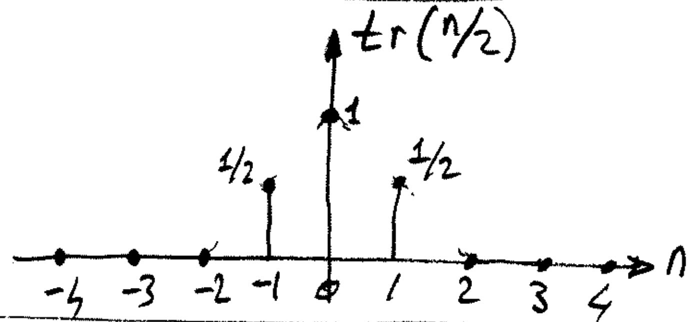
$$x[n] = \text{rect}\left(\frac{n}{2N}\right) = \begin{cases} 1 & |n| \le N \\ 0 & |n| > N \end{cases}$$

$$x(t) = tr(t) = \begin{cases} 1-|t| & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$$

$$x(t) = tr\left(\frac{t}{T}\right) = \begin{cases} 1-|t|/T & |t| \le T \\ 0 & |t| > T \end{cases}$$

$$x[n] = tr\left(\frac{n}{N}\right) = \begin{cases} 1-|n|/N & |n| \le N \\ 0 & |n| > N \end{cases}$$





$$\left.\begin{aligned} \int_{-a}^{a} x(t)\,dt &= 2\int_{0}^{a} x(t)\,dt \\ \sum_{n=-k}^{k} x[n] &= x[0] + 2\sum_{n=1}^{k} x[n] \end{aligned}\right\} \text{sinyal çift}$$

$$\left.\begin{aligned} \int_{-a}^{a} x(t)\,dt &= 0, \quad x(0)=0 \\ \sum_{n=-k}^{k} x[n] &= 0, \quad x[0]=0 \end{aligned}\right\} \text{sinyal tek}$$

$$x(t) = \cos\left(\frac{\pi t}{3}\right) + \sin\left(\frac{\pi t}{4}\right), \quad T=?$$

$$T = \frac{2\pi}{w}$$

$$x_1(t) = \cos\left(\frac{\pi t}{3}\right), \quad w_1 = \frac{\pi}{3} = 2\pi f_1 = \frac{2\pi}{T_1} \rightarrow T_1 = 6 \text{ sn}$$

$$x_2(t) = \sin\left(\frac{\pi t}{4}\right), \quad w_2 = \frac{\pi}{4} = 2\pi f_2 = \frac{2\pi}{T_2} \rightarrow T_2 = 8 \text{ sn}$$

$$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4} \rightarrow 4T_1 = 3T_1 = T = 24 \text{ sn} \quad \text{Peryodik}$$

$$N = \frac{2\pi}{\Omega}$$

$$x[n] = \cos\left(\frac{\pi n}{3}\right) + \sin\left(\frac{\pi n}{4}\right), \quad N=?$$

$$x_1[n] = \cos\left(\frac{\pi n}{3}\right), \quad \Omega_1 = \pi/3 \rightarrow N_1 = \frac{2\pi}{\Omega_1} = 6$$

$$x_2[n] = \sin\left(\frac{\pi n}{4}\right), \quad \Omega_2 = \pi/4 \rightarrow N_2 = \frac{2\pi}{\Omega_2} = 8$$

$$\frac{N_1}{N_2} = \frac{6}{8} = \frac{3}{4} \rightarrow 4N_1 = 3N_2 = N = 24 \quad \text{Peryodik}$$

Doğrusal Zamanla Değişmeyen Sistemler

Sürekli Zamanlı Sinyaller için

$h(t) = T(\delta(t)) \qquad x(t) = \int_{-\infty}^{\infty} x(z)\,\delta(t-z)\,dz$

sistem doğrusal olduğundan

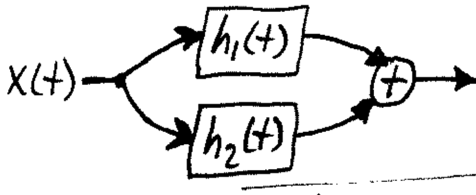
$$y(t) = T(x(t)) = T\left(\int_{-\infty}^{\infty} x(z)\,\delta(t-z)\,dz\right) = \int_{-\infty}^{\infty} x(z)\,T(\delta(t-z))\,dz$$

$$= \int_{-\infty}^{\infty} x(z)\,h(t-z)\,dz, \quad h(t-z) = T(\delta(t-z))$$ sistem zamanla değişmez olduğundan

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t) = h(t) * x(t) = T(x(t))$$

$$= \int_{-\infty}^{\infty} x(z)\,h(t-z)\,dz = \int_{-\infty}^{\infty} h(z)\,x(t-z)\,dz$$

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t) = x(t) * \underbrace{h_1(t) * h_2(t)}, \quad h(t) = h_1(t) * h_2(t)$$



$$y(t) = x(t) * h_1(t) + x(t) * h_2(t)$$

$$= x(t) * (h_1(t) + h_2(t)), \quad h(t) = h_1(t) + h_2(t)$$

$$s(t) = T(u(t)) = h(t) * u(t) = \int_{-\infty}^{\infty} h(z)\,u(t-z)\,dz = \int_{-\infty}^{t} h(z)\,dz$$

$$h(t) = s'(t) = \frac{ds(t)}{dt}$$

Kesikli Zamanlı Sinyaller için

$h[n] = T(\delta[n]), \quad x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$

sistem doğrusal olduğundan

$$y[n] = T(x[n]) = T\left(\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right) = \sum_{k=-\infty}^{\infty} x[k]\,T(\delta[n-k])$$

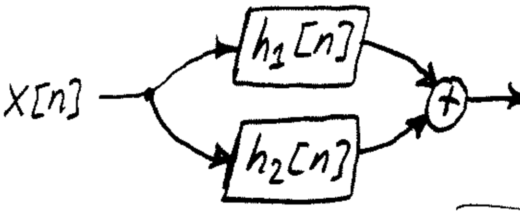
$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k], \quad h[n-k] = T(\delta[n-k])$$ sistem zamanla değişmez olduğundan

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = x[n] * h[n] = h[n] * x[n] = T(x[n])$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{h_2[n]} \longrightarrow y[n] = x[n] * h_1[n] * h_2[n]$$

$$h[n] = h_1[n] * h_2[n]$$



$$y[n] = x[n] * h_1[n] + x[n] * h_2[n]$$

$$= x[n] * (h_1[n] + h_2[n])$$

$$h[n] = h_1[n] + h_2[n]$$

$$s[n] = T(u[n]) = h[n] * u[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\,u[n-k] = \sum_{k=-\infty}^{n} h[k] \qquad h[n] = s[n] - s[n-1]$$

$x(t) * \delta(t) = x(t)$

$x(t) * \delta(t-t_0) = x(t-t_0)$

$x(t) * u(t) = \int_{-\infty}^{t} x(z)\,dz$

$x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$

Sistem Özellikleri

Bellekli ya da Belleksiz Sistemler

Sistemin çıkışı yanlızca o andaki girişe bağlı ise belleksiz, değilse bellekli sistemdir.

$$\left.\begin{array}{l} h(t) = K\delta(t) \longrightarrow y(t) = Kx(t) \\ h[n] = K\delta[n] \longrightarrow y[n] = Kx[n] \end{array}\right\} \text{Belleksiz sistem}$$

$y(t) = Rx(t) \longrightarrow$ direnç belleksiz sistem

$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau \longrightarrow$ kapasitör bellekli sistem

$$\left.\begin{array}{l} y[n] = 5x[n] - 3 \\ y[n] = \left(3x[n] - x^2[n]\right)^2 \\ y[n] = 2^n x[n] \end{array}\right\} \begin{array}{l}\text{Belleksiz} \\ \text{fakat lineer değil.}\end{array}$$

$$\left.\begin{array}{l} y[n] = x[n-5] \\ y[n] = x[n+2] \\ y[n] = 2x[3n] \end{array}\right\} \text{Bellekli sistem}$$

$y[n] = n\,x[n]$ bellekli, lineer

sistem çıkışı o andaki veya daha önceki giriş değerlerine bağlı ise sistem nedenseldir. Belleksiz sistemler de nedenseldir.

Nedensel Sistemler

$$\left.\begin{array}{l} h(t) = 0,\ t<0 \rightarrow y(t) = \int_0^t h(\tau)x(t-\tau)\,d\tau = \int_0^t x(\tau)h(t-\tau)\,d\tau \\ h[n] = 0,\ n<0 \longrightarrow y[n] = \sum_{k=0}^{n} h[k]x[n-k] = \sum_{k=0}^{n} x[k]h[n-k] \end{array}\right\} \text{giriş de nedensel}$$

$x(t)$ nedensel değilse t yerine ∞ yazılır.
$x[n]$ nedensel değilse n yerine ∞ yazılır.

$y[n] = \sum_{k=-\infty}^{n} x[k]$

$y[n] = x[n-2]$

$y[n] = Ax[n] + B$

$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$y(t) = 2\frac{dx(t)}{dt} - 5\frac{d^2x(t)}{dt^2}$

$y[n] = (n+1)^2 x[n]$

$y[n+5] + y[n+3] = x[n+2]$

~~nedensel sistemler~~
işlerinde lineer olmayanlar var.

$y[n] = x[n+2]$

$y[n] = x[n] - x[n+3]$

$y[n] = \sum_{k=-5}^{5} x[n-k]$

$y[n+2] + y[n] = x[n+5]$

$y[n] = x[n+1] - x[n-1]$

nedensel olmayan sistemler

$h[n] = u[n]$

$y[n] = \sum_{k=-\infty}^{n} x[k]$

linear – doğrusal
causal – nedensel
memoryless – belleksiz → statik
with memory – bellekli – dinamik
stable – kararlı
time invariance – zamandan bağımsız

Kararlı Sistemler

$|x(t)| \le M$ için $|y(t)| \le N$ ise sistem kararlıdır.

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \right| \le \int_{-\infty}^{\infty} |h(\tau)| \cdot |x(t-\tau)| \cdot d\tau \le M \int_{-\infty}^{\infty} |h(\tau)| \cdot d\tau < \infty$$

$\int_{-\infty}^{\infty} |h(\tau)| \cdot d\tau < \infty$ sistem kararlı

$|x[n]| \le M$ için $|y[n]| \le N$ ise sistem kararlıdır.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k] x[n-k] \right| \le \sum_{k=-\infty}^{\infty} |h[k]| \cdot |x[n-k]| \le M \sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$ sistem kararlı

$y[n] = 3x[n] - 2x[n-1] \longrightarrow h[n] = 3\delta[n] - 2\delta[n-1]$

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} |3\delta[n] - 2\delta[n-1]| = \sum_{k=0}^{1} |3\delta[n] - 2\delta[n-1]| = 3 + 2 = 5 < \infty$$ sistem kararlı

$$y[n] = \sum_{k=-\infty}^{n} u[k] = (n+1)u[n] = \begin{cases} n+1, & n \ge 0 \\ 0, & n < 0 \end{cases}$$

$n \to \infty$ için $y[n] \to \infty$ kararsız sistem

Doğrusal Sistemler

sabit terim içermemeli, katsayılar n'ye bağlı, katsayılar x ve y'ye ~~bağlı~~ değil.

$$\left.\begin{array}{l} x_1(t) \longrightarrow y_1(t) \\ x_2(t) \longrightarrow y_2(t) \end{array}\right\} a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 y_1(t) + a_2 y_2(t)$$

$$\left.\begin{array}{l} x_1[n] \longrightarrow y_1[n] \\ x_2[n] \longrightarrow y_2[n] \end{array}\right\} a_1 x_1[n] + a_2 x_2[n] \longrightarrow a_1 y_1[n] + a_2 y_2[n]$$

$a_1 x_1 + a_2 x_2 \to 0$ için $a_1 y_1 + a_2 y_2 \to 0$

$x \to 0$ için $y \to 0 \Longrightarrow a_1 x_1 + a_2 x_2 \to 0$

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \qquad \sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$\left.\begin{array}{l} y[n] = x[n] - x[n-2] \\ y(t) = x(t) - \dfrac{d^2 x(t)}{dt^2} \\ y[n] + 3^n y[n-1] = x[n] \\ y(t) + 3^t \dfrac{dy(t)}{dt} = x(t) \end{array}\right\} \text{doğrusal sistemler}$$

$$\left.\begin{array}{l} y[n] = x[n] + 5 \\ y[n] + 3^n = x[n-1] \\ y[n] = x^2[n] \\ y(t) = x^2(t) \end{array}\right\} \text{doğrusal olmayan sistem}$$

Zamandan Bağımsız Sistemler

katsayılar x ve y'ye bağlı, zamana değil.

$y[n] = R\,x[n]$

$y[n] = \sum_{k=-\infty}^{n} x[k]$

$y^2[n] + 3x[n]\,y[n-1] = x[n-1]$

$3^{y[n]} \cdot y[n] = x[n]$

zamandan bağımsız

$y[n] = n\,x[n]$

$y[n] + y[n-1] = x[2n]$

$y[n] + 3^n y[n-1] = x[n]$

zamanla bağımlı

$y[n-k] = T\left(x[n-k]\right)$ zamandan bağımsız.

$t<0 \rightarrow y(t)=0$

$x(t) = u(t)\,,\quad h(t) = e^{-\alpha t}u(t)\,,\quad \alpha>0$

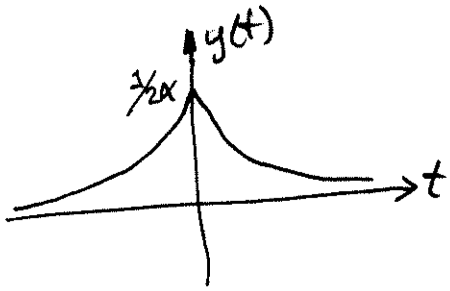
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(z)h(t-z)\,dz = \int_{-\infty}^{\infty} u(z)\cdot e^{-\alpha(t-z)}u(t-z)\,dz \qquad \underline{t\geq 0}$$

$$= e^{-\alpha t}\cdot u(t)\int_0^t e^{\alpha z}\,dz = e^{-\alpha t}u(t)\left.\frac{e^{\alpha z}}{\alpha}\right|_0^t = \frac{1}{\alpha}\left(1-e^{-\alpha t}\right)u(t)$$

$h(t) = e^{-\alpha t}u(t)\,,\quad x(t) = e^{\alpha t}u(-t)\,,\quad \alpha>0$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(z)h(t-z)\,dz = \int_{-\infty}^{\infty} e^{\alpha z}u(-z)\cdot e^{-\alpha(t-z)}u(t-z)\,dz$$

$$= e^{-\alpha t}\int_{-\infty}^{\infty} e^{2\alpha z}u(-z)u(t-z)\,dz$$



$t<0$ için $y(t) = e^{-\alpha t}\int_{-\infty}^{t} e^{2\alpha z}\,dz = \frac{e^{\alpha t}}{2\alpha}$

$t\geq 0$ için $y(t) = e^{-\alpha t}\int_{-\infty}^{0} e^{2\alpha z}\,dz = \frac{e^{-\alpha t}}{2\alpha}$

$$y(t) = \frac{1}{2\alpha}e^{-\alpha|t|}\,,\quad \alpha>0$$

$x(t) = u(t) - u(t-3)\,,\quad h(t) = u(t) - u(t-2) \longrightarrow y(t) = ?$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(z)h(t-z)\,dz = \int_{-\infty}^{\infty}\left(u(z)-u(z-3)\right)\cdot\left(u(t-z)-u(t-2-z)\right)dz$$

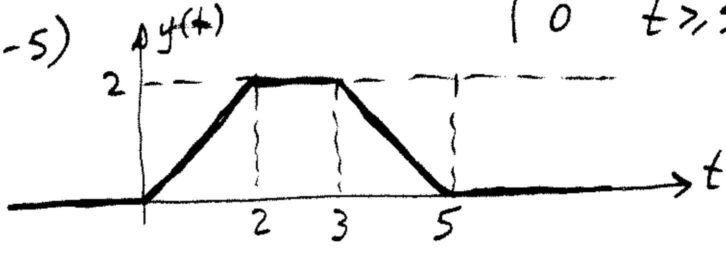
$$= \int_{-\infty}^{\infty} u(z)u(t-z)\,dz - \int_{-\infty}^{\infty} u(z)u(t-2-z)\,dz - \int_{-\infty}^{\infty} u(z-3)u(t-z)\,dz + \int_{-\infty}^{\infty} u(z-3)u(t-2-z)\,dz$$

$$= u(t)\int_0^t dz - u(t-2)\int_0^{t-2} dz - u(t-3)\int_3^t dz + u(t-5)\int_3^{t-2} dz$$

$$= t\,u(t) - (t-2)u(t-2) - (t-3)u(t-3) + (t-5)u(t-5)$$

$$= r(t) - r(t-2) - r(t-3) + r(t-5)$$

$$y(t) = \begin{cases} 0 & t<0 \\ t & 0\leq t<2 \\ 2 & 2\leq t<3 \\ 5-t & 3\leq t<5 \\ 0 & t\geq 5 \end{cases}$$



$s(t) = e^{-t}u(t)$ birim basamak tepkisi

$x(t) = u(t-1) - u(t-3)$

$y(t) = s(t-1) - s(t-3) = e^{1-t}u(t-1) - e^{3-t}u(t-3)$

$h_1(t) = e^{-2t}u(t)$, $h_2(t) = 2e^{-t}u(t)$ $\quad h(t) = ?$

Sistem kararlı mı?

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

$$h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)\cdot 2e^{\tau-t}u(t-\tau)d\tau$$

$$= 2u(t)\int_0^t e^{-\tau-t}d\tau = 2e^{-t}u(t)\int_0^t e^{-\tau}d\tau = 2e^{-t}u(t)\frac{e^{-\tau}}{-1}\Big|_0^t = 2(e^{-t} - e^{-2t})u(t)$$

$$\int_{-\infty}^{\infty}|h(\tau)|\cdot d\tau = 2\int_0^{\infty}(e^{-\tau} - e^{-2\tau})d\tau = 2\int_0^{\infty}e^{-\tau}d\tau - 2\int_0^{\infty}e^{-2\tau}d\tau = 1 < \infty$$ sistem kararlıdır.

$y'(t) + 2y(t) = x(t) + x'(t) \longrightarrow h(t) = ?$

$h'(t) + 2h(t) = \delta(t) + \delta'(t) \longrightarrow h_h(t) = c_1 e^{-2t}u(t)$, $h_p(t) = c_2\delta(t)$

$h(t) = c_1 e^{-2t}u(t) + c_2\delta(t)$ $\xrightarrow{\text{yerine koy}}$ $h(t) = \delta(t) - e^{-2t}u(t)$

$y[n] = x[n] - 2x[n-2] + x[n-3] \longrightarrow h[n] = \delta[n] - 2\delta[n-2] + \delta[n-3]$

$= \{1, 0, -2, 1\}$

$y[n] + 2y[n-1] = x[n] + x[n-1]$

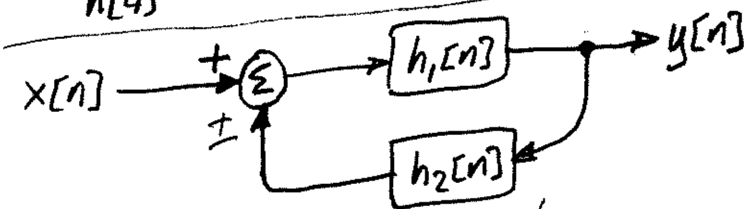
$h[n] + 2h[n-1] = \delta[n] + \delta[n-1]$

$h[n] = -2h[n-1] + \delta[n] + \delta[n-1]$ sistem nedensel olduğundan

$$\left.\begin{aligned}
h[0] &= -2h[-1] + \delta[0] + \delta[-1] = 1\\
h[1] &= -2h[0] + \delta[1] + \delta[0] = -1\\
h[2] &= -2h[1] + \delta[2] + \delta[1] = 2\\
h[3] &= -2h[2] + \delta[3] + \delta[2] = -4\\
h[4] &= -2h[3] + \delta[4] + \delta[3] = 8
\end{aligned}\right\} \quad h[n] = \delta[n] + (-1)^n 2^{n-1}u[n-1]$$

$x[n] = nu[n]$

$$y[n] = \sum_{k=-\infty}^{n} x[n] = \sum_{k=-\infty}^{n} k\,u[k] = \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$



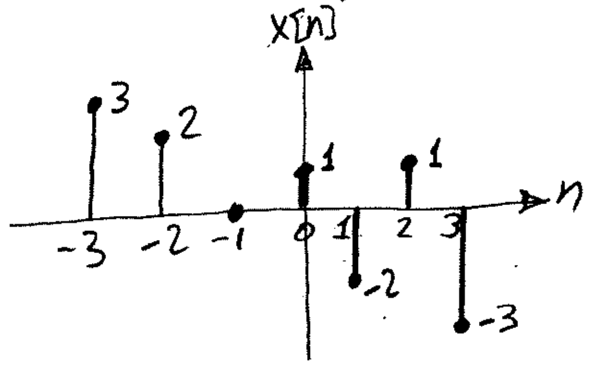
Geri beslemeli sistem

$(\delta[n] \mp h_2[n] * h_1[n]) * y[n] = h_1[n] * x[n]$

$x[n] = a^n u[n]$

$$y[n] = \sum_{k=-\infty}^{n} x[k] = \sum_{k=-\infty}^{n} a^k u[k] = \sum_{k=0}^{n} a^k = \frac{1 - a^{n+1}}{1 - a}$$

$x[n] = \{3, 2, 0, \underset{\uparrow}{1}, -2, 1, -3\}$



$x[-n] = \{-3, 1, -2, \underset{\uparrow}{1}, 0, 2, 3\}$

$x[n+2] = \{3, 2, 0, 1, -2, \underset{\uparrow}{1}, -3\}$

$x[2n] = \{2, \underset{\uparrow}{1}, 1\}$

$x[\frac{2n}{3}] = \{2, 0, 0, \underset{\uparrow}{1}, 0, 0, 1\}$

$x[n] = \{4, 2, 1, \underset{\uparrow}{1}, 3, 5\}$ $\quad x_o[n] = ? \quad x_e[n] = ?$

$x[n] = 4\delta[n+3] + 2\delta[n+2] + \delta[n+1] + \delta[n] + 3\delta[n-1] + 5\delta[n-2]$

$x[-n] = 5\delta[n+2] + 3\delta[n+1] + \delta[n] + \delta[n-1] + 2\delta[n-2] + 4\delta[n-3]$

$x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right) = \{2, -3/2, -1, \underset{\uparrow}{0}, 1, 3/2, -2\}$

$x_e[n] = \frac{1}{2}\left(x[n] + x[-n]\right) = \{2, 7/2, 2, \underset{\uparrow}{1}, 2, 7/2, 2\}$

$\{2, 1, 5, -2, 3\} \xrightarrow{n \to 2n} \{2, 5, 3\} \xrightarrow{n \to n/2} \{2, 0, 5, 0, 3\}$

$x[n] = \begin{cases} |n| & -3 \le n \le 3 \\ 0 & \text{diğer durumlar} \end{cases}$ $\quad y[n] = \sum_{k=-\infty}^{n} x[k] = ?$

$x[n] = \{3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3\}$

$y[n] = \sum_{k=-\infty}^{n} x[k] = \{3, 5, 6, \underset{\uparrow}{6}, 7, 9, 12, 12 \ldots\}$

$h[n] = a^n u[n], \; a < 1$

$x[n] = u[n] - u[n-N]$

$y[n] = h[n] * x[n]$

$= \sum_{k=-\infty}^{\infty} x[k] h[n-k]$

$n < 0 \rightarrow y[n] = 0$

$0 \le n < N \rightarrow y[n] = \sum_{k=0}^{n} a^{n-k} = a^n \sum_{k=0}^{n} \left(\frac{1}{a}\right)^k = a^n \frac{1 - \left(\frac{1}{a}\right)^{n+1}}{1 - \frac{1}{a}} = \frac{a^{n+1} - 1}{a - 1}$

$n \ge N \rightarrow y[n] = \sum_{k=0}^{N-1} a^{n-k} = a^n \sum_{k=0}^{N-1} \left(\frac{1}{a}\right)^k = a^n \frac{1 - \left(\frac{1}{a}\right)^N}{1 - \frac{1}{a}} = \frac{1 - a^{-N}}{1 - a^{-1}} a^n$

$x[n] = \{0, 0, 0, 2, 4, 6\} = 2\delta[n-3] + 4\delta[n-4] + 6\delta[n-5]$

$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$

$\sum_{k=0}^{N} a_k h[n-k] = \sum_{k=0}^{M} b_k \delta[n-k]$

$\sum_{k=0}^{N} a_k \hat{h}[n-k] = \delta[n]$ $\rightarrow h[n] = \sum_{k=0}^{M} b_k \hat{h}(n-k)$

<u>nedensel ise</u>

$\hat{h}[n] = \left(c_1 s_1^n + c_2 s_2^n + \cdots + c_N s_N^n\right) u[n]$

$\hat{h}[0] = 1/a_0, \; \hat{h}[-1] = \hat{h}[-2] = \cdots = \hat{h}[-N+1] = 0$

$y[n] - a y[n-1] = x[n] \longrightarrow h[n] = ?$

$h[n] - a h[n-1] = \delta[n]$

$h[n] = a h[n-1] + \delta[n]$

$h[n]$ nedensel

$h[0] = a h[-1] + \delta[0] = 1$
$h[1] = a h[0] + \delta[1] = a$
$h[2] = a h[1] + \delta[2] = a^2$
$h[3] = a h[2] + \delta[3] = a^3$ } artıyor.

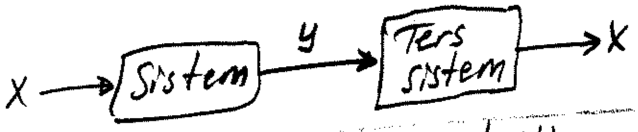
$h[n] = a^n u[n]$ bulunur. $|a| < 1$ için sistem kararlı.

$h[n] - 5h[n-1] + 6h[n-2] = \delta[n]$

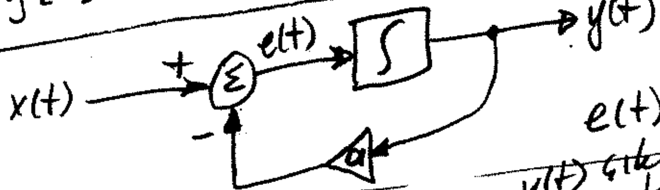
$y(t) = 2x(t) \longrightarrow x(t) = \frac{1}{2} y(t)$

$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \longrightarrow x(t) = \frac{dy(t)}{dt}$

$y[n] = \sum_{k=-\infty}^{n} x[k] \longrightarrow x[n] = y[n] - y[n-1]$ } Tersi alınabilir



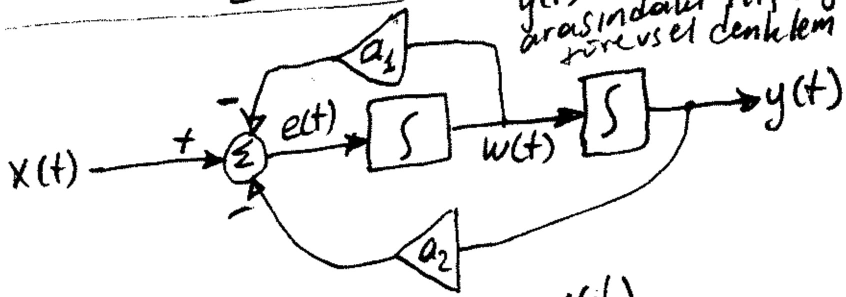
$y(t) = x^2(t)$
$y[n] = n\,x[n]$ } Tersi alınamaz



$y(t) = \int_{-\infty}^{t} e(\tau)\,d\tau \longrightarrow \frac{dy(t)}{dt} = e(t)$

$e(t) = x(t) - a y(t) \longrightarrow \frac{dy(t)}{dt} + a y(t) = x(t)$

~~y(t) çıkışı ile x(t) girişi~~ arasındaki ilişkiyi veren türevsel denklem



$w(t) = \int_{-\infty}^{t} e(\tau)\,d\tau \longrightarrow \frac{dw(t)}{dt} = e(t)$

$y(t) = \int_{-\infty}^{t} w(\tau)\,d\tau \longrightarrow \frac{dy(t)}{dt} = w(t)$

$e(t) = \frac{dw(t)}{dt} = \frac{d^2y(t)}{dt^2}$

$e(t) = x(t) - a_1 w(t) - a_2 y(t)$

$e(t) + a_1 w(t) + a_2 y(t) = x(t) \longrightarrow \frac{d^2y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_2 y(t) = x(t)$

$y[n] = x[2n]$, $n \geq 0$ sistem zamanla bağımlı mı?

$y[n] = x[2n]\,u[n]$

$x_1[n] = x[n-n_0] \longrightarrow y_1[n] = x[2n-2n_0]\,u[n]$

$y[n-n_0] = x[2n-2n_0].\,u[n-n_0]$

} $x[2n-2n_0]u[n] \neq x[2n-2n_0]u[n-n_0]$ zamanla bağımlı

$y[n] = n\,x[n]$ zamanla bağımlı mı?

$x_1[n] = x[n-n_0] \longrightarrow y_1[n] = n\,x[n-n_0]$

$y[n-n_0] = (n-n_0)\,x[n-n_0]$

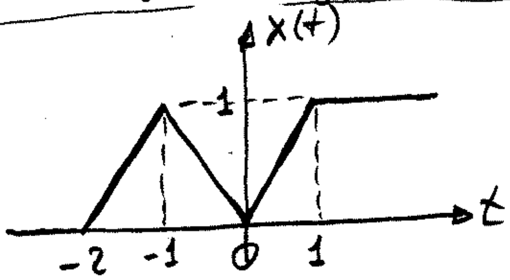
} $n\,x[n-n_0] \neq (n-n_0)\,x[n-n_0]$ zamanla bağımlı

$y[n] = x[n] + 3$ sistem doğrusal mı?

$x[n] = a_1 x_1[n] + a_2 x_2[n] \rightarrow y[n] = a_1 x_1[n] + a_2 x_2[n] + 3$

$y[n] = a_1 y_1[n] + a_2 y_2[n] = a_1 x_1[n] + a_2 x_2[n] + 3a_1 + 3a_2$

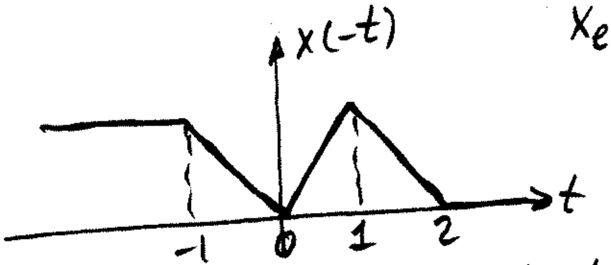
$\neq$ doğrusal değil



$x_e(t) = ?$ $x_o(t) = ?$

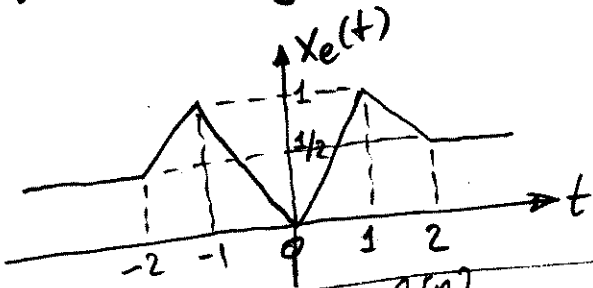
$x(t) = r(t+2) - 2r(t+1) + 2r(t) - r(t-1)$

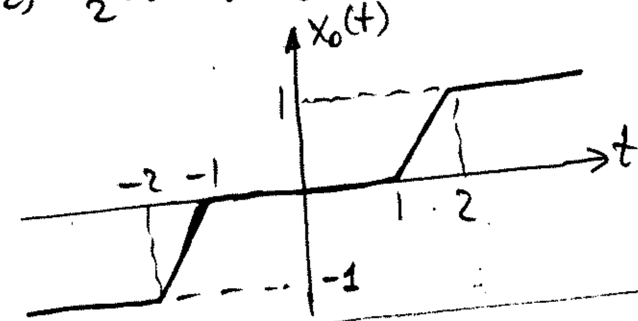
$x(-t) = 1 - r(t+1) + 2r(t) - 2r(t-1) + r(t-2)$

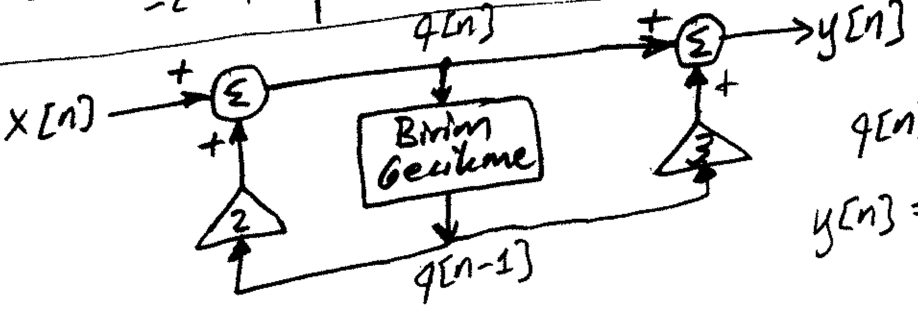


$$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{1}{2} + \frac{1}{2} r(t+2) - \frac{3}{2} r(t+1) + 2r(t) - \frac{3}{2} r(t-1) + \frac{1}{2} r(t-2)$$

$$x_o(t) = \frac{x(t)-x(-t)}{2} = -\frac{1}{2} + \frac{1}{2} r(t+2) - \frac{1}{2} r(t+1) + \frac{1}{2} r(t-1) - \frac{1}{2} r(t-2)$$







$q[n] = 2q[n-1] + x[n]$

$y[n] = q[n] + 3q[n-1]$

$q[n] - 2q[n-1] = x[n]$

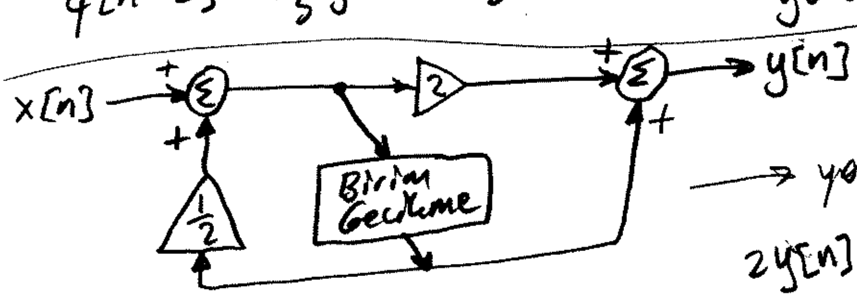
$q[n] + 3q[n-1] = y[n]$

$q[n] = \frac{2}{5} y[n] + \frac{3}{5} x[n] \rightarrow q[n-1] = \frac{2}{5} y[n-1] + \frac{3}{5} x[n-1]$

$q[n-1] = \frac{1}{5} y[n] - \frac{1}{5} x[n]$

Sonuç

$y[n] - 2y[n-1] = x[n] + 3x[n-1]$



$\rightarrow$ yanıt

$2y[n] - y[n-1] = 4x[n] + 2x[n-1]$

$y[n]$ çıkışı ile $x[n]$ girişi arasındaki ilişkiyi veren fark denklemi

Laplace Dönüşümü

$$x(t) \longleftrightarrow X(s) = \mathcal{L}(x(t)) = \int_{-\infty}^{\infty} x(t).e^{-st}.dt \ , \ s = \sigma + j\omega \ , \ s: \text{kompleks frekans}$$

YB: Yakınsama Bölgesi

$$x(t) = \mathcal{L}^{-1}(X(s)) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s).e^{st}.ds$$

$x(t) = \delta(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = \int_{-\infty}^{\infty} \delta(t) e^{-st} dt = \int_{0^-}^{0^+} \delta(t) dt = 1 \ , \ \text{All } s$$

$x(t) = u(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = \int_{0^+}^{\infty} e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_{0^+}^{\infty} = \frac{1}{s} \ , \ Re(s) > 0$$

$x(t) = e^{-at} u(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = \int_{0^+}^{\infty} e^{-(s+a)t} dt = -\frac{1}{s+a} e^{-(s+a)t} \Big|_{0^+}^{\infty} = \frac{1}{s+a} \ , \ Re(s+a) > 0$$

Doğrusallık özelliği

$$\left.\begin{matrix} x_1(t) \longleftrightarrow X_1(s) \ , \ YB = R_1 \\ x_2(t) \longleftrightarrow X_2(s) \ , \ YB = R_2 \end{matrix}\right\} a_1 x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 X_1(s) + a_2 X_2(s) \ , \ R' \supset R_1 \cap R_2$$

$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $x(t-t_0) \longleftrightarrow e^{-st_0} X(s)$, $R' = R$ zamanda öteleme

$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $e^{s_0 t} x(t) \longleftrightarrow X(s - s_0)$, $R' = R + Re(s_0)$ s bölgesinde öteleme

$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $x(at) \longleftrightarrow \frac{1}{|a|} X(s/a)$, $R' = aR$ zamanda ölçekleme

$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $x(-t) \longleftrightarrow X(-s)$, $R' = -R$ zamanda peri dönüş

$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $\frac{dx(t)}{dt} \longleftrightarrow sX(s)$, $R' \supset R$ zaman bölgesinde türev

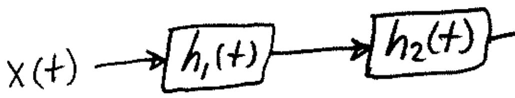
$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $-t x(t) \longleftrightarrow \frac{dX(s)}{ds}$, $R' = R$, s bölgesinde türev

$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $\int_{-\infty}^{t} x(\tau) d\tau \longleftrightarrow \frac{1}{s} X(s)$, $R' = R \cap \{Re(s) > 0\}$ Zaman Bölgesinde Entegral

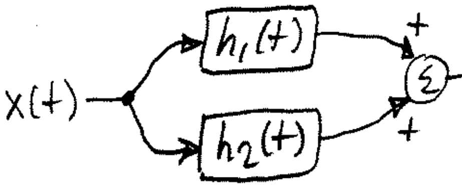
$x(t) \longleftrightarrow X(s)$, $YB = R$ ise $t^k x(t) \longleftrightarrow (-1)^k X^{(k)}(s)$, $R' = R$

$$t\,x(t) \longleftrightarrow -\frac{dX(s)}{ds} \qquad t^2 x(t) \longleftrightarrow \frac{d^2 X(s)}{ds} \qquad \frac{x(t)}{t} \longleftrightarrow \int_{s}^{\infty} F(s) ds$$

$$y(t) = h(t) * x(t) = x(t) * h(t) \Rightarrow Y(s) = H(s).X(s)$$



$$y(t) = \underbrace{h_1(t) * h_2(t)}_{h(t)} * x(t) \Rightarrow Y(s) = \underbrace{H_1(s) H_2(s)}_{H(s)} X(s)$$



$$y(t) = (h_1(t) + h_2(t)) * x(t)$$

$$h(t) = h_1(t) + h_2(t) \qquad H(s) = H_1(s) + H_2(s)$$

$\delta(t) \longleftrightarrow 1$, All s

$u(t) \longleftrightarrow \frac{1}{s}$, $Re(s) > 0$

$-u(-t) \longleftrightarrow \frac{1}{s}$, $Re(s) < 0$

$tu(t) \longleftrightarrow \frac{1}{s^2}$, $Re(s) > 0$

$-tu(-t) \longleftrightarrow \frac{1}{s^2}$, $Re(s) < 0$

$e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}$, $Re(s) > -Re(a)$

$-e^{-at}u(-t) \longleftrightarrow \frac{1}{s+a}$, $Re(s) < -Re(a)$

$te^{-at}u(t) \longleftrightarrow \frac{1}{(s+a)^2}$, $Re(s) > -Re(a)$

$-te^{-at}u(-t) \longleftrightarrow \frac{1}{(s+a)^2}$, $Re(s) < -Re(a)$

$t^k u(t) \longleftrightarrow \frac{k!}{s^{k+1}}$, $Re(s) > 0$

$t^k e^{-at}u(t) \longleftrightarrow \frac{k!}{(s+a)^{k+1}}$, $Re(s) > -Re(a)$

$-t^k u(-t) \longleftrightarrow \frac{k!}{s^{k+1}}$, $Re(s) < 0$

$-t^k e^{-at}u(-t) \longleftrightarrow \frac{k!}{(s+a)^{k+1}}$, $Re(s) < -Re(a)$

$Cos(w_0 t).u(t) \longleftrightarrow \frac{s}{s^2+w_0^2}$, $Re(s) > 0$

$Sin(w_0 t).u(t) \longleftrightarrow \frac{w_0}{s^2+w_0^2}$, $Re(s) < 0$

$e^{-at}.Cos(w_0 t).u(t) \longleftrightarrow \frac{s+a}{(s+a)^2+w_0^2}$, $Re(s) > -Re(a)$

$e^{-at}.Sin(w_0 t).u(t) \longleftrightarrow \frac{w_0}{(s+a)^2+w_0^2}$, $Re(s) < -Re(a)$

$$\cosh(at) = \frac{e^{at}+e^{-at}}{2}$$

$\cosh(at).u(t) \longleftrightarrow \frac{s}{s^2-a^2}$, $-Re(a) < Re(s) < Re(a)$

$$\sinh(at) = \frac{e^{at}-e^{-at}}{2}$$

$\sinh(at).u(t) \longleftrightarrow \frac{a}{s^2-a^2}$, $-Re(a) < Re(s) < Re(a)$

$x(t) = e^{-2t}u(t) + e^{-3t}u(t)$ $\qquad X(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$e^{-2t}u(t) \longleftrightarrow \frac{1}{s+2}$, $Re(s) > -2$ $\qquad = \frac{2s+5}{s^2+5s+6}$, $Re(s) > -2$

$e^{-3t}u(t) \longleftrightarrow \frac{1}{s+3}$, $Re(s) > -3$

$$\left.\begin{array}{l} x_1(t) \longleftrightarrow X_1(s),\ YB = R_1 \\ x_2(t) \longleftrightarrow X_2(s),\ YB = R_2 \end{array}\right\} \quad x_1(t) * x_2(t) \longleftrightarrow X_1(s)\,X_2(s)$$

$$R' \supset R_1 \cap R_2$$

$$\sum_{k=0}^{n} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{m} b_k \frac{d^k x(t)}{dt^k} \rightarrow \sum_{k=0}^{n} a_k s^k Y(s) = \sum_{k=0}^{m} b_k s^k X(s)$$

$$Y(s)\sum_{k=0}^{n} a_k s^k = X(s)\sum_{k=0}^{m} b_k s^k \rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{k=0}^{m} b_k s^k}{\sum_{k=0}^{n} a_k s^k}$$

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2 + \dots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \dots + a_n s^n} = K \cdot \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$m<n$ için

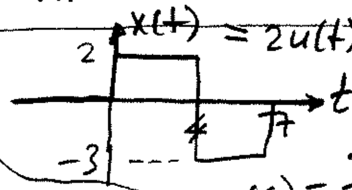
① Basit kutup durumu

$$H(s) = \frac{c_1}{s-p_1} + \frac{c_2}{s-p_2} + \dots + \frac{c_n}{s-p_n}, \quad c_k = (s-p_k)H(s)\Big|_{s=p_k}$$

② Katlı kutup durumu

$$\frac{\lambda_1}{s-p_i} + \frac{\lambda_2}{(s-p_i)^2} + \dots + \frac{\lambda_r}{(s-p_i)^r}$$

$$\lambda_{r-k} = \frac{1}{k!}\frac{d^k}{ds^k}\left((s-p_i)^r H(s)\right)\Big|_{s=p_i}$$



$$x(t) = 2u(t) - 5u(t-4) + 3u(t-7)$$

$$X(s) = \frac{2}{s} - \frac{5}{s}e^{-4s} + \frac{3}{s}e^{-7s} = \frac{1}{s}\left(2 - 5e^{-4s} + 3e^{-7s}\right)$$

$m \geq n$ için

$$H(s) = \frac{Y(s)}{X(s)} = Q(s) + \frac{R(s)}{X(s)}$$

→ $\frac{R(s)}{X(s)}$ kısmı önceki gibi

→ $Q(s)$ kısmı için

$$\frac{d^k \delta(t)}{dt^k} \longleftrightarrow s^k, \quad k=1,2,3,\dots$$

Tek Yanlı Laplace Dönüşümü

$$x(t) \longleftrightarrow X(s) = \int_{0^-}^{\infty} x(t) \cdot e^{-st} \cdot dt, \quad s = \sigma + jw$$

zaman bölgesinde türev

$$\frac{dx(t)}{dt} \longleftrightarrow sX(s) - x(0^-)$$

$$x^{(r)}(0^-) = \frac{d^r x(t)}{dt^r}\Big|_{t=0^-}$$

$$\frac{d^2 x(t)}{dt^2} \longleftrightarrow s^2 X(s) - s x(0^-) - x'(0^-)$$

$$\frac{d^3 x(t)}{dt^3} \longleftrightarrow s^3 X(s) - s^2 x(0^-) - s x'(0^-) - x''(0^-)$$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$$\frac{d^n x(t)}{dt^n} \longleftrightarrow s^n X(s) - \sum_{k=0}^{n-1} s^{n-k-1} \cdot x^{(k)}(0^-)$$

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

Zaman Bölgesinde Entegral

$$\int_{0^-}^{t} x(\tau)d\tau \longleftrightarrow \frac{1}{s}X(s)$$

$$\int_{-\infty}^{t} x(\tau)d\tau \longleftrightarrow \frac{1}{s}X(s) + \int_{-\infty}^{0^-} x(\tau)d\tau$$

$y''(t)+2y'(t)-3y(t)=0$, $y(0^-)=9$, $y'(0^-)=1$ için $y(t)=?$

$$\left(s^2Y(s)-sy(0^-)-y'(0^-)\right)+2\left(sY(s)-y(0^-)\right)-3Y(s)=0$$

$$(s^2+2s-3)Y(s)=sy(0^-)+y'(0^-)+2y(0^-)=9s+19$$

$$Y(s)=\frac{9s+19}{s^2+2s-3}=\frac{a}{s+3}+\frac{b}{s-1} \quad , a=2 , b=7$$

$$Y(s)=\frac{2}{s+3}+\frac{7}{s-1} \Rightarrow y(t)=\left(2e^{-3t}+7e^{t}\right)u(t)$$

$y''(t)-6y'(t)+13y(t)=0$, $y(0^-)=3$, $y'(0^-)=5$ için $y(t)=?$

$$\left(s^2Y(s)-sy(0^-)-y'(0^-)\right)-6\left(sY(s)-y(0^-)\right)+13Y(s)=0$$

$$(s^2-6s+13)Y(s)=sy(0^-)+y'(0^-)-6y(0^-)=3s-13$$

$$Y(s)=\frac{3s-13}{s^2-6s+13}=\frac{3(s-3)-2(2)}{(s-3)^2+4} \Rightarrow y(t)=(3\cos 2t-2\sin 2t)e^{3t}u(t)$$

$y''(t)-3y'(t)+2y(t)=2t^2+3$, $y(0^-)=4$, $y'(0^-)=-1 \Rightarrow y(t)=?$

yok

$$\left(s^2Y(s)-sy(0^-)-y'(0^-)\right)-3\left(sY(s)-y(0^-)\right)+2Y(s)=\frac{4}{s^3}+\frac{3}{s}$$

$$(s^2-3s+2)Y(s)=sy(0^-)+y'(0^-)-3y(0^-)+\frac{4}{s^3}+\frac{3}{s}=4s-13+\frac{3}{s}+\frac{4}{s^3}$$

$$Y(s)=\frac{4s^4-13s^3+3s^2+4}{s^3(s-1)(s-2)}=\frac{a}{s}+\frac{b}{s^2}+\frac{c}{s^3}+\frac{d}{s-1}+\frac{e}{s-2} \quad \begin{array}{l} a=5,\ b=3 \\ c=d=2 \\ e=-3 \end{array}$$

$$Y(s)=\frac{2}{s-1}-\frac{3}{s-2}+\frac{2}{s^3}+\frac{3}{s^2}+\frac{5}{s} \Rightarrow y(t)=\left(2e^{t}-3e^{2t}+t^2+3t+5\right)u(t)$$

$y''(t)+9y(t)=0$, $y(0^-)=2$, $y'(0^-)=3 \Rightarrow y(t)=?$

$$\left(s^2Y(s)-sy(0^-)-y'(0^-)\right)+9Y(s)=0$$

$$(s^2+9)Y(s)=sy(0^-)+y'(0^-)=2s+3$$

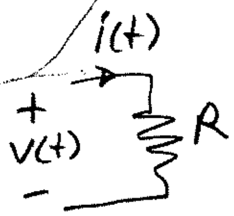
$$Y(s)=\frac{2s+3}{s^2+9} \Rightarrow y(t)=\left(2\cos 3t+\sin 3t\right)u(t)$$

① Sinyal Kaynakları

$v(t) \longleftrightarrow V(s) \qquad i(t) \longleftrightarrow I(s)$

② Direnç R

$v(t) = R\,i(t) \longleftrightarrow V(s) = R\,I(s)$



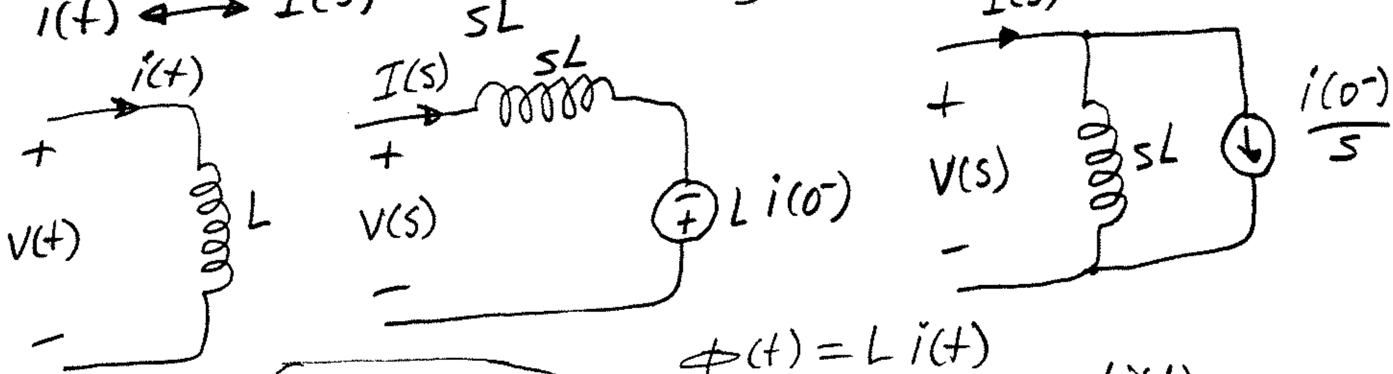
$Z_R = R$

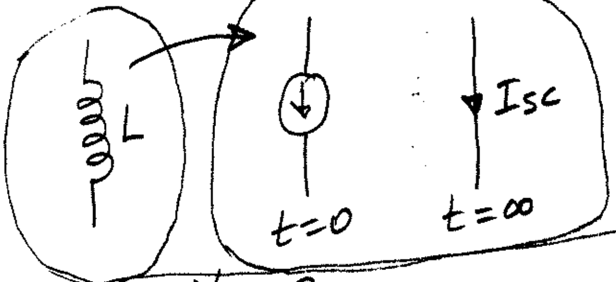
③ Endüktans L

$$v(t) = L\frac{di(t)}{dt} \longleftrightarrow V(s) = sL\,I(s) - L\,i(0^-)$$

$Z_L = sL$

$$i(t) \longleftrightarrow I(s) = \frac{1}{sL}V(s) + \frac{1}{s}i(0^-)$$





$$\phi(t) = L\,i(t)$$

$$v(t) = \frac{d\phi(t)}{dt} = L\frac{di(t)}{dt}$$

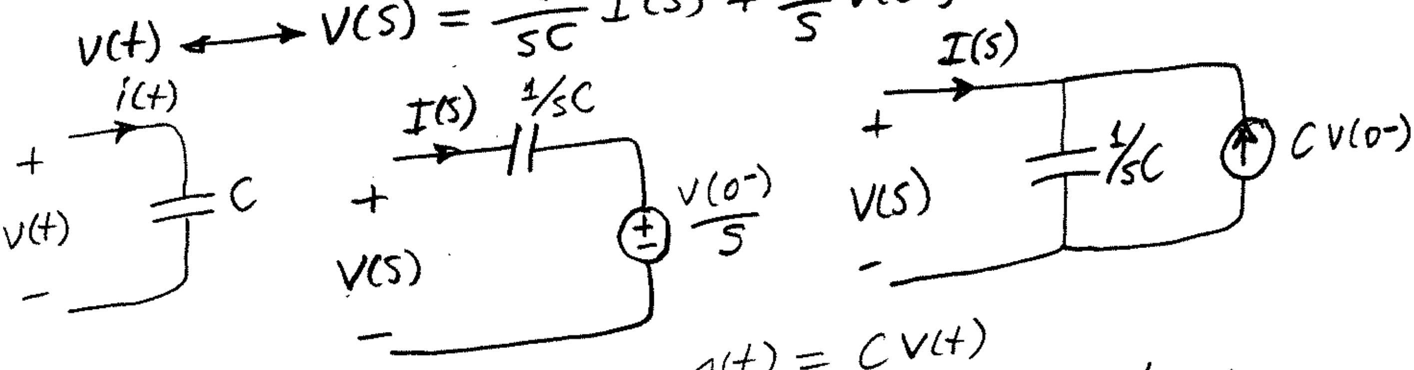
$$i(t) = i(0^-) + \frac{1}{L}\int_{0^-}^{t} v(z)\,dz$$

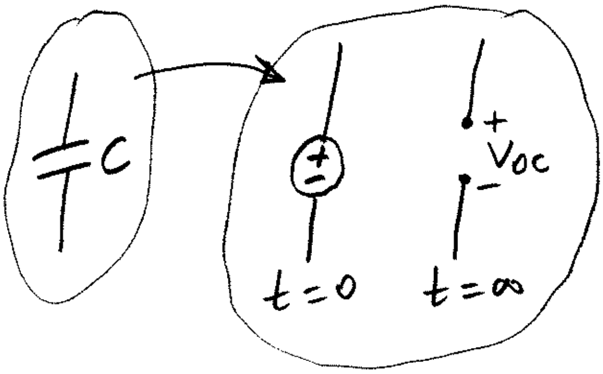
④ Kapasite C

$$i(t) = C\frac{dv(t)}{dt} \longleftrightarrow I(s) = sC\,V(s) - C\,v(0^-)$$

$Z_C = \frac{1}{sC}$

$$v(t) \longleftrightarrow V(s) = \frac{1}{sC}I(s) + \frac{1}{s}v(0^-)$$

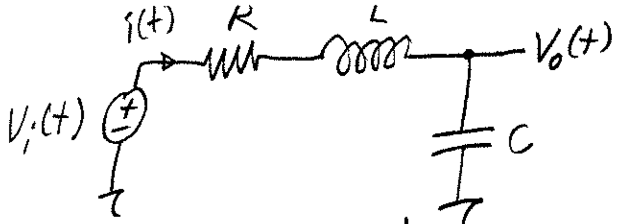




$$q(t) = C\,v(t)$$

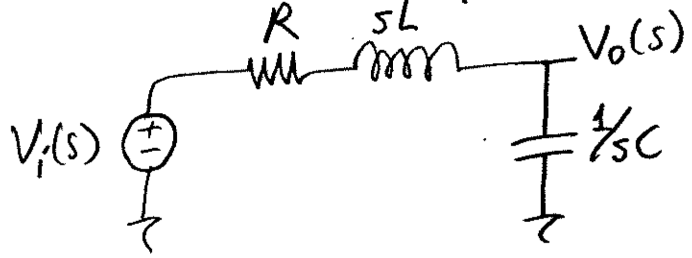
$$i(t) = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt}$$

$$v(t) = v(0^-) + \frac{1}{C}\int_{0^-}^{t} i(z)\,dz$$



$V_i(t) = 7V$, $R = 2\Omega$, $L = 1H$, $C = \frac{1}{50}F$

$i(0^-) = 0A$, $V_0(0^-) = 0 \Rightarrow V_0(t) = ?$



$$I(s) = \frac{V_i(s)}{R + sL + 1/sC}$$

$$V_0(s) = \frac{1}{sC} I(s) = \frac{V_i(s)}{LCs^2 + RCs + 1}$$

$$V_0(s) = \frac{7/s}{\frac{s^2}{50} + \frac{2s}{50} + 1} = \frac{350}{s^3 + 2s^2 + 50} = \frac{a}{s} + \frac{bs + c}{s^2 + 2s + 50} \quad \begin{array}{l} a = 7 \\ b = -7 \\ c = -14 \end{array}$$

$$V_0(s) = \frac{7}{s} - \frac{7s + 14}{(s+1)^2 + 7^2} = \frac{7}{s} - \frac{7(s+1)}{(s+1)^2 + 7^2} - \frac{7}{(s+1)^2 + 7^2}$$

$$V_0(t) = \left(7 - e^{-t}\left(7\cos 7t + \sin 7t\right)\right)u(t)$$

$$\frac{dy(t)}{dt} + 2y(t) = 3\frac{du(t)}{dt} + 2u(t), \quad y(0^-) = 2 \longrightarrow y(t)$$

$$U(s) = 1/s$$

$$sY(s) - y(0^-) + 2Y(s) = 3\left(sU(s) - u(0^-)\right) + 2U(s)$$

$$(s+2)Y(s) = 3sU(s) - \cancel{3u(0^-)} + 2U(s) + y(0^-) = 5 + \frac{2}{s} = \frac{5s+2}{s}$$

$$Y(s) = \frac{5s+2}{s(s+2)} = \frac{a}{s} + \frac{b}{s+2} \quad \begin{array}{l} a = 1 \\ b = 4 \end{array} \quad y(t) = \left(1 + 4e^{-2t}\right)u(t)$$

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = \frac{dx(t)}{dt} - x(t), \quad x(t) = e^{-t}u(t)$$

$y(0^-) = 0$, $y'(0^-) = 0$, $y(t) = ?$, $x(0^-) = 0$

$$s^2Y(s) + 2sY(s) + 5Y(s) = sX(s) - X(s) \qquad X(s) = \frac{1}{s+1}$$

$$Y(s) = \frac{s-1}{(s+1)(s^2+2s+5)} = \frac{a}{s+1} + \frac{bs+c}{s^2+2s+5} \quad a = -1/2,\ b = 1/2,\ c = 3/2$$

$$Y(s) = \frac{-1/2}{s+1} + \frac{1/2\, s + 3/2}{s^2+2s+5} = -\frac{1}{2}\frac{1}{s+1} + \frac{1}{2}\frac{s+1}{(s+1)^2+2^2} + \frac{1}{2}\frac{2}{(s+1)^2+2^2}$$

$$y(t) = -\frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-t}\cos 2t\, u(t) + \frac{1}{2}e^{-t}\sin 2t\, u(t)$$

$$= 0.5\, e^{-t}\left(-1 + \cos 2t + \sin 2t\right) \cdot u(t)$$

$X(s)=\frac{2s+5}{s^2+4s+3}$ veriliyor.

a) $Re(s)>-1$ için $x(t)=?$ b) $Re(-3)<-3$ için $x(t)=?$ c) $-3<Re(s)<-1$ için $x(t)=?$

$$X(s)=\frac{2s+5}{s^2+4s+3}=\frac{2s+5}{(s+1)(s+3)}=\frac{a}{s+1}+\frac{b}{s+3}$$

$$\left.\begin{aligned} a&=(s+1)X(s)\Big|_{s=-1}=\frac{2s+5}{s+3}\Big|_{s=-1}=3/2 \\ b&=(s+3)X(s)\Big|_{s=-3}=\frac{2s+5}{s+1}\Big|_{s=-3}=1/2 \end{aligned}\right\} X(s)=\frac{3}{2}\frac{1}{s+1}+\frac{1}{2}\frac{1}{s+3}$$

a) $Re(s)>-1$ için $x(t)=\left(\frac{3}{2}e^{-t}+\frac{1}{2}e^{-3t}\right).u(t)$, sağ yanlı sinyal

b) $Re(s)<-3$ için $x(t)=-\left(\frac{3}{2}e^{-t}+\frac{1}{2}e^{-3t}\right)u(-t)$, sol yanlı sinyal

c) $-3<Re(s)<-1$ için $x(t)=-\frac{3}{2}e^{-t}u(-t)+\frac{1}{2}e^{-3t}u(t)$, çift yanlı sinyal

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$X(s)=\frac{9s+13}{s(s^2+6s+13)}$, $Re(s)>0$ için $x(t)=?$

$$X(s)=\frac{9s+13}{s(s^2+6s+13)}=\frac{a}{s}+\frac{bs+c}{s^2+6s+13}$$

$$a=sX(s)\Big|_{s=0}=\frac{9s+13}{s^2+6s+13}\Big|_{s=0}=1$$

$$\frac{bs+c}{s^2+6s+13}=\frac{9s+13}{s(s^2+6s+13)}-\frac{1}{s}=\frac{3-s}{s^2+6s+13}$$

$$X(s)=\frac{1}{s}+\frac{3-s}{s^2+6s+13}=\underbrace{\frac{1}{s}}_{Re(s)>0}\underbrace{-\frac{s+3}{(s+3)^2+2^2}+3\frac{2}{(s+3)^2+2^2}}_{Re(s)>-3}$$ sağ taraflı

$$x(t)=\left[1-e^{-3t}(\cos 2t-3\sin 2t)\right]u(t)$$

$X(s)=\frac{s^2+6s+7}{s^2+3s+2}$, $Re(s)>-1$ için $x(t)=?$

$$X(s)=1+\underbrace{\frac{3s+5}{s^2+3s+2}}_{X_1(s)} \qquad X_1(s)=\frac{3s+5}{s^2+3s+2}=\frac{a}{s+1}+\frac{b}{s+2}$$

$$a=(s+1)X_1(s)\Big|_{s=-1}=\frac{3s+5}{s+2}\Big|_{s=-1}=2 \qquad b=(s+2)X_1(s)\Big|_{s=-2}=\frac{3s+5}{s+1}\Big|_{s=-2}=1$$

$$X(s)=1+\frac{2}{s+1}+\frac{1}{s+2}\Rightarrow x(t)=\delta(t)+\left(2e^{-t}+e^{-2t}\right)u(t)$$

$X(s) = \frac{1}{(s+a)^2}$, $Re(s) > -a$ için $x(t) = ?$

$$e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}, \quad Re(s) > -a$$

$$t\,x(t) \longleftrightarrow -\frac{dX(s)}{ds} = -\frac{d}{ds}\left(\frac{1}{s+a}\right) = \frac{1}{(s+a)^2} \Rightarrow x(t) = t e^{-at}u(t)$$

$h(t) = e^{-at}u(t)$, $x(t) = e^{at}u(-t)$, $a>0$ için $y(t) = ?$

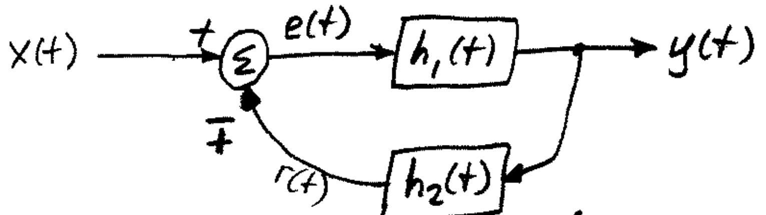
$$y(t) = h(t) * x(t) \longrightarrow Y(s) = H(s)X(s)$$

$$\left.\begin{aligned} H(s) &= \frac{1}{s+a}, \; Re(s) > -a \\ X(s) &= \frac{-1}{s-a}, \; Re(s) < a \end{aligned}\right\} \; Y(s) = -\frac{1}{s^2-a^2}, \quad -a < Re(s) < a$$

$$= \frac{1}{2a}\frac{1}{s+a} - \frac{1}{2a}\frac{1}{s-a}$$

$$y(t) = \frac{1}{2a}e^{-at}u(t) + \frac{1}{2a}e^{at}u(-t) = \frac{1}{2a}e^{-a|t|}$$

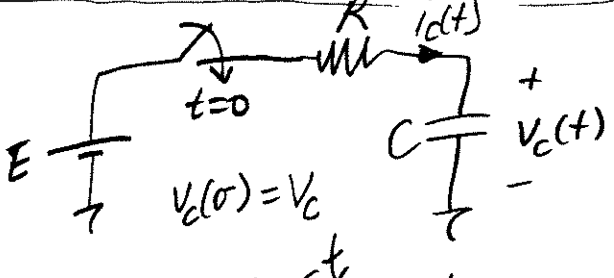
Geribeslemeli Sistem



$$y(t) = h_1(t) * e(t)$$
$$r(t) = h_2(t) * y(t)$$
$$e(t) = x(t) \mp r(t)$$

$$Y(s) = H_1(s)E(s) = H_1(s)\big(X(s) \mp R(s)\big) = H_1(s)\big(X(s) \mp H_2(s)Y(s)\big)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)}{1 \pm H_1(s)H_2(s)}$$



$$R i_c(t) + \frac{1}{C}\int_{-\infty}^{t} i_c(\tau)d\tau = E$$

$$V_c(0^-) = V_c = \frac{1}{C}\int_{-\infty}^{0^-} i_c(\tau)d\tau$$

$$R i_c(t) + \frac{1}{C}\int_{0^-}^{t} i_c(\tau)d\tau = E - V_c$$

$$\left(R + \frac{1}{sC}\right)I_c(s) = \frac{E-V_c}{s} \longrightarrow I_c(s) = \frac{E-V_c}{R}\,\frac{1}{s+1/RC}$$

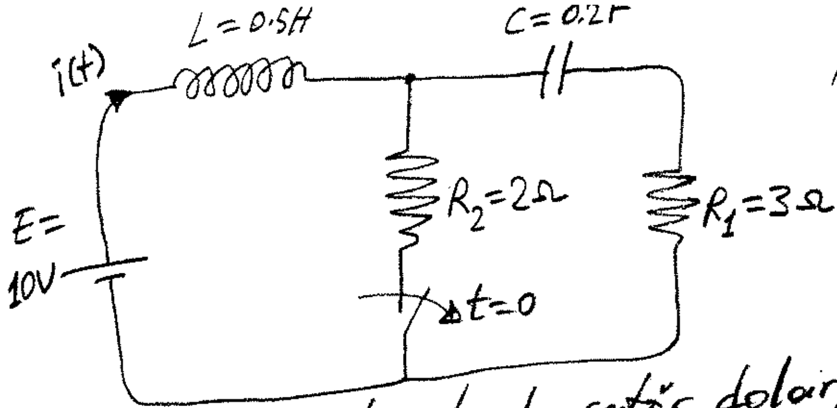
$$i_c(t) = \frac{E-V_c}{R}e^{-t/RC}u(t)$$

$$C\frac{dV_c(t)}{dt} = \frac{E-V_c(t)}{R} \longrightarrow \frac{dV_c(t)}{dt} + \frac{V_c(t)}{RC} = \frac{E}{RC}$$

$$sV_c(s) - V_c(0^-) + \frac{V_c(s)}{RC} = \frac{E}{sRC}$$

$$\left(s + \frac{1}{RC}\right)V_c(s) = V_c + \frac{E}{sRC} \longrightarrow V_c(s) = \frac{V_c}{s+1/RC} + \frac{E}{RC.\,s(s+1/RC)}$$

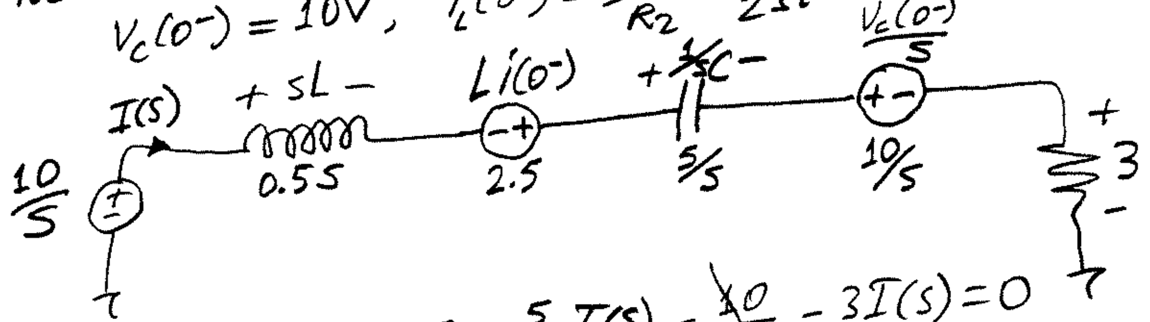
$$V_c(t) = \left(E\left(1 - e^{-t/RC}\right) + V_c e^{-t/RC}\right)u(t)$$



Anahtar uzun müddet kapalı iken $t=0$ anında açılıyor. $i(t)=?$

Anahtar kapalı iken kondansatör dolar ve akım geçirmez. Endüktans ise kısa devre olup üzerinden sabit akım geçer.

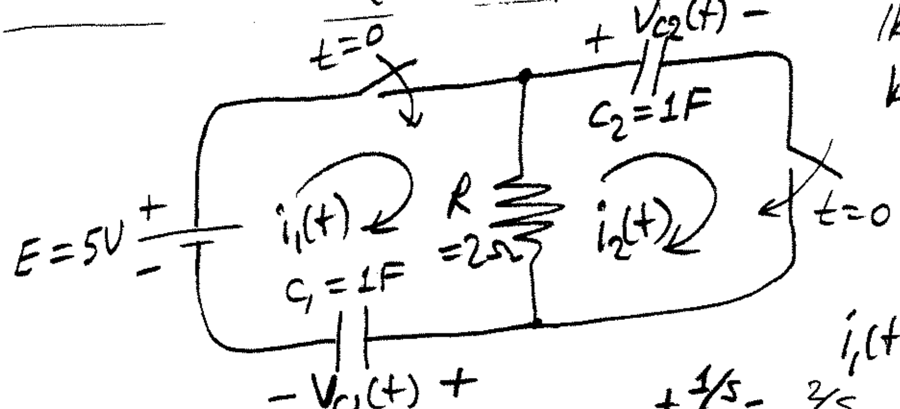
$V_C(0^-)=10V$, $i_L(0^-)=\frac{E}{R_2}=\frac{10V}{2\Omega}=5A=i(0^-)$



$$\cancel{\frac{10}{s}} - 0.5sI(s) + 2.5 - \frac{5}{s}I(s) - \cancel{\frac{10}{s}} - 3I(s) = 0$$

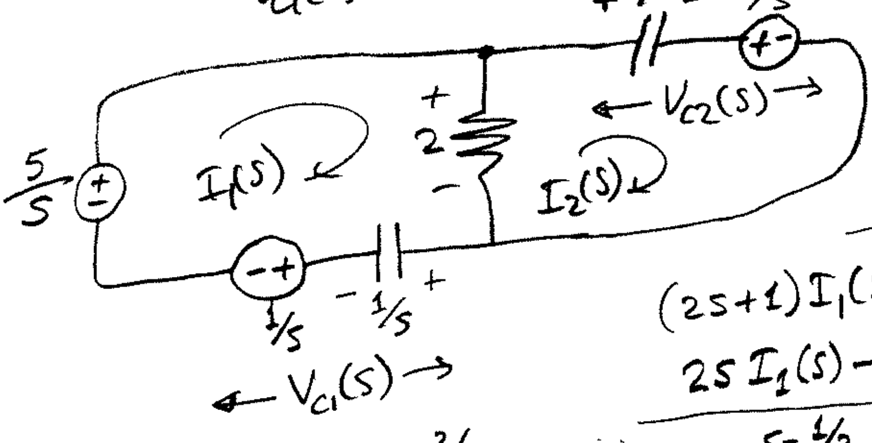
$$\left(0.5s+3+\frac{5}{s}\right)I(s)=2.5 \Rightarrow I(s)=\frac{5s}{s^2+6s+10}=\frac{5(s+3)-15.(1)}{(s+3)^2+1^2}$$

$$i(t)=5e^{-3t}(\cos t-3\sin t)u(t)$$



İki anahtar birden $t=0$ anında kapatılıyor. Anahtar açıkken C_1 ve C_2 kondansatörlerinin gerilimleri sırası ile 1V ve 2V'tur.

$i_1(t)=?$ $i_2(t)=?$ $V_{C1}(0^+)=?$ $V_{C2}(0^+)=?$



$$\left(2+\frac{1}{s}\right)I_1(s)-2I_2(s)=4/s$$

$$-2I_1(s)+\left(2+\frac{1}{s}\right)I_2(s)=-2/s$$

$$(2s+1)I_1(s)-2sI_2(s)=4$$

$$2sI_1(s)-(2s+1)I_2(s)=2$$

$$I_1(s)=\frac{s+1}{s+1/4}=1+\frac{3/4}{s+1/4}$$

$$i_1(t)=\delta(t)+\frac{3}{4}e^{-t/4}u(t)$$

$$I_2(s)=\frac{s-1/2}{s+1/4}=1-\frac{3/4}{s+1/4}$$

$$i_2(t)=\delta(t)-\frac{3}{4}e^{-t/4}u(t)$$

$$V_{C1}(s) = \frac{1}{s} I_1(s) + \frac{1}{s} = \frac{1}{s} \cdot \frac{s+1}{s+1/4} + \frac{1}{s} = \frac{2s+5/4}{s(s+1/4)}$$

$$V_{C2}(s) = \frac{1}{s} I_2(s) + \frac{2}{s} = \frac{1}{s} \frac{s-1/2}{s+1/4} + \frac{2}{s} = \frac{3}{s+1/4}$$

$$V_{C1}(0^+) = \lim_{s\to\infty} sV_{C1}(s) = \lim_{s\to\infty} \frac{2s+5/4}{s+1/4} = 2V \qquad \overset{2}{V_{C1}(0^+)} \neq \overset{1}{V_{C1}(0^-)}$$

$$V_{C2}(0^+) = \lim_{s\to\infty} sV_{C2}(s) = \lim_{s\to\infty} \frac{3s}{s+1/4} = 3V \qquad \overset{3}{V_{C2}(0^+)} \neq \overset{2}{V_{C2}(0^-)}$$

$x(t) = \mathrm{sgn}(t) \longrightarrow X(s) = ?$

$$x(t) = \mathrm{sgn}(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t<0 \end{cases} = u(t) - u(-t)$$

YA aralığı olmadığından $X(s)$ mevcut değil.

$$X(s) = \frac{1}{s(s+1)^2}, \ Re(s) > -1 \Rightarrow x(t) = ?$$

$$X(s) = \underset{(s+1)^2}{\frac{a}{s}} + \underset{s(s+1)}{\frac{b}{s+1}} + \underset{s}{\frac{c}{(s+1)^2}} = \frac{(a+b)s^2 + (2a+b+c)s + a}{s(s+1)^2} \qquad \begin{matrix} a=1 \\ b=c=-1 \end{matrix}$$

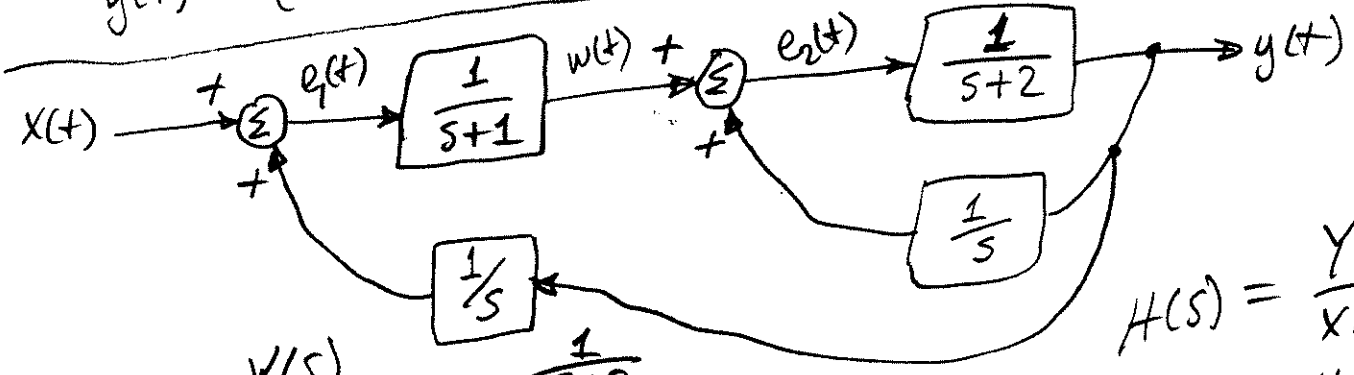
$$X(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} \Rightarrow x(t) = (1 - e^{-t} - te^{-t}).u(t)$$

$$h(t) = e^{-2t}u(t), \ x(t) = e^{-t}u(t) \Rightarrow y(t) = ?$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = h(t) * x(t)$$

$$Y(s) = H(s)X(s) = \frac{1}{s+2} \cdot \frac{1}{s+1} = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = (e^{-t} - e^{-2t})u(t)$$



$$H(s) = \frac{Y(s)}{X(s)}$$

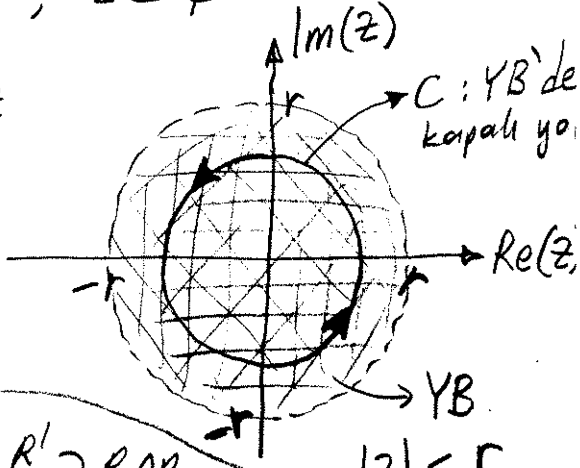
$$H_a(s) = \frac{Y(s)}{W(s)} = \frac{\frac{1}{s+2}}{1 - \frac{1}{s}\frac{1}{s+2}} = \frac{s}{s^2+2s-1} = \frac{H_b(s)}{1 - H_b(s) \cdot 1/s}$$

$$H_b(s) = \frac{Y(s)}{E_1(s)} = \frac{1}{s+1} H_a(s) = \frac{s}{(s+1)(s^2+2s-1)} = \frac{s}{s^3+3s^2+s-2}$$

Z Dönüşümü

$$x[n] \longleftrightarrow X(z) = \mathcal{Z}(x[n]) = \sum_{n=-\infty}^{\infty} x[n] . z^{-n}, \quad z \in \mathbb{C}$$

$$x[n] = \mathcal{Z}^{-1}(X(z)) = \frac{1}{2\pi j} \oint_C X(z) . z^{n-1} . dz$$



$$\frac{1}{2\pi j} \oint_C \frac{f(z)}{z^{n+1}} dz = \frac{\left.\frac{d^n f(z)}{dz^n}\right|_{z=0}}{n!}$$

Cauchy integral formülü

$a_1 x_1[n] + a_2 x_2[n] \longleftrightarrow a_1 X_1(z) + a_2 X_2(z)$, $R' \supset R_1 \cap R_2$ Doğrusallık

$x[n-n_0] \longleftrightarrow z^{-n_0} X(z)$, $R' = R \cap \{0 < |z| < \infty\}$ Zamanda öteleme

$z_0^n x[n] \longleftrightarrow X(z/z_0)$, $R' = |z_0| . R$, z_0^n ile çarpma

$x[-n] \longleftrightarrow X(1/z)$, $R' = 1/R$, Zamanda geri dönüş

$n x[n] \longleftrightarrow -z \dfrac{dX(z)}{dz}$, $R' = R$ n ile çarpma (z'ye göre türev)

$\sum_{k=-\infty}^{n} x[k] \longleftrightarrow \dfrac{z}{z-1} . X(z)$, $R' \supset R \cap \{|z| > 1\}$ Birikim

$x_1[n] * x_2[n] \longleftrightarrow X_1(z) . X_2(z)$, $R' \supset R_1 \cap R_2$ Konvolüsyon

$x[n] = \delta[n]$ ise $X(z) = ?$

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \delta[0] = 1$, tüm z düzleminde yakınsak.

$x[n] = \delta[n+3] - 7\delta[n-2]$ ise $X(z) = ?$ $z=0$ ve $z = \mp\infty$ hariç tüm z düzleminde yakınsak

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = z^3 - 7z^{-2} = \dfrac{z^5 - 7}{z^2}$

$x[n] = \{0, 0, 1, 5, 0, 3\}$ ise $X(z) = ?$ x[n] sağ taraflı sinyal

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = z^{-2} + 5z^{-3} + 3z^{-5} = \dfrac{z^3 + 5z^2 + 3}{z^5}$ $z = 0$ hariç tüm z düz. yakınsak

$\lim_{z \to 0} X(z) = \infty$ $\quad \lim_{z \to \mp\infty} X(z) = 0$

$x[n] = \{3, 0, \underset{\uparrow}{2}, 1, 0, 5\} \Rightarrow X(z) = ?$

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = 3z^2 + 2 + z^{-1} + 5z^{-3} = 3z^2 + 2 + \dfrac{z^2 + 5}{z^3}$

$z = 0$ ve $z = \mp\infty$ hariç tüm z düzleminde yakınsak

$x[n] = u[n] \Rightarrow X(z) = ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} (z^{-1})^n = \frac{1}{1-z^{-1}} = \frac{z}{z-1}, \quad |z| > 1$$

$x[n] = a^n u[n] \Rightarrow X(z) = ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n = \frac{1}{1-a/z} = \frac{z}{z-a}, \quad |z| > |a|$$

$x[n] = -a^n u[-n-1] \Rightarrow X(z) = ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = -\sum_{n=-\infty}^{-1} \left(\frac{a}{z}\right)^n = -\sum_{n=1}^{\infty} \left(\frac{z}{a}\right)^n = 1 - \sum_{n=0}^{\infty} (z/a)^n$$

$$= 1 - \frac{1}{1-z/a} = \frac{z}{z-a}, \quad |z| < |a|$$

$\delta[n] \longleftrightarrow 1$, tüm z düzleminde yakınsak

$\delta[n-m] \longleftrightarrow z^{-m}$, $m>0$ için $z=0$, $m<0$ için $z=\mp\infty$ hariç tüm z düzleminde yakınsak

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$a^n u[n] \longleftrightarrow \dfrac{z}{z-a}, \quad |z| > |a|$

$-a^n u[-n-1] \longleftrightarrow \dfrac{z}{z-a}, \quad |z| < |a|$

$u[n] \longleftrightarrow \dfrac{z}{z-1}, \quad |z| > 1$

$-u[-n-1] \longleftrightarrow \dfrac{z}{z-1}, \quad |z| < 1$

$n a^n u[n] \longleftrightarrow \dfrac{az}{(z-a)^2}, \quad |z| > |a|$

$-n a^n u[-n-1] \longleftrightarrow \dfrac{az}{(z-a)^2}, \quad |z| < |a|$

$$x[n] = \begin{cases} a^n & n \geq 0 \\ b^n & n < 0 \end{cases} \Longrightarrow X(z) = ?, \quad YB = ?$$

$x[n] = a^n u[n] + b^n u[-n-1]$

$$X(z) = \underbrace{\frac{z}{z-a}}_{|z|>|a|} - \underbrace{\frac{z}{z-b}}_{|z|<|b|} = \frac{(a-b)z}{(z-a)(z-b)}$$

$|a| \geq |b|$ ise $YB = \phi$

$|a| < |b|$ ise $YB = \{ z / |a| < |z| < |b| \}$

$x[n]$ nedensel

$$\lim_{z\to\infty} X(z) = x[0]$$

$$\lim_{z\to 1} \left(1 - \frac{1}{z}\right) X(z) = \lim_{z\to\infty} x[n]$$

$X(z) = \dfrac{2z^3 - 8z^2 + 7z + 12}{z^2 - 5z + 6}$, $|z| < 2$ ise $x[n] = ?$

$X(z) = 2z + 1 + \dfrac{6}{(z-2)(z-3)}$, $X_1(z) = \dfrac{6}{(z-2)(z-3)}$

$\dfrac{X_1(z)}{z} = \dfrac{6}{z(z-2)(z-3)} = \dfrac{c_1}{z} + \dfrac{c_2}{z-2} + \dfrac{c_3}{z-3}$

$c_1 = \dfrac{6}{(z-2)(z-3)}\Big|_{z=0} = 1$, $c_2 = \dfrac{6}{z(z-3)}\Big|_{z=2} = -3$, $c_3 = \dfrac{6}{z(z-2)}\Big|_{z=3} = 2$

$\dfrac{X_1(z)}{z} = \dfrac{1}{z} - \dfrac{3}{z-2} + \dfrac{2}{z-3} \Rightarrow X_1(z) = 1 - 3\dfrac{z}{z-2} + 2\dfrac{z}{z-3}$

$X(z) = 2z - 3\dfrac{z}{z-2} + 2\dfrac{z}{z-3} + 2$

$x[n] = 2\delta[n+1] + 3 \times 2^n u[-n-1] - 2 \times 3^n u[-n-1] + 2\delta[n]$

$= 2\delta[n+1] + (3 \times 2^n - 2 \times 3^n) u[-n-1] + 2\delta[n]$

$X(z) = \dfrac{\cancel{z}}{\cancel{z}(z-1)(z-2)^2}$, $|z| > 2$ ise $x[n] = ?$

$\dfrac{X(z)}{z} = \dfrac{1}{(z-1)(z-2)^2} = \dfrac{c_1}{z-1} + \dfrac{c_2}{z-2} + \dfrac{c_3}{(z-2)^2}$

$c_1 = \dfrac{1}{(z-2)^2}\Big|_{z=1} = 1$, $c_3 = \dfrac{1}{z-1}\Big|_{z=2} = 1$

$\dfrac{1}{z-1} + \dfrac{c_2}{z-2} + \dfrac{1}{(z-2)^2} = \dfrac{1}{(z-1)(z-2)^2} \Rightarrow z = 0$ için bak

$-1 - \dfrac{c_2}{2} + \dfrac{1}{4} = -\dfrac{1}{4}$

$c_2 = -1$

$\dfrac{X(z)}{z} = \dfrac{1}{z-1} - \dfrac{1}{z-2} + \dfrac{1}{(z-2)^2}$

$X(z) = \dfrac{z}{z-1} - \dfrac{z}{z-2} + \dfrac{z}{(z-2)^2}$, $|z| > 2$

$na^n u[n] \longleftrightarrow \dfrac{az}{(z-a)^2}$, $|z| > |a|$

$x[n] = u[n] - 2^n u[n] + \dfrac{1}{2} \cdot n \cdot 2^n u[n]$

$= (1 - 2^n + n 2^{n-1}) u[n]$

$X(z) = \ln\left(\frac{z}{z-a}\right)$, $|z|>|a|$ ise $x[n]=?$

$$\ln(1-r) = -\sum_{n=1}^{\infty} \frac{r^n}{n}, \quad |r|<1$$

$$X(z) = \ln\left(\frac{z}{z-a}\right) = \ln\left(\frac{1}{1-az^{-1}}\right) = -\ln(1-az^{-1})$$

$$= \sum_{n=1}^{\infty} \frac{(az^{-1})^n}{n} = \sum_{n=1}^{\infty} \frac{a^n z^{-n}}{n}, \quad |az^{-1}|<1 \Rightarrow |z|>|a|$$

$$x[n] = \frac{1}{n} a^n u[n-1]$$

$X(z) = \ln\left(\frac{a}{a-z}\right)$, $|z|<|a|$ ise $x[n]=?$

$$\ln(1-r) = -\sum_{n=1}^{\infty} \frac{r^n}{n}, \quad |r|<1$$

$$X(z) = \ln\left(\frac{a}{a-z}\right) = \ln\left(\frac{1}{1-a^{-1}z}\right) = -\ln(1-a^{-1}z)$$

$$= \sum_{n=1}^{\infty} \frac{(a^{-1}z)^n}{n} = \sum_{n=-\infty}^{-1} \frac{(a^{-1}z)^{-n}}{-n} = \sum_{n=-\infty}^{-1} \frac{a^n z^{-n}}{-n}, \quad |a^{-1}z|<1 \Rightarrow |z|<|a|$$

$$x[n] = -\frac{1}{n} a^n u[-n-1]$$

$x[n]=u[n]$ iken $y[n] = 2^n u[n]$ ise $h[n]=?$

$x[n] = 3^n u[n]$ iken $y[n]=?$

$$y[n] = h[n] * x[n] \Rightarrow Y(z) = H(z)X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{\frac{z}{z-2}}{\frac{z}{z-1}} = \frac{z-1}{z-2} \Rightarrow \frac{H(z)}{z} = \frac{z-1}{z(z-2)} = \frac{c_1}{z} + \frac{c_2}{z-2}$$

$$c_1 = \left.\frac{z-1}{z-2}\right|_{z=0} = \frac{1}{2}, \quad c_2 = \left.\frac{z-1}{z}\right|_{z=2} = \frac{1}{2} \qquad H(z) = \frac{1}{2} + \frac{1}{2}\frac{z}{z-2}$$

$$h[n] = \frac{1}{2}\delta[n] + \frac{1}{2} \times 2^n u[n] = \frac{\delta[n]}{2} + 2^{n-1} u[n]$$

$$Y(z) = H(z)X(z) = \frac{z-1}{z-2} \times \frac{z}{z-3} \Rightarrow \frac{Y(z)}{z} = \frac{z-1}{(z-2)(z-3)} = \frac{c_1}{z-2} + \frac{c_2}{z-3}$$

$$c_1 = \left.\frac{z-1}{z-3}\right|_{z=2} = -1, \quad c_2 = \left.\frac{z-1}{z-2}\right|_{z=3} = 2 \qquad Y(z) = -\frac{z}{z-2} + 2\frac{z}{z-3}$$

$$y[n] = (2 \times 3^n - 2^n) u[n]$$

$x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n] \Rightarrow X(z) = ?$ ve $YB = ?$

$x_1[n] = \left(\frac{1}{2}\right)^n u[n] \longleftrightarrow X_1(z) = \frac{z}{z - 1/2}, \quad |z| > \frac{1}{2}$

$x_2[n] = \left(\frac{1}{3}\right)^n u[n] \longleftrightarrow X_2(z) = \frac{z}{z - 1/3}, \quad |z| > 1/3$

$X(z) = \frac{z}{z-1/2} + \frac{z}{z-1/3} = \frac{z(2z - 5/6)}{(z-1/2)(z-1/3)}, \quad |z| > 1/2$

$x[n] = a^{|n|}, \; 0 < a < 1 \Rightarrow X(z) = ?$ ve $YB = ?$

$x[n] = a^{|n|} = a^n u[n] + a^{-n} u[-n-1] = x_1[n] - x_2[n]$

$x_1[n] = a^n u[n] \longleftrightarrow X_1(z) = \frac{z}{z-a}, \quad |z| > |a| = a$

$x_2[n] = -a^{-n} u[-n-1] \longleftrightarrow X_2(z) = \frac{z}{z - 1/a} = \frac{az}{az-1}, \quad |z| < \left|\frac{1}{a}\right| = \frac{1}{a}$

$X(z) = X_1(z) - X_2(z) = \frac{z}{z-a} - \frac{az}{az-1}, \quad YB = \{ z \,/\, a < |z| < 1/a \}$

$X(z) = \frac{z^3 - 5z + 9}{z^2 - 3z + 2}, \quad |z| > 2$ ise $x(n) = ?$

$X(z) = z + 3 + \frac{2z+3}{z^2-3z+2} = z + 3 - \frac{5}{z-1} + \frac{7}{z-2} = z + 3 - 5z^{-1}\frac{z}{z-1} + 7z^{-1}\frac{z}{z-2}$

$x[n] = \delta[n+1] + 3\delta[n] - 5u[n-1] + 7 \times 2^{n-1} u[n-1]$

$= \delta[n+1] + 3\delta[n] + \left(7 \times 2^{n-1} - 5\right) u[n-1]$

$X(z) = \frac{az^{-1}}{(1-az^{-1})^2}, \quad |z| > |a| \Rightarrow x[n] = ?$

$X_1(z) = \frac{z}{z-a} = \frac{1}{1-az^{-1}} \longleftrightarrow x_1[n] = a^n u(n), \quad |z| > |a|$

$n x_1[n] \longleftrightarrow -z \frac{dX(z)}{dz} = -z \frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right) = -z \times \frac{-az^{-2}}{(1-az^{-1})^2}$

$= \frac{az^{-1}}{(1-az^{-1})^2}, \quad |z| > |a|$

$x[n] = n x_1[n] = n \cdot a^n \cdot u[n], \quad |z| > |a|$

$x[n]$ nedensel ise $\lim_{z\to\infty} X(z) = x[0]$ olur.

$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$ <28

Tek yanlı z dönüşümü

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} x[n] z^{-n}$$

$$= x[0] + x[1].z^{-1} + x[2].z^{-2} + x[3].z^{-3} + \cdots$$

$z \to \infty$ iken $\frac{1}{z} \to 0$ gider. Dolayısıyla $X(z) \to x[0]$

$$x[n-m] \longleftrightarrow z^{-m} X(z) + \sum_{k=0}^{m-1} z^{k-m+1} x[-k-1] \quad , \quad m \geq 0$$

$$x[n+m] \longleftrightarrow z^{m} X(z) - \sum_{k=0}^{m-1} z^{m-k} x[k] \quad , \quad m \geq 0$$

Tek yanlı z dönüşümü için formüller

$$x[n-4] \longleftrightarrow z^{-4} X(z) + z^{-3} x[-1] + z^{-2} x[-2] + z^{-1} x[-3] + x[-4]$$

$$x[n-3] \longleftrightarrow z^{-3} X(z) + z^{-2} x[-1] + z^{-1} x[-2] + x[-3]$$

$$x[n-2] \longleftrightarrow z^{-2} X(z) + z^{-1} x[-1] + x[-2]$$

$$x[n-1] \longleftrightarrow z^{-1} X(z) + x[-1]$$

$$x[n] \longleftrightarrow X(z)$$

$$x[n+1] \longleftrightarrow z X(z) - z x[0]$$

$$x[n+2] \longleftrightarrow z^{2} X(z) - z^{2} x[0] - z x[1]$$

$$x[n+3] \longleftrightarrow z^{3} X(z) - z^{3} x[0] - z^{2} x[1] - z x[2]$$

$$x[n+4] \longleftrightarrow z^{4} X(z) - z^{3} x[0] - z^{2} x[1] - z x[2] - x[3]$$

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$\sum_{k=0}^{N} a_k z^{-k} Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$x[n]$ ve $y[n]$ nedensel olsun.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

$y[n+2] + 3y[n+1] + 2y[n] = 0$, $y[0] = 2$, $y[1] = 3 \Rightarrow y[n] = ?$

$$\left(z^2 Y(z) - z^2 y[0] - z y[1]\right) + 3\left(z Y(z) - z y[0]\right) + 2Y(z) = 0$$

$$(z^2 + 3z + 2) Y(z) = z^2 y[0] + z y[1] + 3z y[0] = 2z^2 + 9z$$

$$\frac{Y(z)}{z} = \frac{2z+9}{z^2+3z+2} = \frac{a}{z+1} + \frac{b}{z+2} \qquad a = 7 \quad b = -5$$

$$Y(z) = 7\frac{z}{z+1} - 5\frac{z}{z+2}$$

$$y[n] = \left[7 \times (-1)^n - 5(-2)^n\right].u[n]$$

Sr $y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=x[n]$ nedensel ayrık bir sistem.

a) Sistemin dürtü tepkisi ($h[n]=?$)

b) Sistemin basamak tepkisi ($s[n]=?$)

a) $Y(z)-\frac{3}{4}z^{-1}Y(z)+\frac{1}{8}z^{-2}Y(z)=X(z)$

$$\left(1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right)Y(z)=X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{z^2}{z^2-\frac{3}{4}z+\frac{1}{8}}=\frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})}, \quad |z|>\frac{1}{2}$$

$$\frac{H(z)}{z}=\frac{z}{(z-\frac{1}{2})(z-\frac{1}{4})}=\frac{C_1}{z-1/2}+\frac{C_2}{z-1/4}$$

$$C_1=\frac{z}{z-\frac{1}{4}}\Big|_{z=1/2}=2, \quad C_2=\frac{z}{z-1/2}\Big|_{z=1/4}=-1$$

$$H(z)=2\frac{z}{z-1/2}-\frac{z}{z-1/4}, \quad |z|>1/2$$

$$h[n]=\left[2\times\left(\frac{1}{2}\right)^n-\left(\frac{1}{4}\right)^n\right]u[n]$$

b) $x[n]=u[n]$ olursa $X(z)=\frac{z}{z-1}, \quad |z|>1$

$$Y(z)=H(z)X(z)=S(z)=\frac{z^2}{z^2-\frac{3}{4}z+\frac{1}{8}}\times\frac{z}{z-1}, \quad |z|>1$$

$$S(z)=\frac{z^3}{(z-\frac{1}{2})(z-\frac{1}{4})(z-1)}, \quad |z|>1$$

$$\frac{S(z)}{z}=\frac{z^2}{(z-1)(z-1/2)(z-1/4)}=\frac{C_1}{z-1}+\frac{C_2}{z-1/2}+\frac{C_3}{z-1/4}$$

$$C_1=\frac{z^2}{(z-\frac{1}{2})(z-1/4)}\Big|_{z=1}=8/3, \quad C_2=\frac{z^2}{(z-1)(z-1/4)}\Big|_{z=1/2}=-2, \quad C_3=\frac{z^2}{(z-1)(z-1/2)}\Big|_{z=1/4}=1/3$$

$$S(z)=\frac{8}{3}\frac{z}{z-1}-2\frac{z}{z-1/2}+\frac{1}{3}\frac{z}{z-1/4}, \quad |z|>1$$

$$s[n]=\left(\frac{8}{3}-2\left(\frac{1}{2}\right)^n+\frac{1}{3}\left(\frac{1}{4}\right)^n\right)u[n]$$

11 $y[n] - \frac{1}{2} y[n-1] = x[n]$, $x[n] = \left(\frac{1}{3}\right)^n$, $y[-1] = 1$ ise $y[n] = ?$

$$Y(z) - \frac{1}{2}\left(z^{-1}Y(z) + y[-1]\right) = X(z)$$

$$\left(1 - \frac{1}{2}z^{-1}\right)Y(z) = X(z) + \frac{1}{2}y[-1] = \frac{z}{z-1/3} + \frac{1}{2}$$

$$Y(z) = \frac{\frac{z}{z-1/3} + \frac{1}{2}}{\frac{z-1/2}{z}} = \frac{z\left(\frac{3}{2}z - 1/6\right)}{\left(z-\frac{1}{3}\right)\left(z-1/2\right)} \qquad c_1 = \left.\frac{3/2\, z - 1/6}{z - 1/2}\right|_{z=1/3} = -2$$

$$\frac{Y(z)}{z} = \frac{\frac{3}{2}z - 1/6}{\left(z-\frac{1}{3}\right)\left(z-1/2\right)} = \frac{c_1}{z-1/3} + \frac{c_2}{z-1/2} \qquad c_2 = \left.\frac{3/2\, z - 1/6}{z-1/3}\right|_{z=1/2} = 7/2$$

$$Y(z) = \frac{7}{2}\frac{z}{z-1/2} - 2\frac{z}{z-1/3} \Rightarrow y[n] = 7\left(\frac{1}{2}\right)^{n+1} - 2\left(\frac{1}{3}\right)^n, \quad n \geq -1$$

$3y[n] - 4y[n-1] + y[n-2] = x[n]$, $x[n] = \left(\frac{1}{2}\right)^n$

$y[-1] = 1$, $y[-2] = 2$ ise $y[n] = ?$

$$3Y(z) - 4\left(z^{-1}Y(z) + y[-1]\right) + \left(z^{-2}Y(z) + z^{-1}y[-1] + y[-2]\right) = X(z)$$

$$\left(3 - 4z^{-1} + z^{-2}\right)Y(z) = X(z) + 4y[-1] - z^{-1}y[-1] - y[-2]$$

$$\frac{3z^2 - 4z + 1}{z^2}Y(z) = \frac{z}{z-1/2} - \frac{1}{z} + 2$$

$$Y(z) = \frac{\frac{z}{z-1/2} - \frac{1}{z} + 2}{3z^2 - 4z + 1}z^2 = \frac{z^3 - \frac{2}{3}z^2 + \frac{1}{6}z}{(z-1)(z-1/2)(z-1/3)}$$

$$\frac{Y(z)}{z} = \frac{z^2 - \frac{2}{3}z + \frac{1}{6}}{(z-1)\left(z-\frac{1}{2}\right)(z-1/3)} = \frac{c_1}{z-1} + \frac{c_2}{z-1/2} + \frac{c_3}{z-1/3}$$

$$c_1 = \left.\frac{z^2 - \frac{2}{3}z + \frac{1}{6}}{\left(z-\frac{1}{2}\right)(z-1/3)}\right|_{z=1} = 3/2 \qquad c_2 = \left.\frac{z^2 - \frac{2}{3}z + \frac{1}{6}}{(z-1)(z-1/3)}\right|_{z=1/2} = 1$$

$$c_3 = \left.\frac{z^2 - \frac{2}{3}z + 1/6}{(z-1)(z-1/2)}\right|_{z=1/3} = 1/2$$

$$Y(z) = \frac{3}{2}\frac{z}{z-1} + \frac{z}{z-1/2} + \frac{1}{2}\frac{z}{z-1/3}$$

$$y[n] = \frac{3}{2} - \left(\frac{1}{2}\right)^n + \frac{1}{2}\left(\frac{1}{3}\right)^n, \quad n \geq -2$$