Calculus review

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Course objectives

- By the end of this module, you will be able to answer the following question:
  - What is the concept of the derivative?

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What is the concept of the derivative?

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Motivating question

- Do you know the power rule for finding the derivative of a polynomial function?
- The power rule states that:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

- Do you remember the formula without understanding by any chance?
- Where does the formula come from?
- I am going to introduce one example (i.e., aircraft takeoff) to derive the formula!

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Aircraft example

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- How can we calculate the average speed of aircraft during the takeoff phase?

https://www.youtube.com/watch?v=1Lqjy25ZBDc

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Aircraft example

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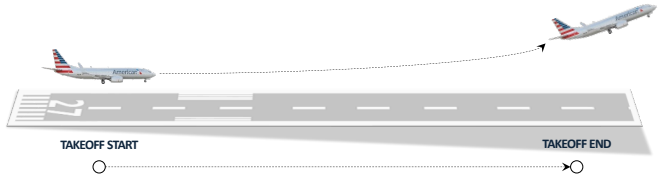
- We may need to specify start and end points on the runway



- We will get speed information by using the formula:

$$Speed = \frac{Distance}{Time}$$

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Aircraft example

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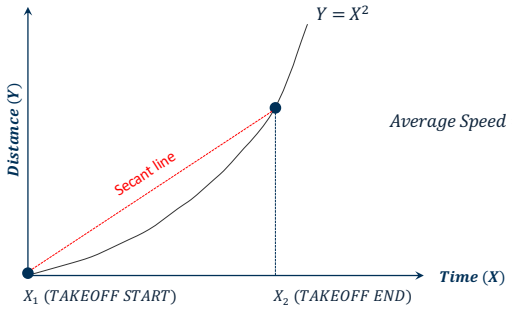
- Assume that we have the following distance-time relationship for the takeoff phase



$$Average\ Speed = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{\Delta Y}{\Delta X}$$

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Aircraft example

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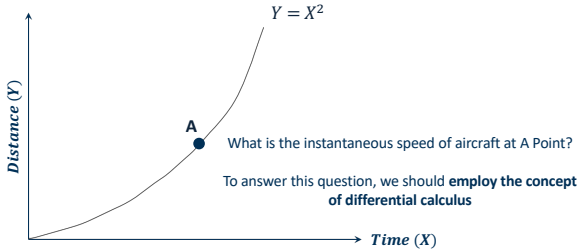
- Did we use the concept of differential calculus to compute the average speed?
- Absolutely not!
- What if we are required to calculate the instantaneous speed of aircraft at a point?



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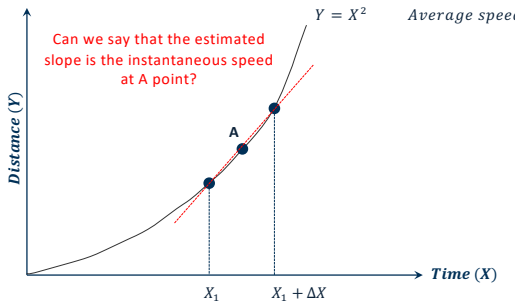
Concept of derivative

- We know how to calculate a slope once two points from a function are given



$$Average\ speed = \frac{\Delta Y}{\Delta X}$$

$$= \frac{(X_1 + \Delta X)^2 - {X_1}^2}{(X_1 + \Delta X) - X_1}$$

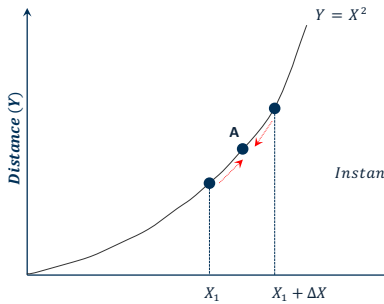
$$= \frac{{X_1}^2 + 2X_1\Delta X + (\Delta X)^2 - {X_1}^2}{\Delta X}$$

$$= \frac{2X_1\Delta X + (\Delta X)^2}{\Delta X}$$

$$= 2X_1 + \Delta X$$

Concept of derivative

- What if we are making the time difference infinitesimally small?



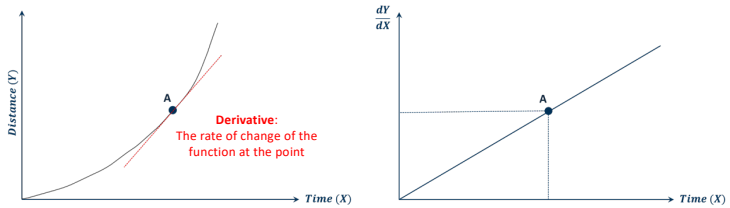
$$Average\ Speed = \frac{\Delta Y}{\Delta X} = 2X_1 + \Delta X$$

$\Delta X \cong 0$

$$Instantaneous\ Speed = \lim_{\Delta X \to 0} \frac{\Delta Y}{\Delta X} = \frac{dY}{dX} = 2X_1 + \Delta X^{0}$$

Concept of derivative

- We can now talk about instantaneous speed information at any points



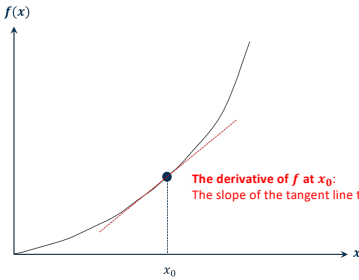
Derivative:
The rate of change of the function at the point

$$Y = X^2 \xrightarrow{\frac{d}{dx}(x^n) = nx^{n-1}} \frac{dY}{dX} = 2X$$

Power rule: one of the differential rules that finds the derivative of a polynomial function

Summary

- In a nutshell
  - Differentiation is a method of finding the derivative of a function
  - Derivative is the rate of change of the function at the point



$$\frac{df(x_0)}{dx} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

The derivative of f at x_0:
The slope of the tangent line to f at the x_0 point

Course summary

- Throughout this module, you have learned:
  - What is the concept of the derivative?

THANK YOU

For more information, please reach out to Prof. Junghyun Kim at
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