HW2 Solutions

Hanggang Zhu*, 3200110457, Suhao Wang, 3200110777, and Lumeng Xu 3200110184

Q4.A solution

Q4.B solution

Q4.C solution

Q4.D solution

Q4.E solution

Q4.F solution

Q4.G solution

Q4.H solution

Q4.I solution

Q4.J solution

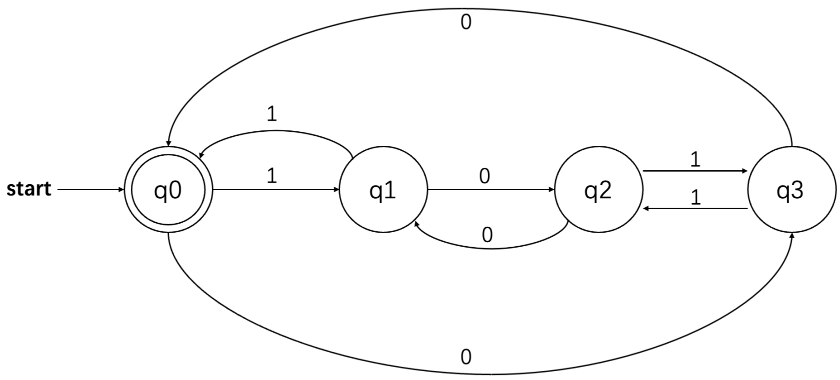
Q4.K solution

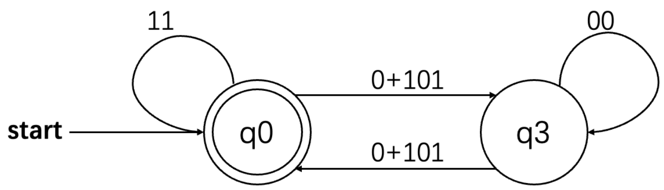
Q5.A solution
The DFA is drawn below.
The DFA accepts all strings that contain an even number of 0s and an even number of 1s. Each state records the number of 0s or 1s so far.

Formal Description:

$$
\begin{gathered}
Q = \{q0, q1, q2, q3\} \\
s = q0 \\
A = \{q0\} \\
\delta\{q0, 0\} = q3 \\
\delta\{q0, 1\} = q1 \\
\delta\{q1, 0\} = q2 \\
\delta\{q1, 1\} = q0 \\
\delta\{q2, 0\} = q1 \\
\delta\{q2, 1\} = q3 \\
\delta\{q3, 0\} = q0 \\
\delta\{q3, 1\} = q2
\end{gathered}
$$

Each state is unique, and $q0$ is the start state and the accepting state, no matter how many number of 0s or 1s are read, the machine can only be accepted when returning to $q0$.

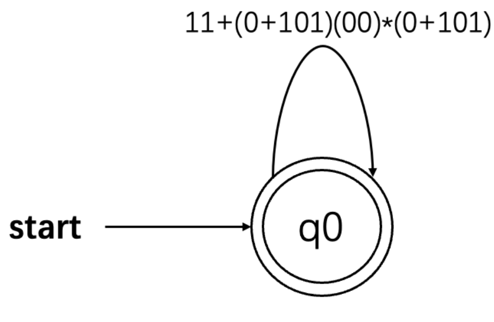




Q5.B solution
According to the state removal method,
if $q1 = \delta(q0, x)$ and $q2 = \delta(q1, y)$, then $q2 = \delta(q1, y) = \delta(\delta(q0, x), y) = \delta(q0, xy)$
After removing the states $q1$ and $q2$, we get the following image:
So,we get the regular expression for the simplified machine : $11 + (0 + 101)(00)^*(0 + 101)$.
Then, after further simplifying, we get the graph below:
So, we get the regular expression of L is $(11 + (0 + 101)(00)^*(0 + 101))^*$.
Then, for the argument that this expression is correct, first, we consider the strings do not contain any consecutive 0s or 1s, that is, the input is the empty string. According to this expression, it is just the accepting state, which satisfies the condition.
What's more, from the regular expression, we can see that it only accepts even number of 0s and even number of 1s, other kinds of numbers are not allowed. So, this expression is correct.

Q6.A solution
For language $L = L_1 \cap L_2 \cap L_3$, formally describe the DFM as $M = (Q, \sum, \delta, s, A)$, where

- $Q = Q_1 \times Q_2 \times Q_3 = \{(q_1, q_2, q_3) \mid q_1 \in Q_1, q_2 \in Q_2, q_3 \in Q_3\}$



- $\delta : Q \times \sum \to Q$,where $\delta((q_1, q_2, q_3), a) = (\delta_1(q_1, a), \delta_2(q_2, a), \delta_3(q_3, a))$ and $a \in \sum$
- $s = (s_1, s_2, s_3)$
- $A = A_1 \times A_2 \times A_3 = \{(q_1, q_2, q_3) \mid q_1 \in A_1, q_2 \in A_2, q_3 \in A_3\}$

Q6.B solution

For $\delta^*(q, w)$, we have the formal definition:

$$\delta^*(q, w) = \begin{cases} q & \text{, if } w = \epsilon, \\ \delta^*(\delta(q, a), x) & \text{, if } w = ax \end{cases}$$

Now we prove by induction.

- **Base case:** Let w be an artibrary string of length 0. we get $w = \epsilon$. Then

$$\begin{aligned} \delta^*((q_1, q_2, q_3), w) &= \delta^*((q_1, q_2, q_3), \epsilon) \\ &= (q_1, q_2, q_3) \\ &= (\delta_1^*(q_1, \epsilon), \delta_2^*(q_2, \epsilon), \delta_3^*(q_3, \epsilon)) \\ &= (\delta_1^*(q_1, w), \delta_2^*(q_2, w), \delta_3^*(q_3, w)) \end{aligned}$$

- **Induction hypothesis:** For any string w of length $n > 0$, we have that

$$\delta^*((q_1, q_2, q_3), w) = (\delta_1^*(q_1, w), \delta_2^*(q_2, w), \delta_3^*(q_3, w))$$

- **Inductive step:** Let w be an artibrary string with length $n > 0$. Assume inductive hypothesis holds for all strings x of length $< n$. Then we have $w = ax$ for some string x with length $< n$ and $a \in \sum$. We have:

$$\begin{aligned} \delta^*((q_1, q_2, q_3), w) &= \delta^*(\delta((q_1, q_2, q_3), a), x) && \text{(definition of } \delta^*(q, w)) \\ &= \delta^*((\delta_1(q_1, a), \delta_2(q_2, a), \delta_3(q_3, a)), x) && \text{(definition of } \delta \text{ in Q6.A)} \\ &= (\delta_1^*(\delta_1(q_1, a), x), \delta_2^*(\delta_2(q_2, a), x), \delta_3^*(\delta_3(q_3, a), x)) && \text{(induction hypothesis)} \\ &= (\delta_1^*(q_1, w), \delta_2^*(q_2, w), \delta_3^*(q_3, w)) && \text{(definition of } \delta^*(q, w)) \end{aligned}$$

So, the equation is proved.

Q6.C solution

Q6.D solution