HW#3 due on Sunday March 12 at 11:59pm **ECE 374, Spring 2023**

Instructions

- **Groups of up to three students can submit joint solutions.** The solutions should be submitted by exactly one student on behalf of the other group members. Please remember to state the names and the IDs of all group members at the beginning of the solutions.
- **Submit your solutions as a single or multiple PDF files to Blackboard**. It is preferred to use LaTeX template available from Blackboard to typeset the solutions, but submitting a scanned handwritten solutions will also be accepted.
- **You may use any source at your disposal** – paper, electronic, or human-but you *must* cite *every* source that you have used, and you *must* write your answers by yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given *automatic zero*, even if the solution is otherwise perfect. Yes, we really mean it. We are not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course materials.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English.
  - Always describe the specific problem your algorithm is supposed to solve.
  - Keep in mind that short **complete** answers are better than long answers. Unnecessarily long answers (which by definition are not perfect) would get zero points. Avoid empty expressions like "in fact", "as anybody, or their uncle, can see if they think about it...", etc.
  - Always give credit to outside sources!
  - Don't be afraid to ask questions on Piazza, in the lectures, or in our office hours.

7 (100 PTS.) NFAs

For each of the following languages over $\Sigma = \{3, 7, 4\}$, draw an NFA that accepts them. Your NFA should have a small number of states (at most say 14 states). Provide a brief explanation for your solution.

7.A. (20 PTS.) $\Sigma^*3\Sigma^*7\Sigma^*4\Sigma^*$

7.B. (20 PTS.) All strings in Σ^* that contain the substrings 374 and 473.

7.C. (20 PTS.) All strings in Σ^* that do not contain 374 as a substring.

7.D. (20 PTS.) All strings in Σ^* that contain the substring 374 and an odd number of 7s.

7.E. (20 PTS.) All strings in Σ^* such that every maximal substring of consecutive 7s is even in size.

8 (100 PTS.) DFAs to NFAs

Given a DFA $M = (\Sigma, Q, \delta, s, A)$ that accepts L, construct an NFA $N = (\Sigma, Q', \delta', s', A')$ that accepts the following languages. You can assume $\Sigma = \{0, 1\}$ in **8.A.** and **8.C.**. Provide a brief explanation for your solution.

8.A. (30 PTS.) $\text{DelOnes}(L) := \left\{0^{\#_0(w)} \;\middle|\; w \in L\right\}$; i.e., removes all 1s from the strings of L.

8.B. (30 PTS.) $\mathit{ThereAndBack}(L) := \left\{xy \;\middle|\; x \in L \text{ and } y^R \in L\right\}$

8.C. (40 PTS.) $\mathit{XOR}(L) := \{z \mid z = \texttt{XOR}(x, y) \text{ for some } x \in L, y \in L, \text{ such that } |x| = |y| = |z|\}$, where $\texttt{XOR}(x, y)$ computes the element-wise XOR of x and y (so for each index i, $z_i = x_i \texttt{ XOR } y_i$).

8.D. (**Not for submission**) Consider, if you must, the language

$$\mathit{Middle}(L) := \{y \in L \mid xyz \in L \text{ for some } x, z \text{ such that } |x| = |y| = |z|\}.$$

Prove that this language is regular.

9 (100 PTS.) Fooling Sets

Prove that the following languages are not regular by providing a fooling set. You need to provide an infinite set and also prove that it is a valid fooling set for the given language.

9.A. (20 PTS.) $L = \left\{ww^Rw \;\middle|\; w \in \{0,1\}^*\right\}$.

9.B. (20 PTS.) $L = \left\{0^i10^j \;\middle|\; i \text{ is divisible by } j\right\}$.

9.C. (20 PTS.) $L = \left\{a^ib^j \;\middle|\; i, j \in \mathbb{N}, \text{ and } j = \log_2 i\right\}$.

9.D. (20 PTS.) $L = \left\{0^i0^j \;\middle|\; i, j \in \mathbb{N}, \text{ and } j = \sqrt{i}\right\}$.

9.E. (20 PTS.) $L = \{wcd^{\#_a(w)} \mid w \in \{a, b\}^*\}$.