Instructions

- please check HW1-3 for detailed instructions (they remain the same)
- please note that HW12 is now optional (for an extra credit)

34 Recall that w^R denotes the reversal of string w; for example, $TURING^R = GNIRUT$. Prove that the following language is undecidable.

$$\textsc{RevAccept} := \left\{\langle M\rangle \mid M \text{ accepts } \langle M\rangle^R\right\}$$

Note that Rice's theorem does *not* apply to this language.

35 Let M be a Turing machine, let w be an arbitrary input string, and let s be an integer. We say that ***M accepts w in space s*** if, given w as input, M accesses only the first s (or fewer) cells on its tape and eventually accepts.

35.A. Sketch a Turing machine/algorithm that correctly decides the following language:

$$\left\{\langle M, w\rangle \,\middle|\, M \text{ accepts } w \text{ in space } |w|^2\right\}$$

35.B. Prove that the following language is undecidable:

$$\left\{\langle M\rangle \,\middle|\, M \text{ accepts at least one string } w \text{ in space } |w|^2\right\}$$

36 Consider the language $\textsc{SometimesHalt} = \{\langle M\rangle \mid M \text{ halts on at least one input string}\}$. Note that $\langle M\rangle \in \textsc{SometimesHalt}$ does not imply that M *accepts* any strings; it is enough that M *halts* on (and possibly rejects) some string.

36.A. Prove that SometimesHalt is undecidable.

36.B. Sketch a Turing machine/algorithm that *accepts* SometimesHalt.

37 For each of the following languages, either prove that the language is decidable, or prove that the language is undecidable.

37.A. $L_0 = \left\{\langle M\rangle \mid \text{given any input string, } M \text{ eventually leaves its start state}\right\}$

37.B. $L_1 = \left\{\langle M\rangle \mid M \text{ decides } L_0\right\}$

37.C. $L_2 = \left\{\langle M\rangle \mid M \text{ decides } L_1\right\}$

37.D. $L_3 = \left\{\langle M\rangle \mid M \text{ decides } L_2\right\}$

37.E. $L_4 = \left\{\langle M\rangle \mid M \text{ decides } L_3\right\}$