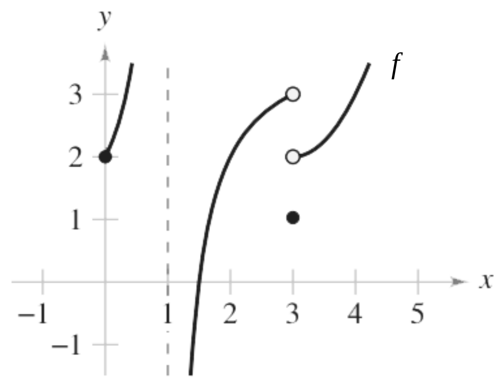
MATH 1950 – Calculus I
Final Exam Practice

Use the graph of f to answer 1-6.



1. $\lim_{x\to 3^-} f(x) =$ _____ 2. $\lim_{x\to 3^+} f(x) =$ _____ 3. $\lim_{x\to 3} f(x) =$ _____

4. $\lim_{x\to 1^+} f(x) =$ _____ 5. $f(3) =$ _____ 6. $\lim_{x\to \infty} f(x) =$ _____

7. Identify all x-coordinates where the graph of f is discontinuous.

Find the limit.

8. $\lim_{x\to 4}\sqrt{x+2}$ 9. $\lim_{x\to 7}(x-4)^3$ 10. $\lim_{x\to 2}\frac{x}{x^2+1}$ 11. $\lim_{x\to 4}\frac{x^2-16}{x-4}$

12. $\lim_{x\to 6^-}\frac{x-6}{x^2-36}$ 13. $\lim_{x\to 4^-}\frac{\sqrt{x}-2}{x-4}$ 14. $\lim_{x\to (1/2)^+}\frac{x}{2x-1}$ 15. $\lim_{x\to 0^+}\frac{\csc 2x}{x}$

16. $\lim_{x\to\infty}\left(8+\frac{1}{x}\right)$ 17. $\lim_{x\to-\infty}\frac{1-4x}{x+1}$ 18. $\lim_{x\to-\infty}\frac{2x}{x^6+9x-7}$ 19. $\lim_{x\to\infty}\frac{2x^2}{3x^2-5}$

Find the derivative.

20. $h(x)=6\sqrt{x}+3\sqrt[3]{x}$ 21. $f(x)=x^{1/2}-x^{-1/2}$ 22. $f(\theta)=4\theta-5\sin\theta$ 23. $g(x)=\frac{8}{5x^4}$

24. $f(x)=x^3-11x^2$ 25. $h(x)=\sqrt{x}\cot x$ 26. $f(x)=\frac{1+\cos x}{1-\cos x}$ 27. $k(x)=\frac{x^2+x-1}{x^2-1}$

28. $g(x)=\ln\sqrt{2x}$ 29. $f(x)=x\sqrt{\ln x}$ 30. $f(x)=\ln\sqrt{\frac{x^2+4}{x^2-4}}$ 31. $k(x)=e^{-x^2/2}$

32. $f(x)=\frac{x^2}{e^x}$ 33. $h(x)=3e^{5x}$

Find $\frac{dy}{dx}$.

34. $x^2+4xy-y^3=6$ 35. $\sqrt{xy}=x-4y$ 36. $x\sin y=y\cos x$

Find the second derivative of each function.

37. $y = 7xe^x$ 38. $f(x) = \sec x$ 39. $y = \sin^2 x$ 40. $y = \dfrac{1}{5x+1}$

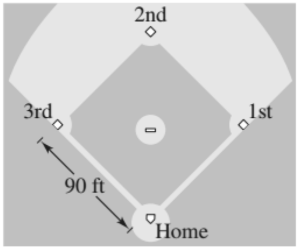
Find $\dfrac{dy}{dx}$ using logarithmic differentiation.

41. $y = \dfrac{6x^2 \sin x}{x+1}$ 42. $y = x\sqrt{x^2+1}$ 43. $y = \dfrac{x(x-1)^{3/2}}{\sqrt{x+1}}$

44. Find the equation of the line tangent to the curve $f(x) = \sqrt{2x^2 - 7}$ at the point (4, 5).

45. Find the equation of the line tangent to the curve $y = (x^2 + 2x)(x+1)$ at the point where $x = 1$.

46. A baseball diamond has the shape of a square with sides 90 feet long (see figure). A player running from second base to third base at a speed of 25 feet per second is 20 feet from third base. At what rate is the player's distance from home plate changing?



47. The radius r of a sphere is increasing at a rate of 3 inches per minute. Find the rate of change of the volume when the radius is 9 inches.

48. Find the length and width of a rectangle with a perimeter of 80 meters that has a maximum area.

49. Find the length and width of a rectangle with an area of 32 square feet with a minimum perimeter.

50. A rectangular page is to contain 30 square inches of print. The margins on each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used.

Find the value of c that satisfies the conclusion of Rolle's Theorem for the given function on the interval.

51. $f(x) = x^2 + 1$, $[-2, 2]$ 52. $f(x) = \sin x$, $[0, \pi]$

Find the value of c that satisfies the conclusion of the Mean Value Theorem for the function on the given interval.

53. $f(x) = x^2 + x$, $[-4, 6]$

54. $f(x) = \sqrt{x+1}$, $[0, 3]$

Find the absolute extrema of each function over the given interval. Fully justify your work.

55. $f(x) = x^3 - \frac{3}{2}x^2$ over [-1, 2].

56. $f(x) = x^3 - 12x$ over $[0, 4]$

For each function, identify the intervals on which the function is increasing, decreasing, concave up, and concave down. Be sure to identify any critical points and inflection points.

57. $f(x) = 5 - 12x + x^3$

58. $f(x) = \dfrac{x}{x^2 + 2}$

Find and identify any local extrema.

59. $f(x) = x^2 - 6x + 5$

60. $f(x) = 4x^3 - 5x$

61. $f(x) = \dfrac{1}{4}x^4 - 8x$

Use left and right endpoints and the given number of rectangles to find two approximations of the area of the region bounded by between the graph of the function and the x-axis over the given interval.

62. $f(x) = 9 - x$, $[2, 4]$, 8 rectangles

63. $f(x) = 2x^2 - x - 1$, $[1, 4]$, 6 rectangles

Find the indefinite integral.

64. $\int (x-6)\, dx$

65. $\int (4x^2 + x - 3)\, dx$

66. $\int (5\cos x - 2\sec^2 x)\, dx$

67. $\int \dfrac{x^4 + 8}{x^3}\, dx$

68. $\int \dfrac{x^2}{\sqrt{x^3 + 3}}\, dx$

69. $\int \dfrac{x+4}{(x^2 + 8x - 7)^2}\, dx$

70. $\int \dfrac{\sin x}{\sqrt{\cos x}}\, dx$

71. $\int 6x^3(7 - 3x^4)^5\, dx$

72. $\int \dfrac{dx}{\sqrt{4 - x^2}}$

73. $\int \dfrac{7}{16 + x^2}\, dx$

74. $\int \dfrac{1}{x\sqrt{x^4 - 4}}\, dx$

75. $\int \dfrac{\sin x}{7 + \cos^2 x}\, dx$

76. $\int \tan 5x\, dx$

77. $\int \sec \dfrac{x}{2}\, dx$

78. $\int (\sec 2x + \tan 2x)\, dx$

Evaluate each definite integral.

79. $\int_0^8 (3+x)\,dx$

80. $\int_2^3 (x^2-1)\,dx$

81. $\int_2^3 (x^4+4x-6)\,dx$

82. $\int_0^{3\pi/4} \sin\theta\,d\theta$

83. $\int_0^1 (3x+1)^5\,dx$

84. $\int_0^1 x^2(x^3-2)^{10}\,dx$

85. $\int_{-\pi/4}^{-\pi/4} \sin 2x\,dx$

86. $\int_0^3 \frac{1}{\sqrt{1+x}}\,dx$

Find the average value of $f(x)$ over the given interval.

87. $f(x)=\cos x$ over $\left[0,\frac{\pi}{6}\right]$

88. $f(x)=2x^3-6x^2$ over [-1, 3]

A particle moves with a velocity of $v(t)$ m/s along an t-axis. Find the displacement and distance traveled by the particle during the given time interval.

89. $v(t)=\sin t;\quad 0\le t\le \pi/2$

90. $s(t)=t^2-4t+6;\quad 0\le t\le 6$

Solve the differential equation.

91. $\frac{dy}{dx}=\frac{y}{x}$

92. $\frac{dy}{dx}=(1+y^2)x^2$

93. $(1+y^2)\frac{dy}{dx}=e^x y$

Find the particular solution to the differential equation using the given initial value.

94. $\frac{dy}{dx}-xy=x$, $y(0)=3$

95. $\frac{dy}{dx}+y=2$, $y(0)=1$

96. $\frac{dy}{dx}=\frac{4x^2}{y+\cos y}$, $y(1)=\pi$

97. The rate of change of V is proportional to V. When $t=0$, $V=20{,}000$ and when $t=4$, $V=12{,}500$. What is the value of V when $t=6$.

98. An isotope of neptunium (Np-240) has a half-life of 65 minutes. What is the decay constant k?

99. Polonium-210 is a highly radioactive element used in space probes. The half-life is 138 days. If a sample is no longer useful after 10% of the radioactive nuclei have disintegrated, how long will the sample be able to be utilized?