

Joint analysis of 21cm observations with normalising flows

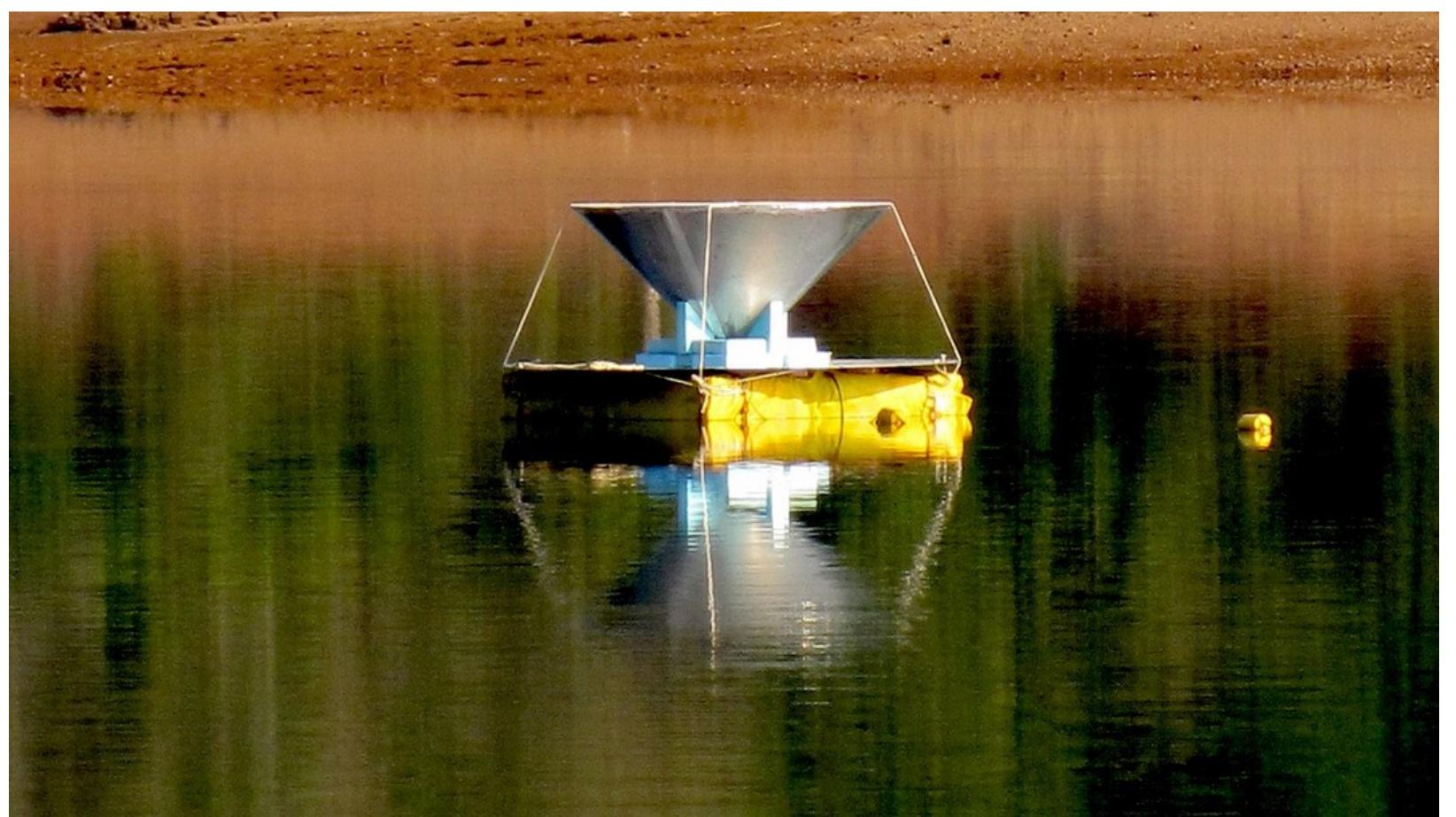
Harry Bevins

With

Stefan Heimersheim, Irene Abril-Cabezas, Anastasia Fialkov, Eloy De Lera Acedo, William Handley, Saurabh Singh, Rennan Barkana

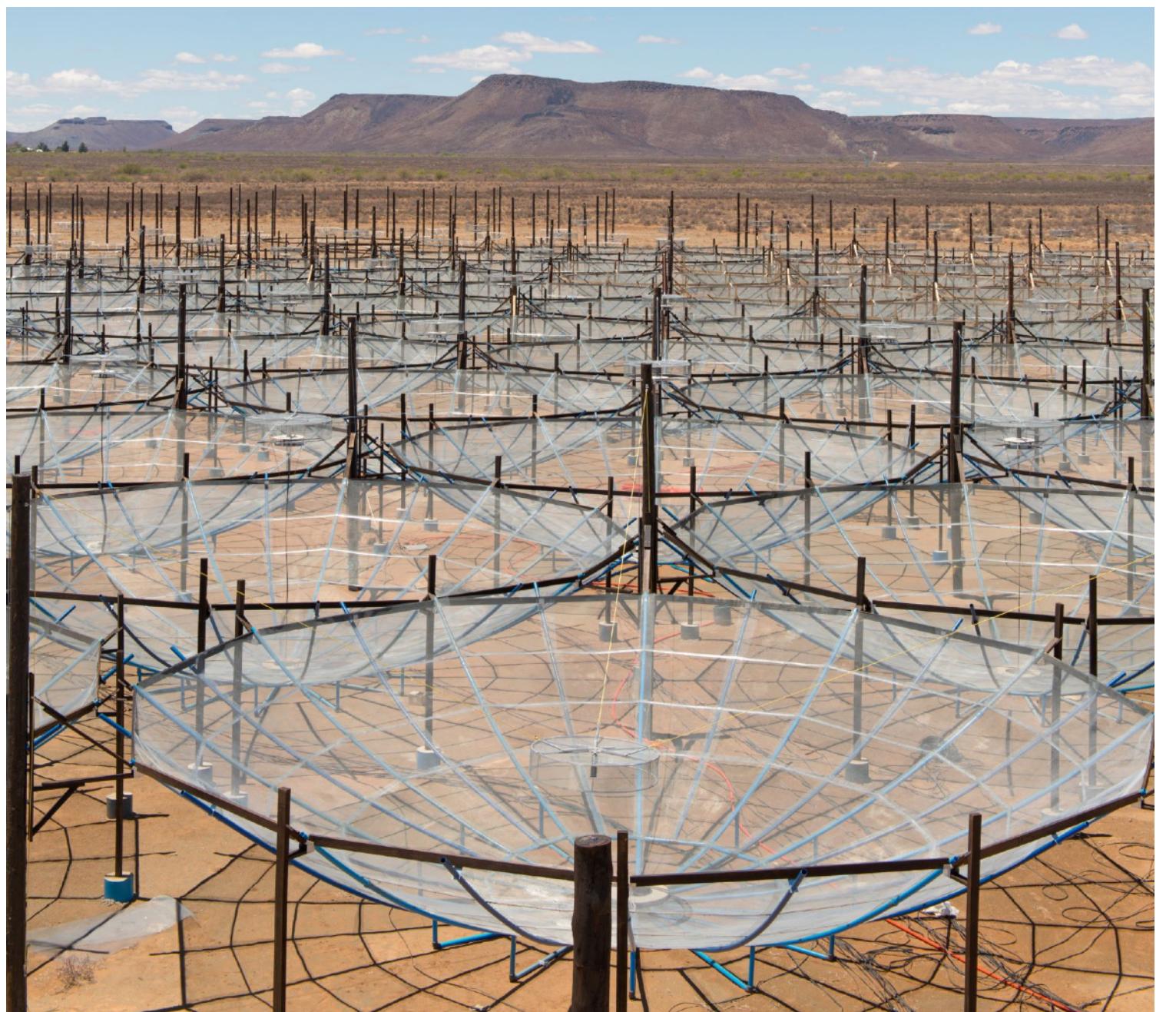
The Plan

1. The Goal of the Work



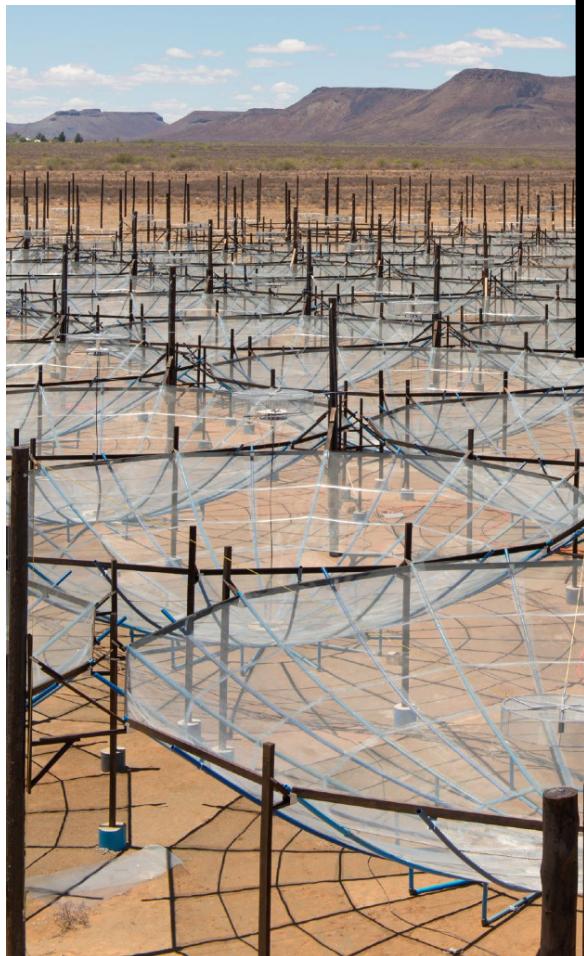
2. The Methodology

3. The Results



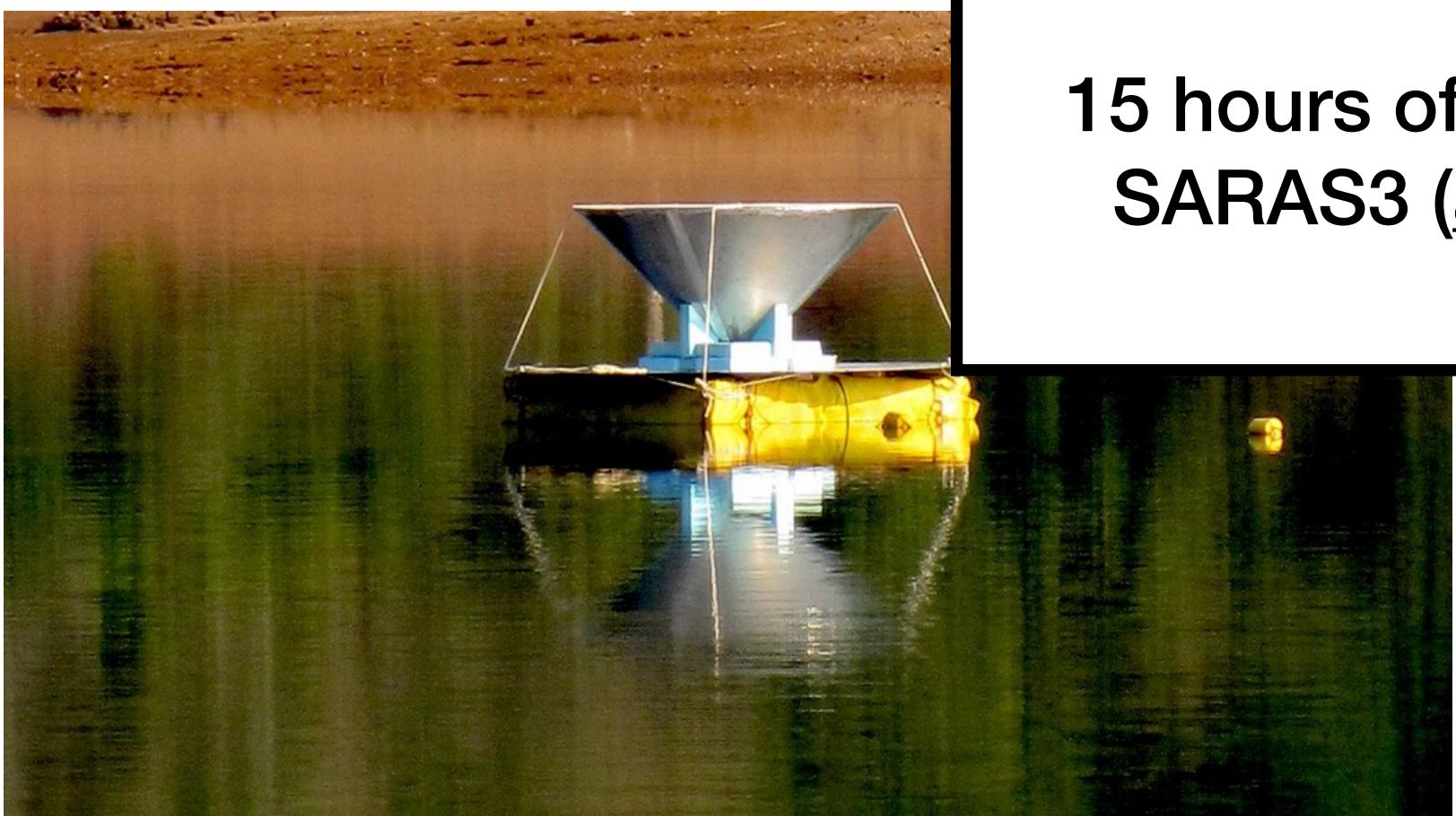
The Goal of the work

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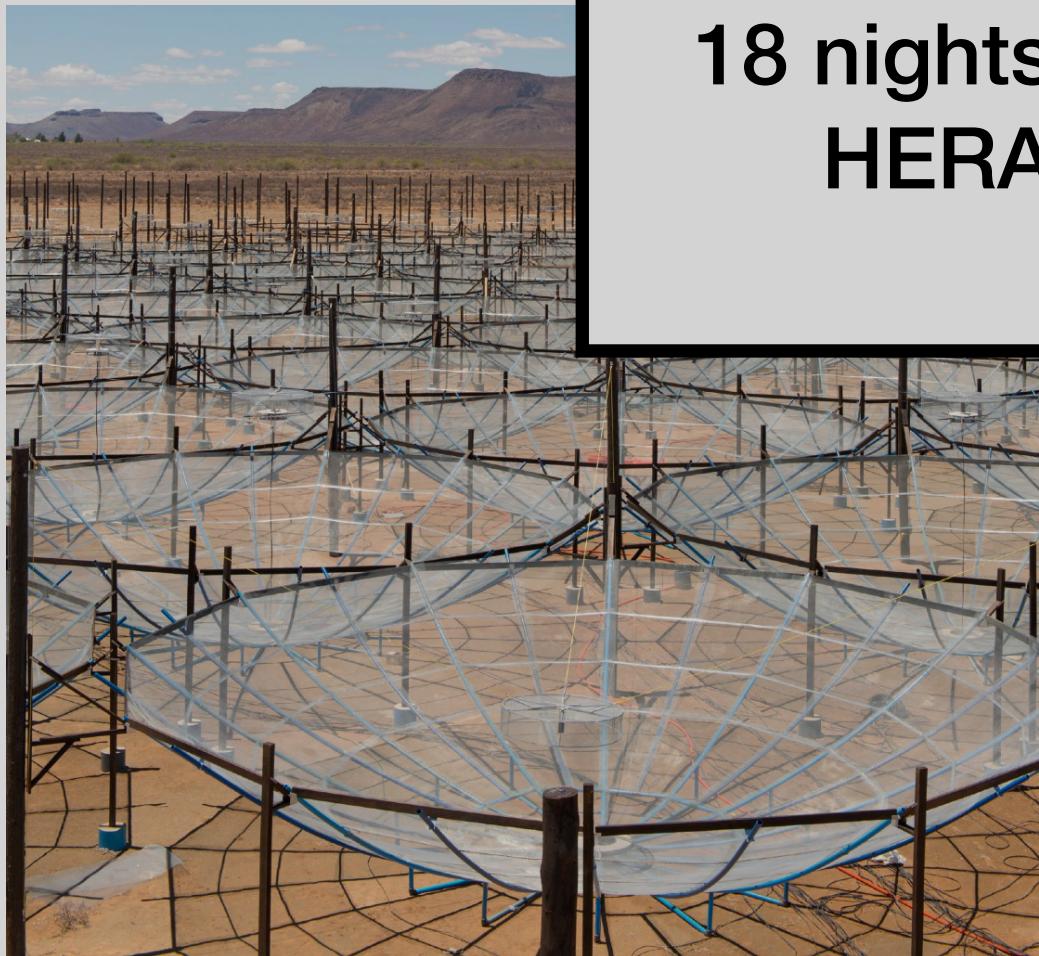
18 nights of observations from
HERA ([arxiv2108.07282](https://arxiv.org/abs/2108.07282))

Joint analysis of this data?
Never been attempted before



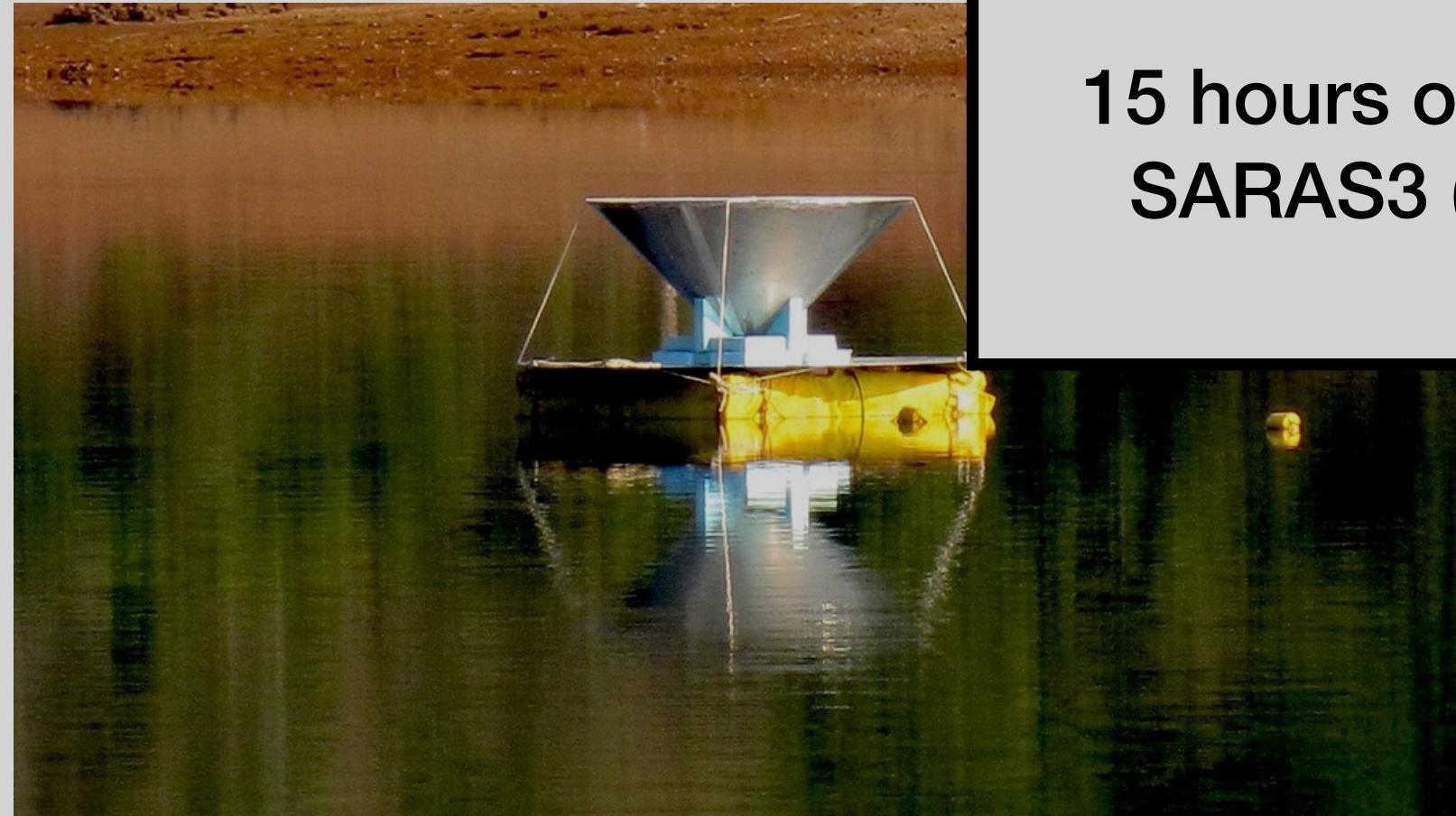
15 hours of observations from
SARAS3 ([arXiv:2112.06778](https://arxiv.org/abs/2112.06778))

The Goal of the Work



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How about adding in LOFAR
([arXiv:1702.08679](https://arxiv.org/abs/1702.08679)), MWA
([arXiv:2002.02575](https://arxiv.org/abs/2002.02575)) and SARAS2
([arXiv:2201.11531](https://arxiv.org/abs/2201.11531))?

The Goal of the Work



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15 hours of observations from
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...and future REACH observations?

The Methodology

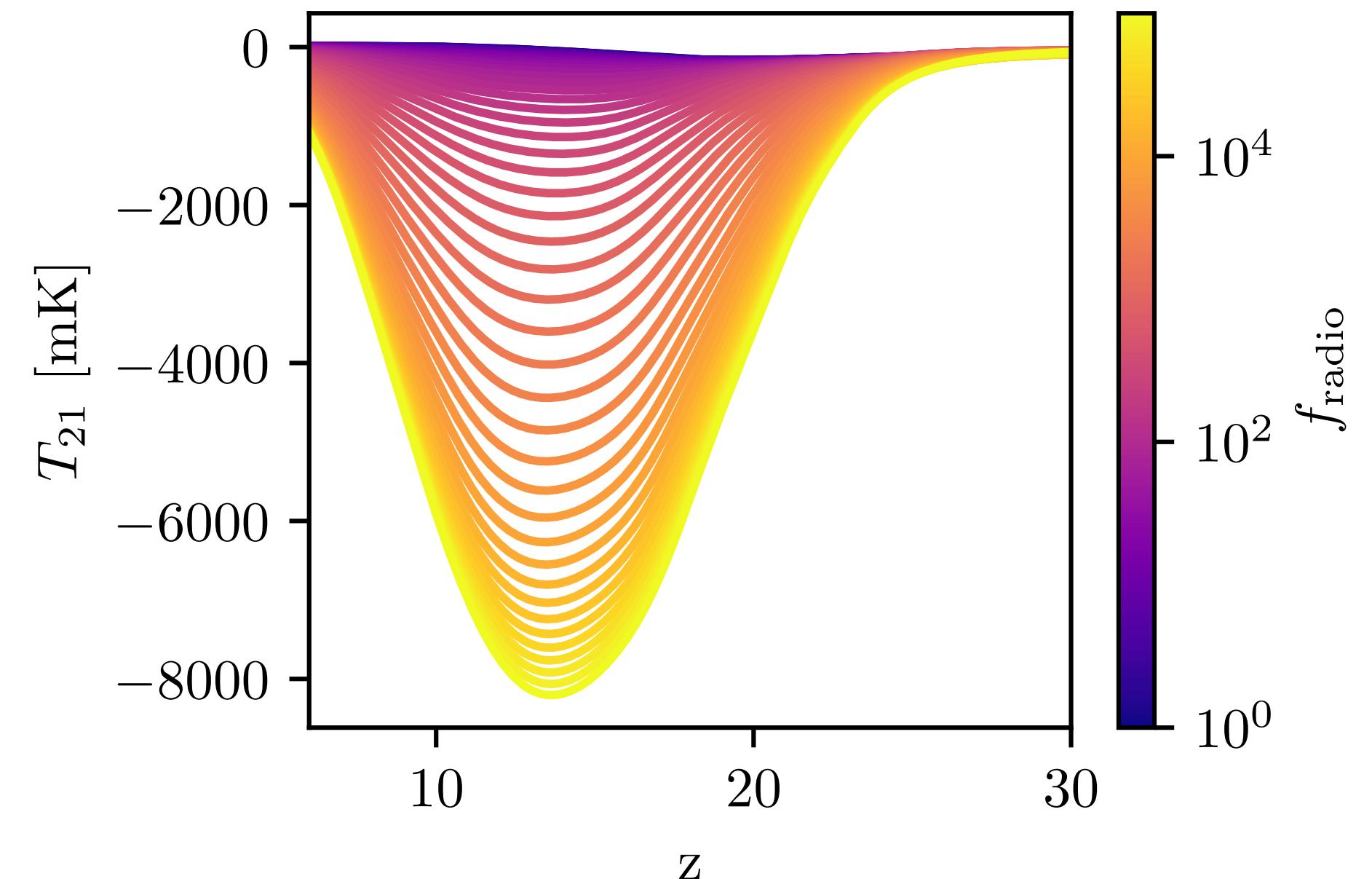
The Methodology: Signal Modelling

Any joint analysis requires consistent modelling of the signal.

Our models (Fialkov et al.) include:

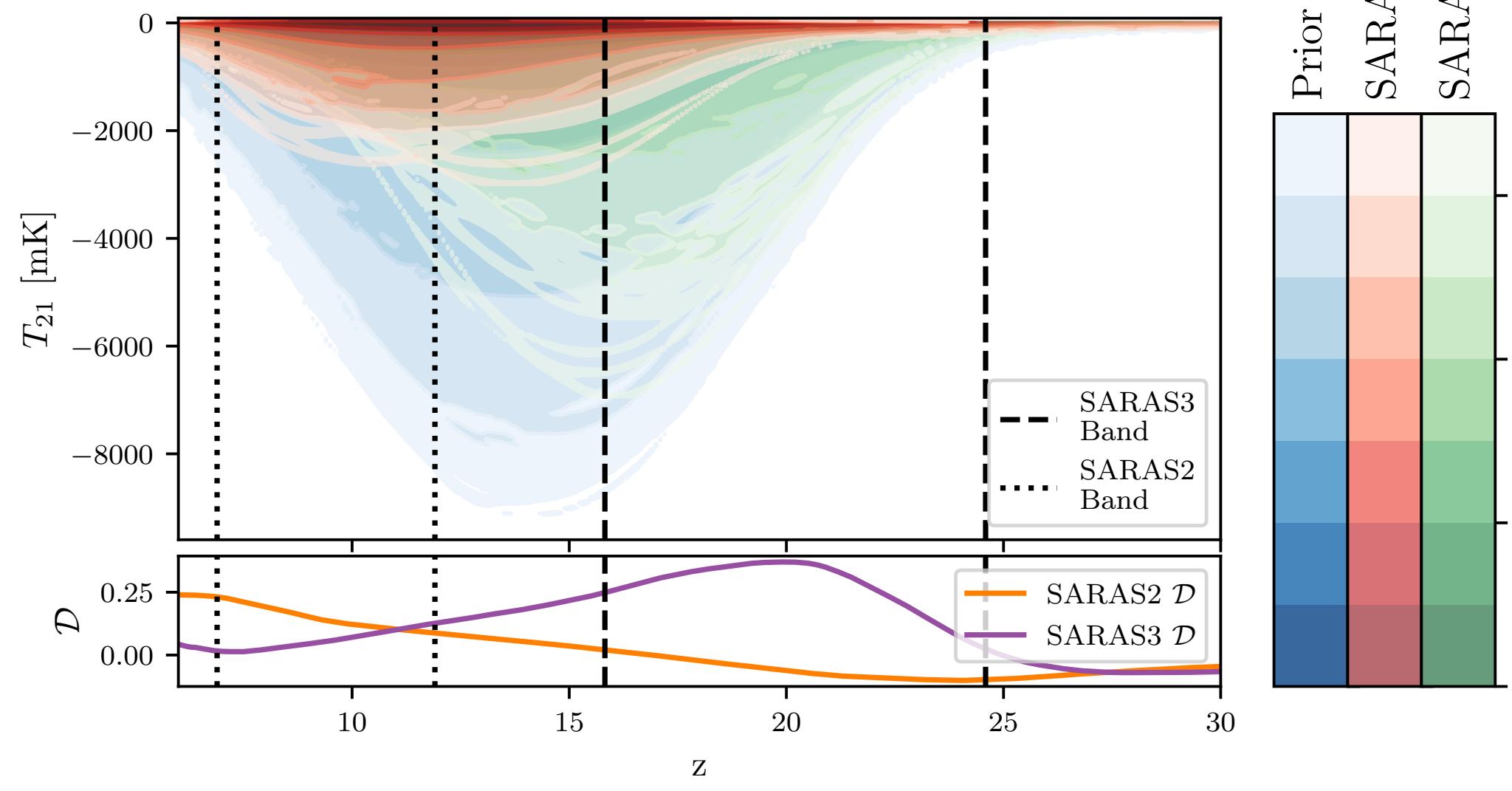
- A contribution to the radio background from high- z galaxies
- Lyman- α heating, CMB heating and X-ray heating
- Multiple scattering of Lyman- α photons

Parameterised by star formation, X-ray and radio production efficiencies ($f_*, f_X, f_{\text{radio}}$), the CMB optical depth (τ) and minimum halo mass for star formation ($M_c \rightarrow V_c$).

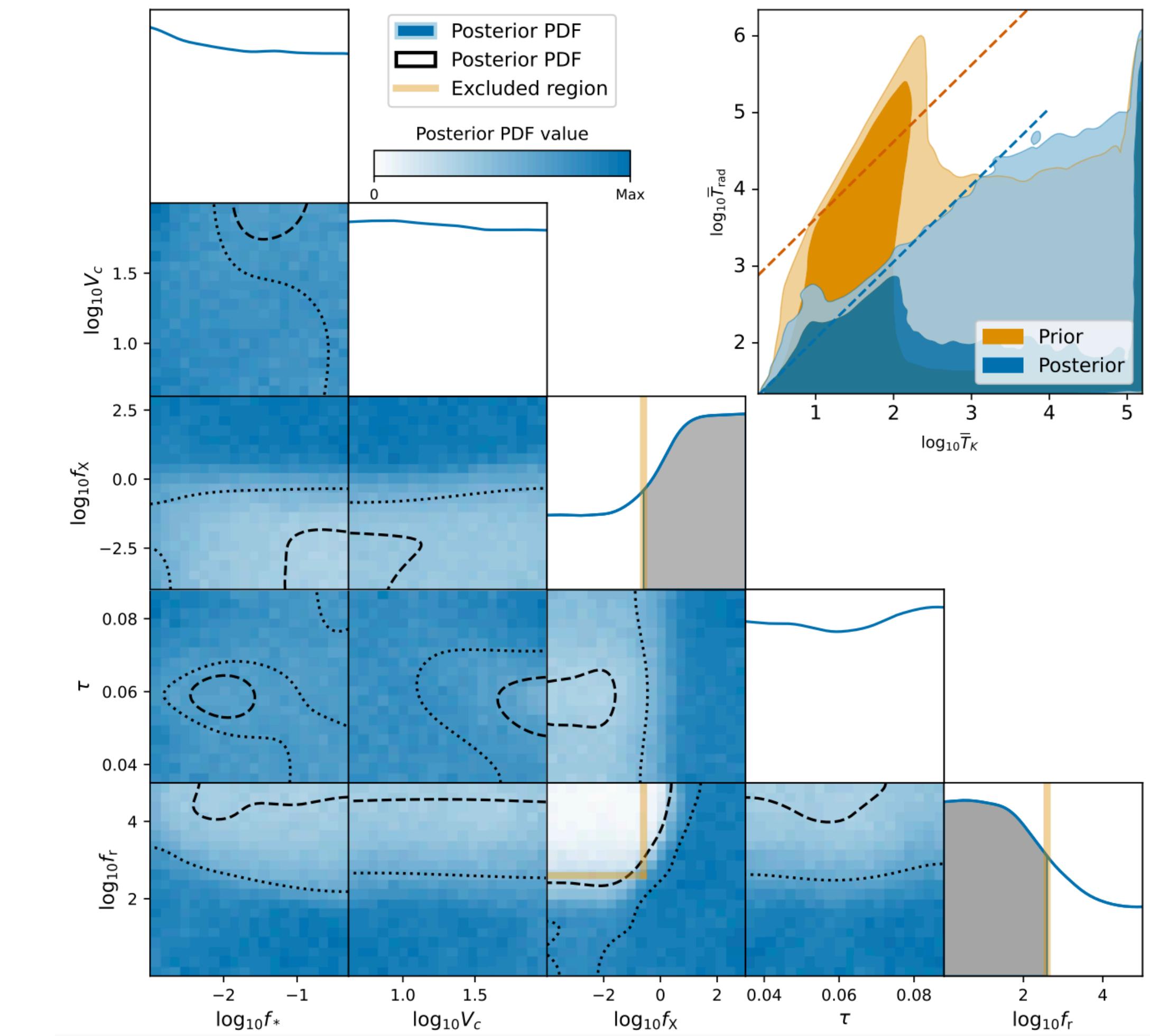


The Methodology: But we have existing constraints?

- We already have constraints from HERA and SARAS3 on this class of models.
- Can we take advantage of this existing body of work?



Bevins et al. 2022 ([arXiv:2112.06778](https://arxiv.org/abs/2112.06778))



The HERA Collaboration, 2022 (2108.07282)

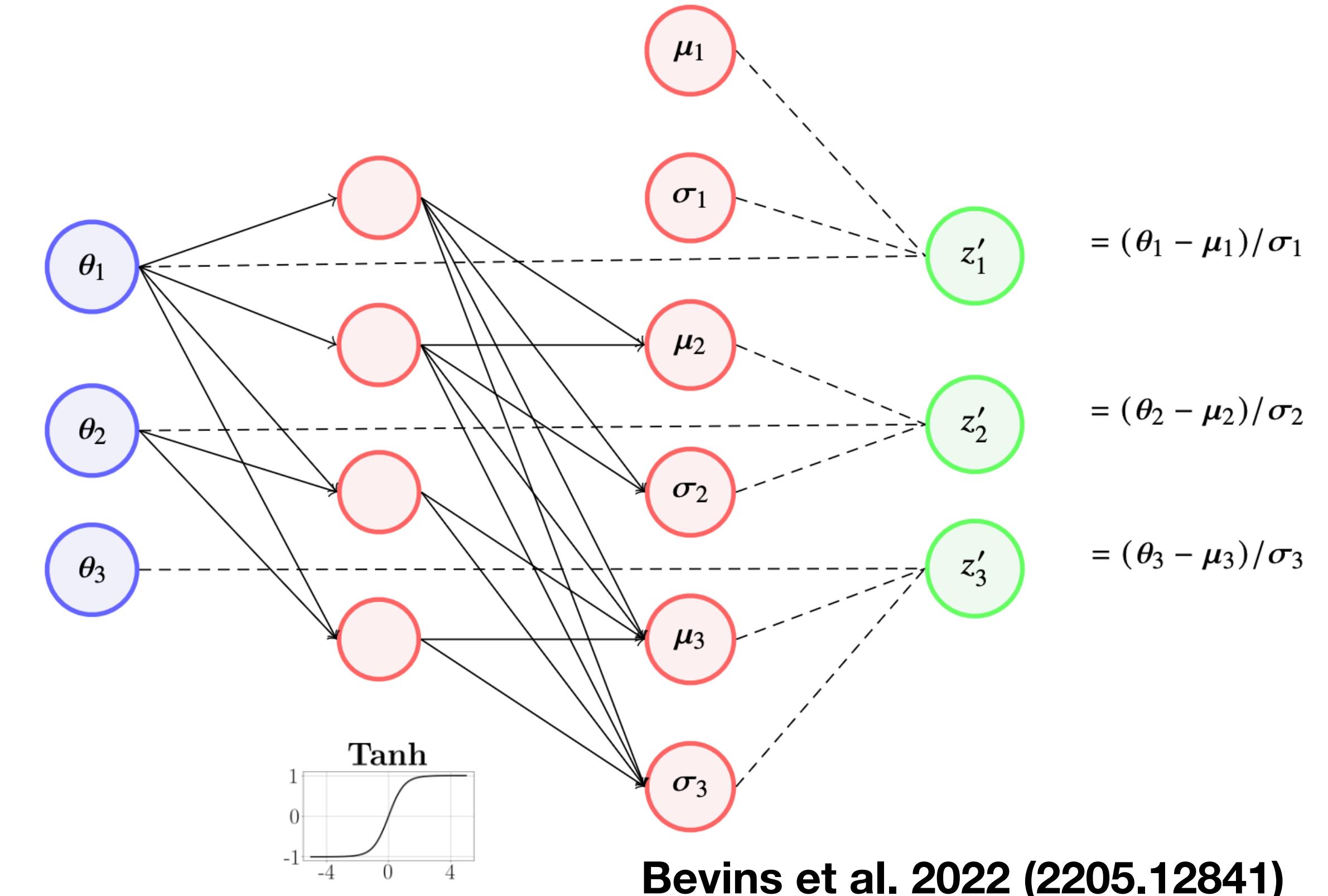
The Methodology: Utilising Normalizing Flows?

Normalizing flows parameterise a transformation from a known base distribution to a target distribution

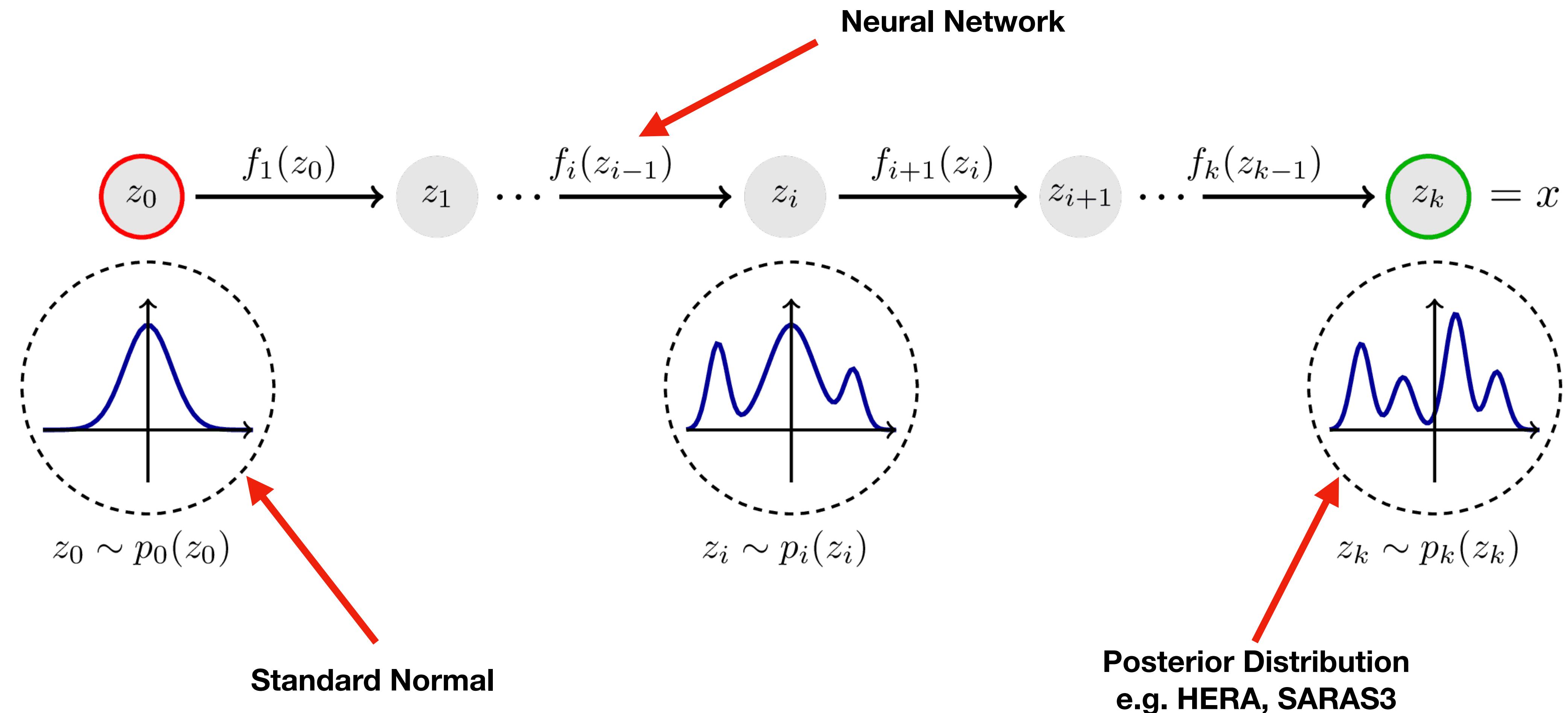
Trainable and bijective density estimators

Bijective property allows us to do lots of cool things!

Using the package *margarine* (<https://github.com/htjb/margarine>)



The Methodology: Utilising Normalizing Flows?

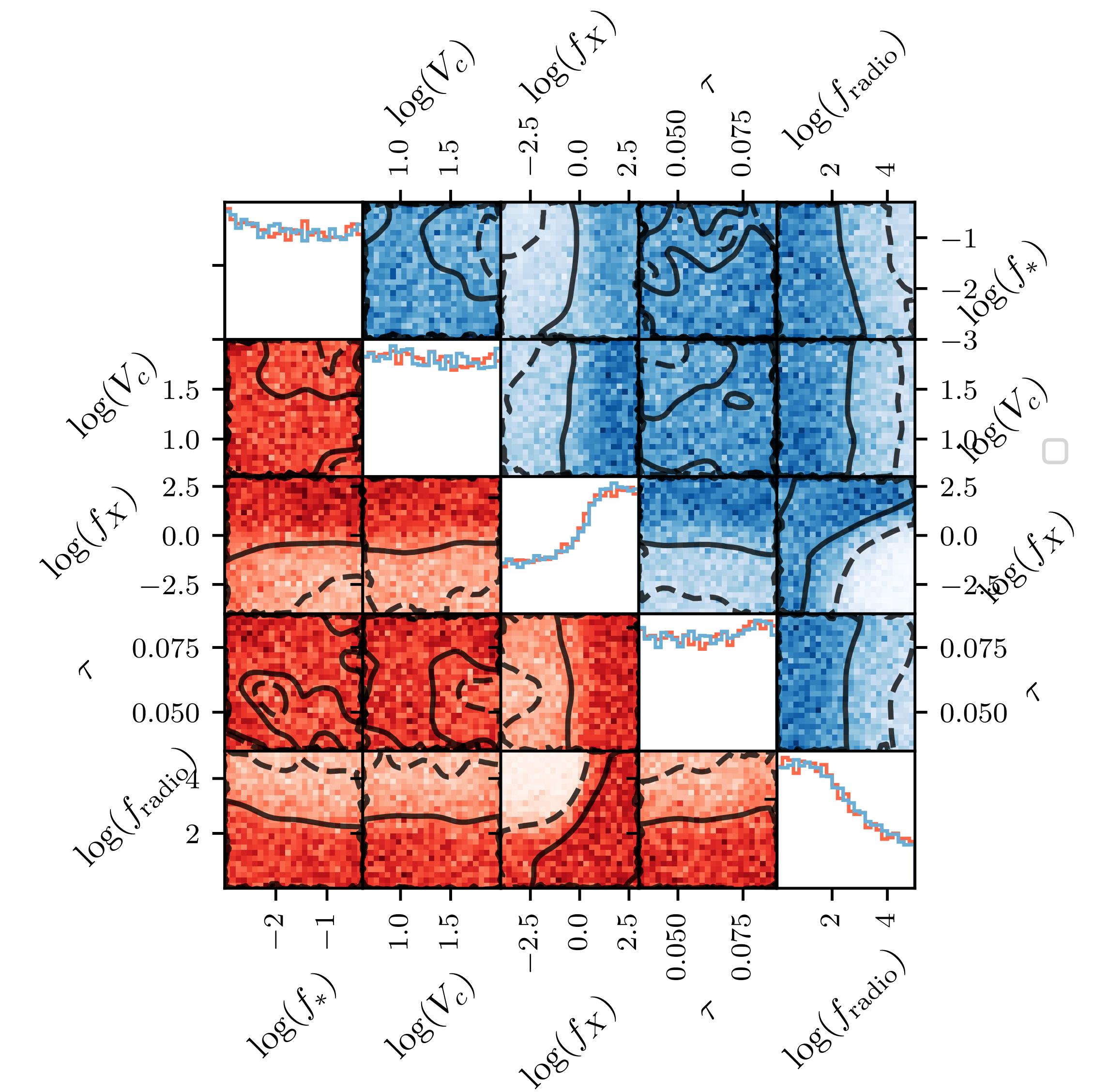


The Methodology: How are the flows useful?

Flows allow us to:

- Draw samples from the target distribution
- Derive non-trivial priors
- Calculate log-probabilities on the target distribution for a set of parameters

See Bevins et al. 2022a,b and 2023
(2205.12841, 2207.11457, 2305.02930)



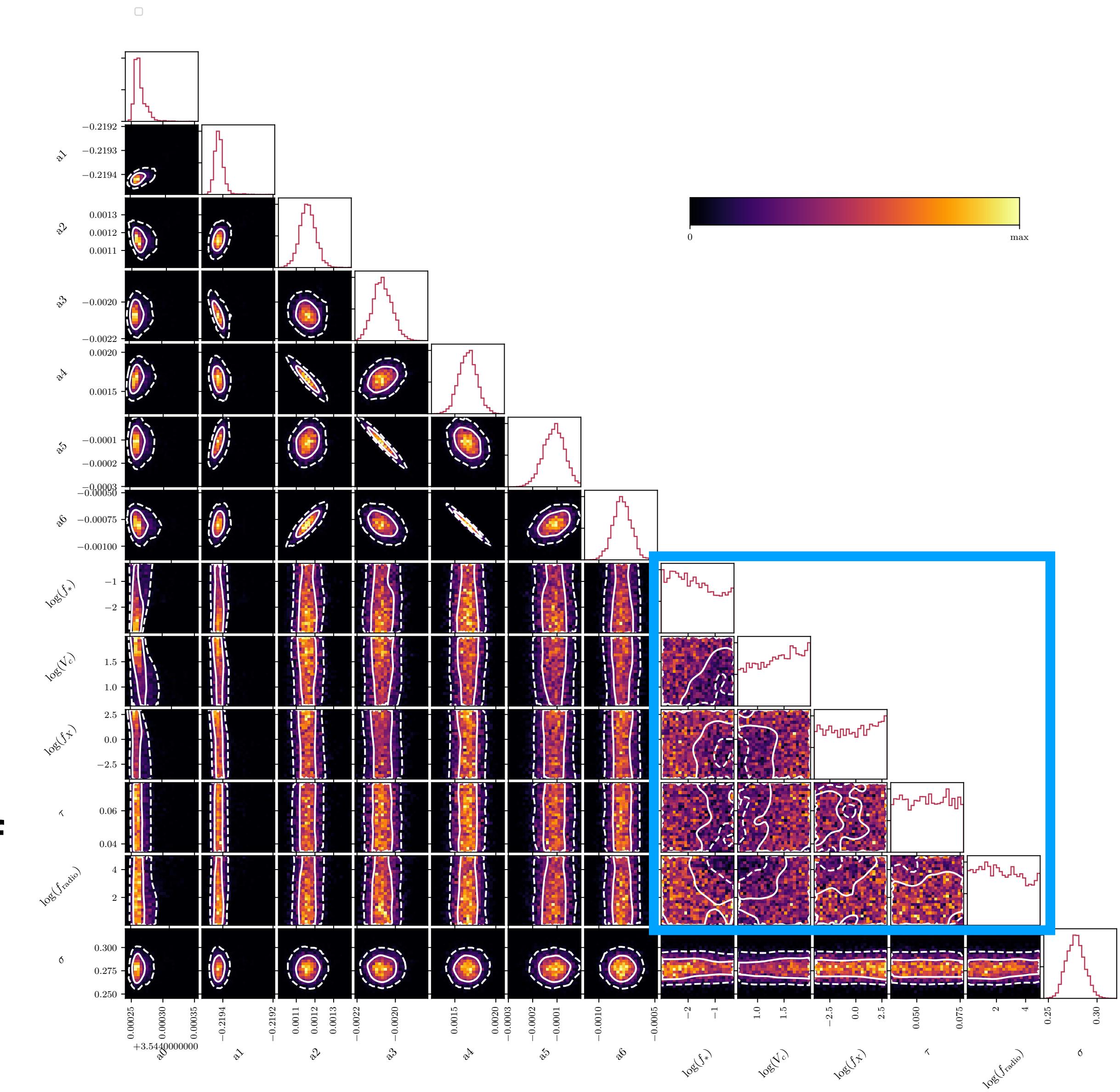
The Methodology: Why bother with normalizing flows?

Why not just run a joint likelihood over all of the parameters in an analytic likelihood?

The SARAS3 model has foreground parameters... but we are not really interested in these!

We refer to these as *nuisance* parameters!

Flows allow us to learn marginal parts of the parameter space



The Methodology: Joint likelihood?

$$\mathcal{P}(\theta|D, \mathcal{M}) = \frac{\mathcal{L}(\theta)\pi(\theta)}{\mathcal{Z}} \quad \theta = \{\theta_I, \theta_{fg}, \theta_{21}\}$$

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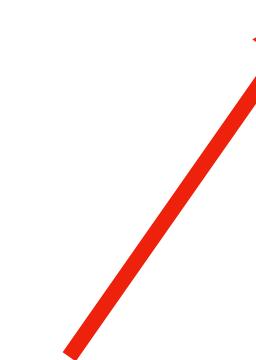
$$\log(\mathcal{L}_{\text{joint}}(\theta_{21})) = \log(\mathcal{L}_{\text{HERA}}(\theta_{21})) + \log(\mathcal{L}_{\text{SARAS3}}(\theta_{21}))$$

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$$\mathcal{L}(\theta_{21}) \equiv \frac{\int \mathcal{L}(\theta_{21}, \alpha)\pi(\theta_{21}, \alpha)d\alpha}{\int \pi(\theta_{21}, \alpha)d\alpha} = \frac{\mathcal{P}(\theta_{21}|D, \mathcal{M})\mathcal{Z}}{\pi(\theta_{21})}$$



Need to be able to evaluate $\mathcal{P}(\theta_{21}|D, \mathcal{M})$ and $\pi(\theta_{21})$ for a set θ_{21}

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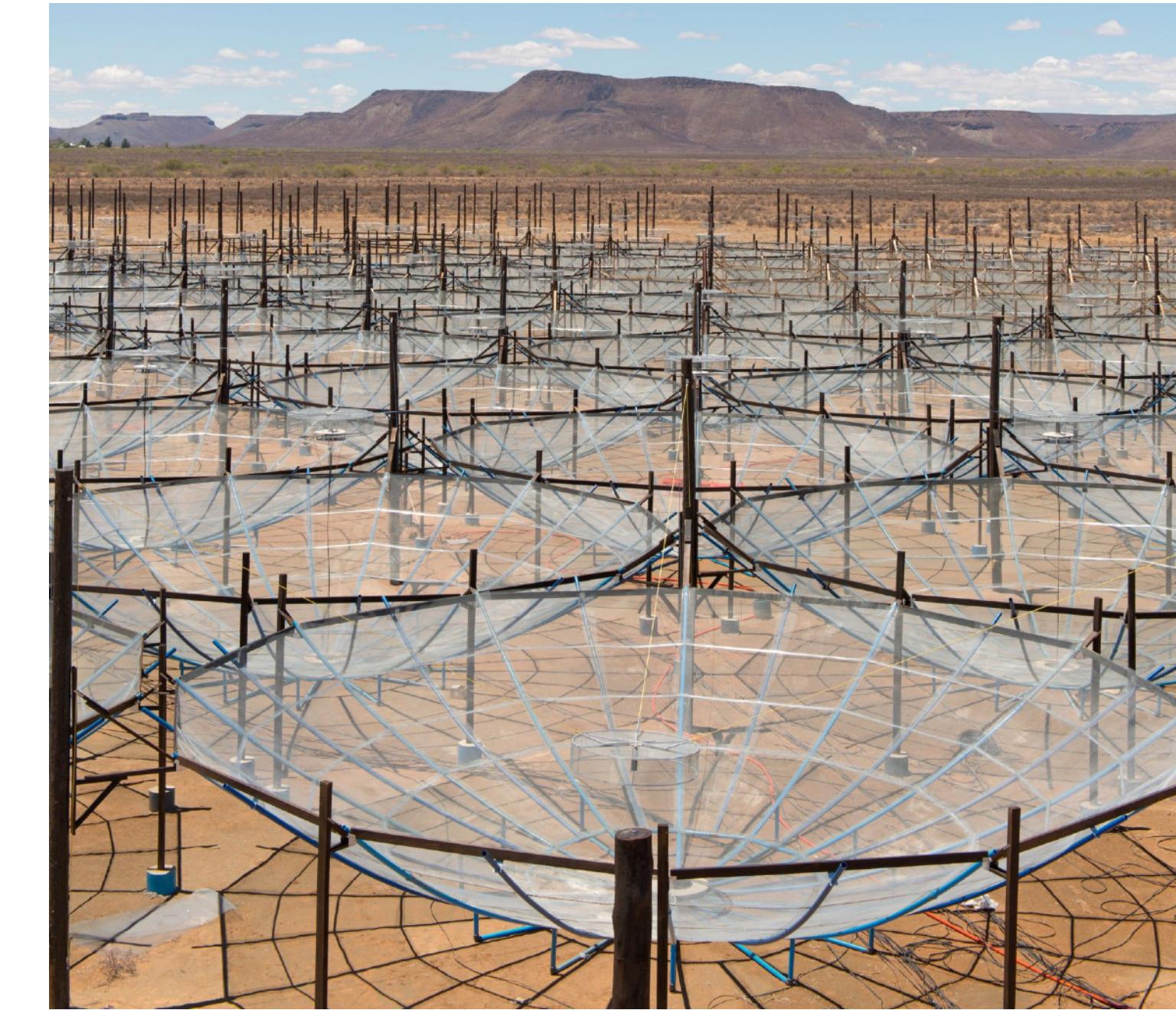
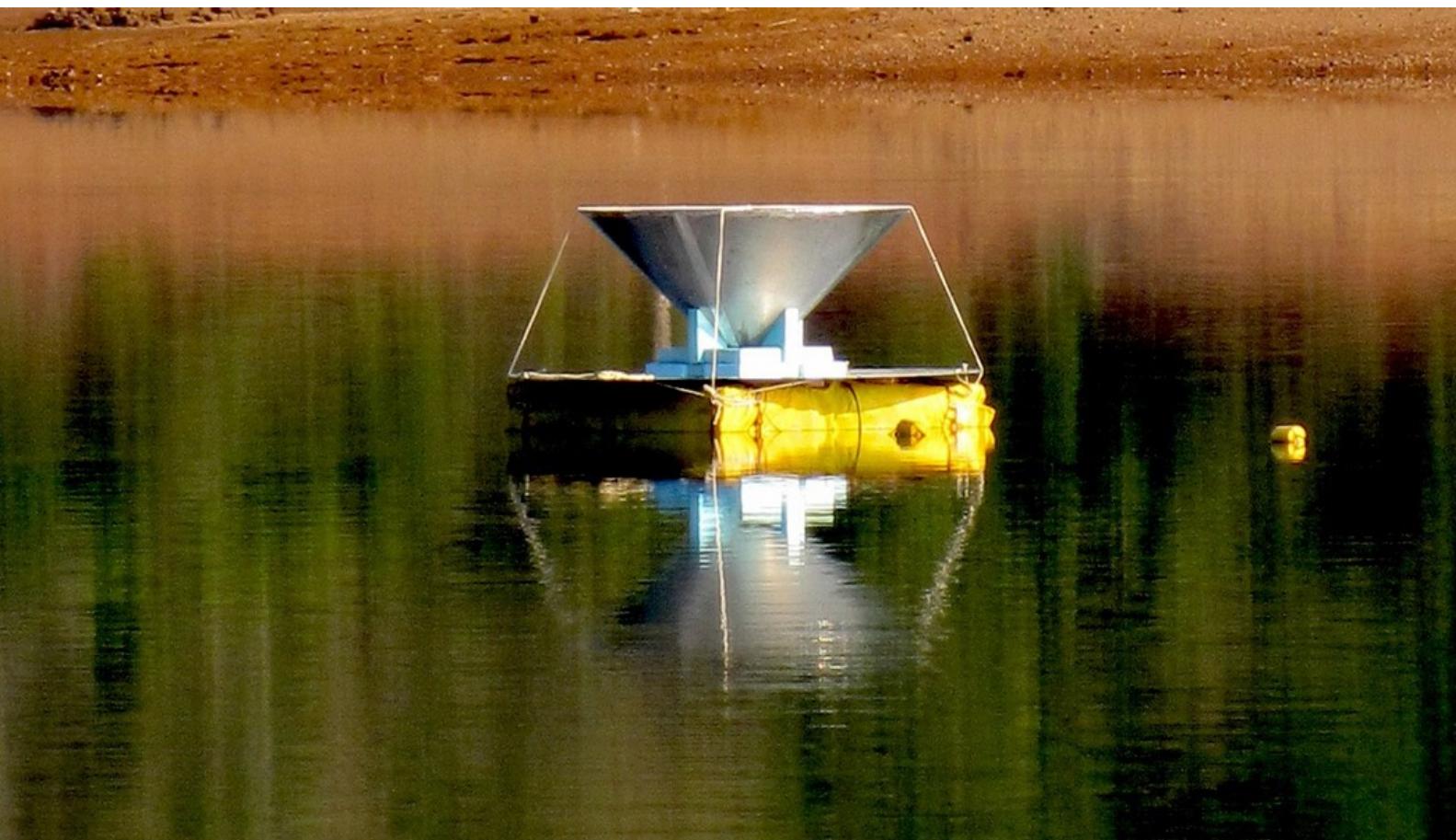
For an individual experiment we have...

$$\theta = \{\theta_I, \theta_{fg}, \theta_{21}\} \rightarrow \{\theta_{21}\} \rightarrow \boxed{\text{MARGARINE}} \rightarrow \log(\mathcal{P}(\theta_{21}|D, \mathcal{M})) \rightarrow \log(\mathcal{L}(\theta_{21}))$$

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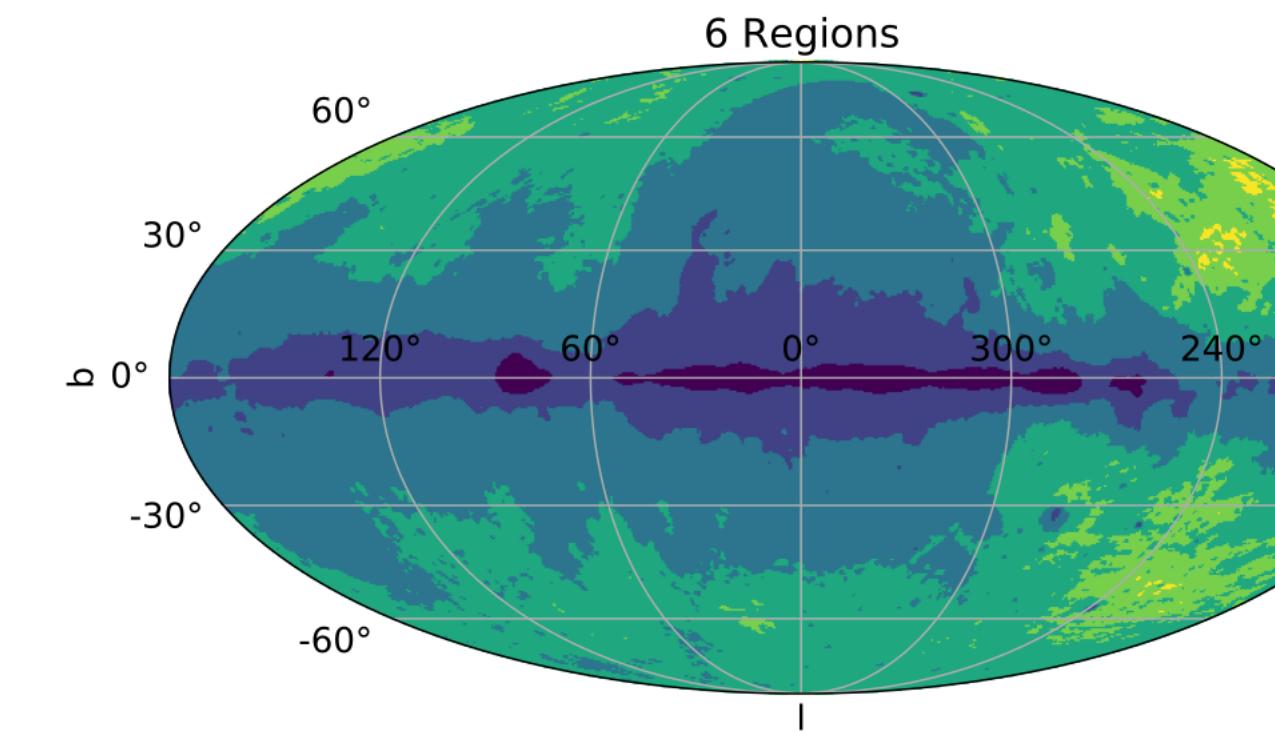
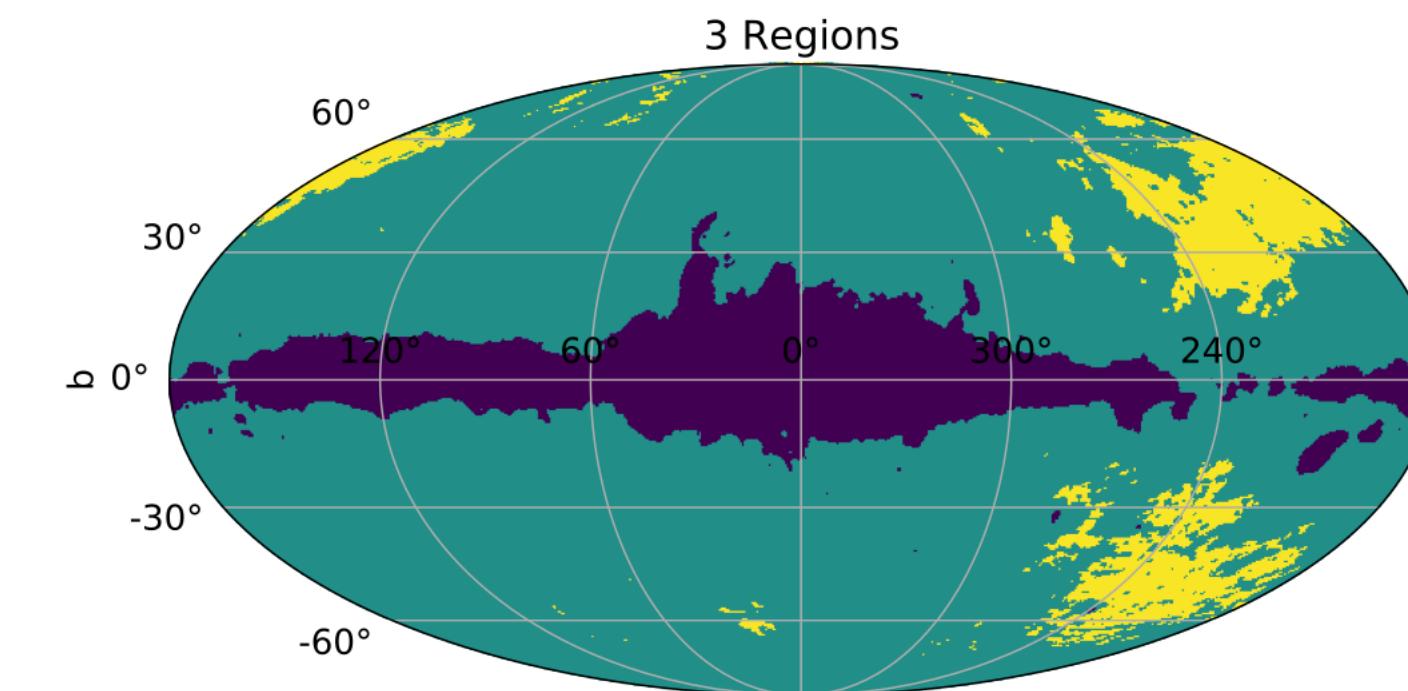
The Methodology: Marginal REACH Likelihoods?

$$\theta = \{\theta_I, \theta_{fg}, \theta_{21}\} \rightarrow \{\theta_{21}\} \rightarrow \text{MARGARINE} \rightarrow \log(\mathcal{P}(\theta_{21}|D, \mathcal{M})) \rightarrow \log(\mathcal{L}(\theta_{21}))$$

REACH likelihood has large numbers of parameters for foreground, signal and potentially the beam (see Carla and John's talks)

Use *margarine* to learn the signal posterior to combine with other probes

See Peter's talk...

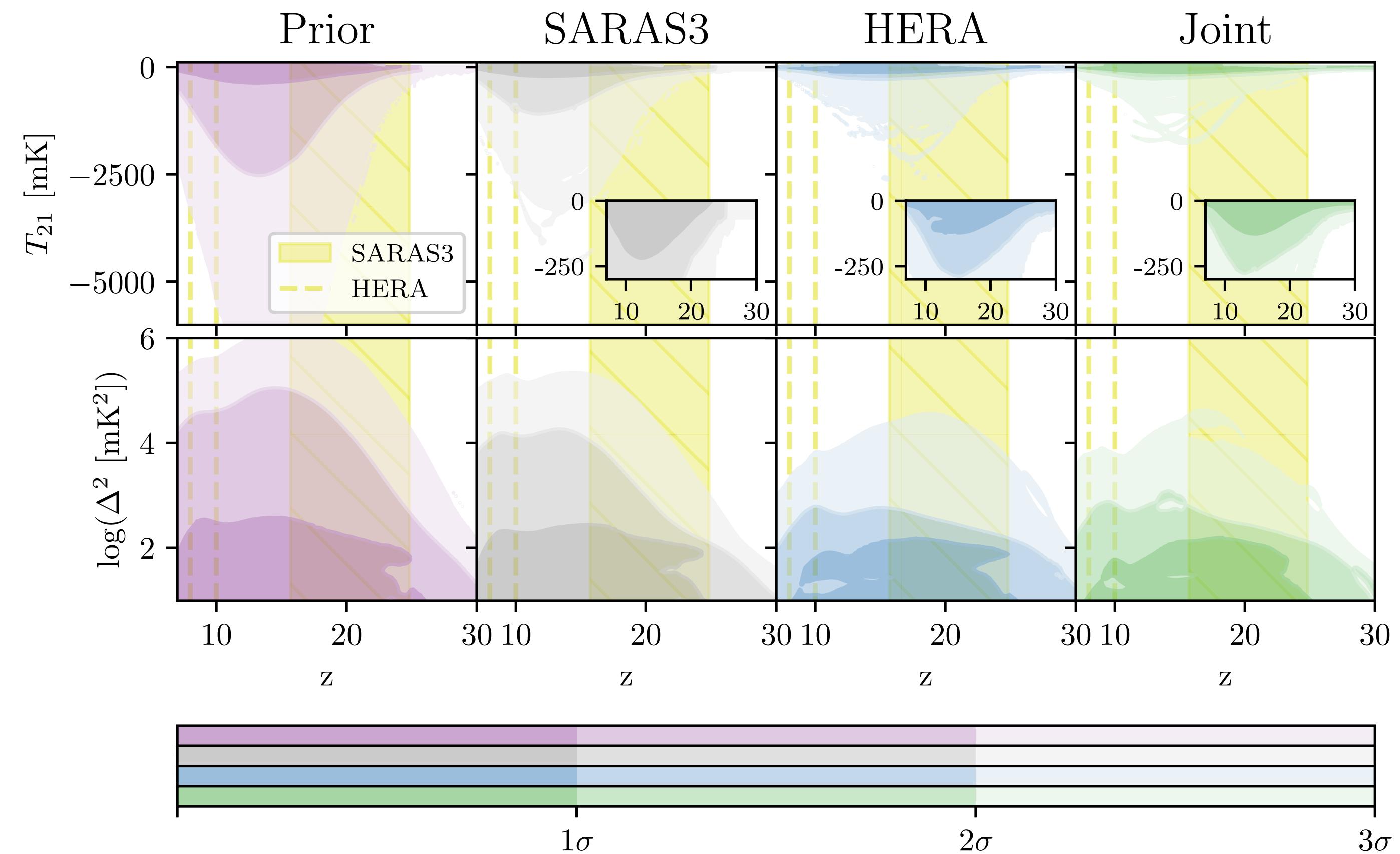


The Results

The Results: Constraints?

We get tighter constraints on the parameter space

Translates to tighter constraints on the 21-cm signal and the power spectrum



The Results: Constraining power?

Quantify constraining power with the *marginal* KL divergence via *margarine*

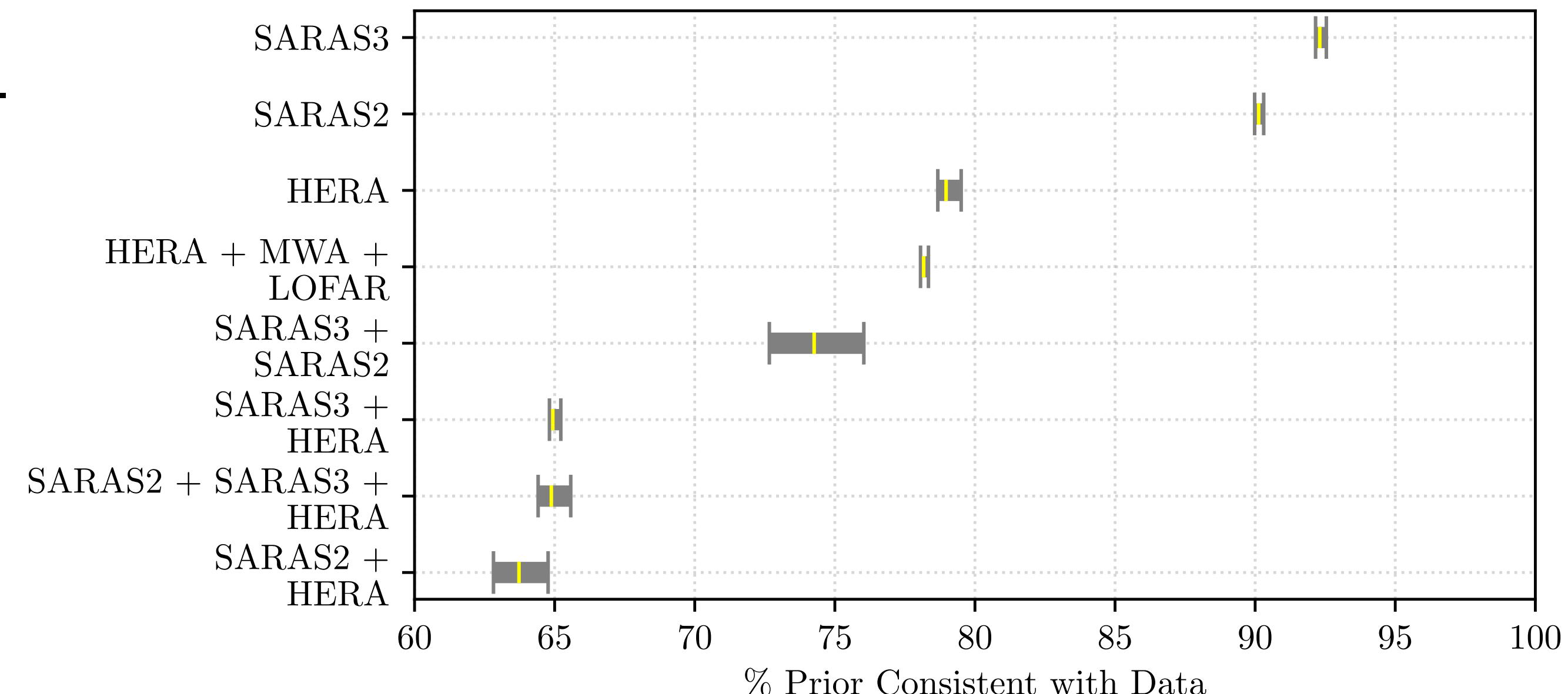
$$\mathcal{D}(P \parallel \pi) = \left\langle \log \frac{P(\theta_{21})}{\pi(\theta_{21})} \right\rangle_{P(\theta_{21})} \approx -\log \frac{V_P}{V_\pi}$$

Contraction of prior on to posterior

SARAS3 $\approx 93\%$

HERA $\approx 78\%$

SARAS3 + HERA $\approx 65\%$



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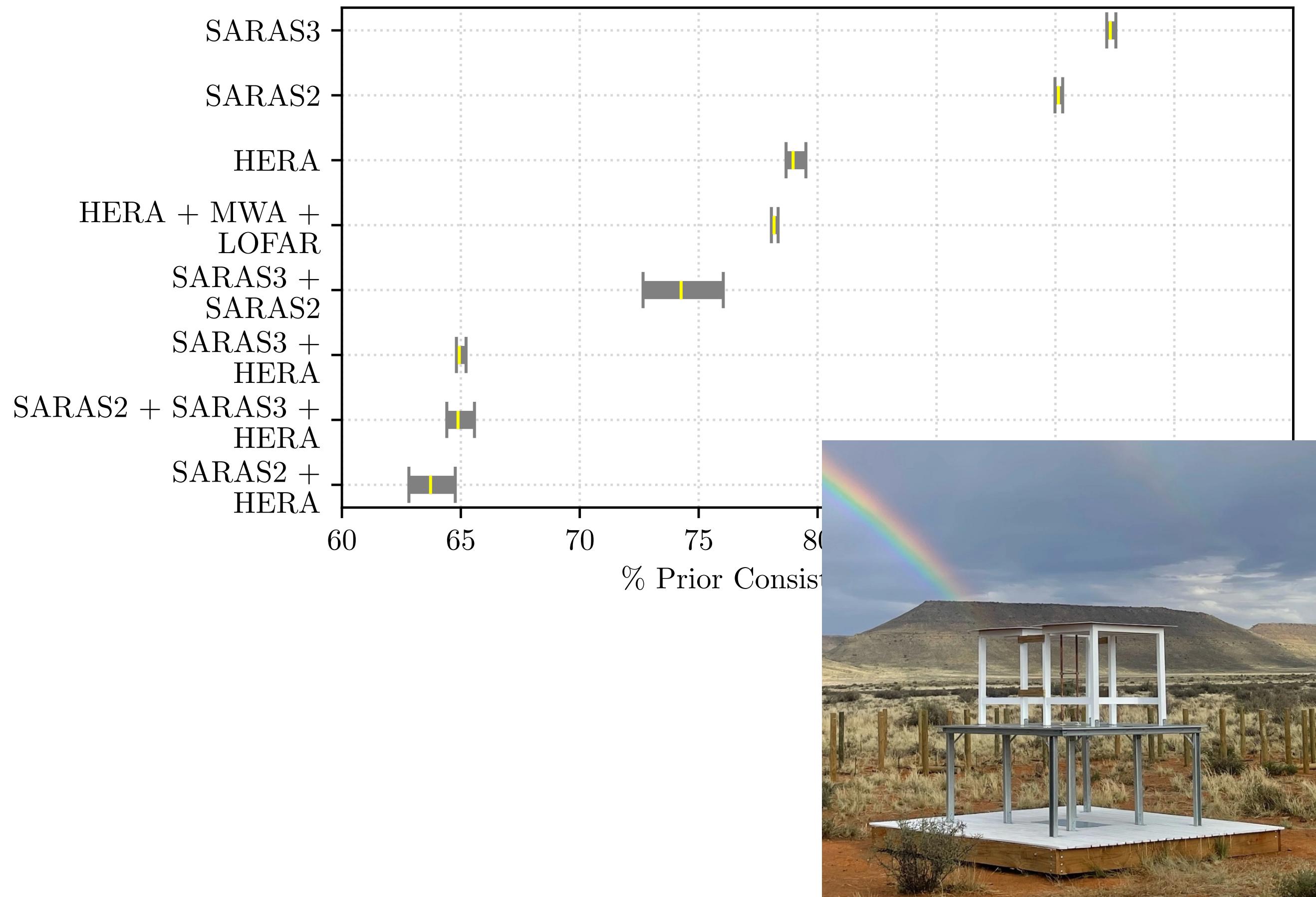
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Where does REACH fit in to this??

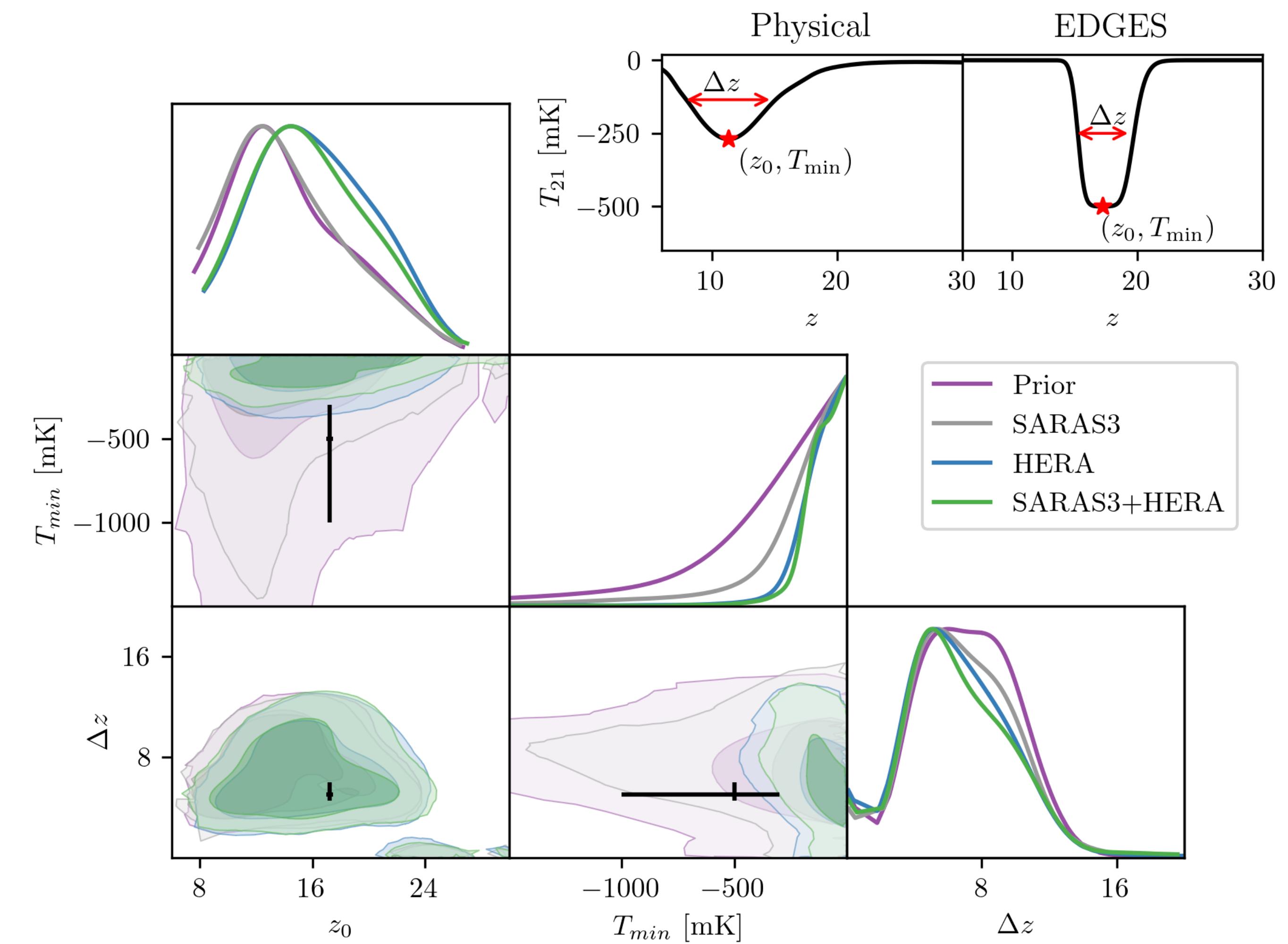


The Results: What can we say about EDGES?

We can take an approximate width, depth and central frequency

Both SARAS3 and HERA allow for physical signals with a depth consistent with EDGES...

... however together they disfavour this class of signals with two sigma confidence!



How about other probes?

Why stop with 21-cm?

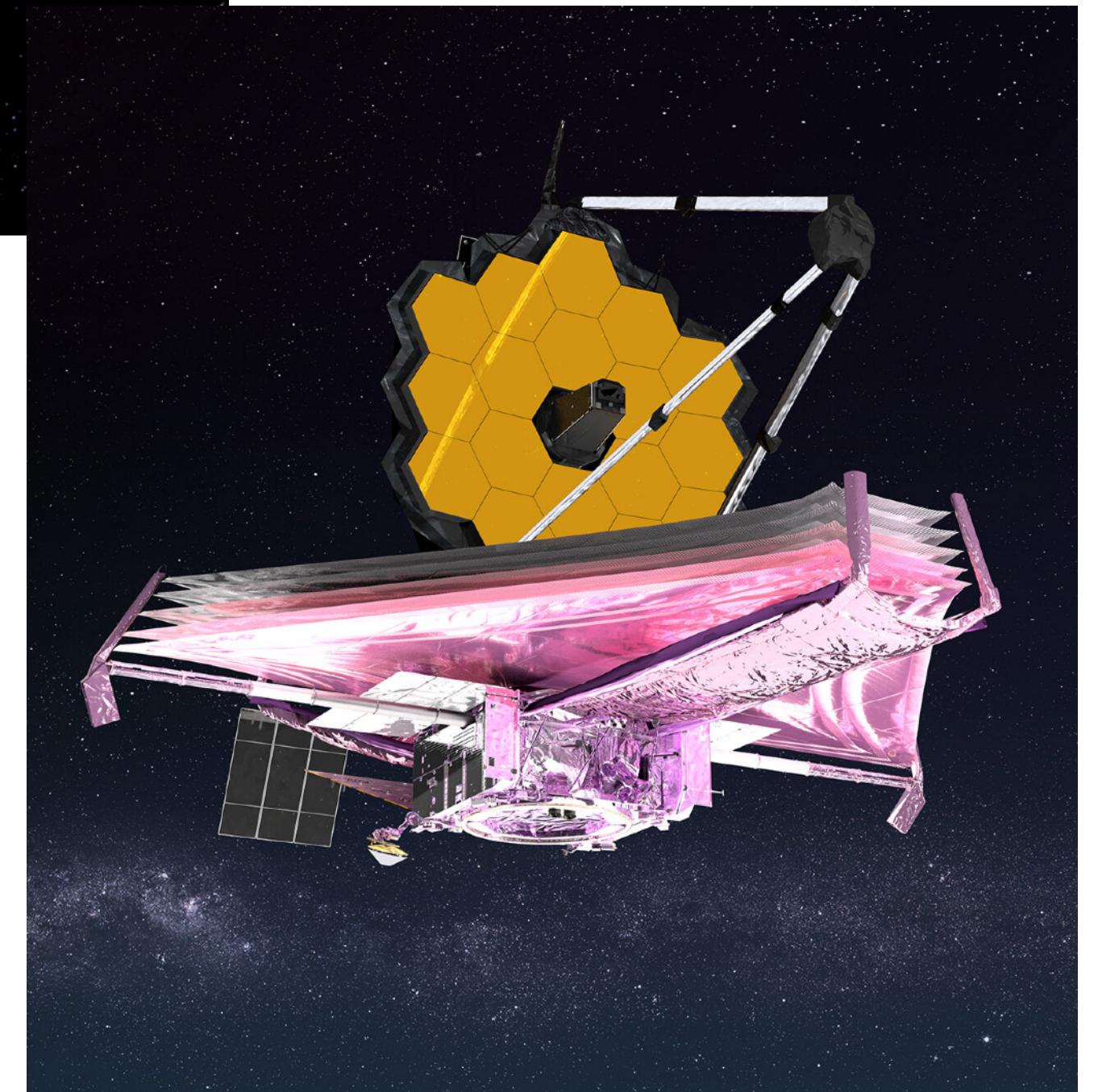
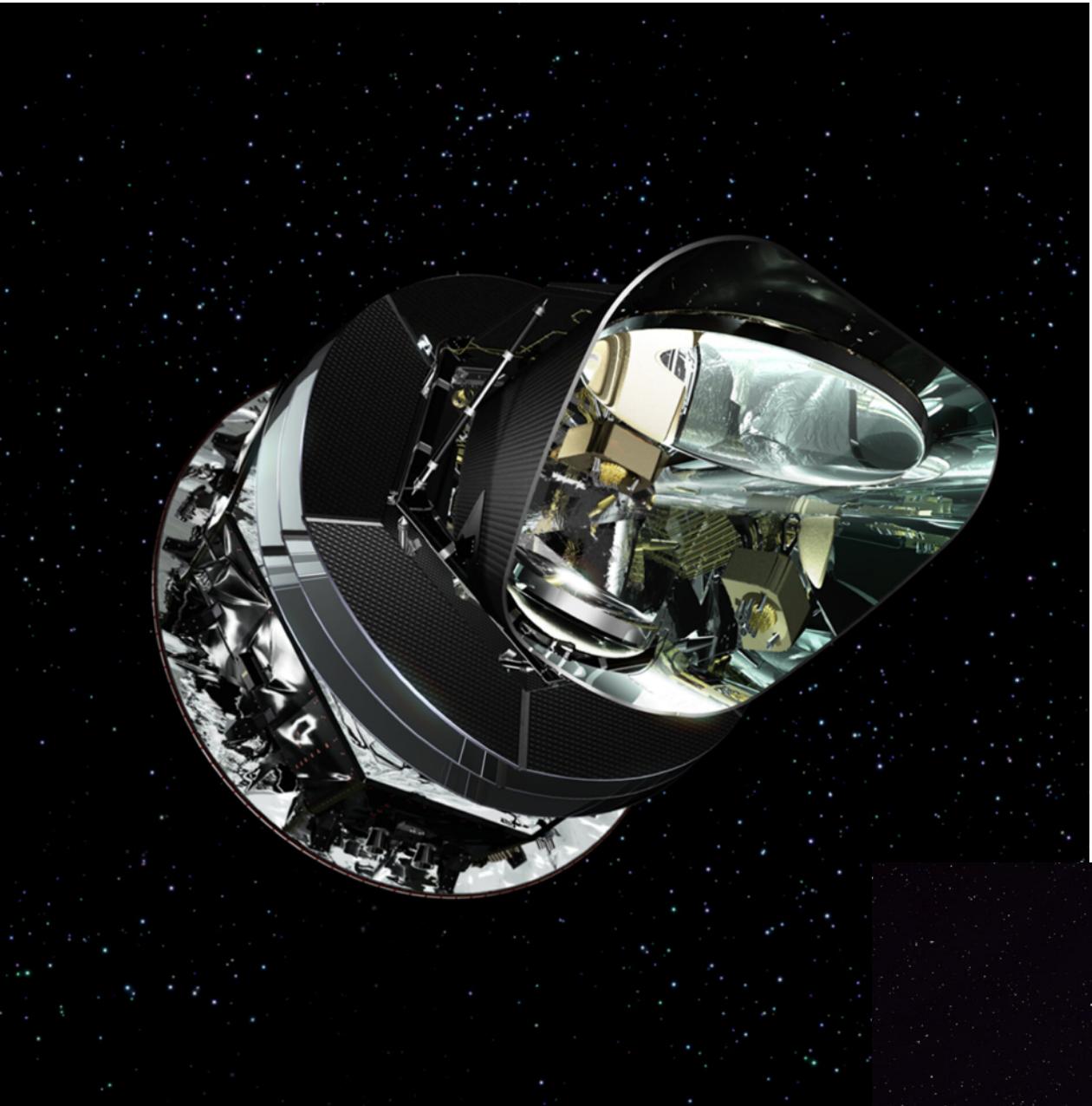
Use normalizing flows to jointly combine other probes with 21-cm

Planck constraints on τ

Constraints on IGM neutral fraction

Luminosity function constraints

See Jiten's talk and Peter's talk next



Conclusions

- Developed a novel method for performing joint analysis of different data sets
- Shown that by combining power spectrum and sky-averaged 21-cm signal data we can get improved constraints

Future Work

- Peter Sims - Looking at applications of the method to REACH
- Thomas Gessey-Jones - joint analysis constraints on Cosmic Strings
- Simon Pochinda - updated constraints with additional information from probes of the X-ray background
- Continued improvements to margarine

Joint Analysis Paper

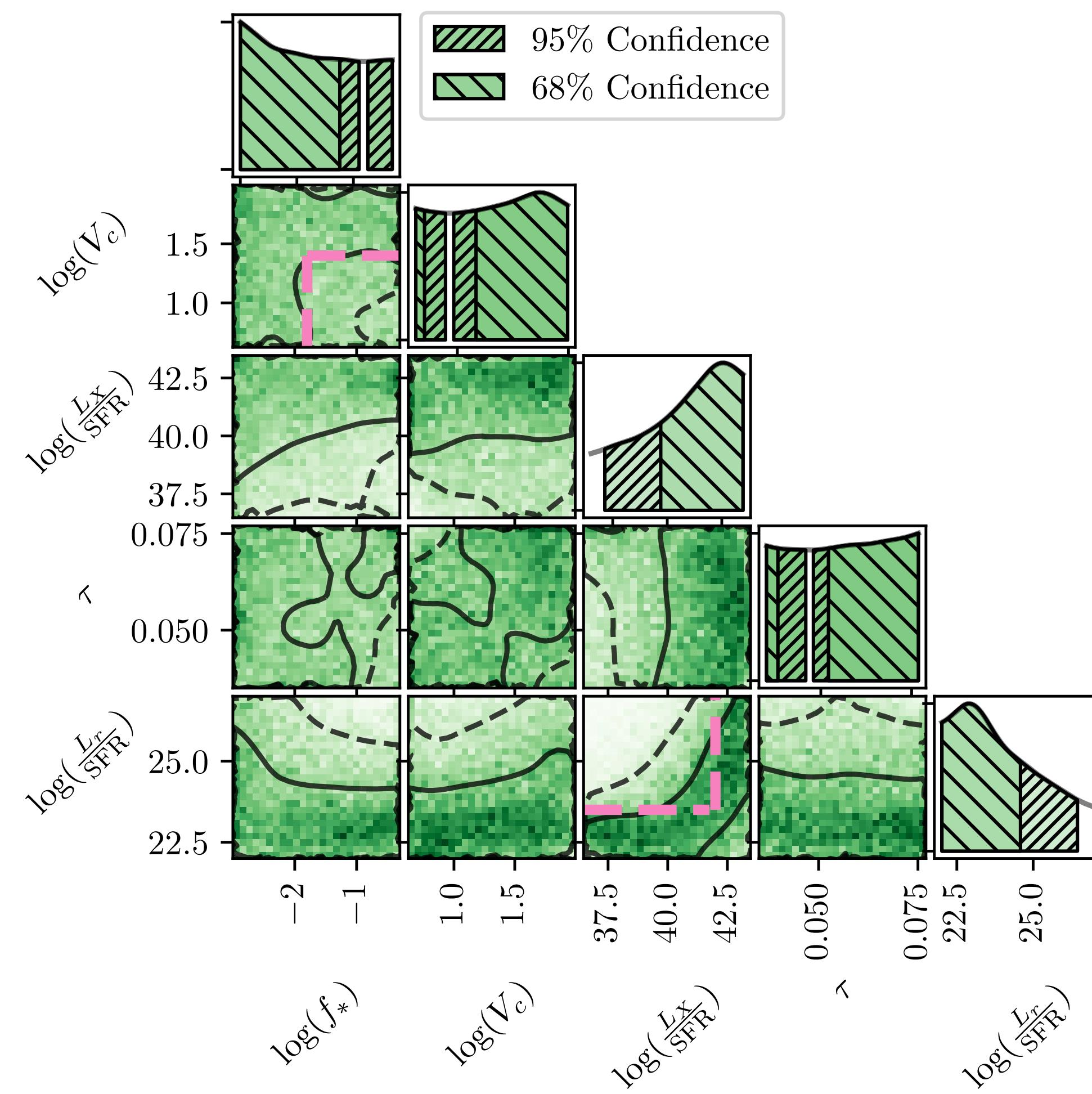


Methods paper

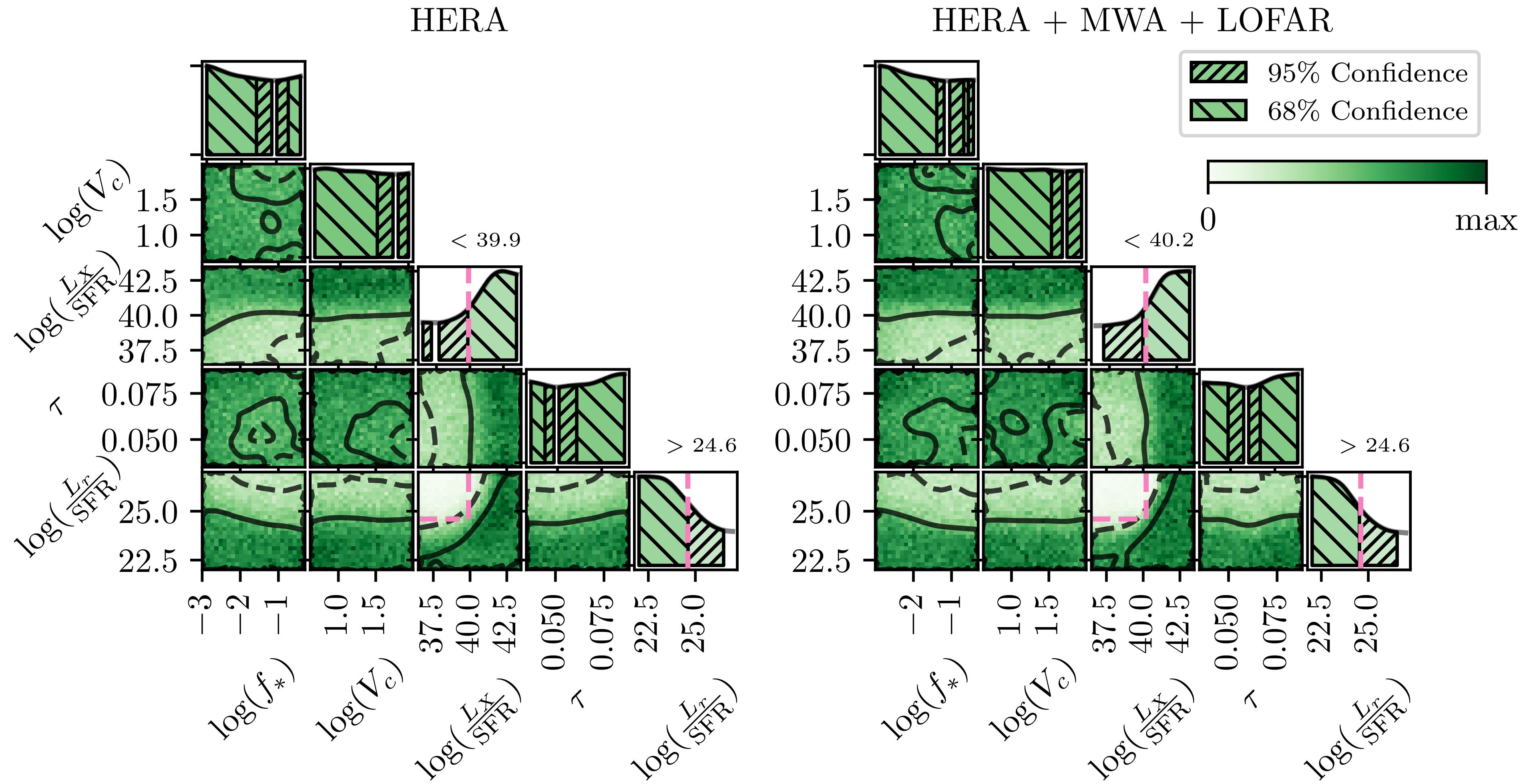


Bonus Slides

The Results: Parameters?



The Results: Parameters?



The Results: Other Global?

