Hamilton's Principle
Classical Dynamics
Problem Sheet

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1. Hamilton's principle basics

(a) State Hamilton's principle and explain how it leads to the Euler-Lagrange equations.

(b) Show explicitly that requiring $\delta S = 0$ for

$$S = \int_{t_1}^{t_2} L(x, \dot{x}, t)\, dt$$

leads to

$$\frac{\delta}{\delta t}\left(\frac{\delta L}{\delta \dot{x}}\right) - \frac{\delta L}{\delta x} = 0.$$

2. Simple harmonic oscillator

A particle of mass m attached to a spring of stiffness k has Lagrangian

$$L = \tfrac{1}{2}m\dot{x}^2 - \tfrac{1}{2}kx^2.$$

(a) Derive the equations of motion using Hamilton's principle.

(b) Verify that $x(t) = A\cos(\omega t) + B\sin(\omega t)$ solves the equation of motion. What is ω?

3. Geodesics on a sphere

A particle constrained to move on the surface of a sphere of radius R moves freely (no potential). Its Kinetic energy is given, in spherical coordinates (r, θ, ϕ), by

$$T = \tfrac{1}{2}m\left(R^2\dot{\theta}^2 + R^2\sin^2\theta\,\dot{\phi}^2\right).$$

(a) Use Hamilton's principle to derive the geodesic equations.

Remember that for motion on the sphere, $r = R$ (constant), so the kinetic energy depends only on θ and ϕ.