Forward Modelling in 21-cm Cosmology

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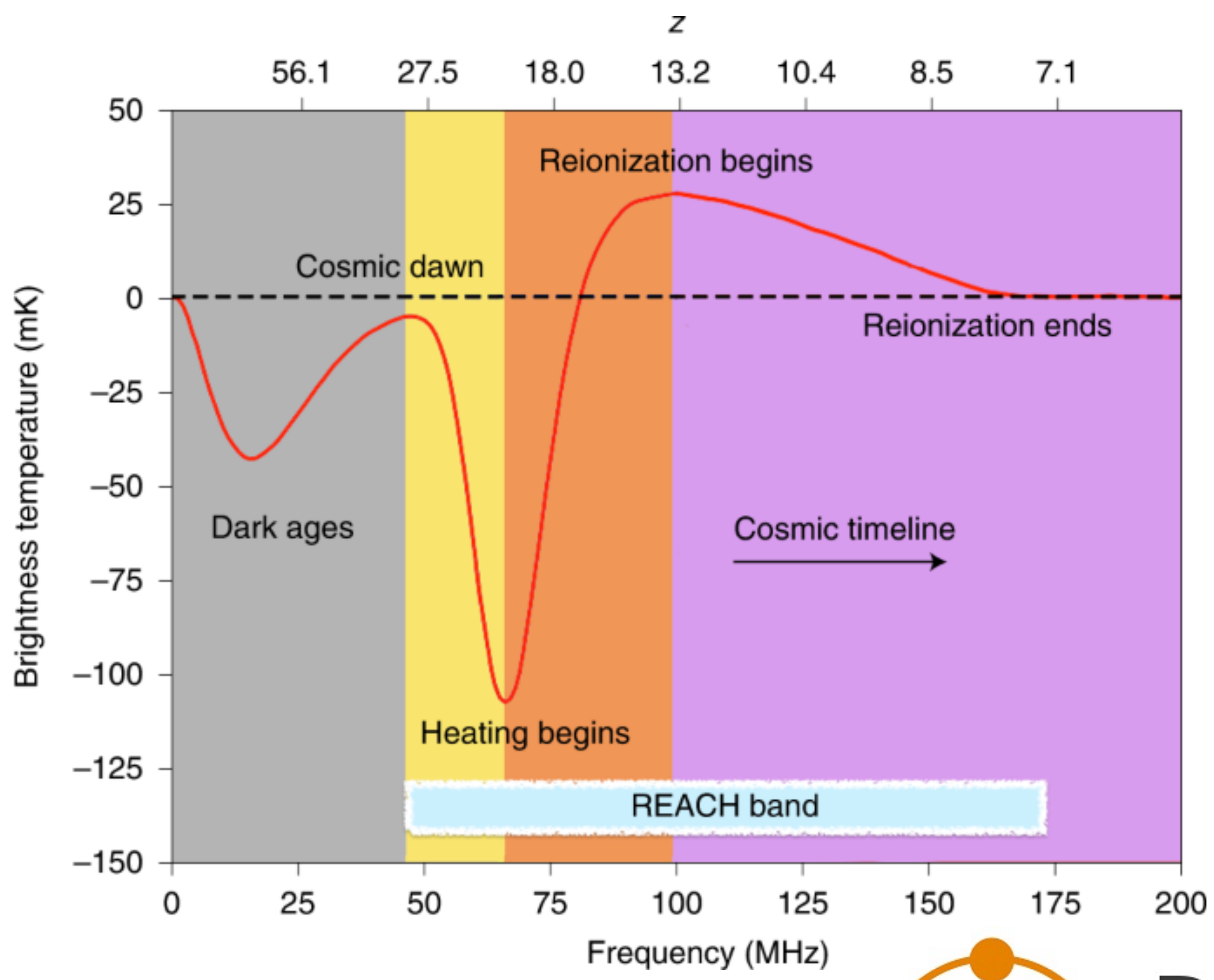
The Plan

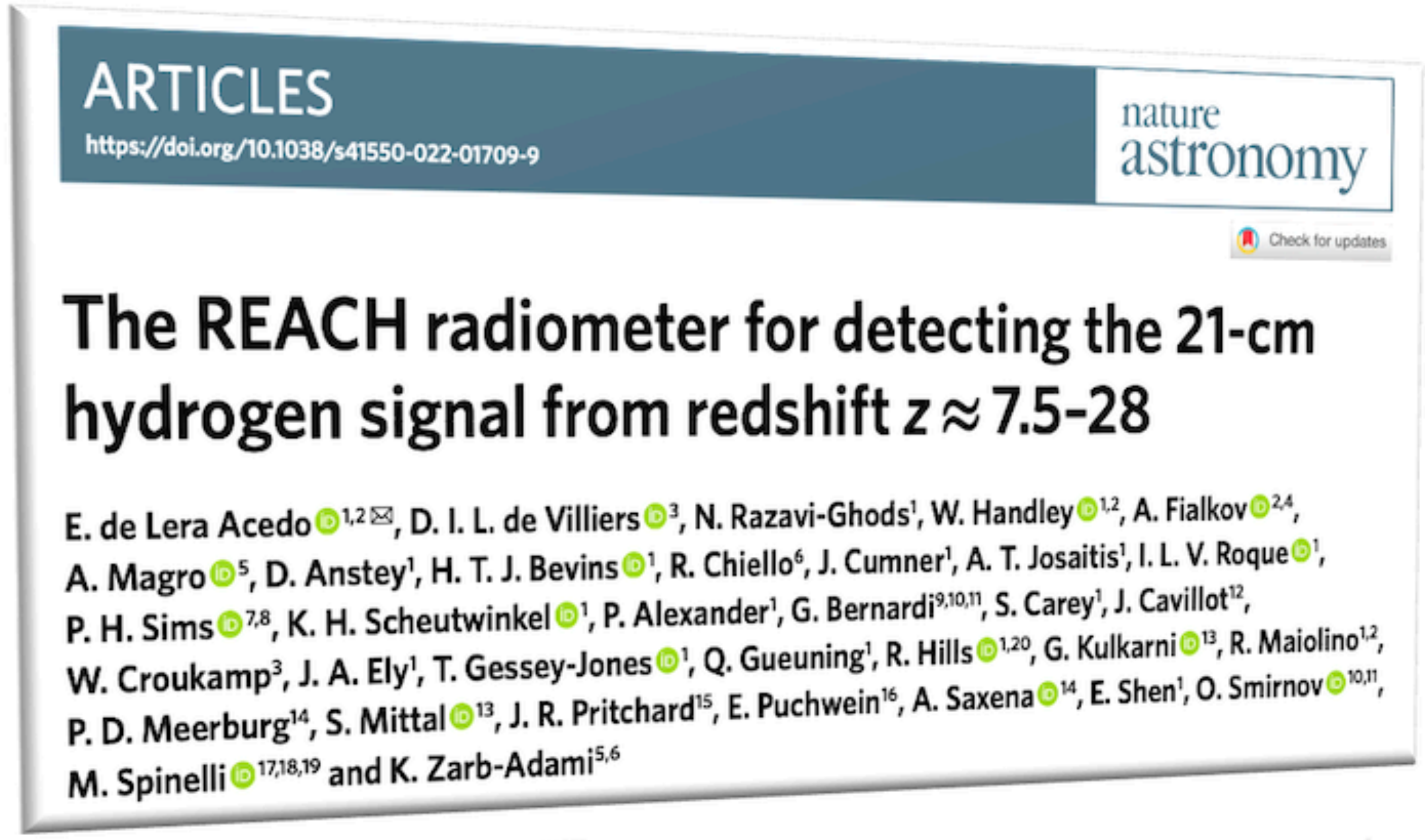
1. Review the data analysis problem

3. Review the state of the art modelling

5. Briefly discuss future work



REACH Collaboration


ARTICLES

https://doi.org/10.1038/s41550-022-01709-9

nature astronomy

Check for updates

The REACH radiometer for detecting the 21-cm hydrogen signal from redshift $z \approx 7.5$-28

E. de Lera Acedo[1,2], D. I. L. de Villiers[3], N. Razavi-Ghods[1], W. Handley[1,2], A. Fialkov[2,4], A. Magro[5], D. Anstey[1], H. T. J. Bevins[1], R. Chiello[6], J. Cumner[1], A. T. Josaitis[1], I. L. V. Roque[1], P. H. Sims[7,8], K. H. Scheutwinkel[1], P. Alexander[1], G. Bernardi[9,10,11], S. Carey[1], J. Cavillot[12], W. Croukamp[3], J. A. Ely[1], T. Gessey-Jones[1], Q. Gueuning[1], R. Hills[1,20], G. Kulkarni[13], R. Maiolino[1,2], P. D. Meerburg[14], S. Mittal[13], J. R. Pritchard[15], E. Puchwein[16], A. Saxena[14], E. Shen[1], O. Smirnov[10,11], M. Spinelli[17,18,19] and K. Zarb-Adami[5,6]

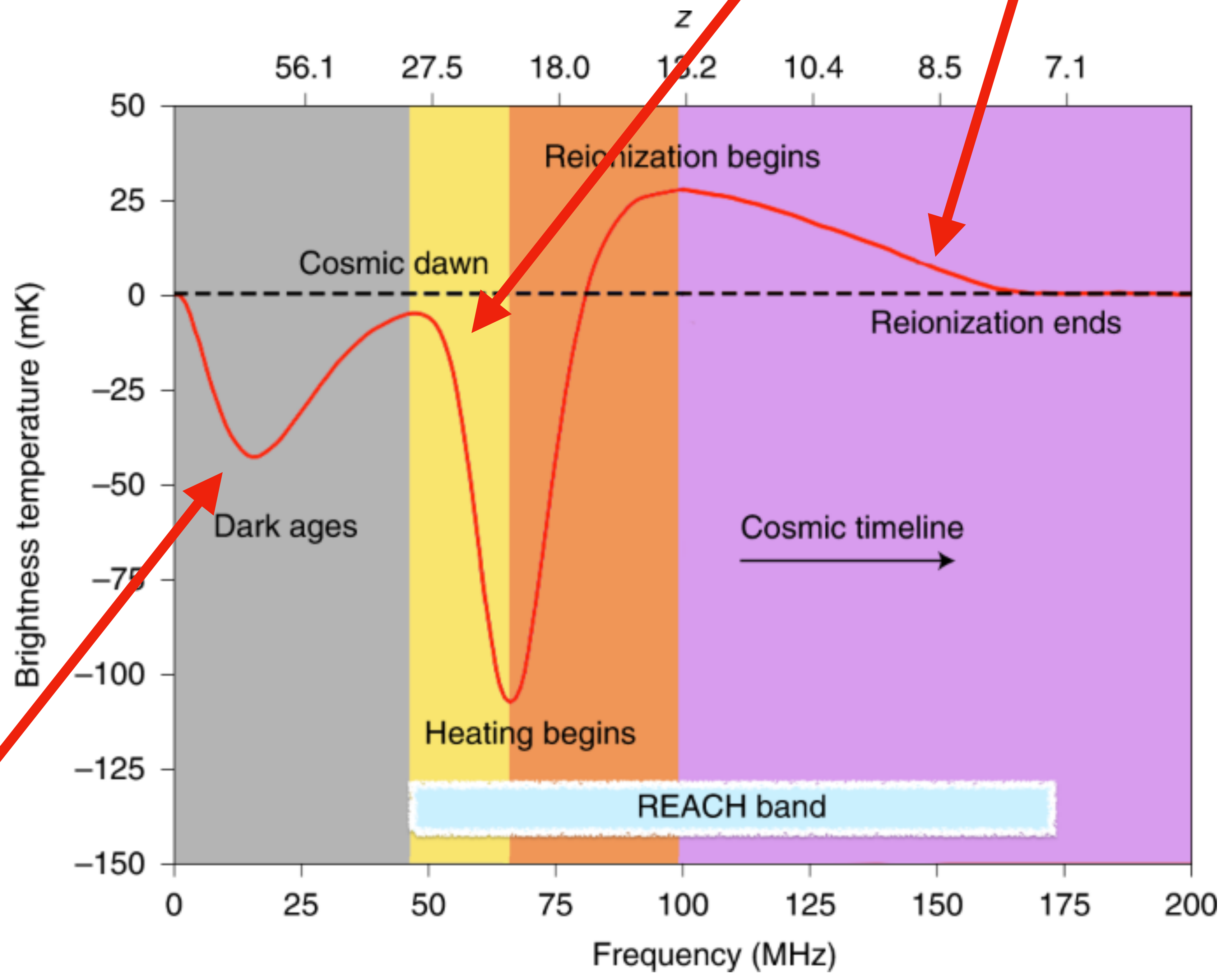
PIs

WP leads

REACH

Why is this important?

Astrophysics - Accessible from Earth and cheap



Cosmology - Need to go to space!

The Problem

Building a likelihood…

- Use Bayes theorem

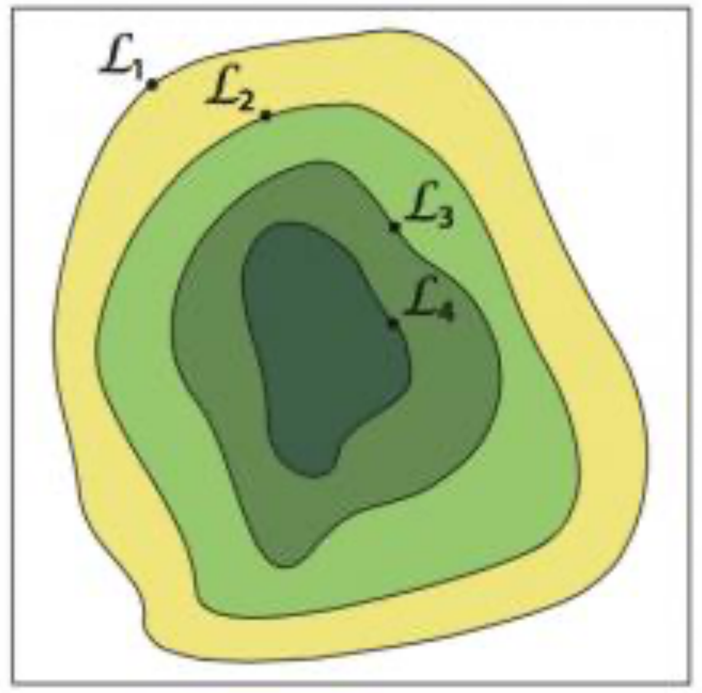
$$P(\theta|D,M) = \frac{P(D|\theta,M)P(\theta|M)}{P(D|M)} = \frac{L(\theta)\pi(\theta)}{Z}$$

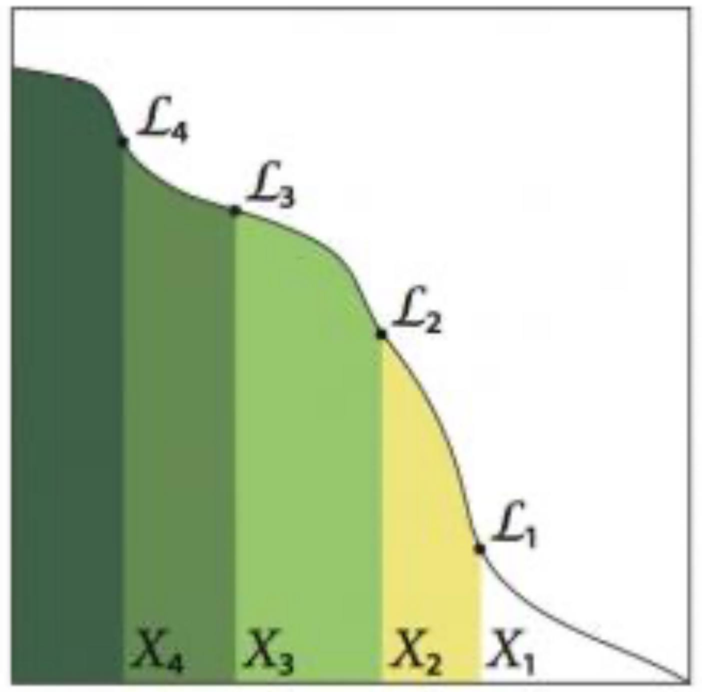
- Need an expression for the likelihood

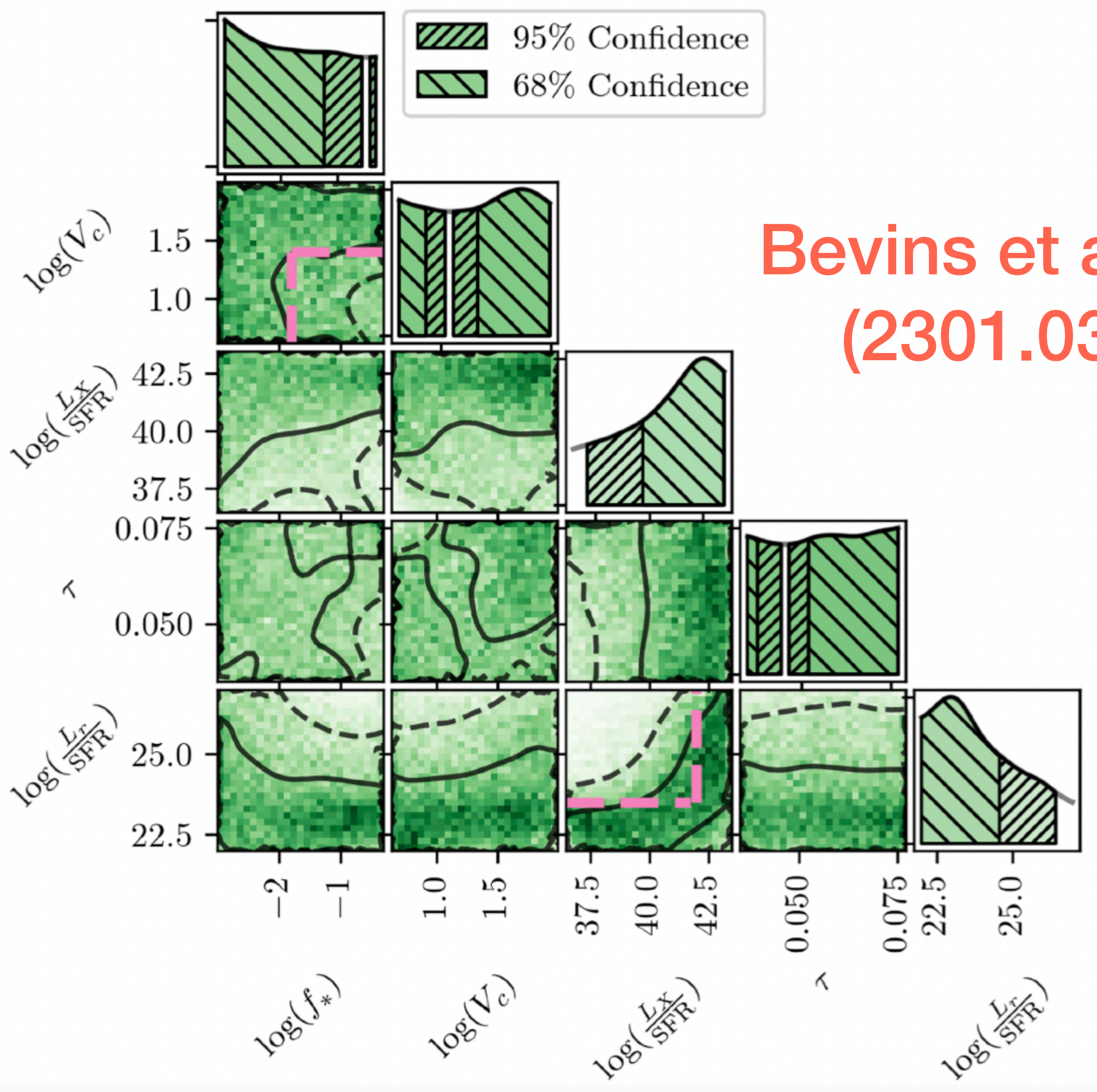
$$\log L(\theta) = -\frac{1}{2}\log 2\pi|\Sigma| - \frac{1}{2}(D - M(\theta))^T\Sigma^{-1}(D - M(\theta))$$

- Other likelihoods are available (see Scheutwinkel **et al. 2022)**

- Turns out writing an expression for $M(\theta)$ is very non-trivial…

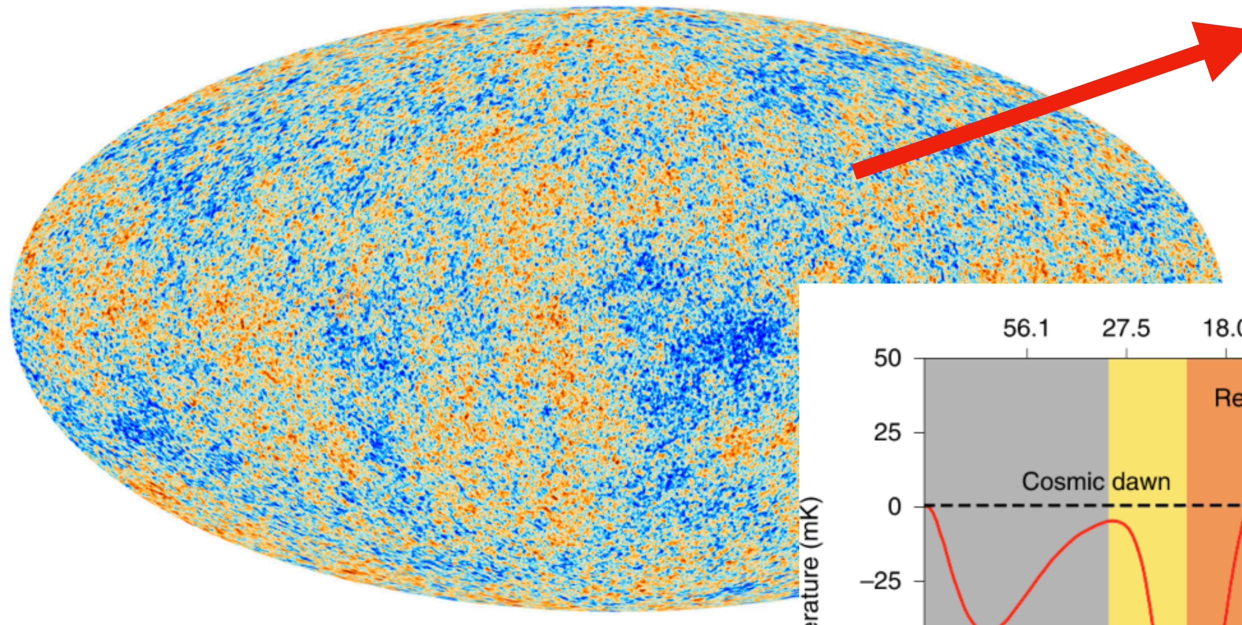




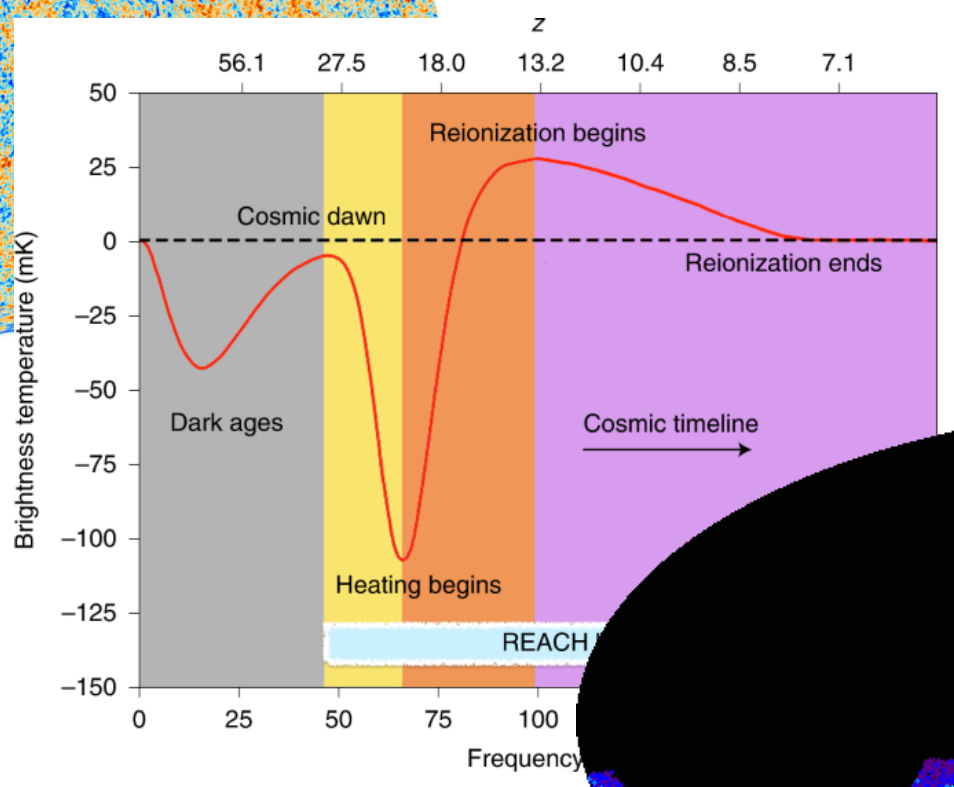


Bevins et al. 2023
(2301.03298)

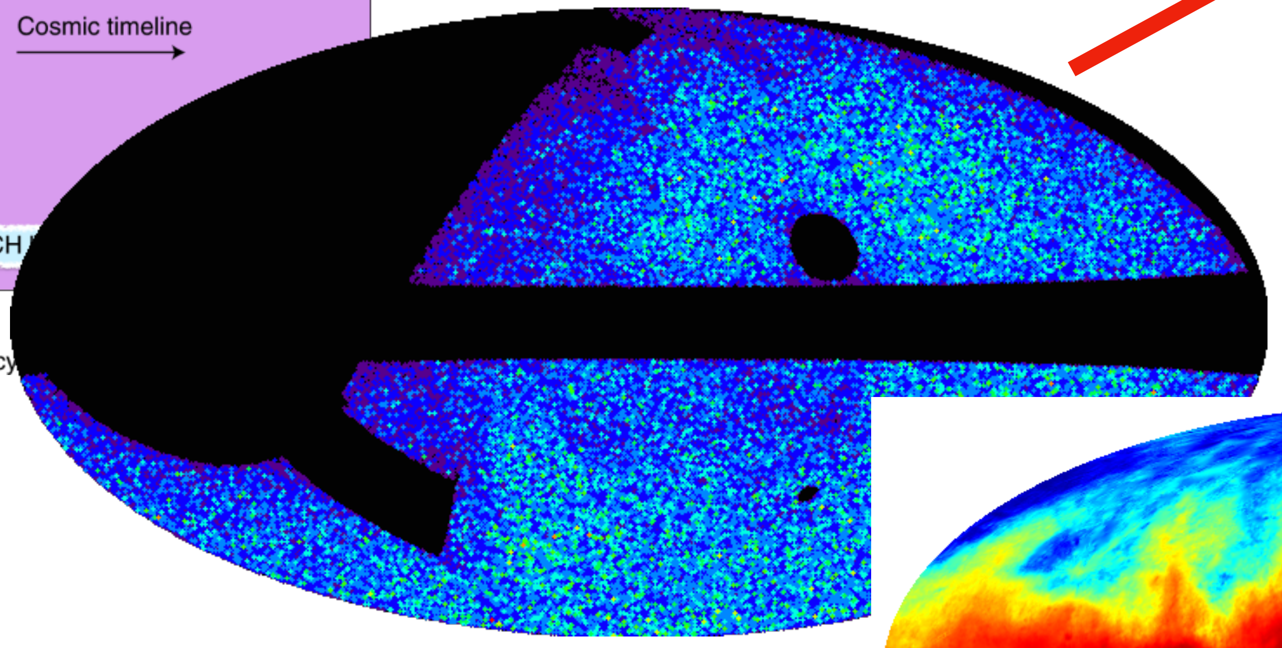
The data…



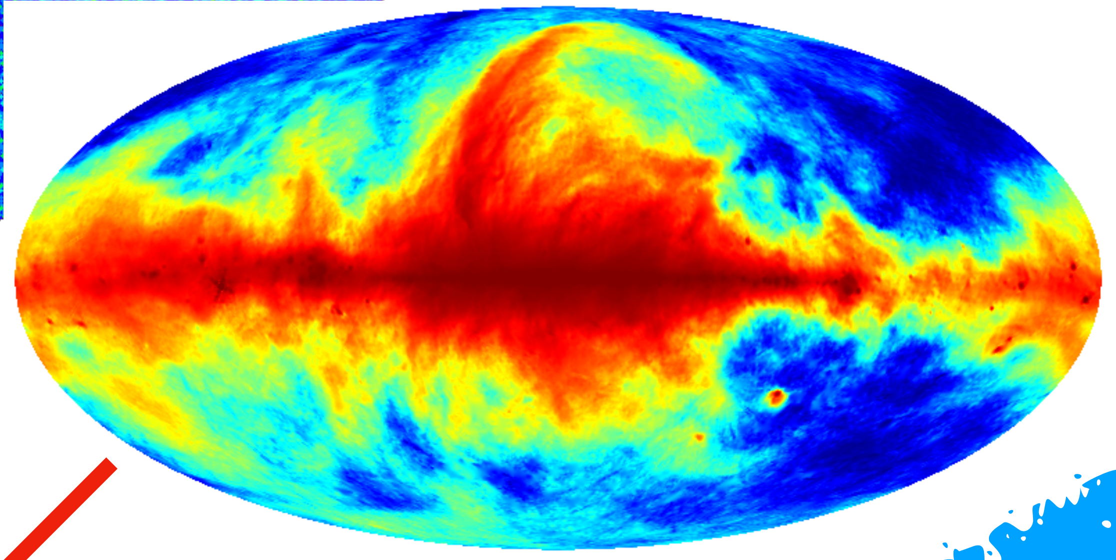
The Cosmic Microwave Background



The 21-cm signal
(What we are interested in!)



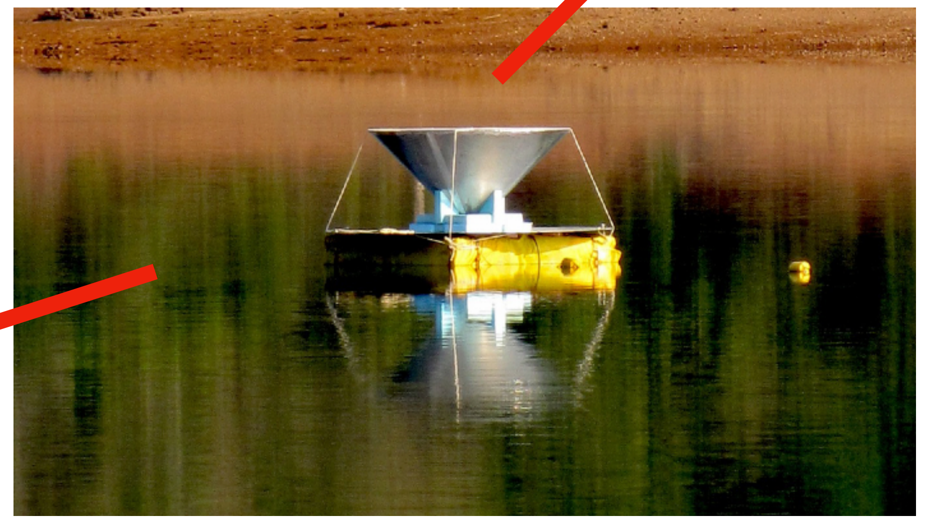
Foreground galaxies



Our own Galaxy

The atmosphere

RFI



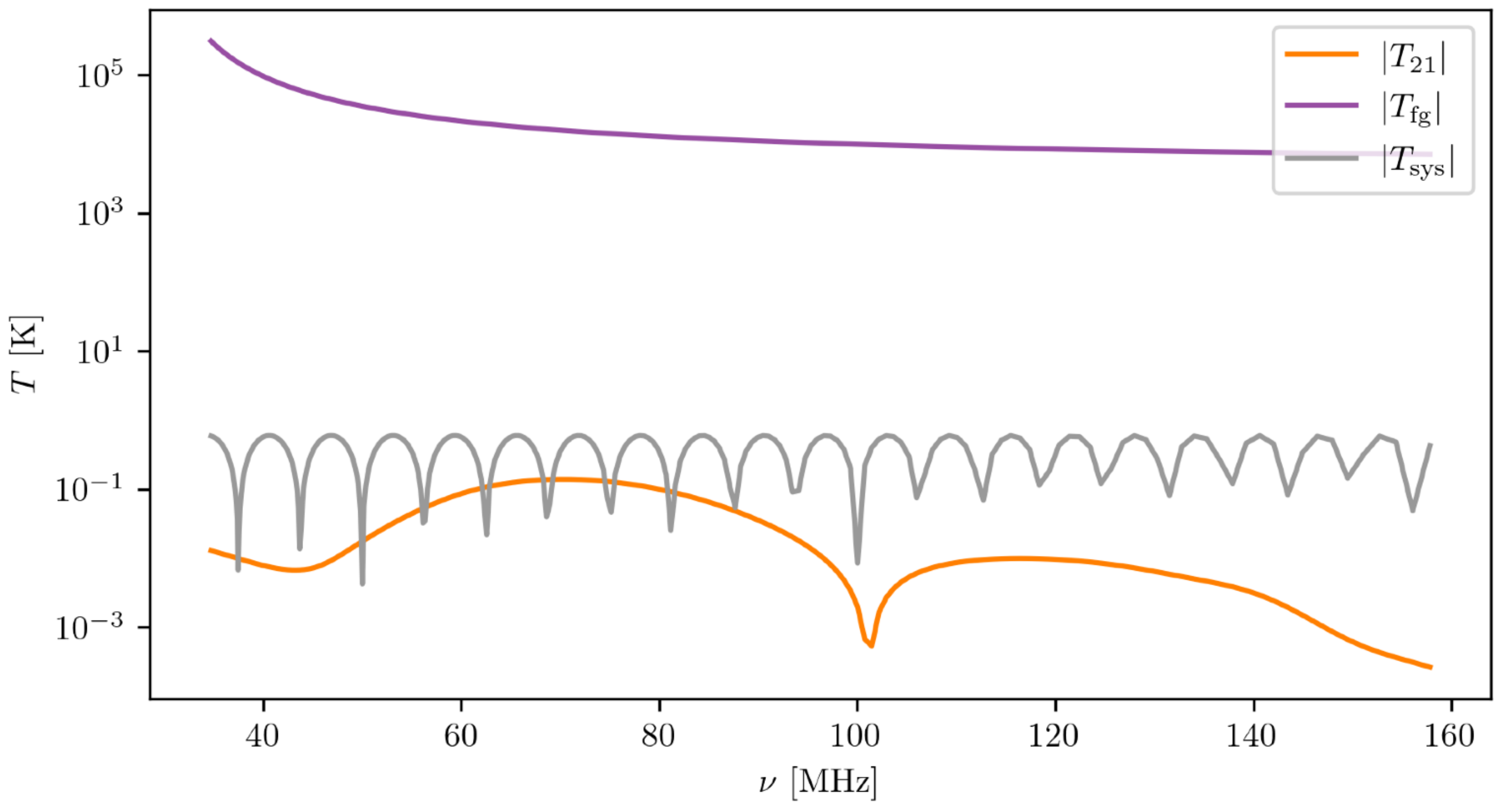
The instrument

The environment around the telescope

The state of the art modelling…

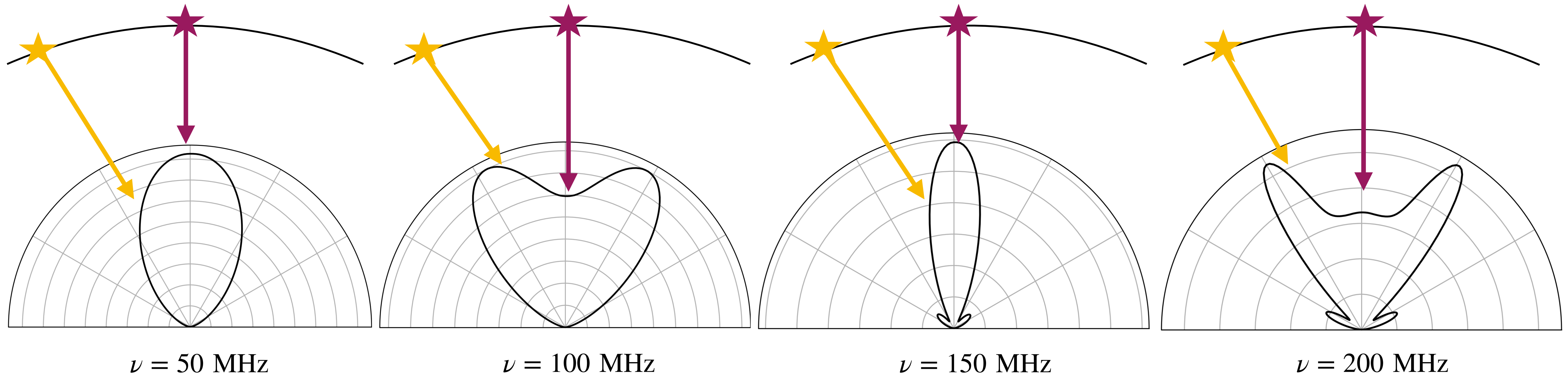
The Foreground

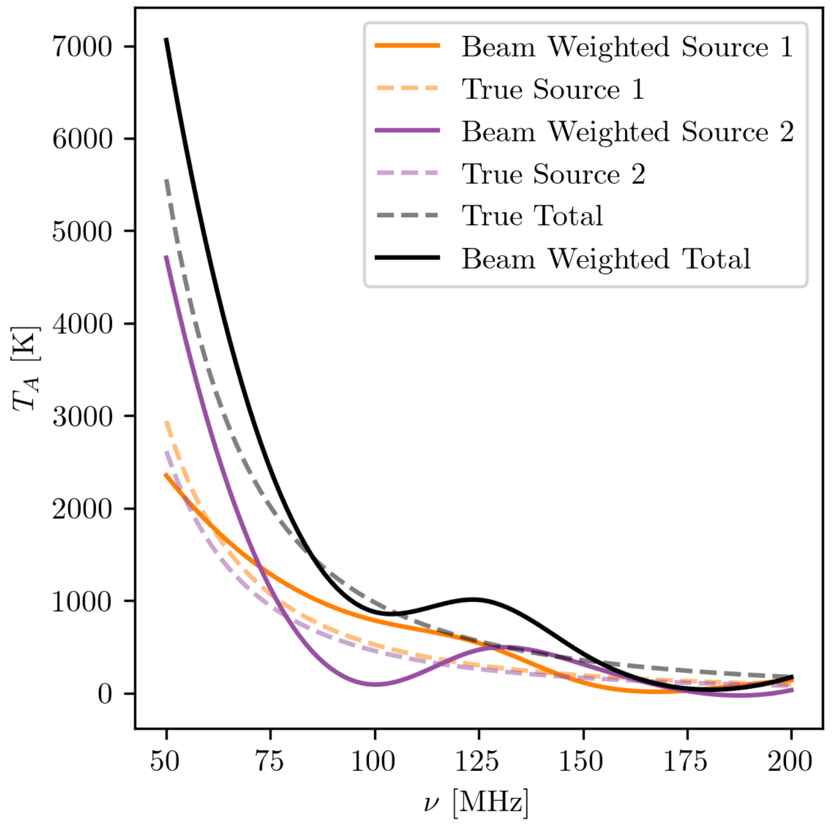
- Dominated by synchrotron and free-free emission from our Galaxy and others
- Approximate $\nu^{-2.5}$ power law
- Model with low order polynomials (e.g. Bevins et al. 2023, Nature Astronomy, 2212.00464)
- Or maximally smooth functions with constrained derivatives (e.g. maxsmooth, Bevins et al 2021, 2007.14970)
- But these tools have limited applications…

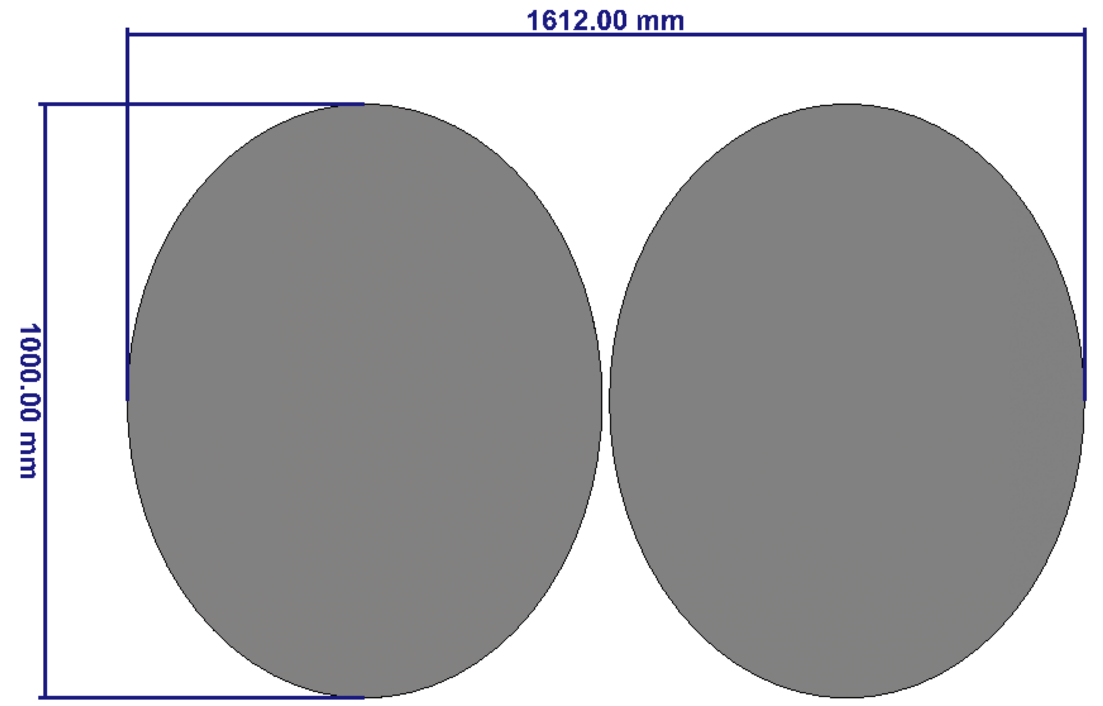


The Instrument

- Instruments generally have a non-smooth response to the sky
- Sky moves through our beam over time
- Distorts that nice smooth power law functions
- Need more complicated models or instruments with smooth responses to sky
- Hard to build an instrument with a very smooth beam (e.g. SARAS2, Bevins et al. 2022, 2201.11531)







Modelling the Foreground and Instrument together

- Approach developed by Dominic Anstey in Cambridge for REACH (Anstey et al. 2020, 2010.09644)
- Sub divide the sky in to regions with similar spectral indices
- Assume a common spectra indices in each region and fit for these as free parameters
- Convolve with the beam of the antenna

$$T_{\text{model}}(\nu) = \left[\sum_{i=1}^{N} K_i(\nu)\left(\frac{\nu}{230}\right)^{-\beta_i}\right] + T_{\text{CMB}},$$

where

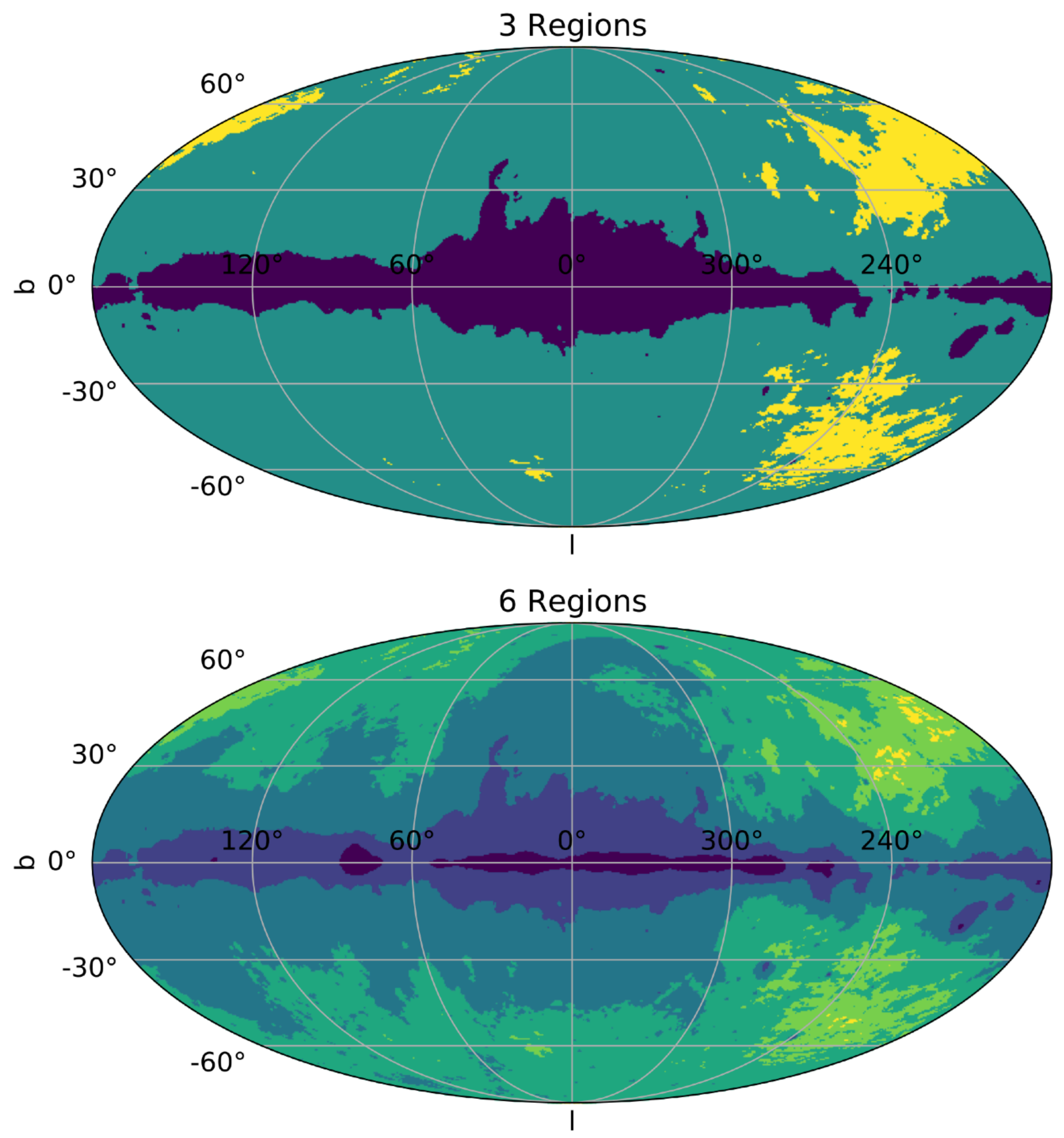
Region Mask

$$K_i(\nu) = \frac{1}{4\pi}\int_0^{4\pi} D(\theta,\phi,\nu)\, M_i(\theta,\phi)$$

$$\times \int_{t_{\text{start}}}^{t_{\text{end}}} \left(T_{230}(\theta,\phi) - T_{\text{CMB}}\right) dt\, d\Omega.$$

Beam Model

Measured Map



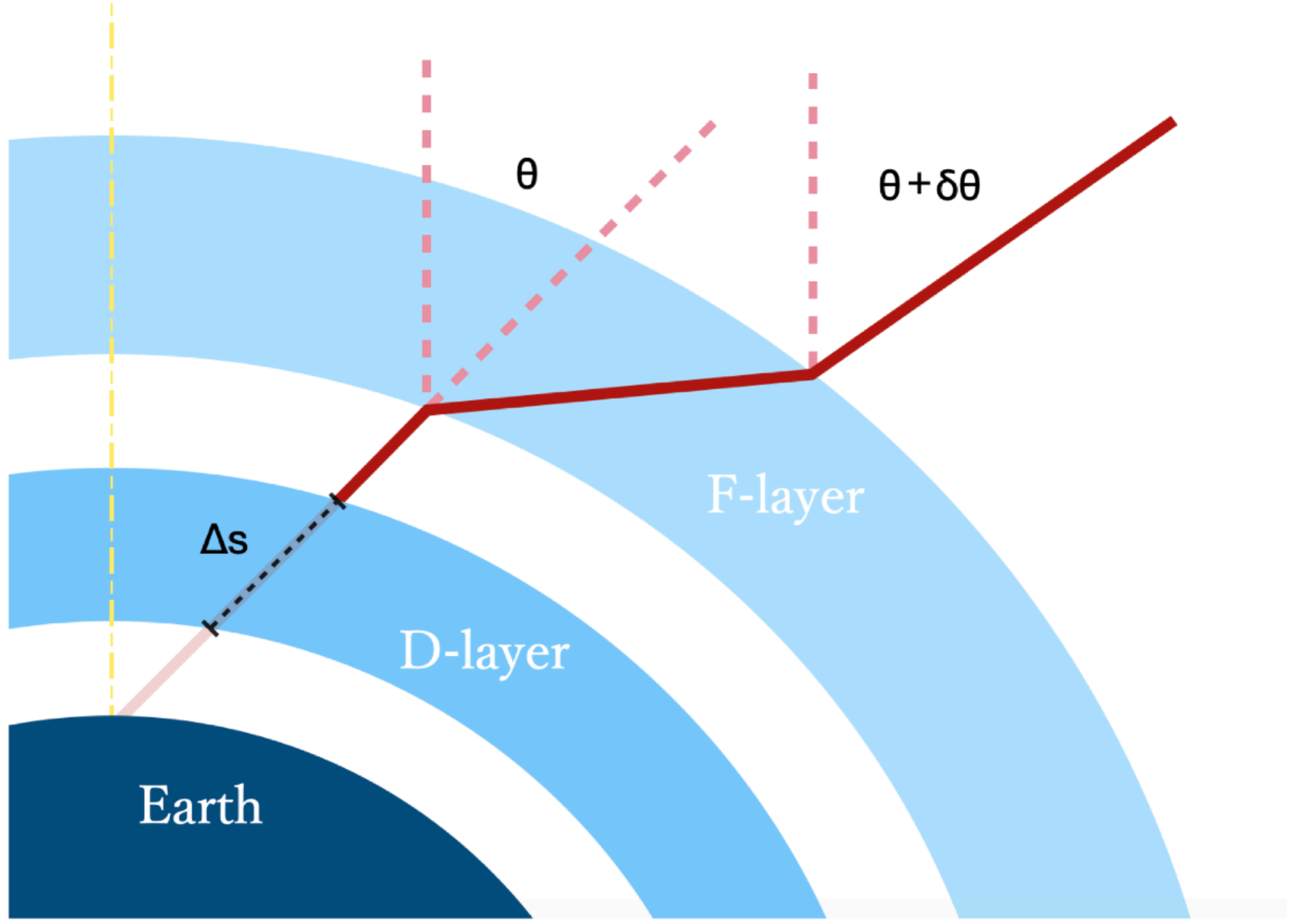
The Ionosphere

- Distorts the apparent position of sources on the sky
- Makes sources appear dimmer
- Modify the beam as was done in Shen et al. 2020 (2011.10517) and Shen et al. 2022 (arXiv:2204.10859)

$$\hat{D}(\nu, \phi, \theta) = D(\nu, \theta + \delta\theta, \phi)\mathscr{L}(\nu, \theta + \delta\theta)$$

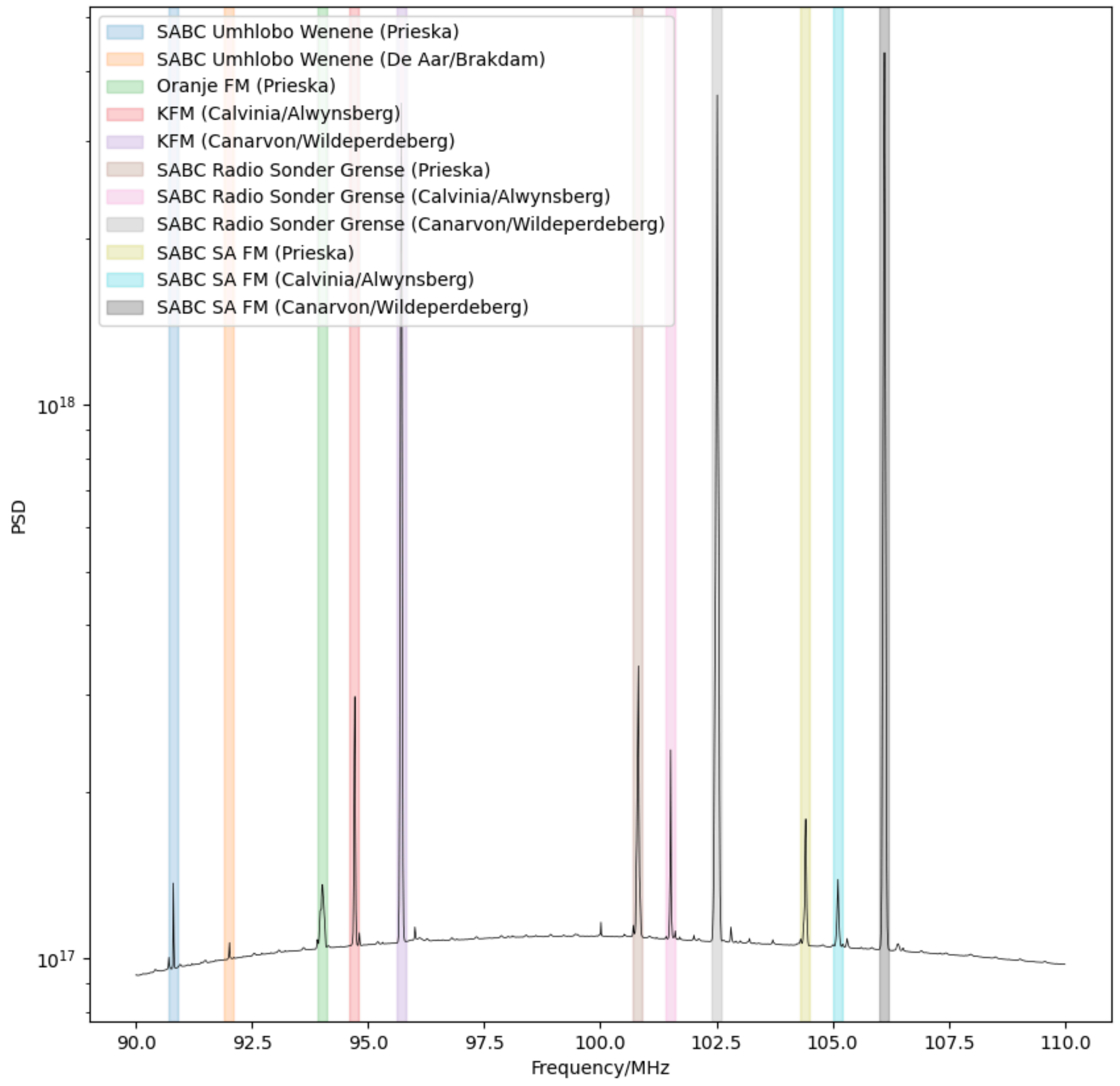
Diffraction

Loss due to absorption



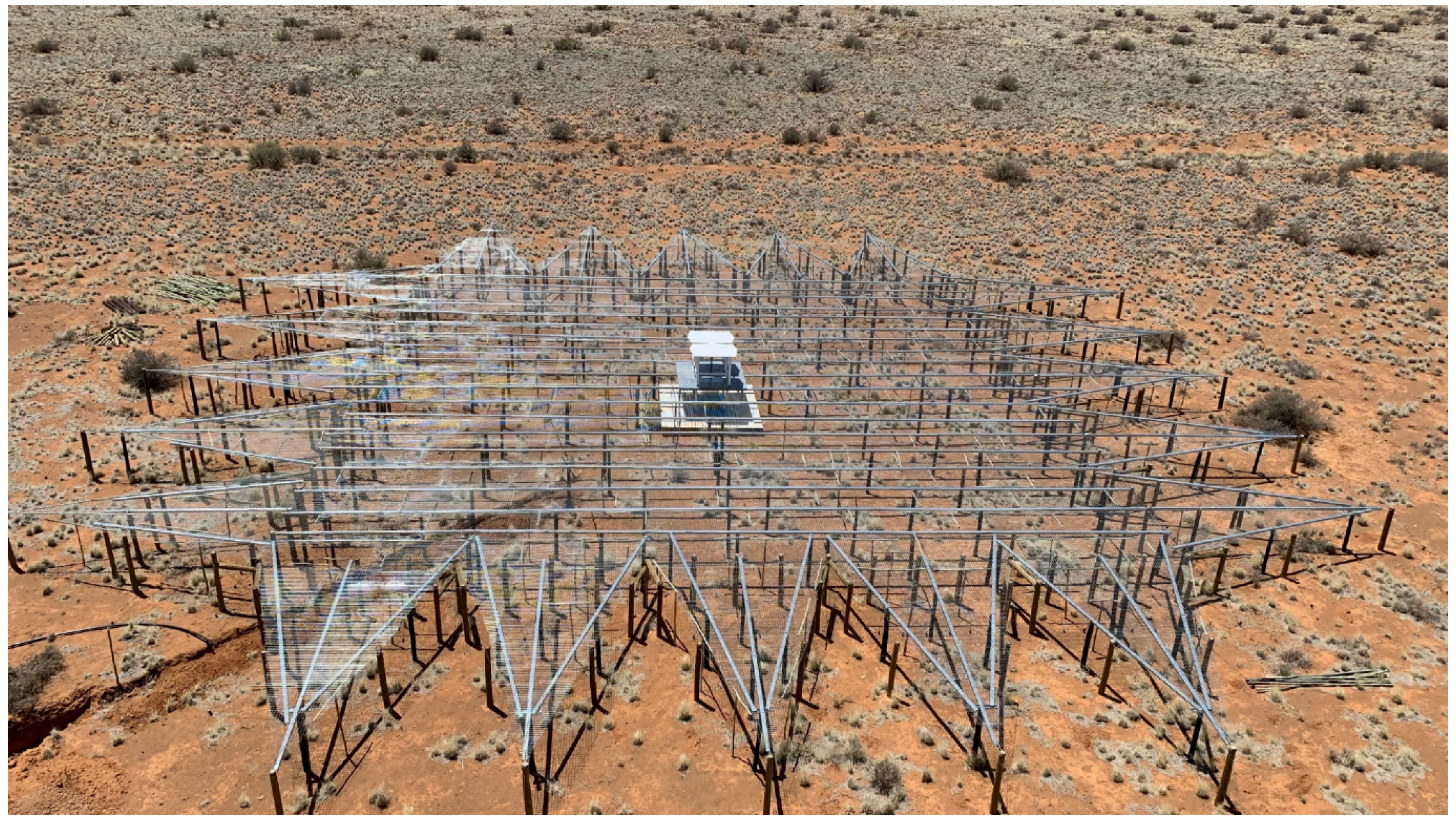
Radio Frequency Interference

- Walkie-talkies, satellites, radio stations, TV, reflections of micro meteorites and planes etc…
- FM radio covers the band 88 - 108 MHz
- Bayesian flagging method (Leeney et al. 2022, 2211.15448)
- Down weight spurious data points in the data when fitting a model

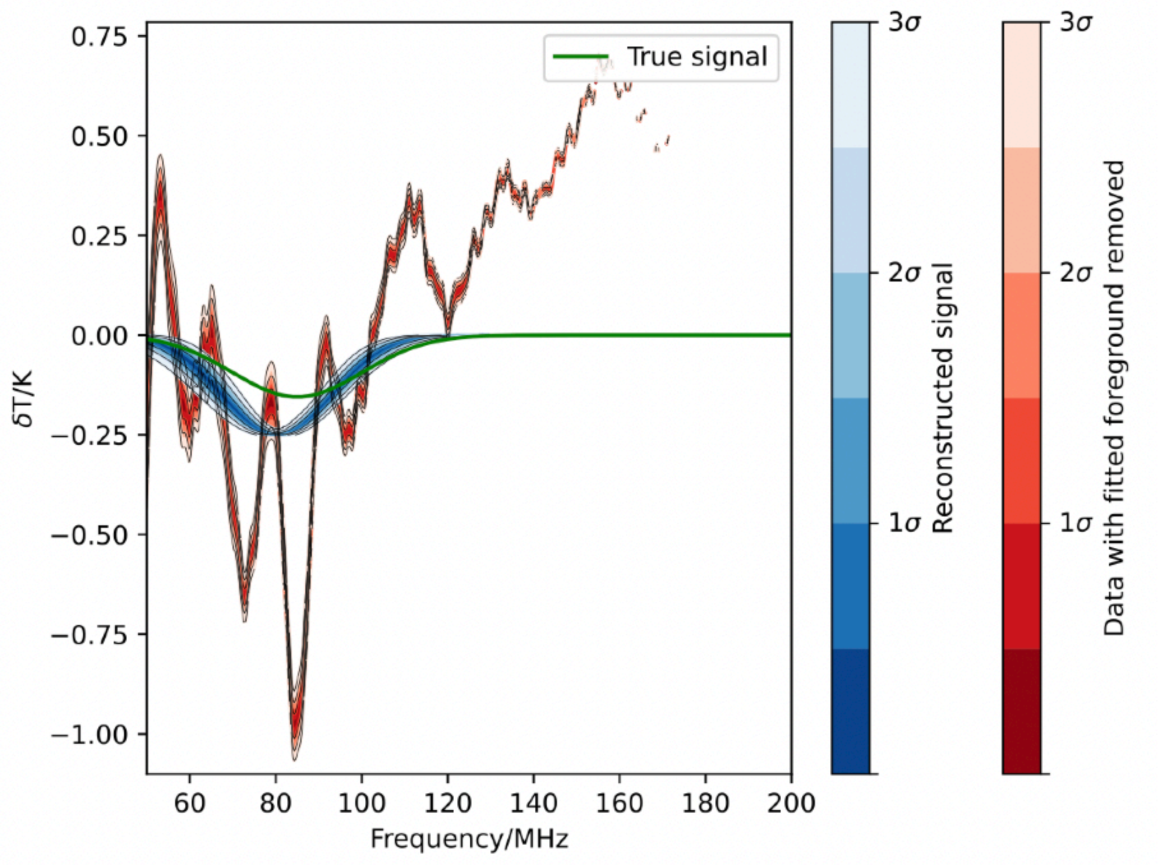


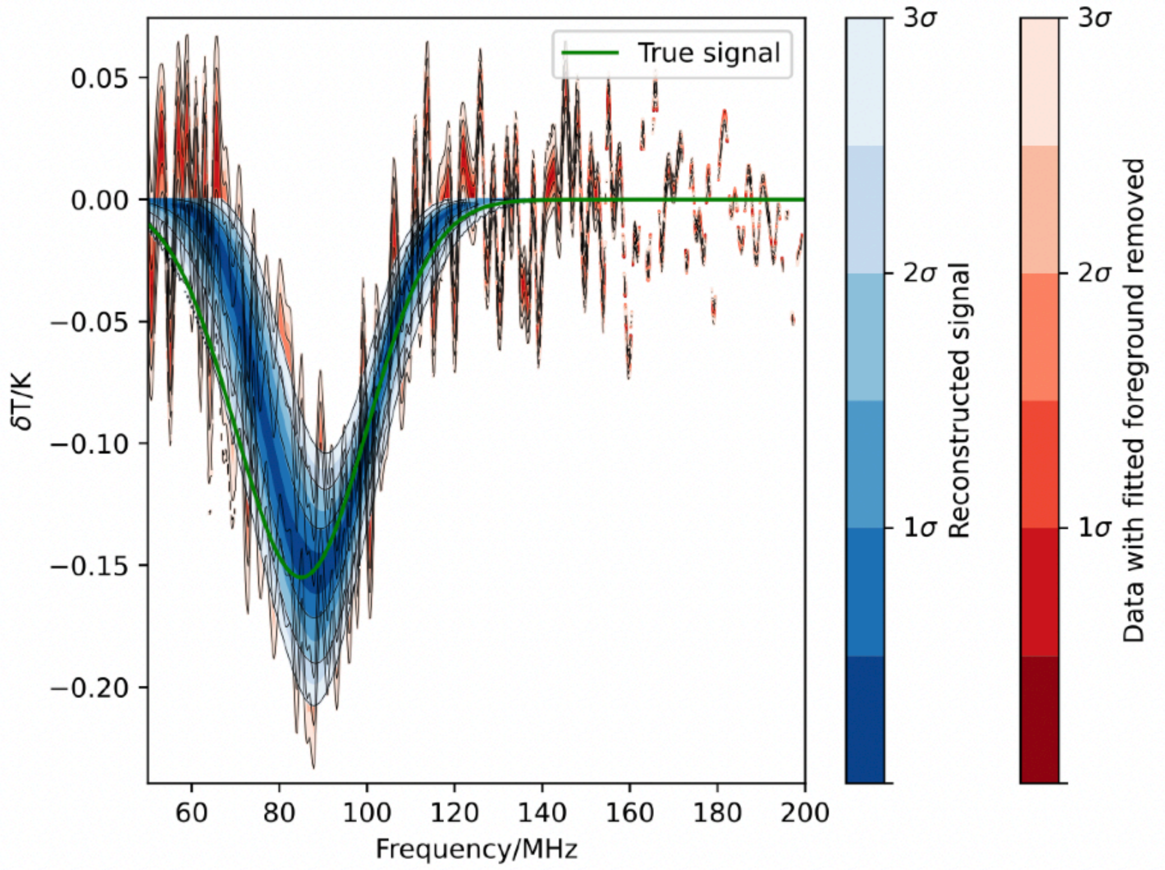
Credit: Dominic Anstey

The Environment



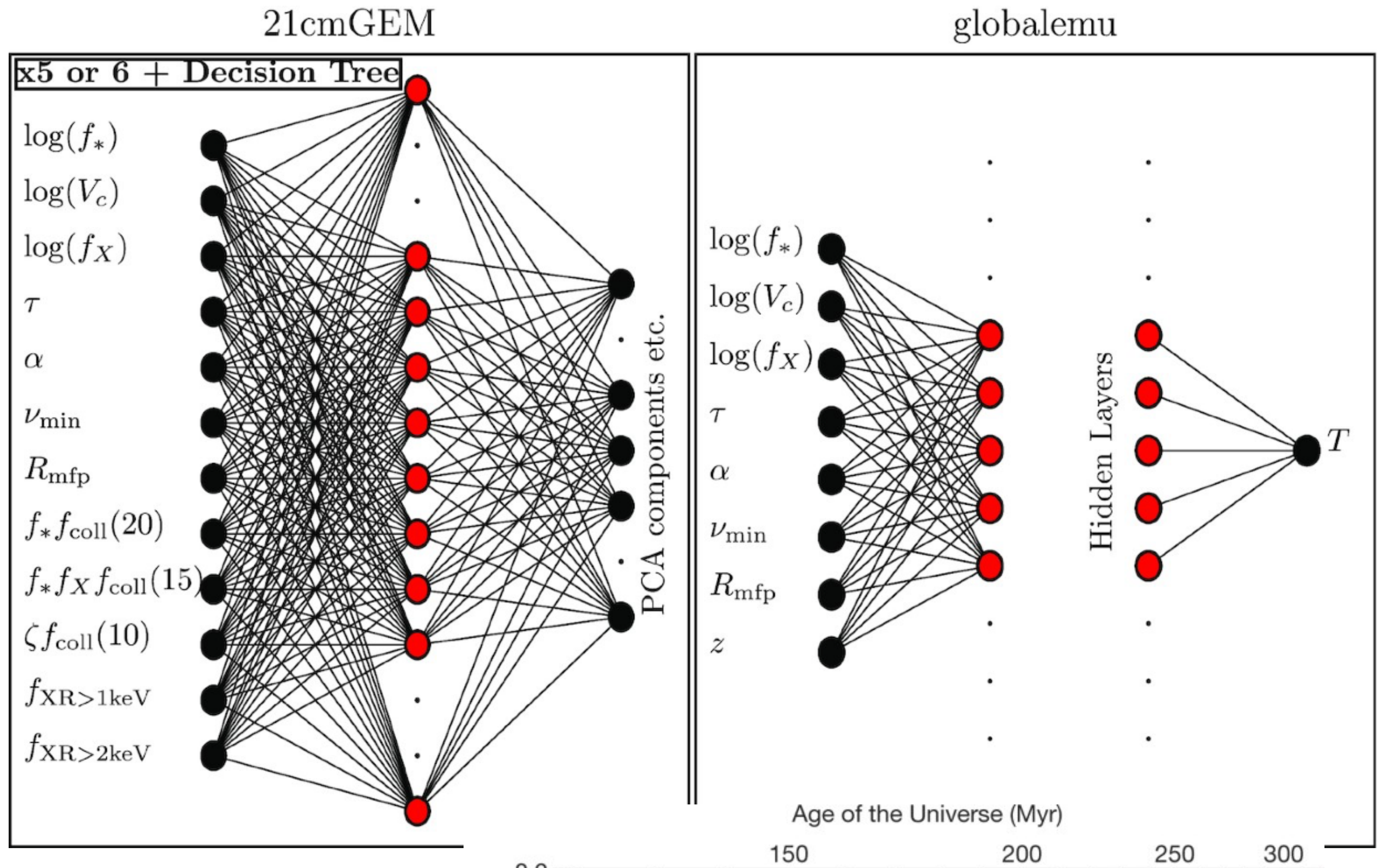
- The REACH instrument sits in a valley and is surrounded by mountains
- Time variable thermal emission from the mountains enters the side lobes of the beam
- And mountains reflect radio waves!
- Pattison et al. 2023 (2307.02908) showed that this can be accounted for in our model

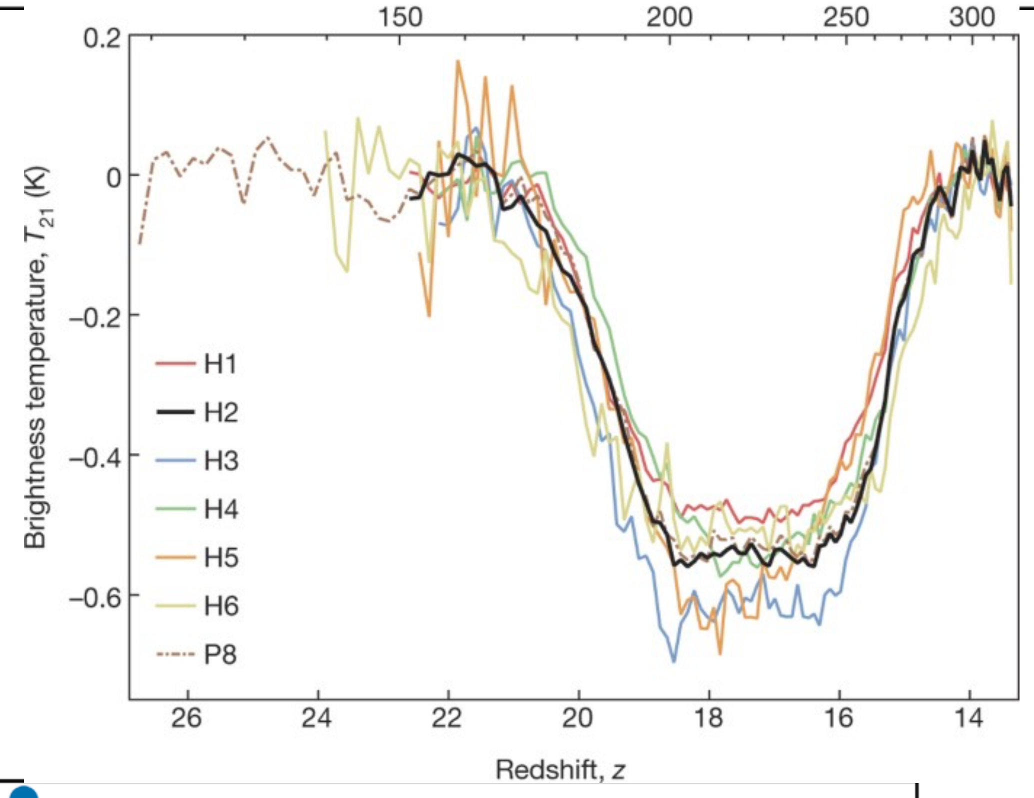


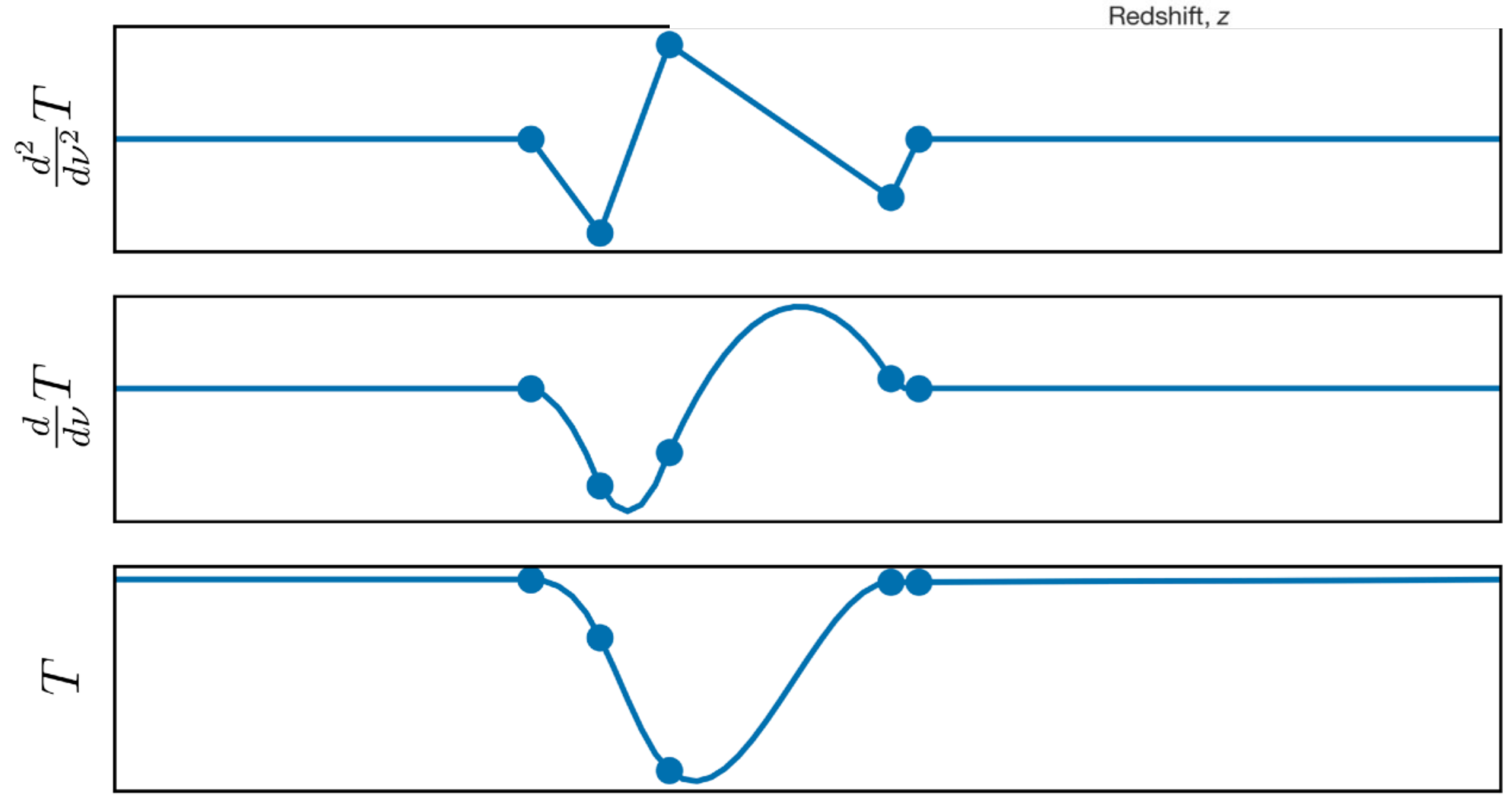


The Signal

- Anastasia introduced the theory this morning
- In practice we have to approximate the signal in our likelihood with;
    - Neural network emulators like globalemu (Bevins et al. 2021, 2104.04336), 21cmVAE (Bye et al. 2021, 2107.05581) and 21cmGEM (Cohen et al. 2020, 1910.06274)
    - Gaussian profiles or flattened Gaussians (e.g. Bowman et al. 2018, 1810.05912 and Bevins et al 2021, 2007.14970)
    - Non-parameteric models like Flexknot and Gaussian Processes (Shen et al. 2023, 2311.14537 and Heimersheim wt al. 2023, 2310.05608)







The future…

Towards ~~Simulation Based~~ Inference
Implicit Likelihood

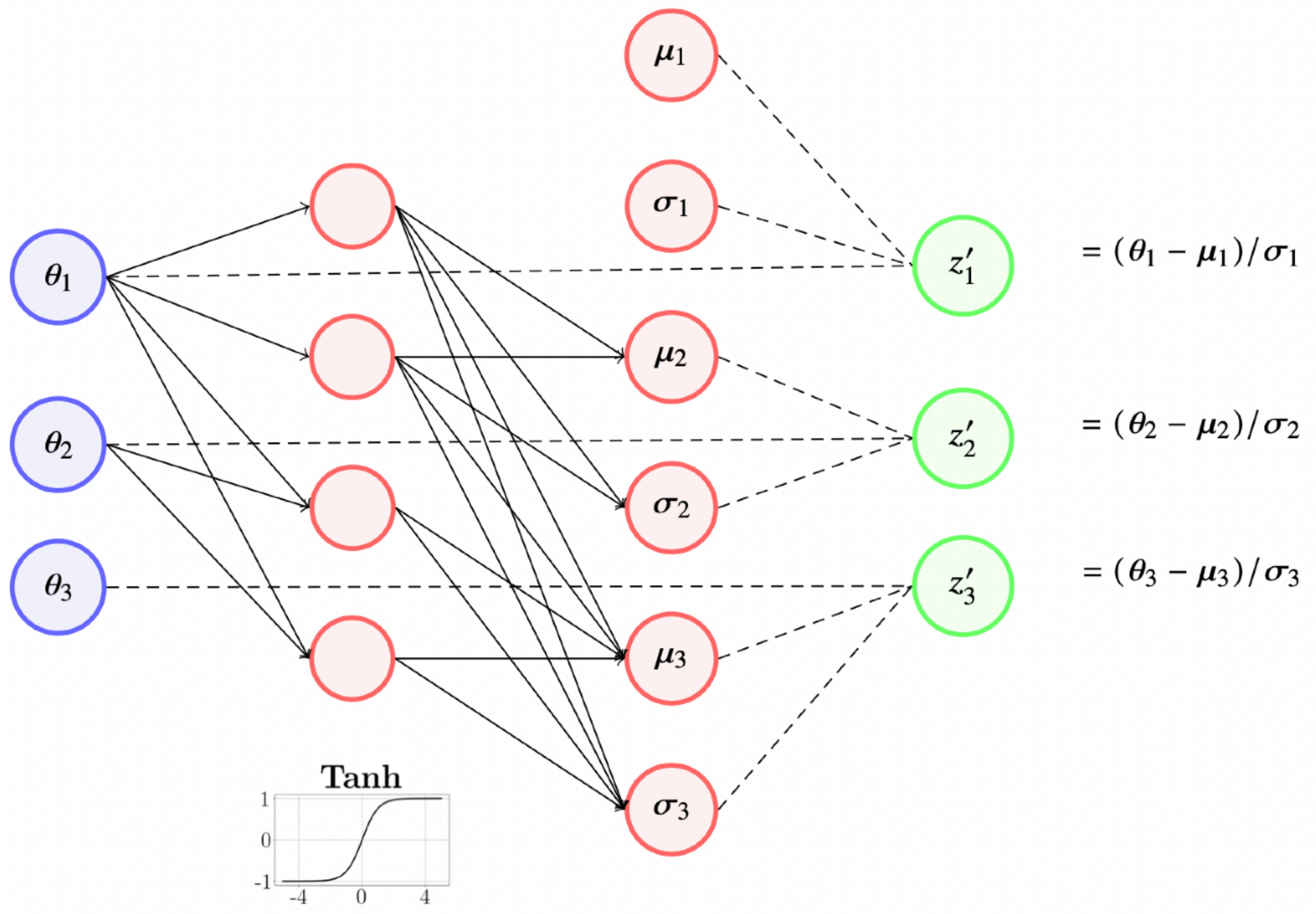
- We've spent a lot of time developing a model for our data

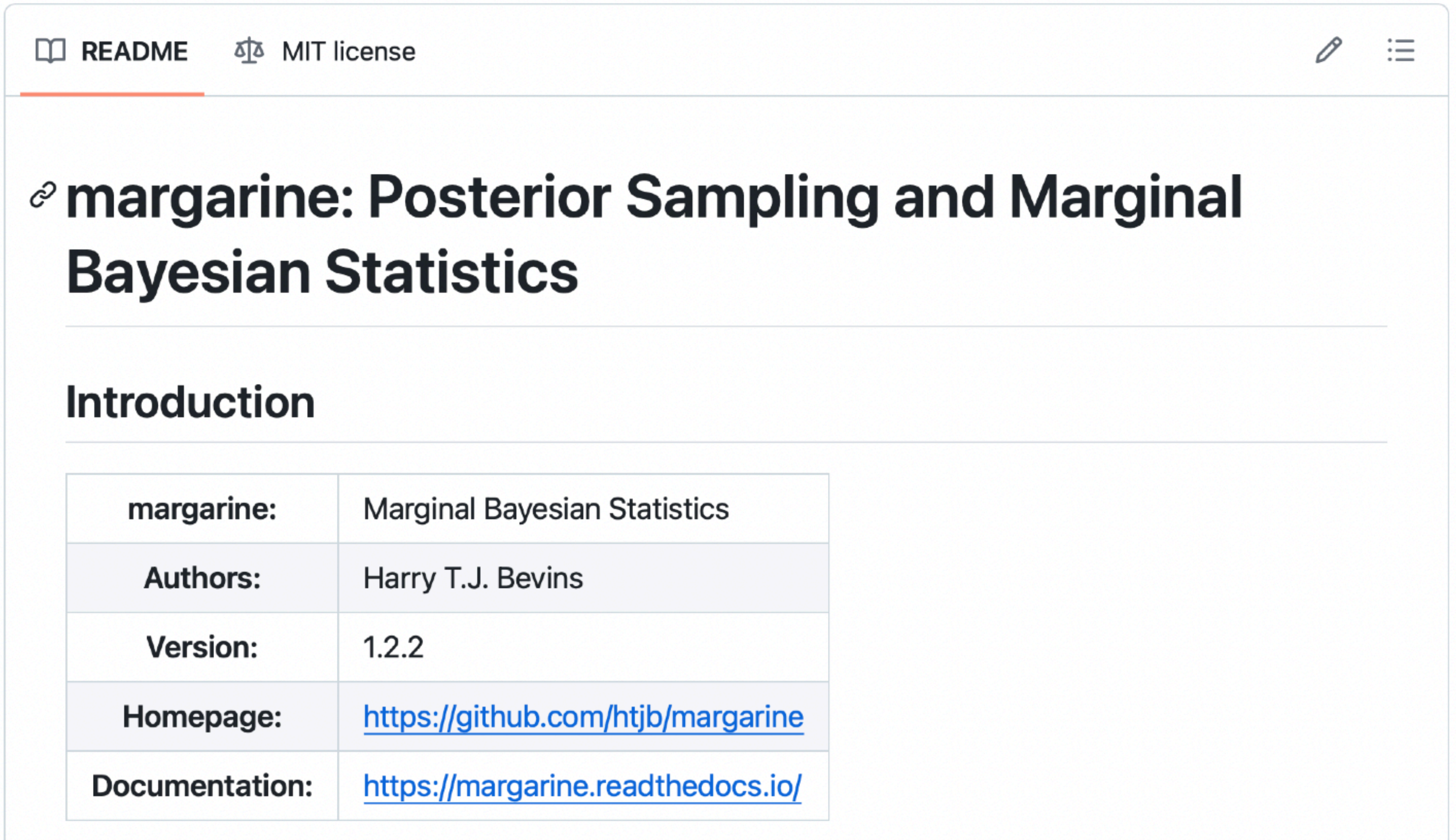
$$\log L(\theta) = -\frac{1}{2}\log 2\pi|\Sigma| - \frac{1}{2}(D - M(\theta))^T\Sigma^{-1}(D - M(\theta))$$

- Draw samples from the likelihood → gives us an observation D for parameters θ given a realisation of radiometric noise Σ

- See also Saxena et al. 2022 (2303.07339) for applications of Neural Ratio Estimation to 21-cm

- Normalising Flows as likelihood and prior emulators (e.g. Bevins et al 2023a, 2205.12841 and Bevins et al. 2023b, 2301.03298)





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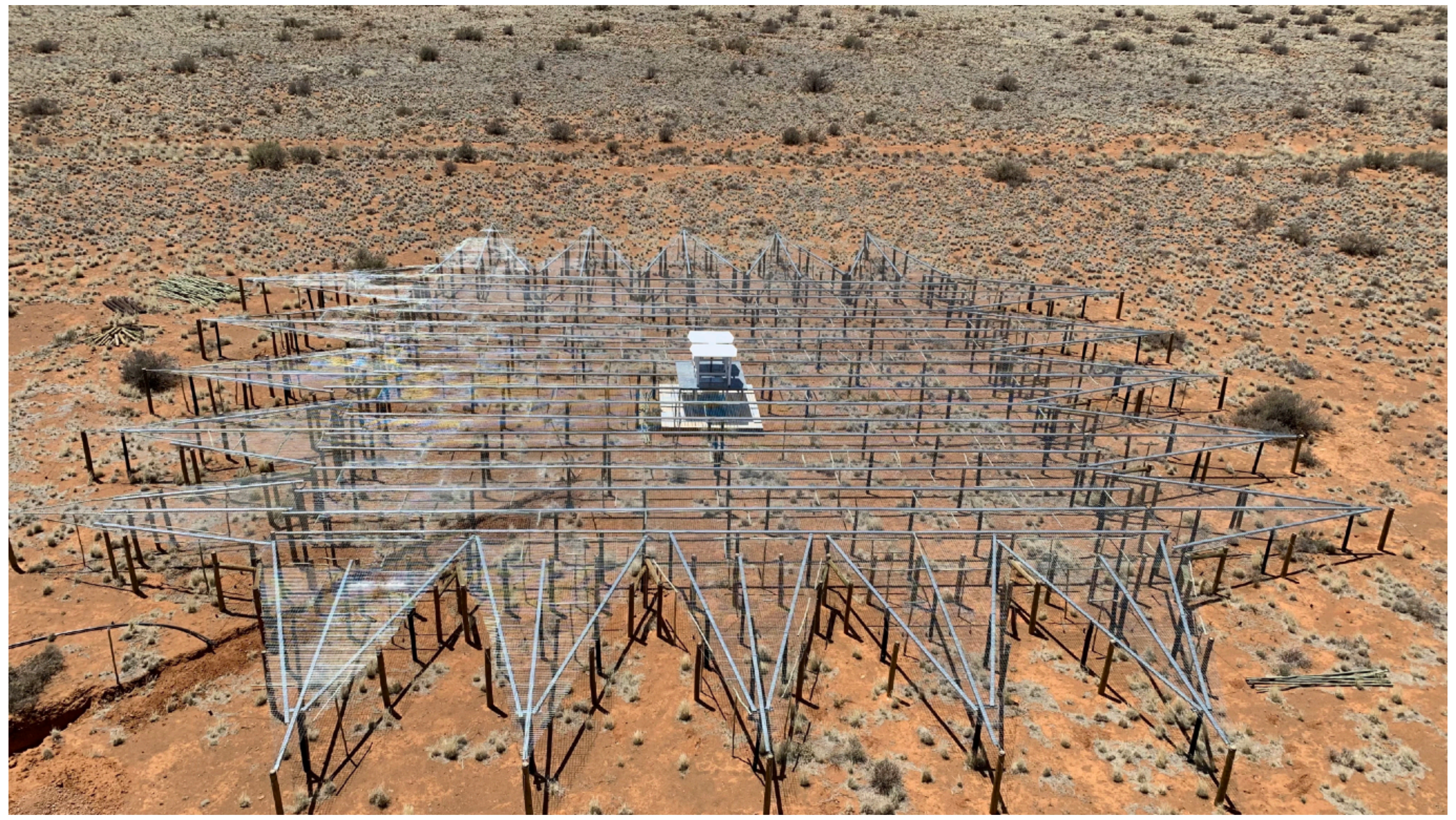
margarine: Posterior Sampling and Marginal Bayesian Statistics

Introduction

margarine:	Marginal Bayesian Statistics
Authors:	Harry T.J. Bevins
Version:	1.2.2
Homepage:	https://github.com/htjb/margarine
Documentation:	https://margarine.readthedocs.io/

Conclusions



- Trying to detect a signal of order 0.1K in foregrounds of order $10^3 - 10^4$ K
- Need physically motivated models for foreground, ionosphere, instrument, RFI, and the 21-cm signal
- If we can get it right for REACH we can apply the same knowledge to instruments looking for the dark ages
- Taking advantage of machine learning tools like emulators and Normalising Flows
- Looking towards a future with implicit likelihood based inference

https://github.com/htjb/Talks