Marginal Bayesian Statistics with Masked Autoregressive Flows and Kernel Density Estimators

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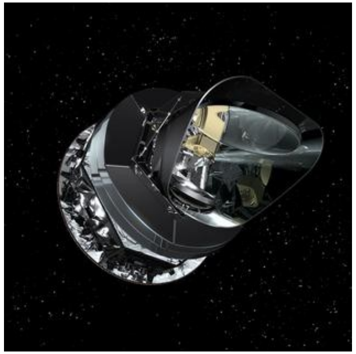
The Problem...

- Combining constraints from experiments probing different aspects of the same physics is computationally expensive.

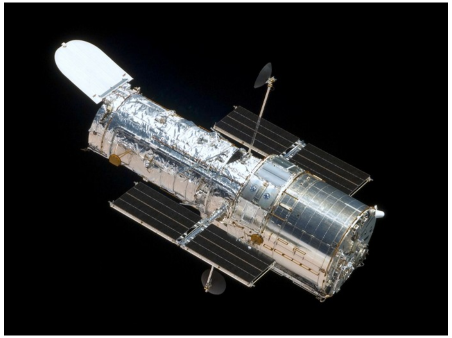
- Often only interested in ~6 cosmological parameters, θ, where each experiment can have an additional ~20 'nuisance' parameters, α, describing systematics and foregrounds.

Planck: $\theta=6$, $\alpha=15$

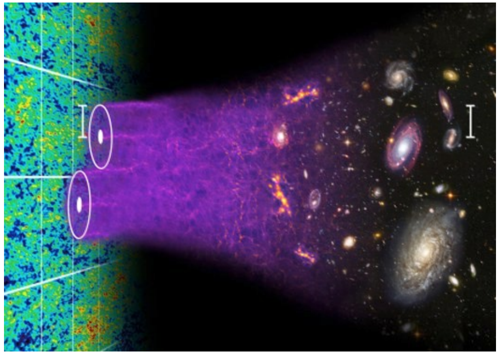


DES: $\theta=6$, $\alpha=20$

SH_0ES: $\theta=6$, $\alpha=0$

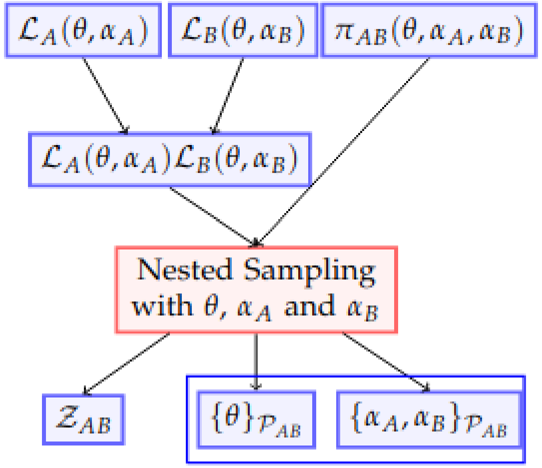


BOSS: $\theta=6$, $\alpha=0$



The Full Nested Sampling Approach

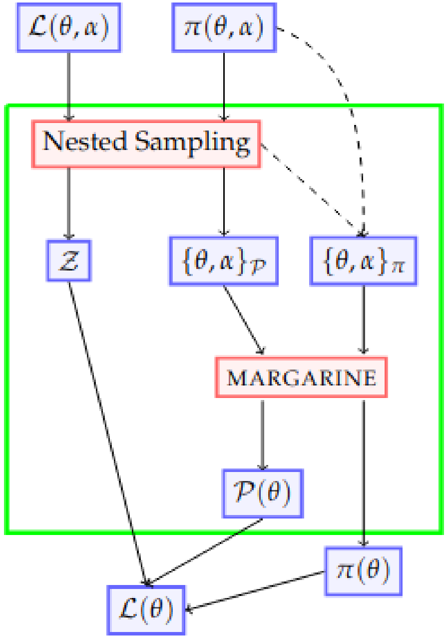
- Typically combination of constraints is performed by sampling over a full joint likelihood.
- Returns the Bayesian Evidence and posterior samples for both nuisance parameters and cosmological parameters.



$$\mathcal{L}_A(\theta, \alpha_A)\mathcal{L}_B(\theta, \alpha_B)\pi_{AB}(\theta, \alpha_A, \alpha_B) = \mathcal{P}_{AB}(\theta, \alpha_A, \alpha_B)\mathcal{Z}_{AB}$$

The *margarine* Approach

- *margarine* uses density estimators to replicate samples output from nested sampling.
- These density estimators are built from known base distributions meaning that we can use them to estimate *P(θ)* and *π(θ)*.
- This can then give us the nuisance-free likelihood along with the KL divergence.



$$\mathcal{L}(\theta) \equiv \frac{\int \mathcal{L}(\theta,\alpha)\pi(\theta,\alpha)d\alpha}{\int \pi(\theta,\alpha)d\alpha} = \frac{\mathcal{P}(\theta)\mathcal{Z}}{\pi(\theta)}$$

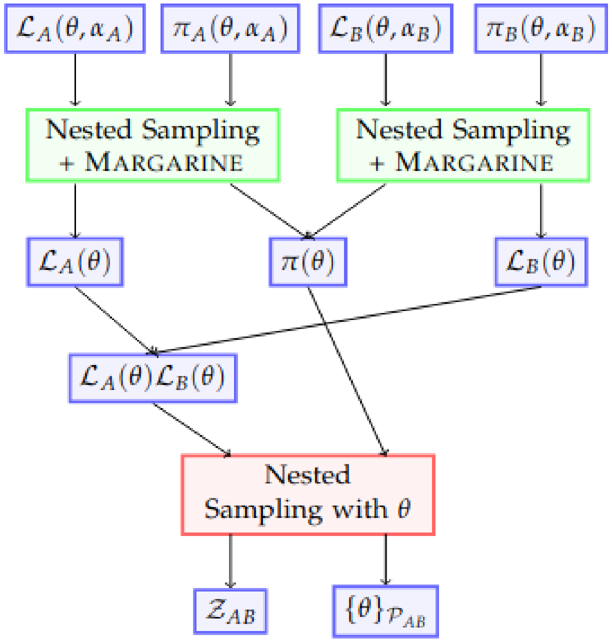
$$\mathcal{D}(\mathcal{P}||\pi) = \int \mathcal{P}(\theta)\log\frac{\mathcal{P}(\theta)}{\pi(\theta)}d\theta$$

The *margarine* Approach

- We can therefore combine nuisance-free likelihoods calculated from samples for experiment A and B.

- We show mathematically that the method is consistent with a full nested sampling run.

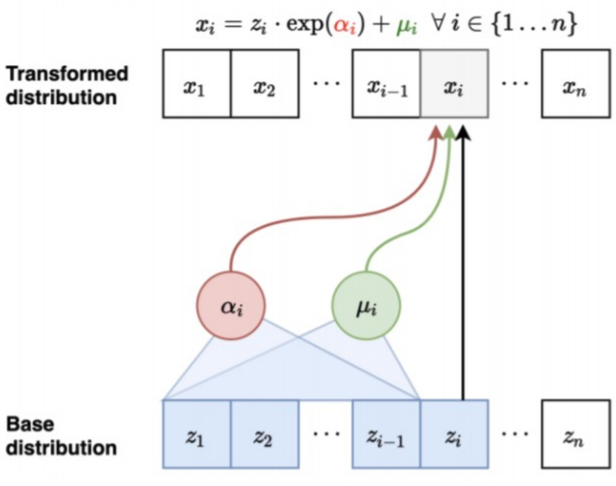
$$\mathcal{L}_A(\theta)\mathcal{L}_B(\theta)\pi(\theta) = \mathcal{P}_{AB}(\theta)\mathcal{Z}_{AB}$$

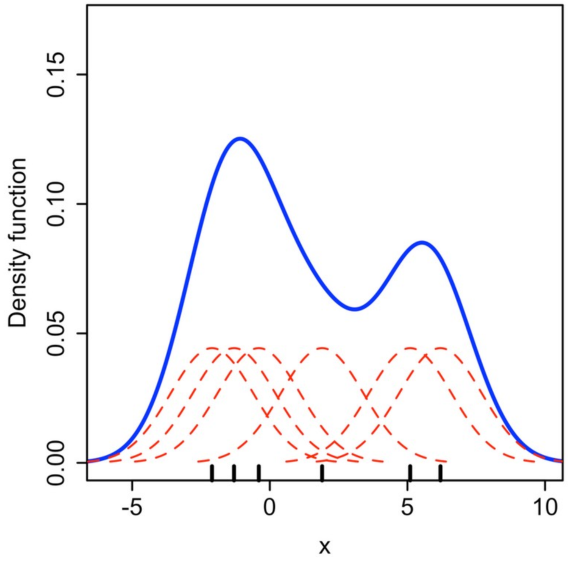


Types of Density Estimators

- **Masked Autoregressive Flows** which shift and scale a base distribution to look like the target posterior or prior.

- **Kernel Density Estimator** which models the posterior or prior distribution using Gaussian kernels.





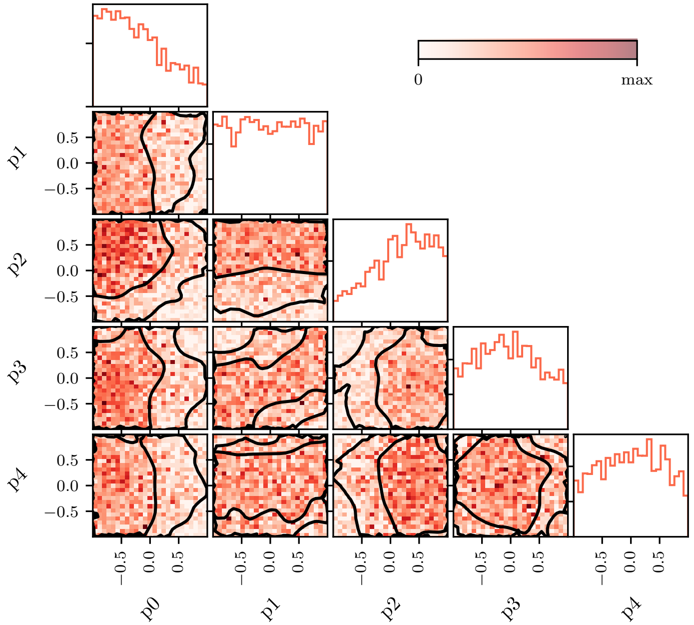
Toy Example

Experiment A:

$$\log \mathcal{L}_A = \sum_i \frac{1}{2}(x_i - \theta_i)^2,$$

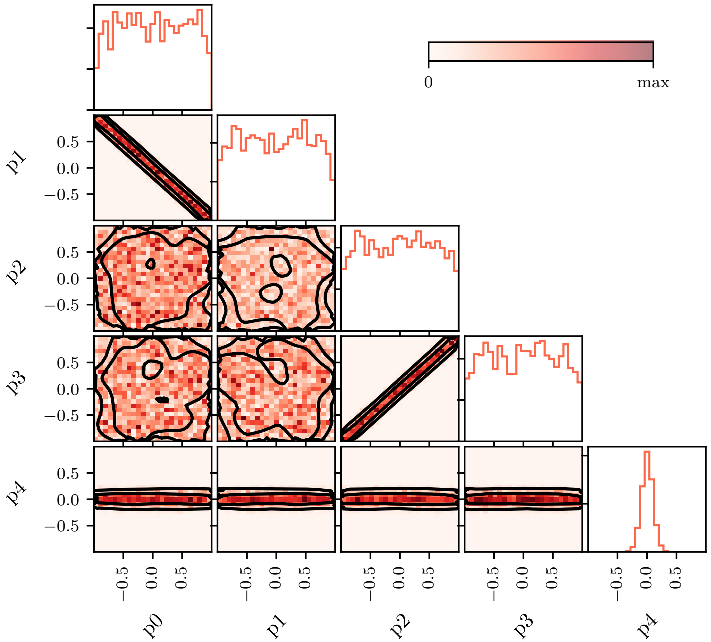
$$x = [-0.8, 0.5, 0.6, -0.2, 0]$$

$$\theta = [p_0,\ p_1 \times p_3,\ p_2,\ p_3,\ p_4]$$



Experiment B:

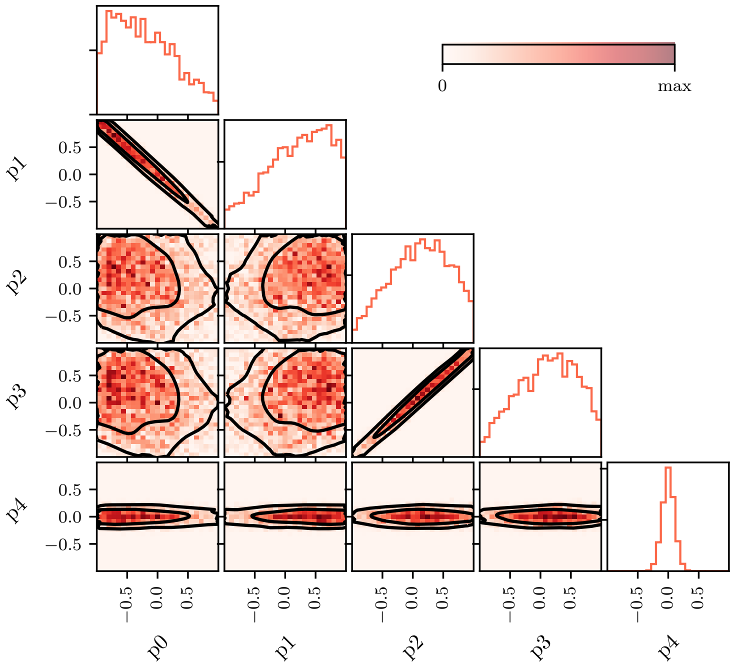
$$\log \mathcal{L}_B = \frac{1}{2}((p_0 + p_1)^2 + (p_2 - p_3)^2 + p_4^2)$$



Toy Example

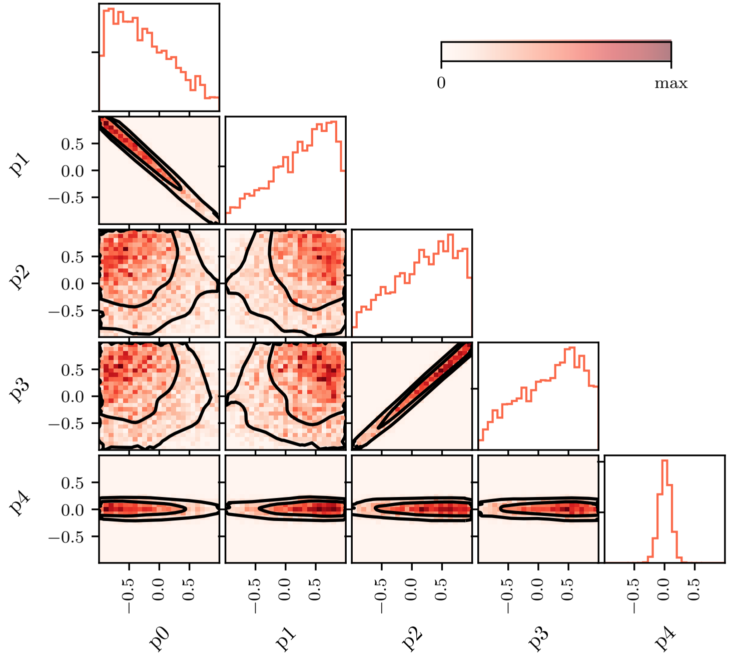
Combined w/ Nested Sampling
log(Z) = -11.7 ± 0.1

$$\log \mathcal{L}_{AB} = \sum \frac{1}{2}(x_i - \theta_i)^2 + \frac{1}{2}((p_0 + p_1)^2 + (p_2 - p_3)^2 + p_4^2)$$



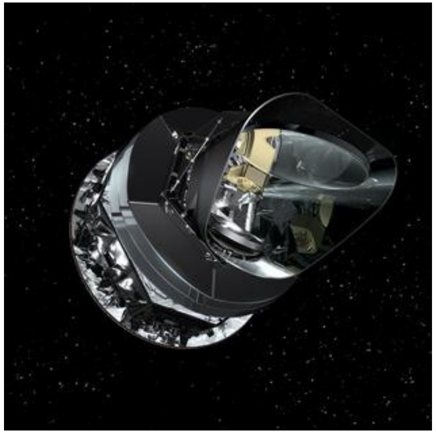
Combined w/ Nested Sampling + *margarine*
log(Z) = -11.8 ± 0.1

$$\log \mathcal{L}_{AB} = \log \mathcal{L}_A^{margarine} + \log \mathcal{L}_B^{margarine}$$



Planck + DES

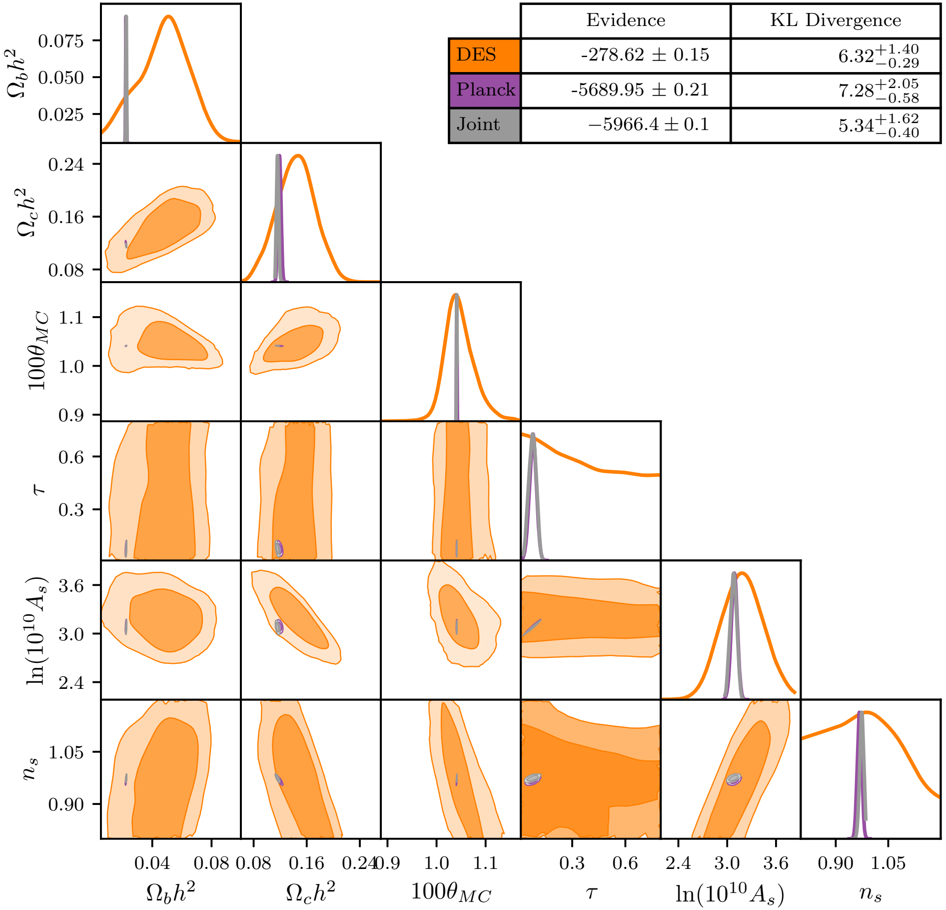
- To model both DES and Planck we have to fit 41 parameters of which 35 are nuisance parameters…
- Previously been done in Handley and Lemos 2019a and 2019b
- $log(Z) = -5965.7 \pm 0.3$
- $D = 6.17 \pm 0.36$



Planck + DES

- *margarine* is much more computationally efficient than fitting for 41 parameters.
- Using nuisance-free likelihoods we can sample over just the six cosmological parameters.
- *log(Z) = -5966.4 ± 0.1*
- *D = 5.34 +(-) 1.62(0.40)*
- We use *margarine* to perform importance sampling.

$$\mathcal{Z}_B = \mathcal{Z}_A \left\langle \frac{\pi_B(\theta)}{\pi_A(\theta)} \right\rangle_{\mathcal{P}_A} \qquad w_B^{(i)} = w_A^{(i)} \frac{\pi_B(\theta^{(i)})}{\pi_A(\theta^{(i)})}$$



	Evidence	KL Divergence
DES	-278.62 ± 0.15	$6.32^{+1.40}_{-0.29}$
Planck	-5689.95 ± 0.21	$7.28^{+2.05}_{-0.58}$
Joint	-5966.4 ± 0.1	$5.34^{+1.62}_{-0.40}$

margarine

https://github.com/htjb/margarine

https://arxiv.org/abs/2205.12841

setup.py — Log probabilities and likelihoods (#5) — 26 days ago

README.rst

margarine: Posterior Sampling and Marginal Bayesian Statistics

Introduction

margarine:	Marginal Bayesian Statistics
Authors:	Harry T.J. Bevins
Version:	0.2.0
Homepage:	https://github.com/htjb/margarine
Documentation:	https://margarine.readthedocs.io/

docs passing | launch binder | astro.IM arXiv:2205.12841

Removing the fat from your posterior samples with MARGARINE

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Conclusions

- *margarine* offers a computationally more efficient path to combined Bayesian analysis that is consistent with a full Nested Sampling run.
- Our approach is lossless in θ as it recovers the same marginal posterior and total evidence.
- The work paves the way for the development of a publicly available library of cosmological likelihood emulators.
- There are applications beyond cosmology.