

Lecture 16

Simulation Based Inference for

Astrophysics and Cosmology

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Overview

- What is Simulation Based Inference?
- Approximate Bayesian Computation
- Neural Posterior Estimation
- Neural Ratio Estimation



What is Simulation Based Inference?



Why not emulate the likelihood?

$$\log_e L(\theta) = \sum_i -\frac{1}{2} \log_e 2\pi |\Sigma| - \frac{1}{2} (D_i - M_i(\theta))^T \Sigma^{-1} (D_i - M_i(\theta))$$

- In the last lecture I discussed how the likelihood defines the noise distribution we expect in our data
- But often this is hard to know and approximations like the above Gaussian don't really describe our data well
- We looked at emulating the model describing our data but we can also think about emulating our likelihood function



Simulation Based Inference (SBI)

- Sometimes referred to as Likelihood-free Inference (not a good name)
- Or Implicit likelihood inference (a better name)
- The idea is to learn an approximation for the likelihood or indeed the posterior via simulations of the data
- Simulations here includes the physics we are interested in, and contaminating signals, the noise and the impact of the instrument we are observing with



Simulation Based Inference (SBI)

- Beneficial when we can not analytically write the likelihood, we do not know the noise distribution in our data or the analytic likelihood is too computationally expensive
- For example, we might want to do field level inference on images from the SKA (analytically hard to define likelihood)
- Or we might want to do quick follow up observations of Gravitational Wave observations (GW analytic likelihoods are very complete but to slow to trigger follow up observations)



Approximate Bayesian Computation



The idea

- ABC originates from the 80s (Rubin 1984) and was one of the first proposed techniques for SBI
- The idea is to generate a set of simulations covering a broad prior parameter space
- Define a distance metric between the data and simulations $\rho(D, S)$
- And define a non-zero tolerance ϵ such that samples are accepted into the posterior based on $\rho(D, S) < \epsilon$



Approximating the posterior

- Effectively we are approximating $P(\theta|D)$ with $P(\theta|\rho(D,S) < \epsilon)$
- For small ϵ this approximation is usually okay
- However, if ϵ is set to large then the approximation will be poor
- What constitutes small and large will be data specific and so setting ϵ is hard

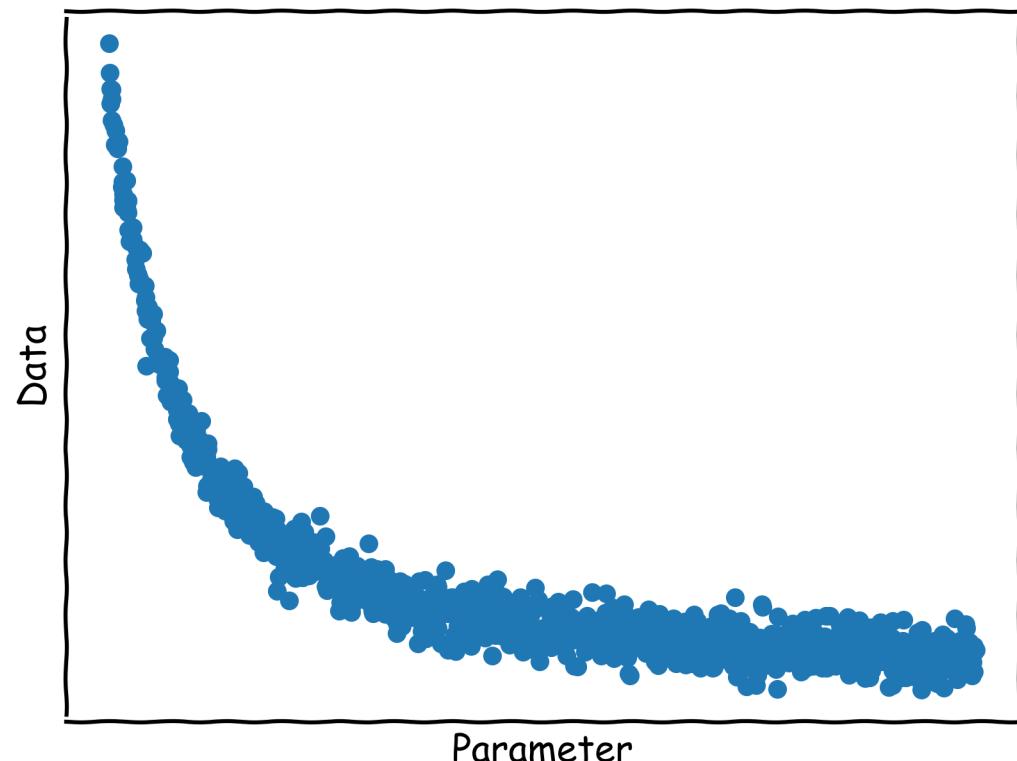


An Example

- Single parameter problem such that our data

$$D = \theta^{1.5} + \sigma$$

- Where σ is Gaussian random noise
- Think about the joint distribution $P(D, \theta)$





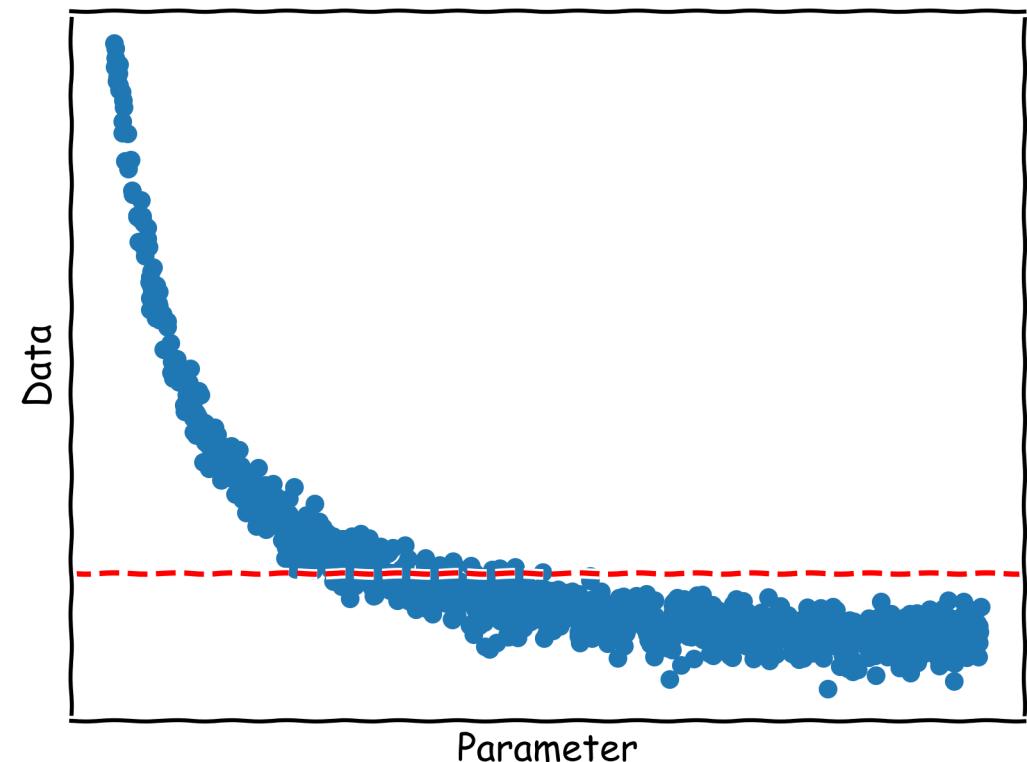
An Example

- Make an observation of our data

- Define a distance metric

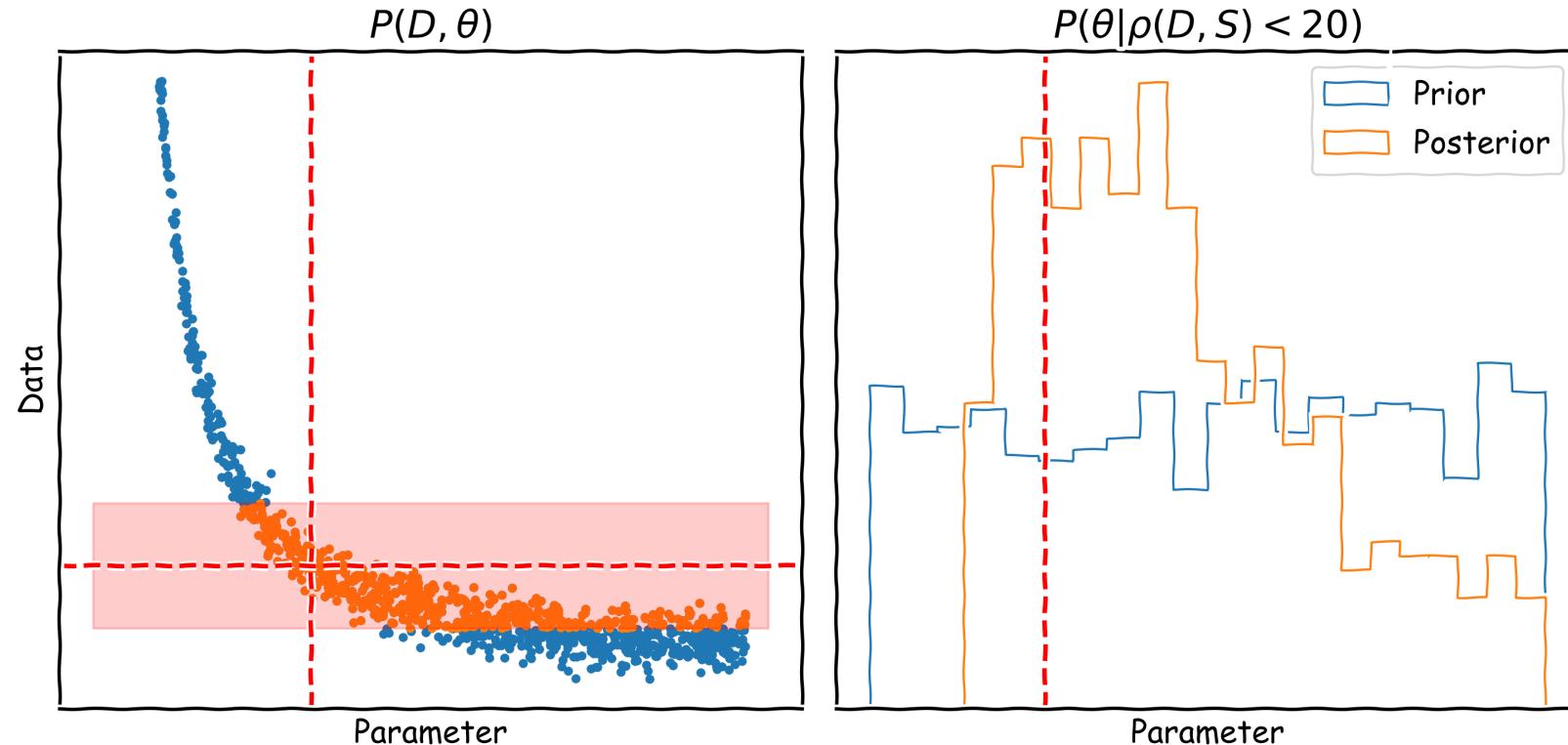
$$\rho(D, S) = |D - S|$$

- Pick a value for epsilon say 20



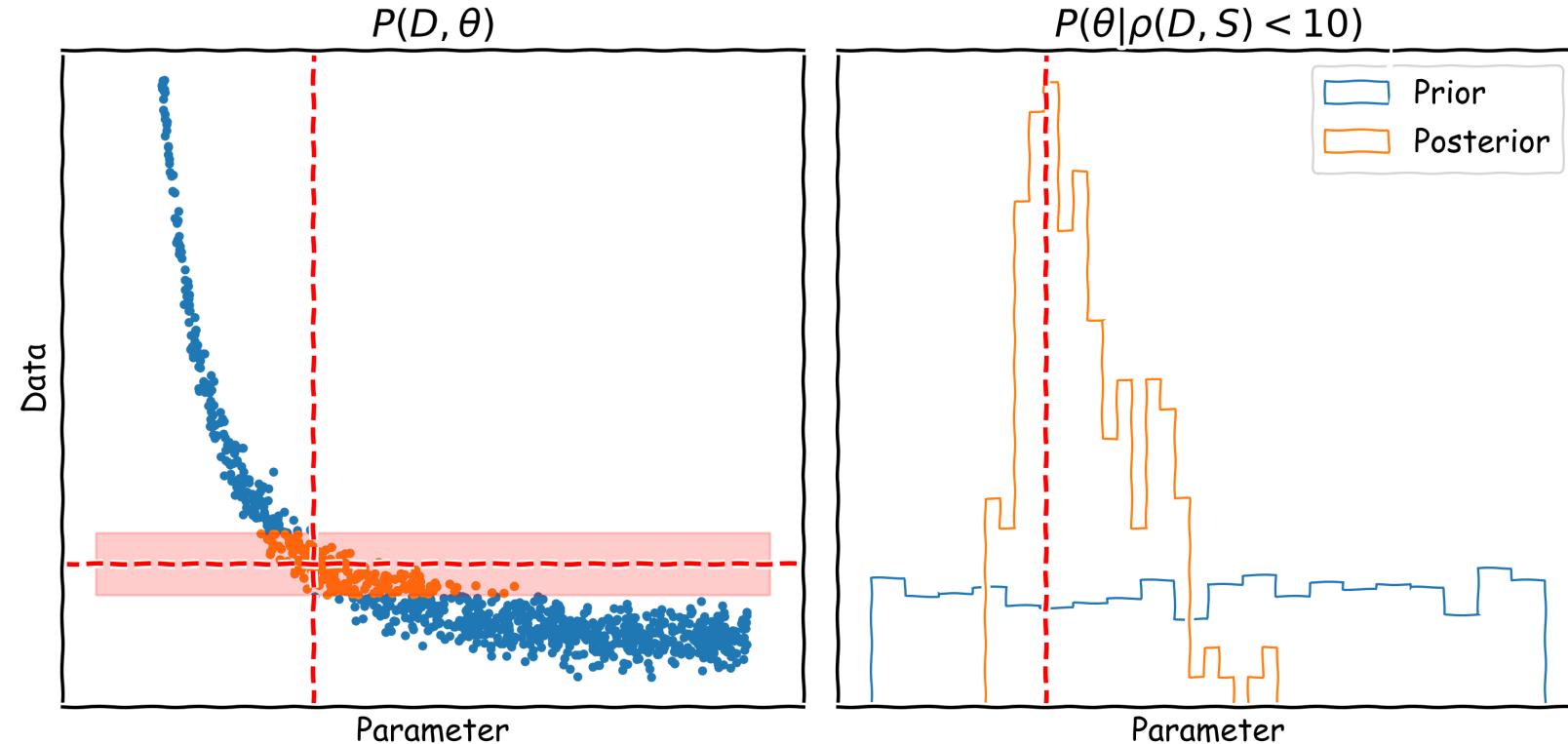


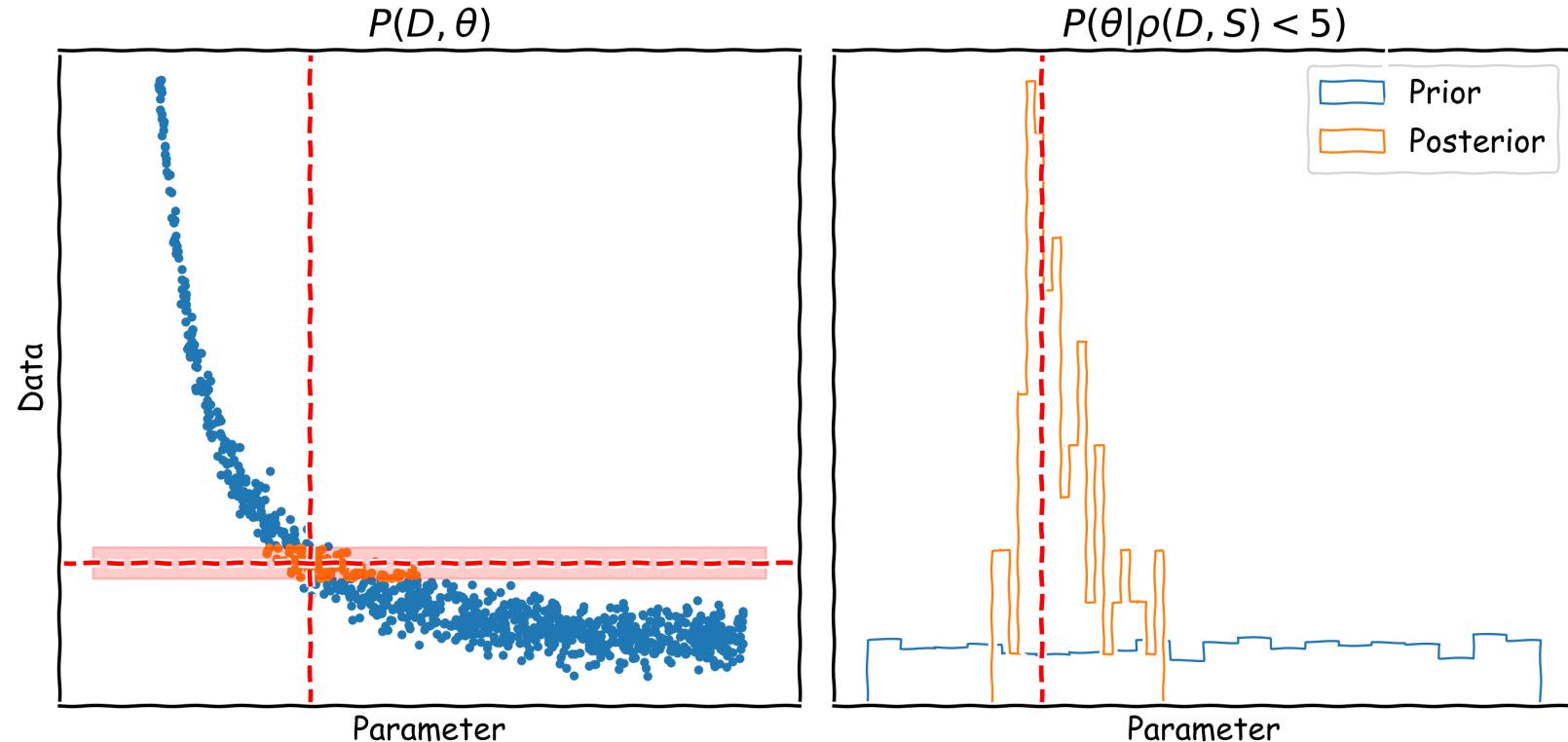
$\epsilon = 20$





$$\epsilon = 10$$



 $\epsilon = 5$ 



What does the algorithm look like?

Distance Metric

Prior samples and simulations

```
def distance(sim_d, d):
    return np.abs(d - sim_d)

def sampling(samples, data, epsilon):

    posterior = []
    accepted_data = []
    for i in range(len(samples)):
        if distance(data[i], true_data) < epsilon:
            posterior.append(samples[i])
            accepted_data.append(data[i])
    posterior = np.array(posterior)
    accepted_data = np.array(accepted_data)
    return posterior, accepted_data
```

$$\rho(D, S) < \epsilon$$

Accepted samples and simulations



No likelihood calls

- We did not make a single call to any likelihood function here
- But there is a cost to generating our simulations (so it might still be slow)
- Typically need fewer simulation calls than NS and MCMC for example
- Likelihood might be intractable anyway so NS and MCMC might not be an option
- We could use emulators to generate the simulations if they are prohibitively costly



Problems with ABC?

- How do we set ϵ ?
- Becomes very expensive when we have many dimensions and big data sets (curse of dimensionality)
- Sensitive to accuracy of simulations (more general for SBI)
- Not amortized? Basically, means that if our data changes we have to repeat most of the process



Neural Posterior Estimation (NPE)



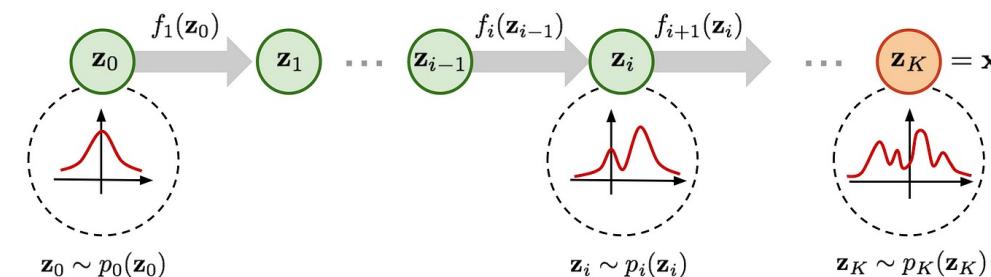
What is NPE?

- Here we use neural networks to approximate the posterior distribution $P(\theta|D)$
- Specifically, we use normalising flows (NFs)
- NPEs are amortized
- After an initial cost to generate simulations and train the NF we can apply the algorithm to many different data sets



Normalizing Flows (NFs)

- Invertible transformation from some known distribution (a multi-variate Gaussian) to a more complex target distribution (a posterior for example)
- Focusing on Masked Autoregressive Flows (MAF) which comprises of a series of Masked Autoencoder for Distribution Estimation (MADE, [Papamakarios et al 2017](#)) neural networks





Normalizing Flows (NFs)

- We choose to represent our data as a series of conditionals where i represents the dimension

$$P(\theta) = \prod_i P(\theta_i | \theta_0, \theta_1, \dots, \theta_{i-1})$$

- And model each conditional as a Gaussian distribution

$$P(\theta_i | \theta_0, \theta_1, \dots, \theta_{i-1}) = N(\mu_i, \sigma_i)$$



Normalizing Flows (NFs)

$$P(\theta_i | \theta_0, \theta_1, \dots \theta_{i-1}) = N(\mu_i, \sigma_i)$$

- Our conditionals are just a shifted and scaled version of the base distribution (standard normal)
- μ_i and σ_i are functions of the neural network hyperparameters and so we have to optimize them with some loss function



Normalizing Flows (NFs)

- Minimize the KL divergence between the probability of some samples on the network $P_\phi(\theta)$ and the true target probability $P(\theta)$

$$D_{KL} = \int P(\theta) \log \frac{P(\theta)}{P_\phi(\theta)} d\theta$$

$$D_{KL} = \int P(\theta) \log P(\theta) d\theta - \int P(\theta) \log P_\phi(\theta) d\theta$$

- The first term is independent of ϕ so we can ignore it and the last term is just

$$-\int P(\theta) \log P_\phi(\theta) d\theta = -\langle \log P_\phi(\theta) \rangle_{P(\theta)} = -\frac{1}{N} \sum_j \log P_\phi(\theta_j)$$



Normalizing Flows (NFs)

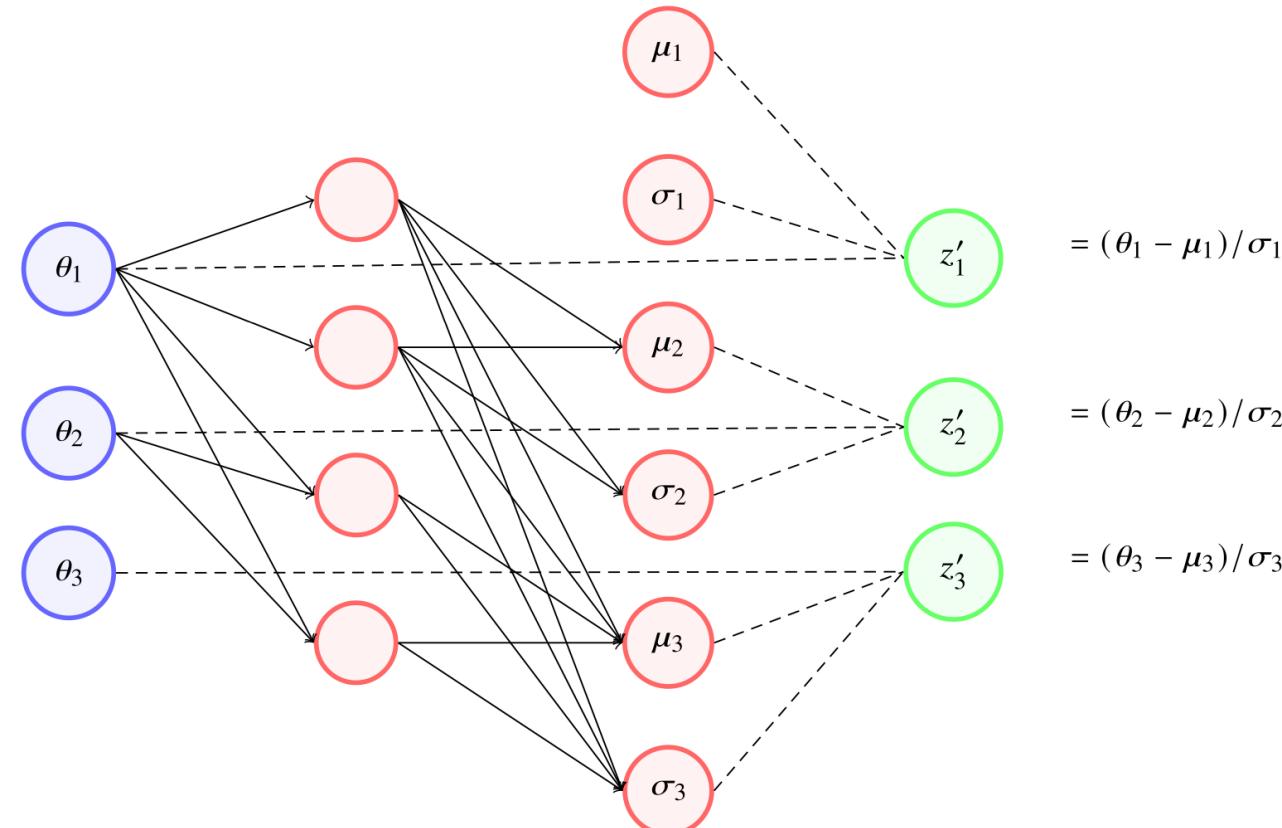
- To evaluate $\log P_\phi(\theta_j)$ we use the change of variables formula between the target and the standard normal distribution

$$\log P_\phi(\theta) = \log P_{Base} \left(f_\phi^{-1}(\theta) \right) + \log \left| \det \left(\frac{df_\phi^{-1}(\theta)}{d\theta} \right) \right|$$

- Here f_ϕ is the network and we invert the network to get samples on the standard normal base distribution which we can then analytically calculate the probability of
- The final term is the jacobian across the network
- It accounts for the change in volume between the base and target and comes for free from tensorflow



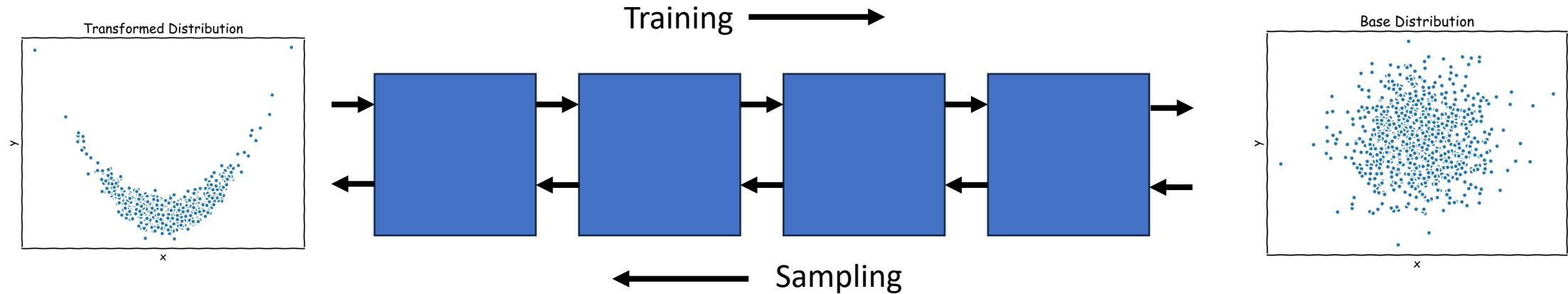
Masked Autoencoder for Distribution Estimation (MADE)



[Bevins et al 2023](#)



Masked Autoregressive Flow (MAF)



- Chain a series of MADEs together to get MAFs
- Idea is that we can learn more complex distributions with more complex architectures
- Essentially shifting and scaling repeatedly the base distribution
- The modelled conditionals in each MADE will be slightly different

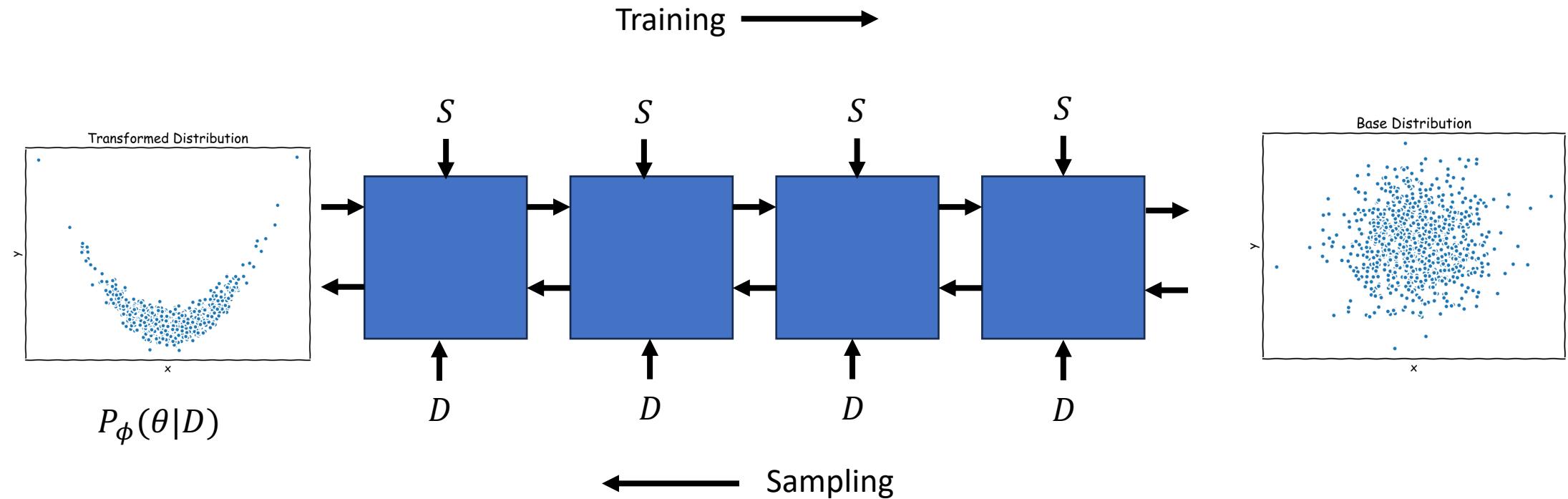


But this is just density estimation?

- NFs estimate the probability of $P(\theta)$ but what we really want is a posterior distribution i.e. $P(\theta|D)$
- To do this we use conditional MAFs
- Essentially the transformation (modelled by the network hyperparameters) from samples on the base distribution to the target becomes conditional on the data



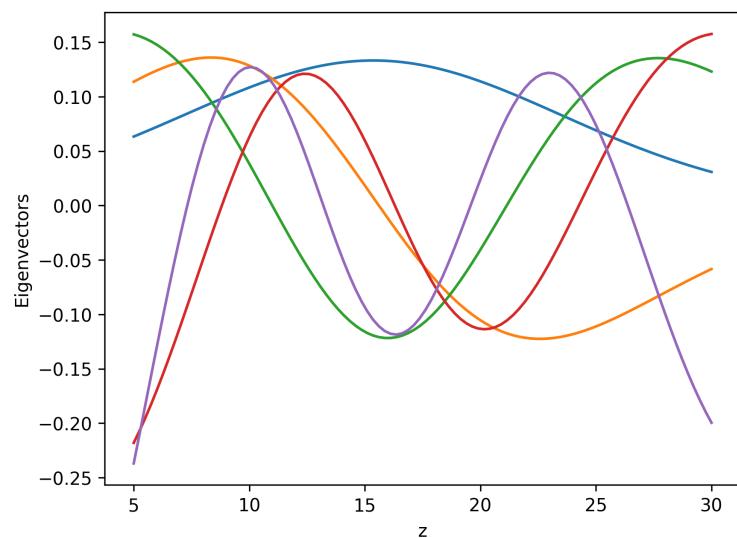
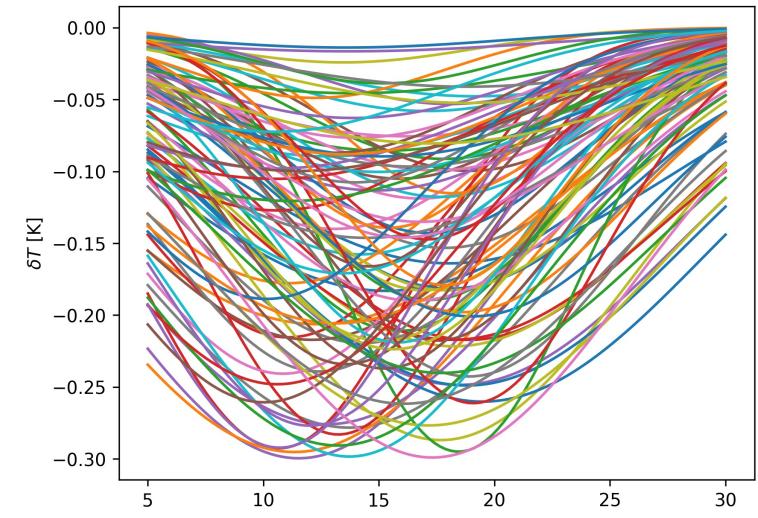
Conditional MAF





An example: 21cm Cosmology

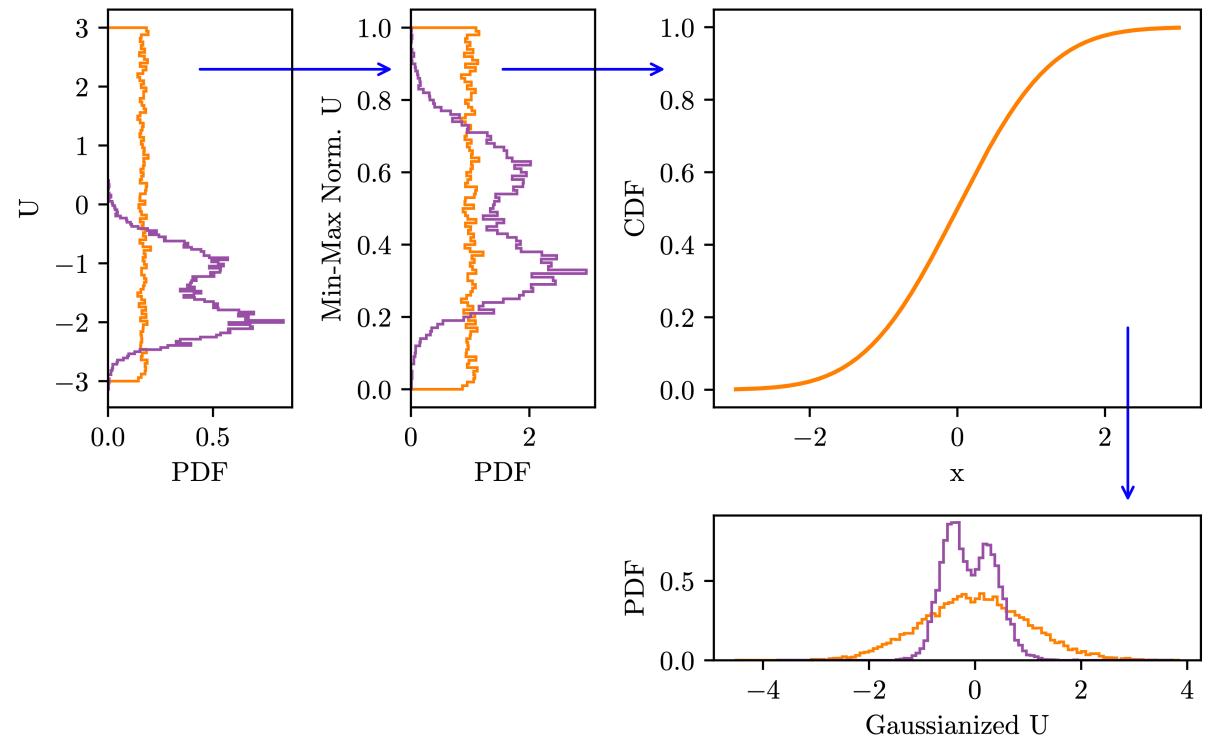
- Think again about sky-averaged 21-cm Cosmology
- Transforming a 3D space of Amplitude, central redshift and width
- No noise for simplicity
- Conditioned on 100D data space!!
- Need to do some compression





An example: 21cm Cosmology

- Example code on
<https://github.com/htjb/Talks>
- Quite non-trivial tensorflow set up but hopefully the basic idea is clear
- We have to do the usual tricks normalising our parameter and data space

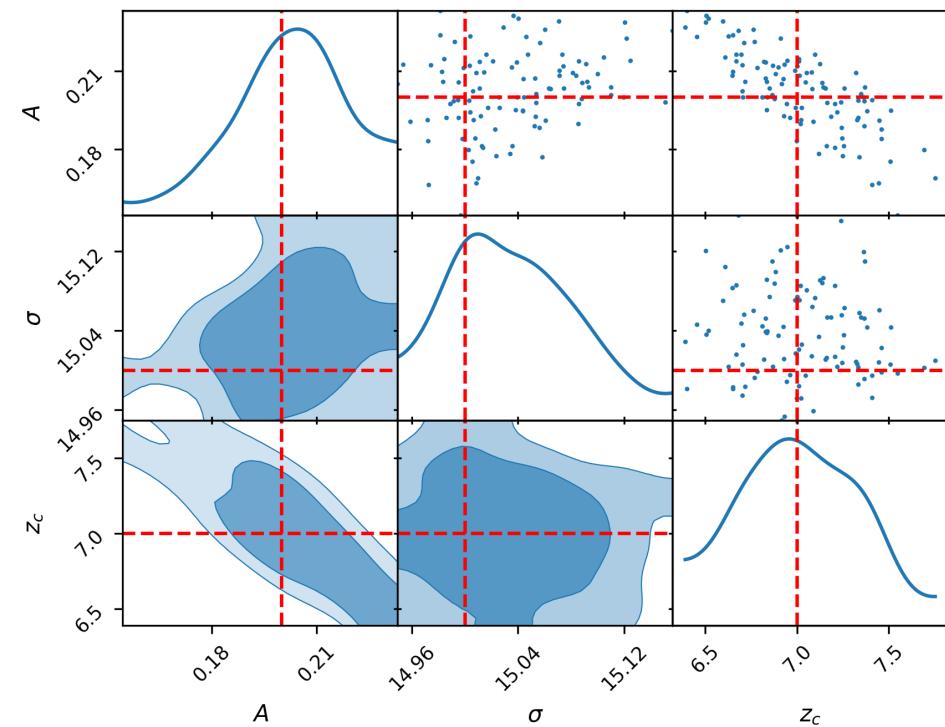


[Bevins et al 2023](#)



An example: 21cm Cosmology

- We train on pairs of parameters and simulated data
- But once trained we pass the network samples from the base distribution and the observed data
- Returns a posterior distribution!
- Very fast and we can show it other data sets without retraining



Neural Ratio Estimation (NRE)



What is NRE?

- NREs are an alternative to ABC and NPE and work by predicting ratios of densities
- Typically we set NREs up to give us the ratio $\frac{P(D|\theta)}{P(D)} = \frac{L(\theta)}{Z}$
- We can sample over our NRE and multiply the ratio with a prior probability to recover a properly normalised posterior



As a classification problem

- NREs are essentially classifiers
- During training we feed the neural network pairs of data and parameters that go together with a label (probability) of 1 and pairs that do not go together with label 0
- Use a binary cross entropy loss function

$$L = \frac{1}{N} \sum_i y_i \log(f(D_i, \theta_i)) + (1 - y_i) \log(1 - f(D_i, \theta_i))$$

- Where $y_i = 1$ (0) for matched (mis-matched) data and parameters and $f(D_i, \theta_i)$ is the predicted probability that the data goes together or not
- Loss is minimum when $y_i = f(D_i, \theta_i)$



As a classification problem

- The network learns to predict the following ratio for pairs of simulations \tilde{D} and parameters θ

$$r = \frac{P(\tilde{D}, \theta)}{P(\tilde{D})P(\theta)}$$

- Which is the ratio of the probability that the simulation and parameters are from the joint distribution or are independent
- We can express this in more familiar language as

$$r = \frac{P(\tilde{D}, \theta)}{P(\tilde{D})P(\theta)} = \frac{P(\tilde{D}|\theta)P(\theta)}{P(\tilde{D})P(\theta)} = \frac{L(\theta)\pi(\theta)}{Z\pi(\theta)} = \frac{L(\theta)}{Z}$$



As a classification problem

- The network learns what kind of data different sets of parameters produce
- We then feed the network the real data that we have observed and samples of θ from a prior to recover a posterior as

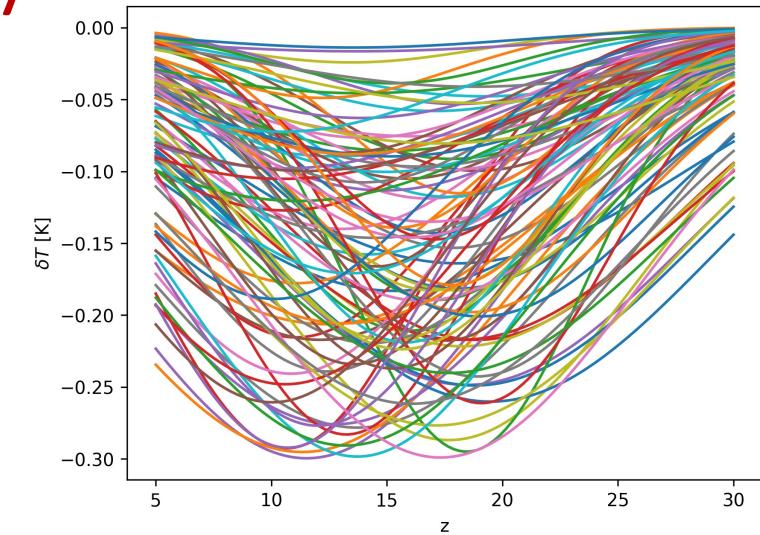
$$P(\theta|D) = r \pi(\theta) = \frac{L(\theta)}{Z} \pi(\theta)$$

- Sampling is best performed by NS or MCMC (watch out some python packages implement their own sampling algorithms!)
- NREs are amortized like NPEs



An Example: 21cm Cosmology

- Lets do the same example again
- Generate a set of example signals combinations, stack them with matching parameters and label them 1
- Stack with mismatched parameters and label 0
- No data compression here (although it is often needed)



```
norm_signals = (signals - signals.mean())/signals.std()
norm_params = (theta - theta.mean(axis=0))/theta.std(axis=0)

norm_data = np.hstack([norm_signals, norm_params])
norm_labels = np.ones(n)

idx = np.arange(0, len(norm_data))
random.shuffle(idx)
shuffled_params = norm_params[idx, :]
shuffled_data = np.hstack([norm_signals, shuffled_params])
shuffled_labels = np.zeros(n)

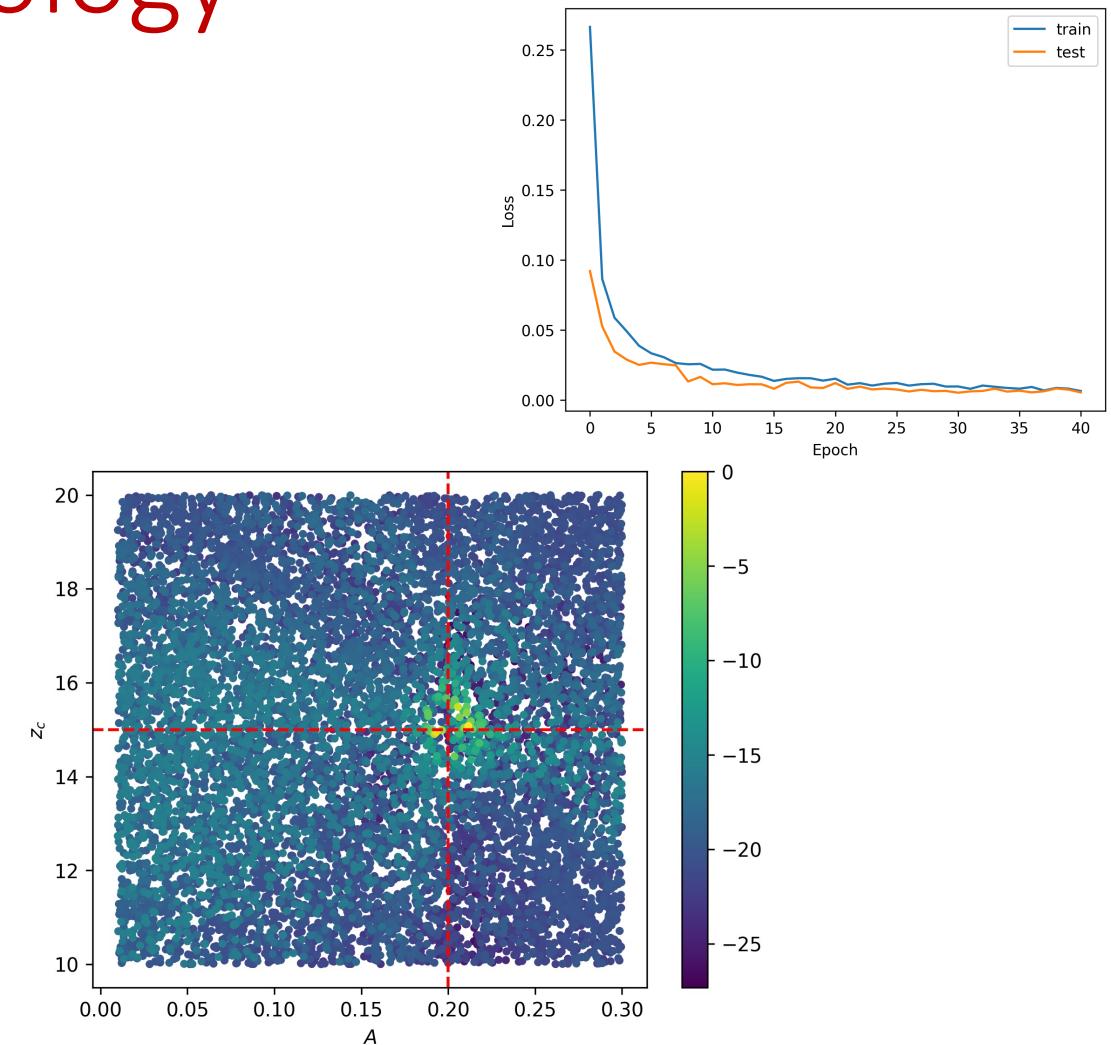
data = np.vstack([shuffled_data, norm_data])
labels = np.hstack([shuffled_labels, norm_labels])

idx = np.arange(0, 2*n)
random.shuffle(idx)
data = data[idx, :]
labels = labels[idx]
```



An Example: 21cm Cosmology

- Train the NRE
- Sample a prior and evaluate $P(\theta|D)$ using the NRE
- Here I am just sampling uniformly across the prior
- But really want to wrap this inside an NS or MCMC implementation





Summary



Simulation Based Inference

- A growing field of interest
- Advantageous when we can not analytically write a likelihood function
- But strongly dependent on the accuracy of our simulations
- We are still learning about how SBI methods scale to higher dimensions and cope with model misspecification



Join the research group!

- You've heard a lot from a lot of people during this course!
- Hopefully it has been exciting and interesting
- We are always interested in prospective PhD students
- The group works on a diverse range of topics so hopefully we are doing something that interests you!
- Remember there are code examples at [https://github.com/htjb/Talks/tree/master/Lectures/
MPhil Data Intensive Science Lectures](https://github.com/htjb/Talks/tree/master/Lectures/MPhil%20Data%20Intensive%20Science%20Lectures)

