IF.05.22 – Theoretical Informatics – Predicate Logic 1.

1. **Expressing in First Ordered Predicate Logic** Translate the following sentences into first ordered predicate logic. Define terms and predicates as needed. You may assume the usual mathematical predicate and function constants, like $<, \leq, =, \ldots +, -, \frac{x}{y}, x^2, \sqrt{x}$, ... Furthermore you may use the usual sets, like $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$, etc.
   (a) If a number divides two numbers, then it also divides their sums.
   (b) No school buses are purple
   (c) Every integer is even or odd
   (d) The only even prime is 2
   (e) Nobody is loved by no none.
   (f) Every citizen of Linz likes Hansi's paternal grandmother

2. **Translate into English/German**
   (a) $\forall x : x \in \mathbb{R} \Rightarrow x^2 \geq 0$
   (b) $\exists t : t \in \mathbb{R} \wedge t > 3 \wedge t^3 > 27$
   (c) $\forall x : x \in \mathbb{N} \Rightarrow (\text{divides}(2, x) \vee \neg\text{divides}(2, x))$ where $\text{divides}(x, y) \Leftrightarrow \exists z : z \in \mathbb{N} \wedge x \cdot z = y$

3. **Syntactic Structure of Formulas** Analyze the syntactic structure of the following formulas by drawing the corresponding syntax trees. Declare which variables are bound and which are free.
   (a) $\forall x : x \in \mathbb{R} \Rightarrow (1 + x)^n < 1 + nx$
   (b) $\forall \varepsilon \exists n \forall i : \varepsilon > 0 \wedge n \in \mathbb{N} \wedge i \geq n \Rightarrow |a_i - a| < \varepsilon$
   (c) $\cos(x)$ is zero, if x is zero.
   (d) Behind every man stands a strong woman. Hint: Translate this first into a predicate logic formula. You may assume the predicates $\text{isMan}(x)$, $\text{isWoman}(x)$, and $\text{standsBehind}(x, y)$. Your universe of discourse is the set of all people.

4. **Define** Define a unary predicate constant called *surjective* in predicate logic based on the following English sentence:

   > f is surjective if and only if every element b of the target set B gets mapped at least one element a of the domain set A.

   You may write the phrase "an element x gets mapped an element y" using the following notation: $f(y) = x$.

   Only the following phrases and definitions are allowed

   - The symbols f, A, B, a, b, and $f(a) = b$ which appear in the sentence above.
   - The binary predicate constant $\in$.
   - Quantifiers and logical composition operators, like $\wedge, \neg, \Rightarrow, \ldots$
   - The symbol ":" (colon).