

1. **Expressing in First Ordered Predicate Logic** Translate the following sentences into first ordered predicate logic. Define terms and predicates as needed. You may assume the usual mathematical predicate and function constants, like $<, \leq, =, \dots, +, -, \frac{x}{y}, x^2, \sqrt{x}, \dots$. Furthermore you may use the usual sets, like $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$, etc.
 - (a) If a number divides two numbers, then it also divides their sums.
 - (b) No school buses are purple
 - (c) Every integer is even or odd
 - (d) The only even prime is 2
 - (e) Nobody is loved by no none.
 - (f) Every citizen of Linz likes Hansi's paternal grandmother
2. **Translate into English/German**
 - (a) $\forall x : x \in \mathbb{R} \Rightarrow x^2 \geq 0$
 - (b) $\exists t : t \in \mathbb{R} \wedge t > 3 \wedge t^3 > 27$
 - (c) $\forall x : x \in \mathbb{N} \Rightarrow (\text{divides}(2, x) \vee \neg \text{divides}(2, x))$ where $\text{divides}(x, y) \Leftrightarrow \exists z : z \in \mathbb{N} \wedge x \cdot z = y$
3. **Syntactic Structure of Formulas** Analyze the syntactic structure of the following formulas. Declare which variables are bound and which are free.
 - (a) $\forall x : x \in \mathbb{R} \Rightarrow (1 + x)^n < 1 + nx$
 - (b) $\forall \varepsilon \exists n \forall i : \varepsilon > 0 \wedge n \in \mathbb{N} \wedge i \geq n \Rightarrow |a_i - a| < \varepsilon$
 - (c) $\cos(x)$ is zero, if x is zero.
 - (d) Behind every man stands a strong woman. Hint: Translate this first into a predicate logic formula. You may assume the predicates $\text{isMan}(x)$, $\text{isWoman}(x)$, and $\text{standsBehind}(x, y)$. Your universe of discourse is the set of all people.
4. **Define** Define a unary predicate constant called *surjective* in predicate logic based on the following English sentence:

f is surjective if and only if every element b of the target set B gets mapped at least one element a of the domain set A .

You may write the phrase “an element x gets mapped an element y ” using the following notation: $f(y) = x$.

Only the following phrases and definitions are allowed

- The symbols f, A, B, a, b , and $f(a) = b$ which appear in the sentence above.
- The binary predicate constant \in .
- Quantifiers and logical composition operators, like $\wedge, \neg, \Rightarrow, \dots$
- The symbol “:” (colon).