3.10

b) $f(x) + x^2 + 2 \ x_0 = 2$

1) $f(x_0) = 2^2 + 2 = 6$

2) $f(x_0 + \Delta x) = (2 + \Delta x)^2 + 2 = (4 + 4\Delta x + \Delta x^2) + 2 = 6 + \underline{\underline{4\Delta x + \Delta x^2}}$

$f(x_0 + \Delta x) - f(x_0) = 6 + 4\Delta x + \Delta x^2 - 6 = 4\Delta x + \Delta x^2$

3) $k = \frac{4\Delta x + \Delta x^2}{\Delta x} = \frac{\Delta x(4 + \Delta x)}{\Delta x} = 4 + \Delta x$

4) $\lim_{\Delta x \to 0} (4 + \Delta x) = 4$

Slg: $y = k \cdot x + d$

$6 = 4 \cdot 2 + d \quad | -8$

$d = 6 - 8 = -2$

$\underline{\underline{y = 4x - 2}}$

c) $f(x) = \frac{x^2}{2} + 1 \ x_0 = 1$

1) $f(x_0) = \frac{1}{2} + 1 = 1\frac{1}{2}$

2) $f(x_0 + \Delta x) = \frac{(1 + \Delta x)^2}{2} + 1 = \frac{(1 + 2\Delta x + \Delta x^2) + 2}{2} = \frac{3 + 2\Delta x + \Delta x^2}{2}$

$f(x_0 + \Delta x) - f(x_0) = \frac{3 + 2\Delta x + \Delta x^2}{2} - \frac{3}{2} = \frac{2\Delta x + \Delta x^2}{2}$

3) $k = \frac{\frac{2\Delta x + \Delta x^2}{2}}{} = \frac{2\Delta x + \Delta x^2}{2\Delta x} = \frac{\Delta x(2 + \Delta x)}{2\Delta x} = \frac{2 + \Delta x}{2}$

4) $\lim_{\Delta x \to 0} \left(\frac{2 + \Delta x}{2}\right) = \lim \left(\frac{2 + 0}{2}\right) = 1$

$y = k \cdot x + d$

$\frac{3}{2} = 1 \cdot 1 + d \quad | -1$

$d = \frac{1}{2}$

$\underline{\underline{y = x + \frac{1}{2}}}$

d) $f(x) = 2x^2 - 4x + 1 \ x_=0 =$

1) $f(x_0) = 2 \cdot 0^2 - 4 \cdot 0 + 1 = 1$

2) $f(x_0 + \Delta x) = 2 \cdot (0 + \Delta x)^2 - 4 \cdot (0 + \Delta x) + 1 = 2\Delta x^2 - 4\Delta x + 1$

$f(x_0 + \Delta x) - f(x_0) = 2\Delta x^2 - 4\Delta x + 1 - 1 = 2\Delta x^2 - 4\Delta x$

3) $k = \frac{2\Delta x^2 - 4\Delta x}{\Delta x} = \frac{\Delta x(2\Delta x - 4)}{\Delta x} = 2\Delta x - 4$

4) $\lim (2\Delta x - 4) = -4$

$y = k \cdot x + d$

$1 = -4 \cdot 0 + d$

$d = 1$

$\underline{\underline{y = x + 1}}$

e) $f(x_0) = (2x + 1)^2 \ x_0 = -1$

1) $f(x_0) = (2(-1)+1)^2 = 1$

2) $f(x_0+\Delta x) = (2\cdot(-1+\Delta x)+1)^2 = (-2+2\Delta x+1)^2 = (2\Delta x-1)^2 = 4\Delta x^2 - 4\Delta x^2+1$

$f(x_0+\Delta x) - f(x_0) = 4\Delta x^2 - 4\Delta x + 1 - 1 = 4\Delta x^2 - 4\Delta x$

3) $k = \frac{4\Delta x^2 - 4\Delta x}{\Delta x} = \frac{\Delta x(4\Delta x - 4)}{\Delta x} = 4\Delta x - 4$

4) $\lim (4\Delta x - 4) = -4$

$y = kx + d$

$1 = +4 + d$

$d = -3$

$y = -4x - 3$

f) $f(x) = x^3 \; x_0 = 1$

1) $f(x_0) = 1$

2) $f(x_0+\Delta x) = (1+\Delta x)^3 = \Delta x^3 + 3\Delta x^2 + 3\Delta x + 1$

$f(x_0+\Delta x) - f(x_0) = \Delta x^3 + 3\Delta x^2 + 3\Delta x + 1 - 1 = \Delta x^3 + 3\Delta x^2 + 3\Delta x$

3) $k = \frac{\Delta x^3 + 3\Delta x^2 + 3\Delta x}{\Delta x} = \frac{\Delta x(\Delta x^2 + 3\Delta x + 3)}{\Delta x} = \Delta x^2 + 3\Delta x + 3$

4) $\lim\limits_{\Delta x \to 0} (\Delta x^2 + 3\Delta x + 3) = 3$

$y = k \cdot x + d$

$1 = 3 + d$

$d = -2$

$y = 3x - 2$