SSerxhs 的 ICPC 模板

SSerxhs

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目录

1 前言

此模板的初衷是个人使用，因此已有的模板可能未列出。建议结合 Heltion 模板和 HDU 模板使用。

模板需要的版本为 cpp17 或 cpp20。

大部分情况下，涉及取模的都需要使用 `unsigned long long`，即使类型名是 `ll`。这是因为值域较大有利于合理减少取模次数。目前，大部分都已经修改为 `ull`。

`optional` 的用法：一个 `optional` 变量 `r` 可以用 `if (r)` 判断其是否有值。取出值的方法是 `*r`。常见于包含无解又包含空集解的代码中，便于区分无解和空集解。

`rev` 宏通常用于数据结构中，正向信息是否与反向信息相同，若定义了宏则表示不同。例如矩阵乘法需要定义，而区间求和则不需要。定义后额外统计信息，会慢一点点。

常见的被漏掉的初始代码：

```cpp
#define all(x) (x).begin(),(x).end()
template<class T1, class T2> bool cmin(T1 &x, const T2 &y) { if (y<x) { x=y; return 1; } return
    0; }
template<class T1, class T2> bool cmax(T1 &x, const T2 &y) { if (x<y) { x=y; return 1; } return
    0; }
```

常见的缺漏算法：

回文自动机。

2 数据结构

2.1 树状数组

支持单点修改、求前缀和、二分前缀和大于等于 x 的第一个位置。

```
template<class T> struct bit
{
    vector<T> a;
    int n;
    bit() { }
    bit(int nn) :n(nn), a(nn + 1) { }
    template<class TT> bit(int nn, TT *b) : n(nn), a(nn + 1)
    {
        for (int i = 1; i <= n; i++) a[i] = b[i];
        for (int i = 1; i <= n; i++) if (i + (i & -i) <= n) a[i + (i & -i)] += a[i];
    }
    void add(int x, T y)
    {
        //cerr<<"add "<<x<<" by "<<y<<endl;
        assert(1 <= x && x <= n);
        a[x] += y;
        while ((x += x & -x) <= n) a[x] += y;
    }
    T sum(int x)
    {
        //cerr<<"sum "<<x;
        assert(0 <= x && x <= n);
        T r = a[x];
        while (x ^= x & -x) r += a[x];
        //cerr<<"= "<<r<<endl;
        return r;
    }
    T sum(int x, int y)
    {
        return sum(y) - sum(x - 1);
    }
    int lower_bound(T x)
    {
        if (n == 0 || x <= 0) return 0;
        int i = __lg(n), j = 0;
        for (; i >= 0; i--) if ((1 << i | j) <= n && a[1 << i | j] < x) j |= 1 << i, x -= a[j];
        return j + 1;
    }
};
```

2.2 线段树

包含标记的线段树，支持线段树上二分，采用左闭右闭。但只支持求左侧第一个符合条件的下标。

要求：具有 info+info，info+=tag，tag+=tag。info，tag 需要有默认构造，但不必有正确的值。

示例的 tag 和 info 是区间加区间覆盖区间历史最大值。

```
template<class info, class tag> struct sgt
```

```
{
    int n, shift;
    info *a;
    info tmp;
    vector<info> s;
    vector<tag> tg;
    vector<int> lz;
    bool flg;
    void build(int x, int l, int r)
    {
        if (l == r)
        {
            s[x] = (flg ? tmp : a[l]);
            return;
        }
        int c = x * 2, m = l + r >> 1;
        build(c, l, m); build(c + 1, m + 1, r);
        s[x] = s[c] + s[c + 1];
    }
    sgt(info *b, int L, int R) :n(R - L + 1), shift(L - 1), a(b + L - 1), s(R - L + 1 << 2), tg(R
        - L + 1 << 2), lz(R - L + 1 << 2)
    {
        flg = 0;
        build(1, 1, n);
    }//[L,R]
    sgt(info b, int L, int R) :n(R - L + 1), shift(L - 1), s(R - L + 1 << 2), tg(R - L + 1 << 2),
        lz(R - L + 1 << 2)
    {
        tmp = b;
        flg = 1;
        build(1, 1, n);
    }//[L,R]
    int z, y;
    info res;
    tag dt;
    bool fir;
private:
    void _modify(int x, int l, int r)
    {
        if (z <= l && r <= y)
        {
            s[x] += dt;
            if (lz[x]) tg[x] += dt; else tg[x] = dt;
            lz[x] = 1;
            return;
        }
        int c = x * 2, m = l + r >> 1;
        if (lz[x])
        {
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            lz[x] = 0;
        }
        if (z <= m) _modify(c, l, m);
        if (m < y) _modify(c + 1, m + 1, r);
```

```
        s[x] = s[c] + s[c + 1];
    }
    void ask(int x, int l, int r)
    {
        if (z <= l && r <= y)
        {
            res = fir ? s[x] : res + s[x];
            fir = 0;
            return;
        }
        int c = x * 2, m = l + r >> 1;
        if (lz[x])
        {
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            lz[x] = 0;
        }
        if (z <= m) ask(c, l, m);
        if (m < y) ask(c + 1, m + 1, r);
    }
    function<bool(info)> check;
    void find_left_most(int x, int l, int r)
    {
        if (r < z || !check(s[x])) return;
        if (l == r) { y = l; res = s[x]; return; }
        int c = x * 2, m = l + r >> 1;
        if (lz[x])
        {
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            lz[x] = 0;
        }
        find_left_most(c, l, m);
        if (y == n + 1) find_left_most(c + 1, m + 1, r);
    }
    void find_right_most(int x, int l, int r)
    {
        if (l > y || !check(s[x])) return;
        if (l == r) { z = l; res = s[x]; return; }
        int c = x * 2, m = l + r >> 1;
        if (lz[x])
        {
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
            lz[c] = 1; s[c] += tg[x]; c ^= 1;
            lz[x] = 0;
        }
        find_right_most(c + 1, m + 1, r);
        if (z == 0) find_right_most(c, l, m);
    }
public:
    void modify(int l, int r, const tag &x)//[l,r]
```

```
    {
        z = l - shift; y = r - shift; dt = x;
        // cerr<<"modify ["<<l<<','<<r<<"] "<<'\n';
        assert(1 <= z && z <= y && y <= n);
        _modify(1, 1, n);
    }
    void modify(int pos, const info &o)
    {
        pos -= shift;
        int l = 1, r = n, m, c, x = 1;
        while (l < r)
        {
            c = x * 2; m = l + r >> 1;
            if (lz[x])
            {
                if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
                lz[c] = 1; s[c] += tg[x]; c ^= 1;
                if (lz[c]) tg[c] += tg[x]; else tg[c] = tg[x];
                lz[c] = 1; s[c] += tg[x]; c ^= 1;
                lz[x] = 0;
            }
            if (pos <= m) x = c, r = m; else x = c + 1, l = m + 1;
        }
        s[x] = o;
        while (x >>= 1) s[x] = s[x * 2] + s[x * 2 + 1];
    }
    info ask(int l, int r)//[l,r]
    {
        z = l - shift; y = r - shift; fir = 1;
        // cerr<<"ask ["<<l<<','<<r<<"] "<<'\n';
        assert(1 <= z && z <= y && y <= n);
        ask(1, 1, n);
        return res;
    }
    pair<int, info> find_left_most(int l, const function<bool(info)> &_check)
    {
        check = _check;
        z = l - shift; y = n + 1;
        assert(1 <= z && z <= n + 1);
        find_left_most(1, 1, n);
        return {y + shift, res};
    }
    pair<int, info> find_right_most(int r, const function<bool(info)> &_check)
    {
        check = _check;
        z = 0; y = r - shift;
        assert(0 <= y && y <= n);
        find_right_most(1, 1, n);
        return {z + shift, res};
    }
};
struct tag
{
    ll hadd, hc, now;
    bool t;
    void operator+=(const tag &o)
    {
```

```
        if (t == 0)
        {
            cmax(hc, o.hc);
            cmax(hadd, now + o.hadd);
        }
        else
            hc = max({hc, now + o.hadd, o.hc});
        if (o.t) now = o.now, t = 1;
        else now += o.now;
    }
};
struct info
{
    ll mx, hmx;
    info operator+(const info &o) const { return {max(mx, o.mx), max(hmx, o.hmx)}; }
    void operator+=(const tag &o)
    {
        hmx = max({hmx, mx + o.hadd, o.hc});
        if (o.t) mx = o.now; else mx += o.now;
    }
};
```

2.3 珂朵莉树

支持区间赋值、单点访问。维护每个连续段的范围和值。

如果希望维护所有连续段的整体信息（如长度的最大值），修改 add 和 del 函数即可，分别表示连续段被加入和被删去。

特别注意一开始 insert 的不会触发 add，只有 modify 会触发。

```
namespace chtholly_tree
{
    using T = int;//可以把 T 修改为任意想要的类型。
    struct node
    {
        int l;
        mutable int r;
        mutable T v;
        int len() const { return r - l + 1; }
        bool operator<(const node &x) const { return l < x.l; }
    };
    void add(const node &a) { }
    void del(const node &a) { }
    class odt : public set<node>
    {
    public:
        typedef odt::iterator iter;
        iter split(int x)
        {
            iter it = lower_bound({x});
            if (it != end() && it->l == x) return it;
            node t = *--it, a = {t.l, x - 1, t.v}, b = {x, t.r, t.v};
            del(*it); add(a); add(b);
            erase(it); insert(a);
            return insert(b).first;
        }
        iter modify(int l, int r, T v)//[l,r]
```

```
        {
            iter lt, rt, it;
            rt = r == rbegin()->r ? end() : split(r + 1); lt = split(l);//[lt,rt)
            while (lt != begin() && (it = prev(lt))->v == v) l = (lt = it)->l;
            while (rt != end() && rt->v == v) r = (rt++)->r;
            for (it = lt; it != rt; it++) del(*it);
            add({l, r, v});
            erase(lt, rt);
            return insert({l, r, v}).first;
        }
        T operator[](const int x) const { return prev(upper_bound({x}))->v; }//直接访问单点
        iter find(int x) const { return prev(upper_bound({x})); }//找到对应的线段
    };
}
using chtholly_tree::node, chtholly_tree::odt;
typedef odt::iterator iter;
int main()
{
    odt s;
    s.insert({0, 5, 1});  // 先 insert({L,R,x}) 表示整个下标范围和初始值。 左闭右闭。
                         // s={1,1,1,1,1,1}
    s.modify(2, 3, 2);   // 左闭右闭。s={1,1,2,2,1,1}
    for (auto [l, r, v] : s)
    {
        //(l,r,v)=(0,1,1)
        //(l,r,v)=(2,3,2)
        //(l,r,v)=(4,5,1)
    }
}
```

2.4 带删堆

本质是额外维护一个堆 q 表示要被删除的元素，当 p 的最值和 q 一样时删除。
需要保证每次 pop 的元素都存在于堆中。
本代码的用法和 `priority_queue` 一致。

```
template<class T, class T1 = vector<T>, class T2 = less<T>> struct heap
{
private:
    priority_queue<T, T1, T2> p, q;
public:
    void push(const T &x)
    {
        if (!q.empty() && q.top() == x)
        {
            q.pop();
            while (!q.empty() && q.top() == p.top()) p.pop(), q.pop();
        }
        else p.push(x);
    }
    void pop()
    {
        p.pop();
        while (!q.empty() && p.top() == q.top()) p.pop(), q.pop();
    }
    void pop(const T &x)
```

```
    {
        if (p.top() == x)
        {
            p.pop();
            while (!q.empty() && p.top() == q.top()) p.pop(), q.pop();
        }
        else q.push(x);
    }
    T top() const { return p.top(); }
    int size() const { return p.size() - q.size(); }
    bool empty() const { return p.empty(); }
    vector<T> to_vector() const
    {
        vector<T> a;
        auto P = p, Q = q;
        while (P.size())
        {
            a.push_back(P.top()); P.pop();
            while (Q.size() && P.top() == Q.top()) P.pop(), Q.pop();
        }
        return a;
    }
};
```

2.5 前 k 大的和

本质是用小根堆维护前 k 大的数，用大根堆维护其余数。

如果需要支持删除，结合前面一个使用，或者直接用 multiset 进行 extract。

为了方便起见，直接给出支持删除的版本，并且使用 long long。如果不需要支持删除，类型改为优先队列并去掉 pop 函数即可。

注意：复杂度为 $O(|k-k'|)$，其中 k' 是上一次询问的 k。也就是说，多组询问时询问的 k 的差值应该尽可能小。

其用法与 priority_queue 保持一致，可以用同样的方法改写成前 k 小。

```
using ll = long long;
template<class T, class T1 = vector<T>, class T2 = less<T>> struct ksum_pop
{
private:
    struct __cmp
    {
        bool operator()(const T &x, const T &y) const
        {
            return x != y && !T2()(x, y);
        }
    };
    heap<T, T1, __cmp> p;
    heap<T, T1, T2> q;
    ll cur;
public:
    ksum_pop() :cur(0) { }
    void push(const T &x)
    {
        if (!q.size() || !T2()(x, q.top())) p.push(x), cur += x; else q.push(x);
    }
    int size() const { return p.size() + q.size(); }
```

```
    void pop(const T &x)
    {
        if (q.size() && !T2()(q.top(), x)) q.pop(x);
        else p.pop(x), cur -= x;
    }
    ll sum(int k)
    {
        while (p.size() < k)
        {
            cur += q.top();
            p.push(q.top());
            q.pop();
        }
        while (p.size() > k)
        {
            cur -= p.top();
            q.push(p.top());
            p.pop();
        }
        return cur;
    }
};
```

2.6 左偏树/可并堆

建议不要使用。pb_ds 可以替代这个功能。我完全没有使用过这个板子。

$O((n+q)\log n)$，$O(n)$。

```
struct left_tree//小根堆，大根堆需要改的地方注释了
{
    int jl[N],v[N],f[N],c[N][2],tf[N],n;//tf只有删非堆顶才用
    bool ed[N];
    void init(const int nn,const int *a)
    {
        jl[0]=-1;n=nn;
        memset(jl+1,0,n<<2);
        memset(tf+1,0,n<<2);//同上
        memset(c+1,0,n<<3);
        memset(ed+1,0,n);
        for (int i=1;i<=n;i++) v[f[i]=i]=a[i];
    }
    int mg(int x,int y)
    {
        if (!(x&&y)) return x|y;
        if (v[x]>v[y]||v[x]==v[y]&&x>y) swap(x,y);//改
        tf[c[x][1]=mg(c[x][1],y)]=x;//同上
        if (jl[c[x][0]]<jl[c[x][1]]) swap(c[x][0],c[x][1]);
        jl[x]=jl[c[x][1]]+1;
        return x;
    }
    int getf(int x)
    {
        if (f[x]==x) return x;
        return f[x]=getf(f[x]);
    }
    int merge(int x,int y)
```

```
    {
        if (ed[x]||ed[y]||(x=getf(x))==(y=getf(y))) return x;
        int z=mg(x,y);return f[x]=f[y]=z;
    }
    int getv(int x)//需要自行判断是否存在
    {
        return v[getf(x)];
    }
    int del(int x)//删除堆内最值
    {
        tf[c[x][0]]=tf[c[x][1]]=0;
        f[c[x][0]]=f[c[x][1]]=f[x]=mg(c[x][0],c[x][1]);
        ed[x]=1;c[x][0]=c[x][1]=tf[x]=0;return f[x];
    }
    int del_all(int x)//删除堆内非最值（没验证过）
    {
        int fa=tf[x];
        if (f[c[x][0]]==x) f[c[x][0]]=getf(tf[x]);
        if (f[c[x][1]]==x) f[c[x][1]]=f[tf[x]];
        tf[x]=tf[c[x][0]]=tf[c[x][1]]=0;
        tf[c[fa][c[fa][1]==x]=mg(c[x][0],c[x][1])]=fa;
        c[x][0]=c[x][1]=0;
        while (jl[c[fa][0]]<jl[c[fa][1]])
        {
            swap(c[fa][0],c[fa][1]);
            jl[fa]=jl[c[fa][1]]+1;
            fa=tf[fa];
        }
    }
    void out(int n)
    {
        for (int i=1;i<=n;i++) printf("%d: c%d&%d f%d v%d\n",i,c[i][0],c[i][1],f[i],v[i]);
    }
};
```

2.7 树状数组区间加区间求和

本质：a_n 区间加等价于差分数组 d_n 的单点加。

$\sum_{i=1}^{m} a_i = \sum_{i=1}^{m}\sum_{j=1}^{i} d_j = \sum_{j=1}^{m} d_j(m-j+1) = ((m+1)\sum_{j=1}^{m} d_j) - (\sum_{j=1}^{m} jd_j)$。

分别维护 d_j 和 jd_j 的前缀和。

$O(n) \sim O(q\log n)$，$O(n)$。

```
template<class T> struct bit
{
    vector<T> a, b;
    int n;
    template<class TT> bit(int n, TT *c) :n(n), a(n + 1), b(n + 1)
    {
        for (int i = 1; i <= n; i++) b[i] = -c[i];
        for (int i = 1; i <= n; i++) if (i + (i & -i) <= n) b[i + (i & -i)] += b[i];
    }
    void add(int l, int r, T d)
    {
        T x;
```

```
        int i;
        for (i = l, x = d * i; i <= n; i += i & -i) a[i] += d, b[i] += x;
        for (i = r + 1, x = d * i; i <= n; i += i & -i) a[i] -= d, b[i] -= x;
    }
    void add(int x, T d)
    {
        for (int i = x; i <= n; i += i & -i) b[i] -= d;
    }
    T sum(int x)
    {
        T r1 = 0, r2 = 0;
        for (int i = x; i; i ^= i & -i) r1 += a[i], r2 += b[i];
        return r1 * (x + 1) - r2;
    }
    T sum(int l, int r)
    {
        return sum(r) - sum(l - 1);
    }
};
```

2.8 二维树状数组矩形加矩形求和

本质还是差分，只不过这次要维护 $d_{i,j}, d_{i,j}i, d_{i,j}j, d_{i,j}ij$。

$O(n^2 + q\log^2 n)$，$O(n^2)$

```
template <class T> struct bit
{
    int n, m;
    vector<vector<T>> a, b, c, d;
private:
    void modify(vector<vector<T>> &a, int x, int y, T z)
    {
        for (int i = x; i <= n; i += i & -i) for (int j = y; j <= m; j += j & -j) a[i][j] += z;
    }
    T ask(const vector<vector<T>> &a, int x, int y) const
    {
        T res = 0; --x; --y;
        for (int i = x; i; i ^= i & -i) for (int j = y; j; j ^= j & -j) res += a[i][j];
        return res;
    }
    void cg(int x, int y, T t)
    {
        if (x > n || y > n) return;
        modify(a, x, y, t);
        modify(b, x, y, x * t);
        modify(c, x, y, y * t);
        modify(d, x, y, x * y * t);
    }
public:
    bit(int n, int m) :n(n), m(m), a(n + 1, vector<T>(m + 1)), b(a), c(a), d(a) { }
    void add(int x1, int y1, int x2, int y2, T t)
    {
        ++x2, ++y2;
        cg(x1, y1, t); cg(x2, y2, t);
        cg(x1, y2, -t); cg(x2, y1, -t);
    }
```

```
    T sum(int x, int y) const
    {
        if (x <= 0 || y <= 0) return 0;
        ++x; ++y;
        return ask(a, x, y) * x * y + ask(d, x, y) - ask(b, x, y) * y - ask(c, x, y) * x;
    }
    T sum(int x1, int y1, int x2, int y2) const
    {
        --x1; --y1;
        return sum(x2, y2) + sum(x1, y1) - sum(x2, y1) - sum(x1, y2);
    }
};
```

2.9 带修莫队

按照 $n^{\frac{2}{3}}$ 分块，排序关键字是 l, r, t 所在的块（t 是版本号，每次修改都会增加一个版本），可以奇偶分块优化。

相比于传统莫队多了一个 modify。这里只给出参考代码，功能是带修区间数颜色。

$O(n^{\frac{5}{3}})$，$O(n)$。

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=1.4e5,M=1e6+2;
int a[N],ans[N],bel[N],cnt[M],sum,z,y,cur;
struct P
{
    int p,v;
};
struct Q
{
    int l,r,t,p;
    bool operator<(const Q &o) const
    {
        if (bel[l]!=bel[o.l]) return bel[l]<bel[o.l];
        if (bel[r]!=bel[o.r]) return (bel[l]&1)^bel[r]<bel[o.r];
        return (bel[r]&1)?t<o.t:t>o.t;
    }
};
Q b[N];
P d[N];
void add(const int &x) {sum+=!(cnt[a[x]]++);}
void del(const int &x) {sum-=!(--cnt[a[x]]);}
void mdf(const int &x)
{
    auto &[p,v]=d[x];
    if (z<=p&&p<=y) del(p);
    swap(a[p],v);
    if (z<=p&&p<=y) add(p);
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,q1=0,q2=0,i,ksiz;
    cin>>n>>m;
```

```
    for (i=1;i<=n;i++) cin>>a[i];
    for (i=1;i<=m;i++)
    {
        char c;
        int l,r;
        cin>>c>>l>>r;
        if (c=='Q') ++q1,b[q1]={l,r,q2,q1};
        else d[++q2]={l,r};
    }
    ksiz=max(1.0,round(cbrt((ll)n*n)));
    for (i=1;i<=n;i++) bel[i]=i/ksiz;
    sort(b+1,b+q1+1);
    z=b[1].l;y=z-1;cur=0;
    for (i=1;i<=q1;i++)
    {
        auto [l,r,t,p]=b[i];
        while (z>l) add(--z);
        while (y<r) add(++y);
        while (z<l) del(z++);
        while (y>r) del(y--);
        while (cur<t) mdf(++cur);
        while (cur>t) mdf(cur--);
        ans[p]=sum;
    }
    for (i=1;i<=q1;i++) cout<<ans[i]<<'\n';
}
```

2.10 二次离线莫队

直接摘录题解，用途不大。

$O(n\sqrt{n})$，$O(n)$。

珂朵莉给了你一个序列 a，每次查询给一个区间 $[l,r]$，查询 $l \le i < j \le r$，且 $a_i \oplus a_j$ 的二进制表示下有 k 个 1 的二元组 (i,j) 的个数。$\oplus$ 是指按位异或。

二次离线莫队，通过扫描线，再次将更新答案的过程离线处理，降低时间复杂度。假设更新答案的复杂度为 $O(k)$，它将莫队的复杂度从 $O(nk\sqrt{n})$ 降到了 $O(nk+n\sqrt{n})$，大大简化了计算。设 x 对区间 $[l,r]$ 的贡献为 $f(x,[l,r])$，我们考虑区间端点变化对答案的影响：以 $[l..r]$ 变成 $[l..(r+k)]$ 为例，$\forall x \in [r+1,r+k]$ 求 $f(x,[l,x-1])$。我们可以进行差分：$f(x,[l,x-1]) = f(x,[1,x-1]) - f(x,[1,l-1])$，这样转化为了一个数对一个前缀的贡献。保存下来所有这样的询问，从左到右扫描数组计算就可以了。但是这样做，空间是 $O(n\sqrt{n})$ 的，不太优秀，而且时间常数巨大。。这样的贡献分为两类：

1. 减号左边的贡献永远是一个前缀和它后面一个数的贡献。这可以预处理出来。2. 减号右边的贡献对于一次移动中所有的 x 来说，都是不变的。我们打标记的时候，可以只标记左右端点。

这样，减小时间常数的同时，空间降为了 $O(n)$ 级别。是一个很优秀的算法了。处理前缀询问的时候，我们利用异或运算的交换律，即 $a \text{ xor } b = c \iff a \text{ xor } c = b$ 开一个桶 t，$t[i]$ 表示当前前缀中与 i 异或有 k 个数位为 1 的数有多少个。则每加入一个数 $a[i]$，对于所有 $\text{popcount}(x) = k$ 的 x，$t[a[i] \text{ xor } x] \leftarrow t[a[i] \text{ xor } x] + 1$ 即可。

```
typedef long long ll;
const int N = 1e5 + 2, M = 1 << 14;
ll f[N], ans[N], ta[N];
int a[N], cnt[M], bel[N], pc[M], st[N];
int n, m, ksiz;
struct Q
{
```

```
    int z, y, wz;
    bool operator<(const Q &x) const { return (bel[z] < bel[x.z]) || (bel[z] == bel[x.z]) && ((y <
        x.y) && (bel[z] & 1) || (y > x.y) && (1 ^ bel[z] & 1)); }
};
Q mq(const int x, const int y, const int z)
{
    Q a;
    a.z = x; a.y = y; a.wz = z;
    return a;
}
Q q[N];
vector<Q> b[N];
int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
    int i, j, k, l = 1, r = 0, tp = 0, x, na;
    cin >> n >> m >> k; ksiz = sqrt(n);
    for (i = 1; i <= n; i++) { cin >> a[i]; bel[i] = (i - 1) / ksiz + 1; }
    if (k == 0) st[++tp] = 0;
    for (i = 1; i < 16384; i++)
    {
        if (i & 1) pc[i] = pc[i >> 1] + 1; else pc[i] = pc[i >> 1];
        if (pc[i] == k) st[++tp] = i;
    }
    for (i = 1; i <= n; i++)
    {
        j = tp + 1; f[i] = f[i - 1];
        while (--j) f[i] += cnt[st[j] ^ a[i]];
        ++cnt[a[i]];
    }
    for (i = 1; i <= m; i++) { cin >> q[i].z >> q[q[i].wz = i].y; }
    sort(q + 1, q + m + 1);
    for (i = 1; i <= m; i++)
    {
        ans[i] = f[q[i].y] - f[r] + f[q[i].z - 1] - f[l - 1];
        if (k == 0) ans[i] += q[i].z - l;
        if (r < q[i].y)
        {
            b[l - 1].push_back(mq(r + 1, q[i].y, -i));
            r = q[i].y;
        }
        if (l > q[i].z)
        {
            b[r].push_back(mq(q[i].z, l - 1, i));
            l = q[i].z;
        }
        if (r > q[i].y)
        {
            b[l - 1].push_back(mq(q[i].y + 1, r, i));
            r = q[i].y;
        }
        if (l < q[i].z)
        {
            b[r].push_back(mq(l, q[i].z - 1, -i));
            l = q[i].z;
        }
```

```
    }
    memset(cnt, 0, sizeof(cnt));
    for (i = 1; i <= n; i++)
    {
        j = tp + 1; x = a[i];
        while (--j) ++cnt[x ^ st[j]];
        for (j = 0; j < b[i].size(); j++)
        {
            na = 0; l = b[i][j].z; r = b[i][j].y;
            for (k = l; k <= r; k++) na += cnt[a[k]];
            if (b[i][j].wz > 0) ans[b[i][j].wz] += na; else ans[-b[i][j].wz] -= na;
        }
    }
    for (i = 2; i <= m; i++) ans[i] += ans[i - 1];
    for (i = 1; i <= m; i++) ta[q[i].wz] = ans[i];
    for (i = 1; i <= m; i++) printf("%lld\n", ta[i]);
}
```

2.11 回滚莫队

不删除的莫队，比如求 max。

做法：块内询问暴力。对于 l 所在块相同的询问，按照 r 升序排序，并且将左指针固定在 l 所在块的最右侧。（由于块内询问暴力，这不会导致左指针更大）

回答每个询问的时候，先右端点右移到 r，然后左端点左移到 l。询问完成后，把左端点移回去。移回去的过程虽然涉及删除，但不需要维护答案变成什么了（因为在左端点左移之前已经求过了）。换句话说，相当于“撤销”而不是删除，完全可以记录移动过程中的所有变化来撤销。

$O(n\sqrt{n})$，$O(n)$。

```
#include "bits/stdc++.h"
using namespace std;
const int N = 2e5 + 2;
int a[N], z[N], y[N], wz[N], b[N], d[N], bel[N], ans[N], st[N][2], pos[N][2];
void qs(int l, int r)
{
    int i = l, j = r, m = bel[z[l + r >> 1]], mm = y[l + r >> 1];
    while (i <= j)
    {
        while ((bel[z[i]] < m) || (bel[z[i]] == m) && (y[i] < mm)) ++i;
        while ((bel[z[j]] > m) || (bel[z[j]] == m) && (y[j] > mm)) --j;
        if (i <= j)
        {
            swap(wz[i], wz[j]);
            swap(z[i], z[j]);
            swap(y[i++], y[j--]);
        }
    }
    if (i < r) qs(i, r);
    if (l < j) qs(l, j);
}
int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
    cin >> n;
    ksiz = sqrt(n);
```

```
    for (i = 1; i <= n; i++) { cin >> a[i]; b[i] = a[i]; bel[i] = (i - 1) / ksiz + 1; }
    sort(b + 1, b + n + 1);
    d[gs = 1] = b[1];
    for (i = 2; i <= n; i++) if (b[i] != b[i - 1]) d[++gs] = b[i];
    for (i = 1; i <= n; i++) a[i] = lower_bound(d + 1, d + gs + 1, a[i]) - d;
    cin >> m; assert(int(n / sqrt(m)));
    for (i = 1; i <= m; i++) cin >> z[i] >> y[wz[i] = i];
    qs(1, m);
    for (i = 1; i <= m; i++)
    {
        if (bel[z[i]] > bel[z[i - 1]])
        {
            while (l <= r) { pos[a[l]][0] = pos[a[l]][1] = 0; ++l; }na = 0;
            if (bel[z[i]] == bel[y[i]])
            {
                for (j = z[i]; j <= y[i]; j++) if (pos[a[j]][0]) na = max(na, j - pos[a[j]][0]);
                    else pos[a[j]][0] = j;
                ans[wz[i]] = na; for (j = z[i]; j <= y[i]; j++) pos[a[j]][0] = 0; na = 0; l = ksiz
                    * bel[z[i]]; r = l - 1;
                continue;
            }
            l = ksiz * bel[z[i]]; r = l - 1; na = 0;
        }
        if (bel[z[i]] == bel[y[i]])
        {
            while (l <= r) { pos[a[l]][0] = pos[a[l]][1] = 0; ++l; }na = 0;
            for (j = z[i]; j <= y[i]; j++) if (pos[a[j]][0]) na = max(na, j - pos[a[j]][0]); else
                pos[a[j]][0] = j;
            ans[wz[i]] = na; for (j = z[i]; j <= y[i]; j++) pos[a[j]][0] = 0;
            l = ksiz * bel[z[i]]; r = l - 1; na = 0;
            continue;
        }
        while (r < y[i])
        {
            x = a[++r]; pos[x][1] = r;
            if (!pos[x][0]) pos[x][0] = r; else na = max(na, r - pos[x][0]);
        }c = na;
        while (l > z[i])
        {
            x = a[--l]; st[++tp][0] = x; st[tp][1] = pos[x][0];
            pos[x][0] = l;
            if (!pos[x][1])
            {
                st[++tp][0] = x + n; st[tp][1] = 0;
                pos[x][1] = l;
            }
            else na = max(na, pos[x][1] - l);
        }
        ans[wz[i]] = na; na = c; ++tp; l = ksiz * bel[z[i]];
        while (--tp) if (st[tp][0] <= n) pos[st[tp][0]][0] = st[tp][1]; else pos[st[tp][0] - n][1]
             = st[tp][1];
    }
    for (i = 1; i <= m; i++) cout << ans[i] << "\n";
}
```

2.12 李超树

题意：插入线段，查询某个 x 的最大 y（输出最小编号）

算法核心：修改时，线段树每个点只维护在中点取值最大的线段，中点取值较小的线段只会在至多一侧有用，递归下去插入，复杂度 $O(\log^2)$。查询时询问线段树上 log 个点的线段中最大的。

```
struct Q
{
    int x0, y0, dx, dy, id;
    Q() :x0(0), y0(-1), dx(1), dy(0), id(-1) { }//y>=0
    Q(int a, int b, int c, int d, int e) :x0(a), y0(b), dx(c), dy(d), id(e) { }
    bool contains(const int &x) const { return x0 <= x && x <= x0 + dx; }
};
bool cmp(const Q &a, const Q &b, int x)//小心数值爆炸
{
    ll A = ((ll)a.y0 * a.dx + (ll)(x - a.x0) * a.dy) * b.dx, B = ((ll)b.y0 * b.dx + (ll)(x - b.x0)
         * b.dy) * a.dx;
    if (A != B) return A < B;
    return a.id > b.id;
}
bool cmp2(const Q &a, const Q &b)
{
    if (a.y0 + a.dy != b.y0 + b.dy) return a.y0 + a.dy < b.y0 + b.dy;
    return a.id > b.id;
}
const int inf = 1e9;
int ans;
namespace seg
{
    const int N = 4e4 + 2, M = N * 4;
    Q s[M], X[N];
    int n, z, y;
    void init(int nn) { n = nn; for (int i = 1; i <= n * 4; i++) s[i] = Q(); }
    void insert(int x, int l, int r, Q dt)
    {
        int c = x * 2, m = l + r >> 1;
        if (z <= l && r <= y)
        {
            if (cmp(s[x], dt, m)) swap(s[x], dt);
            if (l == r) return;
            if (cmp(s[x], dt, l)) insert(c, l, m, dt);
            else if (cmp(s[x], dt, r)) insert(c + 1, m + 1, r, dt);
            return;
        }
        if (z <= m) insert(c, l, m, dt);
        if (y > m) insert(c + 1, m + 1, r, dt);
    }
    void insert(const Q &o)
    {
        z = o.x0; y = z + o.dx;
        assert(1 <= z && z <= y && y <= n);
        if (z == y)
        {
            if (cmp2(X[z], o)) X[z] = o;
            return;
        }
        insert(1, 1, n, o);
    }
```

```
    Q askmax(int p)
    {
        Q ans = s[1].contains(p) ? s[1] : Q();
        int x = 1, l = 1, r = n, c, m;
        while (l < r)
        {
            c = x * 2, m = l + r >> 1;
            if (p <= m) x = c, r = m; else x = c + 1, l = m + 1;
            if (s[x].contains(p) && cmp(ans, s[x], p)) ans = s[x];
        }
        Q o(X[p].x0, X[p].y0 + X[p].dy, 1, 0, 0);
        return cmp(ans, o, p) ? X[p] : ans;
    }
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << setiosflags(ios::fixed) << setprecision(15);
    int n = 4e4, m, i;
    seg::init(n);
    cin >> m;
    while (m--)
    {
        int op;
        cin >> op;
        if (op)
        {
            int x[2], y[2];
            cin >> x[0] >> y[0] >> x[1] >> y[1];
            for (int &v : x) v = (v + ans - 1) % 39989 + 1;
            for (int &v : y) v = (v + ans - 1) % inf + 1;
            if (x[0] > x[1] || x[0] == x[1] && y[0] > y[1]) swap(x[0], x[1]), swap(y[0], y[1]);
            static int id;
            seg::insert({x[0], y[0], x[1] - x[0], y[1] - y[0], ++id});
        }
        else
        {
            int x;
            cin >> x;
            x = (x + ans - 1) % 39989 + 1;
            cout << (ans = max(0, seg::askmax(x).id)) << '\n';
        }
    }
}
```

2.13 李超树（动态开点）

```
struct Q
{
    int k;
    ll b;
    ll y(const int &x) const { return (ll)k * x + b; }
};
const int inf = 1e9;
const ll INF = 1e18;
struct seg//可以析构，不能并行
```

```
{
    const static int N = 4e5 + 2, M = N * 8 * 8 + (1 << 23);
    const static ll npos = 9e18;
    static Q s[M];
    static int c[M][2], id;
    int z, y, L, R;
    seg(int l, int r)
    {
        L = l; R = r; id = 1;
        s[1] = {0, npos};
        assert(L <= R && (ll)R - L < 1ll << 32);
    }
private:
    void insert(int &x, int l, int r, Q o)
    {
        if (!x)
        {
            x = ++id;
            assert(id < M);
            s[x] = {0, npos};
        }
        int m = l + (r - l >> 1);
        if (z <= l && r <= y)
        {
            if (s[x].y(m) > o.y(m)) swap(s[x], o);
            if (s[x].y(l) > o.y(l)) insert(c[x][0], l, m, o);
            else if (s[x].y(r) > o.y(r)) insert(c[x][1], m + 1, r, o);
            return;
        }
        if (z <= m) insert(c[x][0], l, m, o);
        if (y > m) insert(c[x][1], m + 1, r, o);
    }
public:
    void insert(const Q &x, const int &l, const int &r)//[l,r]
    {
        z = l; y = r; int tmp = 1;
        insert(tmp, L, R, x);
        assert(tmp == 1);
    }
    ll askmin(const int &p)
    {
        ll res = s[1].y(p);
        int l = L, r = R, m, x = 1;
        while (l < r)
        {
            m = l + (r - l >> 1);
            if (p <= m) x = c[x][0], r = m; else x = c[x][1], l = m + 1;
            if (!x) return res;
            res = min(res, s[x].y(p));
        }
        return res;
    }
    ~seg()
    {
        ++id;
        while (--id) c[id][0] = c[id][1] = 0;
    }
```

```
};
Q seg::s[seg::M];
int seg::c[seg::M][2], seg::id;
```

2.14 区间线性基

$O((n+q)\log a)$，$O(n\log a)$。

```
template<class T, int M = sizeof(T) * 8> struct base//线性基
{
    array<T, M> a;
    base() :a{ } { }
    bool insert(T x)//线性基插入
    {
        if (x == 0) return 0;
        for (int i = __lg(x); x; i = __lg(x))
        {
            if (!a[i])
            {
                a[i] = x;
                return 1;
            }
            x ^= a[i];
        }
        return 0;
    }
    base &operator+=(const base &o)//合并线性基
    {
        for (T x : o.a) if (x) insert(x);
        return *this;
    }
    base operator+(base o) const { return o += *this; }//合并线性基
    bool contains(T x) const//查询是否能 xor 出 x
    {
        if (x == 0) return 1;
        for (int i = __lg(x); x; i = __lg(x))
        {
            if (!a[i]) return 0;
            x ^= a[i];
        }
        return 1;
    }
    T max(T x = 0) const//查询子集 xor 的最大值。若有传入参数 x，表示子集 xor x 的最大值。
    {
        for (int i = M - 1; i >= 0; i--) if (1 ^ x >> i & 1) x ^= a[i];
        return x;
    }
};
template<class T = ll, int M = sizeof(T) * 8> struct rangebase//[0,...)
{
    vector<array<pair<T, int>, M>> a;
    rangebase() :a{{ }} { }
    rangebase(const vector<T> &b) :a{{ }} { for (T x : b) insert(x); }//直接用一个 vector 构造
    void push_back(T x)//在最后插入 x
    {
        int n = a.size() - 1;
        a.push_back(a.back());
```

```
        if (x == 0) return;
        for (int i = __lg(x); x; i = __lg(x))
        {
            auto &[v, p] = a.back()[i];
            if (v)
            {
                if (n > p)
                {
                    swap(x, v);
                    swap(n, p);
                }
                x ^= v;
            }
            else
            {
                v = x;
                p = n;
                return;
            }
        }
    }
    base<T, M> ask(int l, int r)//查询 $[l,r)$ 元素构成的线性基。下标从 0 开始（同 vector）
    {
        assert(0 <= l && l <= r && r <= a.size());
        base<T, M> res;
        for (int i = 0; i < M; i++)
        {
            auto [v, p] = a[r][i];
            if (v && p >= l) res.a[i] = v;
        }
        return res;
    }
};
```

2.15 splay

指针版。

$O(n)$，$O((n+q)\log n)$。

```
template<class info, class tag> struct splay
{
#define _rev
    struct node
    {
        node *c[2], *f;
        int siz;
        info s, v;
        tag t;
        node() :c{ }, f(0), siz(1), s(), v(), t() { }
        node(info x) :c{ }, f(0), siz(1), s(x), v(x), t() { }
        void operator+=(const tag &o)
        {
            s += o; v += o; t += o;
#ifdef _rev
            if (o.rev) swap(c[0], c[1]);
```

```
#endif
        }
        void pushup()
        {
            if (c[0]) s = c[0]->s + v, siz = c[0]->siz + 1; else s = v, siz = 1;
            if (c[1]) s = s + c[1]->s, siz += c[1]->siz;
        }
        void pushdown()
        {
            for (auto x : c) if (x) *x += t;
            t = { };
        }
        void zigzag()
        {
            node *y = f, *z = y->f;
            int typ = y->c[0] == this;
            if (z) z->c[z->c[1] == y] = this;
            f = z; y->f = this;
            y->c[typ ^ 1] = c[typ];
            if (c[typ]) c[typ]->f = y;
            c[typ] = y;
            y->pushup();
        }
        void splay(node *tar)//不要在 makeroot 以外调用
        {
            for (node *y = f; y != tar; zigzag(), y = f) if (node *z = y->f; z != tar) (z->c[1] ==
                y ^ y->c[1] == this ? this : y)->zigzag();
            pushup();
        }
        void clear()
        {
            for (node *x : c) if (x) x->clear();
            delete this;
        }
    };
    node *rt;
    void debug()
    {
        map<node *, int> id;
        id[0] = 0; id[rt] = 1;
        int cnt = 1;
        function<void(node *)> out = [&](node *x) {
            if (!x) return;
            for (auto y : x->c) if (!id.count(y)) id[y] = ++cnt;
            cerr << id[x] << ' ' << id[x->c[0]] << ' ' << id[x->c[1]] << ' ' << id[x->f] << ' ' <<
                x->siz << '\n';
            for (auto y : x->c) out(y);
        };
        out(rt);
    }
    node *build(info *a, int n)
    {
        if (n == 0) return 0;
        int m = n - 1 >> 1;
        node *x = new node(a[m]);
        x->c[0] = build(a, m);
        x->c[1] = build(a + m + 1, n - 1 - m);
```

```
        for (node *y : x->c) if (y) y->f = x;
        x->pushup();
        return x;
    }
    splay()
    {
        rt = new node;
        rt->c[1] = new node;
        rt->c[1]->f = rt;
        rt->siz = 2;
    }
    int shift;
    splay(info *a, int l, int r)//[l,r)
    {
        shift = l - 1;
        rt = new node;
        rt->c[1] = new node;
        rt->c[1]->f = rt;
        if (l < r)
        {
            rt->c[1]->c[0] = build(a + l, r - l);
            rt->c[1]->c[0]->f = rt->c[1];
        }
        rt->c[1]->pushup();
        rt->pushup();
    }
    void makeroot(node *u, node *tar)
    {
        if (!tar) rt = u;
        u->splay();
    }
    void findnth(int k, node *tar)
    {
        node *x = rt;
        while (1)
        {
            x->pushdown();
            int v = x->c[0] ? x->c[0]->siz : 0;
            if (v + 1 == k) { x->splay(tar); if (!tar) rt = x; return; }
            if (v >= k) x = x->c[0]; else x = x->c[1], k -= v + 1;
        }
    }
    void split(int l, int r)
    {
        assert(1 <= l && r <= rt->siz - 2 && l - 1 <= r);
        findnth(l, 0);
        findnth(r + 2, rt);
    }
#ifdef _rev
    void reverse(int l, int r)
    {
        l -= shift; r -= shift + 1;
        if (l - 1 == r) return;
        assert(1 <= l && l <= r && r <= rt->siz - 2);
        split(l, r);
        *(rt->c[1]->c[0]) += tag(1);
    }
```

```
#endif
    void insert(int pos, info x)//insert before pos
    {
        pos -= shift;
        assert(1 <= pos && pos <= rt->siz - 1);
        split(pos, pos - 1);
        rt->c[1]->c[0] = new node(x);
        rt->c[1]->c[0]->f = rt->c[1];
        rt->c[1]->pushup();
        rt->pushup();
    }
    void insert(int pos, info *a, int n)//insert before pos, [1,n]
    {
        pos -= shift;
        assert(1 <= pos && pos <= rt->siz - 1);
        split(pos, pos - 1);
        rt->c[1]->c[0] = build(a, n);
        rt->c[1]->c[0]->f = rt->c[1];
        rt->c[1]->pushup();
        rt->pushup();
    }
    void erase(int pos)
    {
        pos -= shift;
        assert(1 <= pos && pos <= rt->siz - 2);
        split(pos, pos);
        delete rt->c[1]->c[0];
        rt->c[1]->c[0] = 0;
        rt->c[1]->pushup();
        rt->pushup();
    }
    void erase(int l, int r)
    {
        l -= shift; r -= shift + 1;
        if (l - 1 == r) return;
        assert(1 <= l && l <= r && r <= rt->siz - 2);
        split(l, r);
        rt->c[1]->c[0]->clear();
        rt->c[1]->c[0] = 0;
        rt->c[1]->pushup();
        rt->pushup();
    }
    void modify(int pos, info x)//not checked
    {
        pos -= shift;
        assert(1 <= pos && pos <= rt->siz - 2);
        findnth(pos + 1, 0);
        rt->v = x; rt->pushup();
    }
    void modify(int l, int r, tag w)
    {
        l -= shift; r -= shift + 1;
        if (l - 1 == r) return;
        assert(1 <= l && l <= r && r <= rt->siz - 2);
        split(l, r);
        node *x = rt->c[1]->c[0];
        *x += w;
```

```
        rt->c[1]->pushup();
        rt->pushup();
    }
    info ask(int l, int r)
    {
        l -= shift; r -= shift + 1;
        assert(1 <= l && l <= r && r <= rt->siz - 2);
        split(l, r);
        return rt->c[1]->c[0]->s;
    }
    ~splay() { rt->clear(); }
#undef _rev
};
struct Q
{
    bool rev;
    Q() :rev(0) { }
    Q(bool c) :rev(c) { }
    void operator+=(const Q &o)
    {
        rev ^= o.rev;
    }
};
struct P
{
    ll s;
    void operator+=(const Q &o) const
    { }
    P operator+(const P &o) const { return{s + o.s}; }
};
```

2.16 第 k 大线性基

查询第 k 大/小，即 k-th $\min\limits_{\varnothing\neq T\subset S} r\oplus\bigoplus\limits_{x\in T} x$。

不允许选择空集，如果允许只需要将 `con` 设置为 1。

$O((n+q)\log a)$，$O(\log a)$。

```
const int M = 50;
void ins(ull x)
{
    for (int i = M - 1; x; i--) if (x >> i & 1)
    {
        if (!ji[i]) { ji[i] = x; i = -1; break; }x ^= ji[i];
    }
    if (!x) con = 1;
}
ull kmax(ull x, ull r = 0)
{
    static int a[M];
    int m = 0, i;
    for (i = M - 1; ~i; i--) if (ji[i]) a[++m] = i;
    if (1ll << m <= x - con) return -1;
    x = (1ll << m) - x;
    for (i = 1; i <= m; i++) if ((x >> m - i ^ r >> a[i]) & 1) r ^= ji[a[i]];
    return r;
}
```

```
ull kmin(ull x, ull r = 0)
{
    static int a[M];
    int m = 0, i;
    for (i = M - 1; ~i; i--) if (ji[i]) a[++m] = i;
    x -= con;
    if (1ll << m <= x) return -1;
    for (i = 1; i <= m; i++) if ((x >> m - i ^ r >> a[i]) & 1) r ^= ji[a[i]];
    return r;
}
```

2.17 fhq-treap

洛谷模板：普通平衡树。

$O((n+q)\log n)$，$O(n)$。

```
const int N = 1.1e6 + 2;
int c[N][2], v[N], w[N], s[N];
int n, i, x, y, ds, val, kth, p, q, z, rt, la, m, ans;
void pushup(const int x)
{
    s[x] = s[c[x][0]] + s[c[x][1]] + 1;
}
void split_val(int now, int &x, int &y)//调用外部val,相等归入y
{
    if (!now) return x = y = 0, void();
    if (val <= v[now]) split_val(c[y = now][0], x, c[now][0]);
    else split_val(c[x = now][1], c[now][1], y);
    pushup(now);
}
void split_kth(int now, int &x, int &y)//调用外部kth，左子树大小为 kth
{
    if (!now) return x = y = 0, void();
    if (kth <= s[c[now][0]]) split_kth(c[y = now][0], x, c[now][0]);
    else kth -= s[c[now][0]] + 1, split_kth(c[x = now][1], c[now][1], y);
    pushup(now);
}
int merge(int x, int y)//小根ver.
{
    if (!(x && y)) return x | y;
    if (w[x] < w[y]) { c[x][1] = merge(c[x][1], y); pushup(x); return x; }
    else { c[y][0] = merge(x, c[y][0]); pushup(y); return y; }
}
int main()
{
    cin>>n>>m; srand(998244353);
    for (i = 1; i <= n; i++)
    {
        cin >> x;
        val = v[++ds] = x;
        w[ds] = rand();
        s[ds] = 1;
        split_val(rt, p, q);
        rt = merge(merge(p, ds), q);
    }
    while (m--)
```

```
    {
        cin >> y >> x;
        x ^= la;
        if (y == 4)//找到第 x 小的
        {
            kth = x; split_kth(rt, p, q); x = p;
            while (c[x][1]) x = c[x][1];
            ans ^= (la = v[x]); rt = merge(p, q);
            continue;
        }
        val = x;//注意这一步
        if (y == 1)//插入 x
        {
            v[++ds] = x; w[ds] = rand(); s[ds] = 1;
            split_val(rt, p, q); rt = merge(merge(p, ds), q);
            continue;
        }
        if (y == 2)//删除一个 x
        {
            split_val(rt, p, q); kth = 1; split_kth(q, i, z);
            rt = merge(p, z); continue;
        }
        if (y == 3)//询问 x 的排名（比 x 小的数字个数 +1）
        {
            split_val(rt, p, q); ans ^= (la = s[p] + 1);
            rt = merge(p, q); continue;
        }
        if (y == 5)//询问比 x 小的最大值
        {
            split_val(rt, p, q); x = p;
            while (c[x][1]) x = c[x][1]; ans ^= (la = v[x]);
            rt = merge(p, q); continue;
        }
        ++val; split_val(rt, p, q); x = q;//询问比 x 大的最小值
        while (c[x][0]) x = c[x][0];
        ans ^= (la = v[x]); rt = merge(p, q);
    }
    cout<<ans<<endl;
}
```

2.18 笛卡尔树的线性建树

$p[1, 2, \ldots, n]$ 是原序列，c 表示子结点。

笛卡尔树满足堆性质（权值小于等于子结点权值），并且中序遍历是原序列。

$O(n)$，$O(n)$。

```
int c[N][2], p[N], st[N];
int main()
{
    int i, n, tp = 0;
    cin >> n;
    for (i = 1; i <= n; i++)
    {
        cin >> p[i]; st[tp + 1] = 0;
        while ((tp) && (p[st[tp]] > p[i])) --tp;
        c[c[st[tp]][1] = i][0] = st[tp + 1]; st[++tp] = i;
```

```
    }
}
```

2.19 扫描线

求矩形并的面积和周长（包括内周长）

$O((n+q)\log n)$，$O(n+q)$。

```
using T = ll;
vector<T> fun(vector<tuple<T, T, T, T>> &a)
{
    vector<T> x;
    for (auto [x1, y1, x2, y2] : a)
    {
        x.push_back(x1);
        x.push_back(x2);
    }
    sort(all(x)); x.resize(unique(all(x)) - x.begin());
    for (auto &[x1, y1, x2, y2] : a)
    {
        x1 = lower_bound(all(x), x1) - x.begin();
        x2 = lower_bound(all(x), x2) - x.begin();
    }
    return x;
}
struct sgt
{
    int n, z, y, d;
    vector<T> cnt, &p;
    vector<int> mn, lz;
    void build(int x, int l, int r)
    {
        cnt[x] = p[min(r, n - 1)] - p[l];
        if (l + 1 == r) return;
        int c = x * 2, m = l + r >> 1;
        build(c, l, m); build(c + 1, m, r);
    }
    sgt(vector<T> &p) :n(p.size()), p(p), cnt(n * 4), mn(n * 4), lz(n * 4) { build(1, 0, n); }
    void dfs(int x, int l, int r)
    {
        if (z <= l && r <= y)
        {
            mn[x] += d;
            lz[x] += d;
            return;
        }
        int c = x * 2, m = l + r >> 1;
        if (lz[x])
        {
            lz[c] += lz[x]; lz[c + 1] += lz[x];
            mn[c] += lz[x]; mn[c + 1] += lz[x];
            lz[x] = 0;
        }
        if (z < m) dfs(c, l, m);
        if (m < y) dfs(c + 1, m, r);
        mn[x] = min(mn[c], mn[c + 1]);
```

```
        cnt[x] = cnt[c] * (mn[x] == mn[c]) + cnt[c + 1] * (mn[x] == mn[c + 1]);
    }
    void modify(int l, int r, int dt)
    {
        z = l;
        y = r;
        d = dt;
        dfs(1, 0, n);
    }
};
T area(vector<tuple<T, T, T, T>> a)//[x1,y1,x2,y2], x1<y1, x2<y2
{
    int n = a.size(), i;
    auto X = fun(a);
    vector<tuple<T, int, T, T>> b(n * 2);
    for (i = 0; i < n; i++)
    {
        auto [x1, y1, x2, y2] = a[i];
        b[i] = {y1, -1, x1, x2};
        b[i + n] = {y2, 1, x1, x2};
    }
    sort(all(b), greater<>());
    sgt s(X);
    T lst = 0, ans = 0;
    for (auto [y, d, l, r] : b)
    {
        ans += (lst - y) * (X.back() - X[0] - s.cnt[1]);
        s.modify(l, r, d);
        lst = y;
    }
    return ans;
}
T perimeter_x(vector<tuple<T, T, T, T>> a)
{
    int n = a.size(), i;
    auto X = fun(a);
    vector<tuple<T, int, T, T>> b(n * 2);
    for (i = 0; i < n; i++)
    {
        auto [x1, y1, x2, y2] = a[i];
        b[i] = {y1, -1, x1, x2};
        b[i + n] = {y2, 1, x1, x2};
    }
    sort(all(b), greater<>());
    sgt s(X);
    T lst = s.cnt[1], ans = 0;
    for (auto [y, d, l, r] : b)
    {
        s.modify(l, r, d);
        T cur = s.cnt[1];
        ans += abs(lst - cur);
        lst = cur;
    }
    return ans;
}
T perimeter(vector<tuple<T, T, T, T>> a)//[x1,y1,x2,y2], x1<y1, x2<y2
{
```

```
    T ansx = perimeter_x(a);
    for (auto &[x1, y1, x2, y2] : a)
    {
        swap(x1, y1);
        swap(x2, y2);
    }
    T ansy = perimeter_x(a);
    return ansx + ansy;
}
```

2.20 Segmenttree Beats!

核心是 P（tag）和 Q（info）的维护。线段树部分是套的模板，并非全都有用。

1. l, r, k：对于所有的 $i \in [l, r]$，将 A_i 加上 k（k 可以为负数）。
2. l, r, v：对于所有的 $i \in [l, r]$，将 A_i 变成 $\min(A_i, v)$。
3. l, r：求 $\sum\limits_{i=l}^{r} A_i$。
4. l, r：对于所有的 $i \in [l, r]$，求 A_i 的最大值。
5. l, r：对于所有的 $i \in [l, r]$，求 B_i 的最大值。

其中 B_i 是 A_i 的历史最大值。

```
struct P
{
    ll tg, L, R;
    P(ll a = 0, ll b = -inf, ll c = inf) :tg(a), L(b), R(c) { }
    void operator+=(P o)
    {
        o.L -= tg; o.R -= tg; tg += o.tg;
        if (L >= o.R) L = R = o.R;
        else if (R <= o.L) L = R = o.L;
        else cmax(L, o.L), cmin(R, o.R);
    }
};
struct Q
{
    ll mx0, cmx, mx1, mn0, cmn, mn1, cnt, sum;
    Q() :mx0(-inf), cmx(0), mx1(-inf), mn0(inf), cmn(0), mn1(inf), cnt(0), sum(0) { }
    Q(ll x) :mx0(x), cmx(1), mx1(-inf), mn0(x), cmn(1), mn1(inf), cnt(1), sum(x) { }
    bool operator+=(const P &o)
    {
        if (o.L == o.R)
        {
            ll c = cnt;
            *this = Q(o.L + o.tg);
            cnt = cmx = cmn = c;
            sum = cnt * (o.L + o.tg);
            return 1;
        }
        if (o.L >= mn1 || o.R <= mx1) return 0;
        if (mx0 == mn0)
        {
```

```
            mn0 = min(o.R, max(mx0, o.L));
            sum += cnt * (mn0 - mx0);
            mx0 = mn0;
        }
        else
        {
            if (o.L > mn0)
            {
                sum += (o.L - mn0) * cmn;
                mn0 = o.L;
                cmax(mx1, o.L);
            }
            if (o.R < mx0)
            {
                sum += (o.R - mx0) * cmx;
                mx0 = o.R;
                cmin(mn1, o.R);
            }
        }
        if (o.tg)
        {
            sum += o.tg * cnt;
            mx0 += o.tg;
            mx1 += o.tg;
            mn0 += o.tg;
            mn1 += o.tg;
        }
        return 1;
    }
};
Q operator+(const Q &a, const Q &b)
{
    Q res;
    res.sum = a.sum + b.sum;
    res.cnt = a.cnt + b.cnt;
    res.mx0 = max(a.mx0, b.mx0);
    res.mx1 = max(a.mx1, b.mx1);
    if (res.mx0 == a.mx0) res.cmx += a.cmx; else cmax(res.mx1, a.mx0);
    if (res.mx0 == b.mx0) res.cmx += b.cmx; else cmax(res.mx1, b.mx0);

    res.mn0 = min(a.mn0, b.mn0);
    res.mn1 = min(a.mn1, b.mn1);
    if (res.mn0 == a.mn0) res.cmn += a.cmn; else cmin(res.mn1, a.mn0);
    if (res.mn0 == b.mn0) res.cmn += b.cmn; else cmin(res.mn1, b.mn0);

    return res;
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, q, i;
    cin >> n >> q;
    vector<ll> a(n);
    cin >> a;
    sgt<Q, P> s(a.data(), 0, n - 1);
    while (q--)
```

```
    {
        int op, l, r;
        cin >> op >> l >> r;
        --r;
        if (op == 3)
        {
            ll res = s.ask(l, r).sum;
            cout << res << '\n';
        }
        else
        {
            ll b;
            cin >> b;
            if (op == 0) s.modify(l, r, {0, -inf, b});
            else if (op == 1) s.modify(l, r, {0, b});
            else s.modify(l, r, {b});
        }
    }
}
```

2.21 k-d 树（二进制分组）

均摊 $O(\log^2 n)$ 插入，$O(\sqrt{n})$ 矩形查询。

```
#define tmpl template<class T>
using ll = long long;
tmpl struct P
{
    ll x, y;
    T v;
};
tmpl struct Q
{
    ll x[2], y[2];
    bool t;
    T s;
    Q() { }
    Q(const P<T> &a)
    {
        x[0] = x[1] = a.x;
        y[0] = y[1] = a.y;
        s = a.v;
    }
};
tmpl bool cmp0(const P<T> &a, const P<T> &b) { return a.x < b.x; }
tmpl bool cmp1(const P<T> &a, const P<T> &b) { return a.y < b.y; }
tmpl struct kdt
{
    vector<P<T>> c;
    vector<Q<T>> a;
    ll m, u, d, l, r;
    T ans;
    bool fir;
    void build(int x, P<T> *b, int n)
    {
        if (x == 1)
        {
```

```
            a.resize(m = n << 1);
            a[x].t = 0;
            c.resize(n);
            for (int i = 0; i < n; i++) c[i] = b[i];
        }
        if (n == 1)
        {
            a[x] = Q<T>(b[0]);
            return;
        }
        int mid = n >> 1, c = x << 1;
        nth_element(b, b + mid, b + n, a[x].t ? cmp1<T> : cmp0<T>);
        a[c].t = a[c | 1].t = a[x].t ^ 1;
        build(c, b, mid);
        build(c | 1, b + mid, n - mid);
        a[x].s = a[c].s + a[c | 1].s;
        a[x].x[0] = min(a[c].x[0], a[c | 1].x[0]);
        a[x].x[1] = max(a[c].x[1], a[c | 1].x[1]);
        a[x].y[0] = min(a[c].y[0], a[c | 1].y[0]);
        a[x].y[1] = max(a[c].y[1], a[c | 1].y[1]);
    }
    void find(int x)
    {
        if (x >= m || a[x].x[1]<u || a[x].x[0]>d || a[x].y[1]<l || a[x].y[0]>r) return;
        if (u <= a[x].x[0] && a[x].x[1] <= d && l <= a[x].y[0] && a[x].y[1] <= r)
        {
            ans = fir ? a[x].s : ans + a[x].s;
            fir = 0;
            return;
        }
        find(x << 1); find(x << 1 | 1);
    }
    pair<bool, T> find(ll x1, ll y1, ll x2, ll y2)
    {
        fir = 1;
        ans = { };
        u = x1; d = x2;
        l = y1; r = y2;
        find(1);
        return {!fir, ans};
    }
};
const int N = 2e5 + 2, M = 18;
tmpl struct KDT
{
    kdt<T> s[M];
    P<T> a[N];
    int n, m, i;
    KDT() { n = 0; }
    KDT(int N, ll *x, ll *y, T *w)//[0,n)
    {
        n = N;
        int i, j;
        for (i = 0; i < n; i++) a[i] = {x[i], y[i], w[i]};
        for (i = j = 0; n >> i; i++) if (n >> i & 1) s[i].build(1, a + j, 1 << i), j += 1 << i;
    }
    void insert(ll x, ll y, T w)//插入 (x,y) 的一个数 w
```

```
    {
        a[0] = {x, y, w}; m = 1;
        for (i = 0; n & 1 << i; i++) for (auto u : s[i].c) a[m++] = u;
        s[i].build(1, a, m);
        ++n;
    }
    pair<bool, T> ask(ll x, ll y, ll xx, ll yy)//查询 [x,xx]*[y,yy] 的和
    {
        T ans;
        bool fir = 1;
        for (i = 0; 1 << i <= n; i++) if (1 << i & n)
        {
            auto [_, tmp] = s[i].find(x, y, xx, yy);
            if (!_) continue;
            ans = fir ? tmp : ans + tmp;
            fir = 0;
        }
        return {!fir, ans};
    }
};
int x[N], y[N], w[N];
int main()
{
    ios::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    int n, q, i;
    cin >> n >> q;
    for (i = 0; i < n; i++) cin >> x[i] >> y[i] >> w[i];
    KDT<ll> s(n, x, y, w);
    while (q--)
    {
        int op, x, y, w;
        cin >> op >> x >> y >> w;
        if (op == 0) s.insert(x, y, w); else
        {
            cin >> op;
            cout << s.ask(x, y, w - 1, op - 1) << '\n';
        }
    }
    return 0;
}
```

2.22 双端队列全局查询

对一个支持结合律的信息 T，维护 deque 内信息的和。总复杂度线性。

```
template<class T> struct dq
{
    vector<T> l, sl, r, sr;
    void push_front(const T &o)
    {
        sl.push_back(sl.size() ? o + sl.back() : o);
        l.push_back(o);
    }
    void push_back(const T &o)
    {
        sr.push_back(sr.size() ? sr.back() + o : o);
        r.push_back(o);
```

```
    }
    void pop_front()
    {
        if (l.size()) sl.pop_back(), l.pop_back();
        else
        {
            assert(r.size());
            int n = r.size(), m, i;
            if (m = n - 1 >> 1)
            {
                l.resize(m); sl.resize(m);
                for (i = 1; i <= m; i++) l[m - i] = r[i];
                sl[0] = l[0];
                for (i = 1; i < m; i++) sl[i] = l[i] + sl[i - 1];
            }
            for (i = m + 1; i < n; i++) r[i - (m + 1)] = r[i];
            m = n - (m + 1);
            r.resize(m); sr.resize(m);
            if (m)
            {
                sr[0] = r[0];
                for (i = 1; i < m; i++) sr[i] = sr[i - 1] + r[i];
            }
        }
    }
    void pop_back()
    {
        if (r.size()) sr.pop_back(), r.pop_back();
        else
        {
            assert(l.size());
            int n = l.size(), m, i;
            if (m = n - 1 >> 1)
            {
                r.resize(m); sr.resize(m);
                for (i = 1; i <= m; i++) r[m - i] = l[i];
                sr[0] = r[0];
                for (i = 1; i < m; i++) sr[i] = sr[i - 1] + r[i];
            }
            for (i = m + 1; i < n; i++) l[i - (m + 1)] = l[i];
            m = n - (m + 1);
            l.resize(m); sl.resize(m);
            if (m)
            {
                sl[0] = l[0];
                for (i = 1; i < m; i++) sl[i] = l[i] + sl[i - 1];
            }
        }
    }
    template<class TT> TT ask(TT r)
    {
        if (sl.size()) r = r + sl.back();
        if (sr.size()) r = r + sr.back();
        return r;
    }
    T ask()
    {
```

```
        assert(sl.size() || sr.size());
        if (sl.size() && sr.size()) return sl.back() + sr.back();
        return sl.size() ? sl.back() : sr.back();
    }
};
```

2.23 静态矩形加矩形和

```
const ull p = 998244353;
struct Q
{
    int n, m;
    ull w;
    int typ;
    bool operator<(const Q &o) const
    {
        if (n != o.n) return n < o.n;
        return typ < o.typ;
    }
};
template<class T> struct tork
{
    vector<T> a;
    int n;
    tork(const vector<T> &b) :a(all(b))
    {
        sort(all(a));
        a.resize(unique(all(a)) - a.begin());
        n = a.size();
    }
    tork(const T *first, const T *last) :a(first, last)
    {
        sort(all(a));
        a.resize(unique(all(a)) - a.begin());
        n = a.size();
    }
    void get(T &x) { x = lower_bound(all(a), x) - a.begin() + 1; }
    T operator[](const int &x) { return a[x]; }
};
struct bit
{
    vector<ull> a;
    int n;
    bit() { }
    bit(int nn) :n(nn), a(nn + 1) { }
    template<class T> bit(int nn, T *b) : n(nn), a(nn + 1)
    {
        for (int i = 1; i <= n; i++) a[i] = b[i];
        for (int i = 1; i <= n; i++) if (i + (i & -i) <= n) a[i + (i & -i)] += a[i];
    }
    void add(int x, ull y)
    {
        // cerr<<"add "<<x<<" by "<<y<<endl;
        assert(1 <= x && x <= n);
        if ((a[x] += y) >= p) a[x] -= p;
        while ((x += x & -x) <= n) if ((a[x] += y) >= p) a[x] -= p;
```

```
    }
    ull sum(int x)
    {
        // cerr<<"sum "<<x;
        assert(0 <= x && x <= n);
        ull r = a[x];
        while (x ^= x & -x) r += a[x];
        // cerr<<"= "<<r<<endl;
        return r % p;
    }
    ull sum(int x, int y)
    {
        return (sum(y) + p - sum(x - 1)) % p;
    }
};
struct matrix
{
    int l, d, r, u;
    ull w;
};
vector<ull> rec_add_rec_sum(const vector<matrix> &op, const vector<matrix> &query)
{
    vector<Q> a[4];
    int n = op.size(), m = query.size(), i;
    for (auto &v : a) v.reserve(n + m << 2);
    for (auto [l, d, r, u, w] : op)//[l,r)*[d,u) += w
    {
        a[0].push_back({l, d, w * l % p * d % p, -1});
        a[1].push_back({l, d, w * l % p, -1});
        a[2].push_back({l, d, w * d % p, -1});
        a[3].push_back({l, d, w, -1});
        w = (p - w) % p;
        a[0].push_back({l, u, w * l % p * u % p, -1});
        a[1].push_back({l, u, w * l % p, -1});
        a[2].push_back({l, u, w * u % p, -1});
        a[3].push_back({l, u, w, -1});
        a[0].push_back({r, d, w * r % p * d % p, -1});
        a[1].push_back({r, d, w * r % p, -1});
        a[2].push_back({r, d, w * d % p, -1});
        a[3].push_back({r, d, w, -1});
        w = (p - w) % p;
        a[0].push_back({r, u, w * r % p * u % p, -1});
        a[1].push_back({r, u, w * r % p, -1});
        a[2].push_back({r, u, w * u % p, -1});
        a[3].push_back({r, u, w, -1});
    }
    i = 0;
    for (auto [l, d, r, u, w] : query)//ask sum of [l,r)*[d,u)
    {
        a[0].push_back({l, d, 1, i});
        a[1].push_back({l, d, (p * 2 - d) % p, i});
        a[2].push_back({l, d, (p * 2 - l) % p, i});
        a[3].push_back({l, d, (ull)l * d % p, i});
        a[0].push_back({l, u, p - 1, i});
        a[1].push_back({l, u, u % p, i});
        a[2].push_back({l, u, l % p, i});
        a[3].push_back({l, u, (p * 2 - l) * u % p, i});
```

```
        a[0].push_back({r, u, 1, i});
        a[1].push_back({r, u, (p * 2 - u) % p, i});
        a[2].push_back({r, u, (p * 2 - r) % p, i});
        a[3].push_back({r, u, (ull)u * r % p, i});
        a[0].push_back({r, d, p - 1, i});
        a[1].push_back({r, d, d % p, i});
        a[2].push_back({r, d, r % p, i});
        a[3].push_back({r, d, (p * 2 - d) * r % p, i});
        ++i;
    }
    assert(a[0].size() == n + m << 2);
    vector<ull> ans(m);
    auto cal = [&](vector<Q> a) {
        int n = a.size(), i;
        vector<int> b(n);
        for (i = 0; i < n; i++) b[i] = (a[i].m -= a[i].typ >= 0), a[i].n -= a[i].typ >= 0;
        sort(all(a));
        tork t(b);
        for (i = 0; i < n; i++) t.get(a[i].m);
        int m = t.a.size();
        bit s(m);
        for (auto [n, m, w, typ] : a) if (typ >= 0) ans[typ] = (ans[typ] + s.sum(m) * w) % p; else
            s.add(m, w);
    };
    for (auto &v : a) cal(v);
    return ans;
}
```

2.24 线段树分裂

```
namespace sgt
{
#define ask_kth
    int L = 0, R = 1e9;
    void set_bound(int l, int r) { L = l; R = r; }
    typedef ll info;
    const info E = 0;//找不到会返回 E
    const int N = 8e6 + 5;
#define lc(x) (a[x].lc)
#define rc(x) (a[x].rc)
#define s(x) (a[x].s)
    struct node
    {
        int lc, rc;
        info s;
    };
    node a[N];
    vector<int> id;
    int ids = 0, pos, z, y;
    bool fir;
    info tmp;
    int npt()
    {
        int x;
        if (id.size()) x = id.back(), id.pop_back();
        else x = ++ids;
```

```
    lc(x) = rc(x) = 0;
    return x;
}
void pushup(int &x)
{
    if (lc(x) && rc(x)) s(x) = s(lc(x)) + s(rc(x));
    else if (lc(x)) s(x) = s(lc(x));
    else if (rc(x)) s(x) = s(rc(x));
    else id.push_back(x), x = 0;
}
void insert(int &x, int l, int r)
{
    if (l == r)
    {
        if (!x) x = npt(), s(x) = tmp;
        else s(x) = s(x) + tmp;
        return;
    }
    if (!x) x = npt();
    int mid = l + r >> 1;
    if (pos <= mid)
    {
        insert(lc(x), l, mid);
        if (rc(x)) s(x) = s(lc(x)) + s(rc(x)); else s(x) = s(lc(x));
    }
    else
    {
        insert(rc(x), mid + 1, r);
        if (lc(x)) s(x) = s(lc(x)) + s(rc(x)); else s(x) = s(rc(x));
    }
}
void modify(int &x, int l, int r)
{
    if (!x) x = npt();
    if (l == r)
    {
        s(x) = tmp;
        return;
    }
    int mid = l + r >> 1;
    if (pos <= mid)
    {
        insert(lc(x), l, mid);
        if (rc(x)) s(x) = s(lc(x)) + s(rc(x)); else s(x) = s(lc(x));
    }
    else
    {
        insert(rc(x), mid + 1, r);
        if (lc(x)) s(x) = s(lc(x)) + s(rc(x)); else s(x) = s(rc(x));
    }
}
int merge(int x1, int x2, int l, int r)
{
    if (!(x1 && x2)) return x1 | x2;
    if (l == r) { s(x1) = s(x1) + s(x2); return x1; }
    int mid = l + r >> 1;
    lc(x1) = merge(lc(x1), lc(x2), l, mid);
```

```
        rc(x1) = merge(rc(x1), rc(x2), mid + 1, r);
        pushup(x1);
        return x1;
    }
    void ask(int x, int l, int r)
    {
        if (!x) return;
        if (z <= l && r <= y)
        {
            if (fir) tmp = s(x), fir = 0; else tmp = tmp + s(x);
            return;
        }
        int mid = l + r >> 1;
        if (z <= mid) ask(lc(x), l, mid);
        if (y > mid) ask(rc(x), mid + 1, r);
    }
    void split(int &x1, int &x2, int l, int r)
    {
        assert(!x1);
        if (!x2) return;
        if (z <= l && r <= y) { x1 = x2; x2 = 0; return; }
        x1 = npt();
        int mid = l + r >> 1;
        if (z <= mid) split(lc(x1), lc(x2), l, mid);
        if (y > mid) split(rc(x1), rc(x2), mid + 1, r);
        pushup(x1); pushup(x2);
    }
    info *b;
    void build(int &x, int l, int r)
    {
        x = npt();
        if (l == r) { s(x) = b[l]; return; }
        int mid = l + r >> 1;
        build(lc(x), l, mid); build(rc(x), mid + 1, r);
        s(x) = s(lc(x)) + s(rc(x));
    }
    struct set
    {
        int rt;
        set() :rt(0) { }
        set(info *a) :rt(0) { b = a; build(rt, L, R); }
        void modify(int p, const info &o) { pos = p; tmp = o; sgt::modify(rt, L, R); }
        void insert(int p, const info &o) { pos = p; tmp = o; sgt::insert(rt, L, R); }
        void join(const set &o) { rt = merge(rt, o.rt, L, R); }
        info ask(int l, int r)
        {
            z = l; y = r; fir = 1;
            sgt::ask(rt, L, R);
            return fir ? E : tmp;
        }
        set split(int l, int r)
        {
            z = l; y = r; set p;
            sgt::split(p.rt, rt, L, R);
            return p;
        }
#ifdef ask_kth
```

```
        int kth(info k)
        {
            int x = rt, l = L, r = R, mid;
            if (k > s(x)) return -1;
            s(0) = 0;
            while (l < r)
            {
                mid = l + r >> 1;
                if (s(lc(x)) >= k) x = lc(x), r = mid;
                else k -= s(lc(x)), x = rc(x), l = mid + 1;
            }
            return l;
        }
#endif
    };
#undef lc
#undef rc
#undef s
}
typedef sgt::set tree;
```

2.25 手写 bitset

```
struct Bitset
{
    using ull = unsigned long long;
#define all(x) (x).begin(),(x).end()
    const static ull B = -1llu;
    int n;
    vector<ull> a;
    Bitset() { }
    Bitset(int n) :n(n), a(n + 63 >> 6) { assert(n); }
    bool test(int x) const { assert(x >= 0 && x < n); return a[x >> 6] >> (x & 63) & 1; }
    bool operator[](int x) const { return test(x); }
    void set(int x, bool y) { assert(x >= 0 && x < n); a[x >> 6] = (a[x >> 6] & (B ^ 1llu << (x &
        63))) | ((ull)y << (x & 63)); }
    void set(int x) { assert(x >= 0 && x < n); a[x >> 6] |= 1llu << (x & 63); }
    void set() { memset(a.data(), 0xff, a.size() * sizeof a[0]); if (n & 63) a.back() &= (1llu <<
        (n & 63)) - 1; }
    void reset(int x) { assert(x >= 0 && x < n); a[x >> 6] &= ~(1llu << (x & 63)); }
    void reset() { memset(a.data(), 0, a.size() * sizeof a[0]); }
    int count() const
    {
        int r = 0;
        for (ull x : a) r += __builtin_popcountll(x);
        return r;
    }
    int count(int l, int r) const//[l,r)
    {
        if (l == r) return 0;
        if (l >> 6 == r >> 6) return __builtin_popcountll(a[l >> 6] >> (l & 63) & (1llu << r - l)
            - 1);
        int ans = 0;
        ans += __builtin_popcountll(a[l >> 6] >> (l & 63));
        ++(l >>= 6);
        if (r & 63) ans += __builtin_popcountll(a[r >> 6] & (1llu << (r & 63)) - 1);
```

```
        r >>= 6;
        while (l < r) ans += __builtin_popcountll(a[l++]);
        return ans;
    }
    Bitset &operator|=(const Bitset &o)
    {
        assert(n == o.n);
        for (int i = 0; i < a.size(); i++) a[i] |= o.a[i];
        return *this;
    }
    Bitset operator|(Bitset o) { o |= *this; return o; }
    Bitset &operator&=(const Bitset &o)
    {
        assert(n == o.n);
        for (int i = 0; i < a.size(); i++) a[i] &= o.a[i];
        return *this;
    }
    Bitset operator&(Bitset o) { o &= *this; return o; }
    Bitset &operator^=(const Bitset &o)
    {
        assert(n == o.n);
        for (int i = 0; i < a.size(); i++) a[i] ^= o.a[i];
        return *this;
    }
    Bitset operator^(Bitset o) { o ^= *this; return o; }
    Bitset operator~() const
    {
        auto r = *this;
        for (ull &x : r.a) x = ~x;
        if (n & 63) r.a.back() &= (1ull << (n & 63)) - 1;
        return r;
    }
    Bitset &operator<<=(int x)
    {
        if (x >= n) return reset(), *this;
        assert(x >= 0);
        int y = x >> 6;
        x &= 63;
        if (x == 0)
        {
            for (int i = (int)a.size() - 1; i >= y; i--) a[i] = a[i - y];
            if (n & 63) a.back() &= (1llu << (n & 63)) - 1;
            memset(a.data(), 0, y * sizeof a[0]);
            return *this;
        }
        for (int i = (int)a.size() - 1; i > y; i--) a[i] = a[i - y] << x | a[i - y - 1] >> 64 - x;
        a[y] = a[0] << x;
        memset(a.data(), 0, y * sizeof a[0]);
        // fill_n(a.begin(),y,0);
        if (n & 63) a.back() &= (1llu << (n & 63)) - 1;
        return *this;
    }
    Bitset operator<<(int x)
    {
        auto r = *this;
        r <<= x;
        return r;
```

```
}
Bitset &operator>>=(int x)
{
    if (x >= n) return reset(), *this;
    assert(x >= 0);
    int y = x >> 6, R = (int)a.size() - y - 1;
    x &= 63;
    if (x == 0)
    {
        for (int i = 0; i <= R; i++) a[i] = a[i + y];
        memset(a.data() + R + 1, 0, y * sizeof a[0]);
        return *this;
    }
    for (int i = 0; i < R; i++) a[i] = a[i + y] >> x | a[i + y + 1] << 64 - x;
    a[R] = a.back() >> x;
    memset(a.data() + R + 1, 0, y * sizeof a[0]);
    return *this;
}
Bitset operator>>(int x)
{
    auto r = *this;
    r >>= x;
    return r;
}
void range_set(int l, int r)//[l,r) to 1
{
    if (l == r) return;
    if (l >> 6 == r >> 6)
    {
        a[l >> 6] |= (1llu << r - l) - 1 << (l & 63);
        return;
    }
    if (l & 63)
    {
        a[l >> 6] |= ~((1llu << (l & 63)) - 1);//[l&63,64)
        l += 64;
    }
    if (r & 63) a[r >> 6] |= (1llu << (r & 63)) - 1;
    l >>= 6; r >>= 6;
    memset(a.data() + l, 0xff, (r - l) * sizeof a[0]);
}
void range_reset(int l, int r)//[l,r) to 0
{
    if (l == r) return;
    if (l >> 6 == r >> 6)
    {
        a[l >> 6] &= ~((1llu << r - l) - 1 << (l & 63));
        return;
    }
    if (l & 63)
    {
        a[l >> 6] &= (1llu << (l & 63)) - 1;
        l += 64;
    }
    if (r & 63) a[r >> 6] &= ~((1llu << (r & 63)) - 1);
    l >>= 6; r >>= 6;
    memset(a.data() + l, 0, (r - l) * sizeof a[0]);
```

```
    }
    void range_set(int l, int r, bool x)//[l,r)
    {
        if (x) range_set(l, r);
        else range_reset(l, r);
    }
    int size() const { return n; }
    int _Find_first() const
    {
        for (int i = 0; i < a.size(); i++) if (a[i]) return i * 64 + __lg(a[i] & -a[i]);
        return n;
    }
    int _Find_next(int x) const
    {
        assert(x >= 0 && x < n);
        ++x;
        if (x == n) return n;
        int y = x & 63; x >>= 6;
        if (a[x] >> y) return x * 64 + __lg(a[x] >> y & -(a[x] >> y)) + y;
        ++x;
        while (x < a.size() && !a[x]) ++x;
        return x == a.size() ? n : x * 64 + __lg(a[x] & -a[x]);
    }
    int _Find_last() const
    {
        for (int i = a.size() - 1; i >= 0; i--) if (a[i]) return i * 64 + __lg(a[i]);
        return -1;
    }
    int _Find_prev(int x) const
    {
        assert(x >= 0 && x < n);
        --x;
        if (x == -1) return -1;
        int y = x & 63; x >>= 6;
        if (y < 63)
        {
            if (a[x] & (1llu << y + 1) - 1) return x * 64 + __lg(a[x] & (1llu << y + 1) - 1);
            --x;
        }
        while (x >= 0 && !a[x]) --x;
        return x == -1 ? -1 : x * 64 + __lg(a[x]);
    }
    string to_string() const
    {
        int n = size(), i;
        string s(n, '0');
        for (i = 0; i < n; i++) s[n - i - 1] += test(i);
        return s;
    }
};
istream &operator>>(istream &cin, Bitset &o)
{
    string s;
    cin >> s;
    int n = s.size(), i;
    o.reset();
    assert(n <= o.size());
```

```
    for (i = 0; i < n; i++) o.set(i, s[n - i - 1] - '0');
    return cin;
}
ostream &operator<<(ostream &cout, const Bitset &o) { return cout << o.to_string(); }
```

2.26 区间众数

```
template<class T> struct mode//[0,n)
{
    int n, ksz, m;
    vector<T> b;
    vector<vector<int>> pos, f;
    vector<int> a, blk, id, l;
    mode(const vector<T> &c) :n(c.size()), ksz(max<int>(1, sqrt(n))), m((n + ksz - 1) / ksz), b(c)
        ,
        pos(n), f(m, vector<int>(m)), a(n), blk(n), id(n), l(m + 1)
    {
        int i, j, k;
        sort(all(b)); b.resize(unique(all(b)) - b.begin());
        for (i = 0; i < n; i++)
        {
            a[i] = lower_bound(all(b), c[i]) - b.begin();
            id[i] = pos[a[i]].size();
            pos[a[i]].push_back(i);
        }
        for (i = 0; i < n; i++) blk[i] = i / ksz;
        for (i = 0; i <= m; i++) l[i] = min(i * ksz, n);
        vector<int> cnt(b.size());
        for (i = 0; i < m; i++)
        {
            fill(all(cnt), 0);
            pair<int, int> cur = {0, 0};
            for (j = i; j < m; j++)
            {
                for (k = l[j]; k < l[j + 1]; k++) cmax(cur, pair{++cnt[a[k]], a[k]});
                f[i][j] = cur.second;
            }
        }
    }
    pair<T, int> ask(int L, int R)//返回最大众数
    {
        assert(0 <= L && L < R && R <= n);
        int val = blk[L] == blk[R - 1] ? 0 : f[blk[L] + 1][blk[R - 1] - 1], i;
        int cnt = lower_bound(all(pos[val]), R) - lower_bound(all(pos[val]), L);
        for (i = min(R, l[blk[L] + 1]) - 1; i >= L; i--)
        {
            auto &v = pos[a[i]];
            while (id[i] + cnt < v.size() && v[id[i] + cnt] < R) ++cnt, val = a[i];
            if (a[i] > val && id[i] + cnt - 1 < v.size() && v[id[i] + cnt - 1] < R) val = a[i];
        }
        for (i = max(L, l[blk[R - 1]]); i < R; i++)
        {
            auto &v = pos[a[i]];
            while (id[i] >= cnt && v[id[i] - cnt] >= L) ++cnt, val = a[i];
            if (a[i] > val && id[i] >= cnt - 1 && v[id[i] - cnt + 1] >= L) val = a[i];
        }
```

```
        return {b[val], cnt};
    }
};
```

2.27 表达式树

传入表达式，输出表达式树。

输入的第二个参数是全体括号以外的运算符，每个运算符要记录字符优先级和是否右结合。优先级数字越大，越优先计算，且优先级必须为正整数。

输出的第一个参数是子结点数组，第二个参数是每个结点对应的字符，第三个参数是根。结点编号从 1 开始。

输出的表达式树满足每个结点对应一个字符。若包含数字串，则视为相邻数码之间加一个井号，表示“数码链接”这个运算符。你不需要，也不应该手动加入这个井号。

如果表达式非法，将返回根为 0。不允许一元运算符（负号），不允许省略乘号，不允许出现字母（除非字母是运算符）。

如果需要支持字母作为数字，修改所有包含 `isdigit` 的部分。

由于存在“数码链接”，在 `dfs` 树的时候最好记录一下子树大小，便于链接时计算（你不能在链接时直接看右子树的数字大小，因为有可能有前导 0）。

```
struct Q
{
    char ch;
    int prec;
    bool right;
};
tuple<vector<array<int, 2>>, vector<char>, int> parse_expr(string s, vector<Q> op) {
    static int idx[128];
    int maxp = 0, pos = 0, n, err = 0, i;
    {
        string t;
        for (char c : s)
        {
            if (t.size() && isdigit(t.back()) && isdigit(c)) t += '#';
            t += c;
        }
        swap(s, t);
        n = s.size();
    }
    for (i = 0; i < op.size(); ++i)
    {
        idx[op[i].ch] = i + 1;
        cmax(maxp, op[i].prec);
    }
    op.push_back({'#', ++maxp, 0});
    idx['#'] = op.size();
    vector<array<int, 2>> c(1);
    vector<char> ch(1);
    auto node = [&](char x) {
        c.push_back({0, 0});
        ch.push_back(x);
        return c.size() - 1;
    };
    function<int(int)> parse = [&](int lv) -> int {
        int u;
```

```
        if (lv > maxp)
        {
            if (pos < n && s[pos] == '(')
            {
                pos++;
                u = parse(1);
                if (err |= (pos >= n || s[pos++] != ')')) return 0;
                return u;
            }
            else if (pos < n && isdigit(s[pos])) return u = node(s[pos++]);
            else return err = 1, 0;
        }
        else
        {
            u = parse(lv + 1);
            while (!err && pos < n)
            {
                char ch = s[pos];
                int i = idx[ch] - 1;
                if (i >= 0 && op[i].prec == lv)
                {
                    ++pos;
                    int v = node(ch), w = parse(lv + !op[i].right);
                    c[v] = {u, w};
                    u = v;
                }
                else break;
            }
            return u;
        }
    };
    int root = parse(0);
    for (auto [ch, _, __] : op) idx[ch] = 0;
    if (err || pos != n) return {{ }, { }, 0};
    return {c, ch, root};
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    string s;
    getline(cin, s);
    vector<Q> op = {
        {'|', 1, 0},
        {'&', 2, 0},
    };
    auto [c, ch, root] = parse_expr(s, op);
    assert(root);
    function<array<int, 3>(int)> dfs = [&](int u)->array<int, 3> {
        if (isdigit(ch[u])) return {ch[u] - '0', 0, 0};
        auto [l, r1, r2] = dfs(c[u][0]);
        if (ch[u] == '|')
        {
            if (l) return {1, r1, r2 + 1};
            auto [r, r3, r4] = dfs(c[u][1]);
            return {r, r1 + r3, r2 + r4};
        }
```

```
        else
        {
            if (!l) return {0, r1 + 1, r2};
            auto [r, r3, r4] = dfs(c[u][1]);
            return {r, r1 + r3, r2 + r4};
        }
    };
    auto [r0, r1, r2] = dfs(root);
    cout << r0 << endl << r1 << ' ' << r2 << endl;
}
```

2.28 区间排序区间复合

时空 $O((n+m)\log V)$，其中 V 是排序关键字的值域，要求排序关键字唯一。实际效率较低，10^5 要 1.2s。

构造函数传入排序关键字与信息，以及排序关键字的值域范围。

与其他板子不同的是，所有区间都是左闭右开的，下标从 0 开始。

```
template<class T, class info> struct range_sort
{
    enum type { inc, dec };
    int n;
    T pl, pr;
    vector<int> root, lc, rc, sz;
    vector<info> s, rs;
    struct treap
    {
        int n, rt;
        vector<ui> pr;
        vector<int> sz, num, lc, rc, lz, rev;
        vector<info> v, rv, s, rs;
        void reverse(int x)
        {
            lz[x] ^= 1;
            rev[x] ^= 1;
            swap(lc[x], rc[x]);
            swap(s[x], rs[x]);
            swap(v[x], rv[x]);
        }
        void pushdown(int x)
        {
            if (lz[x])
            {
                if (lc[x]) reverse(lc[x]);
                if (rc[x]) reverse(rc[x]);
                lz[x] = 0;
            }
        }
        void pushup(int x)
        {
            sz[x] = sz[lc[x]] + sz[rc[x]] + num[x];
            s[x] = v[x];
            rs[x] = rv[x];
            if (lc[x])
            {
                s[x] = s[lc[x]] + s[x];
```

```
            rs[x] = rs[x] + rs[lc[x]];
        }
        if (rc[x])
        {
            s[x] = s[x] + s[rc[x]];
            rs[x] = rs[rc[x]] + rs[x];
        }
    }
    int kth;
    void split_kth(int u, int &x, int &y)
    {
        if (!u) return x = y = 0, void();
        pushdown(u);
        if (kth < sz[lc[u]]) split_kth(lc[y = u], x, lc[u]);
        else kth -= sz[lc[u]] + num[u], split_kth(rc[x = u], rc[u], y);
        pushup(u);
    }
    void split_lst(int &x, int &y)
    {
        pushdown(x);
        if (rc[x])
        {
            split_lst(rc[x], y);
            pushup(x);
        }
        else
        {
            y = x;
            x = lc[x];
            lc[y] = 0;
            pushup(y);
        }
    }
    int merge(int x, int y)
    {
        if (!x || !y) return x + y;
        if (pr[x] < pr[y])
        {
            pushdown(x);
            rc[x] = merge(rc[x], y);
            pushup(x);
            return x;
        }
        pushdown(y);
        lc[y] = merge(x, lc[y]);
        pushup(y);
        return y;
    }
    treap(int n) :rt(0), pr(n), sz(n), num(n), lc(n), rc(n), lz(n), v(n), rv(n), s(n), rs(n),
        rev(n)
    {
        static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
        generate(all(pr), rnd);
    }
    void init(const vector<int> &index, const vector<info> &a)
    {
        n = index.size() - 1;
```

```
        for (int i = 1; i <= n; i++)
        {
            s[i] = rs[i] = v[i] = a[index[i]];
            num[i] = sz[i] = 1;
            rt = merge(rt, i);
        }
    }
};
treap t;
int np()
{
    lc.push_back(0);
    rc.push_back(0);
    sz.push_back(0);
    s.push_back({ });
    rs.push_back({ });
    return lc.size() - 1;
}
void pushup(int x)
{
    if (!x) return;
    sz[x] = sz[lc[x]] + sz[rc[x]];
    if (lc[x] && rc[x])
    {
        s[x] = s[lc[x]] + s[rc[x]];
        rs[x] = rs[rc[x]] + rs[lc[x]];
    }
    else if (lc[x]) s[x] = s[lc[x]], rs[x] = rs[lc[x]];
    else if (rc[x]) s[x] = s[rc[x]], rs[x] = rs[rc[x]];
}
void insert(int x, T l, T r, T p, const info &v)
{
    if (l + 1 == r)
    {
        s[x] = rs[x] = v;
        sz[x] = 1;
        return;
    }
    T mid = midpoint(l, r);
    if (p < mid)
    {
        if (!lc[x]) lc[x] = np();
        insert(lc[x], l, mid, p, v);
    }
    else
    {
        if (!rc[x]) rc[x] = np();
        insert(rc[x], mid, r, p, v);
    }
    pushup(x);
}
range_sort(vector<pair<T, info>> a, T pl, T _pr)
    :n(a.size()), pl(pl), pr(_pr - pl), root(n + 1), t(n + 3) {
    np();
    for (int i = 0; i < n; i++)
    {
        a[i].first -= pl;
```

```
        root[i + 1] = np();
        insert(root[i + 1], 0, pr, a[i].first, a[i].second);
    }
    t.init(root, s);
}
int merge(int x, int y, T l, T r)
{
    if (!x || !y) return x + y;
    T mid = midpoint(l, r);
    lc[x] = merge(lc[x], lc[y], l, mid);
    rc[x] = merge(rc[x], rc[y], mid, r);
    pushup(x);
    return x;
}
pair<int, int> split(int x, int k, T l, T r)
{
    if (x == 0) return {0, 0};
    if (l + 1 == r) return {0, x};
    T mid = midpoint(l, r);
    if (k < sz[lc[x]])
    {
        auto [u, v] = split(lc[x], k, l, mid);
        lc[x] = v;
        pushup(x);
        if (!sz[x]) x = 0;
        if (!u) return {0, x};
        int y = np();
        lc[y] = u; pushup(y);
        return {y, x};
    }
    auto [u, v] = split(rc[x], k - sz[lc[x]], mid, r);
    rc[x] = u;
    pushup(x);
    if (!sz[x]) x = 0;
    if (!v) return {x, 0};
    int y = np();
    rc[y] = v; pushup(y);
    return {x, y};
}
void set_treap(int i, int u, bool swp)
{
    root[i] = u;
    if (swp)
    {
        t.s[i] = t.v[i] = rs[u];
        t.rs[i] = t.rv[i] = s[u];
    }
    else
    {
        t.s[i] = t.v[i] = s[u];
        t.rs[i] = t.rv[i] = rs[u];
    }
    t.sz[i] = t.num[i] = sz[u];
    t.lc[i] = t.rc[i] = 0;
    t.rev[i] = swp;
}
pair<int, int> find(int i)
```

```
{
    if (i == t.sz[t.rt]) return {t.rt, 0};
    t.kth = i;
    int x, y, z, r1, r2;
    t.split_kth(t.rt, x, z);
    t.split_lst(x, y);
    int k = i - t.sz[x], tot = t.num[y];
    if (t.rev[y] && k)
    {
        k = t.num[y] - k;
        tie(r1, r2) = split(root[y], k, 0, pr);
        if (r1) set_treap(i, r2, 1);
        set_treap(i + 1, r1, 1);
        x = t.merge(x, r2 ? i : 0);
        z = t.merge(i + 1, z);
        return {x, z};
    }
    else
    {
        tie(r1, r2) = split(root[y], k, 0, pr);
        if (r1) set_treap(i, r1, t.rev[y]);
        set_treap(i + 1, r2, t.rev[y]);
        x = t.merge(x, r1 ? i : 0);
        z = t.merge(i + 1, z);
        return {x, z};
    }
}
tuple<int, int, int> split_range(int l, int r)
{
    auto [_, z] = find(r);
    t.rt = _;
    auto [x, y] = find(l);
    return {x, y, z};
}
void modify(int i, const pair<T, info> &rhs)
{
    assert(rhs.first >= pl && rhs.first < pl + pr);
    auto [x, y, z] = split_range(i, i + 1);
    root[y] = np();
    insert(root[y], 0, pr, rhs.first - pl, rhs.second);
    set_treap(y, root[y], 0);
    t.rt = t.merge(t.merge(x, y), z);
}
info ask(int l, int r)
{
    auto [x, y, z] = split_range(l, r);
    info res = t.s[y];
    t.rt = t.merge(t.merge(x, y), z);
    return res;
}
void merge_dfs(int x, int y)
{
    if (!y) return;
    if (x != y) root[x] = merge(root[x], root[y], 0, pr), root[y] = 0;
    merge_dfs(x, t.lc[y]);
    merge_dfs(x, t.rc[y]);
}
```

```
    void sort(int l, int r, type op)
    {
        auto [x, y, z] = split_range(l, r);
        merge_dfs(y, y);
        set_treap(y, root[y], op == dec);
        t.rt = t.merge(t.merge(x, y), z);
    }
};
const ull p = 998244353;
struct info
{
    ull k, b;
    info operator+(const info &rhs) const { return {(k * rhs.k) % p, (rhs.b + rhs.k * b) % p}; }
    bool operator<(const info &rhs) const { return 0; }
    ull operator()(ull x) const { return (k * x + b) % p; }
};
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, i;
    cin >> n >> m;
    vector<pair<int, info>> a(n);
    for (auto &[x, y] : a)
    {
        auto &[k, b] = y;
        cin >> x >> k >> b;
    }
    range_sort s(a, 0, (int)1e9 + 1);
    while (m--)
    {
        int op;
        cin >> op;
        if (op == 0)
        {
            int i, p;
            ull k, b;
            cin >> i >> p >> k >> b;
            a[i] = {p, {k, b}};
            s.modify(i, {p, {k, b}});
        }
        else
        {
            int l, r;
            cin >> l >> r;
            if (op == 1)
            {
                ull x;
                cin >> x;
                cout << s.ask(l, r)(x) << '\n';
            }
            else s.sort(l, r, op == 2 ? s.inc : s.dec);
        }
    }
}
```

3 数学

3.1 矩阵类（较新）

```
using ull = unsigned long long;
const ull p = 998244353;
ull ksm(ull x, ull y)
{
    ull r = 1;
    while (y)
    {
        if (y & 1) r = r * x % p;
        x = x * x % p; y >>= 1;
    }
    return r;
}
struct matrix;
matrix E(int n);
struct matrix :vector<vector<ull>>
{
    explicit matrix(int n = 0, int m = 0) :vector(n, vector<ull>(m)) { }
    pair<int, int> sz() const { if (size()) return {size(), back().size()}; return {0, 0}; }
    matrix &operator+=(const matrix &b)
    {
        assert(sz() == b.sz());
        auto [n, m] = sz();
        for (int i = 0; i < n; i++) for (int j = 0; j < m; j++) ((*this)[i][j] += b[i][j]) %= p;
        return *this;
    }
    matrix &operator-=(const matrix &b)
    {
        assert(sz() == b.sz());
        auto [n, m] = sz();
        for (int i = 0; i < n; i++) for (int j = 0; j < m; j++) ((*this)[i][j] += p - b[i][j]) %=
            p;
        return *this;
    }
    matrix operator*(const matrix &b) const
    {
        auto [n, m] = sz();
        auto [_, q] = b.sz();
        assert(m == _);
        int i, j, k;
        matrix c(n, q);
        for (k = 0; k < m; k++)
        {
            for (i = 0; i < n; i++) for (j = 0; j < q; j++) c[i][j] += (*this)[i][k] * b[k][j];
            if (!((k ^ q - 1) & 15)) for (auto &v : c) for (ull &x : v) x %= p;
        }
        static_assert(-1llu / p / p > 17);
        return c;
    }
    matrix operator+(const matrix &b) const { auto a = *this; return a += b; }
    matrix operator-(const matrix &b) const { auto a = *this; return a -= b; }
    matrix &operator*=(const matrix &b) { return *this = *this * b; }
    matrix &operator*=(ull k) { for (auto &v : *this) for (ull &x : v) x = x * k % p; return *this
        ; }
```

```
matrix operator*(ull k) const { auto a = *this; return a *= k; }
matrix transpose() const
{
    auto [n, m] = sz();
    matrix res(m, n);
    for (int i = 0; i < n; i++) for (int j = 0; j < m; j++) res[j][i] = (*this)[i][j];
    return res;
}
int rank() const
{
    auto [n, m] = sz();
    vector<vector<ull>> a = n <= m ? *this : transpose();
    if (n > m) ::swap(n, m);
    int i, j, k, l, r = 0;
    for (i = 0, j = 0; i < n && j < m; j++)
    {
        for (k = i; k < n; k++) if (a[k][j]) break;
        if (k == n) continue;
        ::swap(a[i], a[k]);
        ull iv = ksm(a[i][j], p - 2);
        for (k = j; k < m; k++) a[i][k] = a[i][k] * iv % p;
        for (k = i + 1; k < n; k++) for (l = j + 1; l < m; l++) a[k][l] = (a[k][l] + (p - a[k][
            j]) * a[i][l]) % p;
        ++i; ++r;
    }
    return r;
}
vector<ull> poly() const// | kE - A |
{
    auto [n, m] = sz();
    vector<vector<ull>> a = *this;
    assert(n == m);
    int i, j, k;
    for (i = 1; i < n; i++)
    {
        for (j = i; j < n && !a[j][i - 1]; j++);
        if (j == n) continue;
        if (j > i)
        {
            ::swap(a[i], a[j]);
            for (k = 0; k < n; k++) ::swap(a[k][j], a[k][i]);
        }
        ull r = a[i][i - 1];
        for (j = 0; j < n; j++) a[j][i] = a[j][i] * r % p;
        r = ksm(r, p - 2);
        for (j = i - 1; j < n; j++) a[i][j] = a[i][j] * r % p;
        for (j = i + 1; j < n; j++)
        {
            r = a[j][i - 1];
            for (k = 0; k < n; k++) a[k][i] = (a[k][i] + a[k][j] * r) % p;
            r = p - r;
            for (k = i - 1; k < n; k++) a[j][k] = (a[j][k] + a[i][k] * r) % p;
        }
    }
    vector g(n + 1, vector<ull>(n + 1));
    g[0][0] = 1;
    for (i = 0; i < n; i++)
```

```
    {
        ull r = p - 1, rr;
        for (j = i; j >= 0; j--)//第 j 行选第 n 列
        {
            rr = r * a[j][i] % p;
            for (k = 0; k <= j; k++) g[i + 1][k] = (g[i + 1][k] + rr * g[j][k]) % p;
            if (j) r = r * a[j][j - 1] % p;
        }
        for (k = 1; k <= i + 1; k++) (g[i + 1][k] += g[i][k - 1]) %= p;
    }
    auto f = g[n];
    //if (n & 1) for (i = 0; i <= n; i++) if (f[i]) f[i] = p - f[i];
    return f;
}
ull det() const
{
    auto [n, m] = sz();
    vector<vector<ull>> a = *this;
    assert(n == m);
    int i, j, k;
    ull r = 1;
    for (i = 0; i < n; i++)
    {
        for (j = i; j < n; j++) if (a[j][i]) break;
        if (j == n) return 0;
        if (i != j) r = p - r, ::swap(a[i], a[j]);
        (r *= a[i][i]) %= p;
        ull iv = ksm(a[i][i], p - 2);
        for (j = i; j < n; j++) a[i][j] = a[i][j] * iv % p;
        for (j = i + 1; j < n; j++) for (k = i + 1; k < n; k++) a[j][k] = (a[j][k] + (p - a[i][
            k]) * a[j][i]) % p;
    }
    return r % p;
}
tuple<int, vector<ull>, vector<vector<ull>>> gauss(const vector<ull> &b) const//Ax=b, rank of
    base, one sol, base
{
    auto [n, m] = sz();
    if (b.size() != n) return {-1, { }, { }};
    vector<vector<ull>> a = *this;
    int i, j, k, R = m;
    for (i = 0; i < n; i++) a[i].push_back(b[i]);
    vector<int> fix(m, -1);
    for (i = k = 0; i < m; i++)
    {
        for (j = k; j < n; j++) if (a[j][i]) break;
        if (j == n) continue;
        fix[i] = k; --R;
        ::swap(a[k], a[j]);
        auto &u = a[k];
        ull x = ksm(u[i], p - 2);
        for (j = i; j <= m; j++) u[j] = u[j] * x % p;
        for (auto &v : a) if (v.data() != u.data())
        {
            x = p - v[i];
            for (j = i; j <= m; j++) v[j] = (v[j] + x * u[j]) % p;
        }
```

```
            ++k;
        }
        for (i = k; i < n; i++) if (a[i][m]) return {-1, { }, { }};
        vector<ull> r(m);
        vector<vector<ull>> c;
        for (i = 0; i < m; i++) if (fix[i] != -1) r[i] = a[fix[i]][m];
        for (i = 0; i < m; i++) if (fix[i] == -1)
        {
            vector<ull> r(m);
            r[i] = 1;
            for (j = 0; j < m; j++) if (fix[j] != -1) r[j] = (p - a[fix[j]][i]) % p;
            c.push_back(r);
        }
        return {R, r, c};
    }
    optional<matrix> inverse() const
    {
        auto [n, m] = sz();
        assert(n == m);
        vector<int> ih(n, -1), jh(n, -1);
        matrix a = *this;
        int i, j, k;
        for (k = 0; k < n; k++)
        {
            for (i = k; i < n; i++) if (ih[k] == -1) for (j = k; j < n; j++) if (a[i][j])
            {
                ih[k] = i;
                jh[k] = j;
                break;
            }
            if (ih[k] == -1) return { };
            ::swap(a[k], a[ih[k]]);
            for (i = 0; i < n; i++) ::swap(a[i][k], a[i][jh[k]]);
            if (!a[k][k]) return { };
            a[k][k] = ksm(a[k][k], p - 2);
            for (i = 0; i < n; i++) if (i != k) (a[k][i] *= a[k][k]) %= p;
            for (i = 0; i < n; i++) if (i != k) for (j = 0; j < n; j++) if (j != k)
                (a[i][j] += (p - a[i][k]) * a[k][j]) %= p;
            for (i = 0; i < n; i++) if (i != k) (a[i][k] *= p - a[k][k]) %= p;
        }
        for (k = n - 1; k >= 0; k--)
        {
            ::swap(a[k], a[jh[k]]);
            for (i = 0; i < n; i++) ::swap(a[i][k], a[i][ih[k]]);
        }
        return a;
    }
    matrix adjugate() const
    {
        auto [n, m] = sz();
        assert(n == m);
        int R = rank();
        if (n == 1) return E(1);
        if (R == n) return *inverse() * det();
        if (R == n - 1)
        {
            int i, j, k, l;
```

```
            auto [_, x, dx] = gauss(vector<ull>(n));
            auto [__, y, dy] = transpose().gauss(vector<ull>(n));
            if (count(all(x), 0) == n) x = dx[0];
            if (count(all(y), 0) == n) y = dy[0];
            for (k = 0; k < n; k++) if (x[k]) break;
            for (l = 0; l < n; l++) if (y[l]) break;
            assert(k < n && l < n);
            matrix res(n, n), c(n - 1, n - 1);
            for (i = 0; i < n; i++) if (i != l) for (j = 0; j < n; j++) if (j != k) c[i - (i > l)][
                j - (j > k)] = (*this)[i][j];
            for (i = 0; i < n; i++) for (j = 0; j < n; j++) res[i][j] = x[i] * y[j] % p;
            ull t = c.det() * ksm((k + l & 1) ? p - res[k][l] : res[k][l], p - 2) % p;
            assert(res[k][l]);
            assert(c.det());
            assert(t);
            return res * t;
        }
        return matrix(n, n);
    }
};
istream &operator>>(istream &cin, matrix &r) { for (auto &v : r) for (ull &x : v) cin >> x;
    return cin; }
ostream &operator<<(ostream &cout, const matrix &r) { auto [n, m] = r.sz(); for (int i = 0; i < n
    ; i++) for (int j = 0; j < m; j++) cout << r[i][j] << " \n"[j + 1 == m]; return cout; }
matrix E(int n) { matrix r(n, n); for (int i = 0; i < n; i++) r[i][i] = 1; return r; }
matrix pow(matrix a, long long k)
{
    assert(k >= 0);
    auto [n, m] = a.sz();
    assert(n == m);
    matrix r = k & 1 ? a : E(n);
    k >>= 1;
    while (k)
    {
        a *= a;
        if (k & 1) r *= a;
        k >>= 1;
    }
    return r;
}
matrix pow2(matrix a, long long k)
{
    vector<ull> f = a.poly();
    int n = f.size() - 1, i, j;
    if (!n) return matrix();
    if (n == 1) return E(1) * ksm(a[0][0], k);
    assert(f[n] == 1);
    vector<ull> r(n), x(n), t(n * 2);
    r[0] = x[1] = 1;
    for (ull &x : f) x = (p - x) % p;
    reverse(all(f));
    fill(all(t), 0);
    if (k & 1)
    {
        for (i = 0; i < n; i++) for (j = 0; j < n; j++) t[i + j] = (t[i + j] + r[i] * x[j]) % p;
        for (i = n * 2 - 2; i >= n; i--) for (j = 1; j <= n; j++) t[i - j] = (t[i - j] + f[j] * t[
            i]) % p;
```

```
        for (i = 0; i < n; i++) r[i] = t[i];
    }
    k >>= 1;
    while (k)
    {
        fill(all(t), 0);
        for (i = 0; i < n; i++) for (j = 0; j < n; j++) t[i + j] = (t[i + j] + x[i] * x[j]) % p;
        for (i = n * 2 - 2; i >= n; i--) for (j = 1; j <= n; j++) t[i - j] = (t[i - j] + f[j] * t[
            i]) % p;
        for (i = 0; i < n; i++) x[i] = t[i];
        if (k & 1)
        {
            fill(all(t), 0);
            for (i = 0; i < n; i++) for (j = 0; j < n; j++) t[i + j] = (t[i + j] + r[i] * x[j]) % p
                ;
            for (i = n * 2 - 2; i >= n; i--) for (j = 1; j <= n; j++) t[i - j] = (t[i - j] + f[j] *
                 t[i]) % p;
            for (i = 0; i < n; i++) r[i] = t[i];
        }
        k >>= 1;
    }
    matrix res(n, n);
    int b = ceil(sqrt(n));
    vector<matrix> s(b + 1);
    s[0] = E(n); s[1] = a;
    for (i = 2; i <= b; i++) s[i] = s[i - 1] * a;
    for (i = b - 1; i >= 0; i--)
    {
        res *= s[b];
        for (j = min(n, (i + 1) * b) - 1; j >= i * b; j--) res += s[j - i * b] * r[j];
    }
    return res;
}
```

3.2 在线 $O(1)$ 逆元

预处理复杂度为 $O(p^{\frac{2}{3}})$。

```
namespace online_inv
{
    typedef unsigned int ui;
    typedef unsigned long long ull;
    const ull p = 1e9 + 7, n = 1010, m = n * n, N = m + 2;
    static_assert(n *n *n > p);
    int l[N], r[N];
    ull y[N];
    bool s[N];
    ull _inv[N * 2], i, j, k;
    void init_inv()
    {
        _inv[1] = 1;
        for (i = 2; i < m * 2; i++)
        {
            j = p / i;
            _inv[i] = (p - j) * _inv[p - i * j] % p;
        }
        s[0] = y[0] = 1;
```

```
        for (i = 1; i < n; i++) for (j = i; j < n; j++) if (!s[k = i * m / j])
        {
            y[k] = j;
            s[k] = 1;
        }
        l[0] = 1;
        for (i = 1; i <= m; i++) l[i] = s[i] ? y[i] : l[i - 1];
        r[m] = 1;
        for (i = m - 1; ~i; i--) r[i] = s[i] ? y[i] : r[i + 1];
        for (i = 0; i <= m; i++) y[i] = min(l[i], r[i]);
    }
    inline ull inv(const ull &x)
    {
        assert(x && x < p);
        if (x < m * 2) return _inv[x];
        k = x * m / p;
        j = y[k] * x % p;
        return (j < m * 2 ? _inv[j] : p - _inv[p - j]) * y[k] % p;
    }
    bool _ = (init_inv(), 0);
}
using online_inv::inv, online_inv::p;
```

3.3 Strassen 矩阵乘法

没用，不如卡常。$O(n^{\log_2 7})$。

```
#include "bits/stdc++.h"
using namespace std;
typedef unsigned int ui;
typedef unsigned long long ull;
const ui p = 998244353;
const ull fh = 1ull << 31;
struct Q
{
    ui **a;
    int n;
    Q() { n = 0; }
    void clear()
    {
        for (int i = 0; i < n; i++) delete a[i];
        if (n) delete a; n = 0;
    }
    Q(int nn)//不能传入不是 2 的幂的数！
    {
        n = nn;
        assert(n == (n & -n));
        a = new ui * [n];
        for (int i = 0; i < n; i++) a[i] = new ui[n], memset(a[i], 0, n * sizeof a[0][0]);
    }
    const Q &operator=(const Q &b)
    {
        clear(); n = b.n;
        a = new ui * [n];
        for (int i = 0; i < n; i++) a[i] = new ui[n], memcpy(a[i], b.a[i], n * sizeof a[0][0]);
        return *this;
    }
```

```
~Q() { clear(); }
Q operator+(const Q &b)
{
    Q c(n);
    for (int i = 0; i < n; i++) for (int j = 0; j < n; j++) if ((c.a[i][j] = a[i][j] + b.a[i][
        j]) >= p) c.a[i][j] -= p;
    return c;
}
Q operator-(const Q &b)
{
    Q c(n);
    for (int i = 0; i < n; i++) for (int j = 0; j < n; j++) if ((c.a[i][j] = a[i][j] - b.a[i][
        j]) & fh) c.a[i][j] += p;
    return c;
}
Q operator*(Q &b)
{
    Q c(n);
    if (n <= 128)
    {
        for (int i = 0; i < n; i++) for (int k = 0; k < n; k++) for (int j = 0; j < n; j++) c.a
            [i][j] = (c.a[i][j] + (ull)a[i][k] * b.a[k][j]) % p;
        return c;
    }
    Q A[2][2], B[2][2], s[10], p[5];
    n >>= 1;
    int i, j, k, l;
    for (i = 0; i < 2; i++) for (j = 0; j < 2; j++)
    {
        A[i][j] = Q(n);
        for (k = 0; k < n; k++) memcpy(A[i][j].a[k], a[k + i * n] + j * n, n * sizeof a[0][0]);
        B[i][j] = Q(n);
        for (k = 0; k < n; k++) memcpy(B[i][j].a[k], b.a[k + i * n] + j * n, n * sizeof a
            [0][0]);
    }
    s[0] = B[0][1] - B[1][1];
    s[1] = A[0][0] + A[0][1];
    s[2] = A[1][0] + A[1][1];
    s[3] = B[1][0] - B[0][0];
    s[4] = A[0][0] + A[1][1];
    s[5] = B[0][0] + B[1][1];
    s[6] = A[0][1] - A[1][1];
    s[7] = B[1][0] + B[1][1];
    s[8] = A[0][0] - A[1][0];
    s[9] = B[0][0] + B[0][1];
    p[0] = A[0][0] * s[0];
    p[1] = s[1] * B[1][1];
    p[2] = s[2] * B[0][0];
    p[3] = A[1][1] * s[3];
    p[4] = s[4] * s[5];
    A[0][0] = p[4] + p[3] - p[1] + s[6] * s[7];
    A[0][1] = p[0] + p[1];
    A[1][0] = p[2] + p[3];
    A[1][1] = p[4] + p[0] - p[2] - s[8] * s[9];
    for (i = 0; i < 2; i++) for (j = 0; j < 2; j++) for (k = 0; k < n; k++) memcpy(c.a[k + i *
         n] + j * n, A[i][j].a[k], n * sizeof a[0][0]);
    n <<= 1;
```

```
        return c;
    }
};
int main()
{
    int i, j, n, m, k;
    ios::sync_with_stdio(0); cin.tie(0);
    cin >> n >> m >> k;
    int N = 1 << 32 - min({__builtin_clz(n - 1), __builtin_clz(m - 1), __builtin_clz(k - 1)});
    Q a(N), b(N);
    for (i = 0; i < n; i++) for (j = 0; j < m; j++) cin >> a.a[i][j];
    for (i = 0; i < m; i++) for (j = 0; j < k; j++) cin >> b.a[i][j];
    a = a * b;
    for (i = 0; i < n; i++) for (j = 0; j < k; j++) cout << a.a[i][j] << " \n"[j + 1 == k];
}
```

3.4 扩展欧拉定理

求 $a \uparrow\uparrow b \bmod c$。前面的 Prime 命名空间只是求 φ 用的。

特别注意，这里的 `i64` 不是无符号。

```
using i64=long long;
namespace Prime
{
    typedef unsigned int ui;
    typedef unsigned long long ull;
    const int N = 1e6 + 2;
    const ull M = (ull)(N - 1) * (N - 1);
    ui pr[N], mn[N], phi[N], cnt;
    int mu[N];
    void init_prime()
    {
        ui i, j, k;
        phi[1] = mu[1] = 1;
        for (i = 2; i < N; i++)
        {
            if (!mn[i])
            {
                pr[cnt++] = i;
                phi[i] = i - 1; mu[i] = -1;
                mn[i] = i;
            }
            for (j = 0; (k = i * pr[j]) < N; j++)
            {
                mn[k] = pr[j];
                if (i % pr[j] == 0)
                {
                    phi[k] = phi[i] * pr[j];
                    break;
                }
                phi[k] = phi[i] * (pr[j] - 1);
                mu[k] = -mu[i];
            }
        }
        //for (i=2;i<N;i++) if (mu[i]<0) mu[i]+=p;
    }
```

```
    template<class T> T getphi(T x)
    {
        assert(M >= x);
        T r = x;
        for (ui i = 0; i < cnt && (T)pr[i] * pr[i] <= x && x >= N; i++) if (x % pr[i] == 0)
        {
            ui y = pr[i], tmp;
            x /= y;
            while (x == (tmp = x / y) * y) x = tmp;
            r = r / y * (y - 1);
        }
        if (x >= N) return r / x * (x - 1);
        while (x > 1)
        {
            ui y = mn[x], tmp;
            x /= y;
            while (x == (tmp = x / y) * y) x = tmp;
            r = r / y * (y - 1);
        }
        return r;
    }
    template<class T> vector<pair<T, ui>> getw(T x)
    {
        assert(M >= x);
        vector<pair<T, ui>> r;
        for (ui i = 0; i < cnt && (T)pr[i] * pr[i] <= x && x >= N; i++) if (x % pr[i] == 0)
        {
            ui y = pr[i], z = 1, tmp;
            x /= y;
            while (x == (tmp = x / y) * y) x = tmp, ++z;
            r.push_back({y, z});
        }
        if (x >= N)
        {
            r.push_back({x, 1});
            return r;
        }
        while (x > 1)
        {
            ui y = mn[x], z = 1, tmp;
            x /= y;
            while (x == (tmp = x / y) * y) x = tmp, ++z;
            r.push_back({y, z});
        }
        return r;
    }
}
using Prime::pr, Prime::phi, Prime::getw, Prime::getphi;
using Prime::mu, Prime::init_prime;
i64 ksm(i64 x, i64 y, i64 p)
{
    x = (x - p) % p + p;
    i64 r = 1;
    while (y)
    {
        if (y & 1) r = (r * x - p) % p + p;
        x = (x * x - p) % p + p;
```

```
        y >>= 1;
    }
    return r;
}
struct Q
{
    vector<i64> p;
    Q(i64 mod)
    {
        p.push_back(mod);
        while (p.back() > 1) p.push_back(getphi(p.back()));
    }
    i64 operator()(i64 a, i64 b)
    {
        if (!a) return (b + 1 & 1) % p[0];
        i64 r = 1, i = min<i64>(b, p.size());
        while ((--i) >= 0) r = ksm(a, r, p[i]);
        return r % p[0];
    }
};
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, i, T;
    init_prime();
    cin >> T;
    while (T--)
    {
        i64 a, b, c;
        cin >> a >> b >> c;
        cout << Q(c)(a, b) << '\n';
    }
}
```

3.5 exgcd

$O(\log p)$，$O(\log p)$。

递归版：

```
int exgcd(int a, int b, int c)//ax+by=c,return x
{
    if (a == 0) return c / b;
    return (c - (ll)b * exgcd(b % a, a, c)) / a % b;
}
```

递推版：

```
pair<ll, ll> exgcd(ll a, ll b, ll c)//ax+by=c, {-1,-1} 无解, b=0 返回 {c/a,0}, 否则返回最小非负 x
{
    assert(a || b);
    if (!b) return {c / a, 0};
    if (a < 0) a = -a, b = -b, c = -c;
    ll d = gcd(a, b);
    if (c % d) return {-1, -1};
    ll x = 1, x1 = 0, p = a, q = b, k;
    b = abs(b);
    while (b)
```

```
    {
        k = a / b;
        x -= k * x1; a -= k * b;
        swap(x, x1);
        swap(a, b);
    }
    b = abs(q / d);
    x = (c / d) % b * (x % b) % b;
    if (x < 0) x += b;
    return {x, (ll)((c - (lll)p * x) / q)};
}
ll fun(ll a, ll b, ll p)//ax=b(mod p)
{
    return exgcd(a, -p, b).first % p;
}
```

3.6 exCRT

CRT：设 $M=\prod_{i=1}^{n} m_i$，$t_i\times\frac{M}{m_i}\equiv 1\pmod{m_i}$，则 $x\equiv\sum_{i=1}^{n} a_i t_i \frac{M}{m_i}$。

以下为 exCRT，与 CRT 无关。实现了一个类 Q，表示一条方程，支持合并。

```
namespace CRT
{
    pair<ll, ll> exgcd(ll a, ll b, ll c)
    {
        assert(a || b);
        if (!b) return {c / a, 0};
        ll d = gcd(a, b);
        if (c % d) return {-1, -1};
        ll x = 1, x1 = 0, p = a, q = b, k;
        b = abs(b);
        while (b)
        {
            k = a / b;
            x -= k * x1; a -= k * b;
            swap(x, x1);
            swap(a, b);
        }
        b = abs(q / d);
        x = x * (c / d) % b;
        if (x < 0) x += b;
        return {x, (c - p * x) / q};
    }
    struct Q
    {
        ll p, r;//0<=r<p
        Q operator+(const Q &o) const
        {
            if (p == 0 || o.p == 0) return {0, 0};
            auto [x, y] = exgcd(p, -o.p, r - o.r);
            if (x == -1 && y == -1) return {0, 0};
            ll q = lcm(p, o.p);
            return {q, ((r - x * p) % q + q) % q};
        }
    };
}
```

```
using CRT::Q;
```

3.7 exBSGS

$O(\sqrt{n})$。哈希表 ht 可以用 map 代替。

```
namespace BSGS
{
    typedef unsigned int ui;
    typedef unsigned long long ull;
    template<int N,class T,class TT> struct ht//个数，定义域，值域
    {
        const static int p=1e6+7,M=p+2;
        TT a[N];
        T v[N];
        int fir[p+2],nxt[N],st[p+2];//和模数相适应
        int tp,ds;//自定义模数
        ht(){memset(fir,0,sizeof fir);tp=ds=0;}
        void mdf(T x,TT z)//位置，值
        {
            ui y=x%p;
            for (int i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i]=z,void();//若不可能重复不需要 for
            v[++ds]=x;a[ds]=z;
            if (!fir[y]) st[++tp]=y;
            nxt[ds]=fir[y];fir[y]=ds;
        }
        TT find(T x)
        {
            ui y=x%p;
            int i;
            for (i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i];
            return 0;//返回值和是否判断依据要求决定
        }
        void clear()
        {
            ++tp;
            while (--tp) fir[st[tp]]=0;
            ds=0;
        }
    };
    const int N=5e4;
    ht<N,ui,ui> s;
    int exgcd(int a,int b)
    {
        if (a==1) return 1;
        return (1-(long long)b*exgcd(b%a,a))/a;//not ll
    }
    int bsgs(ui a,ui b,ui p)
    {
        s.clear();
        a%=p;b%=p;
        if (!a) return 1-min((int)b,2);//含 -1
        ui i,j,k,x,y;
        x=sqrt(p)+2;
        for (i=0,j=1;i<x;i++,j=(ull)j*a%p)
        {
            if (j==b) return i;
```

```
            s.mdf((ull)j*b%p,i+1);
        }
        k=j;
        for (i=1;i<=x;i++,j=(ull)j*k%p) if (y=s.find(j)) return (ull)i*x-y+1;
        return -1;
    }
    bool isprime(ui p)
    {
        if (p<=1) return 0;
        for (ui i=2;i*i<=p;i++) if (p%i==0) return 0;
        return 1;
    }
    int exbsgs(ui a,ui b,ui p)//a^x=b(mod p)
    {
        //if (isprime(p)) return bsgs(a,b,p);
        a%=p;b%=p;
        ui i,j,k,x,y=__lg(p),cnt=0;
        for (i=0,j=1%p;i<=y;i++,j=(ull)j*a%p) if (j==b) return i;
        y=1;
        while (1)
        {
            if ((x=gcd(a,p))==1) break;
            if (b%x) return -1;//no sol
            ++cnt;
            p/=x;b/=x;
            y=(ull)y*(a/x)%p;
        }
        a%=p;
        b=(ull)b*(p+exgcd(y,p))%p;
        int r=bsgs(a,b,p);
        return r==-1?-1:r+cnt;
    }
}
using BSGS::bsgs,BSGS::exbsgs;
```

3.8 exLucas

求组合数。

```
struct binom
{
    using pa = pair<ui, ui>;
    ull p;
    vector<pa> a;
    vector<vector<ui>> b, ib, pw, pb;
    vector<ui> ph, xs;
    ull ksm(ull x, ll y, ull p)
    {
        ull r = 1;
        while (y)
        {
            if (y & 1) r = r * x % p;
            x = x * x % p;
            y >>= 1;
        }
        return r;
    }
```

```
ull f(ll n, ll m, ll nm, int i)
{
    auto [qi, pi] = a[i];
    ull r = 1;
    ll c1 = 0, c2 = 0;
    while (n)
    {
        r = r * b[i][n % pi] % pi * ib[i][m % pi] % pi * ib[i][nm % pi] % pi;
        c2 += n / pi - m / pi - nm / pi;
        n /= qi, m /= qi, nm /= qi;
        c1 += n - m - nm;
    }
    return 1llu * pw[i][min<int>(c1, pw[i].size() - 1)] * pb[i][c2 % ph[i]] % pi * r % pi;
}
ull operator()(ll n, ll m)
{
    if (m < 0 || n < m) return 0;
    ull r = 0;
    for (int i = 0; i < a.size(); i++) r = (r + xs[i] * f(n, m, n - m, i)) % p;
    return r;
}
binom(ull p) :p(p)
{
    int i, j;
    ull x = p, y, z;
    for (i = 2; i * i <= x; i++) if (x % i == 0)
    {
        z = x; x /= i;
        while (1)
        {
            y = x / i;
            if (i * y == x) x = y; else break;
        }
        a.push_back({i, z / x});
    }
    if (x > 1) a.push_back({x, x});
    int n = a.size();
    b = ib = pw = pb = vector<vector<ui>>(n);
    ph = xs = vector<ui>(n);
    for (i = 0; i < n; i++)
    {
        auto [qi, pi] = a[i];
        ph[i] = pi / qi * (qi - 1);
        xs[i] = ksm(p / pi, ph[i] - 1, p) * (p / pi) % p;
    }
    for (i = 0; i < n; i++)
    {
        auto [qi, pi] = a[i];
        b[i] = ib[i] = vector<ui>(pi, 1);
        for (j = 1; j < pi; j++) b[i][j] = 1llu * b[i][j - 1] * (j % qi == 0 ? 1 : j) % pi;
        ib[i][pi - 1] = ksm(b[i][pi - 1], ph[i] - 1, pi);
        for (j = pi - 1; j; j--) ib[i][j - 1] = 1llu * ib[i][j] * (j % qi == 0 ? 1 : j) % pi;
        pw[i] = {1};
        while (pw[i].back()) pw[i].push_back(1llu * pw[i].back() * qi % pi);
        pb[i].resize(ph[i], 1);
        for (j = 1; j < ph[i]; j++) pb[i][j] = 1llu * pb[i][j - 1] * b[i][pi - 1] % pi;
    }
```

```
    }
};
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int T, p; cin >> T >> p;
    binom s(p);
    while (T--)
    {
        ll n, m;
        cin >> n >> m;
        cout << s(n, m) << '\n';
    }
}
```

3.9 杜教筛

求 $\varphi(n)$ 的前缀和。

核心：构造 g 满足 $h(n) = \sum_{d|n} f(d)g(\frac{n}{d})$ 容易计算，

则有 $\sum_{i=1}^{n} h(i) = \sum_{i=1}^{n} g(i) \sum_{j=1}^{\lfloor n/i \rfloor} f(j)$，

故 $g(1) \sum_{j=1}^{n} f(j) = \sum_{i=1}^{n} h(i) - \sum_{i=2}^{n} g(i) \sum_{j=1}^{\lfloor n/i \rfloor} f(j)$，

则 f 前缀和可以递归求解。

```
namespace du_seive
{
    typedef unsigned int ui;
    typedef unsigned long long ull;
    unordered_map<ull, ui> mp;
    const int N = 1e7 + 2;
    const ui p = 998244353;
    ui pr[N], phi[N];
    ui cnt;
    void init()
    {
        cnt = 0; phi[1] = 1;
        int i, j;
        for (i = 2; i < N; i++)
        {
            if (!phi[i])
            {
                pr[cnt++] = i;
                phi[i] = i - 1;
            }
            for (j = 0; i * pr[j] < N; j++)
            {
                if (i % pr[j] == 0)
                {
                    phi[i * pr[j]] = phi[i] * pr[j];
                    break;
                }
                phi[i * pr[j]] = phi[i] * (pr[j] - 1);
```

```
        }
        if ((phi[i] += phi[i - 1]) >= p) phi[i] -= p;
    }
}
ui get_phi_sum(ull n)
{
    if (n < N) return phi[n];
    if (mp.count(n)) return mp[n];
    ui sum = 0;
    for (ull i = 2, j, k; i <= n; i = j + 1)
    {
        j = n / (k = n / i);
        sum = (sum + (ull)get_phi_sum(k) * (j - i + 1)) % p;
    }
    ui nn = n % p;
    sum = (nn * (nn + 1ll) / 2 + p - sum) % p;
    mp[n] = sum;
    return sum;
}
}
using du_seive::init, du_seive::get_phi_sum;
```

3.10 $\mu^2(n)$ 前缀和

10^{18}，0.46s。

$\mu^2(n) = \sum\limits_{d^2|n} \mu(d)$

```
const int N = 5e7 + 5;
int pr[N / 8], cnt, mu[N];
bool ed[N];
void init()
{
    ui i, j, k;
    mu[1] = 1;
    for (i = 2; i < N; i++)
    {
        if (!ed[i]) pr[++cnt] = i, mu[i] = -1;
        for (j = 1; pr[j] * i < N; j++)
        {
            ed[pr[j] * i] = 1;
            if (i % pr[j] == 0) break;
            mu[pr[j] * i] = -mu[i];
        }
        mu[i] += mu[i - 1];
    }
}
ll sum_mu(ll n)
{
    if (n < N) return mu[n];
    ll r = 1, i, j, k;
    for (i = 2; i <= n; i = j + 1)
    {
        j = n / (k = n / i);
        r -= sum_mu(k) * (j - i + 1);
    }
    return r;
```

```
}
ll sum_mu2(ll n)
{
    ll r = 0, i, j, k, l, s = 0, t;
    for (i = 1; i * i <= n; i = j + 1)
    {
        k = n / (i * i);
        j = sqrtl(n / k);
        t = sum_mu(j);
        r += k * (t - s);
        s = t;
    }
    return r;
}
int main()
{
    ll n;
    init();
    cin >> n;
    cout << sum_mu2(n) << endl;
}
```

3.11 线性规划

用法：构造函数指明目标函数系数，add 函数增加限制。额外的限制是 $x_i \geq 0$。

```
using db = long double;//__float128
struct linear
{
    static const int N = 45;//n+m
    db r[N][N];
    int col[N], row[N];
    const db eps = 1e-10, inf = 1e9;//1e-17
    int n, m;
    template<class T> linear(const vector<T> &a)//target: maximize \sum a(i-1)xi
    {
        memset(r, 0, sizeof r);
        memset(col, 0, sizeof col);
        memset(row, 0, sizeof row);
        n = a.size(); m = 0;
        for (int i = 1; i <= n; i++) r[0][i] = -a[i - 1];
    }
    template<class T> void add(const vector<T> &a, db b)//limit: \sum a(i-1)xi<=b
    {
        assert(a.size() == n);
        ++m;
        for (int i = 1; i <= n; i++) r[m][i] = -a[i - 1];
        r[m][0] = b;
    }
    void pivot(int k, int t)
    {
        swap(row[k + n], row[t]);
        db rkt = -r[k][t];
        int i, j;
        for (i = 0; i <= n; i++) r[k][i] /= rkt;
        r[k][t] = -1 / rkt;
        for (i = 0; i <= m; i++) if (i != k)
```

```
        {
            db rit = r[i][t];
            if (rit >= -eps && rit <= eps) continue;
            for (j = 0; j <= n; j++) if (j != t) r[i][j] += rit * r[k][j];
            r[i][t] = r[k][t] * rit;
        }
    }
    bool init()
    {
        int i;
        for (i = 1; i <= n + m; i++) row[i] = i;
        while (1)
        {
            int q = 1;
            auto b_min = r[1][0];
            for (i = 2; i <= m; i++) if (r[i][0] < b_min) b_min = r[i][0], q = i;
            if (b_min + eps >= 0) return 1;
            int p = 0;
            for (i = 1; i <= n; i++) if (r[q][i] > eps && (!p || row[i] > row[p])) p = i;
            if (!p) break;
            pivot(q, p);
        }
        return 0;
    }
    bool simplex()
    {
        while (1)
        {
            int t = 1, k = 0, i;
            for (i = 2; i <= n; i++) if (r[0][i] < r[0][t]) t = i;
            if (r[0][t] >= -eps) return 1;
            db ratio_min = inf;
            for (i = 1; i <= m; i++) if (r[i][t] < -eps)
            {
                db ratio = -r[i][0] / r[i][t];
                if (!k || ratio<ratio_min || ratio <= ratio_min + eps && row[i]>row[k])
                {
                    ratio_min = ratio;
                    k = i;
                }
            }
            if (!k) break;
            pivot(k, t);
        }
        return 0;
    }
    void solve(int type)
    {
        if (!init())
        {
            cout << "Infeasible\n";
            return;
        }
        if (!simplex())
        {
            cout << "Unbounded\n";
            return;
```

```
        }
        cout << (long double)(-r[0][0]) << '\n';
        if (type)
        {
            int i;
            memset(col + 1, 0, n * sizeof col[0]);
            for (i = n + 1; i <= n + m; i++) col[row[i]] = i;
            for (i = 1; i <= n; i++) cout << (long double)(col[i] ? r[col[i] - n][0] : 0) << " \n"[
                i == n];
        }
    }
};
```

3.12 线性插值（k 次幂和）

$O(m)$，$O(m)$。

```
ull interpolation(vector<ull> a, ull n)
{
    int m = a.size(), i;
    vector<ull> ans(2);
    n %= p;
    if (n < m) return a[n];
    ull k = ifac[m - 1];
    for (i = m - 1;i >= 0;i--)
    {
        (a[i] *= k) %= p;
        (k *= n - i) %= p;
    }
    k = 1;
    for (i = 0;i < m;i++)
    {
        (ans[(m ^ i) & 1] += a[i] * k) %= p;
        k = k * inv[i + 1] % p * (n - i) % p * (m - i - 1) % p;
    }
    return (ans[1] + p - ans[0]) % p;
}
ull sum_of_kth_power(ull n, ull k)
{
    if (n == 0) return 0;
    ull m = min(n + 1, k + 2);
    int i;
    vector<ull> s(m);
    vector<int> pr, ed(m);pr.reserve(m / 4);
    s[1] = 1;
    for (i = 2;i < m;i++)
    {
        if (!ed[i]) s[i] = ksm(i, k);
        for (int j : pr) if (i * j < m)
        {
            s[i * j] = s[i] * s[j] % p;
            if (i % j == 0) break;
        }
        else break;
    }
    for (i = 1;i < m;i++) (s[i] += s[i - 1]) %= p;
    return interpolation(s, n);
```

```
}
```

3.13 原根

你应该传入的是模数 m 和你提供的 `getw` 函数，用于分解质因数。

以下以我的 pollard rho 模板为例。如果在现场赛且范围不大，手动求出更为方便。

定理：只有 $2, 4, p^k, 2p^k$ 有原根。因此可以优化到只分解一次，但没必要。

```
namespace get_root
{
    using ull = unsigned long long;
    using u128 = __uint128_t;
    ull ksm(ull x, ull y, ull p)
    {
        ull r = 1;
        while (y)
        {
            if (y & 1) r = (u128)r * x % p;
            x = (u128)x * x % p; y >>= 1;
        }
        return r;
    }
    template<class T> ll getrt(ull m, T getw)
    {
        assert(m);
        if (m <= 4) return (ll)m - 1;
        ull phi = m;
        auto w = getw(m);
        if (w.size() >= 3 || m % 4 == 0 || w.size() == 2 && w[0] != 2) return -1;
        for (ull x : w) phi = phi / x * (x - 1);
        w = getw(phi);
        for (ull &x : w) x = phi / x;
        for (ull i = 2; i < m; i++) if (gcd(i, m) == 1)
        {
            for (ull x : w) if (ksm(i, x, m) == 1) goto no;
            return i;
        no:;
        }
        return -1;
    }
}
using get_root::getrt;
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    ull p;
    cin >> p;
    cout << getrt(p, [&](ull m) {
        auto ww = pr::getw(m);
        vector<ull> w;
        for (auto [p, k] : ww) w.push_back(p);
        return w;
    }) << '\n';
}
```

3.14 筛全部原根

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
const int N = 1e6 + 2;
int ss[N], mn[N], fmn[N], phi[N];
int t, n, gs, i, d;
bool ed[N], av[N], yg[N], hv[N];
double inv[N];
void getfac(int x, int *a, int &n)
{
    int y = x, z;
    if (1 ^ x & 1)
    {
        a[n = 1] = 2; x >>= 1; while (1 ^ x & 1) x >>= 1;
    }
    while (x > 1)
    {
        x = 1e-9 + (x * inv[a[++n] = z = mn[x]]);
        while (x % z == 0) x = 1e-9 + x * inv[z];
    }
    for (i = 1; i <= n; i++) av[a[i]] = 0, a[i] = 1e-9 + (y * inv[a[i]]);
}
int ksm(int x, int y, int p)
{
    int r = 1;
    while (y)
    {
        if (y & 1) r = (ll)r * x % p;
        x = (ll)x * x % p; y >>= 1;
    }
    return r;
}
bool ck(int x, int *a, int n, int p)
{
    for (int i = 1; i <= n; i++) if (ksm(x, a[i], p) == 1) return 0;
    return 1;
}
void getrt(int x, int d)
{
    if (!hv[x]) return puts("0\n"), void();
    static int a[30];
    int n = 0, y, i, g = 0, c = d; y = phi[x];
    fill(av + 1, av + y + 1, 1);
    getfac(y, a, n);
    for (i = 1; i < x; i++) if (__gcd(i, x) == 1 && ck(i, a, n, x)) break;
    yg[g = i] = 1;//g就是最小原根
    int j = (ll)g * g % x;
    for (i = 2; i < y; i++, j = (ll)j * g % x) yg[j] = av[i] = av[mn[i]] & av[fmn[i]];
    printf("%d\n", phi[y]);
    for (i = 1; i < x; i++) if (yg[i])
    {
        yg[i] = 0;
        if (--c == 0) printf("%d ", i), c = d;
    }puts("");
}
```

```
void init()
{
    int i, j, k, n = N - 1;
    mn[1] = phi[1] = 1;
    for (i = 1; i <= n; i++) inv[i] = 1.0 / i;
    for (i = 2; i <= n; i++)
    {
        if (!ed[i]) phi[mn[i] = ss[++gs] = i] = i - 1, hv[i] = 1;
        for (j = 1; j <= gs && (k = ss[j] * i) <= n; j++)
        {
            ed[k] = 1; mn[k] = ss[j];
            if (i % ss[j] == 0) { phi[k] = phi[i] * ss[j]; hv[k] = hv[i]; break; }
            phi[k] = phi[i] * (ss[j] - 1);
        }
    }
    for (i = n; i; i--) fmn[i] = 1e-9 + (i * inv[mn[i]]), hv[i] |= (1 ^ i & 1) && hv[i >> 1];
    for (i = 8; i <= n; i <<= 1) hv[i] = 0;
}
int main()
{
    init();
    scanf("%d", &t);
    while (t--)
    {
        scanf("%d%d", &n, &d);
        getrt(n, d);
    }
}
```

3.15 高斯消元（浮点数）

$O(n^3)$，$O(n^2)$。

```
namespace Gauss
{
    typedef double db;
    const db eps = 1e-8;
    template<class T> pair<vector<db>, int> solve(const vector<vector<T>> &A)//和为 0。返回秩，负
        数无解
    {
        assert(A.size());
        int n = A.size(), m = A[0].size() - 1, i, j, k, l, r, fg = 1;
        db a[n][m + 1], b;
        for (i = 0; i < n; i++) for (j = 0; j <= m; j++) a[i][j] = A[i][j];
        for (i = l = r = 0; i < n && l < m; i++, l++)
        {
            k = i;
            for (j = i + 1; j < n; j++) if (fabs(a[j][l]) > fabs(a[k][l])) k = j;
            if (fabs(a[k][l]) < eps) { --i; continue; }
            if (i != k) for (j = l; j <= m; j++) swap(a[i][j], a[k][j]);
            b = 1 / a[i][l]; ++r; a[i][l] = 1;
            for (j = l + 1; j <= m; j++) a[i][j] *= b;
            for (j = 0; j < n; j++) if (i != j)
            {
                b = a[j][l]; a[j][l] = 0;
                for (k = l + 1; k <= m; k++) a[j][k] -= b * a[i][k];
            }
```

```
        }
        vector<db> X(m);
        for (j = 0; j < l; j++) for (k = 0; k < i; k++) if (a[k][j] == 1)
        {
            X[j] = -a[k][m];
            break;
        }
        for (j = i; j < n && ~fg; j++)
        {
            b = a[j][m];
            for (k = 0; k < m; k++) b += X[k] * a[j][k];
            if (fabs(b) > eps) fg = -1;
        }
        return {X, r * fg};
    }
}
```

3.16 行列式求值（任意模数）

$O(n^3)$，$O(n^2)$。

原理：辗转相除。注意这个 $\log p$ 并不在 n^3 上。

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
const int N = 502, p = 998244353;
int cal(int a[][N], int n)
{
    int i, j, k, r = 1, fh = 0, l;
    for (i = 1; i <= n; i++)
    {
        k = i;
        for (j = i + 1; j <= n; j++) if (a[j][i]) { k = j; break; }
        if (a[k][i] == 0) return 0;
        if (i != k) { swap(a[k], a[i]); fh ^= 1; }
        for (j = i + 1; j <= n; j++)
        {
            if (a[j][i] > a[i][i]) swap(a[j], a[i]), fh ^= 1;
            while (a[j][i])
            {
                l = a[i][i] / a[j][i];
                for (k = i; k <= n; k++) a[i][k] = (a[i][k] + (ll)(p - l) * a[j][k]) % p;
                swap(a[j], a[i]); fh ^= 1;
            }
        }
        r = (ll)r * a[i][i] % p;
    }
    if (fh) return (p - r) % p;
    return r;
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, i, j;
    static int a[N][N];
    cin >> n;
```

```
    for (i = 1; i <= n; i++) for (j = 1; j <= n; j++) cin >> a[i][j];
    cout << cal(a, n) << endl;
}
```

3.17 BM/稀疏矩阵系列

BM：给定 $\{a\}$，求最短的 $\{r\}$ 满足 $\sum_{j=0}^{m-1} a_{i-j-1}r_j = a_i$。

safe 宏用于验证结果正确性，可不定义。实现了稀疏矩阵的行列式和求解方程组。

```
vector<ui> bm(const vector<ui> &a)
{
    vector<ui> r, lst;
    int n = a.size(), m = 0, q = 0, i, j, k = -1;
    ui D = 0;
    for (i = 0; i < n; i++)
    {
        ui cur = 0;
        for (j = 0; j < m; j++) cur = (cur + (ull)a[i - j - 1] * r[j]) % p;
        cur = (a[i] + p - cur) % p;
        if (!cur) continue;
        if (k == -1)
        {
            k = i;
            D = cur;
            r.resize(m = i + 1);
            continue;
        }
        auto v = r;
        ui x = (ull)cur * ksm(D, p - 2) % p;
        if (m < q + i - k) r.resize(m = q + i - k);
        (r[i - k - 1] += x) %= p;
        ui *b = r.data() + i - k;
        x = (p - x) % p;
        for (j = 0; j < q; j++) b[j] = (b[j] + (ull)x * lst[j]) % p;
        if (v.size() + k < lst.size() + i)
        {
            lst = v;
            q = v.size();
            k = i;
            D = cur;
        }
    }
    return r;
}
#define safe
struct Q
{
    int x, y;
    ui w;
};
mt19937_64 rnd(9980);
vector<ui> minpoly(int n, const vector<Q> &a)//[0,n),max:1
{
    for (auto [x, y, w] : a) assert(min(x, y) >= 0 && max(x, y) < n);
    vector<ui> u(n), v(n), b(n * 2 + 1), tmp(n);
```

```
    int i;
    for (ui &x : u) x = rnd() % p;
    for (ui &x : v) x = rnd() % p;
    assert(*min_element(all(u)) && *min_element(all(v)));
    for (ui &r : b)
    {
        for (i = 0; i < n; i++) r = (r + (ull)u[i] * v[i]) % p;
        fill(all(tmp), 0);
        for (auto [x, y, w] : a) tmp[x] = (tmp[x] + (ull)w * v[y]) % p;
        swap(v, tmp);
    }
    auto r = bm(b);
#ifdef safe
    for (ui &x : u) x = rnd() % p;
    for (ui &x : v) x = rnd() % p;
    for (ui &r : b)
    {
        for (i = 0; i < n; i++) r = (r + (ull)u[i] * v[i]) % p;
        fill(all(tmp), 0);
        for (auto [x, y, w] : a) tmp[x] = (tmp[x] + (ull)w * v[y]) % p;
        swap(v, tmp);
    }
    auto rr = bm(b);
    assert(r == rr);
#endif
    reverse(all(r));
    for (ui &x : r) if (x) x = p - x;
    r.push_back(1);
    return r;
}
ui det(int n, vector<Q> a)//[0,m)
{
    vector<ui> b(n);
    for (ui &x : b) x = rnd() % p;
    assert(*min_element(all(b)));
    for (auto &[x, y, w] : a) w = (ull)w * b[x] % p;
    ui r = minpoly(n, a)[0], tmp = 1;
    for (ui x : b) tmp = (ull)tmp * x % p;
    r = (ull)r * ksm(tmp, p - 2) % p;
#ifdef safe
    for (ui &x : b) x = rnd() % p;
    assert(*min_element(all(b)));
    for (auto &[x, y, w] : a) w = (ull)w * b[x] % p;
    ui rr = minpoly(n, a)[0], tmpp = 1;
    for (ui x : b) tmpp = (ull)tmpp * x % p;
    rr = (ull)rr * ksm(tmpp, p - 2) % p * ksm(tmp, p - 2) % p;
    assert(r == rr);
#endif
    return n & 1 ? (p - r) % p : r;
}
vector<ui> gauss(const vector<Q> &a, vector<ui> v)
{
    int n = v.size(), i, j;
    for (auto [x, y, w] : a) assert(0 <= x && x < n && 0 <= y && y < n);
    vector<ui> u(n), b(2 * n + 1), tmp(n), tv = v;
    for (ui &x : u) x = rnd() % p;
    assert(*min_element(all(u)));
```

```
    for (ui &r : b)
    {
        for (i = 0; i < n; i++) r = (r + (ull)u[i] * v[i]) % p;
        fill(all(tmp), 0);
        for (auto [x, y, w] : a) tmp[x] = (tmp[x] + (ull)w * v[y]) % p;
        swap(v, tmp);
    }
    auto f = bm(b);
    f.insert(f.begin(), p - 1);
    int m = (int)f.size() - 2;
    v = tv; fill(all(u), 0);
    ui x;
    for (i = 0; i <= m; i++)
    {
        x = f[m - i];
        for (j = 0; j < n; j++) u[j] = (u[j] + (ull)v[j] * x) % p;
        fill(all(tmp), 0);
        for (auto [x, y, w] : a) tmp[x] = (tmp[x] + (ull)w * v[y]) % p;
        swap(v, tmp);
    }
    x = ksm((p - f.back()) % p, p - 2);
    for (ui &y : u) y = (ull)y * x % p;
#ifdef safe
    for (auto [x, y, w] : a) tv[x] = (tv[x] + (ull)(p - w) * u[y]) % p;
    assert(!*min_element(all(tv)));
#endif
    return u;
}
```

3.18 Min_25 筛

$f(p^k) = p^k(p^k - 1)$，求 $\sum\limits_{i=1}^{n} f(i)$。这个的原理我了解的不多，因此没有更多注释。

```
const int N = 1e5 + 2, p = 1e9 + 7, i6 = 166666668;
ll fs[N << 1], m;
int ss[N], ys[N << 1], s[N], f[N << 1], g[N << 1], ls[N << 1], cs[N << 1];
int gs, n, i, j, k, cnt, ct, ans, sq;
bool ed[N];
int S(ll n, int x)
{
    int r, i, j, l;
    ll k;
    if (ss[x] >= n) return 0;
    if (n > sq) r = g[ys[m / n]]; else r = g[n];
    if ((r = r - s[x]) < 0) r += p;
    for (i = x + 1; (ll)ss[i] * ss[i] <= n; i++) for (j = 1, k = ss[i]; k <= n; j++, k *= ss[i])
    {
        l = (k - 1) % p;
        r = (r + (ll)l * (l + 1) % p * ((j != 1) + S(n / k, i))) % p;
    }
    return r;
}
int main()
{
    n = 1e5;
    for (i = 2; i <= n; i++)
```

```
{
    if (!ed[i]) ss[++gs] = i;
    for (j = 1; (j <= gs) && (i * ss[j] <= n); j++)
    {
        ed[i * ss[j]] = 1;
        if (i % ss[j] == 0) break;
    }
}ss[gs + 1] = 1e6;
s[1] = ss[1] * ss[1];
for (i = 2; i <= gs; i++) s[i] = (s[i - 1] + (ll)ss[i] * ss[i]) % p;//s 是多项式在素数位置的前
    缀和
memcpy(cs, s, sizeof(s));
ll i, j, k, x, z; scanf("%lld", &m);
sq = n = sqrt(m); while ((ll)(n + 1) * (n + 1) <= m) ++n;
cnt = n - 1;
for (i = n; i <= m; i = j + 1) { j = m / (m / i); ++cnt; }ct = cnt++;
for (i = 1; i <= m; i = j + 1)
{
    j = m / (k = m / i);
    if (k <= n) g[fs[k] = k] = (k * (k + 1) * (k << 1 | 1) / 6 - 1) % p;//这里是多项式前缀和
        （不含1）
    else
    {
        z = k % p;//一样
        g[ys[j] = --cnt] = (z * (z + 1) % p * (z << 1 | 1) % p + p - 6) * i6 % p; fs[cnt] = k;
    }
}
cnt = ct;
for (j = 1; (j <= gs) && (z = (ll)ss[j] * ss[j]); j++) for (i = cnt; z <= fs[i]; i--)
{
    x = fs[i] / ss[j]; if (x > n) x = ys[m / x];
    g[i] = (g[i] + (ll)(p - ss[j]) * ss[j] % p * (g[x] - s[j - 1] + p)) % p;//另一处需要修改的
}
memcpy(ls, g, sizeof(g));
s[1] = ss[1];
for (i = 2; i <= gs; i++) s[i] = s[i - 1] + ss[i];
cnt = n - 1;
for (i = n; i <= m; i = j + 1) { j = m / (m / i); ++cnt; }ct = cnt++;
for (i = 1; i <= m; i = j + 1)
{
    j = m / (k = m / i);
    if (k <= n) g[fs[k] = k] = ((k * (k + 1) >> 1) - 1) % p;
    else
    {
        z = k % p;
        g[ys[j] = --cnt] = (z * (z + 1) - 2 >> 1) % p; fs[cnt] = k;
    }
}
cnt = ct;
for (j = 1; (j <= gs) && (z = (ll)ss[j] * ss[j]); j++) for (i = cnt; z <= fs[i]; i--)
{
    x = fs[i] / ss[j]; if (x > n) x = ys[m / x];
    g[i] = (g[i] + (ll)(p - ss[j]) * (g[x] - s[j - 1] + p)) % p;
}
for (i = 1; i <= cnt; i++) if ((g[i] = ls[i] - g[i]) < 0) g[i] += p;
for (i = 1; i <= gs; i++) if ((s[i] = cs[i] - s[i]) < 0) s[i] += p;
ans = S(m, 0) + 1; if (ans == p) ans = 0; printf("%d", ans);
```

```
}
```

这是一个常数较小的版本，实现的是质数个数。需要注意评测机 double 性能。

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
const int N = 3.2e5 + 2;
ll s[N];
int ss[N], ys[N], gs = 0;
bool ed[N];
ll cal(ll m)
{
    static ll g[N << 1], fs[N << 1];
    ll i, j, k, x;
    int n;
    int p, q, cnt;
    n = round(sqrt(m));
    q = lower_bound(ss + 1, ss + gs + 1, n) - ss;
    memset(g, 0, sizeof(g)); memset(ys, 0, sizeof(ys)); cnt = n - 1;
    for (i = n; i <= m; i = j + 1) { j = m / (m / i); ++cnt; }int ct = cnt++;
    for (i = 1; i <= m; i = j + 1)
    {
        j = m / (k = m / i);
        if (k <= n) g[fs[k] = k] = k - 1; else { g[ys[j] = --cnt] = k - 1; fs[cnt] = k; }
    }cnt = ct;
    for (j = 1; j <= q; j++) for (i = cnt; (ll)ss[j] * ss[j] <= fs[i]; i--)
    {
        x = fs[i] / ss[j]; if (x > n) x = ys[m / x];
        g[i] -= g[x] - j + 1;
    }
    return g[cnt];//这里 g[cnt-i+1] 表示的是 [1,m/i] 的答案
}
int main()
{
    int n, i, j, t;
    n = 3.2e5;
    for (i = 2; i <= n; i++)
    {
        if (!ed[i]) ss[++gs] = i;
        for (j = 1; (j <= gs) && (i * ss[j] <= n); j++)
        {
            ed[i * ss[j]] = 1;
            if (i % ss[j] == 0) break;
        }
    }
    s[1] = ss[1];
    for (i = 2; i <= gs; i++) s[i] = s[i - 1] + ss[i];
    t = 1;
    ll m;
    while (t--) cin >> m, cout << cal(m) << '\n';
}
```

3.19 扩展 min-max 容斥（重返现世）

$$k\text{-th}\max\{S\} = \sum_{T\subseteq S}(-1)^{|T|-k}\binom{|T|-1}{k-1}\min\{T\}$$

```
scanf("%d%d%d", &n, &q, &m); inv[1] = 1; q = n + 1 - q;
for (i = 2; i <= m; i++) inv[i] = p - (ll)p / i * inv[p % i] % p;
for (i = 1; i <= n; i++) scanf("%d", a + i); f[0][0] = 1;
for (j = 1; j <= n; j++) for (i = q; i; i--) for (k = m; k >= a[j]; k--) if ((f[i][k] = f[i][k] +
    f[i - 1][k - a[j]] - f[i][k - a[j]]) >= p) f[i][k] -= p; else if (f[i][k] < 0) f[i][k] += p;
for (i = 1; i <= m; i++) ans = (ans + (ll)f[q][i] * inv[i]) % p;
ans = (ll)ans * m % p; printf("%d", ans);
```

3.20 光速乘

$O(n2^n)$，$O(2^n)$。

```
ll mul(ll x, ll y)
{
    x = x * y - (ll)((ldb)x / p * y + 1e-8) * p;
    if (x < 0) return x + p; return x;
}
```

3.21 二次剩余

```
namespace cipolla
{
    typedef unsigned int ui;
    typedef unsigned long long ull;
    ui p, w;
    struct Q
    {
        ull x, y;
        Q operator*(const Q &o) const { return {(x * o.x + y * o.y % p * w) % p, (x * o.y + y * o.
            x) % p}; }
    };
    ui ksm(ull x, ui y)
    {
        ull r = 1;
        while (y)
        {
            if (y & 1) r = r * x % p;
            x = x * x % p; y >>= 1;
        }
        return r;
    }
    Q ksm(Q x, ui y)
    {
        Q r = {1, 0};
        while (y)
        {
            if (y & 1) r = r * x;
            x = x * x; y >>= 1;
        }
        return r;
    }
    ui mosqrt(ui x, ui P)//0<=x<P
    {
        if (x == 0 || P == 2) return x;
```

```
        p = P;
        if (ksm(x, p - 1 >> 1) != 1) return -1;
        ui y;
        mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
        do y = rnd() % p, w = ((ull)y * y + p - x) % p; while (ksm(w, p - 1 >> 1) <= 1);//not for
            p=2
        y = ksm({y, 1}, p + 1 >> 1).x;
        if (y * 2 > p) y = p - y;//两解取小
        return y;
    }
}
using cipolla::mosqrt;
```

3.22 k 次剩余

```
namespace get_root
{
    typedef unsigned int ui;
    typedef unsigned long long ull;
    bool ied = 0;
    const int N = 1e5 + 5;
    vector<ui> pr;
    bool ed[N];
    void init()
    {
        pr.reserve(N);
        for (ui i = 2; i < N; i++)
        {
            if (!ed[i]) pr.push_back(i);
            for (ui x : pr)
            {
                if (i * x >= N) break;
                ed[i * x] = 1;
                if (i % x == 0) break;
            }
        }
    }
    ui ksm(ui x, ui y, ui p)
    {
        ui r = 1;
        while (y)
        {
            if (y & 1) r = (ull)r * x % p;
            x = (ull)x * x % p; y >>= 1;
        }
        return r;
    }
    vector<ui> getw(ui n)
    {
        vector<ui> w;
        for (ui x : pr)
        {
            if (x * x > n) break;
            if (n % x == 0)
            {
```

```
            w.push_back(x);
            n /= x;
            for (ui i = n / x; n == x * i; i = n / x) n /= x;
        }
    }
    if (n > 1) w.push_back(n);
    return w;
  }
  int getrt(ui n)
  {
    if (n <= 2) return n - 1;
    if (!ed[4]) init();
    auto w = getw(n);
    ui ph = n;
    for (ui x : w) ph = ph / x * (x - 1);
    w = getw(ph);
    for (ui &x : w) x = ph / x;
    for (ui i = 2; i < n; i++) if (gcd(i, n) == 1)
    {
      for (ui x : w) if (ksm(i, x, n) == 1) goto no;
      return i;
    no:;
    }
    return -1;
  }
}
namespace BSGS
{
  typedef unsigned int ui;
  typedef unsigned long long ull;
  template<int N, class T, class TT> struct ht//个数，定义域，值域
  {
    const static int p = 1e6 + 7, M = p + 2;
    TT a[N];
    T v[N];
    int fir[p + 2], nxt[N], st[p + 2];//和模数相适应
    int tp, ds;//自定义模数
    ht() { memset(fir, 0, sizeof fir); tp = ds = 0; }
    void mdf(T x, TT z)//位置，值
    {
      ui y = x % p;
      for (int i = fir[y]; i; i = nxt[i]) if (v[i] == x) return a[i] = z, void();//若不可能重
          复不需要 for
      v[++ds] = x; a[ds] = z;
      if (!fir[y]) st[++tp] = y;
      nxt[ds] = fir[y]; fir[y] = ds;
    }
    TT find(T x)
    {
      ui y = x % p;
      int i;
      for (i = fir[y]; i; i = nxt[i]) if (v[i] == x) return a[i];
      return 0;//返回值和是否判断依据要求决定
    }
    void clear()
    {
      ++tp;
```

```
        while (--tp) fir[st[tp]] = 0;
        ds = 0;
    }
};
const int N = 5e4;
ht<N, ui, ui> s;
int exgcd(int a, int b)
{
    if (a == 1) return 1;
    return (1 - (long long)b * exgcd(b % a, a)) / a;//not ll
}
int bsgs(ui a, ui b, ui p)
{
    s.clear();
    a %= p; b %= p;
    if (!a) return 1 - min((int)b, 2);//含 -1
    ui i, j, k, x, y;
    x = sqrt(p) + 2;
    for (i = 0, j = 1; i < x; i++, j = (ull)j * a % p)
    {
        if (j == b) return i;
        s.mdf((ull)j * b % p, i + 1);
    }
    k = j;
    for (i = 1; i <= x; i++, j = (ull)j * k % p) if (y = s.find(j)) return (ull)i * x - y + 1;
    return -1;
}
bool isprime(ui p)
{
    if (p <= 1) return 0;
    for (ui i = 2; i * i <= p; i++) if (p % i == 0) return 0;
    return 1;
}
int exbsgs(ui a, ui b, ui p)//a^x=b(mod p)
{
    //if (isprime(p)) return bsgs(a,b,p);
    a %= p; b %= p;
    ui i, j, k, x, y = __lg(p), cnt = 0;
    for (i = 0, j = 1 % p; i <= y; i++, j = (ull)j * a % p) if (j == b) return i;
    y = 1;
    while (1)
    {
        if ((x = gcd(a, p)) == 1) break;
        if (b % x) return -1;//no sol
        ++cnt;
        p /= x; b /= x;
        y = (ull)y * (a / x) % p;
    }
    a %= p;
    b = (ull)b * (p + exgcd(y, p)) % p;
    int r = bsgs(a, b, p);
    return r == -1 ? -1 : r + cnt;
}
}
pair<ll, ll> exgcd(ll a, ll b, ll c)//ax+by=c, {-1,-1} 无解, b=0 返回 {c/a,0}, 否则返回最小非负 x
{
    assert(a || b);
```

```
    if (!b) return {c / a, 0};
    if (a < 0) a = -a, b = -b, c = -c;
    ll d = gcd(a, b);
    if (c % d) return {-1, -1};
    ll x = 1, x1 = 0, p = a, q = b, k;
    b = abs(b);
    while (b)
    {
        k = a / b;
        x -= k * x1; a -= k * b;
        swap(x, x1);
        swap(a, b);
    }
    b = abs(q / d);
    x = x * (c / d) % b;
    if (x < 0) x += b;
    return {x, (c - p * x) / q};
}
ll fun(ll a, ll b, ll p)//ax=b(mod p)
{
    return exgcd(-p, a, b).second % p;
}
using get_root::getrt;
using BSGS::bsgs, BSGS::exbsgs;
int nth_root(ui k, ui y, ui p)//x^k=y(mod p)
{
    if (k == 0) return y == 1 ? 0 : -1;
    if (y == 0) return 0;
    ui g = getrt(p);
    ui z = bsgs(g, y, p);
    ll x = fun(k, z, p - 1);
    if (x == -1) return -1;
    return get_root::ksm(g, x, p);
}
```

网上的超快版本

```
#define popcount __builtin_popcount
using namespace std;
typedef long long ll;
//using ll=__int128_t;
typedef pair<ll, int> P;
ll gcd(ll a, ll b){
    if (b==0) return a;
    return gcd(b, a%b);
}
ll powmod(ll a, ll k, ll mod){
    ll ap=a, ans=1;
    while(k){
        if (k&1){
            ans*=ap;
            ans%=mod;
        }
        ap=ap*ap;
        ap%=mod;
        k>>=1;
    }
    return ans;
```

```
}
ll inv(ll a, ll m){
    ll b=m, x=1, y=0;
    while(b>0){
        ll t=a/b;
        swap(a-=t*b, b);
        swap(x-=t*y, y);
    }
    return (x%m+m)%m;
}
vector<P> fac(ll x){
    vector<P> ret;
    for(ll i=2; i*i<=x; i++){
        if (x%i==0){
            int e=0;
            while(x%i==0){
                x/=i;
                e++;
            }
            ret.push_back({i, e});
        }
    }
    if (x>1) ret.push_back({x, 1});
    return ret;
}
//mt19937_64 mt(334);
mt19937 mt(334);
ll solve1(ll p, ll q, int e, ll a){
    int s=0;
    ll r=p-1, qs=1, qp=1;
    while(r%q==0){
        r/=q;
        qs*=q;
        s++;
    }
    for(int i=0; i<e; i++) qp*=q;
    ll d=qp-inv(r%qp, qp);
    ll t=(d*r+1)/qp;
    ll at=powmod(a, t, p), inva=inv(a, p);
    if (e>=s){
        if (powmod(at, qp, p)!=a) return -1;
        else return at;
    }
    //uniform_int_distribution<long long> rnd(1, p-1);
    uniform_int_distribution<> rnd(1, p-1);
    ll rv;
    while(1){
        rv=powmod(rnd(mt), r, p);
        if (powmod(rv, qs/q, p)!=1) break;
    }
    int i=0;
    ll qi=1, sq=1;
    while(sq*sq<q) sq++;
    while(i<s-e){
        ll qq=qs/qp/qi/q;
        vector<P> v(sq);
        ll rvi=powmod(rv, qp*qq*(p-2)%(p-1), p), rvp=powmod(rv, sq*qp*qq, p);
```

```
        ll x=powmod(powmod(at, qp, p)*inva%p, qq*(p-2)%(p-1), p), y=1;
        for(int j=0; j<sq; j++){
            v[j]=P(x, j);
            (x*=rvi)%=p;
        }
        sort(v.begin(), v.end());
        ll z=-1;
        for(int j=0; j<sq; j++){
            int l=lower_bound(v.begin(), v.end(), P(y, 0))-v.begin();
            if (v[l].first==y){
                z=v[l].second+j*sq;
                break;
            }
            (y*=rvp)%=p;
        }
        if (z==-1) return -1;
        (at*=powmod(rv, z, p))%=p;
        i++;
        qi*=q;
        rv=powmod(rv, q, p);
    }
    return at;
}
ll solve0(ll p, ll q, ll r, ll a){
    ll d=q-inv(r%q, q);
    ll t=(d*r+1)/q;
    ll at=powmod(a, t, p), inva=inv(a, p);
    if (powmod(at, q, p)!=a) return -1;
    else return at;
}
ll solve(ll p, ll k, ll a)//p k y
{
    if (k==0)
    {
        if (a==1) return 1;
        return -1;
    }
    if (a==0) return 0;
    if (p==2 || a==1) return 1;
    ll a1=a;
    ll g=gcd(p-1, k);
    ll c=inv(k/g%((p-1)/g), (p-1)/g);
    a=powmod(a, c, p);
    if (g==1){
        if (powmod(a, k, p)==a1) return a;
        else return -1;
    }
    ll g1=gcd(g, (p-1)/g), g2=g;
    vector<P> f1=fac(g1), f;
    for(auto r:f1){
        ll q=r.first;
        int e=0;
        while(g2%q==0){
            g2/=q;
            e++;
        }
        f.push_back({q, e});
```

```
    }
    ll ret=1, gp=1;
    if (g2>1){
        ll x=solve0(p, g2, (p-1)/g2, a);
        if (x==-1) return -1;
        ret=x, gp*=g2;
    }
    for(auto r:f){
        ll qp=1;
        for(int i=0; i<r.second; i++) qp*=r.first;
        ll x=solve1(p, r.first, r.second, a);
        if (x==-1) return -1;
        if (gp==1){
            ret=x, gp*=qp;
            continue;
        }
        ll s=inv(gp%qp, qp), t=(1-gp*s)/qp;
        if (t>=0) ret=powmod(ret, t, p);
        else ret=powmod(ret, p-1+t%(p-1), p);
        if (s>=0) x=powmod(x, s, p);
        else x=powmod(x, p-1+s%(p-1), p);
        (ret*=x)%=p;
        gp*=qp;
    }
    if (powmod(ret, k, p)!=a1) return -1;
    return ret;
}
```

3.23 FWT/子集卷积

$O(n2^n)$，$O(2^n)$。注意全都是无符号的。

```
void fwt_and(vector<ull> &A)//本质：母集和
{
    ull n = A.size(), *a = A.data(), i, j, k, l, *f, *g;
    for (i = 1; i < n; i = l)
    {
        l = i * 2;
        for (j = 0; j < n; j += l)
        {
            f = a + j; g = a + j + i;
            for (k = 0; k < i; k++) f[k] += g[k];
        }
        if (l == n || i == 1 << 10) for (ull &x : A) x %= p;
    }
}
void ifwt_and(vector<ull> &A)
{
    ull n = A.size(), *a = A.data(), i, j, k, l, *f, *g;
    for (i = 1; i < n; i = l)
    {
        l = i * 2;
        for (j = 0; j < n; j += l)
        {
            f = a + j; g = a + j + i;
            for (k = 0; k < i; k++) f[k] += p * i - g[k];
        }
```

```
        if (l == n || i == 1 << 10) for (ull &x : A) x %= p;
    }
}
void fwt_or(vector<ull> &A)//本质：子集和
{
    ull n = A.size(), *a = A.data(), i, j, k, l, *f, *g;
    for (i = 1; i < n; i = l)
    {
        l = i * 2;
        for (j = 0; j < n; j += l)
        {
            f = a + j; g = a + j + i;
            for (k = 0; k < i; k++) g[k] += f[k];
        }
        if (l == n || i == 1 << 10) for (ull &x : A) x %= p;
    }
}
void ifwt_or(vector<ull> &A)
{
    ull n = A.size(), *a = A.data(), i, j, k, l, *f, *g;
    for (i = 1; i < n; i = l)
    {
        l = i * 2;
        for (j = 0; j < n; j += l)
        {
            f = a + j; g = a + j + i;
            for (k = 0; k < i; k++) g[k] += p * i - f[k];
        }
        if (l == n || i == 1 << 10) for (ull &x : A) x %= p;
    }
}
void fwt_xor(vector<ull> &A)
{
    ull n = A.size(), *a = A.data(), i, j, k, l, *f, *g;
    for (i = 1; i < n; i = l)
    {
        l = i * 2;
        for (j = 0; j < n; j += l)
        {
            f = a + j; g = a + j + i;
            for (k = 0; k < i; k++)
            {
                if ((f[k] += g[k]) >= p) f[k] -= p;
                g[k] = (f[k] + 2 * (p - g[k])) % p;
            }
        }
    }
}
void ifwt_xor(vector<ull> &A)
{
    ull n = A.size(), *a = A.data(), i, j, k, l, *f, *g, x = p + 1 >> 1, y = 1;
    for (i = 1; i < n; i = l)
    {
        l = i * 2;
        for (j = 0; j < n; j += l)
        {
            f = a + j; g = a + j + i;
```

```
        for (k = 0; k < i; k++)
        {
            if ((f[k] += g[k]) >= p) f[k] -= p;
            g[k] = (f[k] + 2 * (p - g[k])) % p;
        }
    }
    y = y * x % p;
  }
  for (i = 0; i < n; i++) a[i] = a[i] * y % p;
}
vector<ull> fst(const vector<ull> &s, const vector<ull> &t)
{
  int n = s.size(), m = __builtin_ctz(n), i, j, k;
  vector<ull> a[m + 1], b[m + 1], c[m + 1], r(n);
  for (i = 0; i <= m; i++) a[i].resize(n), b[i].resize(n), c[i].resize(n);
  for (i = 0; i < n; i++)
  {
    k = __builtin_popcount(i);
    a[k][i] = s[i];
    b[k][i] = t[i];
  }
  for (i = 0; i < m; i++) fwt_or(a[i]), fwt_or(b[i]);//如果魔改，上限需改为 m
  for (i = 0; i <= m; i++) for (j = 0; j <= i; j++) for (k = 0; k < n; k++) c[i][k] = (c[i][k] +
      (ull)a[j][k] * b[i - j][k]) % p;
  for (i = 1; i <= m; i++) ifwt_or(c[i]);//如果魔改，下限需改为 0
  for (i = 0; i < n; i++) r[i] = c[__builtin_popcount(i)][i];
  return r;
}
```

3.24 NTT

一种较快的 NTT（尤其是对于卷积以外的用途），但不推荐在不熟悉的情况下直接使用。一般的卷积可以参照字符串部分通配符的字符串匹配，其余的用途可以参照其他板子。

如果确实需要卡常，建议先抄写需要的函数，并递归地找到需要补的内容。

注意事项：所有 `ull` 为无符号。始终保证数组大小为 2^n，不应当使用 `resize` 而应该使用取模来调整长度。三种卷积对应的运算符见注释。

需要特别小心其长度的变化，注意不要越界。如果修改模数，`dft` 和 `hf_dft` 处有一个参数也要修改。

常见函数如下（带 new 的基本上都是较快但较长的）：

卷积 `operator*`，循环卷积 `operator&`，差卷积 `operator^`，求逆 `operator~/`（包含一个较短版，被注释了），分治 `cdq`，对数 `ln`，指数 `exp,exp_cdq,exp_new`，开方 `sqrt,sqrt_new`，幂函数 `pow(Q,ull),pow(Q,string),pow2(Q,ull),pow(Q,ull,Q)`，整除与取模 `div,mod,div_mod`，线性递推 `recurrent,recurrent_new,recurrent_interval`，连乘 `prod,prod_new`，
多点求值 `evaluation,evaluation_new`，阶乘 `factorial`，快速插值 `interpolation`，复合（逆）`comp,comp_inv`，多项式平移 `shift`，区间点值平移 `shift`，Z 变换 `czt`，贝尔数（$[n]$ 划分等价类方案数）`Bell`，斯特林数 `S1_row,S1_column,S2_row,S2_column,signed_S1_row`，伯努利数 `Bernoulli`，划分数 `Partition`，最大公因式 `gcd`，求根 `root`，模多项式意义的逆 `inverse`。

```
#include <optional>
namespace NTT
{
  using ull = unsigned long long;
  const ull g = 3, p = 998244353;
```

```
const int N = 1 << 22;//务必修改
ull inv[N], fac[N], ifac[N];//非必要
void getfac(int n)//非必要
{
    static int pre = -1;
    if (pre == -1) pre = 1, ifac[0] = fac[0] = fac[1] = ifac[1] = inv[1] = 1;
    if (n <= pre) return;
    for (int i = pre + 1, j; i <= n; i++)
    {
        j = p / i;
        inv[i] = (p - j) * inv[p - i * j] % p;
        fac[i] = fac[i - 1] * i % p;
        ifac[i] = ifac[i - 1] * inv[i] % p;
    }
    pre = n;
}
ull w[N];
int r[N];
ull ksm(ull x, ull y)
{
    ull r = 1;
    while (y)
    {
        if (y & 1) r = r * x % p;
        x = x * x % p;
        y >>= 1;
    }
    return r;
}
void init(int n)
{
    static int pr = 0, pw = 0;
    if (pr == n) return;
    int b = __lg(n) - 1, i, j, k;
    for (i = 1; i < n; i++) r[i] = r[i >> 1] >> 1 | (i & 1) << b;
    if (pw < n)
    {
        for (j = 1; j < n; j = k)
        {
            k = j * 2;
            ull wn = ksm(g, (p - 1) / k);
            w[j] = 1;
            for (i = j + 1; i < k; i++) w[i] = w[i - 1] * wn % p;
        }
        pw = n;
    }
    pr = n;
}
int cal(int x) { return 1 << __lg(max(x, 1) * 2 - 1); }
struct Q :vector<ull>
{
    bool flag;
    Q& operator%=(int n) { assert((n & -n) == n); resize(n); return *this; }
    Q operator%(int n) const
    {
        assert((n & -n) == n);
        if (size() <= n)
```

```
        {
            auto f = *this;
            return f %= n;
        }
        return Q(vector(begin(), begin() + n));
    }
    int deg() const
    {
        int n = size() - 1;
        while (n >= 0 && begin()[n] == 0) --n;
        return n;
    }
    explicit Q(int x = 1, bool f = 0) :flag(f), vector<ull>(cal(x)) { }//小心：{}会调用这条而非
        下一条
    Q(const vector<ull>& o, bool f = 0) :Q(o.size(), f) { copy(all(o), begin()); }
    Q(const initializer_list<ull>& o, bool f = 0) :Q(vector(o), f) { }
    ull fx(ull x)
    {
        ull r = 0;
        for (auto it = rbegin(); it != rend(); ++it) r = (r * x + *it) % p;
        return r;
    }
    void dft()
    {
        int n = size(), i, j, k;
        ull y, * f, * g, * wn, * a = data();
        init(n);
        for (i = 1; i < n; i++) if (i < r[i]) ::swap(a[i], a[r[i]]);
        for (k = 1; k < n; k *= 2)
        {
            wn = w + k;
            for (i = 0; i < n; i += k * 2)
            {
                g = (f = a + i) + k;
                for (j = 0; j < k; j++)
                {
                    y = g[j] * wn[j] % p;
                    g[j] = f[j] + p - y;
                    f[j] += y;
                }
            }//此处要求 14*p*p<=2^64。如果调整模数，需要修改 12。
            if (__lg(n / k) % 12 == 1) for (i = 0; i < n; i++) a[i] %= p;
        }
        if (flag)
        {
            y = ksm(n, p - 2);
            for (i = 0; i < n; i++) a[i] = a[i] * y % p;
            reverse(a + 1, a + n);
        }
        flag ^= 1;
    }
    void hf_dft()
    {
        assert(size() >= 2 && flag);
        int n = size() / 2, i, j, k;
        ull x, y, * f, * g, * wn, * a = data();
        init(n);
```

```
        for (i = 1; i < n; i++) if (i < r[i]) ::swap(a[i], a[r[i]]);
        for (k = 1; k < n; k *= 2)
        {
            wn = w + k;
            for (i = 0; i < n; i += k * 2)
            {
                g = (f = a + i) + k;
                for (j = 0; j < k; j++)
                {
                    y = g[j] * wn[j] % p;
                    g[j] = f[j] + p - y;
                    f[j] += y;
                }
            }
            if (__lg(n / k) % 12 == 1) for (i = 0; i < n; i++) a[i] %= p;
        }
        if (flag)
        {
            x = ksm(n, p - 2);
            for (i = 0; i < n; i++) a[i] = a[i] * x % p;
            reverse(a + 1, a + n);
        }
        flag ^= 1;
    }
    Q operator<<(int m) const
    {
        int n = deg(), i;
        Q r(n + m + 1);
        for (i = 0; i <= n; i++) r[i + m] = at(i);
        return r;
    }
    Q operator>>(int m) const
    {
        int n = deg(), i;
        if (n < m) return Q();
        Q r(n + 1 - m);
        for (i = m; i <= n; i++) r[i - m] = at(i);
        return r;
    }
};
Q shrink(Q f) { return f %= cal(f.deg() + 1); }
ostream& operator<<(ostream& cout, const Q& o)
{
    int n = o.deg();
    if (n < 0) return cout << "[0]";
    cout << "[" << o[n];
    for (int i = n - 1; i >= 0; i--) cout << ", " << o[i];
    return cout << "]";
}
Q der(const Q& f)
{
    ull n = f.size(), i;
    Q r(n);
    for (i = 1; i < n; i++) r[i - 1] = f[i] * i % p;
    return r;
}
Q integral(const Q& f)
```

```
{
    ull n = f.size(), i;
    getfac(n);
    Q r(n);
    for (i = 1; i < n; i++) r[i] = f[i - 1] * inv[i] % p;
    return r;
}
Q& operator+=(Q& f, ull x) { (f[0] += x) %= p; return f; }
Q operator+(Q f, ull x) { return f += x; }
Q& operator-=(Q& f, ull x) { (f[0] += p - x) %= p; return f; }
Q operator-(Q f, ull x) { return f -= x; }
Q& operator*=(Q& f, ull x) { for (ull& y : f) (y *= x) %= p; return f; }
Q operator*(Q f, ull x) { return f *= x; }
Q& operator+=(Q& f, const Q& g)
{
    f %= max(f.size(), g.size());
    for (int i = 0; i < g.size(); i++) f[i] = (f[i] + g[i]) % p;
    return f;
}
Q operator+(Q f, const Q& g) { return f += g; }
Q& operator-=(Q& f, const Q& g)
{
    f %= max(f.size(), g.size());
    for (int i = 0; i < g.size(); i++) f[i] = (f[i] + p - g[i]) % p;
    return f;
}
Q operator-(Q f, const Q& g) { return f -= g; }
Q& operator*=(Q& f, Q g)//卷积
{
    if (f.flag | g.flag)
    {
        int n = f.size(), i;
        assert(n == g.size());
        if (!f.flag) f.dft();
        if (!g.flag) g.dft();
        for (i = 0; i < n; i++) (f[i] *= g[i]) %= p;
        f.dft();
    }
    else
    {
        int n = cal(f.size() + g.size() - 1), i, j;
        int m1 = f.deg(), m2 = g.deg();
        if ((ull)m1 * m2 > (ull)n * __lg(n) * 8)
        {
            (f %= n).dft(); (g %= n).dft();
            for (i = 0; i < n; i++) (f[i] *= g[i]) %= p;
            f.dft();
        }
        else
        {
            vector<ull> r(max(0, m1 + m2 + 1));
            for (i = 0; i <= m1; i++) for (j = 0; j <= m2; j++) (r[i + j] += f[i] * g[j]) %= p;
            f = Q(n);
            copy(all(r), f.begin());
        }
    }
    return f;
```

```
}
Q operator*(Q f, const Q& g) { return f *= g; }
Q& operator&=(Q& f, Q g)//循环卷积
{
    assert(f.size() == g.size());
    int n = f.size(), i;
    if (!f.flag) f.dft();
    if (!g.flag) g.dft();
    for (i = 0; i < n; i++) (f[i] *= g[i]) %= p;
    f.dft();
    return f;
}
Q operator&(Q f, const Q& g) { return f &= g; }
Q& operator^=(Q& f, Q g)//差卷积
{
    int n = f.size();
    g %= n;
    reverse(all(g));
    f *= g;
    rotate(f.begin(), n - 1 + all(f));
    return f %= n;
}
Q operator^(Q f, const Q& g) { return f ^= g; }
Q sqr(Q f)
{
    assert(!f.flag);
    int n = f.size() * 2, i;
    (f %= n).dft();
    for (i = 0; i < n; i++) f[i] = f[i] * f[i] % p;
    f.dft();
    return f;
}
/*Q operator~(const Q &f)
{
    Q r;
    r[0]=ksm(f[0],p-2);
    for (int i=1; i<=f.size(); i*=2) r=(-((f%i)*r-2)*r)%i;
    return r;
}//trivial, 5e5 750ms*/
Q operator~(const Q& f)
{
    Q q, r, g;
    int n = f.size(), i, j, k;
    r[0] = ksm(f[0], p - 2);
    for (j = 2; j <= n; j *= 2)
    {
        k = j / 2;
        g = (r %= j) % k;
        r.dft();
        q = f % j * r;
        fill_n(q.begin(), k, 0);
        r *= q;
        copy(all(g), r.begin());
        for (i = k; i < j; i++) r[i] = (p - r[i]) % p;
    }
    return r;
}//5e5 200ms, inv(1 6 3 4 9)=(1 998244347 33 998244169 1020)
```

```
Q& operator/=(Q& f, const Q& g) { int n = f.size(); return (f *= ~g) %= n; }
Q operator/(Q f, const Q& g) { return f /= g; }
void cdq(Q& f, Q& g, int l, int r)//g_0=1,i*g_i=g_{i-j}*f_j,use for cdq
{
    static vector<Q> cd;
    int i, m = l + r >> 1, n = r - l, nn = n >> 1;
    if (r - l == f.size())
    {
        getfac(n - 1);
        g = Q(n);
        cd.clear();
        for (i = 2; i <= n; i *= 2)
        {
            cd.emplace_back(i);
            Q& h = cd.back();
            h %= i;
            copy_n(f.begin(), i, h.begin());
            h.dft();
        }
    }
    if (l + 1 == r)
    {
        g[l] = l ? g[l] * inv[l] % p : 1;
        return;
    }
    cdq(f, g, l, m);
    Q h(n);
    copy_n(g.begin() + l, nn, h.begin());
    h *= cd[__lg(n) - 1];
    for (i = m; i < r; i++) (g[i] += h[i - l]) %= p;
    cdq(f, g, m, r);
}
Q exp_cdq(Q f)
{
    Q g;
    int n = f.size(), i;
    for (i = 1; i < n; i++) f[i] = f[i] * i % p;
    cdq(f, g, 0, n);
    return g;
}//5e5 455ms
Q ln(const Q& f) { return integral(der(f) / f); }
//5e5 330ms, ln(1 2 3 4 5)=(0 2 1 665496236 499122177)
Q exp(Q f)
{
    Q r; r[0] = 1;
    for (int i = 1; i <= f.size(); i *= 2) (r *= f % i - ln(r % i) + 1) %= i;
    return r;
}//5e5 700ms, exp(0 4 2 3 5)=(1 4 10 665496257 665496281)
Q exp_new(Q b)
{
    Q h, f, r, u, v, bj;
    int n = b.size(), i, j, k;
    r[0] = h[0] = 1;
    for (j = 2; j <= n; j *= 2)
    {
        f = bj = der(b % j); k = j / 2; fill(k + all(bj), 0);
        h.dft(); u = der(r) & h;
```

```
        v = (r & h) % j - 1 & bj;
        for (i = 0; i < k; i++) f[i + k] = (p * p + u[i] - v[i] - f[i] - f[i + k]) % p, f[i] =
            0;
        f[k - 1] = (f[j - 1] + v[k - 1]) % p;
        u = (r %= j) & integral(f);
        for (i = k; i < j; i++) r[i] = (p - u[i]) % p;
        if (j < n) h = ~r;
    }
    return r;
}//5e5 420ms
optional<ull> mosqrt(ull x)
{
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static ull W;
    struct P
    {
        ull x, y;
        P operator*(const P& a) const
        {
            return {(x * a.x + y * a.y % p * W) % p, (x * a.y + y * a.x) % p};
        }
    };
    if (x == 0) return {0};
    if (ksm(x, p - 1 >> 1) != 1) return { };
    ull y;
    do y = rnd() % p; while (ksm(W = (y * y % p + p - x) % p, p - 1 >> 1) <= 1);//not for p=2
    y = [&](P x, ull y)
        {
            P r{1, 0};
            while (y)
            {
                if (y & 1) r = r * x;
                x = x * x; y >>= 1;
            }
            return r.x;
        }({y, 1}, p + 1 >> 1);
    return {y * 2 < p ? y : p - y};
}
optional<Q> sqrt(Q f)
{
    const static ull i2 = p + 1 >> 1;
    Q r;
    int n = f.size(), i, l;

    for (i = 0; i < n; i++) if (f[i]) break;
    if (i == n) return f;
    if (i & 1) return { };
    l = i / 2;
    copy(i + all(f), f.begin());
    fill(n - i + all(f), 0);

    auto rt = mosqrt(f[0]);
    if (rt) r[0] = rt.value(); else return { };
    for (i = 2; i <= n; i *= 2) r = (sqr(r) + f % i) / (r % i) % i * i2;

    copy_backward(all(r) - l, r.end());
    fill_n(r.begin(), l, 0);
```

```
    return {r};
}//5e5 530ms, sqrt(0 0 4 2 3)=(0 2 499122177 311951361 171573248)
optional<Q> sqrt_new(Q f)
{
    const static ull i2 = p + 1 >> 1;
    Q q, r;
    int n = f.size(), i, j, k, l;

    for (i = 0; i < n; i++) if (f[i]) break;
    if (i == n) return f;
    if (i & 1) return { };
    l = i / 2;
    copy(i + all(f), f.begin());
    fill(n - i + all(f), 0);

    auto rt = mosqrt(f[0]);
    if (rt) r[0] = rt.value(); else return { };
    for (j = 2; j <= n; j *= 2)
    {
        k = j / 2; (q = r).dft(); (q &= q) %= j;
        for (i = k; i < j; i++) q[i] = (q[i - k] + p * 2 - f[i] - f[i - k]) * i2 % p, q[i - k]
            = 0;
        q &= ~r % j; r %= j;
        for (i = k; i < j; i++) r[i] = (p - q[i]) % p;
    }

    copy_backward(all(r) - l, r.end());
    fill_n(r.begin(), l, 0);

    return {r};
}//5e5 280ms
Q pow(Q b, ull m)//不应传入超过 int 内容
{
    assert(m <= 1llu << 32);
    int n = b.size(), i, j = n, k;
    for (i = 0; i < n; i++) if (b[i]) { j = i; break; }
    if (j == n) return b[0] = !m, b;
    if (j * m >= n) return Q(n);
    copy(j + all(b), b.begin());
    fill(n - j + all(b), 0);
    k = b[0]; j *= m;
    b = exp_new(ln(b * ksm(k, p - 2)) * m) * ksm(k, m);
    copy_backward(all(b) - j, b.end());
    fill_n(b.begin(), j, 0);
    return b;
}
Q pow(Q b, string s)
{
    int n = b.size(), i, j = n, k;
    for (i = 0; i < n; i++) if (b[i]) { j = i; break; }
    if (j == n) return b[0] = s == "0", b;
    if (j && (s.size() > 8 || j * stoll(s) >= n)) return Q(n);
    ull m0 = 0, m1 = 0;
    for (auto c : s) m0 = (m0 * 10 + c - '0') % p, m1 = (m1 * 10 + c - '0') % (p - 1);
    copy(j + all(b), b.begin());
    fill(n - j + all(b), 0);
```

```
    k = b[0]; j *= m0;
    b = exp_new(ln(b * ksm(k, p - 2)) * m0) * ksm(k, m1);
    copy_backward(all(b) - j, b.end());
    fill_n(b.begin(), j, 0);
    return b;
}//5e5 1e18 700ms
Q pow2(Q b, ull m)
{
    int n = b.size();
    Q r(n); r[0] = 1;
    while (m)
    {
        if (m & 1) (r *= b) %= n;
        if (m >>= 1) b = sqr(b) % n;
    }
    return r;
}//5e5 1e18 7425ms
Q div(Q f, Q g)
{
    int n = 0, m = 0, i;
    for (i = f.size() - 1; i >= 0; i--) if (f[i]) { n = i + 1; break; }
    for (i = g.size() - 1; i >= 0; i--) if (g[i]) { m = i + 1; break; }
    assert(m);
    if (n < m) return Q(1);
    reverse(f.begin(), f.begin() + n);
    reverse(g.begin(), g.begin() + m);
    n = n - m + 1; m = cal(n);
    f = (f % m) / (g % m) % m;
    fill(n + all(f), 0);
    reverse(f.begin(), f.begin() + n);
    return f;
}
Q mod(const Q& a, const Q& b)
{
    if (a.deg() < b.deg()) return shrink(a);
    Q r = (a - b * div(a, b));
    return shrink(r %= min(r.size(), b.size()));
}
Q pow(Q x, ull y, Q f)
{
    Q r(1);
    r[0] = 1;
    while (y)
    {
        if (y & 1) r = mod(r * x, f);
        if (y >>= 1) x = mod(sqr(x), f);
    }
    return r;
}
pair<Q, Q> div_mod(const Q& a, const Q& b) { Q q = div(a, b); Q r = (a - b * q); return {q, r
    %= min(r.size(), b.size())}; }
//5e5 430ms (1 2 3 4)=(916755018 427819009)*(5 6 7)+(407446676 346329673)
// Q cdq_inv(const Q &f) { return (~(f-1))*(p-1); }//g_0=1,g_i=g_{i-j}*f_j ?
ull recurrent(const vector<ull>& f, const vector<ull>& a, ull m)//常系数齐次线性递推, find a_m,
    a_n=a_{n-i}*f_i,f_1...k,a_0...k-1
{
    if (m < a.size()) return a[m];
```

```
    assert(f.size() == a.size() + 1 && f[0] == 0);
    int k = a.size(), n = cal(k + 1) * 2, i;
    ull ans = 0;
    Q h(n), g(2);
    for (i = 1; i <= k; i++) h[k - i] = (p - f[i]) % p;
    h[k] = g[1] = 1;
    Q r = pow(g, m, h);
    k = min(k, (int)r.size());
    for (i = 0; i < k; i++) ans = (ans + a[i] * r[i]) % p;
    return ans;
}//1e5 1e18 8500ms
ull recurrent_new(const vector<ull>& f, const vector<ull>& a, ull m)//常系数齐次线性递推, find
    a_m,a_n=a_{n-i}*f_i,f_1...k,a_0...k-1
{
    const static ull i2 = p + 1 >> 1;
    if (m < a.size()) return a[m];
    assert(f.size() == a.size() + 1 && f[0] == 0);
    int k = a.size(), n = cal(k + 1), i;
    Q g(n * 2), h(n * 2);
    for (h[0] = i = 1; i <= k; i++) h[i] = (p - f[i]) % p;
    copy(all(a), g.begin());
    g &= h; fill(k++ + all(g), 0);
    vector<ull> res(n);
    while (m)
    {
        if (m & 1)
        {
            ull x = p - g[0];
            for (i = 1; i < k; i += 2) res[i >> 1] = x * h[i] % p;
            copy_n(g.begin() + 1, k - 1, g.begin());
            g[k - 1] = 0;
        }
        g.dft(); h.dft();
        ull* a = g.data(), * b = h.data(), * c = a + n, * d = b + n;
        for (i = 0; i < n; i++) g[i] = (a[i] * d[i] + b[i] * c[i]) % p * i2 % p;
        for (i = 0; i < n; i++) h[i] = h[i] * h[i ^ n] % p;
        g.hf_dft(); h.hf_dft();
        fill(k + all(g), 0);
        if (m & 1) for (i = 0; i < k; i++) (g[i] += res[i]) %= p;
        fill(k + all(h), 0);
        m >>= 1;
    }
    assert(h[0] == 1);
    return g[0];
}//1e5 1e18 1000ms
vector<ull> recurrent_interval(const vector<ull>& f, const vector<ull>& a, ull L, ull R)//常系
    数齐次线性递推, find a_[L,R),a_n=a_{n-i}*f_i,f_1...k,a_0...k-1
{
    assert(f.size() == a.size() + 1 && f[0] == 0);
    int k = a.size(), n = cal(k + 1) * 2, i, len = R - L;
    ull ans = 0, m = L;
    Q h(n), g(2), r;
    for (i = 1; i <= k; i++) h[k - i] = (p - f[i]) % p;
    h[k] = g[1] = r[0] = 1;
    while (m)
    {
        if (m & 1) r = mod(r * g, h);
```

```
        if (m >>= 1) g = mod(sqr(g), h);
    }
    Q F(f), A(a);
    F[0] = p - 1;
    A *= F;
    A %= cal(k);
    fill(k + all(A), 0);
    n = cal(len + k);
    F %= n;
    A *= ~F;
    r %= cal(k);
    reverse(r.begin(), r.begin() + k);
    r *= A;
    r.erase(r.begin(), r.begin() + k - 1);
    r.resize(len);
    return r;
}//1e5 1e18 5e5 10000ms
Q prod(const vector<Q>& a)
{
    if (!a.size()) return {1};
    function<Q(int, int)> dfs = [&](int l, int r)
        {
            if (r - l == 1) return a[l];
            int m = l + r >> 1;
            return shrink(dfs(l, m) * dfs(m, r));
        };
    return dfs(0, a.size());
}//not check
Q prod_new(const vector<Q>& a)
{
    if (!a.size()) return {1};
    struct cmp
    {
        bool operator()(const Q& f, const Q& g) const { return f.size() > g.size(); }
    };
    priority_queue<Q, vector<Q>, cmp> q(all(a));
    while (q.size() > 1)
    {
        auto f = q.top(); q.pop();
        f = shrink(f * q.top()); q.pop();
        q.push(f);
    }
    return q.top();
}//not check
vector<ull> evaluation(const Q& f, const vector<ull>& X)
{
    int m = X.size(), n = f.size() - 1, i, j;
    vector<Q> pro(m * 4 + 4);
    while (n > 1 && !f[n]) --n;
    vector<ull> y(m);
    function<void(int, int, int)> build = [&](int x, int l, int r)
        {
            if (l + 1 == r)
            {
                pro[x] = Q(vector{(p - X[l]) % p, 1llu});
                return;
            }
```

```
        int mid = l + r >> 1, c = x * 2;
        build(c, l, mid); build(c + 1, mid, r);
        pro[x] = shrink(pro[c] * pro[c + 1]);
    };
    function<void(int, int, int, Q, int)> dfs = [&](int x, int l, int r, Q f, int d)
    {
        const static int limit = 256;
        if (d >= r - l) f = shrink(mod(f, pro[x]));
        if (r - l < limit)
        {
            for (int i = l; i < r; i++) y[i] = f.fx(X[i]);
            return;
        }
        int mid = l + r >> 1, c = x * 2;
        dfs(c, l, mid, f, d);
        dfs(c + 1, mid, r, f, d);
    };
    build(1, 0, m);
    dfs(1, 0, m, f, n);
    return y;
}//131072 880ms
vector<ull> evaluation_new(Q f, const vector<ull>& X)//多项式多点求值
{
    int m = X.size(), i, j;
    vector<ull> y(m);
    if (X.size() <= 10)
    {
        for (i = 0; i < m; i++) y[i] = f.fx(X[i]);
        return y;
    }
    int n = f.size();
    while (n > 1 && !f[n - 1]) --n;
    f.resize(cal(n));
    vector<Q> pro(m * 4 + 4);
    function<void(int, int, int)> build = [&](int x, int l, int r)
    {
        if (l == r)
        {
            pro[x] = Q(vector{1llu, (p - X[l]) % p});
            return;
        }
        int m = l + r >> 1, c = x * 2;
        build(c, l, m); build(c + 1, m + 1, r);
        pro[x] = shrink(pro[c] * pro[c + 1]);
    };
    function<void(int, int, int, Q)> dfs = [&](int x, int l, int r, Q f)
    {
        const static int limit = 30;
        if (r - l + 1 <= limit)
        {
            int m = r - l + 1, m1, m2, mid = l + r >> 1, i, j, k;
            static ull g[limit + 2], g1[limit + 2], g2[limit + 2];
            m1 = m2 = r - l;
            copy_n(f.data(), m, g1);
            copy_n(g1, m, g2);
            for (i = mid + 1; i <= r; i++, --m1) for (k = 0; k < m1; k++) g1[k] = (g1[k] +
                g1[k + 1] * (p - X[i])) % p;
```

```
            for (i = 1; i <= mid; i++, --m2) for (k = 0; k < m2; k++) g2[k] = (g2[k] + g2[k
                + 1] * (p - X[i])) % p;
            for (i = 1; i <= mid; i++)
            {
                copy_n(g1, (m = m1) + 1, g);
                for (j = 1; j <= mid; j++) if (i != j)
                {
                    for (k = 0; k < m; k++) g[k] = (g[k] + g[k + 1] * (p - X[j])) % p;
                    --m;
                }
                y[i] = g[0];
            }
            for (i = mid + 1; i <= r; i++)
            {
                copy_n(g2, (m = m2) + 1, g);
                for (j = mid + 1; j <= r; j++) if (i != j)
                {
                    for (k = 0; k < m; k++) g[k] = (g[k] + g[k + 1] * (p - X[j])) % p;
                    --m;
                }
                y[i] = g[0];
            }
            return;
        }
        int mid = l + r >> 1, c = x * 2, n = f.size();
        f.dft();
        for (auto [x, len] : {pair{c, r - mid}, {c + 1, mid - l + 1}})
        {
            pro[x] %= n;
            reverse(all(pro[x])); pro[x] &= f;
            rotate(all(pro[x]) - 1, pro[x].end());
            pro[x] %= cal(len);
            fill(len + all(pro[x]), 0);
        }
        dfs(c, l, mid, pro[c + 1]);
        dfs(c + 1, mid + 1, r, pro[c]);
    };
    build(1, 0, m - 1);
    pro[1] %= f.size();
    (f ^= ~pro[1]) %= cal(m);
    fill(min(m, n) + all(f), 0);
    dfs(1, 0, m - 1, f);
    return y;
}//131072 460ms
ull factorial(ull n)
{
    if (n >= p) return 0;
    if (n <= 1) return 1 % p;
    ull B = ::sqrt(n), i;
    vector F(B, Q({0, 1}));
    for (i = 0; i < B; i++) F[i][0] = i + 1;
    auto f = prod(F);
    vector<ull> x(B);
    for (i = 0; i < B; i++) x[i] = i * B;
    ull r = 1;
    auto y = evaluation(f, x);
    for (i = 0; i < B; i++) r = r * y[i] % p;
```

```
    for (i = B * B + 1; i <= n; i++) r = r * i % p;
    return r;
}//998244352 170ms
vector<ull> getinvs(vector<ull> a)
{
    int n = a.size(), i;
    if (n <= 2)
    {
        for (i = 0; i < n; i++) a[i] = ksm(a[i], p - 2);
        return a;
    }
    vector<ull> l(n), r(n);
    l[0] = a[0]; r[n - 1] = a[n - 1];
    for (i = 1; i < n; i++) l[i] = l[i - 1] * a[i] % p;
    for (i = n - 2; i; i--) r[i] = r[i + 1] * a[i] % p;
    ull x = ksm(l[n - 1], p - 2);
    a[0] = x * r[1] % p; a[n - 1] = x * l[n - 2] % p;
    for (i = 1; i < n - 1; i++) a[i] = x * l[i - 1] % p * r[i + 1] % p;
    return a;
}
Q interpolation(const vector<ull>& X, const vector<ull>& y)//多项式快速插值
{
    assert(X.size() == y.size());
    int n = X.size(), i, j;
    if (n <= 1) return Q(y);
    if (1)
    {
        auto vv = X; sort(all(vv));
        assert(unique(all(vv)) - vv.begin() == n);
    }
    vector<Q> sum(4 * n + 4), pro(4 * n + 4);
    function<void(int, int, int)> build = [&](int x, int l, int r)
        {
            if (l == r)
            {
                sum[x] = Q(vector{(p - X[l]) % p, 1llu});
                return;
            }
            int mid = l + r >> 1, c = x * 2;
            build(c, l, mid); build(c + 1, mid + 1, r);
            sum[x] = shrink(sum[c] * sum[c + 1]);
        };
    build(1, 0, n - 1);
    auto v = evaluation_new(sum[1] = der(sum[1]), X);
    assert(v.size() == n);
    auto Y = getinvs(v);
    for (i = 0; i < n; i++) Y[i] = Y[i] * y[i] % p;
    function<void(int, int, int)> dfs = [&](int x, int l, int r)
        {
            if (l == r)
            {
                pro[x][0] = Y[l];
                return;
            }
            int c = x * 2, mid = l + r >> 1;
            dfs(c, l, mid); dfs(c | 1, mid + 1, r);
            pro[x] = shrink((pro[c] * sum[c | 1]) + (pro[c | 1] * sum[c]));
```

```
    };
    dfs(1, 0, n - 1);
    return pro[1] %= cal(n);
}//131072 1150ms
Q comp(const Q& f, Q g)//多项式复合 f(g(x))=[x^i]f(x)g(x)^i
{
    int n = f.size(), l = ceil(::sqrt(n)), i, j;
    assert(n >= g.size());//返回 n-1 次多项式
    vector<Q> a(l + 1), b(l);
    a[0] %= n; a[0][0] = 1; a[1] = g;
    g %= n * 2;
    Q u = g, v(n);
    g.dft();
    for (i = 2; i <= l; i++) a[i] = ((u &= g) %= n), u %= n * 2;
    for (i = 2; i < l; i++)
    {
        u.dft(); b[i - 1] = u;
        u &= b[1]; fill(n + all(u), 0);
    }
    u.dft(); b[l - 1] = u;
    for (i = 0; i < l; i++)
    {
        fill(all(v), 0);
        for (j = 0; j < l; j++) if (i * l + j < n) v += a[j] * f[i * l + j];
        if (i == 0) u = v; else u += ((v %= n * 2) &= b[i]) %= n;
    }
    return u;
}//n^2+n\sqrt n\log n, 8000 350ms
Q comp_inv(Q f)//多项式复合逆 g(f(x))=x, 求 g, [x^n]g=([x^{n-1}](x/f)^n)/n, 要求常数 0 一次非 0
{
    assert(!f[0] && f[1]);
    int n = f.size(), l = ceil(::sqrt(n)), i, j, k, m;//l>=2
    rotate(f.begin(), 1 + all(f));
    f = ~f;
    getfac(n * 2);
    vector<Q> a(l + 1), b(l);
    Q u, v;
    u = a[1] = f;
    u %= n * 2; (v = u).dft();
    for (i = 2; i <= l; i++)
    {
        u &= v;
        fill(n + all(u), 0);
        a[i] = u;
    }
    b[0] %= n; b[0][0] = 1; b[1] = u; (v = u).dft();
    for (i = 2; i < l; i++)
    {
        u &= v;
        fill(n + all(u), 0);
        b[i] = u;
    }
    u %= n; u[0] = 0;
    for (i = 0; i < l; i++) for (j = 1; j <= l; j++) if (i * l + j < n)
    {
        m = i * l + j - 1;
        ull r = 0, * f = b[i].data(), * g = a[j].data();
```

```
        for (k = 0; k <= m; k++) r = (r + f[k] * g[m - k]) % p;
        u[m + 1] = r * inv[m + 1] % p;
    }
    return u;
}//8000 200ms
Q shift(Q f, ull c)//get f(x+c),c\in [0,p)
{
    int n = f.size(), i, j;
    Q g(n);
    getfac(n);
    for (i = 0; i < n; i++) (f[i] *= fac[i]) %= p;
    g[0] = 1;
    for (i = 1; i < n; i++) g[i] = g[i - 1] * c % p;
    for (i = 0; i < n; i++) (g[i] *= ifac[i]) %= p;
    f ^= g;
    for (i = 0; i < n; i++) (f[i] *= ifac[i]) %= p;
    return f;
}//5e5 200ms (1 2 3 4 5) 3 -> (547 668 309 64 5)
vector<ull> shift(vector<ull> y, ull c, ull m)//[0,n) 点值 -> [c,c+m) 点值
{
    assert(y.size());
    if (y.size() == 1) return vector(m, y[0]);
    vector<ull> r, res;
    r.reserve(m);
    int n = y.size(), i, j, mm = m;
    while (c < n && m) r.push_back(y[c++]), --m;
    if (c + m > p)
    {
        res = shift(y, 0, c + m - p);
        m = p - c;
    }
    if (!m) { r.insert(r.end(), all(res)); return r; }
    int len = cal(m + n - 1), l = m + n - 1;
    for (i = n & 1; i < n; i += 2) y[i] = (p - y[i]) % p;
    getfac(n);
    for (i = 0; i < n; i++) y[i] = y[i] * ifac[i] % p * ifac[n - 1 - i] % p;
    y.resize(len);
    Q f, g;
    vector<ull> v(m + n - 1);
    c -= n - 1;
    for (i = 0; i < l; i++) v[i] = (c + i) % p;
    f = Q(y); g = Q(getinvs(v)) % len;
    f *= g;
    vector<ull> u(m);
    for (i = n - 1; i < l; i++) u[i - (n - 1)] = f[i];
    v.resize(m);
    for (i = 0; i < m; i++) v[i] = c + i;
    v = getinvs(v); c += n;
    ull tmp = 1;
    for (i = c - n; i < c; i++) tmp = tmp * i % p;
    for (i = 0; i < m; i++) (u[i] *= tmp) %= p, tmp = tmp * (c + i) % p * v[i] % p;
    r.insert(r.end(), all(u));
    r.insert(r.end(), all(res));
    assert(r.size() == mm);
    return r;
}//5e5 430ms, (1 4 9 16) 3 5 -> (16 25 36 49 64)
vector<ull> czt(Q f, ull c, ull m)//求 f(c^[0,m))。核心 ij=C(i+j,2)-C(i,2)-C(j,2)
```

```
{
    const static ull B = 1e5;
    static ull a[B + 2], b[B + 2];
    int i, n = f.size();
    if (n * m < B * 5)
    {
        vector<ull> r(m);
        ull j;
        for (i = 0, j = 1; i < m; i++) r[i] = f.fx(j), j = j * c % p;
        return r;
    }
    auto mic = [&](ull x) { return a[x % B] * b[x / B] % p; };
    ull l = cal(m += n - 1);
    Q g(l);
    assert(B * B > p);
    a[0] = b[0] = g[0] = g[1] = 1;
    for (i = 1; i <= B; i++) a[i] = a[i - 1] * c % p;
    for (i = 1; i <= B; i++) b[i] = b[i - 1] * a[B] % p;
    for (i = 2; i < n; i++) f[i] = f[i] * mic((p * 2 - 2 - i) * (i - 1) / 2 % (p - 1)) % p;
    for (i = 2; i < m; i++) g[i] = mic(i * (i - 1llu) / 2 % (p - 1));
    reverse(all(f)); (f %= l) &= g;
    vector<ull> r(f.begin() + n - 1, f.begin() + m); m -= n - 1;
    for (i = 2; i < m; i++) r[i] = r[i] * mic((p * 2 - 2 - i) * (i - 1) / 2 % (p - 1)) % p;
    return r;
}//luogu 1e6 500ms
vector<ull> Bell(int n)//B(0...n)
{
    ++n;
    getfac(n - 1);
    Q f(n);
    int i;
    for (i = 1; i < n; i++) f[i] = ifac[i];
    f = exp_new(f);
    for (i = 2; i < n; i++) f[i] = f[i] * fac[i] % p;
    return vector<ull>(f.begin(), f.begin() + n);
}//not check
vector<ull> S1_row(int n, int m)//S1(n,0...m),O(nlogn),unsigned
{
    int cm = cal(++m);
    if (n == 0)
    {
        vector<ull> r(m);
        r[0] = 1;
        return r;
    }
    function<Q(int)> dfs = [&](int n)
        {
            if (n == 1)
            {
                Q f(2);
                f[1] = 1;
                return f;
            }
            Q f = dfs(n / 2);
            f *= shift(f, n / 2);
            if (n & 1)
            {
```

```
                f %= cal(n + 1);
                for (int i = n; i; i--) f[i] = f[i - 1];
                // for (int i=1; i<=n; i++) f[i]=f[i-1];
                --n;
                for (int i = 0; i <= n; i++) f[i] = (f[i] + f[i + 1] * n) % p;
            }
            if (f.size() > cm) f %= cm;
            return f;
        };
    Q f = dfs(n);
    if (f.size() < cm) f %= cm;
    return vector<ull>(f.begin(), f.begin() + m);
}
vector<ull> S1_column(int n, int m)//S1(0...n,m),O(nlogn)
{
    if (m == 0)
    {
        vector<ull> r(n + 1);
        r[0] = 1;
        return r;
    }
    Q f(n + 1);
    getfac(max(n, m));
    int i;
    for (i = 1; i <= n; i++) f[i] = inv[i];
    f = pow(f, m);
    for (i = m; i <= n; i++) f[i] = f[i] * fac[i] % p * ifac[m] % p;
    return vector<ull>(f.begin(), f.begin() + n + 1);
}
vector<ull> S2_row(int n, int m)//S2(n,0...m),O(mlogm)
{
    int tm = ++m, i, j, cnt = 0;
    if (n == 0)
    {
        vector<ull> r(m);
        r[0] = 1;
        return r;
    }
    m = min(m, n + 1);
    vector<ull> pr(m), pw(m);
    pw[1] = 1;
    for (i = 2; i < m; i++)
    {
        if (!pw[i]) pr[cnt++] = i, pw[i] = ksm(i, n);
        for (j = 0; i * pr[j] < m; j++)
        {
            pw[i * pr[j]] = pw[i] * pw[pr[j]] % p;
            if (i % pr[j] == 0) break;
        }
    }
    getfac(m - 1);
    Q f(m), g(m);
    for (i = 0; i < m; i += 2) f[i] = ifac[i];
    for (i = 1; i < m; i += 2) f[i] = p - ifac[i];
    // for (i=1; i<m; i++) g[i]=pw[i]*ifac[i]%p;
    for (i = 1; i < m; i++) g[i] = ksm(i, n) * ifac[i] % p;
    f *= g;
```

```
    vector<ull> r(f.begin(), f.begin() + m);
    r.resize(tm);
    return r;
}//5e5 150ms
vector<ull> S2_column(int n, int m)//S2(0...n,m),O(nlogn)
{
    if (m == 0)
    {
        vector<ull> r(n + 1);
        r[0] = 1;
        return r;
    }
    Q f(n + 1);
    getfac(max(n, m));
    int i;
    for (i = 1; i <= n; i++) f[i] = ifac[i];
    f = pow(f, m);
    for (i = m; i <= n; i++) f[i] = f[i] * fac[i] % p * ifac[m] % p;
    return vector<ull>(f.begin(), f.begin() + n + 1);
}//5e5 640ms
vector<ull> signed_S1_row(int n, int m)
{
    auto v = S1_row(n, m);
    for (int i = 1 ^ n & 1; i <= m; i += 2) v[i] = (p - v[i]) % p;
    return v;
}//5e5 190ms
vector<ull> Bernoulli(int n)//B(0...n)
{
    getfac(++n);
    int i;
    Q f(n);
    for (i = 0; i < n; i++) f[i] = ifac[i + 1];
    f = ~f;
    for (i = 0; i < n; i++) f[i] = f[i] * fac[i] % p;
    return vector<ull>(f.begin(), f.begin() + n);
}//5e5 180ms
vector<ull> Partition(int n)//P(0...n), 拆分数
{
    Q f(++n);
    int i, l = 0, r = 0;
    while (--l) if (3 * l * l - l >= n * 2) break;
    while (++r) if (3 * r * r - r >= n * 2) break;
    ++l;
    for (i = l + abs(l) % 2; i < r; i += 2) f[3 * i * i - i >> 1] = 1;
    for (i = l + abs(l + 1) % 2; i < r; i += 2) f[3 * i * i - i >> 1] = p - 1;
    f = ~f;
    return vector<ull>(f.begin(), f.begin() + n);
}//5e5 150ms
struct reg
{
    Q a00, a01, a10, a11;
    reg operator*(const reg& o) const
    {
        return {
            shrink(a00 * o.a00 + a01 * o.a10),
            shrink(a00 * o.a01 + a01 * o.a11),
            shrink(a10 * o.a00 + a11 * o.a10),
```

```
            shrink(a10 * o.a01 + a11 * o.a11)};
    }
    pair<Q, Q> operator*(const pair<Q, Q>& o) const
    {
        const auto& [b0, b1] = o;
        return {shrink(a00 * b0 + a01 * b1), shrink(a10 * b0 + a11 * b1)};
    }
} E = {{vector{1llu}}, Q(), Q(), {vector{1llu}}};
ostream& operator<<(ostream& cout, const reg& o)
{
    return cout << "[" << o.a00 << ",␣" << o.a01 << "]\n"
        << "[" << o.a10 << ",␣" << o.a11 << "]\n";
}
reg hgcd(Q a, Q b)
{
    int m = a.deg() + 1 >> 1;
    if (b.deg() < m) return E;
    reg r = hgcd(a >> m, b >> m);
    auto [c, d] = r * pair{a, b};
    if (d.deg() < m) return r;
    auto [q, e] = div_mod(c, d);
    r.a00 -= shrink(q * r.a10);
    r.a01 -= shrink(q * r.a11);
    swap(r.a00, r.a10);
    swap(r.a01, r.a11);
    if (e.deg() < m) return r;
    int k = 2 * m - d.deg();
    auto s = hgcd(d >> k, e >> k);
    return s * r;
}
Q gcd(Q a, Q b)
{
    if (a.deg() < b.deg()) swap(a, b);
    while (b.deg() >= 0)
    {
        a = mod(a, b);
        swap(a, b);
        auto tmp = hgcd(a, b);
        tie(a, b) = tmp * pair{a, b};
    }
    if (a.deg() == -1) return a;
    ull k = ksm(a[a.deg()], p - 2);
    for (int i = 0; i < a.size(); i++) a[i] = a[i] * k % p;
    return a;
}
vector<ull> root(Q f)
{
    Q x(2);
    x[1] = 1;
    x = pow(x, p, f);
    if (x.size() < 2) x %= 2;
    (x[1] += p - 1) %= p;
    f = gcd(f, x);
    vector<ull> res;
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    function<void(Q)> dfs = [&](Q f)
        {
```

```
            int n = f.deg(), i;
            if (n <= 0) return;
            if (n == 1)
            {
                res.push_back((p - f[0]) % p);
                return;
            }
            Q g(n);
            for (i = 0; i < n; i++) g[i] = rnd() % p;
            g = gcd(pow(g, (p - 1) / 2, f) - 1, f);
            dfs(g); dfs(div(f, g));
        };
        dfs(f);
        sort(all(res));
        assert(unique(all(res)) == res.end());
        return res;
    }//4000 950ms
    optional<Q> inverse(Q a, Q m)
    {
        Q b = m;
        vector<pair<reg, Q>> buf;
        a = mod(a, b);
        swap(a, b);
        while (b.deg() >= 0)
        {
            auto [q, r] = div_mod(a, b);
            swap(a, r); swap(a, b);
            auto tmp = hgcd(a, b);
            tie(a, b) = tmp * pair{a, b};
            buf.emplace_back(move(tmp), q);
        }
        if (a.deg()) return { };
        reg res = E;
        reverse(all(buf));
        for (const auto& [tmp, q] : buf)
        {
            res = res * tmp;
            res.a00 -= shrink(q * res.a01);
            res.a10 -= shrink(q * res.a11);
            swap(res.a00, res.a01);
            swap(res.a10, res.a11);
        }
        return {res.a01 * ksm(a[0], p - 2)};
    }//5e4 950ms
}
using NTT::p;
using poly = NTT::Q;
```

3.25 MTT

如果长度较长，可以考虑将 p_3 替换为 $5 \times 2^{25} + 1$。

```
namespace MTT
{
    template<ull p> constexpr ull ksm(ull x, ull y = p - 2)
    {
        ull r = 1;
```

```
    while (y)
    {
        if (y & 1) r = r * x % p;
        x = x * x % p;
        y >>= 1;
    }
    return r;
}
int cal(int x) { return 1 << __lg(max(x, 1) * 2 - 1); }
const int N = 1 << 22;
const ull p = 1e9 + 7, g = 3,
    p1 = 7 << 26 | 1, p2 = 119 << 23 | 1, p3 = 479 << 21 | 1,//三模，原根都是 3，非常好
    inv_p1 = ksm<p2>(p1), inv_p12 = ksm<p3>(p1 * p2 % p3), _p12 = p1 * p2 % p;//三模，1 关于 2
        逆，1*2 关于 3 逆，1*2 mod 3
int r[N];
struct P
{
    ull v1, v2, v3;
    P operator+(const P &o) const { return {v1 + o.v1, v2 + o.v2, v3 + o.v3}; }
    P operator-(const P &o) const { return {v1 + p1 - o.v1, v2 + p2 - o.v2, v3 + p3 - o.v3}; }
    P operator*(const P &o) const { return {v1 * o.v1, v2 * o.v2, v3 * o.v3}; }
    void operator+=(const P &o) { v1 += o.v1, v2 += o.v2, v3 += o.v3; }
    void operator-=(const P &o) { v1 += p1 - o.v1, v2 += p2 - o.v2, v3 += p3 - o.v3; }
    void operator*=(const P &o) { v1 *= o.v1, v2 *= o.v2, v3 *= o.v3; }
    void mod() { v1 %= p1, v2 %= p2, v3 %= p3; }
};
P w[N];
void init(int n)
{
    static int pr = 0, pw = 0;
    if (pr == n) return;
    int b = __lg(n) - 1, i, j, k;
    for (i = 1; i < n; i++) r[i] = r[i >> 1] >> 1 | (i & 1) << b;
    if (pw < n)
    {
        for (j = 1; j < n; j = k)
        {
            k = j * 2;
            P wn = {ksm<p1>(g, (p1 - 1) / k), ksm<p2>(g, (p2 - 1) / k), ksm<p3>(g, (p3 - 1) / k
                )};
            w[j] = {1, 1, 1};
            for (i = j + 1; i < k; i++) w[i] = w[i - 1] * wn, w[i].mod();
        }
        pw = n;
    }
    pr = n;
}
void dft(vector<P> &a, int o = 0)
{
    int n = a.size(), i, j, k;
    P *f, *g, *wn, *b = a.data(), x, y;
    init(n);
    for (i = 1; i < n; i++) if (i < r[i]) swap(a[i], a[r[i]]);
    for (k = 1; k < n; k *= 2)
    {
        wn = w + k;
        for (i = 0; i < n; i += k * 2)
```

```
            {
                f = b + i; g = b + i + k;
                for (j = 0; j < k; j++)
                {
                    y = g[j] * wn[j];
                    y.mod();
                    g[j] = f[j] - y;
                    f[j] += y;
                }
            }
            if (k * 2 == n || k == 1 << 14) for (P &x : a) x.mod();
        }
        if (o)
        {
            x = {ksm<p1>(n), ksm<p2>(n), ksm<p3>(n)};
            for (P &y : a) y *= x, y.mod();
            reverse(1 + all(a));
        }
    }
    struct Q :vector<ull>
    {
        Q(int x = 1) :vector(x) { }
        Q &operator%=(int n) { resize(n); return *this; }
    };
    Q &operator*=(Q &f, const Q &g)
    {
        int n = f.size() + g.size() - 1, m = cal(n), i;
        vector<P> F(m, {0, 0, 0}), G(m, {0, 0, 0});
        for (i = 0; i < f.size(); i++) F[i] = {f[i] % p1, f[i] % p2, f[i] % p3};
        for (i = 0; i < g.size(); i++) G[i] = {g[i] % p1, g[i] % p2, g[i] % p3};
        dft(F); dft(G);
        for (i = 0; i < m; i++) F[i] *= G[i], F[i].mod();
        dft(F, 1);
        f %= n;
        ull x;
        for (i = 0; i < n; i++)
        {
            auto [r1, r2, r3] = F[i];
            x = (r2 + p2 - r1) * inv_p1 % p2 * p1 + r1;
            f[i] = ((x + p3 - r3) % p3 * (p3 - inv_p12) % p3 * _p12 + x) % p;
        }
        return f;
    }//5e5 440ms
    Q operator*(Q f, const Q &g) { return f *= g; }
}
using MTT::p;
using poly = MTT::Q;
```

3.26 FFT

```
namespace FFT
{
#define all(x) (x).begin(),(x).end()
    typedef double db;
    const int N = 1 << 21;
    const db pi = 3.14159265358979323846;
```

```
struct comp
{
    db x, y;
    comp operator+(const comp &o) const { return {x + o.x, y + o.y}; }
    comp operator-(const comp &o) const { return {x - o.x, y - o.y}; }
    comp operator*(const comp &o) const { return {x * o.x - y * o.y, o.x * y + x * o.y}; }
    comp operator*(const db &o) const { return {x * o, y * o}; }
    void operator*=(const comp &o) { *this = {x * o.x - y * o.y, o.x * y + x * o.y}; }
    void operator*=(const db &o) { x *= o; y *= o; }
    void operator/=(const db &o) { x /= o; y /= o; }
};
long long dtol(const double &x) { return fabs(round(x)); }
const comp I{0, -1};
ostream &operator<<(ostream &cout, const comp &o) { cout << o.x; if (o.y >= 0) cout << '+';
    return cout << o.y << 'i'; }
int r[N];
char c;
comp Wn[N];
void init(int n)
{
    static int preone = -1;
    if (n == preone) return;
    preone = n;
    int b, i;
    b = __builtin_ctz(n) - 1;
    for (i = 1; i < n; i++) r[i] = r[i >> 1] >> 1 | (i & 1) << b;
    for (i = 0; i < n; i++) Wn[i] = {cos(pi * i / n), sin(pi * i / n)};
}
int cal(int x) { return 1u << 32 - __builtin_clz(max(x, 2) - 1); }
struct Q
{
    vector<comp> a;
    int deg;
    comp *pt() { return a.data(); }
    Q(int n = 0)
    {
        deg = n;
        a.resize(cal(n));
    }
    void dft(int xs = 0)//1,0
    {
        int i, j, k, l, n = a.size(), d;
        comp w, wn, b, c, *f = pt(), *g, *a = f;
        init(n);
        if (xs) reverse(a + 1, a + n);//spe
        for (i = 0; i < n; i++) if (i < r[i]) swap(a[i], a[r[i]]);
        for (i = 1, d = 0; i < n; i = l, d++)
        {
            //wn={cos(pi/i),(xs?-1:1)*sin(pi/i)};
            l = i << 1;
            for (j = 0; j < n; j += l)
            {
                //w={1,0};
                f = a + j; g = f + i;
                for (k = 0; k < i; k++)
                {
                    w = Wn[k * (n >> d)];
```

```
                    b = f[k]; c = g[k] * w;
                    f[k] = b + c;
                    g[k] = b - c;
                    //w*=wn;
                }
            }
        }
        if (xs) for (i = 0; i < n; i++) a[i] /= n;
    }
    void operator|=(Q o)
    {
        int n = deg + o.deg - 1, m = cal(n), i;
        a.resize(m); o.a.resize(m);
        dft(); o.dft();
        for (i = 0; i < m; i++) a[i] *= o.a[i];
        dft(1);
        for (i = n; i < m; i++) a[i] = { };
        deg = n;
    }
    Q operator|(Q o) const { o |= *this; return o; }
};
Q mul(Q a, const Q &b)//三次变两次，仅实数，注意精度
{
    int n = a.deg + b.deg - 1, m = cal(n), i;
    a.a.resize(m);
    for (i = 0; i < b.deg; i++) a.a[i] = {a.a[i].x, b.a[i].x};
    a.dft();
    for (i = 0; i < m; i++) a.a[i] *= a.a[i];
    a.dft(1);
    for (i = 0; i < n; i++) a.a[i] = {a.a[i].y * .5};
    for (i = n; i < m; i++) a.a[i] = { };
    a.deg = n;
    return a;
}
void ddt(Q &a, Q &b)//double dft，仅实数，注意精度
{
    comp x, y;
    int n = a.a.size(), i;
    assert(n == b.a.size());
    for (i = 0; i < n; i++) a.a[i] = {a.a[i].x, b.a[i].x};
    a.dft();
    for (i = 0; i < n; i++) b.a[i] = {a.a[i].x, -a.a[i].y};
    reverse(b.pt() + 1, b.pt() + n);
    for (i = 0; i < n; i++)
    {
        x = a.a[i]; y = b.a[i];
        a.a[i] = (x + y) * .5;
        b.a[i] = (y - x) * .5 * I;
    }
}
}
using FFT::dtol;
```

3.27 约数个数和

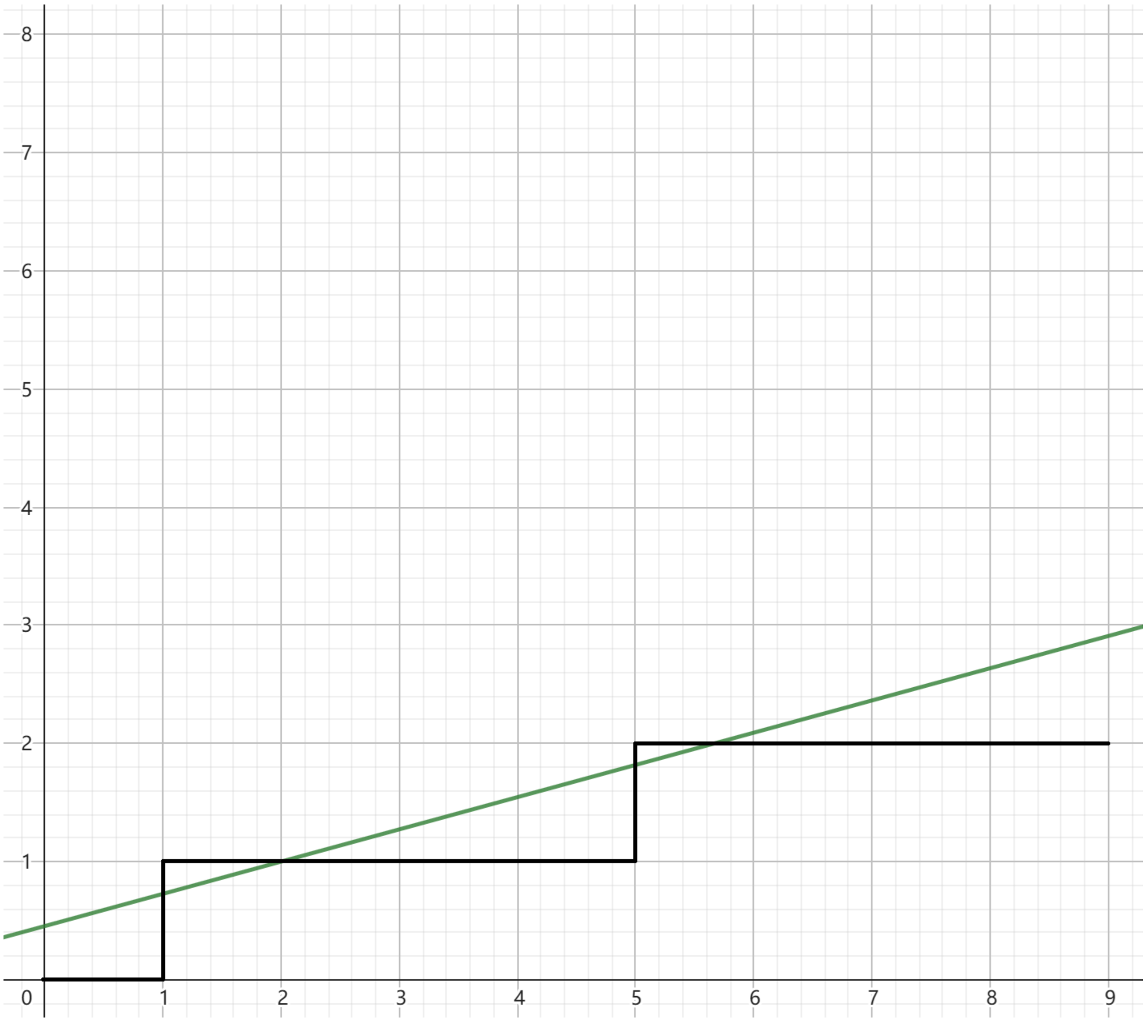
$O(\sqrt[3]{n}\log n)$。

```
#include"bits/stdc++.h"
using ll=long long;
using lll=__int128;
using namespace std;

void myw(lll x){
    if(!x) return;
    myw(x/10);printf("%d",(int)(x%10));
}

struct vec{
    ll x,y;
    vec (ll x0=0,ll y0=0){x=x0,y=y0;}
    vec operator +(const vec b){return vec(x+b.x,y+b.y);}
};

ll N;
vec stk[1000005];int len;
vec P;
vec L,R;

bool ninR(vec a){return N<(lll)a.x*a.y;}
bool steep(ll x,vec a){return (lll)N*a.x<=(lll)x*x*a.y;}

lll Solve(){
    len=0;
    ll cbr=cbrt(N),sqr=sqrt(N);
    P.x=N/sqr,P.y=sqr+1;
    lll ans=0;
    stk[++len]=vec(1,0);stk[++len]=vec(1,1);
    while(1){
        L=stk[len--];
        while(ninR(vec(P.x+L.x,P.y-L.y)))
            ans+=(lll)P.x*L.y+(lll)(L.y+1)*(L.x-1)/2,
            P.x+=L.x,P.y-=L.y;
        if(P.y<=cbr) break;
        R=stk[len];
        while(!ninR(vec(P.x+R.x,P.y-R.y))) L=R,R=stk[--len];
        while(1){
            vec mid=L+R;
            if(ninR(vec(P.x+mid.x,P.y-mid.y))) R=stk[++len]=mid;
            else if(steep(P.x+mid.x,R)) break;
            else L=mid;
        }
    }
    for(int i=1;i<P.y;i++) ans+=N/i;
    return ans*2-sqr*sqr;
}

int T;

int main(){
    scanf("%d",&T);
    while(T--){
        scanf("%lld",&N);
        myw(Solve());printf("\n");
```

```
    }
}
```

3.28 万能欧几里得/min of mod of linear

题意：$\sum_{x=0}^{n-1}\lfloor\frac{ax+b}{m}\rfloor$ （$0\le a,b<m$）



原理：考虑紧贴着斜线的折线的答案。每个 `nd` 表示的是一段折线，你需要实现 `operator+` 来计算出拼接两个折线之后的答案。注意，需要实现默认构造。

你需要传入的 `dx` 和 `dy` 表示向上和向右的折线的答案（也就是边界）。图中对应的折线为 RURRRRURRRR。

以对 x 求和为例，通常的处理手段是在 `dx` 时计算 $x=1$ 的答案，在 `dy` 时更新辅助数组。

注意，如果 $x=0$ 处有答案，你需要手动计算它，并保证传入的 $b<m$。

如果这段向上的竖线对后面有影响，你可以在一开始先加一个 `ksm(dy,...)`，但你仍然可能需要手动计算 $x=0$。

如果答案要取模，特别注意 y 有可能比模数大！不卡时间请使用 `int128`。

```
struct nd
{
    ll x, y, sy;
    nd operator+(const nd &o) const
    {
        return {x + o.x, y + o.y, sy + o.sy + y * o.x};
    }
};
```

```
nd ksm(nd a, ll k)
{
    nd res{ };
    while (k)
    {
        if (k & 1) res = res + a;
        a = a + a; k >>= 1;
    }
    return res;
}
nd sol(int a, int b, int m, int n, nd dx, nd dy)//[0,n] (ax+b)/m 0<=b<m
{
    if (!n) return { };
    if (a >= m) return sol(a % m, b, m, n, ksm(dy, a / m) + dx, dy);
    ll c = ((ll)n * a + b) / m;
    if (!c) return ksm(dx, n);
    ll cnt = n - ((ll)m * c - b - 1) / a;
    return ksm(dx, (m - b - 1) / a) + dy + sol(m, (m - b - 1) % a, a, c - 1, dy, dx) + ksm(dx, cnt
        );
}
ll sum_of_floor_of_linear(int a, int b, int m, int n)//[0,n] sum((ax+b)/m)
{
    nd dx = {1, 0, 0}, dy = {0, 1, 0};
    int nb = (b % m + m) % m;
    return sol(a, nb, m, n, dx, dy).sy + (ll)(b - nb) / m * (n + 1);
}
int min_of_mod_of_linear(int a, int b, int p, int n)//[0,n] min((ax+b) mod p)
{
    ll s = sum_of_floor_of_linear(a, b, p, n);
    int l = 0, r = p - 1, mid;
    while (l < r)
    {
        mid = (l + r + 1) / 2;
        if (sum_of_floor_of_linear(a, b - mid, p, n) >= s) l = mid;
        else r = mid - 1;
    }
    return l;
}
```

3.29 高斯整数类

圆上整点的基础。

```
ll roundiv(ll x,ll y)
{
    return x>=0?(x+y/2)/y:(x-y/2)/y;
}
struct Q
{
    ll x,y;
    Q operator~() const { return {x,-y}; }
    ll len2() const { return x*x+y*y; }
    Q operator+(const Q &o) const { return {x+o.x,y+o.y}; }
    Q operator-(const Q &o) const { return {x-o.x,y-o.y}; }
    Q operator*(const Q &o) const { return {x*o.x-y*o.y,x*o.y+y*o.x}; }
    Q operator/(const Q &o) const
    {
```

```
    Q t=*this*~o;
    ll l=o.len2();
    return {roundiv(t.x,l),roundiv(t.y,l)};
  }
  Q operator%(const Q &o) const { return *this-*this/o*o; }
};
Q gcd(Q a,Q b)
{
  if (a.len2()>b.len2()) swap(a,b);
  while (a.len2())
  {
    b=b%a;
    swap(a,b);
  }
  return b;
}
```

3.30 Miller Rabin/Pollard Rho

1s：200 组 10^{18}。

如果你只需要做 int 以内的分解，你可以改为

```
typedef int ll;
typedef long long lll;
```

```
namespace pr
{
  typedef long long ll;
  typedef __int128 lll;
  typedef pair<ll,int> pa;
  ll ksm(ll x,ll y,const ll p)
  {
    ll r=1;
    while (y)
    {
      if (y&1) r=(lll)r*x%p;
      x=(lll)x*x%p; y>>=1;
    }
    return r;
  }
  namespace miller
  {
    const int p[7]={2,3,5,7,11,61,24251};
    ll s,t;
    bool test(ll n,int p)
    {
      if (p>=n) return 1;
      ll r=ksm(p,t,n),w;
      for (int j=0; j<s&&r!=1; j++)
      {
        w=(lll)r*r%n;
        if (w==1&&r!=n-1) return 0;
        r=w;
      }
      return r==1;
    }
```

```
    bool prime(ll n)
    {
        if (n<2||n==46'856'248'255'981) return 0;
        for (int i=0; i<7; ++i) if (n%p[i]==0) return n==p[i];
        s=__builtin_ctz(n-1); t=n-1>>s;
        for (int i=0; i<7; ++i) if (!test(n,p[i])) return 0;
        return 1;
    }
}
using miller::prime;
mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
namespace rho
{
    void nxt(ll &x,ll &y,ll &p) { x=((lll)x*x+y)%p; }
    ll find(ll n,ll C)
    {
        ll l,r,d,p=1;
        l=rnd()%(n-2)+2,r=l;
        nxt(r,C,n);
        int cnt=0;
        while (l^r)
        {
            p=(lll)p*llabs(l-r)%n;
            if (!p) return gcd(n,llabs(l-r));
            ++cnt;
            if (cnt==127)
            {
                cnt=0;
                d=gcd(llabs(l-r),n);
                if (d>1) return d;
            }
            nxt(l,C,n); nxt(r,C,n); nxt(r,C,n);
        }
        return gcd(n,p);
    }
    vector<pa> w;
    vector<ll> d;
    void dfs(ll n,int cnt)
    {
        if (n==1) return;
        if (prime(n)) return w.emplace_back(n,cnt),void();
        ll p=n,C=rnd()%(n-1)+1;
        while (p==1||p==n) p=find(n,C++);
        int r=1; n/=p;
        while (n%p==0) n/=p,++r;
        dfs(p,r*cnt); dfs(n,cnt);
    }
    vector<pa> getw(ll n)
    {
        w=vector<pa>(0); dfs(n,1);
        if (n==1) return w;
        sort(w.begin(),w.end());
        int i,j;
        for (i=1,j=0; i<w.size(); i++) if (w[i].first==w[j].first) w[j].second+=w[i].second;
            else w[++j]=w[i];
        w.resize(j+1);
        return w;
```

```
        }
        void dfss(int x,ll n)
        {
            if (x==w.size()) return d.push_back(n),void();
            dfss(x+1,n);
            for (int i=1; i<=w[x].second; i++) dfss(x+1,n*=w[x].first);
        }
        vector<ll> getd(ll n)
        {
            getw(n); d=vector<ll>(0); dfss(0,1);
            sort(d.begin(),d.end());
            return d;
        }
    }
    using rho::getw,rho::getd;
    using miller::prime;
}
using pr::getw,pr::getd,pr::prime;
```

4 字符串

4.1 字典树（trie 树）

```
struct trie
{
    const static int N=3e6+2, M=62;
    int c[N][M], sz[N];//sz 维护有多少个以当前字符串为前缀的字符串。
    int cnt;
    void insert(string s)
    {
        int u=0;
        ++sz[u];
        for (char ch:s)
        {
            assert(ch>=0&&ch<M);
            int &v=c[u][ch];
            if (!v) v=++cnt;
            u=v;
            ++sz[u];
        }
        //此时 u 是字符串结束位置。你可以在此存储结点信息。
    }
    int match(string s)//返回字符串结束位置。可能为 0。
    {
        int u=0;
        for (char ch:s)
        {
            assert(ch>=0&&ch<M);
            u=c[u][ch];
            if (!u) return 0;
        }
        return u;
    }
    void clear()
    {
        memset(c, 0, (cnt+1)*sizeof c[0]);
        memset(sz, 0, (cnt+1)*sizeof sz[0]);
        cnt=0;
    }
} s;
```

4.2 AC 自动机

注意 AC 自动机与 trie 不同的地方在于，根必须是 0。

题意：给你一个文本串 S 和 n 个模式串 $T_{1\sim n}$，请你分别求出每个模式串 T_i 在 S 中出现的次数。

```
struct AC
{
    const static int N=3e6+2, M=26;
    int c[N][M], sz[N], pos[N], f[N], app[N];//sz 维护有多少个以当前字符串为前缀的字符串。
    int cnt=0, id=0;
    vector<int> q;
    void insert(string s)
    {
```

```
        int u=0;
        ++sz[u];
        for (char ch:s)
        {
            assert(ch>=0&&ch<M);
            int &v=c[u][ch];
            if (!v) v=++cnt;
            u=v;
            ++sz[u];
        }
        pos[id++]=u;
        //此时 u 是字符串结束位置。你可以在此存储结点信息。
    }
    vector<int> match(string s)//返回答案。复杂度 O(结点数)
    {
        int u=0, i;
        for (char ch:s)
        {
            assert(ch>=0&&ch<M);
            u=c[u][ch];
            ++app[u];
        }
        for (int u:q) app[f[u]]+=app[u];
        vector<int> r(id);
        for (i=0; i<id; i++) r[i]=app[pos[i]];
        memset(app, 0, (cnt+1)*sizeof app[0]);
        return r;
    }
    void clear()
    {
        memset(c, 0, (cnt+1)*sizeof c[0]);
        memset(f, 0, (cnt+1)*sizeof f[0]);
        memset(sz, 0, (cnt+1)*sizeof sz[0]);
        cnt=id=0;
    }
    void build()
    {
        q.clear();
        int ql=0;
        for (int i=0; i<M; i++) if (c[0][i]) q.push_back(c[0][i]);
        while (ql<q.size())
        {
            int u=q[ql++];
            for (int i=0; i<M; i++) if (c[u][i])
            {
                q.push_back(c[u][i]);
                f[c[u][i]]=c[f[u]][i];
            }
            else c[u][i]=c[f[u]][i];
        }
        reverse(all(q));
    }
} s;
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, i;
```

```
    cin>>n;
    while (n--)
    {
        string t;
        cin>>t;
        for (char &c:t) c-='a';
        s.insert(t);
    }
    s.build();
    string t;
    cin>>t;
    for (char &c:t) c-='a';
    auto res=s.match(t);
    for (int x:res) cout<<x<<'\n';
}
```

4.3 hash

在调试时，可以把 base 设置为 10 的幂方便输出。可能建议把第一个模数也设置为 1，但未测试是否有奇怪的问题。但要注意，此时不应当使用接近 10 的幂次的模数。

$O(n)$，$O(n)$。

建议抄 int128 版本。目前来看，int128 版本的既好写又快。

__int128 版本：

```
namespace sh
{
    using ull = unsigned long long;
    using lll = __uint128_t;
    const int N = 1e6 + 5;
    const ull p = (ull)1e18 - 11, b = 137;
    ull m[N];
    int i = []() {
        m[0] = 1;
        for (int i = 1; i < N; i++) m[i] = (lll)m[i - 1] * b % p;
        return 0;
    }();
    struct str
    {
        int n;
        vector<ull> a;
        template<class T> str(const vector<T> &s) :n(s.size()), a(n + 1)
        {
            for (i = 0; i < n; i++) a[i + 1] = ((lll)a[i] * b + s[i]) % p;
        }
        template<class T> str(const basic_string<T> &s) : n(s.size()), a(n + 1)//直接去掉模板换成
            string 也可以
        {
            for (i = 0; i < n; i++) a[i + 1] = ((lll)a[i] * b + s[i]) % p;
        }
        ull getv(int l, int r)//[l,r)
        {
            return (a[r] + (lll)(p - a[l]) * m[r - l]) % p;
        }
        int lcp(int i, int j)
        {
```

```
            if (i == j) return n - i;
            int l = 0, r = n - max(i, j), mid;
            while (l < r)
            {
                mid = (l + r + 1) >> 1;
                if (getv(i, i + mid) == getv(j, j + mid)) l = mid;
                else r = mid - 1;
            }
            return l;
        }
    };
}
using sh::str;
```

双模数版本：

特别注意这里 m 数组预处理的不是幂次，而是幂次的相反数。

其返回值是两个 32 位数拼接而成的，要改动比较麻烦。

```
namespace sh
{
    using ui = unsigned int;
    using ull = unsigned long long;
    const int N = 1e6 + 5;
    const ui p1 = 2'034'452'107, p2 = 2'013'074'419;
    struct pa
    {
        ui v1, v2;
        pa(ui v = 0) :v1(v), v2(v) { }
        pa(ui v1, ui v2) :v1(v1), v2(v2) { }
        pa operator*(const pa &o) const { return pa(1llu * v1 * o.v1 % p1, 1llu * v2 * o.v2 % p2);
             }
    };
    pa fma(const pa &a, const pa &b, const pa &c) { return pa((1llu * a.v1 * b.v1 + c.v1) % p1, (1
        llu * a.v2 * b.v2 + c.v2) % p2); }
    const pa b = {137, 149}, inv = {1'603'801'661, 1'024'053'074};
    pa m[N];
    int i = []() {
        m[0] = {p1 - 1, p2 - 1};
        for (int i = 1; i < N; i++) m[i] = m[i - 1] * b;
        return 0;
    }();
    struct str
    {
        int n;
        vector<pa> a;
        template<class T> str(const vector<T> &s) :n(s.size()), a(n + 1)
        {
            for (i = 0; i < n; i++) a[i + 1] = fma(a[i], b, s[i]);
        }
        template<class T> str(const basic_string<T> &s) : n(s.size()), a(n + 1)//直接去掉模板换成
            string 也可以
        {
            for (i = 0; i < n; i++) a[i + 1] = fma(a[i], b, s[i]);
        }
        ull getv(int l, int r)//[l,r)
        {
            auto [x, y] = fma(a[l], m[r - l], a[r]);
            return (ull)x << 32 | y;
```

```
        }
        int lcp(int i, int j)
        {
            if (i == j) return n - i;
            int l = 0, r = n - max(i, j), mid;
            while (l < r)
            {
                mid = (l + r + 1) >> 1;
                if (getv(i, i + mid) == getv(j, j + mid)) l = mid;
                else r = mid - 1;
            }
            return l;
        }
    };
}
using sh::str;
```

4.4 KMP

$O(n)$, $O(n)$。

```
struct str
{
    vector<int> nxt,s;
    int n;
    str(int *S,int _n)//[1,n]
    {
        n=_n;
        nxt.resize(n+1);
        s=vector<int>(S,S+n+1);
        int i,j=0;
        nxt[1]=0;
        for (i=2;i<=n;i++)
        {
            while (j&&s[i]!=s[j+1]) j=nxt[j];
            nxt[i]=j+=s[i]==s[j+1];
        }
    }
    vector<int> match(int *t,int m)//find s(str) in t (start pos)
    {
        vector<int> r;
        int i,j=0;
        for (i=1;i<=m;i++)
        {
            while (j&&t[i]!=s[j+1]) j=nxt[j];
            if ((j+=t[i]==s[j+1])==n) j=nxt[j],r.push_back(i-n+1);
        }
        return r;
    }
};
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    string s,t;
    cin>>s>>t;
    int n=s.size(),m=t.size(),i;
    vector<int> a(n+1),b(m+1);
```

```
    for (i=1;i<=n;i++) a[i]=s[i-1];
    for (i=1;i<=m;i++) b[i]=t[i-1];
    str q(b.data(),m);
    auto r=q.match(a.data(),n);
    for (int x:r) cout<<x<<'\n';
    for (i=1;i<=m;i++) cout<<q.nxt[i]<<" \n"[i==m];
}
```

4.5 KMP（重构，未验证）

$O(n)$，$O(n)$。

```
struct str//[0,n)
{
    vector<int> nxt,s;
    int n;
    str(const vector<int> &_s):nxt(_s.size(),-1),s(all(_s)),n(_s.size())
    {
        int i,j=-1;
        for (i=1;i<n;i++)
        {
            while (j!=-1&&s[i]!=s[j+1]) j=nxt[j];
            nxt[i]=j+=s[i]==s[j+1];
        }
    }
    vector<int> match(const vector<int> &t)//find s(str) in t (start pos)
    {
        int m=t.size();
        vector<int> r;
        int i,j=-1;
        for (i=0;i<m;i++)
        {
            while (j!=-1&&t[i]!=s[j+1]) j=nxt[j];
            if ((j+=t[i]==s[j+1])==n-1) j=nxt[j],r.push_back(i-n+1);
        }
        return r;
    }
};
```

4.6 manacher

$O(n)$，$O(n)$。

ex[i] 表示以 $i/2$ 为回文中心的回文串长度。

如果 t 可能包含 $#，你需要修改字符。

```
vector<int> manacher(const string &t)
{
    string S = "$#";
    int n = t.size(), i, r = 1, m = 0;
    for (i = 0; i < n; i++) S += t[i], S += '#';
    S += '#';
    char *s = S.data() + 2;
    n = n * 2 - 1;
    vector<int> ex(n);
    ex[0] = 2;
```

```
    for (i = 1; i < n; i++)
    {
        ex[i] = i < r ? min(ex[m * 2 - i], r - i + 1) : 1;
        while (s[i + ex[i]] == s[i - ex[i]]) ++ex[i];
        if (i + ex[i] - 1 > r) r = i + ex[m = i] - 1;
    }
    for (i = 0; i < n; i++) --ex[i];
    return ex;
}
```

4.7 SA

$O((n+\sum)\log n)$，$O(n+\sum)$。
功能：查询两个后缀的 lcp。单次询问复杂度 $O(1)$。
下标从 0 开始。

```
struct SA
{
    int n;
    vector<vector<int>> st;
    vector<int> sa, rk, h;
    int lcp(int x, int y)
    {
        if (x == y) return n - x;
        x = rk[x]; y = rk[y];
        if (x > y) swap(x, y);
        ++x;
        int z = __lg(y - x + 1);
        return min(st[z][x], st[z][y - (1 << z) + 1]);
    }
    SA(vector<int> a) :n(a.size()), st(__lg(n) + 1, vector<int>(n + 1)), sa(n), h(n)
    {
        int i, j, m, cnt;
        m = *min_element(all(a));
        for (int &x : a) x -= m;
        m = *max_element(all(a)) + 1;
        vector<int> id(n), s(max(n, m));
        rk = a;
        for (int i : rk) ++s[i];
        partial_sum(all(s), s.begin());
        for (i = n - 1; i >= 0; i--) sa[--s[rk[i]]] = i;
        for (j = 1; j <= n; j <<= 1)
        {
            fill_n(s.begin(), m, 0);
            cnt = j;
            iota(all(id) - (n - j), n - j);
            for (int i : sa) if (i >= j) id[cnt++] = i - j;
            for (int i : rk) ++s[i];
            partial_sum(all(s), s.begin());
            for (i = n - 1; i >= 0; i--) sa[--s[rk[id[i]]]] = id[i];
            id[sa[0]] = cnt = 0;
            for (i = 1; i < n; i++)
                if (sa[i] + j < n && sa[i - 1] + j < n && rk[sa[i]] == rk[sa[i - 1]] && rk[sa[i] +
                    j] == rk[sa[i - 1] + j])
                    id[sa[i]] = cnt;
                else
```

```
                id[sa[i]] = ++cnt;
            swap(rk, id);
            if ((m = cnt + 1) == n) break;
        }
        j = 0;
        for (i = 0; i < n; i++) if (rk[i])
        {
            cnt = sa[rk[i] - 1];
            while (i + j < n && cnt + j < n && a[i + j] == a[cnt + j]) ++j;
            h[rk[i]] = j;
            if (j) --j;
        }
        st[0] = h;
        for (j = 0; j < __lg(n); j++)
            for (i = 0, m = n - (1 << j + 1); i <= m; i++)
                st[j + 1][i] = min(st[j][i], st[j][i + (1 << j)]);
    }
    template<class T> SA(const T &s) :SA([&]() {
        vector<int> a; a.reserve(s.size());
        for (auto x : s) a.push_back(x);
        return a;
    }()) { }
};
```

4.8 SAM

$O(n\sum)$，$O(2n\sum)$。

```
template<int M> struct sam//M: 字符集大小
{
    vector<array<int,M>> c;
    vector<int> len,fa,ep;
    int np,cd;
    sam():c(2),len(2),fa(2),ep(2),np(1),cd(0) { }
    void insert(int ch)
    {
        int p=np,q,nq;
        np=c.size();
        len.push_back(++cd);
        fa.push_back(0);
        c.push_back({ });
        ep.push_back(cd);
        while (p&&!c[p][ch]) c[p][ch]=np,p=fa[p];
        if (!p)
        {
            fa[np]=1;
            return;
        }
        q=c[p][ch];
        if (len[q]==len[p]+1)
        {
            fa[np]=q;
            return;
        }
        nq=c.size();
        len.push_back(len[p]+1);
        c.push_back(c[q]);
```

```
        fa.push_back(fa[q]);
        ep.push_back(ep[q]);
        fa[np]=fa[q]=nq;
        c[p][ch]=nq;
        while (c[p=fa[p]][ch]==q) c[p][ch]=nq;
    }
    vector<int> match(const string &s)//返回每个前缀最长匹配长度
    {
        vector<int> r;
        r.reserve(s.size());
        int p=1,nl=0;
        for (auto ch:s)
        {
            if (c[p][ch]) ++nl,p=c[p][ch];
            else
            {
                while (p&&c[p][ch]==0) p=fa[p];
                if (p==0) p=1,nl=0; else nl=len[p]+1,p=c[p][ch];
            }
            r.push_back(nl);
        }
        return r;
    }
    array<int,3> max_match(const string &s)//返回长度，结尾（开）
    {
        array<int,3> r{0,0,0};
        int p=1,nl=0,i=0;
        for (auto ch:s)
        {
            if (c[p][ch]) ++nl,p=c[p][ch];
            else
            {
                while (p&&c[p][ch]==0) p=fa[p];
                if (p==0) p=1,nl=0; else nl=len[p]+1,p=c[p][ch];
            }
            cmax(r,array{nl,ep[p],i+1});
            ++i;
        }
        if (r[0]==0) return { };
        return r;
    }
};
```

4.9 ukkonen 后缀树

$O(n)$，$O(2n\sum)$。

```
void dfs(int x,int lf)
{
    if (!fir[x])
    {
        siz[x][1]=1;
        return;
    }
    int i,j;
    for (i=fir[x];i;i=nxt[i])
    {
```

```
        j=c[x][lj[i]];
        if ((f[j]<=m)&&(t[j]>=m)) ++siz[x][0];
        dfs(zd[j],t[j]-f[j]+1);
        siz[x][0]+=siz[zd[j]][0];
        siz[x][1]+=siz[zd[j]][1];
        if ((t[j]==n)&&(f[j]<=m)) --siz[x][1];
    }
    ans+=(ll)siz[x][0]*siz[x][1]*lf;
}
void add(int a,int b,int cc,int d)
{
    zd[++bbs]=b;
    t[bbs]=d;
    c[a][s[f[bbs]=cc]]=bbs;
}
void add(int x,int y)
{
    lj[++bs]=y;
    nxt[bs]=fir[x];
    fir[x]=bs;
}
    s[++m]=26;
    fa[1]=point=ds=1;
    for (i=1;i<=m;i++)
    {
        ad=0;++remain;
        while (remain)
        {
            if (r==0) edge=i;
            if ((j=c[point][s[edge]])==0)
            {
                fa[++ds]=1;
                fa[ad]=point;
                add(ad=point,ds,edge,m);
                add(point,s[edge]);
            }
            else
            {
                if ((t[j]!=m)&&(t[j]-f[j]+1<=r))
                {
                    r-=t[j]-f[j]+1;
                    edge+=t[j]-f[j]+1;
                    point=zd[j];
                    continue;
                }
                if (s[f[j]+r]==s[i]) {++r;fa[ad]=point;break;}
                fa[fa[ad]=++ds]=1;
                add(ad=ds,zd[j],f[j]+r,t[j]);
                add(ds,s[i]);add(ds,s[f[j]+r]);fa[++ds]=1;
                add(ds-1,ds,i,m);
                zd[j]=ds-1;t[j]=f[j]+r-1;
            }
            --remain;
            if ((r)&&(point==1))
            {
                --r;edge=i-remain+1;
            } else point=fa[point];
```

```
        }
    }
    for (i=1;i<=ds;i++) for (j=fir[i];j;j=nxt[j]) {len[j]=t[c[i][lj[j]]]-f[c[i][lj[j]]]+1;lj[j]=zd
        [c[i][lj[j]]];}
```

4.10 ukkonen 后缀树（重构）

```
struct suffixtree
{
    const static int M=27;
    struct P
    {
        int v,w;
    };
    struct Q
    {
        int f,t,v;//t=0: n
    };
    vector<Q> edges;
    vector<vector<P>> e;
    vector<array<int,M>> c;
    vector<int> s,fa,dep,siz;
    int n,point,ds,remain,r,edge;
    bool bd;
    suffixtree():c(2),fa({0,1}),edges(1),e(2)
    {
        n=remain=r=edge=bd=0;
        point=ds=1;
    }
    suffixtree(const string &s):c(2),fa({0,1}),edges(1),e(2)
    {
        n=remain=r=edge=bd=0;
        point=ds=1;
        reserve(s.size());
        for (auto c:s) insert(c-'a');
        insert(26);
    }
    void reserve(int len)
    {
        ++len;
        s.reserve(len);
        len=len*2+2;
        c.reserve(len);
        fa.reserve(len);
        e.reserve(len);
    }
    inline void add(int a,int b,int cc,int d)
    {
        assert(edges.size());
        c[a][s[cc]]=edges.size();
        edges.push_back({cc,d,b});
    }
    void insert(int ch)//[0,|S|)
    {
        assert(ds==fa.size()-1&&ds==c.size()-1&&n==s.size()&&ds==e.size()-1);
        assert(ch>=0&&ch<M);
```

```
    s.push_back(ch);
    int ad=0;
    ++remain;
    while (remain)
    {
        if (!r) edge=n;
        if (int m=c[point][s[edge]];!m)
        {
            assert(!m);
            fa.push_back(1);c.push_back({});e.push_back({});
            fa[ad]=point;
            add(ad=point,++ds,edge,-1);
            e[point].push_back({s[edge]});
            //add(point,s[edge]);
        }
        else
        {
            assert(m);
            auto [f,t,v]=edges[m];
            if (t>=0&&t-f+1<=r)
            {
                assert(t!=n);
                r-=t-f+1;
                edge+=t-f+1;
                point=v;
                continue;
            }
            assert(f+r<=n);
            if (s[f+r]==s[n])
            {
                ++r;
                fa[ad]=point;
                break;
            }
            fa.push_back(1);c.push_back({});e.push_back({});
            fa.push_back(1);c.push_back({});e.push_back({});
            fa[ad]=++ds;
            add(ad=ds,v,f+r,t);
            e[ds].push_back({s[n]});
            e[ds].push_back({s[f+r]});
            //add(ds,s[n]);add(ds,s[f+r]);
            ++ds;add(ds-1,ds,n,-1);
            edges[m]={f,f+r-1,ds-1};
        }
        --remain;
        if (r&&point==1)
        {
            --r;
            edge=n-remain+1;
        } else point=fa[point];
    }
    ++n;
}
void build_edge()
{
    bd=1;
```

```
        //其余信息
        dep.resize(ds+1);
        siz.resize(ds+1);

        int i,j;
        for (i=1;i<=ds;i++) for (auto &[v,w]:e[i])
        {
            j=c[i][v];
            v=edges[j].v;
            w=(edges[j].t>=0?edges[j].t:n-1)-edges[j].f+1;
        }
    }
    void out()
    {
        int i;
        for (i=1;i<=ds;i++) for (int j:c[i]) if (j)
        {
            auto [f,t,v]=edges[j];
            if (t==-1) t=n-1;
            cerr<<i<<' '<<v<<' ';
            //cerr<<i<<" -> "<<v<<": ";
            for (int k=f;k<=t;k++) cerr<<char('a'+s[k]);
            cerr<<endl;
        }
    }
    ll ans;
    void dfs(int u)
    {
        assert(bd);
        ++ans;
        for (auto [v,w]:e[u])
        {
            //dep[v]=dep[u]+w;
            dfs(v);
            ans+=w-1;
        }
    }
    ll fun()
    {
        ans=0;
        build_edge();
        dfs(1);
        return ans-n;
    }
};
```

4.11 Z 函数

表示每个后缀和母串的 lcp。

```
vector<int> Z(const string &s)
{
    int n = s.size(), i, l, r;
    vector<int> z(n);
    z[0] = n;
    for (i = 1, l = r = 0; i < n; i++)
    {
```

```
        if (i <= r && z[i - l] < r - i + 1) z[i] = z[i - l];
        else
        {
            z[i] = max(0, r - i + 1);
            while (i + z[i] < n && s[i + z[i]] == s[z[i]]) ++z[i];
        }
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}
```

4.12 最小表示法

找到一个串的循环同构串中字典序最小的那个，将这个串直接变过去。常见应用：环哈希（基环树哈希）。

如果只需要找到起点下标，在 `rotate` 前返回 $\min\{i, j\}$ 即可。

$O(n)$，$O(1)$。

```
template<class T> void min_order(vector<T>& a)
{
    int n = a.size(), i, j, k;
    a.resize(n * 2);
    for (i = 0;i < n;i++) a[i + n] = a[i];
    i = k = 0;j = 1;
    while (i < n && j < n && k < n)
    {
        T x = a[i + k], y = a[j + k];
        if (x == y) ++k; else
        {
            (x > y ? i : j) += k + 1;
            j += (i == j);
            k = 0;
        }
    }
    a.resize(n);
    //[min(i,j),n)+[0,min(i,j))
    rotate(a.begin(), min(i, j) + all(a));
}
```

4.13 带通配符的字符串匹配

原理：匹配等价于 $\sum(f_i - g_i)^2 = 0$。带通配符等价于 $\sum f_i g_i (f_i - g_i)^2 = 0$，展开即可。

这里也是较为推荐的 NTT 版本，直接实现任意长度的多项式相乘，便于一般情况的运用。不需要提前做任何 init。

```
namespace NTT
{
    typedef unsigned ui;
    typedef unsigned long long ull;
    const int N=1<<22;
    const ui p=998244353, g=3;
    inline ui ksm(ui x, ui y)
    {
        ui ans=1;
        while (y)
```

```
        {
            if (y&1) ans=1llu*ans*x%p;
            y>>=1; x=1llu*x*x%p;
        }
        return ans;
    }
    ui r[N], w[N];
    void ntt(vector<ui> &a)
    {
        int n=a.size(), i, j, k;
        for (i=0; i<n; i++) if (i<r[i]) swap(a[i], a[r[i]]);
        for (k=1; k<n; k<<=1)
        {
            for (i=0; i<n; i+=k<<1)
            {
                for (j=0; j<k; j++)
                {
                    ui x=a[i+j], y=1llu*a[i+j+k]*w[j+k]%p;
                    a[i+j]=(x+y)%p; a[i+j+k]=(x+p-y)%p;
                }
            }
        }
    }
    vector<ui> mul(vector <ui> a, vector <ui> b)
    {
        if (a.size()==0||b.size()==0) return { };
        int m=a.size()+b.size()-1;
        int n=1<<__lg(m*2-1);
        int i, j, base=__lg(n)-1;
        ui inv=ksm(n, p-2);
        for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<base;
        for (j=1; j<n; j<<=1)
        {
            ui wn=ksm(3, (p-1)/(j<<1));
            w[j]=1;
            for (i=1; i<j; i++) w[j+i]=1llu*w[j+i-1]*wn%p;
        }
        a.resize(n); b.resize(n);
        ntt(a); ntt(b);
        for (i=0; i<n; i++) a[i]=1llu*a[i]*b[i]%p;
        ntt(a); reverse(1+all(a)); a.resize(n=m);
        for (i=0; i<n; i++) a[i]=1llu*a[i]*inv%p;
        return a;
    }
}
vector<int> match(const string &s, const string &t)
{
    using NTT::p, NTT::mul;
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static array<ui, 256> c;
    static bool inited=0;
    if (!inited)
    {
        inited=1;
        for (ui &x:c) x=rnd()%NTT::p;
        c['*']=0;//通配符
    }
```

```
    int n=s.size(), m=t.size(), i, j;
    if (n<m) return { };
    vector<int> ans;
    vector<ui> f(n), ff(n), fff(n), g(m), gg(m), ggg(m);
    for (i=0; i<n; i++)
    {
        f[i]=c[s[i]];
        ff[i]=1llu*f[i]*f[i]%p;
        fff[i]=1llu*ff[i]*f[i]%p;
    }
    for (i=0; i<m; i++)
    {
        g[i]=c[t[m-i-1]];
        gg[i]=1llu*g[i]*g[i]%p;
        ggg[i]=1llu*gg[i]*g[i]%p;
    }
    auto fffg=mul(fff, g), ffgg=mul(ff, gg), fggg=mul(f, ggg);
    for (i=0; i<=n-m; i++) if ((fffg[m-1+i]+fggg[m-1+i]+2*(NTT::p-ffgg[m-1+i]))%NTT::p==0) ans.
        push_back(i);
    return ans;
}
```

快一些的版本，手动拆开了多项式乘法。

```
const int N=1<<22;
const ui p=998244353, g=3;
inline ui ksm(ui x, ui y)
{
    ui ans=1;
    while (y)
    {
        if (y&1) ans=1llu*ans*x%p;
        y>>=1; x=1llu*x*x%p;
    }
    return ans;
}
ui r[N], w[N];
void ntt(vector <ui> &a)
{
    int n=a.size(), i, j, k;
    for (k=1; k<n; k<<=1)
    {
        for (i=0; i<n; i+=k<<1)
        {
            for (j=0; j<k; j++)
            {
                ui x=a[i+j], y=1llu*a[i+j+k]*w[j+k]%p;
                a[i+j]=(x+y)%p; a[i+j+k]=(x+p-y)%p;
            }
        }
    }
}
vector<int> match(string s, string t, char ch='*')
{
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static array<ui, 256> c;
    static bool inited=0;
    if (!inited)
```

```
	{
		inited=1;
		for (ui &x:c) x=rnd()%p;
		// for (int i=0; i<256; i++) c[i]=i-96;
		c[ch]=0;//通配符
	}
	int n=s.size(), m=t.size(), i, j;
	if (n<m) return { };
	vector<int> ans;
	int N=1<<__lg(n*2-1), base=__lg(N)-1;
	vector<ui> f(N), ff(N), fff(N), g(N), gg(N), ggg(N);
	reverse(all(t));
	s.resize(N, ch), t.resize(N, ch);
	for (i=0; i<N; i++)
	{
		r[i]=r[i>>1]>>1|(i&1)<<base;
		if (i<r[i])
		{
			swap(s[i], s[r[i]]);
			swap(t[i], t[r[i]]);
		}
	}
	for (j=1; j<N; j<<=1)
	{
		ui wn=ksm(3, (p-1)/(j<<1));
		w[j]=1;
		for (i=1; i<j; i++) w[j+i]=1llu*w[j+i-1]*wn%p;
	}
	for (i=0; i<N; i++)
	{
		f[i]=c[s[i]];
		ff[i]=1llu*f[i]*f[i]%p;
		fff[i]=1llu*ff[i]*f[i]%p;
		g[i]=c[t[i]];
		gg[i]=1llu*g[i]*g[i]%p;
		ggg[i]=1llu*gg[i]*g[i]%p;
	}
	ntt(f); ntt(ff); ntt(fff); ntt(g); ntt(gg); ntt(ggg);
	for (i=0; i<N; i++) f[i]=(1llu*fff[i]*g[i]+1llu*f[i]*ggg[i]+2llu*(p-ff[i])*gg[i])%p;
	for (i=0; i<N; i++) if (i<r[i]) swap(f[i], f[r[i]]);
	ntt(f); reverse(1+all(f));
	for (i=0; i<=n-m; i++) if (f[m+i-1]==0) ans.push_back(i);
	return ans;
}
```

5 图论

5.1 最小生成树相关

5.1.1 切比雪夫距离最小生成树

原理：先转曼哈顿距离，再用曼哈顿的板子。

$O(n\log n)$，$O(n)$。

```
const int N=3e5+2,M=N<<2;
struct P
{
    int u,v,w;
    P(int a=0,int b=0,int c=0):u(a),v(b),w(c){}
    bool operator<(const P &o) const {return w<o.w;}
};
struct Q
{
    int x,y,id;
    Q(int a=0,int b=0,int c=0):x(a),y(b),id(c){}
    bool operator<(const Q &o) const {return x!=o.x?x>o.x:y>o.y;}
};
ll ans;
P lb[M];
Q a[N],b[N];
int f[N],c[N];
int n,m,i,x,y;
struct bit
{
    int a[N],pos[N],n;
    void init(int &nn)
    {
        memset(a+1,0x7f,(n=nn)*sizeof a[0]);
        memset(pos+1,0,n*sizeof pos[0]);
    }
    void mdf(int x,const int y,const int z)
    {
        if (a[x]>y) a[x]=y,pos[x]=z;
        while (x-=x&-x) if (a[x]>y) a[x]=y,pos[x]=z;
    }
    int sum(int x)
    {
        int r=a[x],rr=pos[x];
        while ((x+=x&-x)<=n) if (a[x]<r) r=a[x],rr=pos[x];
        return rr;
    }
};
bit s;
void cal()
{
    int i,x,y;
    s.init(n);
    memcpy(b+1,a+1,sizeof(Q)*n);
    sort(a+1,a+n+1);
    for (i=1;i<=n;i++) c[i]=a[i].y-a[i].x;
    sort(c+1,c+n+1);
    for (i=1;i<=n;i++)
```

```
    {
        if (x=s.sum(y=lower_bound(c+1,c+n+1,a[i].y-a[i].x)-c))
            lb[++m]=P(a[x].id,a[i].id,a[x].x+a[x].y-a[i].x-a[i].y);//谨防 int 爆
        s.mdf(y,a[i].y+a[i].x,i);
    }
    memcpy(a+1,b+1,sizeof(Q)*n);
}
int getf(int x) {return f[x]==x?x:f[x]=getf(f[x]);}
int main()
{
    cin>>n;
    for (i=1;i<=n;i++) {cin>>a[f[i]=a[i].id=i].x>>a[i].y;
        swap(a[i].x,a[i].y);a[i]=Q(a[i].x+a[i].y,a[i].x-a[i].y,i);}
    cal();for (i=1;i<=n;i++) swap(a[i].x,a[i].y);
    cal();for (i=1;i<=n;i++) a[i].y=-a[i].y;
    cal();for (i=1;i<=n;i++) swap(a[i].x,a[i].y);
    cal();sort(lb+1,lb+m+1);
    for (i=1;i<=m;i++) if ((x=getf(lb[i].u))!=(y=getf(lb[i].v))) f[x]=y,ans+=lb[i].w;
    cout<<ans/2<<endl;
}
```

5.1.2 最小乘积生成树

题意：每条边有两个属性 x_i, y_i，你需要最小化 $(\sum x_i)(\sum y_i)$。

你需要实现的是 sol1，即按照 val 为权值的答案。val_i 是根据 x_i, y_i 计算的。

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
const int N = 202, M = 10002;
struct P
{
    int x, y;
    P(int a = 0, int b = 0) :x(a), y(b) { }
    bool operator<(const P &o) const { return (ll)x * y < (ll)o.x * o.y || (ll)x * y == (ll)o.x *
        o.y && x < o.x; }
};
struct Q
{
    int u, v, x, y, val;
    bool operator<(const Q &o) const { return val < o.val; }
};
P ans = P(1e9, 1e9), l, r;
Q a[M];
int f[N];
int n, m, i;
int getf(int x)
{
    if (f[x] == x) return x;
    return f[x] = getf(f[x]);
}
P sol1()
{
    P r = P(0, 0);
    for (i = 1; i <= n; i++) f[i] = i;
    sort(a + 1, a + m + 1);
    for (i = 1; i <= m; i++) if (getf(a[i].u) != getf(a[i].v))
```

```
    {
        f[f[a[i].u]] = f[a[i].v];
        r.x += a[i].x, r.y += a[i].y;
    }
    return r;
}
void sol2(P l, P r)
{
    for (i = 1; i <= m; i++) a[i].val = (r.x - l.x) * a[i].y + (l.y - r.y) * a[i].x;
    P np = sol1();
    ans = min(ans, np);
    if ((ll)(r.x - l.x) * (np.y - l.y) - (ll)(r.y - l.y) * (np.x - l.x) >= 0) return;
    sol2(l, np); sol2(np, r);
}
int main()
{
    cin >> n >> m;
    for (i = 1; i <= m; i++) cin >> a[i].u >> a[i].v >> a[i].x >> a[i].y, ++a[i].u, ++a[i].v;
    for (i = 1; i <= m; i++) a[i].val = a[i].x; l = sol1();
    for (i = 1; i <= m; i++) a[i].val = a[i].y; r = sol1();
    ans = min(ans, min(l, r)); sol2(l, r);
    cout<<ans.x<<' '<<ans.y<<endl;
}
```

5.1.3 最小斯坦纳树

题意：让给定点集连通的最小生成树（不要求全图连通）

$O(3^k n + 2^k m \log m)$。

分为有方案与无方案两个版本，点标号 $[0, n)$。

```
ll steiner(int n, const vector<tuple<int, int, ll>> &eg, vector<int> id)//[0,n)
{
    using pa = pair<ll, int>;
    int m = id.size(), i;
    vector f(1 << m, vector<ll>(n, inf));
    for (i = 0; i < m; i++) f[1 << i][id[i]] = 0;
    vector<vector<pair<int, ll>>> e(n);
    for (auto [u, v, w] : eg) e[u].push_back({v, w}), e[v].push_back({u, w});
    for (int S = 1; S < 1 << m; S++)
    {
        auto &d = f[S];
        priority_queue<pa, vector<pa>, greater<pa>> q;
        for (i = 0; i < n; i++)
        {
            for (int T = S - 1 & S; T; T = T - 1 & S) cmin(d[i], f[T][i] + f[S ^ T][i]);
            if (d[i] != inf) q.push({d[i], i});
        }
        while (q.size())
        {
            auto [_, u] = q.top(); q.pop();
            if (_ != d[u]) continue;
            for (auto [v, w] : e[u]) if (cmin(d[v], d[u] + w)) q.push({d[v], v});
        }
    }
    return *min_element(all(f.back()));
}
```

```
pair<ll, vector<int>> steiner_construct(int n, const vector<tuple<int, int, ll>> &eg, vector<int>
     id)//[0,n)
{
    using pa = pair<ll, int>;
    int m = id.size(), i;
    vector f(1 << m, vector<ll>(n, inf));
    vector pre(1 << m, vector(n, pair{-1, -1}));
    for (i = 0; i < m; i++) f[1 << i][id[i]] = 0;
    vector<vector<tuple<int, ll, int>>> e(n);
    i = 0;
    for (auto [u, v, w] : eg) e[u].push_back({v, w, i}), e[v].push_back({u, w, i++});
    for (int S = 1; S < 1 << m; S++)
    {
        auto &d = f[S];
        priority_queue<pa, vector<pa>, greater<pa>> q;
        for (i = 0; i < n; i++)
        {
            for (int T = S - 1 & S; T; T = T - 1 & S)
                if (cmin(d[i], f[T][i] + f[S ^ T][i]))
                    pre[S][i] = {-2, T};

            if (d[i] != inf) q.push({d[i], i});
        }
        while (q.size())
        {
            auto [_, u] = q.top(); q.pop();
            if (_ != d[u]) continue;
            for (auto [v, w, id] : e[u]) if (cmin(d[v], d[u] + w))
            {
                q.push({d[v], v});
                pre[S][v] = {u, id};
            }
        }
    }
    vector<int> chosen;
    int S = (1 << m) - 1, u = min_element(all(f[S])) - f[S].begin();
    auto dfs = [&](auto &&dfs, int S, int u) {
        auto [x, y] = pre[S][u];
        while (x >= 0)
        {
            u = x;
            chosen.push_back(y);
            tie(x, y) = pre[S][u];
        }
        if (x == -1) return;
        dfs(dfs, y, u); dfs(dfs, S ^ y, u);
    };
    dfs(dfs, S, u);
    sort(all(chosen));
    return {f[S][u], chosen};
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, q, i;
    cin >> n >> m;
```

```
    vector<tuple<int, int, ll>> eg(m);
    cin >> eg >> q;
    vector<int> id(q);
    cin >> id;
    auto [ans, eid] = steiner_construct(n, eg, id);
    cout << ans << '␣' << eid.size() << '\n' << eid << endl;
}
```

5.2 最短路相关

5.2.1 全源最短路与判负环

使用 floyd 实现全源最短路与判负环。注意边权较大时可能需要考虑 int128.

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
typedef pair<int,int> pa;
typedef tuple<int,int,int> tp;
const int N=152;
const ll inf=5e8;
ll dis[N][N],d[N][N];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    while (1)
    {
        int n,m,q,i,j,k;
        cin>>n>>m>>q;
        if (tp(n,m,q)==tp(0,0,0)) return 0;
        for (i=0;i<n;i++) fill_n(dis[i],n,inf*inf);
        for (i=0;i<n;i++) dis[i][i]=0;
        while (m--)
        {
            int u,v,w;
            cin>>u>>v>>w;
            dis[u][v]=min(dis[u][v],(ll)w);
        }
        for (k=0;k<n;k++) for (i=0;i<n;i++) for (j=0;j<n;j++) dis[i][j]=max(min(dis[i][j],dis[i][k
            ]+dis[k][j]),-inf*2);
        for (i=0;i<n;i++) copy_n(dis[i],n,d[i]);
        for (k=0;k<n;k++) for (i=0;i<n;i++) for (j=0;j<n;j++) dis[i][j]=max(min(dis[i][j],dis[i][k
            ]+dis[k][j]),-inf*2);
        while (q--)
        {
            int u,v;
            cin>>u>>v;
            if (d[u][v]>inf) cout<<"Impossible\n"; else if (dis[u][v]!=d[u][v]||d[u][v]<-inf) cout
                <<"-Infinity\n"; else cout<<d[u][v]<<'\n';
        }
        cout<<'\n';
    }
}
```

5.2.2 Dijkstra/SPFA/Johnson

Johnson 不适用于图中存在负环的情况，因为负环不一定是可以经过的。

$O(nm\log m)$，$O(n+m)$。

```
vector<ll> spfa(const vector<vector<pair<int, ll>>> &e, int s)
{
    int n=e.size(), i;
    assert(n);
    queue<int> q;
    vector<int> len(n), ed(n);
    vector<ll> dis(n, inf);
    q.push(s); dis[s]=0;
    while (q.size())
    {
        int u=q.front(); q.pop();
        ed[u]=0;
        for (auto [v, w]:e[u]) if (cmin(dis[v], dis[u]+w))
        {
            len[v]=len[u]+1;
            if (len[v]>n) return { };
            if (!ed[v])
            {
                ed[v]=1;
                q.push(v);
            }
        }
    }
    return dis;
}
vector<ll> spfa(const vector<vector<pair<int, ll>>> &e)
{
    int n=e.size(), i;
    assert(n);
    queue<int> q;
    vector<int> len(n), ed(n, 1);
    vector<ll> dis(n);
    for (i=0; i<n; i++) q.push(i);
    while (q.size())
    {
        int u=q.front(); q.pop();
        ed[u]=0;
        for (auto [v, w]:e[u]) if (cmin(dis[v], dis[u]+w))
        {
            len[v]=len[u]+1;
            if (len[v]>n) return { };
            if (!ed[v])
            {
                ed[v]=1;
                q.push(v);
            }
        }
    }
    return dis;
}
vector<ll> dijk(const vector<vector<pair<int, ll>>> &e, int s)
{
    int n=e.size();
```

```
    using pa=pair<ll, int>;
    vector<ll> d(n, inf);
    vector<int> ed(n);
    priority_queue<pa, vector<pa>, greater<pa>> q;
    d[s]=0; q.push({0, s});
    while (q.size())
    {
        int u=q.top().second; q.pop();
        ed[u]=1;
        for (auto [v, w]:e[u]) if (cmin(d[v], d[u]+w)) q.push({d[v], v});
        while (q.size()&&ed[q.top().second]) q.pop();
    }
    return d;
}
vector<vector<ll>> dijk(const vector<vector<pair<int, ll>>> &e)
{
    vector<vector<ll>> r;
    for (int i=0; i<e.size(); i++) r.push_back(dijk(e, i));
    return r;
}
vector<vector<ll>> john(vector<vector<pair<int, ll>>> e)
{
    int n=e.size(), i, j;
    assert(n);
    auto h=spfa(e);
    if (!h.size()) return { };
    for (i=0; i<n; i++) for (auto &[v, w]:e[i]) w+=h[i]-h[v];
    auto r=dijk(e);
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (r[i][j]!=inf) r[i][j]-=h[i]-h[j];
    return r;
}
```

5.2.3 无向图最小环

原理：floyd 外层循环本质是计算只经过 $\le k$ 的点的最短路。因此枚举环上标号最大的，在做这一轮转移之前正好是不经过它的最短路。

$O(n^3)$，$O(n^2)$。

```
int f[N][N],jl[N][N];
int n,m,c,ans=inf,i,j,k,x,y,z;
int main()
{
    cin>>n>>m;
    memset(f,0x3f,sizeof(f));
    memset(jl,0x3f,sizeof(jl));
    while (m--)
    {
        cin>>x>>y>>z;
        jl[x][y]=jl[y][x]=f[x][y]=f[y][x]=min(f[y][x],z);
    }
    for (k=1;k<=n;k++)
    {
        for (i=1;i<k;i++) if (jl[k][i]!=jl[0][0]) for (j=1;j<i;j++)
            if (jl[k][j]!=jl[0][0]) ans=min(ans,jl[k][i]+jl[k][j]+f[i][j]);
        for (i=1;i<=n;i++) if (i!=k) for (j=1;j<=n;j++)
            if ((j!=i)&&(j!=k)) f[i][j]=min(f[i][j],f[i][k]+f[k][j]);
```

```
    }
    if (ans==inf) cout<<"No␣solution.\n"; else cout<<ans<<endl;
}
```

5.2.4 输出负环

```
#include "bits/stdc++.h"
using namespace std;
const int N=34;
struct Q
{
    int v,w,c;
    Q(){}
    Q(int x,int y,int z):v(x),w(y),c(z){}
};
vector<Q> lj[N];
int dis[N],cnt[N],pt[N],S;
Q pre[N],st[N];
int n,m,ans,tp;
bool ed[N];
int main()
{
    freopen("arbitrage.in","r",stdin);
    freopen("arbitrage.out","w",stdout);
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n>>m;
    while (m--)
    {
        int x,y,z,w;
        cin>>x>>y>>z>>w;
        lj[x].emplace_back(y,w,z);
        lj[y].emplace_back(x,0,-z);
    }
    for (int i=1;i<=n;i++) lj[0].emplace_back(i,1,0);
    while (1)
    {
        memset(dis,-0x3f,sizeof dis);dis[0]=0;
        for (int i=0;i<=n;i++) ed[i]=cnt[i]=0;S=-1;
        queue<int> q;q.push(0);
        while (!q.empty())
        {
            int u=q.front();q.pop();ed[u]=0;
            for (auto &[v,w,c]:lj[u]) if (w&&dis[v]<dis[u]+c)
            {
                dis[v]=dis[u]+c;pre[v]=Q(u,w,c);
                if (!ed[v])
                {
                    if (++cnt[v]>n+1) {S=v;goto aa;}
                    ed[v]=1;q.push(v);
                }
            }
        }
        aa:;
        if (S==-1) break;
        {
            static bool ed[N];
```

```
        memset(ed,0,sizeof ed);
        while (!ed[S]) ed[S]=1,S=pre[S].v;
    }
    st[tp=1]=pre[S];pt[1]=S;
    int x=pre[S].v;
    while (x!=S)
    {
        st[++tp]=pre[x];pt[tp]=x;
        x=pre[x].v;
        assert(tp<=n+5);
    }
    int fl=1e9;
    for (int j=1;j<=tp;j++) fl=min(fl,st[j].w);
    assert(fl);
    for (int j=1;j<=tp;j++)
    {
        ans+=fl*st[j].c;
        int nn=0;
        for (auto &[v,w,c]:lj[st[j].v]) if (v==pt[j]&&st[j].c==c&&st[j].w==w) {++nn;w-=fl;break
            ;}
        for (auto &[v,w,c]:lj[pt[j]]) if (v==st[j].v&&st[j].c+c==0) {++nn;w+=fl;break;}assert(
            nn==2);
    }
  }
  cout<<ans<<endl;
}
```

5.3 二分图与网络流建图

以下约定，若为二分图则 n, m 表示两侧点数，否则仅 n 表示全图点数。

5.3.1 二分图边染色

留坑待填。

结论：$\Delta(G) \le \chi'(G) \le \Delta(G)+1$，二分图时 $\chi'(G)=\Delta(G)$。$\Delta(G)$ 为图的最大度。

5.3.2 二分图最小点集覆盖

$ans = \text{maxmatch}$，方案如下。

```
#include "bits/stdc++.h"
using namespace std;
const int N=5e3+2;
vector<int> e[N];
int ed[N],lk[N],kl[N],flg[N],now;
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
void dfs2(int u)
{
```

```
    for (int v:e[u]) if (!flg[v]) flg[v]=1,dfs2(lk[v]);
}
int main()
{
    int n,m,i,r=0;
    cin>>n>>m;
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
    }
    for (i=1;i<=n;i++) dfs(now=i);
    for (i=1;i<=n;i++) kl[lk[i]]=i;
    for (i=1;i<=n;i++) if (!kl[i]) dfs2(i);
    vector<int> A[2];
    for (i=1;i<=n;i++) if (lk[i])
    {
        if (flg[i]) A[1].push_back(i); else A[0].push_back(lk[i]);
    }
    for (int j=0;j<2;j++)
    {
        cout<<A[j].size();
        for (int x:A[j]) cout<<'␣'<<x;cout<<'\n';
    }
}
```

5.3.3 二分图最大独立集

$ans = n + m - \text{maxmatch}$，方案是最小点集覆盖的补集。

5.3.4 二分图最小边覆盖

$ans = n + m - \text{maxmatch}$，方案是最大匹配加随便一些边（用于覆盖失配点）。无解当且仅当有孤立点，算法会视为单选孤立点（无边）。这个定理对一般图也成立。

5.3.5 有向无环图最小不相交链覆盖

$ans = n - \text{maxmatch}$，其中二分图建图方法是拆入点和出点（实现时直接跑一次二分图就行，不用额外处理），注意不需要传递闭包。方案如下。

```
#include "bits/stdc++.h"
using namespace std;
const int N=152;
vector<int> e[N];
int lk[N],kl[N],ed[N],now;
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
int main()
```

```
{
    int n,m,i;
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n>>m;
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
    }
    int r=0;
    for (i=1;i<=n;i++) r+=dfs(now=i);
    for (i=1;i<=n;i++) kl[lk[i]]=i;
    for (i=1;i<=n;i++) if (ed[i]!=-1&&!lk[i])
    {
        vector<int> ans;
        int u=i;
        while (u)
        {
            ed[u]=-1;
            ans.push_back(u);
            u=kl[u];
        }
        for (int j=0;j<ans.size();j++) cout<<ans[j]<<" \n"[j+1==ans.size()];
    }
    cout<<n-r<<endl;
}
```

5.3.6 有向无环图最大互不可达集

$ans = n - \text{maxmatch}$，其中二分图建图方法是拆入点和出点（实现时直接跑一次二分图就行，不用额外处理），注意需要传递闭包。方案？

5.3.7 最大权闭合子图

若 $v_i > 0$，$s \to i$ 流量 v_i；若 $v_i < 0$，$i \to t$ 流量 $-v_i$。若原图 $u \to v$ 可花费 w 代价违抗，流量 w，否则 $+\infty$ 。答案为 $\sum\limits_{v_i>0} v_i - \text{maxflow}$。方案？

5.4 匹配与网络流相关代码

5.4.1 二分图最大权匹配

```
namespace KM
{
    const int N = 405;//点数
    typedef long long ll;//答案范围
    const ll inf = 1e16;
    int lk[N], kl[N], pre[N], q[N], n, h, t;
    ll sl[N], e[N][N], lx[N], ly[N];
    bool edx[N], edy[N];
    bool ck(int v)
    {
        if (edy[v] = 1, kl[v]) return edx[q[++t] = kl[v]] = 1;
        while (v) swap(v, lk[kl[v] = pre[v]]);
        return 0;
```

```
    }
    void bfs(int u)
    {
        fill_n(sl + 1, n, inf);
        memset(edx + 1, 0, n * sizeof edx[0]);
        memset(edy + 1, 0, n * sizeof edy[0]);
        q[h = t = 1] = u; edx[u] = 1;
        while (1)
        {
            while (h <= t)
            {
                int u = q[h++], v;
                ll d;
                for (v = 1; v <= n; v++) if (!edy[v] && sl[v] >= (d = lx[u] + ly[v] - e[u][v])) if
                    (pre[v] = u, d) sl[v] = d; else if (!ck(v)) return;
            }
            int i;
            ll m = inf;
            for (i = 1; i <= n; i++) if (!edy[i]) m = min(m, sl[i]);
            for (i = 1; i <= n; i++)
            {
                if (edx[i]) lx[i] -= m;
                if (edy[i]) ly[i] += m; else sl[i] -= m;
            }
            for (i = 1; i <= n; i++) if (!edy[i] && !sl[i] && !ck(i)) return;
        }
    }
    template<class TT> ll max_weighted_match(int N, const vector<tuple<int, int, TT>> &edges)//lk
        [[1,n]]->[1,n]
    {
        int i; n = N;
        memset(lk + 1, 0, n * sizeof lk[0]);
        memset(kl + 1, 0, n * sizeof kl[0]);
        memset(ly + 1, 0, n * sizeof ly[0]);
        for (i = 1; i <= n; i++) fill_n(e[i] + 1, n, 0);//若不需保证匹配边最多，置 0 即可，否则 -inf
            /N
        for (auto [u, v, w] : edges) e[u][v] = max(e[u][v], (ll)w);
        for (i = 1; i <= n; i++) lx[i] = *max_element(e[i] + 1, e[i] + n + 1);
        for (i = 1; i <= n; i++) bfs(i);
        ll r = 0;
        for (i = 1; i <= n; i++) r += e[i][lk[i]];
        return r;
    }
}
using KM::max_weighted_match, KM::lk, KM::kl, KM::e;
```

5.4.2 一般图最大匹配

```
namespace blossom_tree
{
    const int N = 1005;
    vector<int> e[N];
    int lk[N], rt[N], f[N], dfn[N], typ[N], q[N];
    int id, h, t, n;
    int lca(int u, int v)
```

```
{
    ++id;
    while (1)
    {
        if (u)
        {
            if (dfn[u] == id) return u;
            dfn[u] = id; u = rt[f[lk[u]]];
        }
        swap(u, v);
    }
}
void blm(int u, int v, int a)
{
    while (rt[u] != a)
    {
        f[u] = v;
        v = lk[u];
        if (typ[v] == 1) typ[q[++t] = v] = 0;
        rt[u] = rt[v] = a;
        u = f[v];
    }
}
void aug(int u)
{
    while (u)
    {
        int v = lk[f[u]];
        lk[lk[u] = f[u]] = u;
        u = v;
    }
}
void bfs(int root)
{
    memset(typ + 1, -1, n * sizeof typ[0]);
    iota(rt + 1, rt + n + 1, 1);
    typ[q[h = t = 1] = root] = 0;
    while (h <= t)
    {
        int u = q[h++];
        for (int v : e[u])
        {
            if (typ[v] == -1)
            {
                typ[v] = 1; f[v] = u;
                if (!lk[v]) return aug(v);
                typ[q[++t] = lk[v]] = 0;
            }
            else if (!typ[v] && rt[u] != rt[v])
            {
                int a = lca(rt[u], rt[v]);
                blm(v, u, a); blm(u, v, a);
            }
        }
    }
}
int max_general_match(int N, vector<pair<int, int>> edges)//[1,n]
```

```
    {
        n = N; id = 0;
        memset(f + 1, 0, n * sizeof f[0]);
        memset(dfn + 1, 0, n * sizeof dfn[0]);
        memset(lk + 1, 0, n * sizeof lk[0]);
        int i;
        for (i = 1; i <= n; i++) e[i].clear();
        mt19937 rnd(114);
        shuffle(all(edges), rnd);
        for (auto [u, v] : edges)
        {
            e[u].push_back(v), e[v].push_back(u);
            if (!(lk[u] || lk[v])) lk[u] = v, lk[v] = u;
        }
        int r = 0;
        for (i = 1; i <= n; i++) if (!lk[i]) bfs(i);
        for (i = 1; i <= n; i++) r += !!lk[i];
        return r / 2;
    }
}
using blossom_tree::max_general_match, blossom_tree::lk;
```

5.4.3 一般图最大权匹配

$n = 400$：UOJ 600ms, Luogu 135ms

```
namespace weighted_blossom_tree
{
#define d(x) (lab[x.u]+lab[x.v]-e[x.u][x.v].w*2)
    const int N = 403 * 2;//两倍点数
    typedef long long ll;//总和大小
    typedef int T;//权值大小
    //均不允许无符号
    const T inf = numeric_limits<int>::max() >> 1;
    struct Q
    {
        int u, v;
        T w;
    } e[N][N];
    T lab[N];
    int n, m = 0, id, h, t, lk[N], sl[N], st[N], f[N], b[N][N], s[N], ed[N], q[N];
    vector<int> p[N];
    void upd(int u, int v) { if (!sl[v] || d(e[u][v]) < d(e[sl[v]][v])) sl[v] = u; }
    void ss(int v)
    {
        sl[v] = 0;
        for (int u = 1; u <= n; u++) if (e[u][v].w > 0 && st[u] != v && !s[st[u]]) upd(u, v);
    }
    void ins(int u) { if (u <= n) q[++t] = u; else for (int v : p[u]) ins(v); }
    void mdf(int u, int w)
    {
        st[u] = w;
        if (u > n) for (int v : p[u]) mdf(v, w);
    }
    int gr(int u, int v)
    {
```

```
    if ((v = find(all(p[u]), v) - p[u].begin()) & 1)
    {
        reverse(1 + all(p[u]));
        return (int)p[u].size() - v;
    }
    return v;
}
void stm(int u, int v)
{
    lk[u] = e[u][v].v;
    if (u <= n) return;
    Q w = e[u][v];
    int x = b[u][w.u], y = gr(u, x), i;
    for (i = 0; i < y; i++) stm(p[u][i], p[u][i ^ 1]);
    stm(x, v);
    rotate(p[u].begin(), y + all(p[u]));
}
void aug(int u, int v)
{
    int w = st[lk[u]];
    stm(u, v);
    if (!w) return;
    stm(w, st[f[w]]);
    aug(st[f[w]], w);
}
int lca(int u, int v)
{
    for (++id; u | v; swap(u, v))
    {
        if (!u) continue;
        if (ed[u] == id) return u;
        ed[u] = id;//?????????v?? 这是原作者的注释，我也不知道是啥
        if (u = st[lk[u]]) u = st[f[u]];
    }
    return 0;
}
void add(int u, int a, int v)
{
    int x = n + 1, i, j;
    while (x <= m && st[x]) ++x;
    if (x > m) ++m;
    lab[x] = s[x] = st[x] = 0; lk[x] = lk[a];
    p[x].clear(); p[x].push_back(a);
    for (i = u; i != a; i = st[f[j]]) p[x].push_back(i), p[x].push_back(j = st[lk[i]]), ins(j)
        ;//复制，改一处
    reverse(1 + all(p[x]));
    for (i = v; i != a; i = st[f[j]]) p[x].push_back(i), p[x].push_back(j = st[lk[i]]), ins(j)
        ;
    mdf(x, x);
    for (i = 1; i <= m; i++) e[x][i].w = e[i][x].w = 0;
    memset(b[x] + 1, 0, n * sizeof b[0][0]);
    for (int u : p[x])
    {
        for (v = 1; v <= m; v++) if (!e[x][v].w || d(e[u][v]) < d(e[x][v])) e[x][v] = e[u][v],
            e[v][x] = e[v][u];
        for (v = 1; v <= n; v++) if (b[u][v]) b[x][v] = u;
    }
```

```
    ss(x);
}
void ex(int u) // s[u] == 1
{
    for (int x : p[u]) mdf(x, x);
    int a = b[u][e[u][f[u]].u], r = gr(u, a), i;
    for (i = 0; i < r; i += 2)
    {
        int x = p[u][i], y = p[u][i + 1];
        f[x] = e[y][x].u;
        s[x] = 1; s[y] = 0;
        sl[x] = 0; ss(y);
        ins(y);
    }
    s[a] = 1; f[a] = f[u];
    for (i = r + 1; i < p[u].size(); i++) s[p[u][i]] = -1, ss(p[u][i]);
    st[u] = 0;
}
bool on(const Q &e)
{
    int u = st[e.u], v = st[e.v], a;
    if (s[v] == -1)
    {
        f[v] = e.u; s[v] = 1;
        a = st[lk[v]];
        sl[v] = sl[a] = s[a] = 0;
        ins(a);
    }
    else if (!s[v])
    {
        a = lca(u, v);
        if (!a) return aug(u, v), aug(v, u), 1;
        else add(u, a, v);
    }
    return 0;
}
bool bfs()
{
    memset(s + 1, -1, m * sizeof s[0]);
    memset(sl + 1, 0, m * sizeof sl[0]);
    h = 1; t = 0;
    int i, j;
    for (i = 1; i <= m; i++) if (st[i] == i && !lk[i]) f[i] = s[i] = 0, ins(i);
    if (h > t) return 0;
    while (1)
    {
        while (h <= t)
        {
            int u = q[h++], v;
            if (s[st[u]] != 1) for (v = 1; v <= n; v++) if (e[u][v].w > 0 && st[u] != st[v])
            {
                if (d(e[u][v])) upd(u, st[v]); else if (on(e[u][v])) return 1;
            }
        }
        T x = inf;
        for (i = n + 1; i <= m; i++) if (st[i] == i && s[i] == 1) x = min(x, lab[i] >> 1);
        for (i = 1; i <= m; i++) if (st[i] == i && sl[i] && s[i] != 1) x = min(x, d(e[sl[i]][i
```

```
            ]) >> s[i] + 1);
        for (i = 1; i <= n; i++) if (~s[st[i]]) if ((lab[i] += (s[st[i]] * 2 - 1) * x) <= 0)
            return 0;
        for (i = n + 1; i <= m; i++) if (st[i] == i && ~s[st[i]]) lab[i] += (2 - s[st[i]] * 4)
            * x;
        h = 1; t = 0;
        for (i = 1; i <= m; i++) if (st[i] == i && sl[i] && st[sl[i]] != i && !d(e[sl[i]][i])
            && on(e[sl[i]][i])) return 1;
        for (i = n + 1; i <= m; i++) if (st[i] == i && s[i] == 1 && !lab[i]) ex(i);
      }
      return 0;
    }
    template<class TT> ll max_weighted_general_match(int N, const vector<tuple<int, int, TT>> &
        edges)//[1,n], 返回权值
    {
      memset(ed + 1, 0, m * sizeof ed[0]);
      memset(lk + 1, 0, m * sizeof lk[0]);
      n = m = N; id = 0;
      iota(st + 1, st + n + 1, 1);
      int i, j;
      T wm = 0;
      ll r = 0;
      for (i = 1; i <= n; i++) for (j = 1; j <= n; j++) e[i][j] = {i, j, 0};
      for (auto [u, v, w] : edges) wm = max(wm, e[v][u].w = e[u][v].w = max(e[u][v].w, (T)w));
      for (i = 1; i <= n; i++) p[i].clear();
      for (i = 1; i <= n; i++) for (j = 1; j <= n; j++) b[i][j] = i * (i == j);
      fill_n(lab + 1, n, wm);
      while (bfs());
      for (i = 1; i <= n; i++) if (lk[i]) r += e[i][lk[i]].w;
      return r / 2;
    }
#undef d
}
using weighted_blossom_tree::max_weighted_general_match, weighted_blossom_tree::lk;
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, m;
    cin >> n >> m;
    vector<tuple<int, int, long long>> edges(m);
    for (auto &[u, v, w] : edges) cin >> u >> v >> w;
    cout << max_weighted_general_match(n, edges) << '\n';
    for (int i = 1; i <= n; i++) cout << lk[i] << " \n"[i == n];
}
```

5.4.4 网络流

输出方案部分几乎没有验证过。

包含多种费用流方案，spfa，dijkstra，以及 capacity scaling 的 spfa。

```
namespace net
{
    const int N = 4e5 + 50;//number of nodes
    namespace flow
    {
        const ll inf = 4e18;
        struct Q
```

```
{
    int v;
    ll w;
    int id;
};
vector<Q> e[N];
vector<Q>::iterator fir[N];
int fc[N], q[N];
int n, s, t;
int bfs()
{
    for (int i = 0; i < n; i++)
    {
        fir[i] = e[i].begin();
        fc[i] = 0;
    }
    int p1 = 0, p2 = 0, u;
    fc[s] = 1; q[0] = s;
    while (p1 <= p2)
    {
        int u = q[p1++];
        for (auto [v, w, id] : e[u]) if (w && !fc[v])
        {
            q[++p2] = v;
            fc[v] = fc[u] + 1;
        }
    }
    return fc[t];
}
ll dfs(int u, ll maxf)
{
    if (u == t) return maxf;
    ll j = 0, k;
    for (auto &it = fir[u]; it != e[u].end(); ++it)
    {
        auto &[v, w, id] = *it;
        if (w && fc[v] == fc[u] + 1 && (k = dfs(v, min(maxf - j, w))))
        {
            j += k;
            w -= k;
            e[v][id].w += k;
            if (j == maxf) return j;
        }
    }
    fc[u] = 0;
    return j;
}
pair<ll, vector<ll>> max_flow(int _n, const vector<tuple<int, int, ll>> &edges, int _s,
    int _t)//[0,n]
{
    s = _s; t = _t; n = _n + 1;
    for (int i = 0; i < n; i++) e[i].clear();
    for (auto [u, v, w] : edges)
    {
        e[u].push_back({v, w, (int)e[v].size()});
        e[v].push_back({u, 0, (int)e[u].size() - 1});
    }
```

```
        ll r = 0;
        while (bfs()) r += dfs(s, inf);
        vector<ll> ans, id(n);
        for (auto [u, v, w] : edges)
        {
            ++id[u];
            ans.push_back(e[v][id[v]++].w);
        }
        return {r, ans};
    }
}
using flow::max_flow, flow::fc;
namespace match
{
    int lk[N], kl[N], ed[N];
    vector<int> e[N];
    int max_match(int n, int m, const vector<pair<int, int>> &edges)//lk[[0,n]]->[0,m]
    {
        ++n; ++m;
        int s = n + m, t = n + m + 1, i;
        vector<tuple<int, int, ll>> eg;
        eg.reserve(n + m + edges.size());
        for (i = 0; i < n; i++) eg.push_back({s, i, 1});
        for (i = 0; i < m; i++) eg.push_back({i + n, t, 1});
        for (auto [u, v] : edges) eg.push_back({u, v + n, 1});
        int r = max_flow(t, eg, s, t).first;
        fill_n(lk, n, -1); fill_n(kl, m, -1);
        for (i = 0; i < n; i++) for (auto [v, w, id] : flow::e[i]) if (v < s && !w)
        {
            lk[i] = v - n;
            kl[v - n] = i;
            break;
        }
        return r;
    }
    void dfs(int u)
    {
        for (int v : e[u]) if (!ed[v]) ed[v] = 1, dfs(kl[v]);
    }
    pair<vector<int>, vector<int>> min_cover(int n, int m, const vector<pair<int, int>> &edges
        )//[0,n]-[0,m]
    {
        max_match(n++, m++, edges);
        fill_n(ed, m, 0);
        int i;
        for (i = 0; i < n; i++) e[i].clear();
        for (auto [u, v] : edges) e[u].push_back(v);
        for (i = 0; i < n; i++) if (lk[i] == -1) dfs(i);
        vector<int> r[2];
        for (i = 0; i < m; i++) if (kl[i] != -1)
        {
            if (ed[i]) r[1].push_back(i); else r[0].push_back(kl[i]);
        }
        sort(all(r[0]));
        return {r[0], r[1]};
    }
}
```

```
using match::max_match, match::min_cover, match::lk, match::kl;
namespace cost_flow
{
    const ll inf = 3e18;
    struct Q
    {
        int v;
        ll w, c;
        int id;
    };
    vector<Q> e[N];
    ll dis[N];
    int pre[N], pid[N], ipd[N];
    bool ed[N];
    int n, s, t;
    pair<ll, lll> spfa()
    {
        queue<int> q;
        fill_n(dis, n, inf);
        memset(ed, 0, n * sizeof ed[0]);
        q.push(s); dis[s] = 0;
        while (q.size())
        {
            int u = q.front(); q.pop(); ed[u] = 0;
            for (auto [v, w, c, id] : e[u]) if (w && cmin(dis[v], dis[u] + c))
            {
                pre[v] = u;
                pid[v] = e[v][id].id;
                ipd[v] = id;
                if (!ed[v]) q.push(v), ed[v] = 1;
            }
        }
        if (dis[t] == inf) return {0, 0};
        ll mw = 9e18;
        for (int i = t; i != s; i = pre[i]) mw = min(mw, e[pre[i]][pid[i]].w);
        for (int i = t; i != s; i = pre[i]) e[pre[i]][pid[i]].w -= mw, e[i][ipd[i]].w += mw;
        return {mw, (lll)mw * dis[t]};
    }
    tuple<ll, lll, vector<ll>> mcmf_spfa(int _n, const vector<tuple<int, int, ll, ll>> &edges,
         int _s, int _t)//[0,n]
    {
        s = _s; t = _t; n = _n + 1;
        for (int i = 0; i < n; i++) e[i].clear();
        for (auto [u, v, w, c] : edges)
        {
            e[u].push_back({v, w, c, (int)e[v].size()});
            e[v].push_back({u, 0, -c, (int)e[u].size() - 1});
        }
        ll w = 0, dw;
        lll c = 0, dc;
        do
        {
            tie(dw, dc) = spfa();
            w += dw, c += dc;
        } while (dw);
        vector<ll> ans, id(n);
        for (auto [u, v, w, c] : edges)
```

```
        {
            ++id[u];
            ans.push_back(e[v][id[v]++].w);
        }
        return {w, c, ans};
    }
    pair<ll, ll> spfa_loop()
    {
        vector<ll> d(n);
        vector<int> ed(n, 1), cnt(n), pre(n), pid(n);
        queue<int> q;
        int i;
        for (i = 0; i < n; i++) q.push(i);
        while (q.size())
        {
            int u = q.front(); q.pop(); ed[u] = 0;
            for (auto [v, w, c, id] : e[u]) if (w && cmin(d[v], d[u] + c))
            {
                pre[v] = u;
                pid[v] = id;
                if (d[v]<-inf * 2 || (cnt[v] = cnt[u] + 1) > n)
                {
                    ll tw = 0, tc = 0;
                    for (u = v; ed[u] <= 1; u = pre[u]) ed[u] = 2;
                    for (; ed[u] == 2; u = v)
                    {
                        v = pre[u];
                        ++e[u][pid[u]].w;
                        --e[v][e[u][pid[u]].id].w;
                        if (e[v][e[u][pid[u]].id].c != -inf) tc += e[v][e[u][pid[u]].id].c;
                        else tw = 1;
                        ed[u] = 3;
                    }
                    return {tw, tc};
                }
                if (!ed[v]) ed[v] = 1, q.push(v);
            }
        }
        return {0, 0};
    }
    tuple<ll, lll, vector<ll>> mcmf_spfa_scaling(int _n, vector<tuple<int, int, ll, ll>> edges
        , int _s, int _t)//[0,n]
    {
        s = _s; t = _t; n = _n + 1;
        for (int i = 0; i < n; i++) e[i].clear();
        ll tot = 1;
        for (auto [u, v, w, c] : edges) tot += w;
        edges.push_back({t, s, tot, -inf});//最后一项 mcmf:-inf, mcf:0
        for (auto [u, v, w, c] : edges)
        {
            e[u].push_back({v, 0, c, (int)e[v].size()});
            e[v].push_back({u, 0, -c, (int)e[u].size() - 1});
        }
        ll w = 0, dw;
        lll c = 0, dc;
        for (int g = __lg(tot); g >= 0; g--)
        {
```

```
            vector<int> id(n);
            for (auto [u, v, w, c] : edges)
            {
                (e[u][id[u]++].w *= 2) += w >> g & 1;
                (e[v][id[v]++].w *= 2);
            }
            w *= 2, c *= 2;
            do
            {
                tie(dw, dc) = spfa_loop();
                w += dw, c += dc;
            } while (dw || dc < 0);
        }
        vector<ll> ans, id(n);
        edges.pop_back();
        e[s].pop_back(), e[t].pop_back();
        for (auto [u, v, w, c] : edges)
        {
            ++id[u];
            ans.push_back(e[v][id[v]++].w);
        }
        return {w, c, ans};
    }
    tuple<ll, lll, vector<ll>> mcmf_dijk(int _n, const vector<tuple<int, int, ll, ll>> &edges,
         int _s, int _t)//[0,n]
    {
        s = _s; t = _t; n = _n + 1;
        for (int i = 0; i < n; i++) e[i].clear();
        for (auto [u, v, w, c] : edges)
        {
            e[u].push_back({v, w, c, (int)e[v].size()});
            e[v].push_back({u, 0, -c, (int)e[u].size() - 1});
        }
        static ll h[N];
        auto get_h = [&]() {
            fill_n(h, n, inf);
            memset(ed, 0, n * sizeof ed[0]);
            queue<int> q;
            q.push(s); h[s] = 0;
            while (q.size())
            {
                int u = q.front(); q.pop(); ed[u] = 0;
                for (auto [v, w, c, id] : e[u]) if (w && h[v] > h[u] + c)
                {
                    assert(c >= 0);
                    h[v] = h[u] + c;
                    if (!ed[v]) q.push(v), ed[v] = 1;
                }
            }
            return;
        };
        auto dijkstra = [&]() -> pair<ll, lll> {
            static int fl[N], zl[N];
            int i;
            memset(ed, 0, n * sizeof ed[0]);
            fill_n(dis, n, inf);
            using pa = pair<ll, int>;
```

```
        priority_queue<pa, vector<pa>, greater<pa>> q;
        dis[s] = 0; q.push({0, s});
        while (q.size())
        {
            int u = q.top().second;
            q.pop(); ed[u] = 1;
            i = 0;
            for (auto [v, w, c, id] : e[u])
            {
                if (w && cmin(dis[v], dis[u] + c))
                {
                    assert(c >= 0);
                    fl[v] = id, zl[v] = i, pre[v] = u;
                    q.push({dis[v], v});
                }
                ++i;
            }
            while (q.size() && ed[q.top().second]) q.pop();
        }
        if (dis[t] == inf) return {0, 0};
        ll tf = numeric_limits<ll>::max();
        for (i = t; i != s; i = pre[i]) tf = min(tf, e[pre[i]][zl[i]].w);
        for (i = t; i != s; i = pre[i]) e[pre[i]][zl[i]].w -= tf, e[i][fl[i]].w += tf;
        for (int u = 0; u < n; u++) for (auto &[v, w, c, id] : e[u]) c += dis[u] - dis[v];
        return {tf, (lll)tf * (h[t] += dis[t])};
    };
    get_h();
    for (int u = 0; u < n; u++) for (auto &[v, w, c, id] : e[u]) c += h[u] - h[v];
    ll w = 0, dw;
    lll c = 0, dc;
    do
    {
        tie(dw, dc) = dijkstra();
        w += dw, c += dc;
    } while (dw);
    vector<ll> ans, id(n);
    for (auto [u, v, w, c] : edges)
    {
        ++id[u];
        assert(e[v][id[v]].v == u);
        ans.push_back(e[v][id[v]++].w);
    }
    return {w, c, ans};
  }
}
using cost_flow::mcmf_spfa, cost_flow::mcmf_spfa_scaling, cost_flow::mcmf_dijk;
namespace bounded_flow
{
    vector<ll> valid_flow(int n, const vector<tuple<int, int, ll, ll>> &edges)
    {//返回空 vector 表示无解。最好保证 edges 非空。
        int i, m = edges.size();
        assert(m);
        ++n;
        ll tot = 0;
        static ll cd[N];
        memset(cd, 0, n * sizeof cd[0]);
        for (auto [u, v, l, r] : edges) cd[u] += l, cd[v] -= l;
```

```
    vector<tuple<int, int, ll>> eg;
    eg.reserve(n + m);
    for (i = 0; i < n; i++) if (cd[i] > 0) eg.push_back({i, n + 1, cd[i]}), tot += cd[i];
    else if (cd[i] < 0) eg.push_back({n, i, -cd[i]});
    for (auto [u, v, l, r] : edges) eg.push_back({u, v, r - l});
    auto [w, ans] = flow::max_flow(n + 1, eg, n, n + 1);
    if (tot != w) return { };
    ans.erase(all(ans) - m);
    for (i = 0; i < m; i++) ans[i] += get<2>(edges[i]);
    return ans;
}
pair<ll, vector<ll>> valid_flow_st(int n, vector<tuple<int, int, ll, ll>> edges, int s,
     int t)
{//返回值 first -1 表示无解
    ll tot = 0;
    for (auto [u, v, l, r] : edges) tot += (u == s) * r;
    edges.push_back({t, s, 0, tot});
    auto ans = valid_flow(n, edges);
    if (!ans.size()) return {-1, { }};
    ans.pop_back();
    assert(flow::e[s].back().v == t);
    assert(flow::e[t].back().v == s);
    return {flow::e[s].back().w, ans};
}
pair<ll, vector<ll>> valid_max_flow(int n, const vector<tuple<int, int, ll, ll>> &edges,
     int s, int t)
{//返回值 first -1 表示无解
    auto [r, _] = valid_flow_st(n, edges, s, t);
    if (r == -1) return {-1, { }};
    flow::s = s, flow::t = t;
    flow::e[s].pop_back(), flow::e[t].pop_back();
    while (flow::bfs()) r += flow::dfs(s, flow::inf);
    int m = edges.size(), i;
    vector<ll> ans(m), id(n + 1);
    for (i = 0; i <= n; i++) id[i] = flow::e[i].size();
    for (i = m - 1; i >= 0; i--)
    {
        auto [u, v, l, r] = edges[i];
        --id[u]; ans[i] = flow::e[v][--id[v]].w + l;
    }
    return {r, ans};
}
pair<ll, vector<ll>> valid_min_flow(int n, const vector<tuple<int, int, ll, ll>> &edges,
     int s, int t)
{
    auto [r, _] = valid_flow_st(n, edges, s, t);
    if (r == -1) return {-1, { }};
    flow::s = t; flow::t = s;
    flow::e[s].pop_back(); flow::e[t].pop_back();
    while (flow::bfs()) r -= flow::dfs(t, flow::inf);
    int m = edges.size(), i;
    vector<ll> ans(m), id(n + 1);
    for (i = 0; i <= n; i++) id[i] = flow::e[i].size();
    for (i = m - 1; i >= 0; i--)
    {
        auto [u, v, l, r] = edges[i];
        --id[u]; ans[i] = flow::e[v][--id[v]].w + l;
```

```
        }
        return {r, ans};
    }//not check
}
using bounded_flow::valid_flow, bounded_flow::valid_flow_st, bounded_flow::valid_max_flow,
    bounded_flow::valid_min_flow;
namespace bounded_cost_flow
{
    tuple<ll, lll, vector<ll>> valid_mcf(int n, const vector<tuple<int, int, ll, ll, ll>> &
        edges, int s, int t)
    {//[u,v,l,r,c],mincost flow
        ++n;
        int ss = n, tt = n + 1, m = edges.size(), i;
        static ll cd[N];
        memset(cd, 0, n * sizeof cd[0]);
        for (auto [u, v, l, r, c] : edges) cd[u] += l, cd[v] -= l;
        vector<tuple<int, int, ll, ll>> e; e.reserve(n + m + 1);
        ll t1 = 0, t2 = 0;
        for (auto [u, v, l, r, c] : edges) e.push_back({u, v, r - l, c});
        for (i = 0; i < n; i++) if (cd[i] > 0) e.push_back({i, tt, cd[i], 0}), t2 += cd[i];
        else if (cd[i] < 0) e.push_back({ss, i, -cd[i], 0});
        for (auto [u, v, w, c] : e) t1 += (u == s) * w;
        e.push_back({t, s, t1, 0});
        auto [tw, tc, _] = mcmf_spfa_scaling(tt, e, ss, tt);//checked spfa/dijk
        if (tw != t2) return {-1, -1, { }};
        tw = cost_flow::e[s].back().w;
        for (auto [u, v, l, r, c] : edges) tc += (lll)l * c;
        vector<ll> ans(m), id(n);
        for (i = 0; i < m; i++)
        {
            auto [u, v, l, r, c] = edges[i];
            assert(cost_flow::e[v][id[v]].v == u);
            ++id[u]; ans[i] = cost_flow::e[v][id[v]++].w + l;
        }
        return {tw, tc, ans};
    }
    tuple<ll, lll, vector<ll>> valid_mcmf(int n, const vector<tuple<int, int, ll, ll, ll>> &
        edges, int s, int t)
    {//[u,v,l,r,c],mincost max_flow, not checked dijk
        auto [tw, tc, _] = valid_mcf(n, edges, s, t);
        if (tw == -1) return {-1, -1, { }};
        cost_flow::e[s].pop_back();
        cost_flow::e[t].pop_back();
        cost_flow::s = s; cost_flow::t = t;
        ll dw;
        lll dc;
        do
        {
            tie(dw, dc) = cost_flow::spfa();
            tw += dw; tc += dc;
        } while (dw);
        int m = edges.size(), i;
        vector<ll> ans(m), id(n + 1);
        for (i = 0; i < m; i++)
        {
            auto [u, v, l, r, c] = edges[i];
            ++id[u]; ans[i] = cost_flow::e[v][id[v]++].w + l;
```

```
        }
        return {tw, tc, ans};
    }
    tuple<ll, lll, vector<ll>> valid_mcmf_scaling(int n, const vector<tuple<int, int, ll, ll,
         ll>> &edges, int s, int t)
    {//[u,v,l,r,c],mincost max_flow, not checked dijk
        using cost_flow::e, cost_flow::spfa_loop;
        auto [tw, tc, _] = valid_mcf(n, edges, s, t);
        if (tw == -1) return {-1, -1, { }};
        e[s].pop_back();
        e[t].pop_back();
        cost_flow::s = s; cost_flow::t = t;
        n = cost_flow::n;
        int m = edges.size(), i, j;
        vector<ll> cur;
        ll tot = 1;
        for (i = 0; i < n; i++) for (auto [v, w, c, id] : cost_flow::e[i]) tot += w;
        for (i = 0; i < n; i++)
        {
            for (auto &[v, w, c, id] : cost_flow::e[i])
                cur.push_back(w), w = 0;
            if (i == t) cur.push_back(tot);
            if (i == s) cur.push_back(0);
        }
        e[t].push_back({s, 0, -cost_flow::inf, (int)e[s].size()});
        e[s].push_back({t, 0, cost_flow::inf, (int)e[t].size() - 1});
        ll w = 0, dw;
        lll c = 0, dc;
        for (int g = __lg(*max_element(all(cur))); g >= 0; g--)
        {
            for (i = j = 0; i < n; i++) for (auto &[v, w, c, id] : e[i]) w = w * 2 + (cur[j++]
                 >> g & 1);
            w *= 2, c *= 2;
            do
            {
                tie(dw, dc) = spfa_loop();
                w += dw, c += dc;
            } while (dw || dc < 0);
        }
        vector<ll> ans(m), id(n + 1);//方案很可能完全不对
        for (i = 0; i < m; i++)
        {
            auto [u, v, l, r, c] = edges[i];
            ++id[u]; ans[i] = cost_flow::e[v][id[v]++].w + l;
        }
        return {tw + w, tc + c, ans};
    }
}
using bounded_cost_flow::valid_mcf, bounded_cost_flow::valid_mcmf, bounded_cost_flow::
    valid_mcmf_scaling;
namespace ne_cost_flow
{
    tuple<ll, lll, vector<ll>> ne_mcmf(int n, const vector<tuple<int, int, ll, ll>> &edges,
         int s, int t)
    {
        vector<tuple<int, int, ll, ll, ll>> e;
        for (auto [u, v, w, c] : edges) if (c >= 0) e.push_back({u, v, 0, w, c}); else
```

```
        {
            e.push_back({u, v, w, w, c});
            e.push_back({v, u, 0, w, -c});
        }
        auto [tw, tc, res] = valid_mcmf_scaling(n, e, s, t);
        int m = edges.size(), i, j;
        vector<ll> ans(m);
        for (i = j = 0; i < m; i++, j++)
        {
            auto [u, v, w, c] = edges[i];
            if (c >= 0) ans[i] = res[j];
            else ans[i] = w - res[++j];
        }
        assert(j == e.size());
        return {tw, tc, ans};
    }
    tuple<ll, lll, vector<ll>> ne_valid_mcf(int n, const vector<tuple<int, int, ll, ll, ll>> &
        edges, int s, int t)
    {
        vector<tuple<int, int, ll, ll, ll>> e;
        for (auto [u, v, l, r, c] : edges) if (c >= 0) e.push_back({u, v, l, r, c}); else
        {
            e.push_back({u, v, r, r, c});
            e.push_back({v, u, 0, r - l, -c});
        }
        auto [tw, tc, res] = valid_mcf(n, e, s, t);
        if (tw == -1) return {-1, -1, { }};
        int m = edges.size(), i, j;
        vector<ll> ans(m);
        for (i = j = 0; i < m; i++, j++)
        {
            auto [u, v, l, r, c] = edges[i];
            if (c >= 0) ans[i] = res[j];
            else ans[i] = r - res[++j];
        }
        assert(j == e.size());
        return {tw, tc, ans};
    }
  }
  using ne_cost_flow::ne_mcmf, ne_cost_flow::ne_valid_mcf;
}
```

5.4.5 假带花树

一种错误的一般图最大匹配算法，但较难卡掉。推荐在时间不足时作为乱搞使用。

```
mt19937 rnd(3214);
vector<int> lj[N];
int lk[N],ed[N];
int n,m,cnt,i,t,x,y,ans,la;
bool dfs(int x)
{
    ed[x]=cnt;int v;
    shuffle(lj[x].begin(),lj[x].end(),rnd);
    for (auto u:lj[x]) if (ed[v=lk[u]]!=cnt)
    {
        lk[v]=0,lk[u]=x,lk[x]=u;
```

```
        if (!v||dfs(v)) return 1;
        lk[v]=u,lk[u]=v,lk[x]=0;
    }
    return 0;
}
int main()
{
    srand(time(0));la=-1;
    cin>>n>>m;
    while (m--) cin>>x>>y,lj[x].push_back(y),lj[y].push_back(x);
    while (la!=ans)
    {
        memset(ed+1,0,n<<2);la=ans;
        for (i=1;i<=n;i++) if (!lk[i]) ans+=dfs(cnt=i);
    }
    cout<<ans<<'\n';
    for (i=1;i<=n;i++) cout<<lk[i]<<" \n"[i==n];
}
```

5.4.6 Stoer-Wagner 全局最小割

无向图 G 的最小割为：一个去掉后可以使 G 变成两个连通分量，且边权和最小的边集的边权和。

$O(n^3)$。可优化到 $O(nm\log n)$。

```
#include "bits/stdc++.h"
using namespace std;
namespace StoerWagner
{
    const int N=602;//点数
    typedef int T;//边权和
    T e[N][N],w[N];
    int ed[N],p[N],f[N];//f 仅输出方案用
    int getf(int u){return f[u]==u?u:f[u]=getf(f[u]);}
    template<class TT> pair<T,vector<int>> mincut(int n,const vector<tuple<int,int,TT>> &edges)//
        [1,n], 返回某一集合
    {
        vector<int> ans;ans.reserve(n);
        int i,j,m;
        T r;
        r=numeric_limits<T>::max();
        for (i=1;i<=n;i++) memset(e[i]+1,0,n*sizeof e[0][0]);
        for (auto [u,v,w]:edges) e[u][v]+=w,e[v][u]+=w;
        fill_n(ed+1,n,0);
        iota(f+1,f+n+1,1);
        for (m=n;m>1;m--)
        {
            fill_n(w+1,n,0);
            for (i=1;i<=n;i++) ed[i]&=2;
            for (i=1;i<=m;i++)
            {
                int x=0;
                for (j=1;j<=n;j++) if (!ed[j]) break;x=j;
                for (j++;j<=n;j++) if (!ed[j]*w[j]>w[x]) x=j;
                ed[p[i]=x]=1;
                for (j=1;j<=n;j++) w[j]+=!ed[j]*e[x][j];
```

```
        }
        int s=p[m-1],t=p[m];
        if (r>w[t])
        {
            r=w[t];ans.clear();
            for (i=1;i<=n;i++) if (getf(i)==getf(t)) ans.push_back(i);
        }
        for (i=1;i<=n;i++) e[i][s]=e[s][i]+=e[t][i];
        ed[t]=2;
        f[getf(s)]=getf(t);
    }
    return {r,ans};
  }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m;
    cin>>n>>m;
    vector<tuple<int,int,int>> e(m);
    for (auto &[u,v,w]:e) cin>>u>>v>>w;
    auto [_,v]=StoerWagner::mincut(n,e);
    cout<<_<<endl;
    static int ed[602];
    for (int x:v) ed[x]=1;
    for (auto [u,v,w]:e) _-=w*(ed[u]^ed[v]);
    assert(!_);
}
```

5.4.7 最小割树

结论：两个点之间的最小割等于最小割树上两点间最小边权。

直接返回任意两点最小割。

```
template<class T> vector<vector<T>> min_cut(int n, const vector<tuple<int, int, T>> &edges)//[0,n
    )
{
    int m=edges.size(), i, s, t, cnt=0;
    vector<int> fir(n, -1), nxt(m*2, -1), fc(n), q(n);
    vector<pair<int, T>> e(m*2);
    vector<tuple<T, int, int>> eg;
    auto add=[&](int u, int v, T w)
        {
            e[cnt]={v, w};
            nxt[cnt]=fir[u];
            fir[u]=cnt++;
        };
    for (auto [u, v, w]:edges) add(u, v, w), add(v, u, w);
    auto E=e;
    auto bfs=[&]()
        {
            fill(all(fc), 0);
            int ql=0, qr=0, u, i;
            fc[q[0]=s]=1;
            while (ql<=qr)
            {
                u=q[ql++];
```

```
            for (int i=fir[u]; i!=-1; i=nxt[i])
                if (auto &[v, w]=e[i]; w&&!fc[v]) fc[q[++qr]=v]=fc[u]+1;
        }
        return fc[t];
    };
    function<T(int, T)> dfs=[&](int u, T maxf)
    {
        if (u==t) return maxf;
        T j=0, k;
        for (int i=fir[u]; i!=-1; i=nxt[i])
            if (auto &[v, w]=e[i]; w&&fc[v]==fc[u]+1&&(k=dfs(v, min(maxf-j, w))))
            {
                j+=k;
                w-=k;
                e[i^1].second+=k;
                if (j==maxf) return j;
            }
        fc[u]=0;
        return j;
    };
    function<void(vector<int>)> solve=[&](vector<int> id)
    {
        static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
        if (id.size()<=1) return;
        vector<int> u(2);
        sample(all(id), u.begin(), 2, rnd);
        s=u[0], t=u[1], e=E;
        T ans=0;
        while (bfs()) ans+=dfs(s, numeric_limits<T>::max());
        auto it=partition(all(id), [&](int u) { return fc[u]; });
        eg.emplace_back(ans, s, t);
        solve(vector(id.begin(), it));
        solve(vector(it, id.end()));
    };
    solve(range(0, n));
    sort(all(eg), greater<>());
    vector<basic_string<int>> ver(n);
    vector ans(n, vector<T>(n));
    vector<int> f(n);
    for (i=0; i<n; i++) ver[i]={f[i]=i};
    function<int(int)> getf=[&](int u) { return f[u]==u?u:f[u]=getf(f[u]); };
    for (auto [w, u, v]:eg)
    {
        u=getf(u);
        v=getf(v);
        for (int w1:ver[u]) for (int w2:ver[v]) ans[w1][w2]=ans[w2][w1]=w;
        ver[u]+=ver[v];
        f[v]=u;
    }
    return ans;
}
```

5.5 缩点相关

5.5.1 双极分解

无向图，图点双连通时对任意 s,t 存在。

含义：确定一个拓扑序，使得按这个拓扑序定向后，入度为 0 的只有 s，出度为 0 的只有 t。

```
vector<int> bipolar_orientation(const vector<pair<int, int>> &edges, int n, int s, int t)//[0,n)
{
    assert(s!=t);
    vector e(n, vector<int>());
    for (auto [u, v]:edges)
    {
        e[u].push_back(v);
        e[v].push_back(u);
    }
    int cur=1, i;
    vector<int> pre(n), low(n), p(n);
    pre[s]=1;
    vector<int> id;
    bool flg=0;
    function<void(int)> dfs=[&](int x)
        {
            pre[x]=++cur;
            low[x]=x;
            for (int y:e[x])
            {
                flg|=y==s;
                if (pre[y]==0)
                {
                    id.push_back(y);
                    dfs(y);
                    p[y]=x;
                    if (pre[low[y]]<pre[low[x]]) low[x]=low[y];
                }
                else if (pre[y]!=0&&pre[y]<pre[low[x]]) low[x]=y;
            }
        };
    dfs(t);
    if (!flg) return { };
    vector<int> sign(n, -1);
    vector<int> l(n), r(n);
    r[s]=t;
    l[t]=s;
    for (int v:id)
    {
        if (sign[low[v]]==-1)
        {
            l[v]=l[p[v]];
            r[l[v]]=v;
            l[p[v]]=v;
            r[v]=p[v];
            sign[p[v]]=1;
        }
        else
        {
            r[v]=r[p[v]];
            l[r[v]]=v;
```

```
            r[p[v]]=v;
            l[v]=p[v];
            sign[p[v]]=-1;
        }
    }
    vector<int> a(n);
    int x;
    for (i=0, x=s; i<n; x=r[x], i++) a[i]=x;
    vector<int> ia(n, -1), rd(n), cd(n);
    for (i=0; i<n; i++) ia[a[i]]=i;
    if (count(all(ia), -1)) return { };
    for (auto [u, v]:edges)
    {
        if (ia[u]>ia[v]) swap(u, v);
        ++cd[u]; ++rd[v];
    }
    for (i=0; i<n; i++) if (i!=s&&i!=t&&(!cd[i]||!rd[i])) return { };
    return a;
}
```

5.5.2 点双

一些结论：

判定一个图里是否有（点不重复）偶环：看其所有点双，若存在点数为偶数的或边数多于点数的点双，则存在偶环。

（无自环时）点双的边不交，边双的点不交。点双内的总点数 $O(n)$，总边数为 m，边双内的总点数为 n，总边数不超过 m。

关于板子本身：

所有标号从 0 开始。

构造函数传入邻接表和边数，其中 `pair` 的 `second` 是边的标号，正反向编号相同，允许不连续但不能大于等于 m。

不能处理有自环的情况。你可以直接在图中删除自环后传入。可以处理有重边的情况。

`bcc_node`：每个点双包含的点（已验证）；`bcc_edge`：每个点双包含的边；`bcc_n`：新图点数；`ct`：是否割点（已验证）；`blk`：边所属点双标号。

```
struct node_bcc
{
    int n, id, tp, bcc_n;
    vector<vector<pair<int, int>>> e;
    vector<vector<int>> bcc_node, bcc_edge;
    vector<int> dfn, low, st, ed, blk, ct;
    node_bcc(const vector<vector<pair<int, int>>> &e, int m) :
        n(e.size()), id(0), tp(0), bcc_n(0), e(e), dfn(n, -1), low(n, -1), st(m), ed(m), blk(m),
            ct(n)
    {
        for (int i = 0; i < n; i++) for (auto [v, w] : e[i])
        {
            assert(v >= 0 && v < n);
            assert(w >= 0 && w < m);
        }
        for (int i = 0; i < n; i++) if (dfn[i] == -1) dfs(i, 1);
        bcc_node.resize(bcc_n);
        for (int i = 0; i < n; i++) for (auto [v, w] : e[i]) bcc_node[blk[w]].push_back(i);
        vector<int> flg(n);
        for (auto &v : bcc_node)
```

```
        {
            vector<int> t;
            for (int x : v) if (!exchange(flg[x], 1)) t.push_back(x);
            swap(t, v);
            for (int x : v) flg[x] = 0;
        }
        for (int i = 0; i < n; i++) if (e[i].size() == 0)
        {
            bcc_node.push_back({i});
            bcc_edge.push_back({ });
            ++bcc_n;
        }
    }
    void dfs(int u, bool rt)
    {
        dfn[u] = low[u] = id++;
        int cnt = 0;
        for (auto [v, w] : e[u]) if (!ed[w])
        {
            st[tp++] = w;
            ed[w] = 1;
            if (dfn[v] == -1)
            {
                dfs(v, 0);
                ++cnt;
                cmin(low[u], low[v]);
                if (dfn[u] <= low[v])
                {
                    ct[u] = cnt > rt;
                    bcc_edge.push_back({ });
                    do
                    {
                        bcc_edge[bcc_n].push_back(st[--tp]);
                        blk[st[tp]] = bcc_n;
                    } while (st[tp] != w);
                    ++bcc_n;
                }
            }
            else cmin(low[u], dfn[v]);
        }
    }
};
int main()
{
    int n, m, i;
    cin >> n >> m;
    vector<vector<pair<int, int>> e(n);
    for (i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v;
        e[u].push_back({v, i});
        e[v].push_back({u, i});
    }
    node_bcc bcc(e, m);
}
```

5.5.3 边双

所有标号从 0 开始。

构造函数传入邻接表和边数，其中 pair 的 second 是边的标号，正反向编号相同，允许不连续但不能大于等于 m。

可以处理有重边和自环的情况。

bcc_node：每个边双包含的点（已验证）；bcc_edge：每个边双包含的边；bcc_n：新图点数；cur_e：新图边表；ct：是否割边；blk：点所属边双标号。

```
struct edge_bcc
{
    int n, id, tp, bcc_n;
    vector<vector<pair<int, int>>> e, cur_e;
    vector<vector<int>> bcc_node, bcc_edge;
    vector<int> dfn, low, st, blk, ct;
    edge_bcc(const vector<vector<pair<int, int>>> &e, int m) :
        n(e.size()), id(0), tp(0), bcc_n(0), e(e), dfn(n, -1), low(n, -1), st(n), blk(n), ct(m)
    {
        for (int i = 0; i < n; i++) for (auto [v, w] : e[i])
        {
            assert(v >= 0 && v < n);
            assert(w >= 0 && w < m);
        }
        for (int i = 0; i < n; i++) if (dfn[i] == -1) dfs(i, -1);
        cur_e.resize(bcc_n);
        for (int i = 0; i < n; i++) for (auto [v, w] : e[i]) if (ct[w]) cur_e[blk[i]].push_back({
            blk[v], w});
        else bcc_edge[blk[i]].push_back(w);
        vector<int> flg(m);
        for (auto &v : bcc_edge)
        {
            vector<int> t;
            for (int x : v) if (!exchange(flg[x], 1)) t.push_back(x);
            swap(t, v);
        }
    }
    void dfs(int u, int fw)
    {
        dfn[u] = low[u] = id++;
        st[tp++] = u;
        for (auto [v, w] : e[u]) if (w != fw)
        {
            if (dfn[v] == -1)
            {
                dfs(v, w);
                cmin(low[u], low[v]);
                ct[w] = (dfn[u] < low[v]);
            }
            else cmin(low[u], dfn[v]);
        }
        if (dfn[u] == low[u])
        {
            bcc_node.push_back({ });
            bcc_edge.push_back({ });
            do
            {
                bcc_node[bcc_n].push_back(st[--tp]);
```

```
            blk[st[tp]] = bcc_n;
        } while (st[tp] != u);
        ++bcc_n;
    }
  }
};
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, m, i;
    cin >> n >> m;
    vector<vector<pair<int, int>>> e(n);
    for (i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v;
        --u, --v;
        e[u].push_back({v, i});
        e[v].push_back({u, i});
    }
    edge_bcc s(e, m);
    cout << s.bcc_n << '\n';
    for (auto &v : s.bcc_node)
    {
        for (int &x : v) ++x;
        cout << v.size() << ' ' << v << '\n';
    }
}
```

5.5.4 Tarjan 强连通分量

未经验证。

所有标号从 0 开始。

构造函数传入邻接表，其中 `pair` 的 `second` 是边的标号。

与点边双不同的是，你可以任意传入 `second`，代码中不会使用它，仅用于新图区分边。

可以处理有重边和自环的情况。

`cc_node`：每个 SCC 包含的点（已验证）；`cc_n`：新图点数；`cur_e`：新图边表；`blk`：点所属 SCC 标号。

```
struct scc
{
    int n, id, tp, cc_n;
    vector<vector<pair<int, int>>> e, cur_e;
    vector<vector<int>> cc_node;
    vector<int> dfn, low, st, ed, blk;
    void dfs(int u)
    {
        dfn[u] = low[u] = id++;
        st[tp++] = u; ed[u] = 1;
        for (auto [v, w] : e[u]) if (dfn[v] == -1)
        {
            dfs(v);
            cmin(low[u], low[v]);
        }
        else if (ed[v]) cmin(low[u], dfn[v]);
        if (low[u] == dfn[u])
```

```
        {
            cc_node.push_back({ });
            do
            {
                int v = st[--tp];
                ed[v] = 0;
                blk[v] = cc_n;
                cc_node[cc_n].push_back(v);
            } while (st[tp] != u);
            cc_n++;
        }
    }
    scc(const vector<vector<pair<int, int>>> &e) :n(e.size()), id(0), tp(0), cc_n(0),
        e(e), cur_e(n), dfn(n, -1), low(dfn), st(n), ed(n), blk(n)
    {
        for (int i = 0; i < n; i++) for (auto [v, w] : e[i]) assert(v >= 0 && v < n);
        for (int i = 0; i < n; i++) if (dfn[i] == -1) dfs(i);
        reverse(all(cc_node));
        for (int &x : blk) x = cc_n - x - 1;
        for (int u = 0; u < n; u++)
            for (auto [v, w] : e[u])
                if (blk[u] != blk[v])
                    cur_e[blk[u]].push_back({blk[v], w});
    }
};
```

5.5.5 动态强连通分量

给出一个加边序列，solve 会返回每个时间进入强连通分量的边。点标号范围是 $[0, n)$

```
struct union_set
{
    vector<int> f;
    int n;
    union_set() { }
    union_set(int nn) :n(nn), f(nn+1)
    {
        iota(all(f), 0);
    }
    int getf(int u) { return f[u]==u ? u : f[u] = getf(f[u]); }
    bool merge(int u, int v)
    {
        u = getf(u); v = getf(v);
        if (u==v) return 0;
        f[u] = v;
        return 1;
    }
    bool connected(int u, int v) { return getf(u)==getf(v); }
};
struct edge
{
    int u, v, t;
};
vector<vector<edge>> solve(int n, const auto& eg)//[0,n)
{
    int m = eg.size(), tp = -1, id = 0, fs = 0;
```

```
vector<vector<edge>> res(m);
vector e(n, vector<int>());
vector<int> dfn(n, -1), low(n, -1), st(n), ed(n), blk(n), node;
union_set s(n-1);
function<void(int)> dfs = [&](int u)
    {
        dfn[u] = low[u] = id++;
        ed[st[++tp] = u] = 1;
        for (int v : e[u]) if (dfn[v]!=-1)
        {
            if (ed[v]) cmin(low[u], dfn[v]);
        }
        else dfs(v), cmin(low[u], low[v]);
        if (dfn[u]==low[u])
        {
            do
            {
                ed[st[tp]] = 0;
                blk[st[tp]] = fs;
            } while (st[tp--]!=u);
            ++fs;
        }
    };
auto ztef = [&](auto ztef, int l, int r, const vector<edge>& q)
    {
        if (eg.size()==0) return;
        if (l+1==r)
        {
            if (l<m)
            {
                res[l].insert(res[l].end(), all(q));
                for (auto [u, v, t]:q) s.merge(u, v);
            }
            return;
        }
        int m = (l+r)/2;
        node.clear();
        for (auto [u, v, t]:q) if (t<m)
        {
            u = s.getf(u);
            v = s.getf(v);
            e[u].push_back(v);
            node.push_back(u);
            node.push_back(v);
        }
        else break;
        for (int u : node) if (dfn[u]==-1) dfs(u);
        vector<vector<edge>> g(2);
        for (auto [u, v, t]:q) g[t<m&&blk[s.f[u]]==blk[s.f[v]]].push_back({u, v, t});
        for (int u : node)
        {
            e[u].clear();
            dfn[u] = low[u] = -1;
        }
        id = fs = 0;
        ztef(ztef, l, m, g[1]);
        ztef(ztef, m, r, g[0]);
```

```
    };
    ztef(ztef, 0, m+1, eg);
    return res;
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout<<fixed<<setprecision(15);
    int n, m, i, j;
    cin>>n>>m;
    vector<ll> x(n);
    cin>>x;
    vector<edge> edges(m);
    for (i = 0;i<m;i++)
    {
        auto& [u, v, t] = edges[i];
        cin>>u>>v;
        t = i;
    }
    auto event = solve(n, edges);
    union_set s(n-1);
    ll ans = 0;
    for (auto e:event)
    {
        for (auto [u, v, t]:e)
        {
            u = s.getf(u);
            v = s.getf(v);
            if (u==v) continue;
            s.f[v] = u;
            (ans += x[u]*x[v]) %= p;
            (x[u] += x[v]) %= p;
        }
        cout<<ans<<'\n';
    }
}
```

5.5.6 圆方树

题意：求仙人掌上两点最短路。

$O(n+m)$，$O(n+m)$。

```
#include "bits/stdc++.h"
using namespace std;
#if !defined(ONLINE_JUDGE)&&defined(LOCAL)
#include "my_header\debug.h"
#else
#define dbg(...); 1;
#endif
typedef unsigned int ui;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=3e4+2,M=3e4+2;//M 包括方点
struct P
{
    int v,w,id;
    P(int a,int b,int c):v(a),w(b),id(c){}
```

```
};
struct Q
{
    int v,w;
    Q(int a,int b):v(a),w(b){}
};
vector<P> e[N];
vector<Q> fe[M];
int dfn[M],low[N],st[N],len[M],top[M],siz[M],hc[M],dep[M],f[M],rb[N];
bool ed[M];//ed,dfn,loop,sum,fe,hc,tp,id,cnt,dep[1] 需初始化（注意倍率），ed 大小为边数
int tp,id,cnt,n;
void dfs1(int u)
{
    dfn[u]=low[u]=++id;
    st[++tp]=u;
    for (auto [v,w,id]:e[u]) if (!ed[id])
    {
        if (dfn[v]) low[u]=min(low[u],dfn[v]),rb[v]=w; else
        {
            ed[id]=1;
            dfs1(v);
            if (dfn[u]>low[v]) low[u]=min(low[u],low[v]),rb[v]=w; else
            {
                int ntp=tp;
                while (st[ntp]!=v) --ntp;
                if (ntp==tp)//圆圆边
                {
                    --tp;
                    fe[u].emplace_back(v,w);
                    f[v]=u;
                    continue;
                }
                ++cnt;f[cnt]=u;
                for (int i=ntp;i<=tp;i++) f[st[i]]=cnt;
                len[st[ntp]]=w;
                for (int i=ntp+1;i<=tp;i++) len[st[i]]=len[st[i-1]]+rb[st[i]];
                len[cnt]=len[st[tp]]+rb[u];
                fe[u].emplace_back(cnt,0);
                for (int i=ntp;i<=tp;i++) fe[cnt].emplace_back(st[i],min(len[st[i]],len[cnt]-len[st
                    [i]]));
                tp=ntp-1;
            }
        }
    }
}
void dfs2(int u)
{
    siz[u]=1;
    for (auto [v,w]:fe[u])
    {
        dep[v]=dep[u]+w;
        dfs2(v);
        siz[u]+=siz[v];
        if (siz[v]>siz[hc[u]]) hc[u]=v;
    }
}
void dfs3(int u)
```

```
{
    dfn[u]=++id;
    if (hc[u])
    {
        top[hc[u]]=top[u];
        dfs3(hc[u]);
        for (auto [v,w]:fe[u]) if (v!=hc[u]) dfs3(top[v]=v);
    }
}
int lca(int u,int v)
{
    while (top[u]!=top[v]) if (dfn[top[u]]>dfn[top[v]]) u=f[top[u]]; else v=f[top[v]];//注意不能用
        dep
    return dfn[u]<dfn[v]?u:v;
}
int find(int u,int v)//u 是根
{
    if (dfn[hc[u]]+siz[hc[u]]>dfn[v]) return hc[u];
    while (f[top[v]]!=u) v=f[top[v]];
    return top[v];
}
int dis(int u,int v)
{
    int o=lca(u,v),r=dep[u]+dep[v];
    if (o<=n) return r-(dep[o]<<1);
    u=find(o,u);v=find(o,v);
    if (len[u]>len[v]) swap(u,v);
    return r+min(len[v]-len[u],len[o]-(len[v]-len[u]))-dep[u]-dep[v];
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int m,q,i;
    cin>>n>>m>>q;cnt=n;
    for (i=1;i<=m;i++)
    {
        int u,v,w;
        cin>>u>>v>>w;
        e[u].emplace_back(v,w,i);
        e[v].emplace_back(u,w,i);
    }
    mt19937 rnd(time(0));
    for (i=1;i<=n;i++) shuffle(all(e[i]),rnd);
    dfs1(1);id=0;
    dfs2(1);
    dfs3(top[1]=1);
    while (q--)
    {
        int u,v;
        cin>>u>>v;
        cout<<dis(u,v)<<'\n';
    }
}
```

5.5.7 广义圆方树

建议使用点双来做这个。你只需要对每个点双建一个虚点，向点双内所有原点连边，就得到了广义圆方树。

```
void dfs(int u)
{
    dfn[u]=low[u]=++id;
    st[++tp]=u;
    for (int v:e[u]) if (dfn[v]) low[u]=min(low[u],dfn[v]); else
    {
        dfs(v);
        low[u]=min(low[u],low[v]);
        if (dfn[u]<=low[v])
        {
            vector cur={u};
            do
            {
                cur.push_back(st[tp]);
            } while (st[tp--]!=v);
            ans.push_back(cur);
        }
    }
}
```

5.5.8 2-sat

支持添加一个条件 add(u,x,v,y)，表示 $a_u = x \Rightarrow a_v = y$。支持设定一个变量的值。

$O(n+m)$，$O(n+m)$。

```
struct sat
{
    vector<vector<int>> e;
    vector<int> dfn, low, st, f, ed;
    int fs, tp, id, n;
    sat(int n) :n(n), e(n * 2), dfn(n * 2, -1), low(n * 2), st(n * 2), f(n * 2, -1), ed(n * 2), fs
        (0), tp(-1), id(0) { }
    void dfs(int u)
    {
        dfn[u] = low[u] = id++;
        ed[u] = 1; st[++tp] = u;
        for (int v : e[u]) if (dfn[v] != -1)
        {
            if (ed[v]) low[u] = min(low[u], dfn[v]);
        }
        else dfs(v), low[u] = min(low[u], low[v]);
        if (dfn[u] == low[u])
        {
            do
            {
                f[st[tp]] = fs;
                ed[st[tp]] = 0;
            } while (st[tp--] != u);
            ++fs;
        }
    }
    void add(int u, bool x, int v, bool y)
```

```
    {
        assert(u >= 0 && u < n && v >= 0 && v < n);
        e[u + x * n].push_back(v + y * n);
        e[v + (y ^ 1) * n].push_back(u + (x ^ 1) * n);
    }
    void set(int u, bool x)
    {
        assert(u >= 0 && u < n);
        e[u + (x ^ 1) * n].push_back(u + x * n);
    }
    vector<int> getans()
    {
        int i;
        for (i = 0; i < n * 2; i++) if (dfn[i] == -1) dfs(i);
        vector<int> r(n);
        for (i = 0; i < n; i++)
        {
            if (f[i] == f[i + n]) return { };
            r[i] = f[i] > f[i + n];
        }
        return r;
    }
};
```

5.5.9 Kosaraju 强连通分量（bitset 优化）

实用意义不大。

$O(\frac{n^2}{w})$，$O(\frac{n^2}{w})$。

```
void dfs1(int x)
{
    int i; ed[x] = 0;
    for (i = (lj[x] & ed)._Find_first(); i <= n; i = (lj[x] & ed)._Find_next(i)) dfs1(i);
    sx[--tp] = x;
}
void dfs2(int x)
{
    int i; ed[x] = 0; tv[f[x] = f[0]] += v[x];
    for (i = (fj[x] & ed)._Find_first(); i <= n; i = (fj[x] & ed)._Find_next(i)) dfs2(i);
}
int main()
{
    cin >> n >> m;
    tp = n + 1;
    for (i = 1; i <= n; i++) { ed[i] = 1; cin >> v[i]; }
    for (i = 1; i <= m; i++)
    {
        cin >> x >> y;
        lj[x][y] = 1; fj[y][x] = 1; lb[i][0] = x; lb[i][1] = y;
    }
    for (i = 1; i <= n; i++) if (ed[i]) dfs1(i);
    ed.set();
    for (i = 1; i <= n; i++) if (ed[sx[i]]) { ++f[0]; dfs2(sx[i]); }
    for (i = 1; i <= m; i++) if (f[lb[i][0]] != f[lb[i][1]])
    {
        flj[f[lb[i][0]]].push_back(f[lb[i][1]]); ++rd[f[lb[i][1]]];
    }
```

```
    for (i = 1; i <= f[0]; i++) if (!rd[i]) dl[++wei] = i;
    while (tou <= wei)
    {
        x = dl[tou++]; g[x] += tv[x];
        for (i = 0; i < flj[x].size(); i++)
        {
            g[flj[x][i]] = max(g[flj[x][i]], g[x]);
            if (--rd[flj[x][i]] == 0) dl[++wei] = flj[x][i];
        }
    }
    for (i = 1; i <= f[0]; i++) ans = max(ans, g[i]); printf("%d", ans);
}
```

5.6 树上问题

5.6.1 轻重链剖分/DFS 序 LCA

首先 `init(n)`，然后正常存边（$[1,n]$），然后 `fun(root)`。

`get_path` 会返回这条路径上的 `dfn` 区间。

```
namespace HLD
{
    const int N = 5e5 + 2;
    vector<int> e[N];
    int dfn[N], nfd[N], dep[N], f[N], siz[N], hc[N], top[N];
    int id, n;
    void dfs1(int u)
    {
        siz[u] = 1;
        for (int v : e[u]) if (v != f[u])
        {
            dep[v] = dep[f[v] = u] + 1;
            dfs1(v);
            siz[u] += siz[v];
            if (siz[v] > siz[hc[u]]) hc[u] = v;
        }
    }
    void dfs2(int u)
    {
        dfn[u] = ++id;
        nfd[id] = u;
        if (hc[u])
        {
            top[hc[u]] = top[u];
            dfs2(hc[u]);
            for (int v : e[u]) if (v != hc[u] && v != f[u]) dfs2(top[v] = v);
        }
    }
    int lca(int u, int v)
    {
        while (top[u] != top[v])
        {
            if (dep[top[u]] < dep[top[v]]) swap(u, v);
            u = f[top[u]];
        }
        if (dep[u] > dep[v]) swap(u, v);
        return u;
```

```
    }
    int dis(int u, int v)
    {
        return dep[u] + dep[v] - (dep[lca(u, v)] << 1);
    }
    void init(int _n)
    {
        n = _n;
        for (int i = 1; i <= n; i++)
        {
            e[i].clear();
            f[i] = hc[i] = 0;
        }
        id = 0;
    }
    void fun(int root)
    {
        dep[root] = 1; dfs1(root); dfs2(top[root] = root);
    }
    vector<pair<int, int>> get_path(int u, int v)//u->v, 注意可能出现 [r>l]（表示反过来走）
    {
        //cerr<<"path from "<<u<<" to "<<v<<": ";
        vector<pair<int, int>> v1, v2;
        while (top[u] != top[v])
        {
            if (dep[top[u]] > dep[top[v]]) v1.push_back({dfn[u], dfn[top[u]]}), u = f[top[u]];
            else v2.push_back({dfn[top[v]], dfn[v]}), v = f[top[v]];
        }
        v1.reserve(v1.size() + v2.size() + 1);
        v1.push_back({dfn[u], dfn[v]});
        reverse(v2.begin(), v2.end());
        for (auto v : v2) v1.push_back(v);
        //for (auto [x,y]:v1) cerr<<"["<<x<<','<<y<<"] ";cerr<<endl;
        return v1;
    }
}
using HLD::e, HLD::dfn, HLD::nfd, HLD::dep, HLD::f, HLD::siz, HLD::get_path;
using HLD::init;//5e5
namespace LCA
{
    using HLD::N, HLD::n;
    int st[__lg(N) + 1][N];
    int cmp(const int &x, const int &y) { return dep[x] < dep[y] ? x : y; }
    void fun(int rt)
    {
        HLD::fun(rt);
        assert(f[rt] == 0);
        for (int i = 1; i <= n; i++) st[0][dfn[i] - 1] = f[i];
        for (int j = 0; j < __lg(n); j++)
            for (int i = 1, k = n - (1 << j + 1); i <= k; i++) st[j + 1][i] = cmp(st[j][i], st[j][i
                 + (1 << j)]);
    }
    int lca(int u, int v)
    {
        if (u == v) return u;
        u = dfn[u], v = dfn[v];
        if (u > v) swap(u, v);
```

```
        int g = __lg(v - u);
        return cmp(st[g][u], st[g][v - (1 << g)]);
    }
    int dis(int u, int v)
    {
        return dep[u] + dep[v] - (dep[lca(u, v)] << 1);
    }
}
using LCA::lca, LCA::fun, LCA::dis;
```

5.6.2 换根树剖

本质是对普通树剖在换根后的子树进行分类讨论。

设预处理的根是 u，当前根是 v，那么 w 的子树如下：

1. $w=v$，`dfn` 区间为 $[1,n]$。
2. w 在 u,v 之间，`dfn` 区间为 $[1,n]$ 去掉 w 前往 v 方向的子树。找到这个子树的方法见 `find` 函数。
3. 其余情况，`dfn` 区间和原来一致。

```
int find(int x,int y)//找到 y 向 x 的子树
{
    while ((top[x]!=top[y])&&(f[top[x]]!=y)) x=f[top[x]];
    if (top[x]==top[y]) return hc[y];
    return top[x];
}
```

$O(n+q\log n)$，$O(n)$。

5.6.3 毛毛虫剖分

毛毛虫剖分，一种由轻重链剖分（HLD）推广而成的树上结点重标号方法，支持修改 / 查询一只毛毛虫的信息，并且可以对毛毛虫的身体和足分别修改 / 查询不同信息.

严格强于树剖，而且复杂度和树剖一样哦！

一些定义（默认在一棵树上）：

毛毛虫：一条链和与这条链邻接的所有结点构成的集合. 虫身（身体）：毛毛虫的链部分. 虫足（足）：毛毛虫除虫身的部分. 重标号方法首先重剖求出重链. DFS，若现在处理到结点 u：若 u 还未被标号，则为其标号. 若 u 是重链头，遍历这条重链，将邻接这条链的结点依次标号. 先递归重儿子，再递归轻儿子. 重标号性质对于重链，除链头外的结点标号连续. 对于任意结点，其轻儿子标号连续. 对于以重链头为根的子树，与这条重链邻接的所有结点标号连续. 这样就可以随便维护毛毛虫信息了，顺便还能维护链信息，子树信息等.

时间复杂度同轻重链剖分.

以 SAM 为例，若我们只保留所有的转移边 (u,v)，满足到达 u 的路径数目大于到达 v 的路径数目一半，且从 v 出发的路径数目大于从 u 出发的路径数目一半，这样剩余的子图显然会形成若干条链，且每个点恰好在一条链上。这样，我们容易证明，从根结点出发的任何一条路径，至多经过 $O(\log n)$ 条不在链上的转移边（也意味着至多经过 $O(\log n)$ 条链）。

以下是一段示例代码，展示了将一条链对应区间取出来的过程

```
vector<int> e[N];
vector<pair<int, int>> seg[N], qu[N];
int ans[Q];
int dfn[N], dep[N], nfd[N], top[N], f[N], sz[N], hc[N], pre[N], fir[N], lst2[N], rt[N];
int
void insert()
void dfs1(int u)
{
    sz[u] = 1;
    for (int v : e[u]) if (v != f[u])
    {
        dep[v] = dep[u] + 1;
        f[v] = u;
        dfs1(v);
        sz[u] += sz[v];
        if (sz[v] > sz[hc[u]]) hc[u] = v;
    }
    if (f[u]) erase(e[u], f[u]);
}
void dfs2(int u)
{
    static int id = 0;
    //dbg(u);
    if (!dfn[u])
    {
        dfn[u] = ++id;
        nfd[id] = u;
    }
    if (top[u] == u)
    {
        vector<int> stk;
        for (int v = u;v;v = hc[v])
        {
            for (int w : e[v]) if (w != hc[v])
            {
                dfn[w] = ++id;
                nfd[id] = w;
                pre[v] = id;
                cmin(fir[v], id);
                lst2[v] = id;
            }
            stk.push_back(v);
        }
        for (int i = (int)stk.size() - 2;i >= 0;i--)
        {
            cmin(fir[stk[i]], fir[stk[i + 1]]);
            cmax(lst2[stk[i]], lst2[stk[i + 1]]);
        }
        for (int i = 1;i < stk.size();i++)
        {
            cmax(pre[stk[i]], pre[stk[i - 1]]);
        }
    }
    //dbg(u);
    top[hc[u]] = top[u];
```

```
    if (hc[u]) dfs2(hc[u]);
    for (int v : e[u]) if (v != hc[u]) dfs2(top[v] = v);
}
mt19937 rnd(245);
int main()
{
    memset(fir, 0x3f, sizeof fir);
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, q, i, j;
    cin >> n >> m >> q;
    for (i = 1;i < n;i++)
    {
        int u, v;
        //cin >> u >> v;
        u = i + 1;
        v = rnd() % i + 1;
        //v = (i + 1) / 2;
        //v = i / 2 + 1;
        //dbg(u, v);
        e[u].push_back(v);
        e[v].push_back(u);
    }
    dfs1(dep[1] = 1);
    //dbg("??");
    dfs2(top[1] = 1);
    //for (i = 1;i <= n;i++) cerr << i << ": " << dfn[i] << endl;
    for (i = 1;i <= m;i++)
    {
        int u, v;
        //cin >> u >> v;
        u = rnd() % n + 1;
        v = rnd() % n + 1;
        int uu = u, vv = v;
        //dbg(uu, vv);
        auto& w = seg[i];
        while (top[u] != top[v])
        {
            if (dep[top[u]] < dep[top[v]]) swap(u, v);
            w.push_back({fir[top[u]], pre[u]});
            //else w.push_back({fir[top[u]], lst2[top[u]]});
            if (hc[u]) w.push_back({dfn[hc[top[u]]], dfn[hc[u]]});
            else if (top[u] != u) w.push_back({dfn[hc[top[u]]], dfn[u]});
            //dbg(u, v, w);
            //[fir[top[u]],lst[u]]
            u = f[top[u]];
        }
        if (dep[u] < dep[v]) swap(u, v);
        w.push_back({fir[v], pre[u]});
        //else if (!hc[u]) w.push_back({fir[v], lst2[v]});
        //dbg(v, lst2[v], fir[v]);
        if (hc[u]) w.push_back({dfn[hc[v]], dfn[hc[u]]});
        else if (u != v) w.push_back({dfn[hc[v]], dfn[u]});
        //dbg(w);
        w.push_back({dfn[v], dfn[v]});
        if (f[v]) w.push_back({dfn[f[v]], dfn[f[v]]});
        erase_if(w, [&](const auto& x) {return x.first > x.second;});
```

```
        //int len = 0;
        //for (auto [l, r] : w) len += r - l + 1;
        //for (auto [l, r] : w)
        //{
        // for (int j = l;j <= r;j++) cerr << nfd[j] << ' ';cerr << " | ";
        //}
        //cerr << endl;
        //int tl = 0;
        //set<int> s = {uu, vv};
        //while (uu != vv)
        //{
        // if (dep[uu] < dep[vv]) swap(uu, vv);
        // s.insert(all(e[uu]));s.insert(f[uu]);uu = f[uu];
        //}
        //s.insert(all(e[uu]));
        //if (f[uu]) s.insert(f[uu]);
        ////dbg(s);
        //assert(len == s.size());
    }
    for (i = 1;i <= q;i++)
    {
        int l, r;
        cin >> l >> r;
        qu[l].push_back({r, i});
    }
    for (i = m;i;i--)
    {

    }
    for (i = 1;i <= q;i++) cout << ans[i] << '\n';
    //cerr << "??\n";
}
```

5.6.4 树上启发式合并，DSU on tree

一种过时的、基于两次 dfs 的写法，在复杂度要求不严时不如直接存储 set。
流程：

1. dfs 轻子树计算答案，并清空全局统计信息。
2. dfs 重子树统计答案和全局信息。
3. dfs 轻子树统计全局信息。

```
void dfs1(int x)
{
    siz[x]=zdep[x]=1;
    int i;
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=f[x])
    {
        dep[lj[i]]=dep[f[lj[i]]=x]+1;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
        zdep[x]=max(zdep[x],zdep[lj[i]]+1);
    }
```

```
}
void cal(int x)
{
    int i;
    dl[tou=wei=1]=x;
    while (tou<=wei)
    {
        ++dp[dep[x=dl[tou++]]];
        if ((dp[dep[x]]>dp[zd])||(dp[dep[x]]==dp[zd])&&(dep[x]<zd)) zd=dep[x];
        for (i=fir[x];i;i=nxt[i]) if (lj[i]!=f[x]) dl[++wei]=lj[i];
    }
}
void dfs2(int x)
{
    if (!hc[x])
    {
        if (++dp[dep[x]]>dp[zd]) zd=dep[x];
        return;
    }
    int i;
    for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x]))
    {
        dfs2(lj[i]);
        memset(dp+dep[lj[i]],0,zdep[lj[i]]<<2);
    }
    dfs2(hc[x]);
    dp[dep[x]]=1;
    if (dp[zd]<=1) zd=dep[x];
    for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x])) cal(lj[i]);
    ans[x]=zd-dep[x];
}
```

5.6.5 长链剖分（k 级祖先）

$O(n\log n+q)$，$O(n)$。

```
void dfs1(int x)
{
    int i;
    for (i = 1; i <= er[dep[x] - 1]; i++) f[x][i] = f[f[x][i - 1]][i - 1]; md[x] = dep[x];
    for (i = fir[x]; i; i = nxt[i]) { dep[lj[i]] = dep[x] + 1; dfs1(lj[i]); if (md[lj[i]] > md[dc[
        x]]) dc[x] = lj[i]; }
    if (dc[x]) md[x] = md[dc[x]];
}
void dfs2(int x)
{
    int i;
    if (dc[x])
    {
        top[dc[x]] = top[x];
        dfs2(dc[x]);
        for (i = fir[x]; i; i = nxt[i]) if (lj[i] != dc[x]) dfs2(top[lj[i]] = lj[i]);
    }
    if (x == top[x])
    {
        c = md[x] - dep[x]; y = x; up[x].push_back(x); down[x].push_back(x);
        for (i = 1; (i <= c) && (y = f[y][0]); i++) up[x].push_back(y); y = x;
```

```
        for (i = 1; i <= c; i++) down[x].push_back(y = dc[y]);
    }
}
int main()
{
    int n, q, ans = 0, x, y, c, i;
    ll ta = 0;
    cin >> n >> q >> s;
    for (i = 1; i <= n; i++) { cin >> f[i][0]; if (f[i][0] == 0) rt = i; else add(f[i][0], i); }
    for (i = 2; i <= n; i++) er[i] = er[i >> 1] + 1; dep[rt] = 1;
    dfs1(rt); dfs2(top[rt] = rt);
    for (i = 1; i <= q; i++)
    {
        x = (get(s) ^ ans) % n + 1; y = (get(s) ^ ans) % dep[x];
        //此时计算 x 的 y 级祖先。结果在 ans 中。
        if (y == 0) { ans = x; ta ^= (ll)i * ans; continue; }
        c = dep[x] - y; x = top[f[x][er[y]]];
        if (dep[x] > c) ans = up[x][dep[x] - c]; else ans = down[x][c - dep[x]];
        ta ^= (ll)i * ans;
    }
    cout << ta << endl;
}
```

5.6.6 长链剖分（dp 合并）

一种常见的实现方法是用指针指向同一片数组区域，使得从链头到链尾正好指向连续的一段数组，就不需要计算偏移量了。

$O(n)$，$O(n)$。

```
void dfs1(int x)
{
    top[x]=1;
    for (int i=fir[x];i;i=nxt[i]) if (!top[lj[i]])
    {
        dfs1(lj[i]);
        if (len[lj[i]]>len[hc[x]]) hc[x]=lj[i];
    }
    len[x]=len[hc[x]]+1;top[hc[x]]=0;
}
void dfs2(int x)
{
    *f[x]=1;gs[x]=1;
    if (!hc[x]) return;
    ed[x]=1;f[hc[x]]=f[x]+1;
    for (int i=fir[x];i;i=nxt[i]) if (!ed[lj[i]]) dfs2(lj[i]);
    ans[x]=ans[hc[x]]+1;gs[x]=gs[hc[x]];
    if (gs[x]==1) ans[x]=0;
    for (int i=fir[x];i;i=nxt[i]) if ((!ed[lj[i]])&&(lj[i]!=hc[x]))
    {
        int v=lj[i],*p;
        for (int j=0;j<len[v];j++)
        {
            *(p=f[x]+j+1)+=*(f[v]+j);
            if (j+1==ans[x]) {gs[x]=*p;continue;}
            if ((*p>gs[x])||(*p==gs[x])&&(j+1<ans[x])) {gs[x]=*p;ans[x]=j+1;}
        }
```

```
    }
    gs[x]=*(f[x]+ans[x]);
    ed[x]=0;
}
```

5.6.7 LCT

$O(n\log n)$，$O(n)$。

makeroot 会变根，split 会把 y 变根，findroot 会把根变根，link 会把 x,y 变根（y 是新的），cut 会把 x,y 变根（x 是新的），注意 swap 子节点可能要 pushup。

代码为动态割边割点。

```
#include "bits/stdc++.h"
using namespace std;
template<class info, class tag> struct lct
{
    vector<array<int, 2>> c;
    vector<int> f, rev, lz, st;
    vector<info> s, v;
    vector<tag> tg;
#ifdef Rev
    vector<info> rs;
#endif
    lct(int n) :f(n + 1), c(n + 1), s(n + 1), v(n + 1), tg(n + 1), rev(n + 1), lz(n + 1), st(n +
        1)
#ifdef Rev
        , rs(n + 1)
#endif
    {}

    bool nroot(int x) const
    {
        return c[f[x]][0] == x || c[f[x]][1] == x;
    }
    void pushup(int x)
    {
        int lc = c[x][0], rc = c[x][1];
        s[x] = v[x];
#ifdef Rev
        rs[x] = v[x];
#endif
        if (lc)
        {
            s[x] = s[lc] + s[x];
#ifdef Rev
            rs[x] = rs[x] + rs[lc];
#endif
        }
        if (rc)
        {
            s[x] = s[x] + s[rc];
#ifdef Rev
            rs[x] = rs[rc] + rs[x];
#endif
        }
    }
```

```
    void swp(int x)
    {
        swap(c[x][0], c[x][1]);
#ifdef Rev
        swap(s[x], rs[x]);
#endif
        rev[x] ^= 1;
    }
    void pushdown(int x)
    {
        if (rev[x])
        {
            for (int y : c[x]) if (y) swp(y);
            rev[x] = 0;
        }
        if (lz[x])
        {
            for (int y : c[x]) if (y)
            {
                if (lz[y]) tg[y] += tg[x]; else tg[y] = tg[x], lz[y] = 1;
                s[y] += tg[x];
            }
            lz[x] = 0;
        }
    }
    void zigzag(int x)
    {
        int y = f[x], z = f[y], typ = (c[y][0] == x);
        if (nroot(y)) c[z][c[z][1] == y] = x;
        f[x] = z; f[y] = x;
        if (c[x][typ]) f[c[x][typ]] = y;
        c[y][typ ^ 1] = c[x][typ]; c[x][typ] = y;
        pushup(y);
    }
    void splay(int x)
    {
        int y, tp;
        st[tp = 1] = y = x;
        while (nroot(y)) st[++tp] = y = f[y];
        while (tp) pushdown(st[tp--]);
        for (; nroot(x); zigzag(x)) if (nroot(y = f[x])) zigzag((c[y][0] == x) ^ (c[f[y]][0] == y)
             ? x : f[x]);
        pushup(x);
    }
    int access(int x)
    {
        int y = 0;
        for (; x; x = f[y = x]) splay(x), c[x][1] = y, pushup(x);
        return y;
    }
    int findroot(int x)//splay 根为树根，splay 维护树根到 x 的链
    {
        access(x); splay(x); pushdown(x);
        while (c[x][0]) pushdown(x = c[x][0]);
        splay(x); return x;
    }
    void split(int x, int y)//x 为树新根，y 为 splay 新根
```

```
    {
        makeroot(x); access(y); splay(y);
    }
    void makeroot(int x)//x 为树、splay 新根
    {
        access(x); splay(x); swp(x);
    }
    void modify(int x, const info &o)
    {
        makeroot(x); v[x] = o; pushup(x);
    }
    void modify(int x, int y, const tag &o)
    {
        split(x, y); s[y] += o;
        if (lz[y]) tg[y] += o; else tg[y] = o, lz[y] = 1;
    }
    info ask(int x, int y) { split(x, y); return s[y]; }
    bool connected(int x, int y)//注意会改变形态
    {
        makeroot(x); return findroot(y) == x;
    }
    void link(int x, int y)//y 为新根
    {
        if (!connected(x, y)) makeroot(f[x] = y);
    }
    void cut(int x, int y)
    {
        if (connected(x, y))//可能本不连通
        {
            pushdown(x);
            if (c[x][1] == y && !c[y][0] && !c[y][1])//可能连通但无边
            {
                c[x][1] = f[y] = 0;
                pushup(x);
            }
        }
    }
    int lca(int x, int y) { access(x); return access(y); }
    vector<int> res;
    void dfs(int x)
    {
        if (!x) return;
        pushdown(x);
        dfs(c[x][0]); res.push_back(x); dfs(c[x][1]);
    }
    vector<int> get_path(int x, int y)
    {
        res.clear(); split(x, y); dfs(y);
        if (res[0] != x) return { };
        return res;
    }
};
const int N = 2e5 + 5, M = 4e5 + 5;
struct tag
{
    void operator+=(const tag &o) const { }
};
```

```
void operator+=(int &x, const tag &o) { x = 0; }
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, m, i, r = 0;
    cin >> n >> m;
    lct<int, tag> s(n * 2), t(n + m);
    for (i = 1; i <= n; i++) s.modify(i + n, 1), t.modify(i, 1);
    int bs = n, ds = n;
    while (m--)
    {
        int op, u, v;
        cin >> op >> u >> v;
        u ^= r; v ^= r;
        if (op == 1)
        {
            if (s.connected(u, v))
            {
                s.modify(u, v, { });
                auto c = t.get_path(u, v);
                for (i = 1; i < c.size(); i++) t.cut(c[i - 1], c[i]);
                ++ds;
                for (int x : c) t.link(ds, x);
            }
            else
            {
                s.link(++bs, u);
                s.link(bs, v);
                t.link(++ds, u);
                t.link(ds, v);
            }
        }
        else
        {
            if (!s.connected(u, v))
            {
                cout << "-1\n";
                continue;
            }
            r = op == 2 ? s.ask(u, v) : t.ask(u, v);
            cout << r << '\n';
        }
    }
}
```

5.6.8 带子树的 LCT

$O(n \log n)$，$O(n)$。
你需要实现的是 info 类的 +,+=,-=。

1. info 维护的是从上往下的一条链以及这条链挂着的所有轻子树的信息。
2. a+b 表示两条链合并后的信息，其中 a 接近根。
3. a+=b 表示一条链合并了一个轻子树，即 b 是 a 链尾的子结点。
4. a-=b 表示一条链移除了一个轻子树，即 b 是 a 链尾的子节点。

如果有边权，将边 (u,v) 当成点并与 u,v 分别连边。
代码对应题意：
对于满足 $0 \le e \le N-2$ 的整数 e，定义 $f_e(x) = b_e x + c_e$。
设 $e_0, e_1, \ldots, e_k$ 为从顶点 x 到顶点 y 的简单路径上的边，按顺序排列，并定义

$$P(x,y) = f_{e_0}(f_{e_1}(\ldots f_{e_k}(a_y)\ldots)).$$

按给定顺序处理 Q 个查询。查询有两种类型：

- `0 w x r`：将 a_w 更新为 x，然后输出

$$\left(\sum_{v=0}^{N-1} P(r,v)\right) \mod 998244353.$$

- `1 e y z r`：将 (b_e, c_e) 更新为 (y,z)，然后输出

$$\left(\sum_{v=0}^{N-1} P(r,v)\right) \mod 998244353.$$

```
template<class info> struct lct
{
    int n;
    vector<info> sum, sum_rev, val, sum_oth;
    vector<int> f, lz;
    vector<array<int, 2>> c;
    lct(int _n, const info &o) :n(_n + 1), sum(n, o), sum_rev(n, o), val(n, o), sum_oth(n, o), f(n
        ), lz(n), c(n) { }
    bool nroot(int x) const
    {
        return c[f[x]][0] == x || c[f[x]][1] == x;
    }
    void pushup(int x)
    {
        sum[x] = val[x];
        sum[x] += sum_oth[x];
        sum_rev[x] = sum[x];
        sum[x] = sum[c[x][0]] + sum[x] + sum[c[x][1]];
        sum_rev[x] = sum_rev[c[x][1]] + sum_rev[x] + sum_rev[c[x][0]];
    }
    void rev(int x)
    {
        if (x)
        {
            swap(c[x][0], c[x][1]);
            swap(sum[x], sum_rev[x]);
            lz[x] ^= 1;
        }
    }
    void pushdown(int x)
    {
        if (lz[x])
        {
            rev(c[x][0]);
            rev(c[x][1]);
```

```
        lz[x] = 0;
    }
}
void zigzag(int x)
{
    int y = f[x], z = f[y], typ = (c[y][0] == x);
    if (nroot(y)) c[z][c[z][1] == y] = x;
    f[x] = z; f[y] = x;
    if (c[x][typ]) f[c[x][typ]] = y;
    c[y][typ ^ 1] = c[x][typ]; c[x][typ] = y;
    pushup(y);
}
void splay(int x)
{
    static vector<int> st(n);
    int y, tp;
    st[tp = 1] = y = x;
    while (nroot(y)) st[++tp] = y = f[y];
    while (tp) pushdown(st[tp--]);
    for (; nroot(x); zigzag(x)) if (!nroot(f[x])) continue; else zigzag((c[f[x]][0] == x) ^ (c
        [f[f[x]]][0] == f[x]) ? x : f[x]);
    pushup(x);
}
void access(int x)
{
    for (int y = 0; x; x = f[y = x])
    {
        splay(x);
        sum_oth[x] -= sum[y];
        sum_oth[x] += sum[c[x][1]];
        c[x][1] = y; pushup(x);
    }
}
int findroot(int x)
{
    access(x); splay(x); pushdown(x);
    while (c[x][0]) pushdown(x = c[x][0]);
    splay(x);
    return x;
}
void split(int x, int y)
{
    makeroot(x);
    access(y);
    splay(y);
}
void makeroot(int x)
{
    access(x);
    splay(x);
    rev(x);
}
void link(int x, int y)
{
    makeroot(x);
    if (x != findroot(y))//可能已经连通
    {
```

```
            makeroot(y); f[x] = y;
            sum_oth[y] += sum[x];
            pushup(y);
        }
    }
    void cut(int x, int y)
    {
        makeroot(x);
        if (x == findroot(y))//可能本不连通
        {
            pushdown(x);
            if (c[x][1] == y && !c[y][0] && !c[y][1])//可能连通但无边
            {
                c[x][1] = f[y] = 0;
                pushup(x);
            }
        }
    }
    void set(int x, info y)
    {
        makeroot(x);
        val[x] = y;
        pushup(x);
    }
};
const ull p = 998244353;
struct Q
{
    ull k, b, sum, sz;
    Q operator+(const Q &o) const
    {
        return {k * o.k % p, (b + k * o.b) % p, (sum + k * o.sum + b * o.sz) % p, sz + o.sz};
    }
    void operator+=(const Q &o)
    {
        (sum += k * o.sum + b * o.sz) %= p;
        sz += o.sz;
    }
    void operator-=(const Q &o)
    {
        (sum += p * p * 2 - k * o.sum - b * o.sz) %= p;
        sz -= o.sz;
    }
};
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, m, i;
    cin >> n >> m;
    vector<Q> a(n * 2 + 1);
    for (i = 1; i <= n; i++)
    {
        ull x;
        cin >> x;
        a[i] = {1, 0, x, 1};
    }
    vector<pair<int, int>> edges(n);
```

```
    for (i = 1; i < n; i++)
    {
        auto &[u, v] = edges[i];
        ull k, b;
        cin >> u >> v >> k >> b;
        ++u, ++v;
        a[i + n] = {k, b, 0, 0};
    }
    lct<Q> s(n * 2 - 1, Q{1, 0, 0, 0});
    for (i = 1; i < n * 2; i++) s.set(i, a[i]);
    for (i = 1; i < n; i++)
    {
        auto [u, v] = edges[i];
        s.link(u, n + i);
        s.link(v, n + i);
    }
    while (m--)
    {
        int op;
        cin >> op;
        if (op == 0)
        {
            int u;
            ull x;
            cin >> u >> x;
            ++u;
            a[u] = {1, 0, x, 1};
            s.set(u, a[u]);
        }
        else
        {
            int id;
            ull k, b;
            cin >> id >> k >> b;
            ++id;
            a[id + n] = {k, b, 0, 0};
            s.set(id + n, a[id + n]);
        }
        int rt;
        cin >> rt;
        ++rt;
        s.makeroot(rt);
        cout << s.sum[rt].sum << '\n';
    }
}
```

5.6.9 动态 dp（全局平衡二叉树）

意义不大。

$O((n+q)\log n)$，$O(n)$。

```
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <fstream>
using namespace std;
const int N=1e6+2,M=6e7+2,INF=-1e9;
```

```
struct matrix
{
    int a[2][2];
};
matrix s[N],js;
matrix operator *(matrix x,matrix y)
{
    js.a[0][0]=max(x.a[0][0]+y.a[0][0],x.a[0][1]+y.a[1][0]);
    js.a[0][1]=max(x.a[0][0]+y.a[0][1],x.a[0][1]+y.a[1][1]);
    js.a[1][0]=max(x.a[1][0]+y.a[0][0],x.a[1][1]+y.a[1][0]);
    js.a[1][1]=max(x.a[1][0]+y.a[0][1],x.a[1][1]+y.a[1][1]);
    return js;
}
int st[N],c[N][2],hc[N],lj[N<<1],nxt[N<<1],fir[N],siz[N],v[N],g[N][2],fa[N],f[N],val[N];
int n,m,i,j,x,y,z,dtp,stp,tp,fh,bs,rt,aaa,la;
char dr[M+5],sc[M];
void pushup(int x)
{
    s[x].a[0][0]=s[x].a[0][1]=g[x][0];
    s[x].a[1][0]=g[x][1];s[x].a[1][1]=INF;
    if (c[x][0]) s[x]=s[c[x][0]]*s[x];
    if (c[x][1]) s[x]=s[x]*s[c[x][1]];
}
void add(int x,int y)
{
    lj[++bs]=y;
    nxt[bs]=fir[x];
    fir[x]=bs;
    lj[++bs]=x;
    nxt[bs]=fir[y];
    fir[y]=bs;
}
bool nroot(int x)
{
    return ((c[f[x]][0]==x)||(c[f[x]][1]==x));
}
void dfs1(int x)
{
    siz[x]=1;
    int i;
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=fa[x])
    {
        fa[lj[i]]=x;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
    }
}
int build(int l,int r)
{
    if (l>r) return 0;
    int i,tot=0,upn=0;
    for (i=l;i<=r;i++) tot+=val[i];tot>>=1;
    for (i=l;i<=r;i++)
    {
        upn+=val[i];
        if (upn>=tot)
```

```
        {
            f[c[st[i]][0]=build(l,i-1)]=st[i];
            f[c[st[i]][1]=build(i+1,r)]=st[i];
            pushup(st[i]);
            ++aaa;
            return st[i];
        }
    }
}
int dfs2(int x)
{
    int i,j;
    for (i=x;i;i=hc[i]) for (j=fir[i];j;j=nxt[j]) if ((lj[j]!=fa[i])&&(lj[j]!=hc[i]))
    {
        f[y=dfs2(lj[j])]=i;
        g[i][0]+=max(s[y].a[0][0],s[y].a[1][0]);
        g[i][1]+=s[y].a[0][0];
    }
    tp=0;
    for (i=x;i;i=hc[i]) st[++tp]=i;
    for (i=1;i<tp;i++) val[i]=siz[st[i]]-siz[st[i+1]];
    val[tp]=siz[st[tp]];
    return build(1,tp);
}
void change(int x,int y)
{
    g[x][1]+=y-v[x];v[x]=y;
    while (f[x])
    {
        if (nroot(x)) pushup(x);
        else
        {
            g[f[x]][0]-=max(s[x].a[0][0],s[x].a[1][0]);
            g[f[x]][1]-=s[x].a[0][0];
            pushup(x);
            g[f[x]][0]+=max(s[x].a[0][0],s[x].a[1][0]);
            g[f[x]][1]+=s[x].a[0][0];
        }
        x=f[x];
    }
    pushup(x);
}
int main()
{
    scanf("%d%d",&n,&m);
    fread(dr+1,1,min(M,n*20+m*20),stdin);
    for (i=1;i<=n;i++)
    {
        read(g[i][1]);
        v[i]=g[i][1];
    }
    for (i=1;i<n;i++)
    {
        read(x);read(y);
        add(x,y);
    }
    dfs1(1);
```

```
    rt=dfs2(1);tp=0;
    while (m--)
    {
        read(x);read(y);
        change(x^la,y);
        x=la=max(s[rt].a[0][0],s[rt].a[1][0]);
        while (x)
        {
            st[++tp]=x%10;
            x/=10;
        }
        while (tp) sc[++stp]=st[tp--]|48;
        sc[++stp]=10;
    }
    fwrite(sc+1,1,stp,stdout);
}
```

5.6.10 全局平衡二叉树（修改版）

$O((n+q)\log n)$，$O(n)$。

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
typedef pair<int, int> pa;
const int N = 1e6 + 2, M = 1e6 + 2;
ll ans;
pa w[N];
int c[N][2], f[N], fa[N], v[N], s[N], lz[N], lj[M], nxt[M], siz[N], hc[N], fir[N], st[N];
int a[N], top[N];
int n, i, x, y, z, bs, tp, rt;
void add()
{
    lj[++bs] = y; nxt[bs] = fir[x]; fir[x] = bs;
    lj[++bs] = x; nxt[bs] = fir[y]; fir[y] = bs;
}
void pushup(int &x)
{
    s[x] = min(v[x], min(s[c[x][0]], s[c[x][1]]));
}
void pushdown(int &x)
{
    if (lz[x] < 0)
    {
        int cc = c[x][0];
        if (cc)
        {
            lz[cc] += lz[x]; s[cc] += lz[x]; v[cc] += lz[x];
        }
        cc = c[x][1];
        if (cc)
        {
            v[cc] += lz[x]; lz[cc] += lz[x]; s[cc] += lz[x];
        }lz[x] = 0;
        return;
    }
}
```

```
bool nroot(int &x) { return c[f[x]][0] == x || c[f[x]][1] == x; }
bool cmp(pa &o, pa &p) { return o > p; }
void dfs1(int x)
{
   siz[x] = 1;
   for (int i = fir[x]; i; i = nxt[i]) if (lj[i] != fa[x])
   {
      fa[lj[i]] = x; dfs1(lj[i]); siz[x] += siz[lj[i]];
      if (siz[hc[x]] < siz[lj[i]]) hc[x] = lj[i];
   }
}
int build(int l, int r)
{
   if (l > r) return 0;
   if (l == r)
   {
      l = st[l]; s[l] = v[l] = siz[l] >> 1;
      return l;
   }
   int x = lower_bound(a + l, a + r + 1, a[l] + a[r] >> 1) - a, y = st[x];
   c[y][0] = build(l, x - 1);
   c[y][1] = build(x + 1, r);
   v[y] = siz[y] >> 1;
   if (c[y][0]) f[c[y][0]] = y;
   if (c[y][1]) f[c[y][1]] = y;
   pushup(y);
   return y;
}
void dfs2(int x)
{
   if (!hc[x]) return;
   int i;
   top[hc[x]] = top[x];
   if (top[x] == x)
   {
      st[tp = 1] = x;
      for (i = hc[x]; i; i = hc[i]) st[++tp] = i;
      for (i = 1; i <= tp; i++) a[i] = siz[st[i]] - siz[hc[st[i]]] + a[i - 1];
      f[build(1, tp)] = fa[x];
   }
   dfs2(hc[x]);
   for (i = fir[x]; i; i = nxt[i]) if (lj[i] != fa[x] && lj[i] != hc[x]) dfs2(top[lj[i]] = lj[i])
       ;
}
void mdf(int x)
{
   int y = x;
   st[tp = 1] = x;
   while (y = f[y]) st[++tp] = y; y = x;
   while (tp) pushdown(st[tp--]);
   while (x)
   {
      --v[x]; --lz[c[x][0]]; --v[c[x][0]]; --s[c[x][0]];
      while (c[f[x]][0] == x) x = f[x]; x = f[x];
   }
   pushup(y);
   while (y = f[y]) pushup(y);
```

```
}
int ask(int x)
{
    int y = x;
    st[tp = 1] = x;
    while (y = f[y]) st[++tp] = y;
    while (tp) pushdown(st[tp--]);
    int r = v[x];
    while (x)
    {
        r = min(r, min(v[x], s[c[x][0]]));
        while (c[f[x]][0] == x) x = f[x]; x = f[x];
    }
    return r;
}
signed main()
{
    cin >> n; s[0] = 1e9;
    for (i = 1; i <= n; i++) cin >> w[w[i].second = i].first;
    for (i = 1; i < n; i++) cin >> x >> y, add();
    sort(w + 1, w + n + 1, cmp); dfs1(1); dfs2(top[1] = 1); rt = 1; while (f[rt]) rt = f[rt];
    for (i = 1; i <= n && v[rt]; i++) if (ask(w[i].second)) mdf(w[i].second), ans += w[i].first;
    cout << ans << endl;
}
```

5.6.11 虚树

传入点标号列表，返回虚树边表。自动认为 1 是根，标号从 1 开始。

需要注意的是：在清空的时候需要同时考虑点列表和边表，都清空一下。

你需要提供的是：dep,lca,dfn。

$O(n+\sum k\log n)$，$O(n)$。

```
vector<pair<int, int>> get_tree(vector<int> a)
{
    vector<pair<int, int>> edges;
    sort(all(a), [&](int u, int v) { return dfn[u]<dfn[v]; });
    vector<int> st(a.size()+2);
    int tp=0;
    auto ins=[&](int u)
        {
            if (tp==0)
            {
                st[tp=1]=u;
                return;
            }
            int v=lca(st[tp], u);
            while (tp>1&&dep[v]<dep[st[tp-1]])
            {
                edges.emplace_back(st[tp-1], st[tp]);
                --tp;
            }
            if (dep[v]<dep[st[tp]]) edges.emplace_back(v, st[tp--]);
            if (!tp||st[tp]!=v) st[++tp]=v;
            st[++tp]=u;
        };
    if (a[0]!=1) st[tp=1]=1;//先行添加根节点
```

```
    for (int u:a) ins(u);
    if (tp) while (--tp) edges.emplace_back(st[tp], st[tp+1]);//回溯
    return edges;
}
```

5.6.12 点分治

点分治板子的参考意义不大。

$O(n\log n)$，$O(n)$。

```
int siz[N], dep[N];
int n, ksiz, md, rt, mn;
bool ed[N];
void find(int u)
{
    ed[u] = 1; siz[u] = 1;
    int mx = 0;
    for (int v : e[u]) if (!ed[v])
    {
        find(v);
        siz[u] += siz[v];
        mx = max(mx, siz[v]);
    }
    mx = max(mx, ksiz - siz[u]);
    if (mn > mx) mn = mx, rt = u;
    ed[u] = 0;
}
void cal(int u)
{
    md = max(md, dep[u]);
    ed[u] = 1; ++cnt[dep[u]];
    for (int v : e[u]) if (!ed[v])
    {
        dep[v] = dep[u] + 1;
        cal(v);
    }
    ed[u] = 0;
}
void solve(int u)
{
    mn = 1e9;
    find(u);
    ed[rt] = 1;
    vector<int> c;
    for (int v : e[rt]) if (!ed[v])
    {
        c.push_back(v);
        if (siz[v] >= siz[rt]) siz[v] = siz[u] - siz[rt];
    }
    sort(all(c), [&](const int &a, const int &b) {return siz[a] < siz[b]; });
    NTT::Q a(vector<ui>{1});
    NT::Q b(vector<ui>{1});
    for (int v : c)
    {
        md = 0; dep[v] = 1;
        cal(v); ++md;
        vector<ui> d(cnt, cnt + md);
```

```
        NTT::Q e(d);
        NT::Q f(d);
        auto g = e & a;
        auto h = f & b;
        for (int i = 0; i < g.a.size(); i++) r1[i] = (r1[i] + g.a[i]) % NTT::p;
        for (int i = 0; i < h.a.size(); i++) r2[i] = (r2[i] + h.a[i]) % NT::p;
        a += e; b += f;
        fill_n(cnt, md, 0);
    }
    for (int v : c)
    {
        ksiz = siz[v];
        solve(v);
    }
}
```

5.6.13 点分树

核心结论：点分树上 lca 出现在原树路径上。

$O(n\log^2 n)$，$O(n\log n)$。

```
template<class typC> struct bit
{
    vector<typC> a;
    int n;
    bit() { }
    bit(int nn) :n(nn), a(nn + 1) { }
    template<class T> bit(int nn, T *b) : n(nn), a(nn + 1)
    {
        for (int i = 1; i <= n; i++) a[i] = b[i - 1];
        for (int i = 1; i <= n; i++) if (i + (i & -i) <= n) a[i + (i & -i)] += a[i];
    }
    void add(int x, typC y)
    {
        //cerr<<"add "<<x<<" by "<<y<<endl;
        ++x;
        x = clamp(x, 1, n + 1);
        if (x > n) return;
        assert(1 <= x && x <= n);
        a[x] += y;
        while ((x += x & -x) <= n) a[x] += y;
    }
    typC sum(int x)
    {
        //cerr<<"sum "<<x;
        ++x;
        x = clamp(x, 0, n);
        assert(0 <= x && x <= n);
        typC r = a[x];
        while (x ^= x & -x) r += a[x];
        //cerr<<"= "<<r<<endl;
        return r;
    }
    typC sum(int x, int y)
    {
        return sum(y) - sum(x - 1);
    }
```

```
    int lower_bound(typC x)
    {
        if (n == 0) return 0;
        int i = __lg(n), j = 0;
        for (; i >= 0; i--) if ((1 << i | j) <= n && a[1 << i | j] < x) j |= 1 << i, x -= a[j];
        return j + 1;
    }
};
namespace DFS
{
    typedef long long ll;
    const int N = 1e5 + 5, M = 18;
    ll a[N];
    int st[M][N * 2], lg[N * 2];
    int dep[N], dfn[N], siz[N], f[N], szp[N], szn[N];
    vector<int> e[N], c[N], rg[N];
    bool ed[N];
    int n, ksiz, rt, mn, id;
    int lca(int u, int v)
    {
        u = dfn[u]; v = dfn[v];
        if (u > v) swap(u, v);
        int z = lg[v - u + 1];
        return dep[st[z][u]] < dep[st[z][v - (1 << z) + 1]] ? st[z][u] : st[z][v - (1 << z) + 1];
    }
    int dis(int u, int v)
    {
        return dep[u] + dep[v] - dep[lca(u, v)] * 2;
    }
    void findroot(int u)
    {
        ed[u] = siz[u] = 1;
        int mx = 0;
        for (int v : e[u]) if (!ed[v])
        {
            findroot(v);
            siz[u] += siz[v];
            mx = max(mx, siz[v]);
        }
        mx = max(mx, ksiz - siz[u]);
        ed[u] = 0;
        if (mn > mx) mn = mx, rt = u;
    }
    int dfs(int u)
    {
        mn = 1e9;
        findroot(u);
        u = rt;
        ed[u] = 1;
        for (int v : e[u]) if (!ed[v] && siz[v] > siz[u]) siz[v] = ksiz - siz[u];
        for (int v : e[u]) if (!ed[v])
        {
            ksiz = siz[v];
            c[u].push_back(dfs(v));
            f[c[u].back()] = u;
        }
        return u;
```

```
}
void pre_dfs(int u)
{
    st[0][dfn[u] = ++id] = u;
    ed[u] = 1;
    for (int v : e[u]) if (!ed[v])
    {
        dep[v] = dep[u] + 1;
        pre_dfs(v);
        st[0][++id] = u;
    }
    ed[u] = 0;
}
void init(int _n)
{
    n = _n; id = 0;
    int i;
    for (int i = 1; i <= n; i++)
    {
        e[i].clear();
        a[i] = f[i] = ed[i] = 0;
    }
}
void new_dfs(int u)
{
    siz[u] = 1;
    for (int v : c[u]) new_dfs(v), siz[u] += siz[v];
    vector<int> &q = rg[u];
    q = {u};
    int ql = 0;
    while (ql < q.size())
    {
        int x = q[ql++];
        for (int v : c[x]) q.push_back(v);
    }
}
void fun()
{
    pre_dfs(1);
    int i, j;
    for (i = 2; i <= id; i++) lg[i] = lg[i >> 1] + 1;
    for (j = 0; j < lg[id]; j++)
    {
        int R = id - (2 << j) + 1;
        for (i = 1; i <= R; i++) st[j + 1][i] = dep[st[j][i]] < dep[st[j][i + (1 << j)]] ? st[j
            ][i] : st[j][i + (1 << j)];
    }
    ksiz = n;
    rt = dfs(1);
    new_dfs(rt);
}
vector<int> get(int u)
{
    vector<int> st = {u};
    while (u = f[u]) st.push_back(u);
    return st;
}
```

```
}
using DFS::init, DFS::fun, DFS::e, DFS::dis, DFS::rg, DFS::get;
```

圆环修改和单点查询：

```
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, i;
    cin >> n >> m;
    vector<int> a(n + 1);
    for (i = 1; i <= n; i++) cin >> a[i];
    DFS::init(n);
    for (i = 1; i < n; i++)
    {
        int u, v;
        cin >> u >> v;
        ++u; ++v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    DFS::fun();
    vector<bit<ll>> inc(n + 1), dec(n + 1);
    for (i = 1; i <= n; i++)
    {
        int mx = 0;
        for (int v : rg[i]) cmax(mx, dis(i, v));
        inc[i] = bit<ll>(mx + 1);
        if (i != DFS::rt)
        {
            mx = 0;
            for (int v : rg[i]) cmax(mx, dis(DFS::f[i], v));
            dec[i] = bit<ll>(mx + 1);
        }
    }
    while (m--)
    {
        int op, u;
        cin >> op >> u; ++u;
        if (op == 0)
        {
            int l, r, x;
            cin >> l >> r >> x;
            auto v = get(u);
            int m = v.size();
            for (i = 0; i < m; i++)
            {
                inc[v[i]].add(l - dis(v[i], u), x);
                inc[v[i]].add(r - dis(v[i], u), -x);
            }
            for (i = 0; i + 1 < m; i++)
            {
                dec[v[i]].add(l - dis(v[i + 1], u), x);
                dec[v[i]].add(r - dis(v[i + 1], u), -x);
            }
        }
        else
```

```
        {
            ll res = a[u];
            auto v = get(u);
            int m = v.size();
            for (i = 0; i < m; i++) res += inc[v[i]].sum(dis(v[i], u));
            for (i = 0; i + 1 < m; i++) res -= dec[v[i]].sum(dis(v[i + 1], u));
            cout << res << '\n';
        }
    }
}
```

单点修改和圆环查询：

```
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, i;
    cin >> n >> m;
    vector<int> a(n + 1);
    for (i = 1; i <= n; i++) cin >> a[i];
    DFS::init(n);
    for (i = 1; i < n; i++)
    {
        int u, v;
        cin >> u >> v;
        ++u; ++v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    DFS::fun();
    vector<bit<ll>> inc(n + 1), dec(n + 1);
    vector<ll> tmp(n + 1);
    for (i = 1; i <= n; i++)
    {
        int mx = 0;
        for (int v : rg[i])
        {
            int d = dis(i, v);
            cmax(mx, d);
            tmp[d] += a[v];
        }
        inc[i] = bit<ll>(mx + 1, tmp.data());
        fill_n(tmp.begin(), mx + 1, 0);
        if (i != DFS::rt)
        {
            mx = 0;
            for (int v : rg[i])
            {
                int d = dis(DFS::f[i], v);
                cmax(mx, d);
                tmp[d] += a[v];
            }
            dec[i] = bit<ll>(mx + 1, tmp.data());
            fill_n(tmp.begin(), mx + 1, 0);
        }
    }
    while (m--)
```

```
    {
        int op, u;
        cin >> op >> u; ++u;
        if (op == 0)
        {
            int x;
            cin >> x;
            auto v = get(u);
            int m = v.size();
            for (i = 0; i < m; i++) inc[v[i]].add(dis(v[i], u), x);
            for (i = 0; i + 1 < m; i++) dec[v[i]].add(dis(v[i + 1], u), x);
        }
        else
        {
            int l, r;
            cin >> l >> r;
            --r;
            ll res = 0;
            auto v = get(u);
            int m = v.size();
            for (i = 0; i < m; i++) res += inc[v[i]].sum(l - dis(v[i], u), r - dis(v[i], u));
            for (i = 0; i + 1 < m; i++) res -= dec[v[i]].sum(l - dis(v[i + 1], u), r - dis(v[i +
                1], u));
            cout << res << '\n';
        }
    }
}
```

5.6.14 （基环）树哈希

有根树返回每个子树的哈希值，无根树返回树的哈希值（长度至多为 2 的 vector），基环树返回图的哈希值（长度等于环长的 vector）。

```
vector<int> tree_hash(const vector<vector<int>>& e, int root)//[0,n)
{
    int n = e.size();
    static map<vector<int>, int> mp;
    static int id = 0;
    vector<int> h(n), ed(n);
    function<void(int)> dfs = [&](int u)
        {
            ed[u] = 1;
            vector<int> c;
            for (int v : e[u]) if (!ed[v])
            {
                dfs(v);
                c.push_back(h[v]);
            }
            sort(all(c));
            if (!mp.count(c)) mp[c] = id++;
            h[u] = mp[c];
        };
    dfs(root);
    return h;
}
vector<int> tree_hash(const vector<vector<int>>& e)//[0,n)
{
```

```
    int n = e.size();
    if (n == 0) return { };
    vector<int> sz(n), mx(n);
    function<void(int)> dfs = [&](int u)
        {
            sz[u] = 1;
            for (int v : e[u]) if (!sz[v])
            {
                dfs(v);
                sz[u] += sz[v];
                cmax(mx[u], sz[v]);
            }
            cmax(mx[u], n - sz[u]);
        };
    dfs(0);
    int m = *min_element(all(mx)), i;
    vector<int> rt;
    for (i = 0;i < n;i++) if (mx[i] == m) rt.push_back(i);
    for (int& u : rt) u = tree_hash(e, u)[u];
    sort(all(rt));
    return rt;
}
template<class T> void min_order(vector<T>& a)
{
    int n = a.size(), i, j, k;
    a.resize(n * 2);
    for (i = 0;i < n;i++) a[i + n] = a[i];
    i = k = 0;j = 1;
    while (i < n && j < n && k < n)
    {
        T x = a[i + k], y = a[j + k];
        if (x == y) ++k; else
        {
            (x > y ? i : j) += k + 1;
            j += (i == j);
            k = 0;
        }
    }
    a.resize(n);
    //[min(i,j),n)+[0,min(i,j))
    rotate(a.begin(), min(i, j) + all(a));
}
vector<int> pseudotree_hash(const vector<vector<int>>& e)//[0,n)
{
    int n = e.size();
    static map<vector<int>, int> mp;
    static int id = 0;
    vector<int> f(n), ed(n), h(n);
    pair lp{-1, -1};
    function<void(int)> dfs = [&](int u)
        {
            ed[u] = 1;
            for (int v : e[u]) if (!ed[v])
            {
                f[v] = u;
                dfs(v);
            }
```

```
        else if (v != f[u]) lp = {u, v};
    };
    dfs(0);
    auto [x, y] = lp;
    vector<int> node = {y};
    do node.push_back(y = f[y]); while (y != x);
    fill(all(ed), 0);
    for (int u : node) ed[u] = 1;
    dfs = [&](int u)
        {
            ed[u] = 1;
            vector<int> c;
            for (int v : e[u]) if (!ed[v])
            {
                dfs(v);
                c.push_back(h[v]);
            }
            sort(all(c));
            if (!mp.count(c)) mp[c] = id++;
            h[u] = mp[c];
        };
    vector<int> r0;
    for (int u : node)
    {
        dfs(u);
        r0.push_back(h[u]);
    }
    auto r1 = r0;
    reverse(all(r1));
    min_order(r0);
    min_order(r1);
    return min(r0, r1);
}
```

5.7 欧拉路相关

5.7.1 构造：字典序最小

```
#include "bits/stdc++.h"
using namespace std;
#define all(x) (x).begin(),(x).end()
const int N=1e5+2;
vector<int> e[N];
int rd[N],cd[N];
vector<int> ans;
void dfs(int u)
{
    while (e[u].size())
    {
        int v=e[u].back();
        e[u].pop_back();
        dfs(v);
        ans.push_back(v);
    }
}
int main()
```

```
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,i,x=0;
    cin>>n>>m;ans.reserve(m);
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
        ++cd[u];++rd[v];
    }
    for (i=1;i<=n;i++) if (cd[i]!=rd[i])
    {
        if (abs(cd[i]-rd[i])>1) goto no;
        ++x;
    }
    if (x>2) goto no;x=1;
    for (i=1;i<=n;i++) if (cd[i]>rd[i]) {x=i;break;}
    for (i=1;i<=n;i++) sort(all(e[i])),reverse(all(e[i]));
    dfs(x);ans.push_back(x);reverse(all(ans));
    for (i=0;i<ans.size();i++) cout<<ans[i]<<" \n"[i+1==ans.size()];
    return 0;
    no:cout<<"No"<<endl;
}
```

5.7.2 回路/通路构造

$O(n+m)$，$O(n+m)$。

```
optional<vector<int>> undirected_euler_cycle(int n, const vector<pair<int, int>> &edges)//[1,n
    ]/[1,m]，正数表示正向，负数表示反向
{
    int i = 0;
    vector<int> rd(n + 1), ed(edges.size() + 1), r;
    vector<vector<pair<int, int>>> e(n + 1);
    for (auto [u, v] : edges)
    {
        ++rd[u], ++rd[v];
        e[u].push_back({v, ++i});
        e[v].push_back({u, -i});
    }
    for (i = 1; i <= n; i++) if (rd[i] & 1) return { };
    function<void(int)> dfs = [&](int u) {
        while (e[u].size())
        {
            auto [v, w] = e[u].back();
            e[u].pop_back();
            if (ed[abs(w)]) continue;
            ed[abs(w)] = 1;
            dfs(v);
            r.push_back(w);
        }
    };
    for (i = 1; i <= n; i++) if (rd[i]) { dfs(i); break; }
    reverse(all(r));
    if (r.size() != edges.size()) return { };
    return {r};
```

```
}
optional<vector<int>> directed_euler_cycle(int n, const vector<pair<int, int>> &edges)//[1,n]/[1,
    m]
{
    int i = 0;
    vector<int> rd(n + 1), cd(n + 1), r;
    vector<vector<pair<int, int>>> e(n + 1);
    for (auto [u, v] : edges)
    {
        ++cd[u], ++rd[v];
        e[u].push_back({v, ++i});
    }
    for (i = 1; i <= n; i++) if (rd[i] != cd[i]) return { };
    function<void(int)> dfs = [&](int u) {
        while (e[u].size())
        {
            auto [v, w] = e[u].back();
            e[u].pop_back();
            dfs(v);
            r.push_back(w);
        }
    };
    for (i = 1; i <= n; i++) if (cd[i]) { dfs(i); break; }
    reverse(all(r));
    if (r.size() != edges.size()) return { };
    return {r};
}
optional<vector<int>> undirected_euler_trail(int n, const vector<pair<int, int>> &edges)//[1,n
    ]/[1,m], 正数表示正向, 负数表示反向
{
    int i = 0;
    vector<int> rd(n + 1), ed(edges.size() + 1), r;
    vector<vector<pair<int, int>>> e(n + 1);
    for (auto [u, v] : edges)
    {
        ++rd[u], ++rd[v];
        e[u].push_back({v, ++i});
        e[v].push_back({u, -i});
    }
    int odd = 0;
    for (i = 1; i <= n; i++) odd += rd[i] & 1;
    if (odd > 2) return { };
    function<void(int)> dfs = [&](int u) {
        while (e[u].size())
        {
            auto [v, w] = e[u].back();
            e[u].pop_back();
            if (ed[abs(w)]) continue;
            ed[abs(w)] = 1;
            dfs(v);
            r.push_back(w);
        }
    };
    for (i = 1; i <= n; i++) if (rd[i] & 1) { dfs(i); break; }
    if (i > n)
    {
        for (i = 1; i <= n; i++) if (rd[i]) { dfs(i); break; }
```

```
    }
    reverse(all(r));
    if (r.size() != edges.size()) return { };
    return {r};
}
optional<vector<int>> directed_euler_trail(int n, const vector<pair<int, int>> &edges)//[1,n]/[1,
    m]
{
    int i = 0;
    vector<int> rd(n + 1), cd(n + 1), r;
    vector<vector<pair<int, int>>> e(n + 1);
    for (auto [u, v] : edges)
    {
        ++cd[u], ++rd[v];
        e[u].push_back({v, ++i});
    }
    int diff = 0;
    for (i = 1; i <= n; i++)
    {
        if (abs(rd[i] - cd[i]) > 1) return { };
        if (rd[i] != cd[i]) ++diff;
    }
    if (diff > 2) return { };
    function<void(int)> dfs = [&](int u) {
        while (e[u].size())
        {
            auto [v, w] = e[u].back();
            e[u].pop_back();
            dfs(v);
            r.push_back(w);
        }
    };
    for (i = 1; i <= n; i++) if (cd[i] > rd[i]) { dfs(i); break; }
    if (i > n)
    {
        for (i = 1; i <= n; i++) if (cd[i]) { dfs(i); break; }
    }
    reverse(all(r));
    if (r.size() != edges.size()) return { };
    return {r};
}
optional<vector<int>> mixed_euler_cycle(int n, const vector<tuple<int, int, bool>> &edges)//true:
     单向, false: 双向
{
    int i = 0, j, m = edges.size();
    vector<int> rd(n + 1), cd(n + 1), r, rev(m + 1);
    vector<vector<pair<int, int>>> e(n + 1);
    for (auto [u, v, d] : edges) ++cd[u], ++rd[v];
    for (i = 1; i <= n; i++) if (cd[i] + rd[i] & 1) return { };
    vector<tuple<int, int, ll>> eg;
    for (auto [u, v, d] : edges) if (!d) eg.push_back({u, v, 1});
    ll sum = 0;
    for (i = 1; i <= n; i++)
    {
        int d = (cd[i] - rd[i]) / 2;
        if (d > 0) eg.push_back({0, i, d}), sum += d;
        else if (d < 0) eg.push_back({i, n + 1, -d});
```

```
    }
    auto [w, res] = net::max_flow(n + 1, eg, 0, n + 1);
    if (w != sum) return { };
    vector<pair<int, int>> G(m);
    for (i = j = 0; i < m; i++)
    {
        auto [u, v, d] = edges[i];
        if (d || !res[j++]) G[i] = {u, v};
        else G[i] = {v, u}, rev[i + 1] = 1;
    }
    auto ans = directed_euler_cycle(n, G);
    if (!ans) return ans;
    for (int &x : *ans) if (rev[x]) x = -x;
    return ans;
}
```

5.7.3 回路计数（BEST 定理）/生成树计数

$O(n^3)$，$O(n^2)$。

以 u 为起点的欧拉回路个数 $sum = T(u) \times \prod_{v=1}^{n}(out(v)-1)!$，其中 $T(u)$ 是以 u 为根的内向树个数（出度矩阵-邻接矩阵），$out(v)$ 是 v 的出度。若允许循环同构（如 $1 \to 2 \to 1 \to 3 \to 1$ 与 $1 \to 3 \to 1 \to 2 \to 1$），还需多乘 $out(u)$。

这里的部分代码是未经验证的。

```
ull det(vector<vector<ull>> b)
{
    ull r=1;
    int n=b.size(), i, j, k;
    for (i=0; i<n; i++)
    {
        for (j=i; j<n; j++) if (b[j][i]) break;
        if (j==n) return 0;
        swap(b[j], b[i]);
        if (j!=i) r=(p-r)%p;
        r=r*b[i][i]%p;
        b[i][i]=ksm(b[i][i], p-2);
        for (j=n-1; j>=i; j--) b[i][j]=b[i][j]*b[i][i]%p;
        for (j=i+1; j<n; j++) for (k=n-1; k>=i; k--) b[j][k]=(b[j][k]+(p-b[j][i])*b[i][k])%p;
    }
    return r;
}
ull euler_path_count(vector<vector<int>> a, int s, int t)
{
    int n=a.size(), i, j, k;
    ++a[t][s]; s=t;
    vector<int> rd(n), cd(n);
    for (i=0; i<n; i++) for (j=0; j<n; j++) cd[i]+=a[i][j], rd[j]+=a[i][j];
    for (i=0; i<n; i++) if (cd[i]!=rd[i]) return 0;
    vector<int> f(n);
    iota(all(f), 0);
    function<int(int)> getf=[&](int u) { return f[u]==u?u:f[u]=getf(f[u]); };
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (a[i][j]) f[getf(i)]=getf(j);
    ull r=1;
    vector<int> id;
    for (i=0; i<n; i++) if (cd[i])
```

```
    {
        if (getf(i)!=getf(s)) return 0;
        r=r*fac[cd[i]-1]%p;
        if (i!=s) id.push_back(i);
    }
    n=id.size();
    vector b(n, vector<ull>(n));
    for (i=0; i<n; i++)
    {
        b[i][i]=cd[id[i]]-a[id[i]][id[i]];
        for (j=0; j<n; j++) if (i!=j) b[i][j]=(p-a[id[i]][id[j]])%p;
    }
    return r*det(b)%p;
}
ull euler_path_count(vector<vector<int>> a)
{
    int n=a.size(), i, j, s=-1, t=-1;
    vector<int> rd(n), cd(n), d(n);
    for (i=0; i<n; i++) for (j=0; j<n; j++) cd[i]+=a[i][j], rd[j]+=a[i][j];
    if (count(all(cd), 0)==n) return 1;
    for (i=0; i<n; i++) d[i]=cd[i]-rd[i];
    s=max_element(all(d))-d.begin();
    t=min_element(all(d))-d.begin();
    ull r=0;
    if (s==t)
    {
        for (i=0; i<n; i++) if (cd[i]) r+=euler_path_count(a, i, i);
    }
    else r=euler_path_count(a, s, t);
    return r%p;
}
ull euler_circuit_count(vector<vector<int>> a)
{
    int n=a.size(), i, j;
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (a[i][j]) return euler_path_count(a, i, i)*ksm(
        accumulate(all(a[i]), 0llu)%p, p-2)%p;
    return 1;
}
ull directed_spanning_tree_count(vector<vector<int>> a, int s)
{
    int n=a.size(), i, j;
    vector b(n-1, vector<ull>(n-1));
    for (i=0; i<n; i++) a[i][i]=0;
    for (i=0; i<n; i++) if (i!=s) for (j=0; j<n; j++) if (j!=s&&i!=j) b[i-(i>s)][j-(j>s)]=(p-a[i][
        j])%p;
    for (i=0; i<n; i++) if (i!=s) for (j=0; j<n; j++) (b[i-(i>s)][i-(i>s)]+=a[j][i])%=p;
    return det(b);
}//外向
ull undirected_spanning_tree_count(vector<vector<int>> a)
{
    int n=a.size(), i, j;
    --n;
    vector b(n, vector<ull>(n));
    for (i=0; i<n; i++) a[i][i]=0;
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (i!=j) b[i][j]=(p-a[i][j])%p;
    for (i=0; i<n; i++) b[i][i]=reduce(all(a[i]), 0llu)%p;
    return det(b);
```

```
}
```

5.8 三/四元环计数

不能处理有重边和自环的情况。

$O(m\sqrt{m})$，$O(n+m)$。

注意四元环数的是边四元环。点四元环需要去掉四点完全图个数 *2，似乎不太能做？

三元环是可以枚举的，你可以在 ans 改变时记录三元环 (i,u,v)。

```
ll triple(const vector<pair<int,int>> &edges)//start from 0
{
    int n=0,i;
    for (auto [u,v]:edges) n=max({n,u,v});
    ++n;
    vector<int> d(n),id(n),rk(n),cnt(n);
    vector<vector<int>> e(n);
    for (auto [u,v]:edges) ++d[u],++d[v];
    iota(all(id),0); sort(all(id),[&](int x,int y) { return d[x]<d[y]; });
    for (i=0; i<n; i++) rk[id[i]]=i;
    for (auto [u,v]:edges)
    {
        if (rk[u]>rk[v]) swap(u,v);
        e[u].push_back(v);
    }
    ll ans=0;
    for (i=0; i<n; i++)
    {
        for (int u:e[i]) cnt[u]=1;
        for (int u:e[i]) for (int v:e[u]) ans+=cnt[v];
        for (int u:e[i]) cnt[u]=0;
    }
    return ans;
}
ll quadruple(const vector<pair<int,int>> &edges)
{
    int n=0,i;
    for (auto [u,v]:edges) n=max({n,u,v});
    ++n;
    vector<int> d(n),id(n),rk(n),cnt(n);
    vector<vector<int>> e(n),lk(n);
    for (auto [u,v]:edges) ++d[u],++d[v];
    iota(all(id),0); sort(all(id),[&](int x,int y) { return d[x]<d[y]; });
    for (i=0; i<n; i++) rk[id[i]]=i;
    for (auto [u,v]:edges)
    {
        if (rk[u]>rk[v]) swap(u,v);
        e[u].push_back(v);
        lk[u].push_back(v);
        lk[v].push_back(u);
    }
    ll ans=0;
    for (i=0; i<n; i++)
    {
        for (int u:lk[i]) for (int v:e[u]) if (rk[v]>rk[i]) ans+=cnt[v]++;
        for (int u:lk[i]) for (int v:e[u]) cnt[v]=0;
    }
```

```
    return ans;
}
map<pair<int, int>, ll> quadruple(vector<pair<int, int>> edges)
{
    int n = 0, i;
    for (auto [u, v] : edges) n = max({n, u, v});
    ++n;
    map<pair<int, int>, int> ec;
    for (auto [u, v] : edges)
    {
        if (u > v) swap(u, v);
        ++ec[{u, v}];
    }
    vector<ll> c;
    edges.clear();
    for (auto [_, cc] : ec) edges.push_back(_), c.push_back(cc);
    vector d(n, 0), id(d), rk(d);
    vector<ll> cnt(n);
    vector<vector<pair<int, int>>> e(n), lk(n);
    for (auto [u, v] : edges) ++d[u], ++d[v];
    iota(all(id), 0); sort(all(id), [&](int x, int y) { return d[x] < d[y]; });
    for (i = 0; i < n; i++) rk[id[i]] = i;
    i = 0;
    for (auto [u, v] : edges)
    {
        if (rk[u] > rk[v]) swap(u, v);
        e[u].push_back({v, i});
        lk[u].push_back({v, i});
        lk[v].push_back({u, i});
        ++i;
    }
    int m = edges.size();
    vector<ll> ans(m);
    for (i = 0; i < n; i++)
    {
        for (auto [u, w1] : lk[i]) for (auto [v, w2] : e[u]) if (rk[v] > rk[i])
        {
            cnt[v] += c[w1] * c[w2];
        }
        for (auto [u, w1] : lk[i]) for (auto [v, w2] : e[u]) if (rk[v] > rk[i])
        {
            ans[w1] += (cnt[v] - c[w1] * c[w2]) * c[w2];
            ans[w2] += (cnt[v] - c[w1] * c[w2]) * c[w1];
        }
        for (auto [u, w1] : lk[i]) for (auto [v, w2] : e[u]) if (rk[v] > rk[i]) cnt[v] = 0;
    }
    map<pair<int, int>, ll> mp;
    for (i = 0;i < m;i++) mp[edges[i]] = ans[i];
    return mp;
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, i;
    cin >> n >> m;
    vector<pair<int, int>> eg(m);
```

```
    cin >> eg;
    auto mp = quadruple(eg);
    for (i = 0;i < m;i++)
    {
        auto [u, v] = eg[i];
        if (u > v) swap(u, v);
        cout << mp[{u, v}] << "␣\n"[i + 1 == m];
    }
}
```

5.9 支配树

u 支配 v 指的是从 S 到 v 的路径必然经过 u。支配树是保持支配关系不变的树，其中 s 是根，$idom[u]$ 是 u 的父节点。

DAG 版：$O(m\log n)$，$O(n\log n)$。

```
int lca(int x, int y)
{
    int i;
    if (dep[x] < dep[y]) swap(x, y);
    for (i = lm[x]; dep[x] != dep[y]; i--) if (dep[f[x][i]] >= dep[y]) x = f[x][i];
    if (x == y) return x;
    for (i = lm[x]; f[x][0] != f[y][0]; i--) if (f[x][i] != f[y][i])
    {
        x = f[x][i]; y = f[y][i];
    }
    return f[x][0];
}
void dfs(int x)
{
    s[x] = 1;
    int i;
    for (i = sfir[x]; i; i = snxt[i])
    {
        dfs(slj[i]);
        s[x] += s[slj[i]];
    }
}
int main()
{
    dep[0] = -1;
    cin >> n;
    for (i = 1; i <= n; i++)
    {
        cin >> x;
        while (x)
        {
            add(x, i);
            cin >> x;
        }
    }
    dl[tou = wei = 1] = ++n;
    for (i = 1; i < n; i++) if (!rd[i]) add(n, i);
    while (tou <= wei)
    {
        for (i = fir[x = dl[tou++]]; i; i = nxt[i]) if (--rd[lj[i]] == 0) dl[++wei] = lj[i];
```

```
        if (i = ffir[x])
        {
            y = flj[i];
            while (i = fnxt[i]) y = lca(y, flj[i]);
            f[x][0] = y;
        }
        else y = 0;
        sadd(y, x);
        f[x][0] = y;
        for (i = 1; i <= 16; i++) if (0 == (f[x][i] = f[f[x][i - 1]][i - 1]))
        {
            lm[x] = i;
            break;
        }
        dep[x] = dep[y] + 1;
    }
    dfs(n);
    for (i = 1; i < n; i++) printf("%d\n", s[i] - 1);
}
```

一般图：标号从 1 开始。

```
vector<int> dom_tree(vector<vector<int>> e, int s)
{
    int n = e.size() - 1, i, id = 0;
    vector<vector<int>> buc(n + 1), ie(n + 1);
    vector<int> mn(n + 1), f(n + 1), sdom(n + 1, n + 1), idom(n + 1), dfn(n + 1), nfd(n + 2), pv(n
         + 1), ed(n + 1), cf(n + 1);
#define cmp(x,y) (sdom[x] < sdom[y] ? x : y)
    auto getf = [&](auto &&getf, int u) ->void {
        if (f[u] == u) return;
        getf(getf, f[u]);
        mn[u] = cmp(mn[u], mn[f[u]]);
        f[u] = f[f[u]];
    };
    for (i = 1; i <= n; i++) mn[i] = f[i] = i;
    {
        auto dfs = [&](auto &&dfs, int u) ->void {
            ed[u] = 1;
            for (int v : e[u]) if (!ed[v]) dfs(dfs, v);
        };
        dfs(dfs, s);
    }
    for (i = 1; i <= n; i++) if (ed[i]) erase_if(e[i], [&](int v) { return !ed[v]; });
    else e[i].clear();
    for (i = 1; i <= n; i++) for (int v : e[i]) ie[v].push_back(i);
    auto dfs = [&](auto &&dfs, int u) ->void {
        nfd[dfn[u] = ++id] = u;
        for (int v : e[u]) if (!dfn[v]) dfs(dfs, v), cf[v] = u;
    };
    dfs(dfs, s); dfn[0] = n + 1;
    for (i = id; i; i--)
    {
        int u = nfd[i], w = 0;
        for (int v : ie[u])
        {
            cmin(sdom[u], dfn[v]);
            if (dfn[v] > dfn[u])
```

```
        {
            getf(getf, v);
            w = cmp(w, mn[v]);
        }
    }
    cmin(sdom[u], sdom[w]);
    buc[nfd[sdom[u]]].push_back(u);
    for (int v : buc[u]) getf(getf, v), pv[v] = mn[v];
    for (int v : e[u]) if (cf[v] == u) f[v] = u, mn[v] = cmp(mn[v], mn[u]);
  }
  for (i = 1; i <= n; i++) idom[nfd[i]] = (sdom[pv[nfd[i]]] == sdom[nfd[i]]) ? nfd[sdom[nfd[i]]]
      : idom[pv[nfd[i]]];
  idom[s] = s;
  return idom;
#undef cmp
}
int main()
{
  int n, m, s;
  cin >> n >> m >> s; ++s;
  vector<vector<int>> e(n + 1);
  for (int i = 1; i <= m; i++)
  {
    int u, v;
    cin >> u >> v; ++u; ++v;
    e[u].push_back(v);
  }
  auto r = dom_tree(e, s);
  for (int i = 1; i <= n; i++) cout << r[i] - 1 << " \n"[i == n];
}
```

5.10 prufer 与树的互相转化

$O(n)$，$O(n)$。

```
vector<int> edges_to_prufer(const vector<pair<int, int>> &eg)//[1,n], 定根为 n
{
  int n = eg.size() + 1, i, j, k;
  vector<int> fir(n + 1), nxt(n * 2 + 1), e(n * 2 + 1), rd(n + 1);
  int cnt = 0;
  for (auto [u, v] : eg)
  {
    e[++cnt] = v; nxt[cnt] = fir[u]; fir[u] = cnt; ++rd[v];
    e[++cnt] = u; nxt[cnt] = fir[v]; fir[v] = cnt; ++rd[u];
  }
  for (i = 1; i <= n; i++) if (rd[i] == 1) break;
  int u = i;
  vector<int> r; r.reserve(n - 2);
  for (j = 1; j < n - 1; j++)
  {
    for (k = fir[u], u = rd[u] = 0; k; k = nxt[k]) if (rd[e[k]])
    {
      r.push_back(e[k]);
      if ((--rd[e[k]] == 1) && (e[k] < i)) u = e[k];
    }
    if (!u) { while (rd[i] != 1) ++i; u = i; }
  }
```

```
    return r;
}
vector<pair<int, int>> prufer_to_edges(const vector<int> &p)//[1,n], 定根为 n
{
    int n = p.size(), i, j, k;
    int m = n + 3;
    vector<int> cs(m);
    for (i = 0; i < n; i++) ++cs[p[i]];
    i = 0;
    while (cs[++i]);
    int u = i, v;
    vector<pair<int, int>> r;
    r.reserve(n - 2);
    for (j = 0; j < n; j++)
    {
        cs[u] = 1e9;
        r.push_back({u, v = p[j]});
        if ((--cs[v] == 0) && (v < i)) u = v;
        if (v != u) { while (cs[i]) ++i; u = i; }
    }
    r.push_back({u, n + 2});
    return r;
}
```

5.11 最小密度环

求所有环中边权和除以边数最少的，$O(nm)$。更常用的做法是二分 spfa。

```
#include "bits/stdc++.h"
using namespace std;
const int N=3e3+5,M=1e4+5;
const double inf=1e18;
int u[M],v[M];
double f[N][N],w[M];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(8);
    int n,m,i,j;
    cin>>n>>m;
    for (i=1;i<=m;i++) cin>>u[i]>>v[i]>>w[i];
    ++n;
    for (i=1;i<=n;i++)
    {
        fill_n(f[i]+1,n,inf);
        for (j=1;j<=m;j++) f[i][v[j]]=min(f[i][v[j]],f[i-1][u[j]]+w[j]);
    }
    double ans=inf;
    for (i=1;i<n;i++) if (f[n][i]!=inf)
    {
        double r=-inf;
        for (j=1;j<n;j++) r=max(r,(f[n][i]-f[j][i])/(n-j));
        ans=min(ans,r);
    }
    cout<<ans<<endl;
}
```

5.12 点染色

结论：$\chi(G) \le \Delta(G)+1$，其中 $\Delta(G)$ 是图的最大度。只有奇圈和完全图取等。
构造方案只能爆搜。

```
vector<int> chromatic_number(int n,const vector<pair<int,int>> &edges)//[0,n)
{
    vector r(n,-1),cur(n,-1);
    vector<vector<int>> e(n);
    int ans=0,i;
    for (auto [u,v]:edges) e[u].push_back(v),e[v].push_back(u);
    for (i=0;i<n;i++) ans=max(ans,(int)e[i].size());
    ans+=2;
    vector p(n,vector(ans,0));
    function<void(int)> dfs=[&](int u)
    {
        int col=u?*max_element(cur.begin(),cur.begin()+u)+1:0;
        if (col>=ans) return;
        if (u==n)
        {
            r=cur;
            ans=col;
            return;
        }
        int i;
        for (int i=0;i<=col;i++) if (!p[u][i])
        {
            cur[u]=i;
            for (int v:e[u]) ++p[v][i];
            dfs(u+1);
            for (int v:e[u]) --p[v][i];
        }
    };
    dfs(0);
    return r;
}
```

5.13 最大独立集

爆搜。

```
vector<int> indep_set(int n,const vector<pair<int,int>> &edges)//[0,n)
{
    vector<vector<int>> e(n);
    mt19937 rnd(998);
    vector<int> p(n),q(n),ed(n);
    iota(all(p),0);
    shuffle(all(p),rnd);
    for (int i=0;i<n;i++) q[p[i]]=i;
    for (auto [u,v]:edges)
    {
        e[p[u]].push_back(p[v]);
        e[p[v]].push_back(p[u]);
    }
    vector<int> r,cur;
    function<void(int)> dfs=[&](int u)
    {
```

```
        if (cur.size()+n-u<=r.size()) return;
        if (u==n)
        {
            r=cur;
            return;
        }
        if (!ed[u])
        {
            cur.push_back(u);
            for (int v:e[u]) ++ed[v];
            dfs(u+1);
            for (int v:e[u]) --ed[v];
            cur.pop_back();
        }
        if (ed[u]||e[u].size()) dfs(u+1);
    };dfs(0);
    for (int &x:r) x=q[x];
    sort(all(r));
    return r;
}
```

5.14 弦图

单纯点：v 和 v 邻点构成团。

完美消除序列：v_i 在 $\{v_i, v_{i+1}, \cdots, v_n\}$ 为单纯点。

$N(v_i)=\{v_j|j>i\wedge(v_i,v_j)\in E\}$，$next(v_i)$ 为 $N(v_i)$ 最靠前的点。

极大团一定是 $\{v\}\cup N(v)$ 。

最大团大小等于色数。

弦图判定：等价于是否存在完美消除序列。首先求出一个完美消除序列，然后判定是否合法。

判定方法：设 $v_{i+1}, \cdots, v_n$ 中与 v_i 相邻的依次为 $v'_1, \cdots, v'_m$。只需判断是否 v'_1 与 $v'_2, \cdots, v'_m$ 相邻。

LexBFS 算法（我不会写）

每个点有一个字符串 label，初始为 0。从 $i=n$ 到 $i=1$ 确定，选 label 字典序最大的 u，再把 u 邻点的 label 后面接一个 i。

最大势算法：从 v_n 求到 v_1，设 $label_i$ 表示 i 与多少个已选点相邻，每次选 $label_i$ 最大的点。

弦图极大团：$\{v|\forall next(w)=v, |N(v)|\geq|N(w)|\}$。选出的集合为基本点，按上述极大团构造。

弦图染色：从 v_n 到 v_1 依次选最小可染的色。

最大独立集：从 v_1 到 v_n 能选就选。

最小团覆盖：设最大独立集为 $\{p_m\}$，最小团覆盖为 $\{\{p_i\}\cup N(p_i)\}$。

区间图：两个区间有边当且仅当交集非空。

区间图是弦图。

代码如下：

```
namespace chordal_graph//下标从 1 开始
{
    const int N=1e5+2;//点数
    bool ed[N];
    vector<int> e[N];
    int n;
    void init(const vector<pair<int,int>> &edges)
    {
        n=0;
        for (auto [u,v]:edges) n=max({n,u,v});
```

```
    for (int i=1;i<=n;i++) e[i].clear();
    for (auto [u,v]:edges) e[u].push_back(v),e[v].push_back(u);
}
vector<int> perfect_seq(const vector<pair<int,int>> &edges)//MCS
{
    init(edges);
    static int d[N];
    static vector<int> buc[N];
    int i,mx=0;
    memset(d+1,0,n*sizeof d[0]);
    memset(ed+1,0,n*sizeof ed[0]);
    for (i=1;i<=n;i++) buc[i].clear();
    buc[0].resize(n);
    iota(all(buc[0]),1);
    vector<int> r(n);
    for (i=n-1;i>=0;i--)
    {
        int u=0;
        while (!u)
        {
            while (buc[mx].size()) if (ed[buc[mx].back()]) buc[mx].pop_back();
            else
            {
                ed[u=buc[mx].back()]=1;
                buc[mx].pop_back();
                goto yes;
            }
            --mx;
        }
        yes:;
        r[i]=u;
        for (int v:e[u]) if (!ed[v]) buc[++d[v]].push_back(v),mx=max(mx,d[v]);
    }
    return r;
}
bool check_perfect_seq(vector<int> a)
{
    static bool ee[N];
    memset(ed+1,0,n*sizeof ed[0]);
    memset(ee+1,0,n*sizeof ee[0]);
    reverse(all(a));
    for (int u:a)
    {
        ed[u]=1;
        int w=0;
        for (int v:e[u]) if (ed[v]) {w=v;break;}
        if (!w) continue;
        ee[w]=1;
        for (int v:e[w]) ee[v]=1;
        for (int v:e[u]) if (ed[v]&&!ee[v]) return 0;
        ee[w]=0;
        for (int v:e[w]) ee[v]=0;
    }
    return 1;
}
bool check_chordal(const vector<pair<int,int>> &edges) {return check_perfect_seq(perfect_seq(
    edges));}
```

```
    vector<int> color(int _n,const vector<pair<int,int>> &edges)//返回长度为 _n+1。其中 0 无意义
    {
        auto a=perfect_seq(edges);
        reverse(all(a));
        memset(ed+1,0,n*sizeof ed[0]);
        vector<int> r(_n+1);
        for (int u:a)
        {
            for (int v:e[u]) ed[r[v]]=1;
            int x=1;
            while (ed[x]) ++x;
            r[u]=x;
            for (int v:e[u]) ed[r[v]]=0;
        }
        for (int i=n+1;i<=_n;i++) r[i]=1;
        return r;
    }
    vector<int> max_independent(int _n,const vector<pair<int,int>> &edges)//注意有孤立点这种奇怪东
        西
    {
        auto a=perfect_seq(edges);
        memset(ed+1,0,n*sizeof ed[0]);
        vector<int> r;
        for (int u:a) if (!ed[u])
        {
            r.push_back(u);
            for (int v:e[u]) ed[v]=1;
        }
        for (int i=n+1;i<=_n;i++) r.push_back(i);
        return r;
    }
}
using chordal_graph::check_chordal,chordal_graph::color,chordal_graph::max_independent;
```

6 计算几何

6.1 自适应 simpson 法

sim(l,r) 计算 $\int_l^r f(x)\,\mathrm{d}x$

```
const db eps=1e-7;
db sl,sr,sm,a;
db f(db x)
{
    return pow(x,a/x-x);
}
db g(db l,db r)
{
    db mid=(l+r)*0.5;
    return (f(l)+f(r)+f(mid)*4)/6*(r-l);
}
db sim(db l,db r)
{
    db mid=(l+r)*0.5;
    sl=g(l,mid);sr=g(mid,r);sm=g(l,r);
    if (abs(sl+sr-sm)<eps) return sl+sr;
    return sim(l,mid)+sim(mid,r);
}
```

6.2 计算几何全

功能其实比较少，因为实际遇到的几何题不多。最有用的可能是闵可夫斯基和合并凸包，和常规的线段判交之类的。其余功能最好直接使用 HDU 板。

```
namespace geo//不要用 int!
{
#define tmpl template<class T>
    using ll = long long;
    using lll = __int128;
    using db = long double;
    tmpl using up = conditional_t<std::is_same_v<T, ll>, lll,
        conditional_t<std::is_same_v<T, db>, db, void>>;
    const db eps = 1e-7, pi = 3.14159265358979323846264 34;
#define all(x) (x).begin(),(x).end()
    inline int sgn(const ll &x)
    {
        if (x < 0) return -1;
        return x > 0;
    }
    inline int sgn(const lll &x)
    {
        if (x < 0) return -1;
        return x > 0;
    }
    inline int sgn(const db &x)
    {
        if (abs(x) < eps) return 0;
        return x > 0 ? 1 : -1;
    }
    tmpl struct vec//* 为叉乘，dot 为点乘，只允许使用 long double 和 ll
    {
```

```
    mutable T x, y;
    vec() { }
    vec(T a, T b) :x(a), y(b) { }
    operator vec<ll>() const { return vec<ll>(x, y); }
    operator vec<db>() const { return vec<db>(x, y); }
    vec<T> operator+(const vec<T> &o) const { return vec(x + o.x, y + o.y); }
    vec<T> operator-(const vec<T> &o) const { return vec(x - o.x, y - o.y); }
    vec<T> operator*(const T &k) const { return vec(x * k, y * k); }
    vec<T> operator/(const T &k) const { return vec(x / k, y / k); }
    T operator*(const vec<T> &o) const { return x * o.y - y * o.x; }
    T dot(const vec<T> &o) const { return x * o.x + y * o.y; }
    void operator+=(const vec<T> &o) { x += o.x; y += o.y; }
    void operator-=(const vec<T> &o) { x -= o.x; y -= o.y; }
    void operator*=(const T &k) { x *= k; y *= k; }
    void operator/=(const T &k) { x /= k; y /= k; }
    bool operator==(const vec<T> &o) const { return x == o.x && y == o.y; }
    bool operator!=(const vec<T> &o) const { return x != o.x || y != o.y; }
    db len() const { return sqrt(len2()); }//模长
    T len2() const { return x * x + y * y; }
    vec<db> rotate(db angle)
    {
        db c = cos(angle), s = sin(angle);
        return vec<db>(x * c - y * s, x * s + y * c);
    }
    vec<T> rotate_90() { return vec<T>(-y, x); }
};
const vec<db> npos = vec<db>(514e194, 9810e191), apos = vec<db>(145e174, 999e180), O = vec<db
    >(0, 0);
tmpl int quad(const vec<T> &o)//坐标轴归右上象限，返回值 [1,4]
{
    const static int d[4] = {1, 2, 4, 3};
    return d[(sgn(o.y) < 0) * 2 + (sgn(o.x) < 0)];
}
tmpl bool angle_cmp(const vec<T> &a, const vec<T> &b)
{
    int c = quad(a), d = quad(b);
    if (c != d) return c < d;
    return a * b > 0;
}
tmpl db dis(const vec<T> &a, const vec<T> &b) { return (a - b).len(); }
tmpl T dis2(const vec<T> &a, const vec<T> &b) { return (a - b).len2(); }
tmpl vec<T> operator*(const T &k, const vec<T> &o) { return vec<T>(k * o.x, k * o.y); }
tmpl bool operator<(const vec<T> &a, const vec<T> &b)
{
    int s = sgn(a * b);
    return s > 0 || s == 0 && sgn(a.len2() - b.len2()) < 0;
}
void read(db &x) { static string s; cin >> s; x = stod(s); }
template<typename T, typename... Args> void read(T &first, Args&... args) { read(first); read(
    args...); }
istream &operator>>(istream &cin, vec<ll> &o) { return cin >> o.x >> o.y; }
istream &operator>>(istream &cin, vec<db> &o)
{
    static string s, t;
    cin >> s >> t;
    o = vec<db>(stod(s), stod(t));
    return cin;
```

```
}
tmpl ostream &operator<<(ostream &cout, const vec<T> &o)
{
    if ((vec<db>)o == apos) return cout << "all position";
    if ((vec<db>)o == npos) return cout << "no position";
    return cout << '(' << o.x << ',' << o.y << ')';
}
tmpl struct line
{
    vec<T> o, d;
    line() { }
    line(const vec<T> &a, const vec<T> &b);
    line(db a, db b, db c);
    bool operator!=(const line<T> &m) { return !(*this == m); }
};
template<> line<ll>::line(const vec<ll> &a, const vec<ll> &b) :o(a), d(b - a)
{
    ll tmp = gcd(d.x, d.y);
    assert(tmp);
    if (d.x < 0 || d.x == 0 && d.y < 0) tmp = -tmp;
    d.x /= tmp; d.y /= tmp;
}
template<> line<db>::line(const vec<db> &a, const vec<db> &b) :o(a), d(b - a)
{
    int s = sgn(d.x);
    if (s < 0 || !s && d.y < 0) d.x = -d.x, d.y = -d.y;
    assert(sgn(d.x) || sgn(d.y));
}
template<> line<db>::line(db a, db b, db c) : o(abs(a) > abs(b) ? vec<db>(-c / a, 0) : vec<db
    >(0, -c / b)), d(-b, a) { }//ax+by+c=0
tmpl db get_angle(const vec<T> &m, const vec<T> &n) { return asin(clamp<db>((m * n) / (m.len()
     * n.len()),-1,1)); }
tmpl bool operator<(const line<T> &m, const line<T> &n)
{
    int s = sgn(m.d * n.d);
    return s ? s > 0 : m.d * m.o < n.d * n.o;
}
tmpl bool operator==(const line<T> &m, const line<T> &n) { return sgn(m.d - n.d) == 0 && sgn((
    m.o - n.o) * m.d) == 0; }
tmpl ostream &operator<<(ostream &cout, const line<T> &o) { return cout << '(' << o.d.x << "k
    + " << o.o.x << " , " << o.d.y << "k + " << o.o.y << ")"; }
tmpl vec<db> intersect(const line<T> &m, const line<T> &n)
{
    if (!sgn(m.d * n.d)) return (!sgn(m.d * (n.o - m.o))) ? apos : npos;
    return (vec<db>)m.o + (n.o - m.o) * n.d / (db)(m.d * n.d) * (vec<db>)m.d;
}
tmpl db dis(const line<T> &m, const vec<T> &o) { return abs(m.d * (o - m.o) / m.d.len()); }
tmpl db dis(const vec<T> &o, const line<T> &m) { return abs(m.d * (o - m.o) / m.d.len()); }
struct circle
{
    vec<db> o;
    db r;
    circle() { }
    circle(const vec<db> &O, const db &R = 0) :o(vec<db>((db)O.x, (db)O.y)), r(R) { }//圆心半径
        构造
    circle(const vec<db> &a, const vec<db> &b) { o = (a + b) * 0.5; r = dis(b, o); }//直径构造
    circle(const vec<db> &a, const vec<db> &b, const vec<db> &c)//三点构造外接圆（非最小圆）
```

```
    {
        auto A = (b + c) * 0.5, B = (a + c) * 0.5;
        o = intersect(line(A, A + (c - A).rotate_90()), line(B, B + (c - B).rotate_90()));
        r = dis(o, c);
    }
    circle(vector<vec<db>> a)
    {
        int n = a.size(), i, j, k;
        mt19937 rnd(75643);
        shuffle(all(a), rnd);
        *this = circle(a[0]);
        for (i = 1; i < n; i++) if (!cover(a[i]))
        {
            *this = circle(a[i]);
            for (j = 0; j < i; j++) if (!cover(a[j]))
            {
                *this = circle(a[i], a[j]);
                for (k = 0; k < j; k++) if (!cover(a[k])) *this = circle(a[i], a[j], a[k]);
            }
        }
    }
    circle(const vector<vec<ll>> &b)
    {
        vector<vec<db>> a(b.size());
        int n = a.size(), i, j, k;
        for (i = 0; i < a.size(); i++) a[i] = (vec<db>)b[i];
        *this = circle(a);
    }
    tmpl bool cover(const vec<T> &a) { return sgn(dis((vec<db>)a, o) - r) <= 0; }
};
tmpl struct segment
{
    vec<T> a, b;
    segment() { }
    segment(const vec<T> &o, const vec<T> &p) :a(o), b(p)
    {
        int s = sgn(a.x - b.x);
        if (s > 0 || !s && a.y > b.y) swap(a, b);
    }
    bool cover(const vec<T> &o) const { return sgn((o - a) * (b - a)) == 0 && sgn((o - a).dot(
        o - b)) <= 0; }
};
tmpl bool intersect(const segment<T> &m, const segment<T> &n)
{
    auto a = n.b - n.a, b = m.b - m.a;
    auto d = n.a - m.a;
    if (sgn(n.b.x - m.a.x) < 0 || sgn(m.b.x - n.a.x) < 0) return 0;
    if (sgn(max(n.a.y, n.b.y) - min(m.a.y, m.b.y)) < 0 || sgn(max(m.a.y, m.b.y) - min(n.a.y, n
        .b.y)) < 0) return 0;
    return sgn(b * d) * sgn((n.b - m.a) * b) >= 0 && sgn(a * d) * sgn((m.b - n.a) * a) <= 0;
}
tmpl bool intersect(const segment<T> &m, const line<T> &n) { return sgn(n.d * (m.a - n.o)) *
    sgn(n.d * (m.b - n.o)) <= 0; }
tmpl bool intersect(const line<T> &n, const segment<T> &m) { return intersect(m, n); }
tmpl db dis(const vec<T> &o, const segment<T> &l)
{
    if (sgn((l.b - l.a).dot(o - l.a)) < 0 || sgn((l.a - l.b).dot(o - l.b)) < 0) return min(dis
```

```
            (o, l.a), dis(o, l.b));
    return dis(o, line(l.a, l.b));
}
tmpl db dis(const segment<T> &l, const vec<T> &o) { return dis(o, l); }
tmpl struct polygon
{
    vector<vec<T>> p;
    polygon(const vector<vec<T>> &a = { }) :p(a) { }
    db peri() const//周长
    {
        int i, n = p.size();
        if (n == 0) return 0;
        db C = (p[n - 1] - p[0]).len();
        for (i = 1; i < n; i++) C += (p[i - 1] - p[i]).len();
        return C;
    }
    db area() const { return area2() * 0.5; }//面积
    T area2() const//两倍面积
    {
        int i, n = p.size();
        if (n == 0) return 0;
        T S = p[n - 1] * p[0];
        for (i = 1; i < n; i++) S += p[i - 1] * p[i];
        return abs(S);
    }
    int cover(const vec<T> &o) const//点是否在多边形内，-1 外 0 上 1 内
    {
        static mt19937 rnd(75643);
        static uniform_int_distribution<ll> gen(1.2e9, 2e9);
        vec<T> t;
        t.x = gen(rnd); t.y = gen(rnd);
        segment<T> s(o, t);
        int i, n = p.size(), r = 0;
        for (i = 0; i < n; i++)
        {
            if (segment(p[i], p[(i + 1) % n]).cover(o)) return 0;
            r ^= intersect(s, segment(p[i], p[(i + 1) % n]));
        }
        return r ? 1 : -1;
    }
};
tmpl struct convex : polygon<T>
{
    convex(vector<vec<T>> a)
    {
        auto &p = this->p;
        int n = a.size(), i;
        if (!n) return;
        p = a;
        for (i = 1; i < n; i++) if (p[i].x < p[0].x || p[i].x == p[0].x && p[i].y < p[0].y)
            swap(p[0], p[i]);
        a.resize(0); a.reserve(n);
        for (i = 1; i < n; i++) if (p[i] != p[0]) a.push_back(p[i] - p[0]);
        sort(all(a));
        for (i = 0; i < a.size(); i++) a[i] += p[0];
        vec<T> *st = p.data() - 1;
        int tp = 1;
```

```
        for (auto &v : a)
        {
            while (tp > 1 && sgn((st[tp] - st[tp - 1]) * (v - st[tp - 1])) <= 0) --tp;
            st[++tp] = v;
        }
        p.resize(tp);
    }
    int cover(const vec<T> &o) const//点是否在凸包内，-1 外 0 上 1 内
    {
        const auto &p = this->p;
        if (sgn(o.x - p[0].x) < 0 || sgn(o.x - p[0].x) == 0 && sgn(o.y - p[0].y) < 0) return
            -1;
        if (o == p[0]) return 0;
        if (p.size() == 1) return -1;
        int tmp = sgn((o - p[0]) * (p.back() - p[0]));
        if (tmp == 0) return sgn(dis2(o, p[0]) - dis2(p.back(), p[0])) <= 0 ? 0 : -1;
        if (tmp < 0 || p.size() == 2) return -1;
        int x = upper_bound(1 + all(p), o, [&](const vec<T> &a, const vec<T> &b) { return sgn((
            a - p[0]) * (b - p[0])) > 0; }) - p.begin() - 1;
        tmp = sgn((o - p[x]) * (p[x + 1] - p[x]));
        if (tmp > 0) return -1;
        return tmp < 0;
    }
    convex<T> operator+(const convex<T> &A) const
    {
        auto &p = this->p;
        int n = p.size(), m = A.p.size(), i, j;
        vector<vec<T>> a(n), b(m), c;
        for (i = 0; i + 1 < n; i++) a[i] = p[i + 1] - p[i];
        a[n - 1] = p[0] - p[n - 1];
        for (i = 0; i + 1 < m; i++) b[i] = A.p[i + 1] - A.p[i];
        b[m - 1] = A.p[0] - A.p[m - 1];
        c.reserve(n + m);
        c.push_back(p[0] + A.p[0]);
        for (i = j = 0; i < n && j < m;)
        {
            int t = sgn(a[i] * b[j]);
            if (t == 0) c.push_back(c.back() + a[i] + b[j]), ++i, ++j;
            else c.push_back(c.back() + (t > 0 ? a[i++] : b[j++]));
        }
        while (i < n) c.push_back(c.back() + a[i++]);
        while (j < m) c.push_back(c.back() + b[j++]);
        c.pop_back();
        convex<T> t({ });
        t.p = c;
        return t;
    }
};
tmpl struct half_plane//默认左侧
{
    vec<T> o, d;
    operator half_plane<ll>() const { return {(vec<ll>)o, (vec<ll>)(o + d)}; }
    operator half_plane<db>() const { return {(vec<db>)o, (vec<db>)(o + d)}; }
    half_plane() { }
    half_plane(const vec<T> &a, const vec<T> &b) :o(a), d(b - a) { }
    bool operator<(const half_plane<T> &a) const
    {
```

```
        int p = quad(d), q = quad(a.d);
        if (p != q) return p < q;
        p = sgn(d * a.d);
        if (p) return p > 0;
        return sgn(d * (a.o - o)) > 0;
    }
};
tmpl ostream &operator<<(ostream &cout, half_plane<T> &m) { return cout << m.o << " | " << m.d
    ; }
tmpl vec<db> intersect(const half_plane<T> &m, const half_plane<T> &n)
{
    if (!sgn(m.d * n.d))
    {
        if (!sgn(m.d * (n.o - m.o))) return apos;
        return npos;
    }
    return (vec<db>)m.o + (n.o - m.o) * n.d / (db)(m.d * n.d) * (vec<db>)m.d;
}
const db inf = 1e18;
convex<db> intersect(vector<half_plane<db>> a)
{
    db I = inf;
    a.push_back({{-I, -I}, {I, -I}});
    a.push_back({{I, -I}, {I, I}});
    a.push_back({{I, I}, {-I, I}});
    a.push_back({{-I, I}, {-I, -I}});
    sort(all(a));
    int n = a.size(), i, h = 0, t = -1;
    vector<half_plane<db>> q(n);
    vector<vec<db>> p(n);
    for (i = 0; i < n; i++) if (i == n - 1 || sgn(a[i].d * a[i + 1].d))
    {
        auto x = (half_plane<db>)a[i];
        while (h < t && sgn((p[t - 1] - x.o) * x.d) >= 0) --t;
        while (h < t && sgn((p[h] - x.o) * x.d) >= 0) ++h;
        q[++t] = x;
        if (h < t) p[t - 1] = intersect(q[t - 1], q[t]);
    }
    while (h < t && sgn((p[t - 1] - q[h].o) * q[h].d) >= 0) --t;
    if (h == t) return convex<db>(vector<vec<db>>(0));
    p[t] = intersect(q[h], q[t]);
    return convex<db>(vector<vec<db>>(p.begin() + h, p.begin() + t + 1));
}
pair<ll, ll> __sqrt(ll x)
{
    ll y = sqrtl(x);
    return {y, y + (y * y < x)};
}
pair<db, db> __sqrt(db x)
{
    db y = sqrtl(x);
    return {y - eps, y + eps};
}
tmpl pair<int, int> closest_pair(const vector<vec<T>> &a)
{
    int n = a.size(), i;
    assert(n >= 2);
```

```
    vector<pair<vec<T>, int>> b(n);
    for (i = 0; i < n; i++) b[i] = {a[i], i};
    sort(all(b), [&](auto u, auto v) {
        if (u.first.x != v.first.x) return u.first.x < v.first.x;
        return u.first.y < v.first.y;
    });
    tuple<T, int, int> ans = {dis2(a[0], a[1]), 0, 1};
    set<pair<T, int>> s;
    int j = 0;
    for (auto [v, i] : b)
    {
        auto [x, y] = v;
        T d = __sqrt(get<0>(ans)).first;
        if (d == 0) break;
        for (auto it = s.lower_bound({y - d, 0}); it != s.end() && it->first <= y + d; ++it)
            cmin(ans, tuple{dis2(a[it->second], v), i, it->second});
        s.emplace(v.y, i);
        while (b[j].first.x < v.x - d) s.erase({b[j].first.y, b[j].second}), ++j;
    }
    return {get<1>(ans), get<2>(ans)};
}
tmpl pair<int, int> furthest_pair(const vector<vec<T>> &a)
{
    int n = a.size(), i, j;
    assert(n >= 2);
    auto b = convex(a).p;
    int m = b.size();
    if (m == 1) return {0, 1};
    b.push_back(b[0]);
    tuple<T, int, int> ans{dis2(b[0], b[1]), 0, 1};
    for (i = 0, j = 1; i < m; i++)
    {
        while (abs((b[i + 1] - b[i]) * (b[j] - b[i])) < abs((b[i + 1] - b[i]) * (b[(j + 1) % m]
             - b[i]))) j = (j + 1) % m;
        cmax(ans, tuple{dis2(b[i], b[j]), i, j});
        cmax(ans, tuple{dis2(b[i + 1], b[j]), i + 1, j});
    }
    auto [_, j1, j2] = ans;
    int i1, i2;
    for (i1 = 0; i1 < n; i1++) if (a[i1] == b[j1]) break;
    for (i2 = 0; i2 < n; i2++) if (i2 != i1 && a[i2] == b[j2]) break;
    return {i1, i2};
}
tmpl array<vec<db>, 4> rectangle_cover(const vector<vec<T>> &a)
{
    const auto &p = convex(a).p;
    int n = p.size(), m = n * 4, i, j, k, l;
    if (n <= 2) return {0, 0, 0, 0};
    vector<vec<T>> b(m);
    for (i = 0; i < m; i++) b[i] = p[i % n];
    tuple<db, int, int, int, int> tmp{inf, 0, 0, 0, 0};
    for (i = j = k = l = 0; i < n * 2; i++)
    {
        cmax(j, i + 1);
        auto d = b[i + 1] - b[i];
        while (d.dot(b[j] - b[i]) < d.dot(b[j + 1] - b[i])) ++j;
        while (j > i && d.dot(b[j] - b[i]) < d.dot(b[j - 1] - b[i])) --j;
```

```
        cmax(k, j);
        while (abs(d * (b[k] - b[i])) < abs(d * (b[k + 1] - b[i]))) ++k;
        while (k > j && abs(d * (b[k] - b[i])) < abs(d * (b[k - 1] - b[i]))) --k;
        cmax(l, k);
        while (d.dot(b[l] - b[i]) > d.dot(b[l + 1] - b[i])) ++l;
        while (l > k && d.dot(b[l] - b[i]) > d.dot(b[l - 1] - b[i])) --l;
        assert(l + 1 < m);
        if (i >= n) cmin(tmp, tuple{(db)(b[j] - b[l]).dot(d) * abs((b[k] - b[i]) * d) / d.len2
            (), i, j, k, l});
    }
    tie(ignore, i, j, k, l) = tmp;
    auto d = b[i + 1] - b[i], rd = d.rotate_90();
    line l1(b[i], b[i] + d), l2(b[j], b[j] + rd), l3(b[k], b[k] + d), l4(b[l], b[l] + rd);
    return {intersect(l1, l2), intersect(l2, l3), intersect(l3, l4), intersect(l4, l1)};
}
tmpl vector<line<T>> convex_up(vector<line<T>> a)
{
    for (auto &t : a) t.d.y = -t.d.y;
    a = convex_down(a);
    for (auto &t : a) t.d.y = -t.d.y;
    return a;
}
tmpl vector<line<T>> convex_down(vector<line<T>> a)
{
    sort(all(a), [&](const auto &u, const auto &v) {
        int t = sgn(u.d * v.d);
        return t ? t > 0 : sgn((u.o - v.o) * v.d) > 0;
    });
    vector<line<T>> b;
    int tp = -1;
    for (auto t : a)
    {
        while (tp >= 0 && sgn(b[tp].d * t.d) == 0) --tp, b.pop_back();
        while (tp >= 1 && sgn((up<T>)((b[tp].o - t.o) * b[tp].d) * (t.d * b[tp - 1].d) - (up<T
            >)((b[tp - 1].o - t.o) * b[tp - 1].d) * (t.d * b[tp].d)) <= 0)
            --tp, b.pop_back();
        ++tp; b.push_back(t);
    }
    return b;
}
tmpl vector<vec<T>> convex_down(vector<vec<T>> a)
{
    sort(all(a), [&](const auto &u, const auto &v) {
        int t = sgn(u.x - v.x);
        if (t) return t < 0;
        return u.y > v.y;
    });
    vector<vec<T>> b;
    int tp = -1;
    for (auto t : a)
    {
        while (tp >= 0 && sgn(b[tp].x - t.x) == 0) --tp, b.pop_back();
        while (tp >= 1 && sgn((t - b[tp]) * (t - b[tp - 1])) >= 0)
            --tp, b.pop_back();
        ++tp; b.push_back(t);
    }
    return b;
```

```
}
tmpl vector<vec<T>> convex_up(vector<vec<T>> a)
{
    for (auto &t : a) t.d.y = -t.d.y;
    a = convex_down(a);
    for (auto &t : a) t.d.y = -t.d.y;
    return a;
}
tmpl vector<vec<db>> to_vec(const vector<line<T>> &a)
{
    int n = a.size(), i;
    vector<vec<db>> b(n - 1);
    for (i = 0; i < n - 1; i++) b[i] = intersect(a[i], a[i + 1]);
    return b;
}
tmpl T find_max(const vector<vec<T>> &a, T kx, T ky)//要求函数凸
{
    vec<T> p = {kx, ky};
    int l = 0, r = a.size() - 1, mid;
    while (l < r)
    {
        mid = (l + r) / 2;
        if (a[mid].dot(p) < a[mid + 1].dot(p)) l = mid + 1;
        else r = mid;
    }
    return max({a[l].dot(p), a[0].dot(p), a.back().dot(p)});
}
tmpl T find_min(const vector<vec<T>> &a, T kx, T ky)//要求函数凸
{
    vec<T> p = {kx, ky};
    int l = 0, r = a.size() - 1, mid;
    while (l < r)
    {
        mid = (l + r) / 2;
        if (a[mid].dot(p) > a[mid + 1].dot(p)) l = mid + 1;
        else r = mid;
    }
    return min({a[l].dot(p), a[0].dot(p), a.back().dot(p)});
}
tmpl T max_subset_sum(const vector<vec<T>> &a)
{
    int n = a.size(), i;
    function<convex<T>(int, int)> dfs = [&](int l, int r) {
        if (l + 1 == r) return convex<T>({vec<T>(0, 0), a[l]});
        int mid = (l + r) / 2;
        return dfs(l, mid) + dfs(mid, r);
    };
    const auto &p = dfs(0, n).p;
    T ans = 0;
    for (auto t : p) ans = max(ans, t.len2());
    return ans;
}
template<class T> struct dynamic_convex//下凸
{
    set < vec<T>, decltype([](const vec<T> &a, const vec<T> &b) {
        return sgn(a.x - b.x) < 0;
    }) > s;
```

```
        void check(auto it)
        {
            decltype(it) jt, kt;
            if (it != s.begin())
            {
                jt = prev(it);
                if (jt != s.begin())
                {
                    kt = prev(jt);
                    while (sgn((*it - *kt) * (*jt - *kt)) >= 0)
                    {
                        s.erase(jt);
                        if (kt == s.begin()) break;
                        jt = kt--;
                    }
                }
            }
            jt = next(it);
            if (jt != s.end())
            {
                kt = next(jt);
                while (kt != s.end() && sgn((*kt - *it) * (*jt - *it)) >= 0)
                    s.erase(jt), jt = kt++;
            }
        }
        void insert(const vec<T> &p)
        {
            auto it = s.lower_bound(p);
            if (it == s.end() || sgn(s.begin()->x - p.x) > 0) it = s.insert(it, p);
            else if (sgn(it->x - p.x) == 0) cmin(it->y, p.y);
            else
            {
                auto l = *prev(it);
                if (sgn((p - l) * (*it - l)) > 0) it = s.insert(it, p);
            }
            check(it);
        }
        int cover(const vec<T> &p)//在凸壳区域以上返回 1，在凸壳上返回 0，其余返回 -1。
        {
            if (s.size() == 0) return -1;
            if (sgn(p.x - s.begin()->x) < 0 || sgn(p.x - s.rbegin()->x) > 0) return -1;
            auto it = s.lower_bound(p);
            if (sgn(it->x - p.x) == 0) return sgn(p.y - it->y);
            auto l = *prev(it);
            return sgn((*it - l) * (p - l));
        }
    };
#undef tmpl
}
using geo::vec, geo::line, geo::circle, geo::convex, geo::polygon, geo::half_plane;
using geo::eps, geo::pi, geo::segment, geo::read, geo::sgn, geo::dynamic_convex;
using Q = vec<ll>;
```

另附一份支持二分斜率的代码。这里维护的是 (x, y)，其中 $y = C + x^2$，因此 x 整体增加会影响 y。

```
bool FLAG = 0;
```

```
ll K;
struct Q
{
    ll x;
    mutable ll y;
    mutable decltype(set<Q>().begin()) r;
    Q operator-(const Q &o) const { return {x - o.x, y - o.y}; }
    lll operator*(const Q &o) const
    {
        return (lll)x * o.y - (lll)y * o.x;
    }
};
set<Q>::iterator END;
bool operator<(const Q &a, const Q &b)
{
    if (FLAG)
    {
        assert(b.x == 0 && b.y == 0);
        if (a.r == END) return 0;
        return (lll)a.x * K + a.y < (lll)a.r->x * K + a.r->y;
    }
    return a.x < b.x;
}
struct convex
{
    set<Q> s;
    ll tagx = 0, tagy = 0;
    void check(auto it)
    {
        decltype(it) jt, kt;
        if (it != s.begin())
        {
            jt = prev(it);
            if (jt != s.begin())
            {
                kt = prev(jt);
                while ((*it - *kt) * (*jt - *kt) <= 0)
                {
                    s.erase(jt);
                    if (kt == s.begin()) break;
                    jt = kt--;
                }
            }
        }
        jt = next(it);
        if (jt != s.end())
        {
            kt = next(jt);
            while (kt != s.end() && (*kt - *it) * (*jt - *it) <= 0)
                s.erase(jt), jt = kt++;
        }
    }
    void insert(Q p)
    {
        p.y -= tagy + p.x * p.x;
        p.x -= tagx;
        p.y += p.x * p.x;
```

```
        auto it = s.lower_bound(p);
        if (it == s.end() || s.begin()->x - p.x > 0) it = s.insert(it, p);
        else if (it->x == p.x) cmax(it->y, p.y);
        else
        {
            auto l = *prev(it);
            if ((p - l) * (*it - l) < 0) it = s.insert(it, p);
        }
        check(it);
        it->r = next(it);
        if (it != s.begin()) prev(it)->r = it;
    }
    void add(ll X, ll Y) { tagx += X; tagy += Y; }
    ll query(ll k)
    {
        k += tagx * 2;
        FLAG = 1; K = k; END = s.end();
        auto it = s.lower_bound({0, 0});
        FLAG = 0;
        return k * (it->x + tagx) + it->y + tagy - tagx * tagx;
    }
    void fun(ll &x, ll &y) const
    {
        y -= x * x;
        x += tagx;
        y += tagy + x * x;
    }
};
```

7 杂项

7.1 枚举大小为 k 的集合

思路：通过进位创造 1，再把一串 1 移到最后。

```
for (int s=(1<<k)-1,t;s<1<<n;t=s+(s&-s),s=(s&~t)>>__lg(s&-s)+1|t)
{}
```

7.2 min plus 卷积

计算 $c_i = \min_{j=0}^{i} a_j + b_{i-j}$。

要求 b 是凸的，即 $b_{i+1} - b_i$ 不降。

```
template <class T> vector<T> min_plus_convolution(const vector<T> &a, const vector<T> &b)
{
    int n = a.size(), m = b.size(), i;
    vector<T> c(n + m - 1);
    function<void(int, int, int, int)> dfs = [&](int l, int r, int ql, int qr) {
        if (l > r) return;
        int mid = l + r >> 1;
        while (ql + m <= l) ++ql;
        while (qr > r) --qr;
        int qmid = -1;
        c[mid] = inf;
        for (int i = ql; i <= qr; i++) if (mid - i >= 0 && mid - i < m && cmin(c[mid], a[i] + b[
            mid - i])) qmid = i;
        dfs(l, mid - 1, ql, qmid);
        dfs(mid + 1, r, qmid, qr);
    };
    dfs(0, n + m - 2, 0, n - 1);
    return c;
}
```

7.3 所有区间 GCD

需要自定义 fun，如 gcd，and，or。

```
template<class T> struct GCD
{
    vector<pair<int, T>> res;
    GCD(const vector<T> &a) :res(n)
    {
        int n = a.size(), i, j;
        vector<ll> v(n);
        vector<int> l(n);
        for (i = 0; i < n; i++)
        {
            for (v[i] = a[i], j = l[i] = i; j >= 0; j = l[j] - 1)
            {
                v[j] = fun(v[j], a[i]);
                while (l[j] && fun(a[i], v[l[j] - 1]) == fun(a[i], v[j])) l[j] = l[l[j] - 1];
                //[l[j]..j,i]区间内的值求 fun 均为 v[j]
            }
        }
```

```
    }
};
```

7.4 整体二分（区间 k-th）

$O((n+q)\log a)$，$O(n+q)$。

```
struct cz
{
    int x, y, kth, pos, typ;
};
cz q[M], st1[M], st2[M];
int a[N], b[N], d[N], ans[N], s[N];
int n, m, t1, t2, i, j, c, gs;
int lb(int x)
{
    return x & (-x);
}
void add(int x, int y)
{
    for (; x <= n; x += lb(x)) s[x] += y;
}
int sum(int x)
{
    int ans = 0;
    for (; x; x -= lb(x)) ans += s[x];
    return ans;
}
void ztef(int ql, int qr, int l, int r)
{
    if (ql > qr) return;
    int mid = l + r >> 1, i, midd;
    t1 = t2 = 0;
    if (l == r)
    {
        for (i = ql; i <= qr; i++) if (q[i].typ) ans[q[i].pos] = d[l];
        return;
    }
    for (i = ql; i <= qr; i++) if (q[i].typ)
    {
        midd = sum(q[i].y) - sum(q[i].x - 1);
        if (midd >= q[i].kth) st1[++t1] = q[i]; else
        {
            st2[++t2] = q[i];
            st2[t2].kth -= midd;
        }
    }
    else if (q[i].pos <= mid)
    {
        add(q[i].x, 1);
        st1[++t1] = q[i];
    }
    else st2[++t2] = q[i];
    for (i = 1; i <= t1; i++) if (!st1[i].typ) add(st1[i].x, -1);
    for (i = 1; i <= t1; i++) q[i + ql - 1] = st1[i];
    midd = ql + t1 - 1;
    for (i = 1; i <= t2; i++) q[i + midd] = st2[i];
```

```
    ztef(ql, midd, l, mid); ztef(midd + 1, qr, mid + 1, r);
}
int main()
{
    cin >> n >> m;
    for (i = 1; i <= n; i++)
    {
        cin >> a[i];
        b[i] = a[i];
    }
    sort(b + 1, b + n + 1);
    d[gs = 1] = b[1];
    for (i = 2; i <= n; i++) if (b[i] != b[i - 1]) d[++gs] = b[i];
    for (i = 1; i <= n; i++) a[i] = lower_bound(d + 1, d + gs + 1, a[i]) - d;
    for (i = 1; i <= n; i++)
    {
        q[i].x = i; q[i].pos = a[i]; q[i].typ = 0;
    }
    for (i = 1; i <= m; i++)
    {
        cin >> q[i + n].x >> q[i + n].y >> q[i + n].kth;
        q[i + n].pos = i; q[i + n].typ = 1;
    }
    ztef(1, n + m, 1, gs);
    for (i = 1; i <= m; i++) printf("%d\n", ans[i]);
}
```

7.5 高精度

除法和取模有点问题，但 gcd 是对的。

```
struct bigint;
int cmp(const bigint &a, const bigint &b);
struct bigint
{
    using ull = unsigned long long;
    using lll = unsigned __int128;
    const static ull sign = 1llu << 63;
    const static lll p = 4'179'340'454'199'820'289;
    const static lll g = 3;
    const static ull base = 1e6;
    const static int output_base = 10;
    const static int length = round(log(bigint::base) / log(output_base));
    static_assert(output_base == 10 || output_base == 16, "output base must be 10 or 16");
    static_assert(round(pow(output_base, length)) == base);
    const static int N = 1 << 23;
    static int r[N];
    static lll w[N];
    bool neg;
    vector<ull> a;
private:
    static lll ksm(lll x, ull y)
    {
        lll r = 1;
        while (y)
        {
            if (y & 1) r = r * x % p;
```

```
        x = x * x % p; y >>= 1;
    }
    return r;
}
static void init(int n)
{
    static int pr = 0, pw = 0;
    if (pr == n) return;
    int b = __lg(n) - 1, i, j, k;
    for (i = 1; i < n; i++) r[i] = r[i >> 1] >> 1 | (i & 1) << b;
    if (pw < n)
    {
        for (j = 1; j < n; j = k)
        {
            k = j * 2;
            ull wn = ksm(g, (p - 1) / k);
            w[j] = 1;
            for (i = j + 1; i < k; i++) w[i] = w[i - 1] * wn % p;
        }
        pw = n;
    }
    pr = n;
}
static void dft(vector<lll> &a, int o = 0)
{
    int n = a.size(), i, j, k;
    lll y, *f, *g, *wn, *A = a.data();
    init(n);
    for (i = 1; i < n; i++) if (i < r[i]) swap(A[i], A[r[i]]);
    static const int T = 12;
    static_assert(T + 2 <= numeric_limits<lll>::max() / (p * p));
    for (k = 1; k < n; k *= 2)
    {
        wn = w + k;
        for (i = 0; i < n; i += k * 2)
        {
            f = A + i; g = A + i + k;
            for (j = 0; j < k; j++)
            {
                y = g[j] * wn[j] % p;
                g[j] = f[j] + p - y;
                f[j] += y;
            }
        }
        if (__lg(n / k) % T == 1) for (lll &x : a) x %= p;
    }
    if (o)
    {
        y = ksm(n, p - 2);
        for (lll &x : a) x = x * y % p;
        reverse(1 + all(a));
    }
}
ull &operator[](const int &x) { return a[x]; }
const ull &operator[](const int &x) const { return a[x]; }
static void plus_by(vector<ull> &a, const vector<ull> &b)
{
```

```
        int n = a.size(), m = b.size(), i, j;
        cmax(n, m);
        a.resize(++n);
        for (i = 0; i < m; i++) if ((a[i] += b[i]) >= base) a[i] -= base, ++a[i + 1];
        for (i = m; i < n && a[i] >= base; i++) a[i] -= base, ++a[i + 1];
        if (a[n - 1] == 0) a.pop_back();
    }
    static void minus_by(vector<ull> &a, const vector<ull> &b)
    {
        int n = a.size(), m = b.size(), i, j;
        for (i = 0; i < m; i++) if (!(a[i] & sign) && a[i] >= b[i]) a[i] -= b[i];
        else --a[i + 1], a[i] += base - b[i];
        for (; i < n && (a[i] & sign); i++) a[i] += base, --a[i + 1];
        while (a.size() > 1 && !a.back()) a.pop_back();
    }
    static bool less(const vector<ull> &a, const vector<ull> &b)
    {
        if (a.size() != b.size()) return a.size() < b.size();
        for (int i = a.size() - 1; i >= 0; i--) if (a[i] != b[i]) return a[i] < b[i];
        return 0;
    }
    static int cal(int x) { return 1 << __lg(max(x, 1) * 2 - 1); }
public:
    bigint &operator+=(const bigint &o)
    {
        if (neg == o.neg) plus_by(a, o.a);
        else if (neg)
        {
            if (less(o.a, a)) minus_by(a, o.a);
            else
            {
                neg = 0;
                auto t = o.a;
                swap(a, t);
                minus_by(a, t);
            }
        }
        else
        {
            if (less(a, o.a))
            {
                neg = 1;
                auto t = o.a;
                swap(a, t);
                minus_by(a, t);
            }
            else minus_by(a, o.a);
        }
        return *this;
    }
    bigint &operator-=(const bigint &o)
    {
        neg ^= 1;
        *this += o;
        neg ^= 1;
        if (a == vector<ull>{0}) neg = 0;
        return *this;
```

```
	}
	bigint &operator*=(const bigint &o)
	{
		neg ^= o.neg;
		int n = a.size(), m = o.a.size(), i, j;
		assert(min(n, m) <= p / ((base - 1) * (base - 1)));
		if (min(n, m) <= 64 && 0)
		{
			vector<ull> c(n + m);
			for (i = 0; i < n; i++) for (j = 0; j < m; j++) c[i + j] += a[i] * o[j];
			for (i = 0; i < n + m - 1; i++)
			{
				c[i + 1] += c[i] / base;
				c[i] %= base;
			}
			swap(a, c);
			while (a.size() > 1 && !a.back()) a.pop_back();
			if (a == vector<ull>{0}) neg = 0;
			return *this;
		}
		int len = cal(n + m);
		vector<lll> f(len), g(len);
		copy_n(a.begin(), n, f.begin());
		copy_n(o.a.begin(), m, g.begin());
		dft(f); dft(g);
		for (i = 0; i < len; i++) f[i] = f[i] * g[i] % p;
		dft(f, 1);
		a.resize(n + m);
		copy_n(f.begin(), n + m - 1, a.begin());
		for (i = n + m - 2; i >= 0; i--)
		{
			a[i + 1] += a[i] / base;
			a[i] %= base;
		}
		for (i = 0; i < n + m - 1; i++)
		{
			a[i + 1] += a[i] / base;
			a[i] %= base;
		}
		while (a.size() > 1 && !a.back()) a.pop_back();
		if (a == vector<ull>{0}) neg = 0;
		return *this;
	}
	bigint &operator/=(long long x)//to zero
	{
		if (x < 0) x = -x, neg ^= 1;
		for (int i = a.size() - 1; i; i--)
		{
			a[i - 1] += a[i] % x * base;
			a[i] /= x;
		}
		a[0] /= x;
		while (a.size() > 1 && !a.back()) a.pop_back();
		if (a == vector<ull>{0}) neg = 0;
		return *this;
	}
	bigint operator+(bigint o) const { return o += *this; }
```

```
    bigint operator-(bigint o) const { o -= *this; if (o.a != vector<ull>{0}) o.neg ^= 1; return o
        ; }
    bigint operator*(bigint o) const { return o *= *this; }
    bigint operator/(long long x) const { auto res = *this; return res /= x; }
    long long operator%(long long x) const
    {
        bool flg = neg;
        if (x < 0) flg ^= 1, x = -x;
        ull res = 0;
        for (int i = (base % x == 0 ? 0 : a.size() - 1); i >= 0; i--) res = (res * base + a[i]) %
            x;
        return (long long)res * (flg ? -1 : 1);
    }
    bigint(long long x = 0) :neg(0)
    {
        if (x < 0) x = -x, neg = 1;
        a.push_back(x % base);
        while (x /= base) a.push_back(x % base);
    }
    bool operator<(const bigint &o) const { return cmp(*this, o) < 0; }
    bool operator>(const bigint &o) const { return cmp(*this, o) > 0; }
    bool operator<=(const bigint &o) const { return cmp(*this, o) <= 0; }
    bool operator>=(const bigint &o) const { return cmp(*this, o) >= 0; }
    bool operator==(const bigint &o) const { return cmp(*this, o) == 0; }
    bool operator!=(const bigint &o) const { return cmp(*this, o) != 0; }
};
int cmp(const bigint &a, const bigint &b)
{
    if (a.neg != b.neg) return a.neg ? -1 : 1;
    if (a.neg) return -cmp(b, a);
    if (a.a.size() != b.a.size()) return a.a.size() < b.a.size() ? -1 : 1;
    for (int i = a.a.size() - 1; i >= 0; i--) if (a.a[i] != b.a[i]) return a.a[i] < b.a[i] ? -1 :
        1;
    return 0;
}
istream &operator>>(istream &cin, bigint &x)
{
    x.neg = 0;
    x.a.clear();
    string s;
    cin >> s;
    const static int length = bigint::length;
    static int mp[128], _ = [&]() {
        for (int i = '0'; i <= '9'; i++) mp[i] = i - '0';
        for (int i = 'a'; i <= 'z'; i++) mp[i] = i - 'a' + 10;
        for (int i = 'A'; i <= 'Z'; i++) mp[i] = i - 'A' + 10;
        return 0;
    }();
    reverse(all(s));
    if (s.back() == '-') x.neg = 1, s.pop_back();
    ull base = 1;
    for (int i = 0; i < s.size(); i++)
    {
        if (i % length == 0) x.a.push_back(0), base = 1;
        x.a.back() += mp[s[i]] * base;
        base *= bigint::output_base;
    }
```

```
    return cin;
}
ostream &operator<<(ostream &cout, const bigint &x)
{
    if (x.neg) cout << "-";
    const static int length = bigint::length;
    if (bigint::output_base == 10)
    {
        cout << setfill('0') << x.a.back();
        for (int i = (int)x.a.size() - 2; i >= 0; i--) cout << setw(length) << x.a[i];
    }
    else if (bigint::output_base == 16)
    {
        cout << hex << uppercase << setfill('0') << x.a.back();
        for (int i = (int)x.a.size() - 2; i >= 0; i--) cout << setw(length) << x.a[i];
        cout << dec;
    }
    else assert(0);
    return cout;
}
bigint abs(bigint x)
{
    x.neg = 0;
    return x;
}
bigint gcd(bigint x, bigint y)
{
    x.neg = y.neg = 0;
    if (x == bigint(0)) return y;
    if (y == bigint(0)) return x;
    int c1 = 0, c2 = 0;
    while (x % 2 == 0) x /= 2, ++c1;
    while (y % 2 == 0) y /= 2, ++c2;
    cmin(c1, c2);
    if (x > y) swap(x, y);
    while (x != y)
    {
        y -= x;
        y /= 2;
        while (y % 2 == 0) y /= 2;
        if (x > y) swap(x, y);
    }
    while (c1--) y *= bigint(2);
    return y;
}
bigint::lll bigint::w[bigint::N];
int bigint::r[bigint::N];
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(0);
    int T; cin >> T;
    while (T--)
    {
        bigint a, b;
        cin >> a >> b;
        cout << (a *= b) << '\n';
```

```
    }
}
```

7.6 分散层叠算法（Fractional Cascading）

$O(n+q(k+\log n))$，$O(n)$。

给出 k 个长度为 n 的有序数组。

现在有 q 个查询：给出数 x，分别求出每个数组中大于等于 x 的最小的数（非严格后继）。

若后继不存在，则定义为 0。你需要在线地回答这些询问。

```
int a[M][N], b[M][N << 1], c[M][N << 1][2], len[M], ans[M];
int n, m, qs, p, q, d, i, j, x, y, la;
int main()
{
    cin >> n >> m >> qs >> d;
    for (j = 1; j <= m; j++) for (i = 0; i < n; i++) cin >> a[j][i];
    for (j = 1; j <= m; j++) a[j][n] = inf + j; ++n;
    for (i = 0; i < n; i++) b[m][i] = a[m][i], c[m][i][0] = i;
    len[m] = n;
    for (j = m - 1; j; j--)
    {
        p = 0, q = 1;
        while (p < n && q < len[j + 1])
            if (a[j][p] < b[j + 1][q]) b[j][len[j]] = a[j][p], c[j][len[j]][0] = p++, c[j][len[j
                ]++][1] = q;
            else b[j][len[j]] = b[j + 1][q], c[j][len[j]][0] = p, c[j][len[j]++][1] = q, q += 2;
        while (p < n) b[j][len[j]] = a[j][p], c[j][len[j]][0] = p++, c[j][len[j]++][1] = q;
        while (q < len[j + 1]) b[j][len[j]] = b[j + 1][q], c[j][len[j]][0] = p, c[j][len[j]++][1]
            = q, q += 2;
    }
    for (int ii = 1; ii <= qs; ii++)
    {
        cin >> x; x ^= la;
        y = lower_bound(b[1], b[1] + len[1], x) - b[1];
        ans[1] = a[1][c[1][y][0]]; y = c[1][y][1];//下标是c[1][y][0]
        for (j = 2; j <= m; j++)
        {
            if (y && b[j][y - 1] >= x) --y;
            ans[j] = a[j][c[j][y][0]];//下标是c[j][y][0]
            y = c[j][y][1];
        }
        la = 0;
        for (i = 1; i <= m; i++) la ^= ans[i] > inf ? 0 : ans[i];
        if (ii % d == 0) cout << la << '\n';
    }
}
```

7.7 圆上整点（二平方和定理）

$x^2+y^2=n$ 的整数解的数目的四分之一 $f(n)$ 是积性数论函数，且对于**素数幂**有：$f(p^k)=$

$$\begin{cases} 1 & p=2 \\ k+1 & p\equiv 1 \pmod 4 \\ (k+1) \bmod 2 & p\equiv 3 \pmod 4 \end{cases}$$

以下代码给出所有的非负整数解。注意非负整数解个数不等于 $f(n)$。

时间复杂度为 $O(n^{\frac{1}{4}}+f(n))$，其中 $O(n^{\frac{1}{4}})$ 是 pollard-rho 的复杂度。

$f(n)$ 的量级不好分析，但不会超过约数个数 $O(d(n))\approx O(n^{\frac{1}{3}})$，且可以推测不能达到。实践上 10^{18} 以内 $f(n)\leq 3072$。

```
namespace pr
{
    typedef long long ll;
    typedef __int128 lll;
    typedef pair<ll, int> pa;
    ll ksm(ll x, ll y, const ll p)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=(lll)r*x%p;
            x=(lll)x*x%p; y>>=1;
        }
        return r;
    }
    namespace miller
    {
        const int p[7]={2, 3, 5, 7, 11, 61, 24251};
        ll s, t;
        bool test(ll n, int p)
        {
            if (p>=n) return 1;
            ll r=ksm(p, t, n), w;
            for (int j=0; j<s&&r!=1; j++)
            {
                w=(lll)r*r%n;
                if (w==1&&r!=n-1) return 0;
                r=w;
            }
            return r==1;
        }
        bool prime(ll n)
        {
            if (n<2||n==46'856'248'255'981ll) return 0;
            for (int i=0; i<7; ++i) if (n%p[i]==0) return n==p[i];
            s=__builtin_ctz(n-1); t=n-1>>s;
            for (int i=0; i<7; ++i) if (!test(n, p[i])) return 0;
            return 1;
        }
    }
    using miller::prime;
    mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
    namespace rho
    {
        void nxt(ll &x, ll &y, ll &p) { x=((lll)x*x+y)%p; }
        ll find(ll n, ll C)
        {
            ll l, r, d, p=1;
            l=rnd()%(n-2)+2, r=l;
            nxt(r, C, n);
            int cnt=0;
            while (l^r)
```

```
        {
            p=(lll)p*llabs(l-r)%n;
            if (!p) return gcd(n, llabs(l-r));
            ++cnt;
            if (cnt==127)
            {
                cnt=0;
                d=gcd(llabs(l-r), n);
                if (d>1) return d;
            }
            nxt(l, C, n); nxt(r, C, n); nxt(r, C, n);
        }
        return gcd(n, p);
    }
    vector<pa> w;
    vector<ll> d;
    void dfs(ll n, int cnt)
    {
        if (n==1) return;
        if (prime(n)) return w.emplace_back(n, cnt), void();
        ll p=n, C=rnd()%(n-1)+1;
        while (p==1||p==n) p=find(n, C++);
        int r=1; n/=p;
        while (n%p==0) n/=p, ++r;
        dfs(p, r*cnt); dfs(n, cnt);
    }
    vector<pa> getw(ll n)
    {
        w=vector<pa>(0); dfs(n, 1);
        if (n==1) return w;
        sort(w.begin(), w.end());
        int i, j;
        for (i=1, j=0; i<w.size(); i++) if (w[i].first==w[j].first) w[j].second+=w[i].second;
            else w[++j]=w[i];
        w.resize(j+1);
        return w;
    }
    void dfss(int x, ll n)
    {
        if (x==w.size()) return d.push_back(n), void();
        dfss(x+1, n);
        for (int i=1; i<=w[x].second; i++) dfss(x+1, n*=w[x].first);
    }
    vector<ll> getd(ll n)
    {
        getw(n); d=vector<ll>(0); dfss(0, 1);
        sort(d.begin(), d.end());
        return d;
    }
  }
  using rho::getw, rho::getd;
  using miller::prime;
}
using pr::getw, pr::getd, pr::prime;
lll roundiv(lll x, lll y)
{
  return x>=0?(x+y/2)/y:(x-y/2)/y;
```

```
}
struct G
{
    lll x, y;
    G operator~() const { return {x, -y}; }
    lll len2() const { return x*x+y*y; }
    G operator+(const G &o) const { return {x+o.x, y+o.y}; }
    G operator-(const G &o) const { return {x-o.x, y-o.y}; }
    G operator*(const G &o) const { return {x*o.x-y*o.y, x*o.y+y*o.x}; }
    G operator/(const G &o) const
    {
        G t=*this*~o;
        lll l=o.len2();
        return {roundiv(t.x, l), roundiv(t.y, l)};
    }
    G operator%(const G &o) const { return *this-*this/o*o; }
};
G gcd(G a, G b)
{
    if (a.len2()>b.len2()) swap(a, b);
    while (a.len2())
    {
        b=b%a;
        swap(a, b);
    }
    return b;
}
namespace cipolla
{
    typedef unsigned long long ull;
    typedef __uint128_t ll;
    ull p, w;
    struct Q
    {
        ulll x, y;
        Q operator*(const Q &o) const { return {(x*o.x+y*o.y%p*w)%p, (x*o.y+y*o.x)%p}; }
    };
    ull ksm(ulll x, ull y)
    {
        ulll r=1;
        while (y)
        {
            if (y&1) r=r*x%p;
            x=x*x%p; y>>=1;
        }
        return r;
    }
    Q ksm(Q x, ull y)
    {
        Q r={1, 0};
        while (y)
        {
            if (y&1) r=r*x;
            x=x*x; y>>=1;
        }
        return r;
    }
```

```
    ull mosqrt(ull x, ull P)//0<=x<P
    {
        if (x==0||P==2) return x;
        p=P;
        if (ksm(x, p-1>>1)!=1) return -1;
        ull y;
        mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
        do y=rnd()%p, w=((ulll)y*y+p-x)%p; while (ksm(w, p-1>>1)<=1);//not for p=2
        y=ksm({y, 1}, p+1>>1).x;
        if (y*2>p) y=p-y;//两解取小
        return y;
    }
}
using cipolla::mosqrt;
vector<pair<ll, ll>> two_sqr_sum(ll n)//只会返回非负解，按照字典序排序
{
    if (n<0) return { };
    if (n==0) return {{0, 0}};
    ll m=__lg(n&-n), d=1<<m/2, i;
    n>>=m;
    auto w=getw(n);
    vector<G> r((m&1)?vector{G{1, 1}}:vector{G{0, 1}, G{1, 0}});
    for (auto [p, k]:w) if (p%4==1)
    {
        vector<G> pw(k+1);
        pw[0]={1, 0};
        pw[1]=gcd(G(p, 0), G(mosqrt(p-1, p), 1));
        assert(pw[1].len2()==p);
        for (i=2; i<=k; i++) pw[i]=pw[i-1]*pw[1];
        vector<G> rr; rr.reserve(r.size()*(k+1));
        for (i=0; i<=k; i++)
        {
            G x=pw[i]*~pw[k-i];
            for (G y:r) rr.push_back(x*y);
        }
        swap(r, rr);
    }
    else
    {
        if (k%2) return { };
        k/=2;
        while (k--) d*=p;
    }
    vector<pair<ll, ll>> ans;
    ans.reserve(r.size());
    for (auto [x, y]:r) ans.push_back({abs((ll)x*d), abs((ll)y*d)});
    sort(all(ans));
    ans.resize(unique(all(ans))-ans.begin());
    return ans;
}
```

7.8 快速取模

```
__uint128_t brt=((__uint128_t)1<<64)/mod;
for(int i=1;i<=n;i++)
{
```

```
    ans*=i;
    ans=ans-mod*(brt*ans>>64);
    while(ans>=mod) ans-=mod;//可以替换为 if，但据说会变慢。如果循环展开则需要替换
}

struct barret{
    ll p,m; //p 表示上面的模数，m 为取模参数
    int c=0;
    inline void init(ll t){
        c=48+log2(t),p=t;
        m=(ll((ulll(1)<<c)/t));
    }
    friend inline ll operator % (ll n,const barret &d) { // get n % d
        return n-((ulll(n)*d.m)>>d.c)*d.p;
    }
}modp;
```

7.9 IO 优化

```
class fast_iostream
{
private:
#define SIGNED
    const static int MAXBF = 1 << 20; FILE *inf, *ouf;
    char *inbuf, *inst, *ined;
    char *oubuf, *oust, *oued;
    inline void _flush() { fwrite(oubuf, 1, oued - oust, ouf); }
    inline char _getchar() {
        if (inst == ined) inst = inbuf, ined = inbuf + fread(inbuf, 1, MAXBF, inf);
        return inst == ined ? EOF : *inst++;
    }
    inline void _putchar(char c) {
        if (oued == oust + MAXBF) _flush(), oued = oubuf;
        *oued++ = c;
    }
public:
    fast_iostream(FILE *_inf = stdin, FILE *_ouf = stdout)
        :inbuf(new char[MAXBF]), inf(_inf), inst(inbuf), ined(inbuf),
        oubuf(new char[MAXBF]), ouf(_ouf), oust(oubuf), oued(oubuf) {
    }
    ~fast_iostream() { _flush(); delete inbuf; delete oubuf; }
    fast_iostream &operator >> (char &c) {
        while (isspace(c = _getchar()));
        return *this;
    }
    fast_iostream &operator >> (string &s) {
        static char c;
        while (isspace(c = _getchar()));
        s = c;
        while (!isspace(c = _getchar())) s += c;
        return *this;
    }
    template <class Int>
    fast_iostream &operator >> (Int &n) {
        static char c;
#ifdef SIGNED
```

```
        bool neg = 0;
        while ((c = _getchar()) < '0' || c > '9') neg |= c == '-';
#else
        while ((c = _getchar()) < '0' || c > '9');
#endif
        n = c - '0';
        while ((c = _getchar()) >= '0' && c <= '9') n = n * 10 + c - '0';
#ifdef SIGNED
        if (neg) n = -n;
#endif
        return *this;
    }
    template <class Int>
    fast_iostream &operator << (Int n) {
        if (n < 0) _putchar('-'), n = -n; static char S[20]; int t = 0;
        do { S[t++] = '0' + n % 10, n /= 10; } while (n);
        for (int i = 0; i < t; ++i) _putchar(S[t - i - 1]);
        return *this;
    }
    fast_iostream &operator << (char c) { _putchar(c); return *this; }
    fast_iostream &operator << (const char *s) {
        for (int i = 0; s[i]; ++i) _putchar(s[i]); return *this;
    }
}fio;
```

7.10 手动开栈

一种写法是文件开头放，但部分 OJ 会失效。

```
#pragma comment(linker, "/STACK:102400000,102400000")
```

另一种写法是在 main 开头写，但必须以 exit(0) 结束程序。

以下两个应该有一个是对的，不对会 CE。

```
{
    static int OP = 0;
    if (OP++ == 0)
    {
        int size = 256 << 20; // 256MB
        char *p = (char *)malloc(size) + size;
        __asm__("movl %0, %%esp\n" :: "r"(p));
    }
}
{
    static int OP=0;
    if (OP++==0)
    {
        int size=128<<20;//128MB
        char* p=new char[size]+size;
        __asm__ __volatile__("movq %0, %%rsp\n""pushq $exit\n""jmp main\n"::"r"(p));
    }
}
```

7.11 德扑

solve 返回按照出现次数排序的 vector<int>（0 下标处为牌型），这样就可以字典序比较了。

```
struct Q
{
    int suit, rank;
    bool operator<(const Q &o) const { return pair{rank, suit}<pair{o.rank, o.suit}; }
    bool operator==(const Q &o) const { return pair{rank, suit}==pair{o.rank, o.suit}; }
};
auto solve=[&](vector<Q> a)
{
    vector<int> res;
    vector<int> cnt(15);
    for (auto [s, r]:a) ++cnt[r];
    sort(all(a));
    int i;
    bool is_flush=1, is_str=0;
    for (i=1; i<5; i++) is_flush&=a[i].suit==a[0].suit;
    is_str=*max_element(all(cnt))==1&&a[0].rank+4==a[4].rank;
    vector<int> b(6);
    for (i=1; i<6; i++) b[i]=a[i-1].rank;
    sort(1+all(b), [&](int x, int y)
        {
            return pair{cnt[x], x}>pair{cnt[y], y};
        });
    if (b==vector{0, 12, 3, 2, 1, 0}) is_str=1, b[1]=0;
    if (is_flush&&is_str) return b[0]=9, b;
    if (cnt[b[1]]==4) return b[0]=8, b;
    if (cnt[b[1]]==3&&cnt[b[4]]==2) return b[0]=7, b;
    if (is_flush) return b[0]=6, b;
    if (is_str) return b[0]=5, b;
    if (cnt[b[1]]==3) return b[0]=4, b;
    if (cnt[b[1]]==2&&cnt[b[3]]==2) return b[0]=3, b;
    if (cnt[b[1]]==2) return b[0]=2, b;
    return b;
};
auto turn=[&](string s)
{
    Q res=Q{"SHDC"s.find(s[0]), "23456789TJQKA"s.find(s[1])};
    return res;
};
```

7.12 约数个数表

n	n 前第一个质数	n 后第一个质数	$\max\{\omega(n)\}$	$\max\{d(n)\}$	$\pi(n)$
10^1	$n-3$	$n+1$	2	$d(6)=4$	4
10^2	$n-3$	$n+1$	3	$d(60)=12$	25
10^3	$n-3$	$n+13$	4	$d(840)=32$	168
10^4	$n-27$	$n+7$	5	$d(7560)=64$	1229
10^5	$n-9$	$n+3$	6	$d(83160)=128$	9592
10^6	$n-17$	$n+3$	7	$d(720720)=240$	$7.9\cdot10^4$
10^7	$n-9$	$n+19$	8	$d(8648640)=448$	$6.7\cdot10^5$
10^8	$n-11$	$n+7$	8	$d(73513440)=768$	$5.8\cdot10^6$
10^9	$n-63$	$n+7$	9	$d(735134400)=1344$	$5.1\cdot10^7$
10^{10}	$n-33$	$n+19$	10	$d(6983776800)=2304$	$4.6\cdot10^8$
10^{11}	$n-23$	$n+3$	10	$d(97772875200)=4032$	$4.2\cdot10^8$
10^{12}	$n-11$	$n+39$	11	$d(963761198400)=6720$	$3.8\cdot10^9$
10^{13}	$n-29$	$n+37$	12	$d(9316358251200)=10752$	$3.5\cdot10^{10}$
10^{14}	$n-27$	$n+31$	12	$d(97821761637600)=17280$	$3.3\cdot10^{11}$
10^{15}	$n-11$	$n+37$	13	$d(866421317361600)=26880$	$3\cdot10^{12}$
10^{16}	$n-63$	$n+61$	13	$d(8086598962041600)=41472$	$2.8\cdot10^{13}$
10^{17}	$n-3$	$n+3$	14	$d(74801040398884800)=64512$	
10^{18}	$n-11$	$n+3$	15	$d(897612484786617600)=103680$	
10^{19}	$n-39$	$n+51$	16	$d(9200527969062830400)=161280$	

7.13 NTT 质数

$p = r \times 2^k + 1$	r	k	g（最小原根）	位数
17	1	4	3	2
97	3	5	5	2
193	3	6	5	3
257	1	8	3	3
7681	15	9	17	4
12289	3	12	11	5
40961	5	13	3	5
65537	1	16	3	5
786433	3	18	10	6
5767169	11	19	3	7
7340033	7	20	3	7
23068673	11	21	3	8
104857601	25	22	3	9
167772161	5	25	3	9
469762049	7	26	3	9
998244353	119	23	3	10
1004535809	479	21	3	10
2013265921	15	27	31	10
2281701377	17	27	3	10
3221225473	3	30	5	10
75161927681	35	31	3	11
77309411329	9	33	7	11
206158430209	3	36	22	12
2061584302081	15	37	7	13
2748779069441	5	39	3	13
6597069766657	3	41	5	13
39582418599937	9	42	5	14
79164837199873	9	43	5	14
263882790666241	15	44	7	15
1231453023109121	35	45	3	16
1337006139375617	19	46	3	16
3799912185593857	27	47	5	16
4222124650659841	15	48	19	16
7881299347898369	7	50	6	16
31525197391593473	7	52	3	17
180143985094819841	5	55	6	18
1945555039024054273	27	56	5	19
4179340454199820289	29	57	3	19

7.14 公式

向上取整的整除分块 $[i, \lfloor \frac{n-1}{\lceil \frac{n}{i} \rceil - 1} \rfloor]$

n 个点 k 个连通块的生成树方案 $n^{k-2}\prod\limits_{i=1}^{k} siz_i$

(x,y) 曼哈顿距离 $\to (x+y,x-y)$ 切比雪夫距离

(x,y) 切比雪夫距离 $\to (\dfrac{x+y}{2},\dfrac{x-y}{2})$ 曼哈顿距离

Kummer's Theorem: $\binom{n+m}{n}$ 含 p ($p\in \text{prime}$) 的次数是 $n+m$ 在 p 进制下的进位数

$\ln(1-x^V)=-\sum\limits_{i\ge 1}\dfrac{x^{Vi}}{i}$

$x^{\bar{n}}=\sum\limits_{i} S_1(n,i)x^i$

$$\begin{cases} x\equiv a_1 \pmod{m_1} \\ x\equiv a_2 \pmod{m_2} \\ \cdots \\ x\equiv a_n \pmod{m_n} \end{cases}$$

$V-E+F=2$，$S=n+\frac{s}{2}-1$。（n 为内部，s 为边上）

用途：对于相邻的不相等的值，在中间画一条线（最外也画），连通块个数 $=1+E-V+$ 内部框个数

注意全都是不含矩形边界上的。

卡特兰三角：由 n 个 -1 和 m 个 1 组成一个序列，满足所有前缀和小于 k 的方案数（$k\le m\le n+k-1$）：$\binom{n+m}{m}-\binom{n+m}{m-k}$

贝尔数（划分集合方案数）EGF：$\exp(e^x-1)$，$B_n=\sum\limits_{i=0}^{n}S_2(n,i)$，伯努利数 EGF：$\dfrac{x}{e^x-1}$

$S_1(i,m)$ EGF：$\dfrac{(\sum\limits_{i\ge 0}\dfrac{x^i}{i})^m}{m!}$，$S_2(i,m)$ EGF：$\dfrac{(e^x-1)^m}{m!}$。$S_2(n,m)=[x^n]x^m(\prod\limits_{i=1}^{m}(1-ix))^{-1}$，$S_2(n,m)\equiv [n-m\&\lfloor\frac{m-1}{2}\rfloor=0] \pmod 2$。

多项式牛顿迭代：如果已知 $G(F(x))\equiv 0 \pmod{x^{2n}}$，$G(F_*(x))\equiv 0 \pmod{x^n}$，则有 $F(x)\equiv F_*(x)-\frac{G(F_*(x))}{G'(F_*(x))} \pmod{x^{2n}}$。求导时孤立的多项式视为常数。

$\int_0^1 t^a(1-t)^b\mathrm{d}t=\dfrac{a!b!}{(a+b+1)!}$，$\sum\limits_{i=0}^{n-1} i^{\underline{k}}=\dfrac{n^{\underline{k+1}}}{k+1}$

Burnside 引理：等价类数量为 $\sum\limits_{g\in G}\dfrac{X^g}{|G|}$，$X^g$ 表示 g 变换下不动点的数量。

Polya 定理：染色方案数为 $\sum\limits_{g\in G}\dfrac{m^{c(g)}}{|G|}$，其中 $c(g)$ 表示 g 变换下环的数量。

假设已经只保留了一个牛人酋长，其名字为 $A=a_1a_2\cdots a_l$。

假设王国旁边开了一座赌场，每单位时间（就称为“秒”吧）会有一个赌徒带着 1 铜币进入赌场。

赌场规则很简单：支付 x 铜币赌下一秒会唱出 y，如果猜对了就返还 nx 铜币，否则钱就没了。

每个赌徒会如下行动：支付 1 铜币赌下一秒会唱出 a_1，如果赌对了就支付得到的 n 铜币赌下一秒会唱出 a_2，如果还对了就支付得到的 n^2 铜币赌下一秒会唱出 a_3，等等，以此类推，最后支付 n^{l-1} 铜币赌下一秒会唱出 a_l。

一旦连续唱出了 $a_1a_2\cdots a_l$，赌场老板就会认为自己亏大了而关门，并驱散所有赌徒。

那么关门前发生了什么呢？以 $A=\{1,4,1,5,1,1,4,1\},n=5$ 为例：

- 最后一位赌徒拿着 5 铜币离开；- 倒数第三位赌徒拿着 5^3 铜币离开；- 倒数第八位赌徒拿着 5^8 铜币离开；- 其他所有赌徒空手而归。

我们可以发现 $1,3$ 恰好是原序列的所有 border 的长度，而且对于其他的名字也有这样的规律。

这时候最神奇的一步来了：由于这个赌博游戏是公平的，因此赌场应该期望下不赚不赔，因此关门时期望来了 $5+5^3+5^8$ 个赌徒，因此期望需要 $5+5^3+5^8$ 单位时间唱出这个名字。

同理，即可知道对于一般的 A，答案为：

$$\sum_{a_1a_2\cdots a_c=a_{l-c+1}a_{l-c+2}\cdots a_l} n^c$$

8 语言基础

8.1 Makefile

```
%:%.cpp %.in
	g++ $< -o $@ -std=c++20 -DLOCAL -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC -fsanitize=undefined
	./$@ < $@.in
```

8.2 debug.h

```cpp
template <class T, size_t size = tuple_size<T>::value> string to_dbg(T, string s = "") requires(not ranges::range<T>);
string to_dbg(auto x) requires requires(ostream &os) { os << x; }
{
    ostringstream os;
    os << x;
    return os.str();
}
string to_dbg(ranges::range auto x, string s = "") requires(not is_same_v<decltype(x), string>)
{
    for (auto xi : x) s += ", " + to_dbg(xi);
    return "[" + s.substr(2 * !!s.size()) + "]";
}
template <class T, size_t size> string to_dbg(T x, string s) requires(not ranges::range<T>)
{
    [&] <size_t... I>(index_sequence<I...>) { ((s += ", " + to_dbg(get<I>(x))), ...); } (make_index_sequence<size>());
    return "(" + s.substr(2 * !!s.size()) + ")";
}
#define dbg(...) cerr << __LINE__ << ": (" #__VA_ARGS__ ") = " << to_dbg(tuple(__VA_ARGS__)) << "\n"
```

8.3 初始代码

```cpp
#include "bits/stdc++.h"
using namespace std;
using ll = long long;
#define all(x) (x).begin(),(x).end()
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
}
```

抄写完 debug.h 后，补充于 `using namespace std;` 下一行：

```cpp
#ifdef LOCAL
#include "debug.h"
#endif
```

8.4 bitset

```
#include "bits/stdc++.h"
using namespace std;
bitset<10> f(12);
char s2[]="100101";
bitset<10> g(s2);
string s="100101";//reverse 了
bitset<10> h(s);
int main()
{
    for (int i=0;i<=9;i++) cout<<f.test(i);cout<<endl;
    for (int i=0;i<=9;i++) cout<<g.test(i);cout<<endl;
    for (int i=0;i<=9;i++) cout<<h.test(i);cout<<endl;
    cout<<h<<endl;
    foo.count();//1的个数
    foo.flip();//全部翻转
    foo.set();//变1
    foo.reset();//变0
    foo.to_string();
    foo.to_ulong();
    foo.to_ullong();
    foo._Find_first();
    foo._Find_next();
    //位运算: << 变大, >> 变小
}
```

输出:

```
0011000000
1010010000
1010010000
0000100101
```

8.5 pb_ds 和一些奇怪的用法

```
#pragma GCC optimize("Ofast")
#pragma GCC target("popcnt","sse3","sse2","sse","avx","sse4","sse4.1","sse4.2","ssse3","f16c","
    fma","avx2","xop","fma4")
#pragma GCC optimize("inline","fast-math","unroll-loops","no-stack-protector")
#include "bits/stdc++.h"
#include "ext/pb_ds/assoc_container.hpp"
#include "ext/pb_ds/tree_policy.hpp" //balanced tree
#include "ext/pb_ds/hash_policy.hpp" //hash table
#include "ext/pb_ds/priority_queue.hpp" //priority_queue
using namespace __gnu_pbds;
using namespace std;
template <typename T> using rbt = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
cc_hash_table<string,int>mp1;//拉链法
gp_hash_table<string,int>mp2;//查探法
rbt<int> s1,s2;//注意是不可重的
//null_type无映射(低版本g++为null_mapped_type)
//less<int>从小到大排序
//插入t.insert();
//删除t.erase();
//求有多少个数比 k 小:t.order_of_key(k);
```

```
//求树中第 k+1 小:t.find_by_order(k);
//a.join(b) b并入a, 前提是两棵树的 key 的取值范围不相交, b 会清空但迭代器没事, 如不满足会抛出异常。我
    听说复杂度是线性???
//a.split(v,b) key 小于等于 v 的元素属于 a, 其余的属于 b
template <typename T> using heap = __gnu_pbds::priority_queue<T, greater<T>, pairing_heap_tag>;
//join(priority_queue &other) //合并两个堆,other会被清空
//split(Pred prd,priority_queue &other) //分离出两个堆
//modify(point_iterator it,const key) //修改一个节点的值
int main()
{
    __builtin_clz();//前导 0
    __builtin_ctz();//后面的 0
    ios::sync_with_stdio(0);cin.tie(0);
    mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    cout<<fixed<<setprecision(15);
    rbtree::iterator it;
    string s="abc",t="dabce";
    boyer_moore_horspool_searcher S(all(s));
    if (search(all(t),S)!=t.end())
    {
        cout<<"find\n";
    }
    uniform_real_distribution<> a(1,2);
    numeric_limits<int>::max();
}
```

8.6 python 使用方法

注意事项：python 容易爆栈，且引用与赋值较为混乱。注意局部变量的 global 怎么写（如果需要修改全局内容）。

文件操作

```
fi = open("discuss.in", "r")
fo = open("discuss.out", "w")
n=int(fi.readline())
fo.write(str(ans))
```

类的构造，重载运算符

```
class Q:
    def __init__(self,x,y):
        self.x=x
        self.y=y
    def __add__(self,o):
        r=Q(self.x+o.x,self.y+o.y)
        return r
    def __sub__(self,o):
        r=Q(self.x-o.x,self.y-o.y)
        return r
    def __mul__(self,o):
        return self.x*o.y-self.y*o.x
    def __lt__(self,o):
        if self.x!=o.x:
            return self.x<o.x
        return self.y<o.y
n,m=map(int,input().split())
c=list(map(int,input().split()))
```

```
print(*c)
a=Q(0,0)
b=Q(1,1)
if a<b-a:
    pass
```

9 其他人的板子（补充）

9.1 MTT+exp

```
#include"bits/stdc++.h"
using namespace std;
typedef long long ll;
typedef double db;
int read(){
    int res=0;
    char c=getchar(),f=1;
    while(c<48||c>57){if(c=='-')f=0;c=getchar();}
    while(c>=48&&c<=57)res=(res<<3)+(res<<1)+(c&15),c=getchar();
    return f?res:-res;
}

const int L=1<<19,mod=1e9+7;
const db pi2=3.141592653589793*2;
int inc(int x,int y){return x+y>=mod?x+y-mod:x+y;}
int dec(int x,int y){return x-y<0?x-y+mod:x-y;}
int mul(int x,int y){return (ll)x*y%mod;}
int qpow(int x,int y){
    int res=1;
    for(;y;y>>=1)res=y&1?mul(res,x):res,x=mul(x,x);
    return res;
}
int inv(int x){return qpow(x,mod-2);}

struct cp{
    db x,y;
    cp(){}
    cp(db a,db b){x=a,y=b;}
    cp operator+(const cp& p)const{return cp(x+p.x,y+p.y);}
    cp operator-(const cp& p)const{return cp(x-p.x,y-p.y);}
    cp operator*(const cp& p)const{return cp(x*p.x-y*p.y,x*p.y+y*p.x);}
    cp conj(){return cp(x,-y);}
}w[L];
int re[L];
int getre(int n){
    int len=1,bit=0;
    while(len<n)++bit,len<<=1;
    for(int i=1;i<len;++i)re[i]=(re[i>>1]>>1)|((i&1)<<(bit-1));
    return len;
}
void getw(){
    for(int i=0;i<L;++i)w[i]=cp(cos(pi2/L*i),sin(pi2/L*i));
}
void fft(cp* a,int len,int m){
    for(int i=1;i<len;++i)if(i<re[i])swap(a[i],a[re[i]]);
    for(int k=1,r=L>>1;k<len;k<<=1,r>>=1)
        for(int i=0;i<len;i+=k<<1)
            for(int j=0;j<k;++j){
                cp &L=a[i+j],&R=a[i+j+k],t=w[r*j]*R;
                R=L-t,L=L+t;
            }
    if(!~m){
        reverse(a+1,a+len);
```

```
        cp tmp=cp(1.0/len,0);
        for(int i=0;i<len;++i)a[i]=a[i]*tmp;
    }
}
void mul(int* a,int* b,int* c,int n1,int n2,int n){
    static cp f1[L],f2[L],f3[L],f4[L];
    int len=getre(n1+n2-1);
    for(int i=0;i<len;++i){
        f1[i]=i<n1?cp(a[i]>>15,a[i]&32767):cp(0,0);
        f2[i]=i<n2?cp(b[i]>>15,b[i]&32767):cp(0,0);
    }
    fft(f1,len,1),fft(f2,len,1);
    cp t1=cp(0.5,0),t2=cp(0,-0.5),r=cp(0,1);
    cp x1,x2,x3,x4;
    for(int i=0;i<len;++i){
        int j=(len-i)&(len-1);
        x1=(f1[i]+f1[j].conj())*t1;
        x2=(f1[i]-f1[j].conj())*t2;
        x3=(f2[i]+f2[j].conj())*t1;
        x4=(f2[i]-f2[j].conj())*t2;
        f3[i]=x1*(x3+x4*r);
        f4[i]=x2*(x3+x4*r);
    }
    fft(f3,len,-1),fft(f4,len,-1);
    ll c1,c2,c3,c4;
    for(int i=0;i<n;++i){
        c1=(ll)(f3[i].x+0.5)%mod,c2=(ll)(f3[i].y+0.5)%mod;
        c3=(ll)(f4[i].x+0.5)%mod,c4=(ll)(f4[i].y+0.5)%mod;
        c[i]=((((c1<<15)+c2+c3)<<15)+c4)%mod;
    }
}
void inv(int* a,int* b,int n){
    if(n==1){b[0]=1;return;}
    static int c[L];
    int l=(n+1)>>1;
    inv(a,b,l);
    mul(a,b,c,n,l,n);
    for(int i=0;i<n;++i)c[i]=mod-c[i];
    c[0]+=2;
    mul(b,c,b,n,n,n);
}
void der(int* a,int n){
    for(int i=1;i<n;++i)a[i-1]=mul(a[i],i);
    a[n-1]=0;
}
void its(int* a,int n){
    for(int i=n-1;i;--i)a[i]=mul(a[i-1],inv(i));
    a[0]=0;
}
void ln(int* a,int* b,int n){
    static int c[L];
    for(int i=0;i<n;++i)c[i]=a[i];
    der(c,n);
    inv(a,b,n);
    mul(b,c,b,n,n,n);
    its(b,n);
}
```

```
void exp(int* a,int* b,int n){
    if(n==1){b[0]=1;return;}
    static int c[L];
    int l=(n+1)>>1;
    exp(a,b,l);
    ln(b,c,n);
    for(int i=0;i<n;++i)c[i]=dec(a[i],c[i]);
    ++c[0];
    mul(b,c,b,l,n,n);
    for(int i=0;i<n;++i)c[i]=0;
}

int n,k,a[L],f[L],g[L];
int main(){
    getw();
    n=read(),k=read();
    for(int i=1;i<=k;++i)a[i]=inv(i);
    for(int i=2;i<=n;++i)
        for(int j=1;i*j<=k;++j)
            f[i*j]=inc(f[i*j],a[j]);
    for(int i=1;i<=k;++i)f[i]=mod-f[i];
    for(int i=1;i<=k;++i)f[i]=inc(f[i],mul(n-1,a[i]));
    exp(f,g,k+1);
    printf("%d\n",g[k]);
}
```

9.2 半平面交

```
const int N=305;
const db inf=1e15,eps=1e-10;
int sign(db x){
    if(fabs(x)<eps)return 0;
    return x>0?1:-1;
}

struct vec{
    db x,y;
    vec(){}
    vec(db a,db b){x=a,y=b;}
    vec operator+(const vec& p)const{
        return vec(x+p.x,y+p.y);
    }
    vec operator-(const vec& p)const{
        return vec(x-p.x,y-p.y);
    }
    db operator*(const vec& p)const{
        return x*p.y-y*p.x;
    }
    vec operator*(const db& p)const{
        return vec(x*p,y*p);
    }
}p1[N],p2[N];

struct line{
    vec s,t;
    line(){}
```

```
    line(vec a,vec b){s=a,t=b;}
}a[N],q[N];
db ang(vec v){
    return atan2(v.y,v.x);
}
db ang(line l){
    return ang(l.t-l.s);
}
bool cmp(line x,line y){
    int s=sign(ang(x)-ang(y));
    return s?s<0:sign((x.t-x.s)*(y.t-x.s))>0;
}

vec inter(line x,line y){
    vec a=y.s-x.s,b=x.t-x.s,c=y.t-y.s;
    return y.s+c*((a*b)/(b*c));
}
bool out(line l,vec p){
    return sign((l.t-l.s)*(p-l.s))<0;
}

int n,tot=0;
db ans=inf;
int main(){
    scanf("%d",&n);
    for(int i=1;i<=n;++i)scanf("%lf",&p1[i].x);
    for(int i=1;i<=n;++i)scanf("%lf",&p1[i].y);
    for(int i=1;i<n;++i)a[i]=line(p1[i],p1[i+1]);
    a[n]=line(vec(p1[1].x,inf),vec(p1[1].x,p1[1].y));
    a[n+1]=line(vec(p1[n].x,p1[n].y),vec(p1[n].x,inf));

    sort(a+1,a+n+2,cmp);
    for(int i=1;i<=n;++i){
        if(!sign(ang(a[i])-ang(a[i+1])))continue;
        a[++tot]=a[i];
    }a[++tot]=a[n+1];

    int l=1,r=0;
    q[++r]=a[1],q[++r]=a[2];
    for(int i=3;i<=tot;++i){
        while(l<r&&out(a[i],inter(q[r],q[r-1])))--r;
        while(l<r&&out(a[i],inter(q[l],q[l+1])))++l;
        q[++r]=a[i];
    }
    while(l<r&&out(q[l],inter(q[r],q[r-1])))--r;
    while(l<r&&out(q[r],inter(q[l],q[l+1])))++l;
//......
}
```

9.3 多项式复合 (yurzhang)

$O(n\log n\sqrt{n\log n})$，奇慢无比，慎用

```
#pragma GCC optimize("Ofast,inline")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.1,sse4.2,popcnt,abm,mmx,avx,avx2,tune=native")
#include <cstdio>
#include <cstring>
```

```
#include <cmath>
#include <algorithm>

#define MOD 998244353
#define G 332748118
#define N 262210
#define re register
#define gc pa==pb&&(pb=(pa=buf)+fread(buf,1,100000,stdin),pa==pb)?EOF:*pa++
typedef long long ll;
static char buf[100000],*pa(buf),*pb(buf);
static char pbuf[3000000],*pp(pbuf),st[15];
int read() {
    re int x(0);re char c(gc);
    while(c<'0'||c>'9')c=gc;
    while(c>='0'&&c<='9')
        x=x*10+c-48,c=gc;
    return x;
}
void write(re int v) {
    if(v==0)
        *pp++=48;
    else {
        re int tp(0);
        while(v)
            st[++tp]=v%10+48,v/=10;
        while(tp)
            *pp++=st[tp--];
    }
    *pp++=32;
}

int pow(re int a,re int b) {
    re int ans(1);
    while(b)
        ans=b&1?(ll)ans*a%MOD:ans,a=(ll)a*a%MOD,b>>=1;
    return ans;
}

int inv[N],ifac[N];
void pre(re int n) {
    inv[1]=ifac[0]=1;
    for(re int i(2);i<=n;++i)
        inv[i]=(ll)(MOD-MOD/i)*inv[MOD%i]%MOD;
    for(re int i(1);i<=n;++i)
        ifac[i]=(ll)ifac[i-1]*inv[i]%MOD;
}

int getLen(re int t) {
    return 1<<(32-__builtin_clz(t));
}

int lmt(1),r[N],w[N];
void init(re int n) {
    re int l(0);
    while(lmt<=n)
        lmt<<=1,++l;
    for(re int i(1);i<lmt;++i)
```

```
        r[i]=(r[i>>1]>>1)|((i&1)<<(l-1));
    re int wn(pow(3,(MOD-1)/lmt));
    w[lmt>>1]=1;
    for(re int i((lmt>>1)+1);i<lmt;++i)
        w[i]=(ll)w[i-1]*wn%MOD;
    for(re int i((lmt>>1)-1);i;--i)
        w[i]=w[i<<1];
}

void DFT(int*a,re int l) {
    static unsigned long long tmp[N];
    re int u(__builtin_ctz(lmt)-__builtin_ctz(l)),t;
    for(re int i(0);i<l;++i)
        tmp[i]=(a[r[i]>>u])%MOD;
    for(re int i(1);i<l;i<<=1)
        for(re int j(0),step(i<<1);j<l;j+=step)
            for(re int k(0);k<i;++k)
                t=(ll)w[i+k]*tmp[i+j+k]%MOD,
                tmp[i+j+k]=tmp[j+k]+MOD-t,
                tmp[j+k]+=t;
    for(re int i(0);i<l;++i)
        a[i]=tmp[i]%MOD;
}

void IDFT(int*a,re int l) {
    std::reverse(a+1,a+l);DFT(a,l);
    re int bk(MOD-(MOD-1)/l);
    for(re int i(0);i<l;++i)
        a[i]=(ll)a[i]*bk%MOD;
}

int n,m;
int a[N],b[N],c[N];

void getInv(int*a,int*b,int deg) {
    if(deg==1)
        b[0]=pow(a[0],MOD-2);
    else {
        static int tmp[N];
        getInv(a,b,(deg+1)>>1);
        re int l(getLen(deg<<1));
        for(re int i(0);i<l;++i)
            tmp[i]=i<deg?a[i]:0;
        DFT(tmp,l),DFT(b,l);
        for(re int i(0);i<l;++i)
            b[i]=(2ll-(ll)tmp[i]*b[i]%MOD+MOD)%MOD*b[i]%MOD;
        IDFT(b,l);
        for(re int i(deg);i<l;++i)
            b[i]=0;
    }
}

void getDer(int*a,int*b,int deg) {
    for(re int i(0);i+1<deg;++i)
        b[i]=(ll)a[i+1]*(i+1)%MOD;
    b[deg-1]=0;
}
```

```
void getComp(int*a,int*b,int k,int m,int&n,int*c,int*d) {
    if(k==1) {
        for(re int i(0);i<m;++i)
            c[i]=0,d[i]=b[i];
        n=m,c[0]=a[0];
    } else {
        static int t1[N],t2[N];
        int nl(n),nr(n),*cl,*cr,*dl,*dr;
        getComp(a,b,k>>1,m,nl,cl=c,dl=d);
        getComp(a+(k>>1),b,(k+1)>>1,m,nr,cr=c+nl,dr=d+nl);
        n=std::min(n,nl+nr-1);
        re int _l(getLen(nl+nr));
        for(re int i(0);i<_l;++i)
            t1[i]=i<nl?dl[i]:0;
        for(re int i(0);i<_l;++i)
            t2[i]=i<nr?cr[i]:0;
        DFT(t1,_l),DFT(t2,_l);
        for(re int i(0);i<_l;++i)
            t2[i]=(ll)t1[i]*t2[i]%MOD;
        IDFT(t2,_l);
        for(re int i(0);i<n;++i)
            c[i]=((i<nl?cl[i]:0)+t2[i])%MOD;
        for(re int i(0);i<_l;++i)
            t2[i]=i<nr?dr[i]:0;
        DFT(t2,_l);
        for(re int i(0);i<_l;++i)
            t2[i]=(ll)t1[i]*t2[i]%MOD;
        IDFT(t2,_l);
        for(re int i(0);i<n;++i)
            d[i]=t2[i];
    }
}

void getComp(int*a,int*b,int*c,int deg) {
    static int ts[N],ps[N],c0[N],_t1[N],idM[N];
    int M(std::max((int)ceil(sqrt(deg/log2(deg))*2.5),2)),_n(deg+deg/M);
    getComp(a,b,deg,M,_n,c0,_t1);
    re int _l(getLen(_n+deg));
    for(re int i(_n);i<_l;++i)
        c0[i]=0;
    for(re int i(0);i<_l;++i)
        ps[i]=i==0;
    for(re int i(0);i<_l;++i)
        ts[i]=M<=i&&i<deg?b[i]:0;
    getDer(b,_t1,M);
    for(re int i(M-1);i<deg;++i)
        _t1[i]=0; /// Important!!!
    getInv(_t1,idM,deg);
    for(int i=deg;i<_l;++i)
        idM[i]=0;
    DFT(ts,_l),DFT(idM,_l);
    for(re int t(0);t*M<deg;++t) {
        for(re int i(0);i<_l;++i)
            _t1[i]=i<deg?c0[i]:0;
        DFT(ps,_l),DFT(_t1,_l);
        for(re int i(0);i<_l;++i)
```

```
            _t1[i]=(ll)_t1[i]*ps[i]%MOD,
            ps[i]=(ll)ps[i]*ts[i]%MOD;
        IDFT(ps,_l),IDFT(_t1,_l);
        for(re int i(deg);i<_l;++i)
            ps[i]=0;
        for(re int i(0);i<deg;++i)
            c[i]=((ll)_t1[i]*ifac[t]+c[i])%MOD;
        getDer(c0,c0,_n);
        for(re int i(_n-1);i<_l;++i)
            c0[i]=0;
        DFT(c0,_l);
        for(re int i(0);i<_l;++i)
            c0[i]=(ll)c0[i]*idM[i]%MOD;
        IDFT(c0,_l);
        for(re int i(_n-1);i<_l;++i)
            c0[i]=0;
    }
}

int main() {
    n=read(),m=read();
    for(re int i(0);i<=n;++i)
        a[i]=read();
    for(re int i(0);i<=m;++i)
        b[i]=read();

    m=(n>m?n:m)+1;
    pre(m);init(m*5);
    getComp(a,b,c,m);

    for(re int i(0);i<=n;++i)
        write(c[i]);
    fwrite(pbuf,1,pp-pbuf,stdout);
    return 0;
}
```

9.4 下降幂多项式乘法

$O(n\log n)$。

```
#include<cstdio>
#include<algorithm>
const int N=524288,md=998244353,g3=(md+1)/3;
typedef long long LL;
int n,m,A[N],B[N],fac[N],iv[N],rev[N],C[N],g[20][N],lim,M;
int pow(int a,int b){
    int ret=1;
    for(;b;b>>=1,a=(LL)a*a%md)if(b&1)ret=(LL)ret*a%md;
    return ret;
}
void upd(int&a){a+=a>>31&md;}
void init(int n){
    int l=-1;
    for(lim=1;lim<n;lim<<=1)++l;M=l+1;
    for(int i=1;i<lim;++i)
    rev[i]=((rev[i>>1])>>1)|((i&1)<<l);
}
```

```
void NTT(int*a,int f){
    for(int i=1;i<lim;++i)if(i<rev[i])std::swap(a[i],a[rev[i]]);
    for(int i=0;i<M;++i){
        const int*G=g[i],c=1<<i;
        for(int j=0;j<lim;j+=c<<1)
        for(int k=0;k<c;++k){
            const int x=a[j+k],y=a[j+k+c]*(LL)G[k]%md;
            upd(a[j+k]+=y-md),upd(a[j+k+c]=x-y);
        }
    }
    if(!f){
        const int iv=pow(lim,md-2);
        for(int i=0;i<lim;++i)a[i]=(LL)a[i]*iv%md;
        std::reverse(a+1,a+lim);
    }
}
int main(){
    scanf("%d%d",&n,&m);++n,++m;
    for(int i=0;i<20;++i){
        int*G=g[i];
        G[0]=1;
        const int gi=G[1]=pow(3,(md-1)/(1<<i+1));
        for(int j=2;j<1<<i;++j)G[j]=(LL)G[j-1]*gi%md;
    }
    for(int i=0;i<n;++i)scanf("%d",A+i);
    for(int i=0;i<m;++i)scanf("%d",B+i);
    for(int i=*fac=1;i<N;++i)
    fac[i]=fac[i-1]*(LL)i%md;
    iv[N-1]=pow(fac[N-1],md-2);
    for(int i=N-2;~i;--i)iv[i]=(i+1LL)*iv[i+1]%md;
    init(n+m<<1);
    for(int i=0;i<n+m-1;++i)C[i]=iv[i];
    NTT(A,1),NTT(B,1),NTT(C,1);
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*C[i]%md,B[i]=(LL)B[i]*C[i]%md;
    NTT(A,0),NTT(B,0);
    for(int i=0;i<lim;++i)C[i]=0;
    for(int i=0;i<n+m-1;++i)
    C[i]=(i&1)?md-iv[i]:iv[i];
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*B[i]%md*fac[i]%md;
    for(int i=n+m-1;i<lim;++i)A[i]=0;
    NTT(A,1),NTT(C,1);
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*C[i]%md;
    NTT(A,0);
    for(int i=0;i<n+m-1;++i)printf("%d%c",A[i]," \n"[i==n+m-2]);
    return 0;
}
```

9.5 弦图找错

```
#include "bits/stdc++.h"
using namespace std;
const int MAXN = 200005;
using lint = long long;
using pi = pair<int, int>;
// the algorithm may be wrong. if you have any ideas for proving / disproving this, please
    contact me.
```

```
vector<int> gph[MAXN];
int n, m, cnt[MAXN], idx[MAXN];
int mark[MAXN], vis[MAXN], par[MAXN];
void report(int x, int y){
    gph[x].erase(find(gph[x].begin(), gph[x].end(), y));
    gph[y].erase(find(gph[y].begin(), gph[y].end(), x));
    for(int i=1; i<=n; i++){
        if(binary_search(gph[i].begin(), gph[i].end(), x) &&
            binary_search(gph[i].begin(), gph[i].end(), y)){
            mark[i] = 1;
        }
    }
    queue<int> que;
    vis[x] = 1;
    que.push(x);
    while(!que.empty()){
        int x = que.front(); que.pop();
        for(auto &i : gph[x]){
            if(!mark[i] && !vis[i]){
                par[i] = x;
                vis[i] = 1;
                que.push(i);
            }
        }
    }
    assert(vis[y]);
    vector<int> v;
    while(y){
        v.push_back(y);
        y = par[y];
    }
    printf("NO\n%d\n", v.size());
    for(auto &i : v) printf("%d␣", i-1);
}

int main(){
    scanf("%d␣%d",&n,&m);
    for(int i=0; i<m; i++){
        int s, e; scanf("%d␣%d",&s,&e);
        s++, e++;
        gph[s].push_back(e);
        gph[e].push_back(s);
    }
    for(int i=1; i<=n; i++) sort(gph[i].begin(), gph[i].end());
    priority_queue<pi> pq;
    for(int i=1; i<=n; i++) pq.emplace(cnt[i], i);
    vector<int> ord;
    while(!pq.empty()){
        int x = pq.top().second, y = pq.top().first;
        pq.pop();
        if(cnt[x] != y || idx[x]) continue;
        ord.push_back(x);
        idx[x] = n + 1 - ord.size();
        for(auto &i : gph[x]){
            if(!idx[i]){
                cnt[i]++;
                pq.emplace(cnt[i], i);
```

```
            }
        }
    }
    reverse(ord.begin(), ord.end());
    for(auto &i : ord){
        int minBef = 1e9;
        for(auto &j : gph[i]){
            if(idx[j] > idx[i]) minBef = min(minBef, idx[j]);
        }
        minBef--;
        if(minBef < n){
            minBef = ord[minBef];
            for(auto &j : gph[i]){
                if(idx[j] > idx[minBef] && !binary_search(gph[minBef].begin(), gph[minBef].end(), j
                    )){
                    report(minBef, i);
                    return 0;
                }
            }
        }
    }
    puts("YES");
    for(auto &i : ord) printf("%d␣", i-1);
}
```

9.6 最长公共子序列

复杂度 $O(\frac{nm}{\omega})$。

```
/*
 * Author : _Wallace_
 * Source : https://www.cnblogs.com/-Wallace-/
 * Problem : LOJ #6564. 最长公共子序列
 * Standard : GNU C++ 03
 * Optimal : -Ofast
 */
#include <algorithm>
#include <cstddef>
#include <cstdio>
#include <cstring>

typedef unsigned long long ULL;

const int N = 7e4 + 5;
int n, m, u;

struct bitset {
  ULL t[N / 64 + 5];

  bitset() {
    memset(t, 0, sizeof(t));
  }
  bitset(const bitset &rhs) {
    memcpy(t, rhs.t, sizeof(t));
  }

  bitset& set(int p) {
```

```
    t[p >> 6] |= 1llu << (p & 63);
    return *this;
  }
  bitset& shift() {
    ULL last = 0llu;
    for (int i = 0; i < u; i++) {
      ULL cur = t[i] >> 63;
      (t[i] <<= 1) |= last, last = cur;
    }
    return *this;
  }
  int count() {
    int ret = 0;
    for (int i = 0; i < u; i++)
      ret += __builtin_popcountll(t[i]);
    return ret;
  }

  bitset& operator = (const bitset &rhs) {
    memcpy(t, rhs.t, sizeof(t));
    return *this;
  }
  bitset& operator &= (const bitset &rhs) {
    for (int i = 0; i < u; i++) t[i] &= rhs.t[i];
    return *this;
  }
  bitset& operator |= (const bitset &rhs) {
    for (int i = 0; i < u; i++) t[i] |= rhs.t[i];
    return *this;
  }
  bitset& operator ^= (const bitset &rhs) {
    for (int i = 0; i < u; i++) t[i] ^= rhs.t[i];
    return *this;
  }

  friend bitset operator - (const bitset &lhs, const bitset &rhs) {
    ULL last = 0llu; bitset ret;
    for (int i = 0; i < u; i++){
      ULL cur = (lhs.t[i] < rhs.t[i] + last);
      ret.t[i] = lhs.t[i] - rhs.t[i] - last;
      last = cur;
    }
    return ret;
  }
} p[N], f, g;

signed main() {
  scanf("%d%d", &n, &m), u = n / 64 + 1;
  for (int i = 1, c; i <= n; i++)
    scanf("%d", &c), p[c].set(i);
  for (int i = 1, c; i <= m; i++) {
    scanf("%d", &c), (g = f) |= p[c];
    f.shift(), f.set(0);
    ((f = g - f) ^= g) &= g;
  }
  printf("%d\n", f.count());
  return 0;
```

```
}
```

另一个实现

```
#include "bits/stdc++.h"
#pragma GCC target("popcnt,bmi")

using namespace std;
using ull = uint64_t;

const int N = 70005, M = 1136;

int n, m;
ull g[N][M], f[M];

int read() {
    const int M = 1e6;
    static streambuf *in = cin.rdbuf();
#define gc (p1 == p2 && (p2 = (p1 = buf) + in -> sgetn(buf, M), p1 == p2) ? -1 : *p1++)
    static char buf[M], *p1, *p2;
    int c = gc, r = 0;

    while (c < 48)
        c = gc;

    while (c > 47)
        r = r * 10 + (c & 15), c = gc;

    return r;
}
int main() {
    cin.tie(0)->sync_with_stdio(0);
    cin >> n >> m;

    for (int i = 0; i < n; i++)
        g[read()][i / 62] |= 1ULL << (i % 62);

    int lim = (n - 1) / 62;

    for (int i = 0; i < m; i++) {
        int c = 1;
        auto can = g[read()];

        for (int j = 0; j <= lim; j++) {
            ull x = f[j], y = x | can[j];
            x += x + c + (~y & (1ULL << 62) - 1);
            f[j] = x & y, c = x >> 62;
        }
    }

    int ans = 0;

    for (int i = 0; i <= lim; i++)
        ans += __builtin_popcountll(f[i]);

    cout << ans;
}
```

9.7 区间 LIS（排列）

在线：

```
/*

References

[1] Tiskin, A. (2008).Semi-local string comparison: Algorithmic techniques
     and applications. Mathematics in Computer Science, 1(4), 571-603.

[2] Claude, F., Navarro, G., & Ordónez, A. (2015). The wavelet matrix: An
     efficient wavelet tree for large alphabets. Information Systems, 47,
     15-32.

*/

// #pragma GCC target("popcnt")

#include <algorithm>
#include <cassert>
#include <climits>
#include <numeric>
#include <utility>
#include <vector>

namespace noshi91 {

namespace range_lis_query_impl {

namespace wavelet_matrix_impl {

using uint = unsigned int;
using ll = long long;

static constexpr int w = CHAR_BIT * sizeof(uint);

int popcount(uint x) {
#ifdef __GNUC__
  return __builtin_popcount(x);
#else
  static_assert(w == 32, "");
  x -= x >> 1 & 0x55555555;
  x = (x & 0x33333333) + (x >> 2 & 0x33333333);
  x = x + (x >> 4) & 0x0F0F0F0F;
  return x * 0x01010101 >> 24 & 0x3F;
#endif
}

class bit_vector {
  class node_type {
  public:
    uint bit;
    int sum;

    node_type() : bit(0), sum(0) {}
  };
```

```
  std::vector<node_type> v;

public:
  bit_vector(const uint n) : v(n / w + 1) {}

  void set(const uint i) {
    v[i / w].bit |= uint(1) << i;
    v[i / w].sum += 1;
  }

  void build() {
    for (int i = 1; i < int(v.size()); i++) {
      v[i].sum += v[i - 1].sum;
    }
  }

  int rank(const uint i) const {
    return v[i / w].sum - popcount(v[i / w].bit & ~uint(0) << i % w);
  }

  int one() const { return v.back().sum; }
};

class wavelet_matrix {
private:
  template <class I> static bool test(const I x, const int k) {
    return (x & I(1) << k) != I(0);
  }

  std::vector<bit_vector> mat;

public:
  template <class I>
  wavelet_matrix(const int bit_length, std::vector<I> a)
      : mat(bit_length, bit_vector(a.size())) {
    const int n = a.size();
    std::vector<I> a0;
    a0.reserve(n);
    for (int p = bit_length - 1; p >= 0; p--) {
      bit_vector &v = mat[p];
      auto itr = a.begin();
      for (int i = 0; i < n; i++) {
        if (test(a[i], p)) {
          v.set(i);
          *itr = a[i];
          itr++;
        } else {
          a0.push_back(a[i]);
        }
      }
      v.build();
      std::copy(a0.begin(), a0.end(), itr);
      a0.clear();
    }
  }

  int count_less_than(int l, int r, const ll key) const {
```

```
    int ret = r - l;
    for (int p = mat.size() - 1; p >= 0; p--) {
      const bit_vector &v = mat[p];
      const int rank_l = v.rank(l);
      const int rank_r = v.rank(r);
      if (test(key, p)) {
        l = rank_l;
        r = rank_r;
      } else {
        ret -= rank_r - rank_l;
        const int o = v.one();
        l += o - rank_l;
        r += o - rank_r;
      }
    }
    return ret - (r - l);
  }
};

} // namespace wavelet_matrix_impl

using wavelet_matrix_impl::wavelet_matrix;

using vi = std::vector<int>;
using iter = typename vi::iterator;

static constexpr int none = -1;

vi inverse(const vi &p) {
  const int n = p.size();
  vi q(n, none);
  for (int i = 0; i < n; i++) {
    if (p[i] != none) {
      q[p[i]] = i;
    }
  }
  return q;
}

void unit_monge_dmul(const int n, iter stack, const iter a, const iter b) {
  if (n == 1) {
    stack[0] = 0;
    return;
  }
  const iter c_row = stack;
  stack += n;
  const iter c_col = stack;
  stack += n;

  const auto map = [=](const int len, const auto f, const auto g) {
    const iter a_h = stack + 0 * len;
    const iter a_m = stack + 1 * len;
    const iter b_h = stack + 2 * len;
    const iter b_m = stack + 3 * len;
    const auto split = [=](const iter v, iter v_h, iter v_m) {
      for (int i = 0; i < n; i++) {
        if (f(v[i])) {
```

```
        *v_h = g(v[i]);
        ++v_h;
        *v_m = i;
        ++v_m;
      }
    }
  };
  split(a, a_h, a_m);
  split(b, b_h, b_m);
  const iter c = stack + 4 * len;
  unit_monge_dmul(len, c, a_h, b_h);
  for (int i = 0; i < len; i++) {
    const int row = a_m[i];
    const int col = b_m[c[i]];
    c_row[row] = col;
    c_col[col] = row;
  }
};
const int mid = n / 2;
map(mid, [mid](const int x) { return x < mid; },
    [](const int x) { return x; });
map(n - mid, [mid](const int x) { return x >= mid; },
    [mid](const int x) { return x - mid; });

class d_itr {
public:
  int delta;
  int col;
  d_itr() : delta(0), col(0) {}
};
int row = n;
const auto right = [&](d_itr &it) {
  if (b[it.col] < mid) {
    if (c_col[it.col] >= row) {
      it.delta += 1;
    }
  } else {
    if (c_col[it.col] < row) {
      it.delta += 1;
    }
  }
  it.col += 1;
};
const auto up = [&](d_itr &it) {
  if (a[row] < mid) {
    if (c_row[row] >= it.col) {
      it.delta -= 1;
    }
  } else {
    if (c_row[row] < it.col) {
      it.delta -= 1;
    }
  }
};
d_itr neg, pos;
while (row != 0) {
  while (pos.col != n) {
```

```
      d_itr temp = pos;
      right(temp);
      if (temp.delta == 0) {
        pos = temp;
      } else {
        break;
      }
    }
    row -= 1;
    up(neg);
    up(pos);
    while (neg.delta != 0) {
      right(neg);
    }
    if (neg.col > pos.col) {
      c_row[row] = pos.col;
    }
  }
}

vi subunit_monge_dmul(vi a, vi b) {
  const int n = a.size();
  vi a_inv = inverse(a);
  vi b_inv = inverse(b);
  std::swap(b, b_inv);
  vi a_map, b_map;
  for (int i = n - 1; i >= 0; i--) {
    if (a[i] != none) {
      a_map.push_back(i);
      a[n - a_map.size()] = a[i];
    }
  }
  std::reverse(a_map.begin(), a_map.end());
  {
    int cnt = 0;
    for (int i = 0; i < n; i++) {
      if (a_inv[i] == none) {
        a[cnt] = i;
        cnt += 1;
      }
    }
  }
  for (int i = 0; i < n; i++) {
    if (b[i] != none) {
      b[b_map.size()] = b[i];
      b_map.push_back(i);
    }
  }
  {
    int cnt = b_map.size();
    for (int i = 0; i < n; i++) {
      if (b_inv[i] == none) {
        b[cnt] = i;
        cnt += 1;
      }
    }
  }
```

```
  vi c([](int n) {
    int ret = 0;
    while (n > 1) {
      ret += 2 * n;
      n = (n + 1) / 2;
      ret += 4 * n;
    }
    ret += 1;
    return ret;
  }(n));
  unit_monge_dmul(n, c.begin(), a.begin(), b.begin());

  vi c_pad(n, none);
  for (int i = 0; i < int(a_map.size()); i++) {
    const int t = c[n - a_map.size() + i];
    if (t < int(b_map.size())) {
      c_pad[a_map[i]] = b_map[t];
    }
  }
  return c_pad;
}

vi seaweed_doubling(const vi &p) {
  const int n = p.size();
  if (n == 1) {
    return vi({none});
  }
  const int mid = n / 2;
  vi lo, hi;
  vi lo_map, hi_map;
  for (int i = 0; i < n; i++) {
    const int e = p[i];
    if (e < mid) {
      lo.push_back(e);
      lo_map.push_back(i);
    } else {
      hi.push_back(e - mid);
      hi_map.push_back(i);
    }
  }
  lo = seaweed_doubling(lo);
  hi = seaweed_doubling(hi);
  vi lo_pad(n), hi_pad(n);
  std::iota(lo_pad.begin(), lo_pad.end(), 0);
  std::iota(hi_pad.begin(), hi_pad.end(), 0);
  for (int i = 0; i < mid; i++) {
    if (lo[i] == none) {
      lo_pad[lo_map[i]] = none;
    } else {
      lo_pad[lo_map[i]] = lo_map[lo[i]];
    }
  }
  for (int i = 0; mid + i < n; i++) {
    if (hi[i] == none) {
      hi_pad[hi_map[i]] = none;
    } else {
      hi_pad[hi_map[i]] = hi_map[hi[i]];
```

```cpp
    }
  }
  return subunit_monge_dmul(std::move(lo_pad), std::move(hi_pad));
}

bool is_permutation(const vi &p) {
  const int n = p.size();
  std::vector<bool> used(n, false);
  for (const int e : p) {
    if (e < 0 || n <= e || used[e]) {
      return false;
    }
    used[e] = true;
  }
  return true;
}

wavelet_matrix convert(const vi &p) {
  assert(is_permutation(p));
  int n = p.size();
  vi row;
  if (n != 0) {
    row = seaweed_doubling(vi(p.begin(), p.end()));
  }
  for (int &e : row) {
    if (e == none) {
      e = n;
    }
  }
  int bit_length = 0;
  while (n > 0) {
    bit_length += 1;
    n /= 2;
  }
  return wavelet_matrix(bit_length, std::move(row));
}

class range_lis_query {
  int n;
  wavelet_matrix wm;

public:
  range_lis_query() : range_lis_query(std::vector<int>()) {}
  explicit range_lis_query(const std::vector<int> &p)
      : n(p.size()), wm(convert(p)) {}

  int query(const int left, const int right) const {
    assert(0 <= left && left <= right && right <= n);
    return (right - left) - wm.count_less_than(left, n, right);
  }
};

} // namespace range_lis_query_impl

using range_lis_query_impl::range_lis_query;

} // namespace noshi91
```

```cpp
#include <iostream>
#include <vector>

int main() {
  std::ios::sync_with_stdio(false);
  std::cin.tie(nullptr);

  int N, Q;
  std::cin >> N >> Q;
  std::vector<int> P(N);
  for (int &p : P) {
    std::cin >> p;
  }

  const noshi91::range_lis_query rlq(P);

  for (int i = 0; i < Q; i++) {
    int l, r;
    std::cin >> l >> r;
    std::cout << rlq.query(l, r) << "\n";
  }

  return 0;
}
```

离线：

```cpp
//http://10.49.18.71/submission/164696
#include<bits/stdc++.h>
using namespace std;
constexpr int M=1e6+5;
int n,q,a[M],tr[M],ans[M];
vector<pair<int,int>>qry[M];
int blk,bel[M],L[M],R[M],val[M];
priority_queue<int>Q[M];
priority_queue<int,vector<int>,greater<int>>P[M];
int read(){
    int x=0;char ch=getchar();
    while (!isdigit(ch)) ch=getchar();
    while (isdigit(ch)) x=x*10+ch-48,ch=getchar();
    return x;
}
void update(int x){while (x) tr[x]++,x-=x&-x;}
int query(int x){int res=0;while (x<=n) res+=tr[x],x+=x&-x;return res;}
int main(){
    n=read();q=read();
    for (int i=1;i<=n;i++) a[i]=read()+1;
    blk=(int)ceil(sqrt(n));
    for (int i=1;i<=n;i++) bel[i]=(i-1)/blk+1;
    for (int i=1;i<=bel[n];i++) L[i]=R[i-1]+1,R[i]=R[i-1]+blk; R[bel[n]]=n;
    auto push_back=[&](int x){
        const int p=a[x],B=bel[p];
        if (!P[B].empty()){
            for (int i=L[B];i<=R[B];i++)
                if (val[i]){
                    P[B].push(val[i]);
                    val[i]=P[B].top();
```

```
                P[B].pop();
            }
            while (!P[B].empty()) P[B].pop();
        }
        val[p]=x;Q[B].push(x);
        int tmp=0;bool flag=0;
        for (int i=p+1;i<=R[B];i++)
            if (tmp<val[i]) swap(tmp,val[i]),flag=1;
        if (flag){
            while (!Q[B].empty()) Q[B].pop();
            for (int i=L[B];i<=R[B];i++)
                if (val[i]) Q[B].push(val[i]);
        }
        for (int i=B+1;i<=bel[n];i++)
            if (!Q[i].empty()&&tmp<Q[i].top()){
                P[i].push(tmp);
                if (tmp) Q[i].push(tmp);
                tmp=Q[i].top(),Q[i].pop();
            }
        update(tmp);
    };
    for (int i=1;i<=q;i++){
        int l=read()+1,r=read();
        ans[i]=r-l+1;
        if(l<=r)qry[r].emplace_back(l,i);
    }
    for (int i=1;i<=n;i++){
        push_back(i);
        for (auto [x,id]:qry[i])
            ans[id]-=query(x);
    }
    for (int i=1;i<=q;i++) printf("%d\n",ans[i]);
    return 0;
}
```

9.8 区间 LCS

$s_{[0,a)}$ 和 $t_{[b,c)}$ 的 LCS

```
#include"bits/stdc++.h"
using namespace std;
//dengyaotriangle!

const int maxn=1005;
const int maxq=500005;
int n,m,q;
char a[maxn],b[maxn];
struct qryt{
    int x,nxt;
}z[maxq];
int qry[maxn][maxn];
int ans[maxq];
int r[maxn];
int bit[maxn];

int main(){
    ios::sync_with_stdio(0);cin.tie(0);
```

```
    cin>>q>>b>>a;n=strlen(a);m=strlen(b);
    //q,s,t
    for(int i=1;i<=q;i++){
        int a,b,c;
        cin>>a>>b>>c;
        if(a){
            ans[i]=c-b;
            z[i].x=b;z[i].nxt=qry[a][c];
            qry[a][c]=i;
        }
    }
    for(int i=0;i<n;i++)r[i]=i;
    for(int i=0;i<m;i++){
        int lp=-1;
        for(int j=0;j<n;j++)if(a[j]==b[i]){lp=j;break;}
        if(lp!=-1){
            for(int j=lp+1;j<n;j++){
                if(a[j]!=b[i]){
                    if(r[j-1]<r[j])swap(r[j-1],r[j]);
                }
            }
            for(int i=n-1;i>lp;i--)r[i]=r[i-1];
            r[lp]=-1;
        }
        for(int i=0;i<=n;i++)bit[i]=0;
        for(int j=0;j<n;j++){
            if(r[j]!=-1){
                for(int p=n-r[j];p<=n;p+=p&-p)bit[p]++;
            }
            for(int y=qry[i+1][j+1];y;y=z[y].nxt){
                for(int p=n-z[y].x;p;p-=p&-p)ans[y]-=bit[p];
            }
        }
    }
    for(int i=1;i<=q;i++)cout<<ans[i]<<'\n';
    return 0;
}
```