Reduction from SIR model to SSSP

November 2020

Suppose that $G = (V, E)$ is the underlying, undirected contact network. Suppose that the source of infection is some node $s \in V$ and infection spreads according to the SIR model with parameters α and β. Here α is the probability that node u will transition from state S (susceptible) to state I (infected) at the end of time step t, given a contact (edge) with a node v who is in state I in time step t. And β is the probability that node u will transition from I to R (recovered) during each time step that starts with u being in I.

To turn this into an instance of SSSP with source s, the following sampling steps need to be performed:

1. Visit each node u and sample a positive integer quantity i_u from the probability distribution

   $$\text{Prob}[I_u = i] = \beta \cdot (1 - \beta)^{i-1}.$$

   Therefore, i_u serves as the amount of time u will stay infected, *conditioned on u becoming infected.* Note that it is quite possible that the infection does not reach u.

2. Consider each undirected edge $\{u, v\}$ and view this each as two directed edges (u, v) and (v, u). For directed edge (u, v), sample a positive integer quantity j_{uv} from the following probability distribution:

   $$\text{Prob}[J_{uv} = j] = \alpha \cdot (1 - \alpha)^{j-1}.$$

   The quantity j_{uv} represents the delay in infection traveling from u to v after u got infected. Using i_u and j_{uv}, define a weight $w(u, v)$ for directed edge (u, v) as follows:

   $$w(u, v) = \begin{cases} j_{uv} & \text{if} j_{uv} \leq i_u \\ \infty & \text{otherwise} \end{cases}$$

   Repeat this for edge (v, u).

The process described above gives us an edge-weighted, directed graph. (We drop any edge assigned weight ∞.) It is not too hard to show that (i) the nodes reachable from s via directed edges in this directed graph are the ones that become infected and (ii) the distance from s to u in this graph is the time at which u became infected.