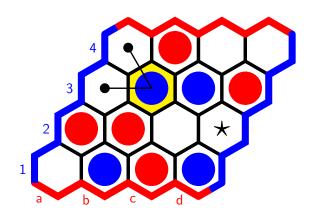
hexboard v1.0

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1 Background

hexboards is a package for LaTeX that provides functionality for drawing Hex boards and games. The aim is a clean, clear design with flexibility for drawing different sorts of Hex diagrams.

2 Usage notes

2.1 The game Hex

In this document it is assumed you know how Hex is played. The players are referred to as A (playing between bottom and top) and B (playing between left and right).

2.2 hexpicture

Hex boards are drawn inside an environment called hexpicture. The basic structure for drawing with this package is

2.3 Coordinate labels

Various commands use coordinates on the hexagonal board as a pair $\{x\}\{y\}$ using column x and row y. In this, the column x may be specified either as a number or using the column letter. For example, the following two commands are equivalent.

In this document, letters are used to match the board labels.

Note that this does not apply to the size of a board, where both dimensions are given as numbers. For example, you ask for a 5 by 7 board, not an e by 7 board.

2.4 Order of commands

New elements are placed on top of the diagram, so some care should be taken that elements do not obscure each other by placing the command for the bottom element first. This includes counters, shaded cells, dots and lines.

For example, it is best to draw shaded cells early so that they appear behind other elements. For example, the counter is hidden here:

\begin{hexpicture}

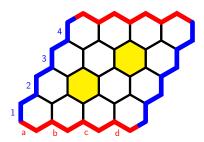
\hexboard{4}

\hexcounter{b}{2}{B}

\hexcellshaded{b}{2}

\hexcellshaded{c}{3}

\end{hexpicture}



Whereas the hidden counter appears if the commands are reordered:

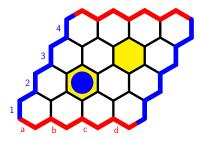
\begin{hexpicture}

\hexboard{4}

\hexcellshaded{b}{2}

\hexcellshaded{c}{3}

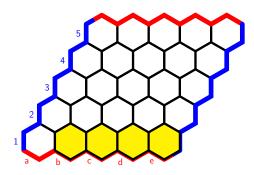
\hexcounter{b}{2}{B}



Indeed, if cells at the edge of the board are to be shaded, do this before drawing the board otherwise the shading will cover the edge border. (On the game board, unshaded cells are transparent.)

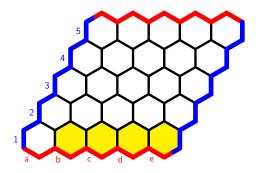
For example, if this happens:

\begin{hexpicture}
 \hexboard{5}
 \hexshadedsubrow{b}{e}{1}
\end{hexpicture}



try this instead:

\begin{hexpicture}
 \hexshadedsubrow{b}{e}{1}
 \hexboard{5}
\end{hexpicture}



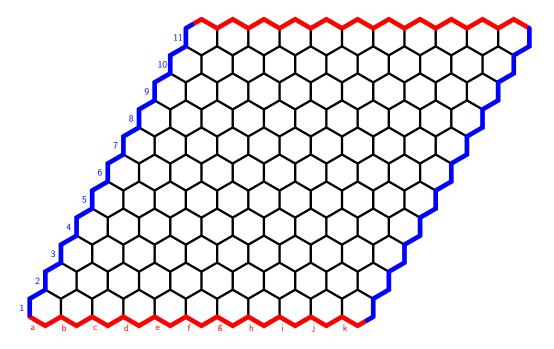
3 Basic usage

3.1 Drawing Hex boards

The basic usage is via \hexboard{num} which draws a Hex board of size num by num. For example, the command

\begin{hexpicture}
 \hexboard{11}
\end{hexpicture}

Produces a Hex board like this



3.2 Drawing Hex games

The environments hexgame and hexgamelabels are used to draw Hex games. These starts a hexpicture and also initialise a counter to keep track of moves. The difference is that hexgamelabels draws a number on each counter to indicate the order in which pieces were played.

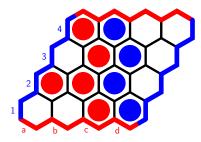
An optional parameter controls the size of the board, with default 11.

Within a hexgame or hexgamelabels, game moves are indicated in the order they are played by hexmove{x}{y}. The colouring of pieces is handled by the counter, so odd-numbered moves are player A and even-numbered moves are player B.

For example, here is a Hex game on a 4 by 4 board without labelling using hexgame:

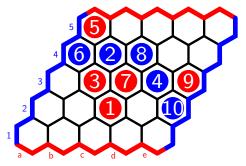
\begin{hexgame}[4]

\hexmove{a}{2}
\hexmove{c}{3}
\hexmove{b}{2}
\hexmove{c}{2}
\hexmove{c}{1}
\hexmove{d}{1}
\hexmove{b}{3}
\hexmove{b}{4}
\hexmove{a}{4}
\end{hexgame}



Here is a Hex game on a 5 by 5 board with move labelling using hexgamelabels:

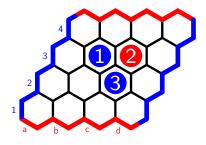
\begin{hexgamelabels}[5]
 \hexmove{c}{2}
 \hexmove{b}{4}
 \hexmove{b}{3}
 \hexmove{d}{3}
 \hexmove{a}{5}
 \hexmove{a}{4}
 \hexmove{c}{3}
 \hexmove{c}{4}
 \hexmove{e}{3}
 \hexmove{e}{2}
 \hexmove{e}{2}



By default, player A moves first. A command **\hexskipmove** is used to increment the move counter without placing a piece, which can be used to skip player A's first move if

player B is to move first (including, for example, if the swap rule has been used but you wish to maintain labelling that shows the order in which pieces were placed).

\begin{hexgamelabels}[4]
 \hexskipmove
 \hexmove{b}{3}
 \hexmove{c}{3}
 \hexmove{c}{2}
\end{hexgamelabels}



3.3 Placing Hex counters

Within a hexpicture, individual counters can be placed using \hexcounter{x}{y}{player}. For example,

\begin{hexpicture}

\hexboard{5}

\hexcounter{b}{1}{B}

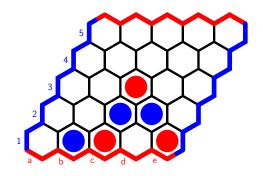
\hexcounter{c}{2}{B}

\hexcounter{d}{2}{B}

\hexcounter{c}{1}{A}

\hexcounter{e}{1}{A}

\hexcounter{c}{3}{A}



3.4 Drawing a shaded cell

The command $\hexcellshaded\{x\}\{y\}$ is used to draw a shaded cell. For example

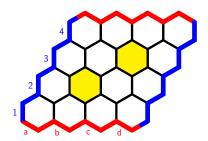
\begin{hexpicture}

\hexboard{4}

\hexcellshaded{b}{2}

\hexcellshaded{c}{3}

\end{hexpicture}



The default shading colour is yellow. An optional parameter allows you to change this. For example

\begin{hexpicture}

\hexboard{4}

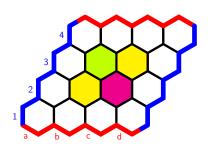
\hexcellshaded{b}{2}

\hexcellshaded{c}{3}

\hexcellshaded[magenta]{c}{2}

\hexcellshaded[lime]{b}{3}

\end{hexpicture}



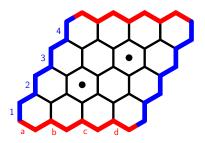
3.5 Drawing a small dot

To indicate a position on the board using a small dot, use $\mbox{hexdot}\{x\}\{y\}$. For example

\begin{hexpicture}

\hexboard{4}

\hexdot{b}{2} \hexdot{c}{3} \end{hexpicture}



3.6 Drawing lines

The command $\ensuremath{\text{hexconnect}\{x1\}\{y1\}\{x2\}\{y2\}}\ draws a line from cell (x1,y1) to cell (x2,y2).$ For example

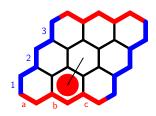
\begin{hexpicture}

\hexboard{3}

\hexcounter{b}{1}{A}

 $\hexconnect{b}{1}{b}{2}$

\end{hexpicture}



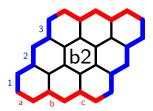
3.7 Putting text or other content in a cell

The command $\text{mexcontent}\{x\}\{y\}\{\text{content}\}\$ puts content into cell (x,y). For example, here we write "b2" in cell b2.

\begin{hexpicture}

\hexboard{3}

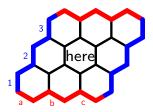
\hexcontent{b}{2}{b2}



By default, the text is \scriptsize (and then scaled by \hexscale, see 4.1). This can be overridden.

For example

```
\begin{hexpicture}
    \hexboard{3}
    \hexcontent{b}{2}{\tiny here}
\end{hexpicture}
```



The content can contain other LATEX commands. For example

```
\usepackage{rotating}
```

. . .

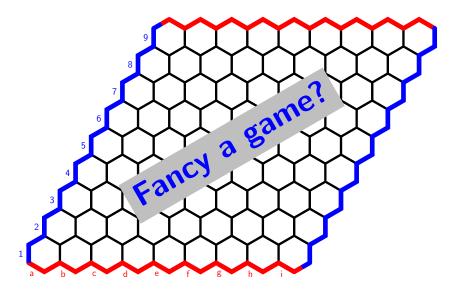
\begin{hexpicture}

\hexboard{9}

\hexcontent{e}{5}{\begin{turn}{30}}

 $\verb|\colorbox{lightgray}{\color{blue}{\normalsize \textbf{Fancy a game?}}}|$

\end{turn}}



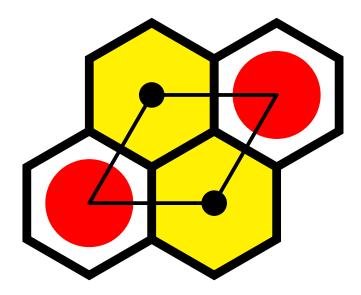
4 User configuration

4.1 Scaling images

The basic unit of the diagrams can be scaled using \hexscale{number}. The default is 2. For example \hexscale{8} changes this



into this



Fractional values are possible. For example

 $\hexscale{0.5}$

\begin{hexpicture}

\hexboard{2}

 $\hexapsum {a}{1}{A}$

\hexcounter{b}{2}{B}

\end{hexpicture}



and

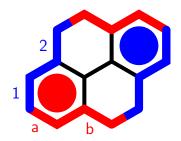
 $\hexscale{3.5}$

\begin{hexpicture}

\hexboard{2}

\hexcounter{a}{1}{A}

\hexcounter{b}{2}{B}
\end{hexpicture}



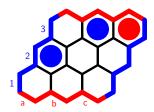
4.2 Colours

Player A's colour can be set using \setcolorA{color} and player B's colour can be set using \setcolorB{color}. The defaults are red for player A and blue for player B.

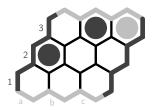
For example, for a greyscale diagram use

\setcolorA{lightgray}
\setcolorB{darkgray}

This changes this



into this



To use the current user colours elsewhere in the document, access these as \colorA and \colorB . For example

{\color{\colorA} This text is player A's colour} and {\color{\colorB} This text is player B's colour}.

\setcolorA{teal}
\setcolorB{magenta}
{\color{\colorA} This text is player A's colour} and

{\color{\colorB} This text is player B's colour}.

This text is player A's colour and This text is player B's colour. This text is player A's colour and This text is player B's colour.

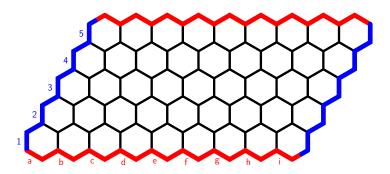
5 Advanced use

5.1 Drawing an irregular board

The command \hexgrid{columns}{rows} is used to draw a grid of that size. In fact, the command \hexboard is provided for convenience and simply calls \hexgrid with the same number of rows and columns.

For example

\begin{hexpicture}
 \hexgrid{9}{5}
\end{hexpicture}

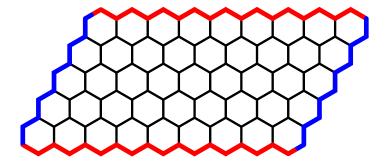


5.2 Drawing a board with no edge labels

The command \hexgridnolabel{columns}{rows} is used to draw a grid of that size without edge labels.

For example

\begin{hexpicture}
 \hexgridnolabel{9}{5}
\end{hexpicture}



5.3 Drawing a single cell

An individual cell is drawn using $\mbox{hexcell{x}{y}}$. For example

\begin{hexpicture}

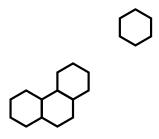
 $\hexcell{a}{1}$

\hexcell{b}{1}

 $\hexcell{b}{2}$

 $\hexcell{c}{4}$

\end{hexpicture}



5.4 Drawing a sub-row

The command \hexsubrow{start}{end}{row} draws a partial row of cells from column start to column end on row row.

For example

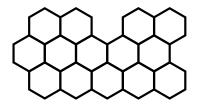
\begin{hexpicture}

\hexsubrow{b}{f}{1}

\hexsubrow{a}{e}{2}

 $\hexsubrow{a}{b}{3}$

\hexsubrow{d}{e}{3}



5.5 Shading a sub-row

The command \hexshadedsubrow{start}{end}{row} draws a partial row of shaded cells from column start to column end on row row.

For example

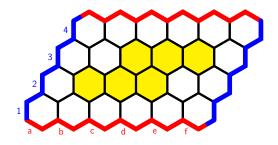
\begin{hexpicture}

 $\hexgrid{6}{4}$

\hexshadedsubrow{b}{d}{2}

\hexshadedsubrow{c}{e}{3}

\end{hexpicture}



5.6 Drawing a partial bottom border

The command \hexbottomsubborder{start}{end} draws a bottom border from column start to column end.

For example

\begin{hexpicture}

\hexsubrow{b}{f}{1}

 $\hexsubrow{a}{e}{2}$

 $\hexsubrow{a}{d}{3}$

\hexbottomsubborder{b}{f}



5.7 Drawing a reducing part-board

The command \hexreducing{columns}{rows} draws a grid with a bottom border only that starts at columns wide but decreases width by 1 each row until it has drawn rows rows.

For example

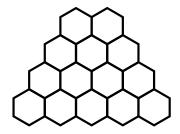
\begin{hexpicture}
 \hexreducing{6}{3}
\end{hexpicture}



You can also draw a reducing board without a bottom coloured border using \hexreducingnoborder{columns}{rows}.

For example

\begin{hexpicture}
 \hexreducingnoborder{5}{4}
\end{hexpicture}



5.8 A note about board size

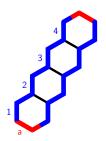
A 1 by 1 Hex board is possible.

\begin{hexpicture}
 \hexboard{1}
\end{hexpicture}

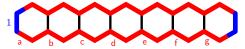


By extension, irregular boards which are a single column or row are possible.

\begin{hexpicture}
 \hexgrid{1}{4}
\end{hexpicture}



\begin{hexpicture}
 \hexgrid{7}{1}
\end{hexpicture}



Theoretically there is no maximum size of the board, though note the board will happily move beyond the size of the paper, and references to cells beyond column \mathbf{z} are by number only, both when referring to coordinates and in the bottom edge labels.

For example

\hexscale{1.4}
\begin{hexpicture}
 \hexgrid{27}{4}
 \hexcounter{z}{4}{A}
 \hexcounter{27}{4}{B}
\end{hexpicture}



5.9 Use with tikz

The package uses tikz and \usetikzlibrary{shapes.geometric}. The definition of hexpicture (also used by hexgame) is

\newenvironment{hexpicture}{\begin{tikzpicture}[hex]}{\end{tikzpicture}} In theory, then, any tikz commands that will work in this context can be used.

Note that the diagrams were made based around one cell being 1em across. This leads to quite small diagrams, hence the default for \hexscale being set to 2. This means that

it is best to give tikz commands in ems, and has unfortunate consequences for example on text size. In the following example, the label is positioned using (14em, 3em) and the text is \tiny for this reason.

\begin{hexpicture}

\hexshadedsubrow{b}{h}{1}

\hexshadedsubrow{b}{g}{2}

\hexshadedsubrow{b}{f}{3}

 $\hexebox{exshadedsubrow{c}{d}{4}}$

\hexreducing{9}{5}

\hexcounter{c}{4}{A}

 $\hext{bexdot{c}{3}}$

\hexdot{e}{3}

 $\hexconnect{c}{4}{c}{3}$

 $\hexconnect{c}{4}{d}{3}$

 $\hexconnect{d}{3}{e}{3}$

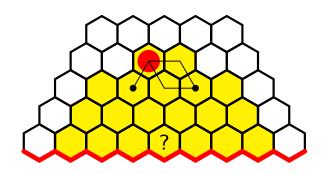
 $\hexconnect{c}{4}{4}{4}$

 $\hexconnect{d}{4}{e}{3}$

\hexcontent{e}{1}{?}

\node at (14em,3em) {\parbox{5em}{\tiny Consider what happens if blue puts a
piece in the hex marked ``?''.}};

\end{hexpicture}



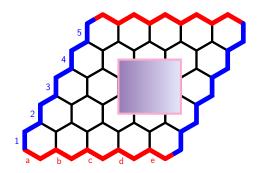
Consider what happens if blue puts a piece in the hex marked "?".

If you wish to draw tikz items relative to the grid, it may be useful to know that the centre of any cell on the grid can be determined using $\ensuremath{\mbox{hexcoord}\{x\}\{y\}}$. This command sets $\ensuremath{\mbox{xchor}}$ and $\ensuremath{\mbox{ychor}}$ to be the centre of the cell at (x,y), then these coordinates can be used in further commands.

So if you for some reason wanted a pinkish-purple box aligned with certain cells of your Hex grid, you could do this.

\definecolor{lavendermist}{rgb}{0.9, 0.9, 0.98} \definecolor{lavenderpink}{rgb}{0.98, 0.68, 0.82} \definecolor{lavenderpurple}{rgb}{0.59, 0.48, 0.71} \begin{hexpicture}

```
\hexboard{5}
  \hexcoord{c}{2}
  \node (n1) at (\xchor em,\ychor em) {};
  \hexcoord{d}{4}
  \node (n2) at (\xchor em,\ychor em) {};
  \draw[ultra thick,lavenderpink,left color=lavenderpurple,right color=lavendermist] (n1) rectangle (n2);
  \end{hexpicture}
```



5.10 Internal

Various internal commands are used:

hexedge (counter)

hexmovecount (counter)

\xchor

\ychor

\colorA

\colorB

\hexlinewidth

\hexedgewidth

\hexthinline

\hexcoord

\hexsize

\Ia

\Ib

\Iaend

\leftofIa

\belowIa

\leftofxchor

6 Some examples

\begin{hexpicture}

 $\hexcell{a}{1}$

\hexcellshaded{b}{1}

 $\hexcellshaded{a}{2}$

\hexcell{b}{2}

 $\hexapsum {1}{A}$

\hexcounter{b}{2}{A}

 $\hext{a}{2}$

 $\hexardon{b}{1}$

 $\hexconnect{a}{1}{a}{2}$

 $\hexconnect{a}{1}{b}{1}$

 $\hexconnect{a}{2}{b}{2}$

 $\hexconnect{b}{1}{b}{2}$

\end{hexpicture}



\begin{hexpicture}

\hexshadedsubrow{b}{c}{1}

\hexreducing{4}{2}

\hexcounter{b}{2}{A}

\hexdot{b}{1}

 $\hext{c}{1}$

 $\hexconnect{b}{2}{b}{1}$

 $\hexconnect{b}{2}{c}{1}$

\end{hexpicture}



\begin{hexpicture}

\hexshadedsubrow{b}{e}{1}

\hexshadedsubrow{b}{d}{2}

\hexshadedsubrow{b}{c}{3}

\hexreducing{6}{4}

\hexcounter{b}{3}{A}

 $\hexdot{b}{2}$

 $\hext{d}{2}$

 $\hexconnect{b}{3}{b}{2}$

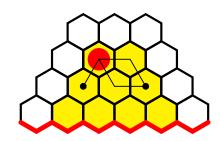
 $\hexconnect{b}{3}{c}{2}$

 $\hexconnect{c}{2}{d}{2}$

 $\hexconnect{b}{3}{c}{3}$

 $\hexconnect{c}{3}{d}{2}$

\end{hexpicture}



\begin{hexpicture}

\hexshadedsubrow{b}{c}{1}

\hexshadedsubrow{e}{f}{1}

\hexshadedsubrow{b}{e}{2}

\hexshadedsubrow{b}{d}{3}

\hexreducing{7}{4}

\hexcounter{c}{3}{A}

\hexdot{b}{2}

 $\hext{e}{2}$

 $\hexconnect{b}{2}{c}{2}$

 $\hexconnect{b}{3}{b}{2}$

 $\hexconnect{c}{3}{b}{3}$

 $\hexconnect{c}{3}{c}{2}$

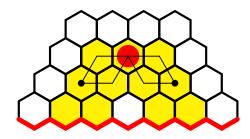
\hexconnect(c) $\{3\}\{d\}\{2\}$

 $\hexconnect{d}{2}{e}{2}$

 $\hexconnect{c}{3}{d}{3}$

 $\hexconnect{d}{3}{e}{2}$

\end{hexpicture}



\begin{hexpicture}

 $\verb|\hexshadedsubrow{b}{i}{i}{1}|$

\hexshadedsubrow{b}{d}{2}

 $\hexebox{hexshadedsubrow{f}{h}{2}}$

\hexshadedsubrow{b}{g}{3}

\hexshadedsubrow{c}{e}{4}

\hexreducing{10}{5}

\hexcounter{d}{4}{A}

\hexdot{c}{3}

 $\hext{f}{3}$

 $\hexconnect{c}{3}{d}{3}$

 $\hexconnect{c}{4}{c}{3}$

 $\hexconnect{d}{4}{c}{4}$

 $\hexconnect{d}{4}{d}{3}$

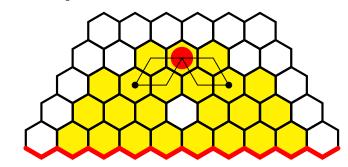
 $\hexconnect{d}{4}{e}{3}$

 $\hexconnect{e}{3}{f}{3}$

 $\hexconnect{d}{4}{e}{4}$

 $\hexconnect{e}{4}{f}{3}$

\end{hexpicture}



\begin{hexpicture}

\hexsubrow{b}{f}{1}

\hexsubrow{a}{e}{2}

 $\hexard {a}{d}{3}$

\hexsubrow{a}{c}{4}

\hexshadedsubrow{c}{e}{1}

\hexshadedsubrow{b}{d}{2}

\hexshadedsubrow{b}{c}{3}

\hexbottomsubborder{b}{f}

\hexcounter{b}{3}{A}

\hexcounter{c}{3}{A}



\begin{hexpicture}

\hexsubrow{b}{g}{1}

 $\hexarder{b}{f}{2}$

\hexsubrow{a}{e}{3}

 $\hexsubrow{a}{d}{4}$

\hexshadedsubrow{c}{f}{1}

\hexshadedsubrow{c}{e}{2}

\hexshadedsubrow{b}{d}{3}

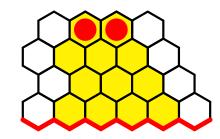
\hexshadedsubrow{b}{c}{4}

\hexbottomsubborder{b}{g}

\hexcounter{b}{4}{A}

\hexcounter{c}{4}{A}

\end{hexpicture}



\begin{hexpicture}

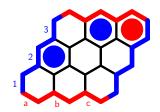
\hexboard{3}

\hexcounter{a}{2}{B}

\hexcounter{b}{3}{B}

\hexcounter{c}{3}{A}

\end{hexpicture}

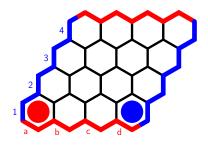


\begin{hexpicture}

\hexboard{4}

\hexcounter{d}{1}{B}

\hexcounter{a}{1}{A}



\begin{hexpicture}

\hexboard{5}

\hexcounter{b}{1}{B}

\hexcounter{c}{2}{B}

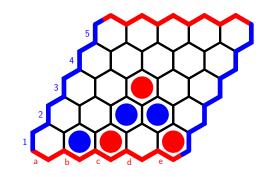
\hexcounter{d}{2}{B}

\hexcounter{c}{1}{A}

\hexcounter{e}{1}{A}

\hexcounter{c}{3}{A}

\end{hexpicture}



\begin{hexpicture}

\hexboard{11}

\hexcounter{a}{4}{B}

\hexcounter{b}{4}{B}

\hexcounter{c}{4}{B}

\hexcounter{d}{4}{B}

\hexcounter{e}{4}{B}

\hexcounter{f}{4}{B}

\hexcounter{g}{4}{B}

\hexcounter{h}{4}{A}

\hexcounter{i}{4}{B}

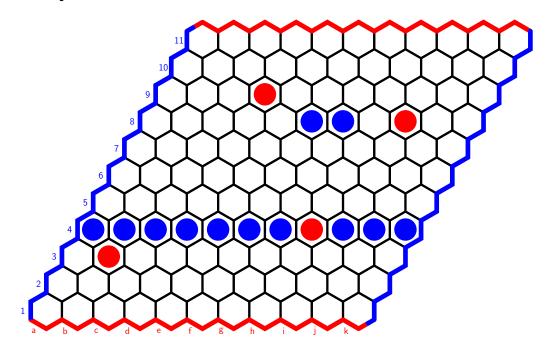
\hexcounter{j}{4}{B}

\hexcounter{k}{4}{B} \hexcounter{b}{3}{A}

\hexcounter{f}{8}{B}

\hexcounter{g}{8}{B}

\hexcounter{i}{8}{A} \hexcounter{d}{9}{A} \end{hexpicture}



\begin{hexpicture}

\hexsubrow{b}{c}{1}

\hexsubrow{a}{c}{2}

 $\hexsubrow{a}{b}{3}$

 $\hexsubrow{f}{g}{1}$

\hexsubrow{e}{g}{2}

\hexsubrow{e}{f}{3}

 $\hexsubrow{j}{k}{1}$

 $\hexsubrow{i}{k}{2}$

\hexsubrow{i}{j}{3}

\hexcounter{b}{2}{A}

\hexcounter{b}{1}{B}

\hexcounter{c}{1}{A}

\hexcounter{f}{2}{A}

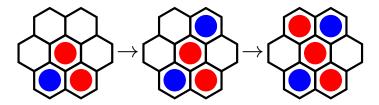
\hexcounter{f}{1}{B}

```
\hexcounter{g}{1}{A}
\hexcounter{f}{3}{B}

\hexcountent{h}{2}{\(\to\)}

\hexcounter{j}{2}{A}
\hexcounter{j}{1}{B}
```

\hexcounter{j}{1}{B}
\hexcounter{k}{1}{A}
\hexcounter{j}{3}{B}
\hexcounter{i}{3}{A}
\end{hexpicture}

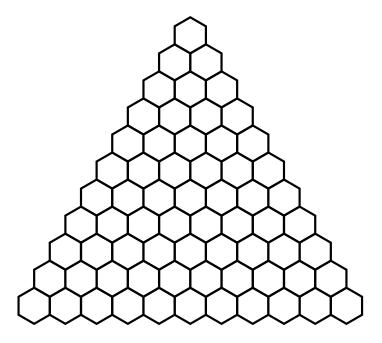


7 Other games

The target of this package is Hex. However, notice that some games on a hexagonal grid can be created using the commands provided here with cells shaded and counters, dots and lines placed in the usual way.

For example, Y is played on a reducing hexagonal grid that can be created using **\hexagonal proborder**. For example

\begin{hexpicture}
 \hexreducingnoborder{11}{11}
\end{hexpicture}



Similarly, games such as Cross, Pentalath, Yavalath and Three Player Hex can be played on a hexagonal board without coloured borders which can be created via use of **\hexsubrow**.

\begin{hexpicture}

 $\hexsubrow{a}{g}{13}$

 $\hexard {h}{12}$

\hexsubrow{a}{i}{11}

 $\hexsubrow{a}{j}{10}$

 $\hexsubrow{a}{k}{9}$

 $\hexsubrow{a}{1}{8}$

 $\hexsubrow{a}{m}{7}$

\hexsubrow{b}{m}{6}

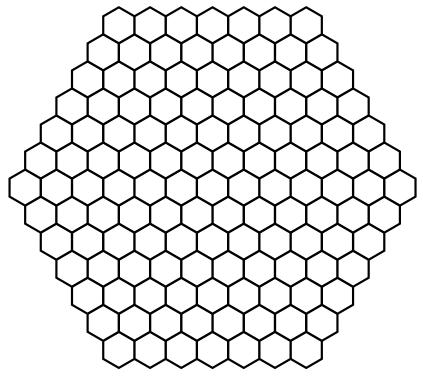
\hexsubrow{c}{m}{5}

 $\hexsubrow{d}{m}{4}$

\hexsubrow{e}{m}{3}

 $\mbox{hexsubrow}{f}{m}{2}$

 $\mbox{hexsubrow}\{g\}\{m\}\{1\}$



If desired, certain cells can be shaded in the usual way.

\begin{hexpicture}

 $\hexsubrow{a}{g}{13}$

 $\hexsubrow{a}{h}{12}$

 $\hexsubrow{a}{i}{11}$

\hexsubrow{a}{j}{10}

(HORDGOTOW (d) (J) (10)

 $\hexsubrow{a}{k}{9}$

 $\hexsubrow{a}{1}{8}$

 $\hexsubrow{a}{m}{7}$

\hexsubrow{b}{m}{6}

\hexsubrow{c}{m}{5}

 $\hexaptor(d){m}{4}$

\hexsubrow{e}{m}{3}

 $\hexsubrow{f}{m}{2}$

\hexsubrow{g}{m}{1}

\hexshadedsubrow{g}{1}{2}

 $\hexellshaded{f}{3}$

\hexcellshaded{1}{3}

\hexcellshaded{e}{4}

\hexcellshaded{1}{4}

 $\mbox{\ hexcellshaded{d}{5}}$

\hexcellshaded{1}{5}

\hexcellshaded{c}{6}

```
\hexcellshaded{1}{6}
\hexcellshaded{b}{7}
\hexcellshaded{b}{8}
\hexcellshaded{k}{8}
\hexcellshaded{k}{8}
\hexcellshaded{b}{9}
\hexcellshaded{j}{9}
\hexcellshaded{j}{9}
\hexcellshaded{j}{10}
\hexcellshaded{i}{10}
\hexcellshaded{h}{11}
\hexcellshaded{h}{11}
\hexcellshaded{h}{12}
```

