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a) $\vec{x} \times \vec{v} = (f\vec{x}_0 + g\vec{v}_0) \times (\dot{f}\vec{x}_0 + \dot{g}\vec{v}_0) = \cancel{f\dot{f}(\vec{x}_0 \times \vec{x}_0)} + fg(\vec{x}_0 \times \vec{v}_0) + g\dot{f}(\vec{v}_0 \times \vec{x}_0) \cancel{+ g\dot{g}(\vec{v}_0 \times \vec{v}_0)}$

$= fg(\vec{x}_0 \times \vec{v}_0) + g\dot{f}(\vec{v}_0 \times \vec{x}_0)$

$= (fg - g\dot{f})(\vec{x}_0 \times \vec{v}_0)$

has to be 1

b) $f = \frac{a}{r_0}(\cos E - 1) + 1 \qquad g = \frac{1}{n}(M + \sin E - E)$

$\dot{f} = -\frac{a^2}{r r_0} n \sin E \qquad \dot{g} = \frac{a}{r}(\cos E - 1) - 1$

$f\dot{g} - g\dot{f} = \frac{a}{r r_0}\left((\cos E - 1) + \frac{r_0}{a}\right)\left((\cos E - 1) + \frac{r}{a}\right) + \sin E\,(M + \sin E - E)$

$= \frac{a}{r r_0}\left((\cos E - 1 + 1 - e)(\cos E - 1 + 1 - e\cos E) + \sin E\,(M + \sin E - E)\right)$

$= \frac{a}{r r_0}\left((\cos E - e)(\underbrace{\cos E - e\cos E}) + \sin E\,(\sin E - e\sin E)\right)$

$= \frac{a}{r r_0}\left((1-e)(\cos E - e)\cos E + (1-e)\sin^2 E\right)$

$= \frac{a}{r r_0}(1-e)\left(\overbrace{\cos^2 E - e\cos E + \sin^2 E}\right)$

$= \frac{a}{r r_0}(1-e)(1 - e\cos E) = \frac{a}{r r_0}\frac{r_0}{a}\frac{r}{a} = 1$