EE194/BIO196: Modeling Biological Systems
HW 1: Simple model of population growth

In this homework, you will write a Python program that models a particular population and projects its initial population forwards.

We will be dealing with simple age-based categories. We're studying a population of animals with the following characteristics:

- They can be reasonably grouped into four categories: 0-1 years old, 1-2 years old, 2-3 and 3-4 years old. Any animal that reaches their fourth birthday immediately dies.
- We will model only the females.
- We will describe birth rate with 4 variables. m_0 is the average number of female offspring that an individual will have when they are between 0 and 1. Similarly, m_0, m_1, m_2 and m_3 are the average number of female offspring expected for the other three age categories. We will use a birth-pulse model as discussed in class, where all of the offspring are born in one pulse just after entering the new age group, and just before the population is counted. You may assume that m_0 is always 0.
- We will describe survival rates with another 4 variables. The fraction of newborn animals that survive to their 1st birthday is p_0. Similarly, p_1 is the fraction of animals who, once reaching their 1st birthday, survive to their 2nd. p_2 and p_3 are defined similarly. As we have noted above, p_3 will always be zero.

You should use another four variables n_0, n_1, n_2 and n_3 to model the current population in the four age stages at any time. Note that for any simulation, m_0-m_3 and p_0-p_3 will remain constant, since the vital rates are not changing. However, n_0-n_3 will potentially change at every simulation step, since the population itself may grow or shrink.

In this homework and also in homework #2, fractional individuals are fine; no need to round anything.

***Problem #1*.**

For the first two problems, use the following values for birth rates: m_0=0, m_1=1, m_2=5/6, and m_3=0. For survival rates, use p_0=0.6, p_1=0.8, p_2=2/3 and p_3=0. Assume that the initial-population size is n_0=0, n_1=1200, n_2=900 and n_3=900 (and as usual, this is only the females).

Your first task is to simulate the initial population forwards by one year. Your code should run the simulation for one year, and then print out the number of females in each of the 4 age groups. As a double check, at the end of the year you should see n_0=800, n_1=0, n_2=960 and n_3=600.

***Problem #2*.**

Using exactly the same vital rates and initial population sizes as in problem #1, your next task is to simulate 70 years. In principle, you could try to make 70 copies of your code from part 1. However, it will be much more clear to use a loop. You should use the same parameters as in problem #1. After each year, you should print out the year number, the number of females in each of the 4 age groups, and the total female population.

***Problem #3*.**

Use the same code as in problem #2 and the same initial population. Use all of the same vital rates as well, except that this time you should set m_2=.9 (where previously it was 5/6). Rerun the same 70 years of population growth as in problem 2, again printing out the population in each age group every year.

Discussion:

- Check your answer for problem #1 manually. Did the computer get the right answer?
- Look at the last few iterations for problem number 2. Has the population reached any kind of pattern? For example, has the number of individuals in each age stage roughly stabilized (and to what numbers, if so)? Or has it achieved exponential growth (and if so, with what growth rate)?
- Same question, but this time for problem #3.
- Any time we simulate a model, it's a good idea to sanity-check our results. First, note that problem #1 and #2 use the same initial population and vital rates. Should the first-year simulation in problem #2 match that of problem #1? If so, does it?
- Going from problem #2 to problem #3, we have changed m_2 from roughly .866 to .9. Would we expect that change to make our population growth faster or slower? Is that what you actually see in your simulation?

Extra credit

An early version of this homework had physically improper values for p_0-p_3. Let's explore constraints on their values a bit deeper. Many populations eventually do reach a stable exponential growth, where the ratios between the different age-group sizes stay constant.

Given a set of vital rates p_0-p_3 and m_0-m_3, what are the stable ratios n_1/n_0, n_2/n_1 and n_3/n_2 that the age-group populations will satisfy? To make it easier, assume that we eventually approach a stable situation with an exponential growth rate of exactly zero; i.e., the population eventually stabilizes to a constant value (without this assumption, we have an eigenvector problem).

Exponential growth cannot last forever; the world cannot support an infinitely-big population! Resources are finite; there is only a finite amount of food, shelter area and other resources to go around. As the population gets larger, it is thus reasonable to expect the vital rates to decrease.

Let's say we wanted to keep our survival rates unchanged, but alter our birth rates such that the environment only supported a total population of K individuals (often called the *carrying capacity*). How might you do this? Remember that when a population reaches its carrying capacity, it will stay at that same size forever – so you can use the result that you just derived above. You might also consider using the expression N/K to adjust the birth rates so that, as the total population reaches the carrying capacity, the birth rate becomes just what it needs to feed the downstream age classes stably.

Logistics:

- Use any lab PC system to write your code. You may use your own laptop if you prefer.
- The due date for this assignment is on the class calendar
- Submit your project at https://www.ece.tufts.edu/ee/194MSO/provide.cgi, which is also accessible from the course web page. You should submit:

- HW1.py. You should use comments to clearly mark which code is for problem #1, #2 and #3.
- a copy of your report, for the discussion questions. You may use whatever format is convenient, as long as we can reasonably read it.
- If you do the extra credit, you may choose to either include it with the discussion questions, or just come in during office hours and explain it to me.