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Discussion

1. The computer get the right answer for Problem 1.
2. The population reached a stable constant in the end: $n_0 = 571$ for 0-1 years age group, $n_1 = 342$ for 1-2 years age group, $n_2 = 274$ for 2-3 years age group, $n_3 = 182$ for 3-4 years age group, $n_{total} = 1371$ for total population.
3. The population achieved exponential growth in Problem 3. The growth rate is 1.022 for each age group and the total population.
4. Yes, they should be the same.
5. Yes, population should grow faster.
6. (Extra Credit)

   What are the stable ratios n_1/n_0, n_2/n_1 and n_3/n_2 that the age-group populations will satisfy?

   The stable ratios equals to the survival rate. $n_1/n_0 = p_0$, $n_2/n_1 = p_1$ and $n_3/n_2 = p_2$

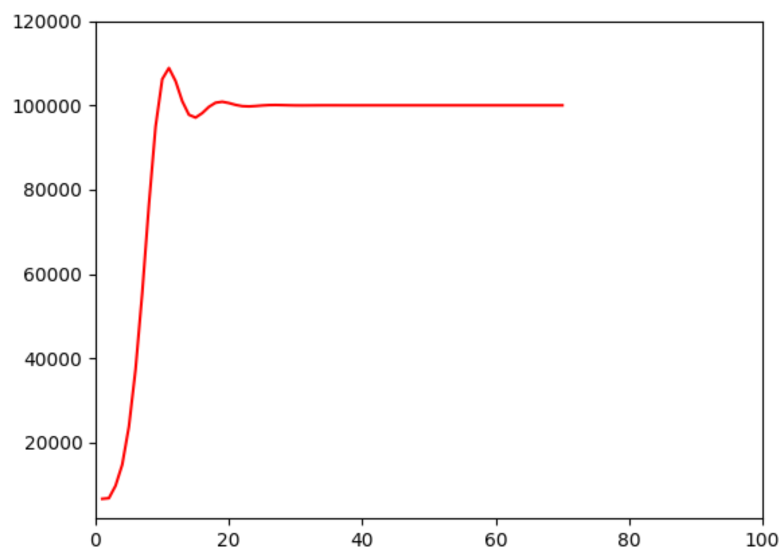

Fig.1 Total population when k=100000

Suppose $k = 100000$. Set the birth rate keep decreasing with the increase of total population. When the total population reach the capacity, then birth rate also remains unchanged. Such birth rate should satisfy the requirement to keep population at a stable size.

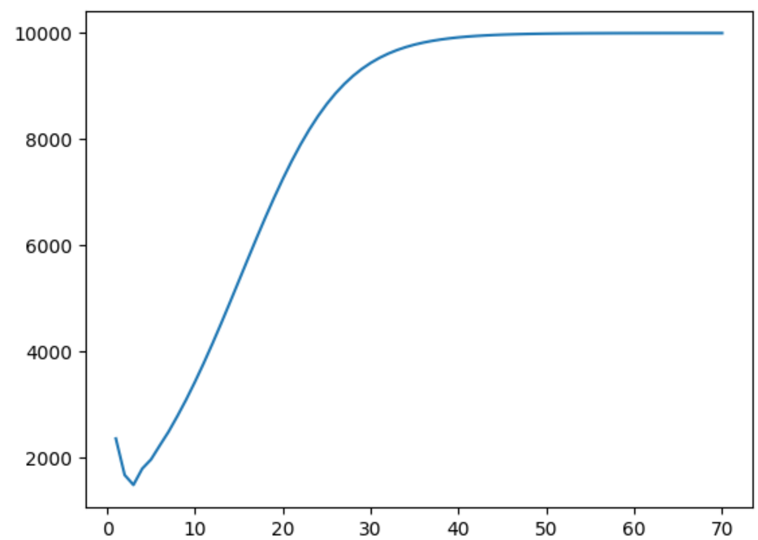

Fig.2 Total population when k=10000

If set $k = 10000$, and keep birth rate change at a slow speed. The total population will change like fig.2.

$$n_1 = n_0 \cdot p_0$$
$$n_2 = n_1 \cdot p_1$$
$$n_3 = n_2 \cdot p_2$$
$$n_0 = n_1 m_1 + n_2 m_2 + n_3 m_3 \qquad m_3 = 0$$

$$\begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ n_3 \end{bmatrix}^{(k+1)} = \underbrace{\begin{bmatrix} p_0 m_1 & p_1 m_2 & \cancel{p_2 m_3}\ 0 & 0 \\ p_0 & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & p_2 & 0 \end{bmatrix}}\begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ n_3 \end{bmatrix}^{(k)}$$

$$\Rightarrow n_0^{k+1} = n_0^k p_0 \cdot m_1 + n_1^k p_1 m_2 \cancel{+ n_2}.$$

$$A = \begin{bmatrix} p_0 m_1 & p_1 m_2 & 0 & 0 \\ p_0 & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & p_2 & 0 \end{bmatrix}$$

max Eigenvalues of $A \leq 1 \Rightarrow$ system ~~stab~~ stable.

The four eigenvalues of A is

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                                            0
                                            0
(3*m1)/10 - ((9*m1^2)/25 + (48*m2)/25)^(1/2)/2
(3*m1)/10 + ((9*m1^2)/25 + (48*m2)/25)^(1/2)/2
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So the sable birth rate should meet the requirement as $\frac{3*m_1}{10} + \frac{1}{2}\sqrt[2]{\frac{9*m_1^2}{25} + \frac{48*m_2}{25}} \leq 1$