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MATH 320 - Lab 6 Report

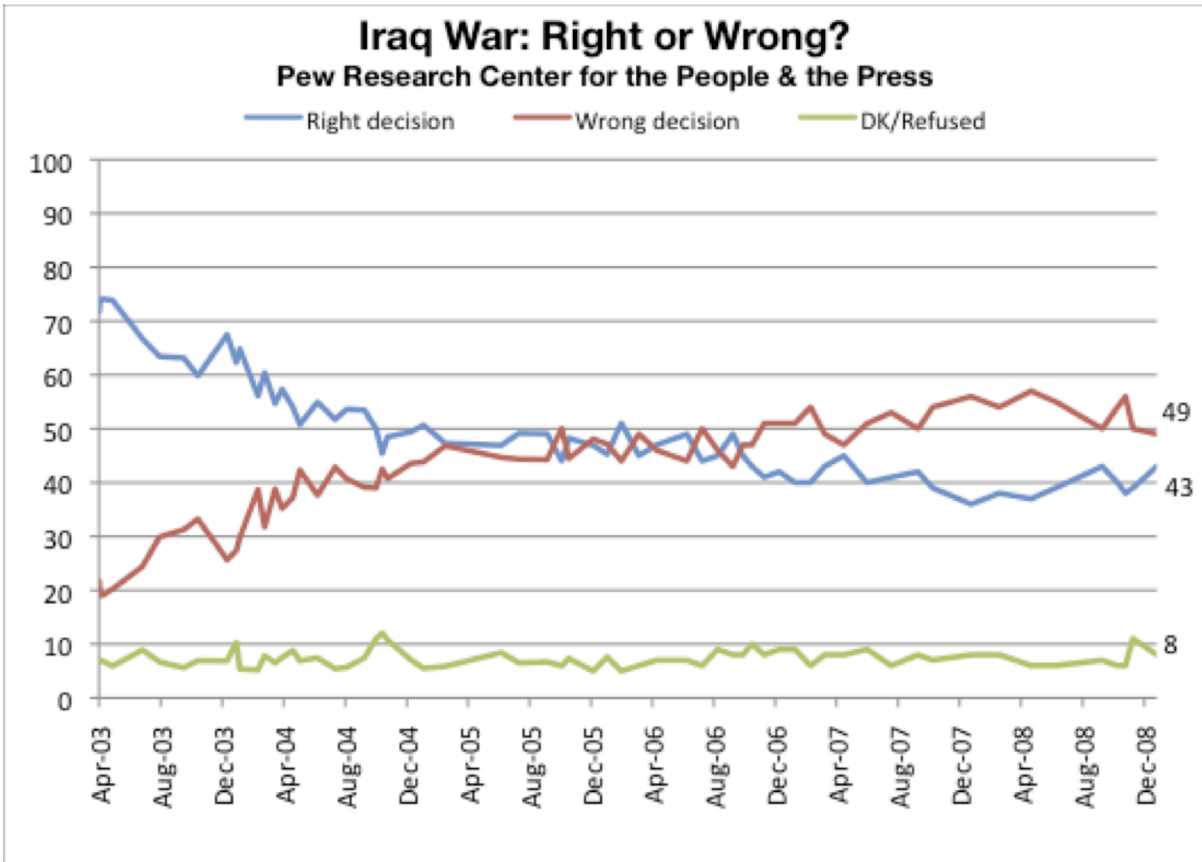
1. Objective:
The objective of this lab is to develop two differential equation models for the rate of change in percentage of people in their political positions on the Iraq War, with respect to time, which are the SIS model and the SIZS model.

2. Methodology and Results and interpretation/conclusion(s)
We are going to use two different models to measure the rate of change in percentage of people in their political positions on the Iraq War over time as follows:

(a) The SIS model
The SIS model does not take into account of the people who have no opinion or neutral opinion on the matter and only take into account of the people who are either against or in favor of the Iraq War. For this model, the rate of change of percentage in people in favor of and against Iraq War, with respect to time/years, can be modeled by the following differential equations:

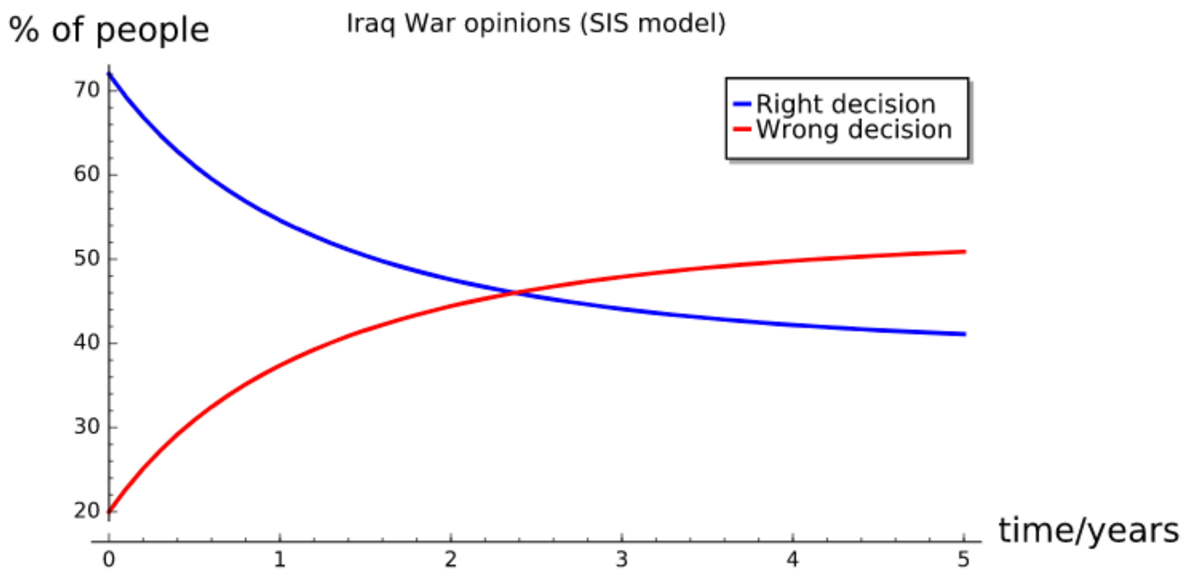
$$S' = -\beta\frac{SI}{N} + \gamma I$$
$$I' = \beta\frac{SI}{N} - \gamma I$$

where S and I, respectively, denote the percentage of people against the Iraq War and in favor of the Iraq War in a population of constant size N; S', I' denote the rate of change of people against and in favor of Iraq War, respectively, with respect to time t and β, γ are parameters. Since we are considering the percentage of people, let N = 100.



The Pew data shows that the data for the percentage of people against the war and in favor of the war converges towards a specific non-zero range after 5 years (around 50-55% of people in favor of the war and around 38-43% of people against the war). Thus, since SIS model admits non-zero, stable equilibrium solutions, it may be suitable to simulate the Pew data.

According to the data, we take the average equilibrium percentage of people in favor of the war to be 40.5 (I), and the average equilibrium percentage of people against the war to be 52.5 (S). At equilibrium, $S' = I' = 0$. Thus, we have $0 = -\beta \dfrac{52.5(40.5)}{100} + \gamma(40.5)$. Solve this equation, we have $\dfrac{\beta}{\gamma} = 1.9048$. Let the initial value of S and I to be 20 and 72, respectively. After trying for some values of γ, we estimate $\gamma = 0.65, \beta = 0.65\ (1.9048) = 1.23812$. Using Sage, we have the following graph to model the Pew data:



(b) The SIZS model

The SIZS model is based on the assumption that any members of any group can directly move into any other group. For this model, the rate of change of percentage in people in terms of their opinion on the Iraq War, with respect to time, can be modeled by the following differential equations:

$$S' = -\beta_i \frac{(S+Z)I}{N} - \beta_z \frac{(S+I)Z}{N} + \gamma_i I + \gamma_z Z$$
$$I' = \beta_i \frac{(S+Z)I}{N} - \gamma_i I$$
$$Z' = \beta_z \frac{(S+I)Z}{N} - \gamma_z Z$$

where S, I, Z, respectively, denote the percentage of people who have no opinion on, in favor of and against the Iraq War in a population of constant size N; S', I', Z' denote the rate of change of people who have no opinion on, in favor of and against the Iraq War, respectively, with respect to time t; the rest are parameters.

According to the data, S seems to be pretty constant throughout the year at 8%. We take the average equilibrium percentage of people in favor of the war (I) to be 40.5, and the average equilibrium percentage of people against the war (Z) to be 52.5. At equilibrium, $I' = Z' = 0$. Thus, we have $0 = \beta_i \frac{(52.5+8)(40.5)}{100} - \gamma_i(52.5)$. Solve this equation, we have $\frac{\beta_i}{\gamma_i} = 1.6529$. We also have $0 = \beta_z \frac{(40.5+8)(52.5)}{100} - \gamma_z(52.5)$. Solve this equation, we have $\frac{\beta_z}{\gamma_z} = 2.062$. Let the initial value of I and Z to be 72 and 20 respectively.

After trying for some values of $\gamma_i,\ \gamma_z,$ we estimate $\gamma_i = 0.5, \beta_i = 0.5$ (1.6529), and estimate $\gamma_z = 1.2,\ \beta_z = 1.2\ (2.062)$.

Using Sage, we have the following graph to model the Pew data: