grassroots_politics_model

November 24, 2017

In [0]:
```
%hide
%html
<H3  ALIGN="CENTER"><FONT COLOR="#030856">
DE models for the spread
of grassroots political views
</FONT></H3>

<H5>
    Introduction
</H5>


Differential equation models for the spread of infectious
diseases have been widely extended and adapted to a
variety of other application areas.  In this lab we explore a
class of similar models for studying the spread of
political views.  As with infectious diseases, the overall
strategy consists of splitting the entire population into
distinct, non-overlapping groups, whose numbers change
as a function of time.
<P></P>

Consider, for example, the problem of modeling a political
issue on which there are only two sides.  Let $N$ denote the
population of interest and let $I$ represent the number of
individuals in
the population who support the political idea under
consideration.  Then the number who
oppose the idea would be $S=N-I$.  This provides a natural
framework for adapting an SIS (Susceptible-Infected-Susceptible)
type of infectious disease
model.
<P></P>

<TABLE ALIGN="CENTER" WIDTH="80%" BORDER="0"  CELLSPACING="0"  CELLPADDING="8">
  <TR VALIGN="TOP">
    <TD ALIGN="CENTER">
```

```
      <IMG SRC="./sis_picture.png" width="400">
      </IMG>
    </TD>
  </TR>
</TABLE>


As seen in the compartmental sketch, the only difference
between the disease and the political ideas model is
in how we interpret the contents of each compartment.
However, the options get more interesting, and diverse,
if we consider models with 3 or more compartments.
For instance, the compartmental sketch below shows an
SIRS (Susceptible-Infected-Recovered-Susceptible)
schematic for diseases and for political ideas.
<P></P>

<TABLE ALIGN="CENTER" WIDTH="80%" BORDER="0"  CELLSPACING="0"  CELLPADDING="8">
  <TR VALIGN="TOP">
    <TD ALIGN="CENTER">
      <IMG SRC="./sirs_picture.png" width="500">
      </IMG>
    </TD>
  </TR>
</TABLE>

Unlike the infectious disease situation, in the case of
political ideas, members of any group
can directly move into any other group.  Thus, the model
terms must be modified to account for these additional
possibilities.
```

```
In [0]: %hide
        %html

        <H5>
            Lab project
        </H5>

        We will develop an SIS and SIR type of model for the spread
        of political ideas.  This will involve model development,
        analysis, and parameter estimation.  To help us think
        concretely through the details of model development, we
        will work with historical data from past political events.
        The figure below shows a 5-year history of U.S. public opinion
        about the U.S. war against Iraq, compiled by the Pew Research
        Center.
        <P></P>
```

```
<TABLE ALIGN="CENTER" WIDTH="80%" BORDER="0"  CELLSPACING="0"  CELLPADDING="8">
  <TR VALIGN="TOP">
    <TD ALIGN="CENTER">
      <IMG SRC="./pew_iraqwar.png"  width="700">
      </IMG>
    </TD>
  </TR>
</TABLE>


Although the results are shown in the form of a graph,
the details are sufficient to estimate approximate values
for the parameters, and to fit approximate
models to the data.  We will study two models for
simulating these data.
<P> </P>


<OL>
    <LI><B>The SIS model:</B>
From the epidemiology literature, this model has the form
\[
  S^\prime = -\beta \frac{S I}{N} + \gamma I  \\
  I^\prime = \beta \frac{S I}{N} - \gamma I
\]
where $S$ and $I$, respectively, denote the number of
susceptibles and infectives in a population of constant size $N$, and
the rest of the quantities are parameters.  In adapting this
model to political ideas/opinion, we would need to associate
$S$ and $I$ with the number of people on either side of a
political position.  This model doesn't permit more than two
sides for the political position.  We will do our best to fit
it to the Iraq war data.  To keep things simple, let $N=100$.
Then $S$, $I$, will directly correspond to percentages
of the population.  Here are some items to explore and address </LI>
<SUP> </SUP>
<UL>
    <LI>The SIS model admits non-zero, stable equilibrium
solution(s).  Why does that suggest it may be
suitable to simulate the Pew data (if we focus on two variables)? </LI>

    <LI>Make<SUP> </SUP>suitable choices for the two variables, e.g., by
ignoring one variable or combining it with one of the others. </LI>

    <LI>Using<SUP> </SUP>the Pew data, estimate the values of the
parameters in the model.  Use data from the graphs, together
```

```
with your understanding of equilibrium solutions and stability
to get the best parameter values.  Compute solutions (using Sage
or other methods) and show how well your model works. </LI>
</UL>

<P> </P>


    <LI><B>The SIZS model:</B>
This is an extension of the
SIR model from epidemiology.  The key difference is that it
allows members of any group to directly move into any
other group.  The model has the form
\[
  S^\prime = -\beta_i \frac{(S+Z) I}{N} - \beta_z \frac{(S+I) Z}{N}
        + \gamma_i I + \gamma_z Z  \\
  I^\prime =  \beta_i \frac{(S+Z) I}{N} - \gamma_i I \\
  Z^\prime =  \beta_z \frac{(S+I) Z}{N}  - \gamma_z Z
\]
In this model, the variables would have the following
interpretation:  <BR></BR>
    $S(t) =$ number of susceptible people at time t
(e.g., no opinion on the Iraq war) <BR></BR>
    $I(t) = $ number of people "infected" with the idea  <BR></BR>
    $Z(t) = $ number of people opposed to the idea $I(t)$ above  <BR></BR>

Here are the tasks to consider with this model
 </LI>
<SUP> </SUP>
<UL>
    <LI>Make suitable choices for the variables. </LI>
    <LI>Using the Pew data, estimate the values of the
parameters in the model.  Think about equilibrium
solutions and stability to help in this task. </LI>
    <LI>Here is one strategy that works, though there may
be others that work even better.  The segment below is
reproduced from my rough notes, so use it at your own risk! <BR></BR>
There are 4 parameters in this model.
Suppose we know the long-term (steady-state) values
of S, I, Z.  In other words, we know $S(\infty)$, $I(\infty)$,
and $Z(\infty)$.  Then, we can compute the following two ratios:
<BR></BR>
        $\gamma_i/\beta_i = (S + Z) / N$,
        $\gamma_z/\beta_z = (S + I) / N$
<BR></BR>
Next, suppose we can find a way to estimate
$I^\prime(0)$ and $Z^\prime(0)$, together with the values
```

```
$S(0), I(0), Z(0)$.  From the model ODEs we get
<BR></BR>
        $\gamma_i = \beta_i [S(0)+Z(0)] / N  - I^\prime(0)/I(0)$,
        $\gamma_z = \beta_z [S(0)+I(0)] / N  - Z^\prime(0)/Z(0)$
<BR></BR>
Then $\gamma_i, \gamma_z$ are known in terms of
$\beta_i, \beta_z$ respectively.  These can be plugged into the
previous $\gamma_i/\beta_i$, $\gamma_z/\beta_z$ ratios.
 </LI>

    <LI>Compute solutions (using Sage
or other methods) and show how well your model works. </LI>

</UL>

</OL>
<P> </P>
Shown below is sample sage code to solve a system of 3 ODEs
and plot each component of the solution on a common axis.
```

In [0]:
```
# Example showing num soln of the SIR diff eqn model
# Systems is: S'= - b*S*I; I' = b*S*I -g*I; R'= g*I

s, i, r, t = var('s i r t')
b = 5e-6
g = 1/2
s0 = 1e6
i0 = 5
r0 = 0
de1 = - b*s*i
de2 = b*s*i - g*i
de3 = g*i
P = desolve_system_rk4 ([de1, de2, de3], [s, i, r], ics=[0, s0, i0, r0], ivar=t, end_poi
Q = [ [i,j] for i, j, k, l in P]
P1 = line(Q)
Q = [ [i,k] for i, j, k, l in P]
P2 = line(Q, color='green')
Q = [ [i,l] for i, j, k, l in P]
P3 = line(Q, color='red')
show(P1+P2+P3)
```