Huong Phan, Son Tran, Shana Weisman

Age-structured Female Bobcats Modelling

1. Introduction

Bobcats, or *Lynx rufus,* is one of the most common predators in North America, with significant influence to lower levels of the food chain despite its scattered distribution (Campbell, 2017). They balance the ecosystem as it preys on a large scale of species, preventing overpopulation of species at lower levels of the food chain and keeping food sources at a sustainable level (Campbell, 2017). Bobcats usually breed in their first year of age i.e. their first reproductive season (Crowe, 1975), or their second year (Fritts and Sealander, 1978). Reproduction of bobcats is restricted to one breeding cycle per year, thus a deterministic age-distribution model can be established for bobcats (Crowe, 1975).

However, reproduction and survival of bobcats in various age class depends on different factors, such as habitat, level of exploitation and prey availability. Especially, the survival of newborn bobcats depends greatly on the abundance of preys, and they have higher mortality rate due to susceptibility to predation (Crowe, 1975). This is because female bobcats do not receive male assistance while rearing their newborns, so food abundance significantly affect their reproduction success and the newborn's survival chances (Ferguson et al, 2009). Thus, in the event of catastrophe, the food source availability may be exhausted, leading to a lower reproductive rate for female bobcats. A stochastic age-structured model that takes into account of a lower reproductive rate for all age class in the years of scarcer food availability due to a disaster may be considered.

Our project aims to build a deterministic matrix model for distribution of female bobcats by age in the long term based on survival rate and reproduction rate as seen in the life table above, as well as a stochastic model that take into account of lower reproductive rate due to possibility of catastrophe.

2. Methodology

Our model is based on the age class data from Cox et al., 1994

Age Class	Survival Worst Case	Survival Best Case	Reproduction Worst Case	Reproduction Best Case
1	0.32	0.34	0.60	0.63
2	0.68	0.71	0.60	0.63

3	0.68	0.71	1.15	1.20
4	0.68	0.71	1.15	1.20
5	0.68	0.71	1.15	1.20
⋮	⋮	⋮	⋮	⋮
15	0.68	0.71	1.15	1.20
16	0.68	0.71	1.15	1.20

Assuming that the maximum attainable age class of bobcats population is 16, we constructed state diagrams with 16 states representing 16 age classes. The reproduction rates are represented by arrows flowing from each state to state 1, while the survival rates are depicted by arrows flowing from one state to the subsequent state. From the state diagram, we constructed Leslie matrix (Leslie, 1945) to predict the long-term proportion of age-structured female population of bobcats, assuming initial equal proportions of female bobcats population in different age classes. The Leslie matrix is one of the most commonly used matrices for modeling population growth. It models a population that looks only at the female population and does not take into account other factors such as prey abundance or male population. This model divides the population over age classes, each age class has a best case and worst case rate for reproduction and survival. These values are what make up each Leslie matrix. One matrix represents the worst case, and the other represents the best case. This approach looks at each age class in the population over time in the long run.

$$\begin{bmatrix} b_1 & b_2 & b_3 & \dots & b_n \\ S_{12} & 0 & 0 & \dots & 0 \\ 0 & S_{23} & 0 & \dots & 0 \\ 0 & 0 & S_{3,4} & \dots & 0 \\ 0 & 0 & \dots & S_{i,i+1} & 0 \end{bmatrix}$$

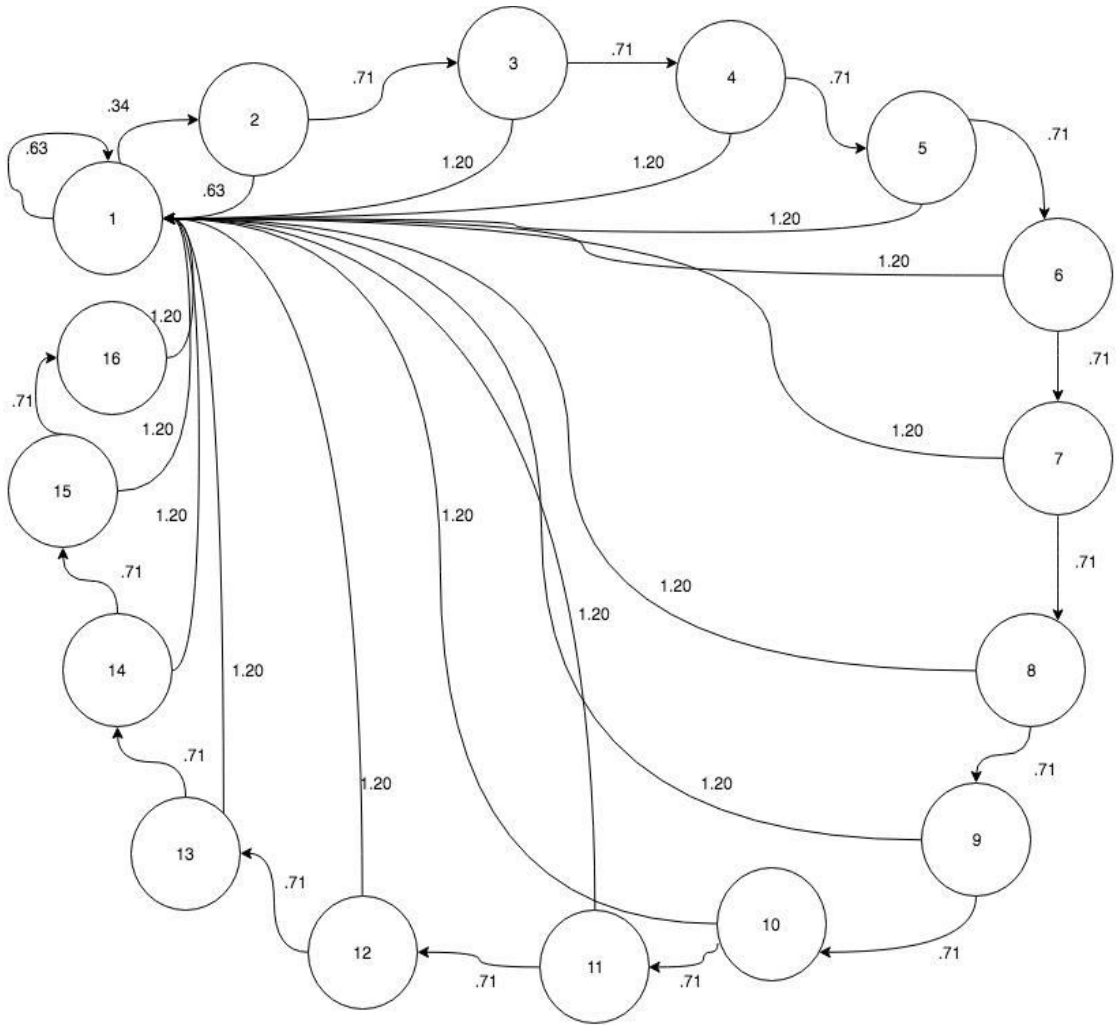
The above is the layout of the Leslie matrix, in which b_n denotes the reproduction rate for each of the n classes. $S_{i,\,i+1}$ represents the survival rate of each age group. The dominant eigenvalue and eigenvector of each Leslie matrix represents the stabilizing point of the population. Using Sage, we found the dominant eigenvalue and its corresponding right eigenvector to determine the stable age-structured population of female bobcats.

Using Excel, starting with unequal initial proportions of each age class in the population, we found the age class distribution after 10 years. We also plotted line graphs of proportion of each age class in the female bobcats population initially and in the long term, with initial uniform and non-uniform age class distribution for the best case and worst case

We also constructed a stochastic age-structured model for Bobcat population in a ten-year period, in which catastrophe would lower reproductive rate by 30% in 25 years on average. Using the same initial population (0, 100, 50, 50, 25, 10, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), we ran 100 simulations over a ten-year period. We used Excel's function NORMINV(RAND(), 25, 10) to generate the set of 100 catastrophe years.

3. Results

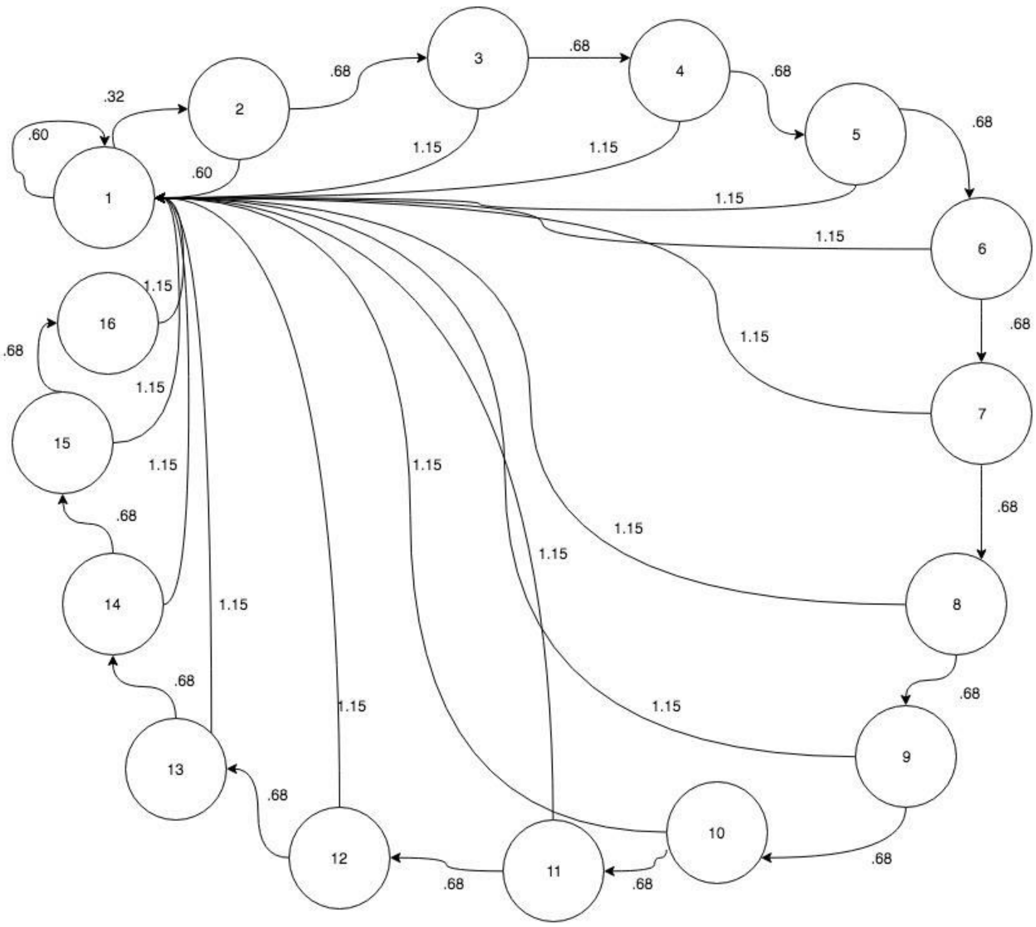
a. Deterministic age-distribution model



$$
\begin{bmatrix}
0.63 & 0.63 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 & 1.20 \\
0.34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & 0
\end{bmatrix}
$$

Figure 1. State Diagram and the Leslie matrix for the best case of survival rate and reproduction rate.

The dominant eigenvalue is 1.24144, with its corresponding eigenvector (1, 0.27388, 0.15663, 0.08958, 0.05123, 0.02930, 0.01676, 0.00958, 0.00548, 0.00313, 0.00179, 0.00103, 0.00059, 0.00034, 0.00019, 0.00011). Normalizing the vector would give us (0.60990, 0.16704, 0.09553, 0.05464, 0.03125, 0.01787, 0.01022, 0.00585, 0.00334, 0.0019, 0.00109, 0.00063, 0.00036, 0.0002, 0.00012, 0.00007). This normalized vector is the best case long term proportion of each age class population with its initial equal proportion.



$$
\begin{bmatrix}
0.60 & 0.60 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 & 1.15 \\
0.32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0
\end{bmatrix}
$$

Figure 2. State Diagram and the Leslie matrix for the worst case of survival rate and reproduction rate.

The dominant eigenvalue is 1.18285, with its corresponding eigenvector (1, 0.27053, 0.15552, 0.08941, 0.05140, 0.02955, 0.01699, 0.00977, 0.00561, 0.00323, 0.00186, 0.00107, 0.00061, 0.00035, 0.00020, 0.00012). Normalizing the vector would give us (0.61117, 0.16534, 0.09505, 0.05464, 0.03141, 0.01806, 0.01038, 0.00597, 0.00343, 0.00197, 0.00114, 0.00065, 0.00037, 0.00022, 0.00012, 0.00007). This normalized vector is the worst case long term proportion of each age class population with its initial equal proportion.

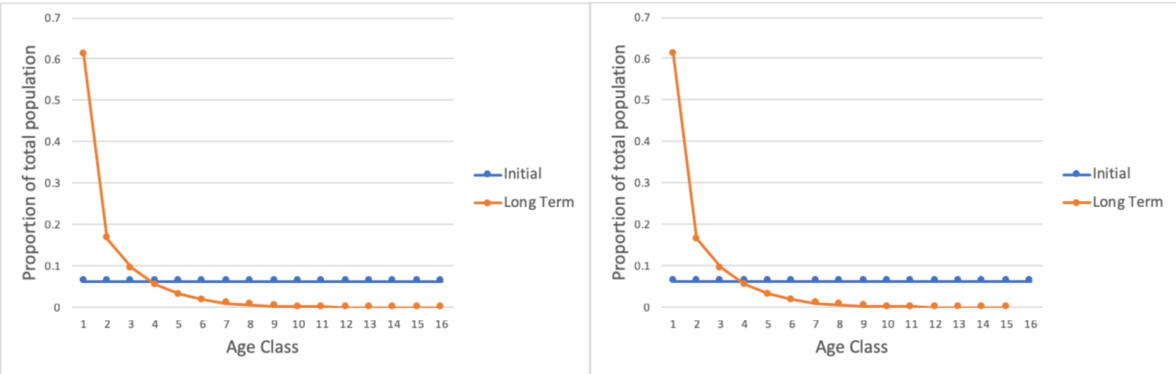


Figure 3. Line graphs of proportion of each age class in the female bobcats population initially and in the long term, assuming uniform distribution for the best case (left) and worst case (right).

However, bobcats population may not have the same initial proportion for each age class. If we start from a non-uniform distribution of age class population, represented by the population vector (0, 100, 50, 50, 25, 10, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), the worst case distribution of each age class population after 10 years is (6.99686, 0.00352, 1.55436, 1.55436, 0.77718, 0.31087, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), while the best case distribution of each age class population after 10 years is (9.83206, 0.00607, 2.29243, 2.29243, 1.14621, 0.45849, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0). Therefore, the worst case proportion of age-structured bobcats population is (0.62488, 0.00031, 0.13882, 0.13882, 0.06941, 0.02777, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), and the best case proportion of age-structured bobcats population is (0.61344, 0.00038, 0.14303, 0.14303, 0.07151, 0.02861, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0).

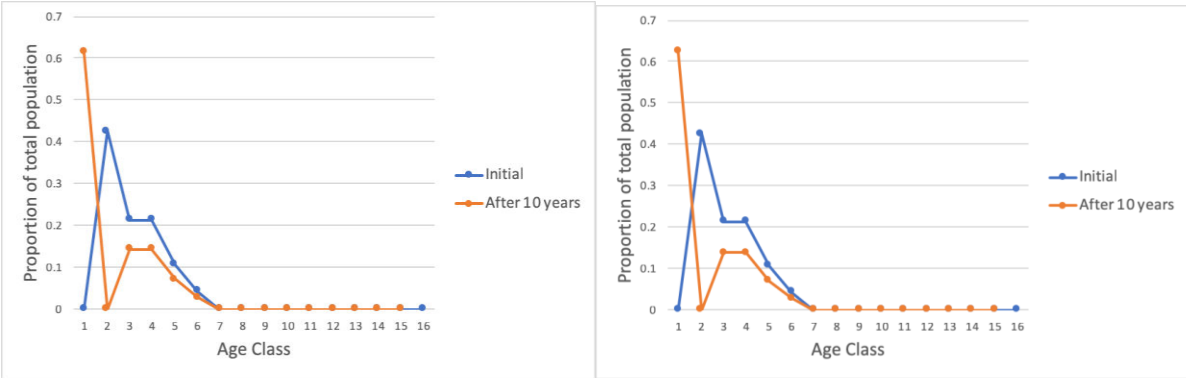


Figure 3. Line graphs of proportion of each age class in the female bobcats population initially and in the long term, assuming non-uniform distribution for the best case (left) and worst case (right).

b. Stochastic model

In this part, we construct a stochastic age-structured model for Bobcat population in a ten-year period given the population's average catastrophe timeline is every 25 years. Our assumptions are: i. the population's average catastrophe time is every 25 years; ii. the distribution of catastrophe time follows normal distribution model.; iii. the population standard deviation for average catastrophe time is 10 years.

Simulation No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Catastrophe year	24	6	32	7	33	30	23	28	31	19	26	23	38	18	11	23	21	15	26	23
Simulation No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Catastrophe year	22	21	31	18	25	31	10	21	23	23	30	25	16	28	13	45	32	20	23	10
Simulation No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Catastrophe year	31	18	37	19	27	8	29	9	33	27	15	20	33	37	45	18	22	24	24	46
Simulation No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Catastrophe year	6	26	28	30	4	26	43	25	29	15	9	19	36	19	6	18	22	2	21	32
Simulation No.	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
Catastrophe year	24	40	39	32	27	18	30	38	29	6	19	34	8	17	49	44	11	28	27	7

Figure 4. The catastrophe years in 100 simulations

Since we are looking into a ten year period only, all the simulation with catastrophe year larger or equal to 10 will produce the same result in the concerned period. Therefore, we will group them together into one line in the graph called "Others". After this modification, the catastrophe year table is as follows:

Simulation No.	**46**	**48**	**61**	**65**	**71**	**75**	**78**	**90**	**93**	**100**	**Others**
Catastrophe year	8	9	6	4	9	6	2	6	8	7	Others

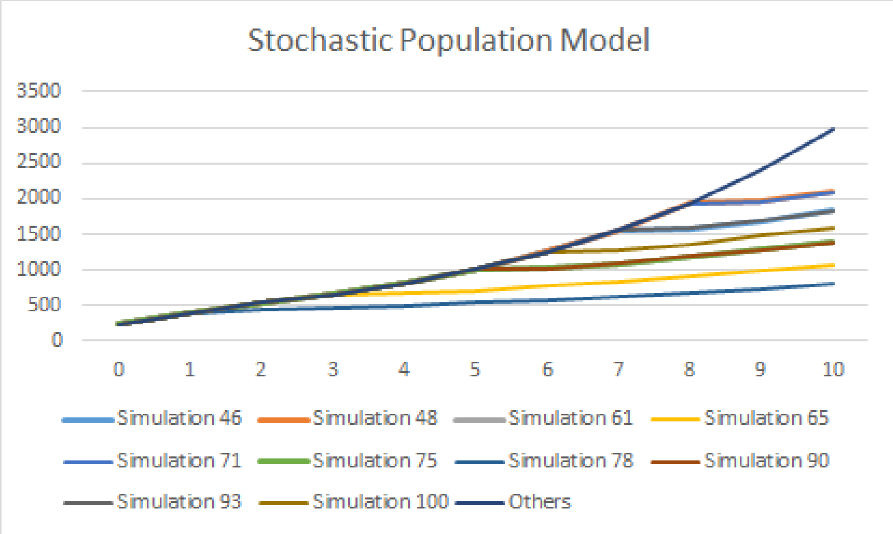
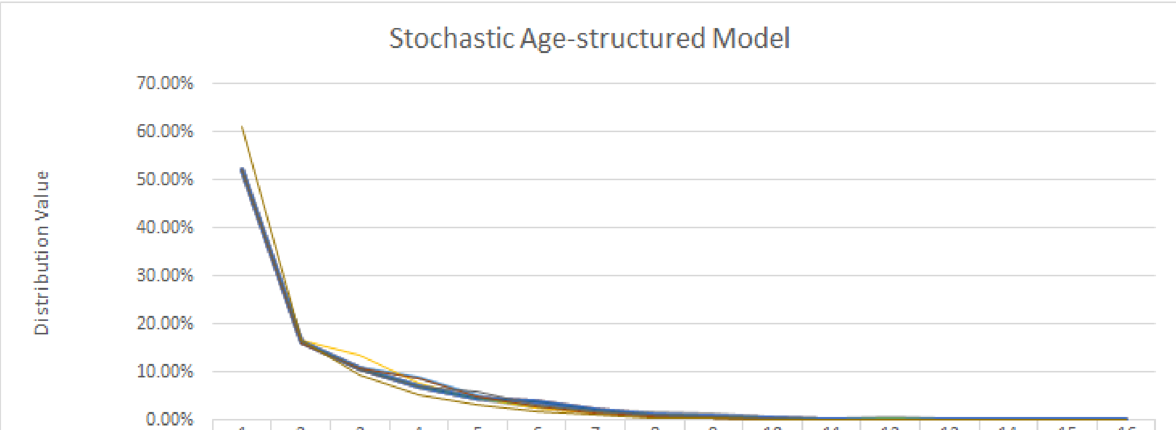


Figure 6. Multi-line graph representing bobcat total population over a ten year period



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Simulation 46	52.24%	16.13%	10.88%	8.89%	5.08%	2.90%	1.66%	0.96%	0.58%	0.27%	0.00%	0.18%	0.09%	0.09%	0.04%	0.02%
Simulation 61	52.28%	16.44%	10.75%	6.93%	4.68%	3.81%	2.18%	1.27%	0.76%	0.35%	0.00%	0.23%	0.12%	0.12%	0.06%	0.02%
Simulation 65	52.28%	16.42%	10.77%	7.07%	4.63%	2.98%	2.00%	1.67%	1.00%	0.47%	0.00%	0.31%	0.15%	0.15%	0.08%	0.03%
Simulation 71	51.56%	16.66%	13.61%	7.78%	4.45%	2.54%	1.45%	0.84%	0.51%	0.24%	0.00%	0.16%	0.08%	0.08%	0.04%	0.02%
Simulation 75	52.28%	16.44%	10.75%	6.93%	4.68%	3.81%	2.18%	1.27%	0.76%	0.35%	0.00%	0.23%	0.12%	0.12%	0.06%	0.02%
Simulation 78	52.28%	16.42%	10.77%	7.06%	4.64%	3.04%	1.98%	1.30%	0.93%	0.62%	0.00%	0.41%	0.20%	0.20%	0.10%	0.04%
Simulation 90	52.28%	16.44%	10.75%	6.93%	4.68%	3.81%	2.18%	1.27%	0.76%	0.35%	0.00%	0.23%	0.12%	0.12%	0.06%	0.02%
Simulation 93	52.24%	16.13%	10.88%	8.89%	5.08%	2.90%	1.66%	0.96%	0.58%	0.27%	0.00%	0.18%	0.09%	0.09%	0.04%	0.02%
Simulation 100	52.33%	16.38%	10.56%	7.13%	5.82%	3.32%	1.90%	1.10%	0.66%	0.31%	0.00%	0.20%	0.10%	0.10%	0.05%	0.02%
Others	60.99%	16.70%	9.55%	5.46%	3.12%	1.78%	1.02%	0.59%	0.35%	0.17%	0.00%	0.11%	0.05%	0.05%	0.03%	0.01%

Figure 7. Multi-line graph representing Bobcat age-structured population distribution over a ten year period

The results indicate that the catastrophe alter the age-structured population distribution. The most noticeable change is in the percentage that age group 1 accounts for in the overall population. This group's stake in the population drop more than 8% due to the catastrophe.

4. Discussion

Our results show that starting with an even proportion for each age group, the proportion of each age class in female bobcat population for the worst and best case cases are very similar in the long run, with age class 1 being the most abundant at approximately 61% of the whole female population. With each subsequent age class, its proportion in the population becomes increasingly lower for both the worst case and the best case. However, if we start from an unequal distribution of age-structured bobcat population, after 10 years the proportion of age class 1 also become the greatest with approximately 61-62%, which is similar to that if we start with initial uniform distribution of age classes. Age class 2 becomes the least abundant (~0.03%) among the age classes with non-zero proportions, despite starting with the greatest proportion (42%).

As telling as the models used in this project are, there are many more variables to be examined in the consideration of the bobcat population. Future work includes taking into account predator and prey population (Maehr 1986). Urbanization along with migration and immigration affects bobcat populations as well (Ruell, 2012). Furthermore, there are many different bobcat population in different regions in North America, modeling each individual population would more accurately depict each one. A population model that takes into account both sexes is another area to be examined in the future (Fritts 1978).

References

Campbell, K., The Ecosystems of Bobcats. *Sciencing,* 25 April 2017, retrieved 31 March 2019, https://sciencing.com/ecosystem-bobcats-22254.html

Cox, J., Kautz, R., MacLaughlin, M. and Gilbert, T., 1994. Closing the gaps in Florida's wildlife habitat conservation system. *Office of Environmental Services, Florida Game and Fresh Water Fish Commission, Tallahassee, Florida*.

Crowe, D.M., 1975. A model for exploited bobcat populations in Wyoming. *The Journal of Wildlife Management*, pp. 408-415.

Fritts, S.H. and Sealander, J.A., 1978. Reproductive biology and population characteristics of bobcats (Lynx rufus) in Arkansas. Journal of Mammalogy, 59(2), pp. 347-353.

Ferguson, A.W., Currit, N.A. and Weckerly, F.W., 2009. Isometric scaling in home-range size of male and female bobcats (Lynx rufus). *Canadian Journal of Zoology*, *87*(11), pp. 1052-1060.

Leslie, P.H., 1945. On the use of matrices in certain population mathematics. *Biometrika*, pp.183-212.

Maehr, D.S. and Brady, J.R., 1986. Food habits of bobcats in Florida. *Journal of Mammalogy*, *67*(1), pp.133-138.

Ruell, E.W., Riley, S.P.D., Douglas, M.R., Antolin, M.F., Pollinger, J.R., Tracey, J.A., Lyren, L.M., Boydston, E.E., Fisher, R.N. and Crooks, K.R., 2012. Urban habitat fragmentation and genetic population structure of bobcats in coastal southern California. *The American Midland Naturalist*, *168*(2), pp. 265-281.