S-05

2. Using mesh analysis, determine the voltage across the 10 kΩ resistor at terminals *a-b* of the circuit shown in Fig. 2.58. **[2.65 V]** *(Elect. Technology, Indore Univ.)*

3. Apply loop current method to find loop currents I_1, I_2 and I_3 in the circuit of Fig. 2.59. **[I_1 = 3.75 A, I_2 = 0, I_3 = 1.25 A]**

2.12. Nodal Analysis With Sources

The node-equation method is based directly on Kirchhoff's current law unlike loop-current method which is based on Kirchhoff's voltage law. However, like loop current method, nodal method also has the advantage that a minimum number of equations need be written to determine the unknown quantities. Moreover, it is particularly suited for networks having many parallel circuits with common ground connected such as electronic circuits.

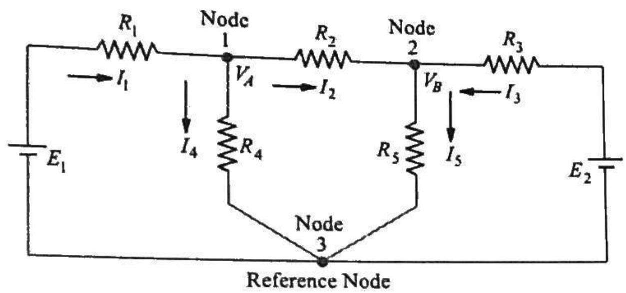


Fig. 2.60

For the application of this method, every junction in the network where three or more branches meet is regarded a node. One of these is regarded as the reference node or datum node or zero-potential node. Hence the number of simultaneous equations to be solved becomes $(n-1)$ where n is the number of independent nodes. These node equations often become simplified if all voltage sources are converted into current sources (Art. 2.12).

(i) First Case

Consider the circuit of Fig. 2.60 which has three nodes. One of these *i.e.* node 3 has been taken in as the reference node. V_A represents the potential of node 1 with reference to the datum node 3. Similarly, V_B is the potential difference between node 2 and node 3. Let the current directions which have been chosen arbitrary be as shown.

For node 1, the following current equation can be written with the help of *KCL*.

$$I_1 = I_4 + I_2 \qquad ...(i)$$

Now $\quad I_1R_1 = E_1 - V_A \quad \therefore \; I_1 = (E_1 - V_A)/R_1$

Obviously, $\quad I_4 = V_A/R_4 \quad$ Also, $I_2R_2 = V_A - V_B \quad (\because V_A > V_B)$

$\therefore \quad I_2 = (V_A - V_B)/R_2$

Substituting these values in Eq. (*i*) above, we get,

$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2}$$

Simplifying the above, we have

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0 \qquad ...(ii)$$

The current equation for node 2 is $I_5 = I_2 + I_3$

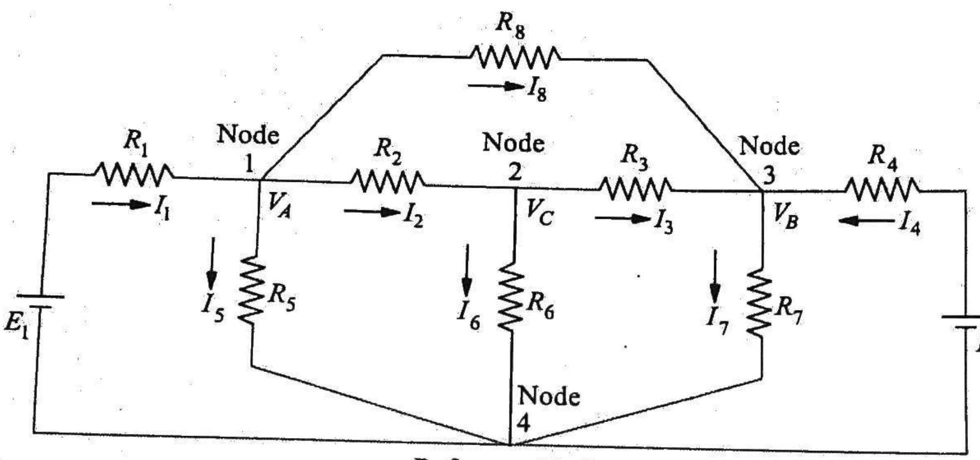
or $$\frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3} \qquad ...(iii)$$

or $$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0 \qquad ...(iv)$$

Though the above nodal equations (*ii*) and (*iii*) seem to be complicated, they employ a very simple and systematic arrangement of terms which can be written simply by inspection. Eq. (*ii*) at node 1 is represented by

1. The product of node potential V_A and $(1/R_1 + 1/R_2 + 1/R_4)$ *i.e.* the sum of the reciprocals of the branch resistance connected to this node.
2. ***Minus*** the ratio of adjacent potential V_B and the interconnecting resistance R_2.
3. ***Minus*** ratio of adjacent battery (or generator) voltage E_1 and interconnecting resistance R_1.
4. All the above set to zero.

Same is the case with Eq. (*iii*) which applies to node 2.



Reference Node

Fig. 2.61

Using conductances instead of resistances, the above two equations may be written as

$$V_A(G_1 + G_2 + G_4) - V_BG_2 - E_1G_1 = 0 \quad \text{...}(iv)$$

$$V_B(G_2 + G_3 + G_5) - V_AG_2 - E_2G_3 = 0 \quad \text{...}(v)$$

To emphasize the procedure given above, consider the circuit of Fig. 2.61.

The three node equations are

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_8}\right) - \frac{V_C}{R_2} - \frac{V_B}{R_8} - \frac{E_1}{R_1} = 0 \quad \text{(node 1)}$$

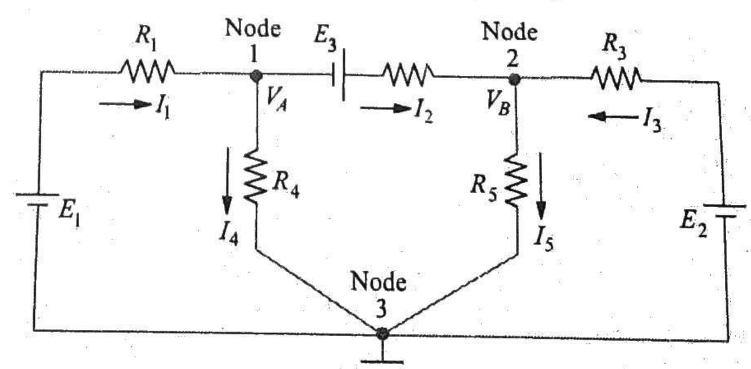
$$V_C\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6}\right) - \frac{V_A}{R_2} - \frac{V_B}{R_3} = 0 \quad \text{(node 2)}$$

$$V_B\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_7} + \frac{1}{R_8}\right) - \frac{V_C}{R_3} - \frac{V_A}{R_8} - \frac{E_4}{R_4} = 0 \quad \text{(node 3)}$$

After finding different node voltages, various currents can be calculated by using Ohm's law.

(*ii*) Second Case

Now, consider the case when a third battery of e.m.f. E_3 is connected between nodes 1 and 2 as shown in Fig. 2.62.



Reference Node

Fig. 2.62

It must be noted that as we travel from node 1 to node 2, we go from the –ve terminal of E_3 to its +ve terminal. Hence, according to the sign convention given in Art. 2.3, E_3 must be taken as *positive.* However, if we travel from node 2 to node 1, we go from the +ve to the –ve terminal of E_3. Hence, ***when viewed from node 2, E_3 is taken negative.***

For node 1

$$I_1 - I_4 - I_2 = 0 \text{ or } I_1 = I_4 + I_1 \text{ —as per KCL}$$

Now, $$I_1 = \frac{E_1 - V_A}{R_1};\ I_2 = \frac{V_A + E_3 - V_B}{R_2};\ I_4 = \frac{V_A}{R_4}$$

$\therefore$ $$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A + E_3 - V_B}{R_2}$$

or $$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{E_1}{R_1} - \frac{V_B}{R_2} + \frac{E_2}{R_2} = 0 \qquad ...(i)$$

It is exactly the same expression as given under the First Case discussed above except for the additional term involving E_3. This additional term is taken as $+E_3/R_2$ (and not as $-E_3/R_2$) because this third battery is so connected that when viewed from mode 1, it represents a *rise* in voltage. Had it been connected the other way around, the additional term would have been taken as $-E_3/R_2$.

For node 2

$$I_2 + I_3 - I_5 = 0 \quad \text{or} \quad I_2 + I_3 = I_5 \quad \text{—as per } KCL$$

Now, as before, $$I_2 = \frac{V_A + E_3 - V_B}{R_2},\ I_3 = \frac{E_2 - V_B}{R_3},\ I_5 = \frac{V_B}{R_5}$$

$\therefore$ $$\frac{V_A + E_3 - V_B}{R_2} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_5}$$

On simplifying, we get $$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0 \qquad ...(ii)$$

As seen, the additional terms is $-E_3/R_2$ (and not $+ E_3/R_2$) because as viewed from this node, E_3 represents a *fall* in potential.

It is worth repeating that the additional term in the above Eq. (*i*) and (*ii*) can be either $+E_3/R_2$ or $-E_3/R_2$ depending on whether it represents a rise or fall of potential when viewed from ***the node under consideration.***

Example 2.33. *Using Node voltage method, find the current in the 3Ω resistance for the network shown in Fig. 2.63.* **(Elect. Tech. Osmania Univ.)**

Solution. As shown in the figure node 2 has been taken as the reference node. We will now find the value of node voltage V_1. Using the technique developed in Art. 2.10, we get

$$V_1\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{2}\right) - \frac{4}{2} - \left(\frac{4+2}{5}\right) = 0$$

The reason for adding the two battery voltages of 2 V and 4 V is because they are connected in additive series. Simplifying above, we get $V_1 = 8/3$ V. The current flowing through the 3 Ω resistance towards node 1 is $= \dfrac{6 - (8/3)}{(3+2)} = \dfrac{2}{3}$ A

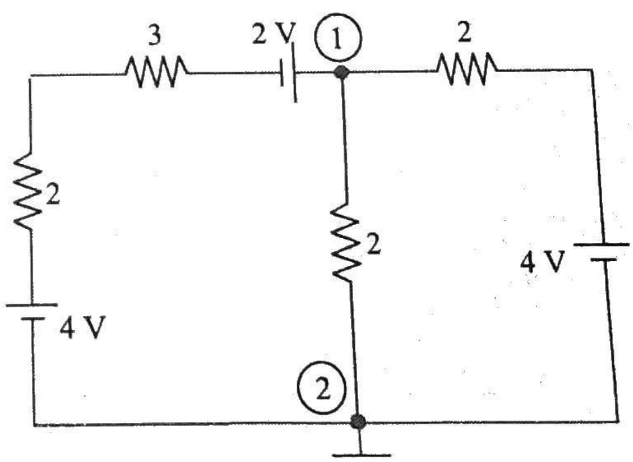


Fig. 2.63

Alternatively

$$\frac{6 - V_1}{5} + \frac{4}{2} - \frac{V_1}{2} = 0$$

$$12 - 2V_1 + 20 - 5V_1 = 0$$

$$7V_1 = 32$$

Also
$$\frac{6-V_1}{5} + \frac{4-V_1}{2} = \frac{V_1}{2}$$
$$12 - 2V_1 + 20 - 5V_1 = 5\,V_1$$
$$12V_1 = 32;\ V_1 = 8/3$$

Example 2.34. *Frame and solve the node equations of the network of Fig. 2.64. Hence, find the total power consumed by the passive elements of the network.* **(Elect. Circuits Nagpur Univ.)**

Solution. The node equation for node 1 is

$$V_1\left(1+1+\frac{1}{0.5}\right) - \frac{V_2}{0.5} - \frac{15}{1} = 0$$

or $4V_1 - 2V_2 = 15$ ...(*i*)

Similarly, for node 2, we have

$$V_1\left(1+\frac{1}{2}+\frac{1}{0.5}\right) - \frac{V_2}{0.5} - \frac{20}{1} = 0$$

or $4V_1 - 7V_2 = -40$ ...(*ii*)

$\therefore$ $V_2 = 11$ volt and $V_1 = 37/4$ volt

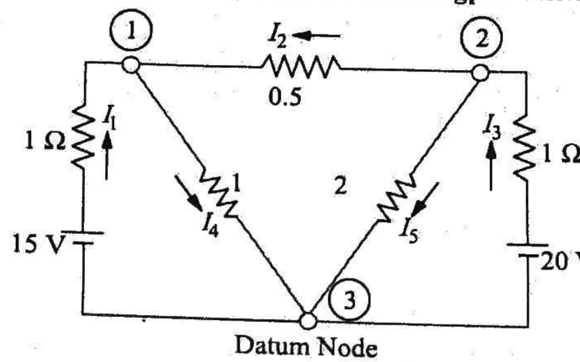


Fig. 2.64

Now,

$$I_1 = \frac{15-37/4}{I} = \frac{23}{4}\text{A} = 5.75\text{ A};\ I_2 = \frac{11-37/4}{0.5} = 3.5\text{ A}$$

$$I_4 = 5.75 + 3.5 = 9.25\text{ A};\ I_3 = \frac{20-11}{1} = 9\text{ A};\ I_5 = 9 - 3.5 = 5.5\text{ A}$$

The passive elements of the network are its five resistances. Total power consumed by them is $= 5.75^2 \times 1 + 3.5^2 \times 0.5 + 9^2 \times 1 + 9.25^2 \times 1 + 5.5^2 \times 2 = \mathbf{266.25}$

Example 2.35. *Find the branch currents in the circuit of Fig. 2.65 by using (i) nodal analysis and (ii) loop analysis.*

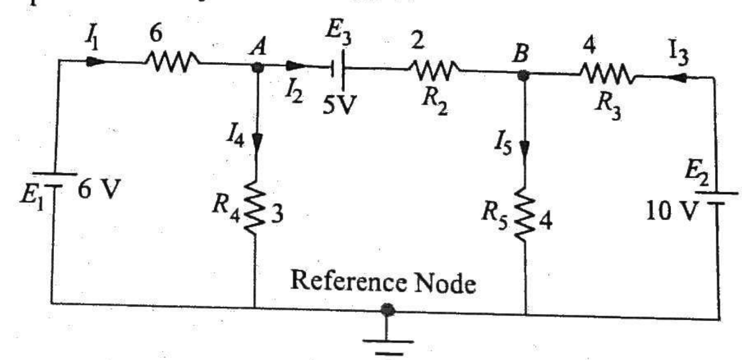


Fig. 2.65

Solution. (*i*) **Nodal Method**

The equation for node A can be written by inspection as explained in Art. 2-12.

$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right) - \frac{E_1}{R_1} - \frac{V_B}{R_2} + \frac{E_3}{R_2} = 0$$

Substituting the given data, we get,

$$V_A\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{3}\right) - \frac{6}{6} - \frac{V_B}{2} + \frac{5}{2} = 0 \quad \text{or } 2\,V_A - V_B = -3 \qquad \text{...(i)}$$

For node B, the equation becomes

$$V_B\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0$$

$$\therefore \qquad V_B\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right) - \frac{10}{4} - \frac{V_A}{2} - \frac{5}{2} = 0 \quad \therefore\ V_B - \frac{V_A}{2} = 5 \qquad \text{...(ii)}$$

From Eq. (*i*) and (*ii*), we get,

$$V_A = \frac{4}{3}V, V_B = \frac{17}{3}V$$

$$I_1 = \frac{E_1 \quad V_A}{R_1} \quad \frac{6 \quad 4/3}{6} \quad \frac{7}{9}\text{ A}$$

$$I_2 = \frac{V_A \quad E_3 \quad V_B}{R_2} \quad \frac{(4/3) \quad 5 \quad (17/3)}{2} \quad \frac{1}{3}\text{ A}$$

$$I_3 = \frac{E_2 \quad V_B}{R_3} \quad \frac{10 \quad 17/3}{4} \quad \frac{13}{12}\text{ A}$$

$$I_4 = \frac{V_A}{R_4} \quad \frac{4/3}{3} \quad \frac{4}{9}\text{ A}, \; I_5 \quad \frac{V_B}{R_5} \quad \frac{17/3}{4} \quad \frac{17}{12}\text{ A}$$

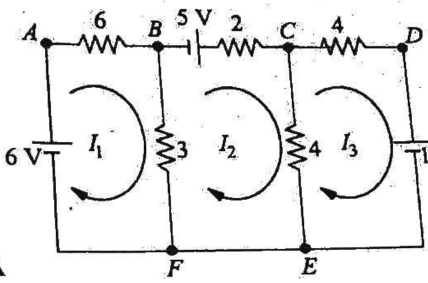


Fig. 2.66

(ii) Loop Current Method

Let the direction of flow of the three loop currents be as shown in Fig. 2.66.

Loop ABFA :

$$-6I_1 - 3(I_1 - I_2) + 6 = 0$$

or $\qquad 3I_1 - I_2 = 2 \qquad$ ...(*i*)

Loop BCEFB :

$$+5 - 2I_2 - 4(I_2 - I_3) - 3(I_2 - I_1) = 0$$

or $\qquad 3I_1 - 9I_2 + 4I_3 = -5 \qquad$ ...(*ii*)

Loop CDEC : $\qquad$ ...(*iii*)

$$-4I_3 - 10 - 4(I_3 - I_2) = 0 \quad \text{or} \quad 2I_2 - 4I_3 = 5$$

The matrix form of the above three simultaneous equations is

$$\begin{bmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix}; \Delta = \begin{bmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{bmatrix} = 84 - 12 - 0 = 72$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & 0 \\ -5 & -9 & 4 \\ 5 & 2 & -4 \end{vmatrix} = 56; \Delta_2 = \begin{vmatrix} 3 & 2 & 0 \\ 3 & -5 & 4 \\ 0 & 5 & -4 \end{vmatrix} = 24; \Delta_3 = \begin{vmatrix} 3 & -1 & 2 \\ 3 & -9 & -5 \\ 0 & 2 & 5 \end{vmatrix} = -78$$

$$\therefore \quad I_1 = \Delta_1/\Delta = 56/72 = 7/9\text{ A}; \; I_2 = \Delta_2/\Delta = 24/72 = 1/3\text{ A}$$

$$I_3 = \Delta_3/\Delta = -78/72 = \mathbf{-13/12\ A}$$

The negative sign of I_3 shows that it is flowing in a direction opposite to that shown in Fig. 2.64 *i.e.* it flows in the CCW direction. The actual directions are as shown in Fig. 2.67.

The various branch currents are as under :

$$I_{AB} \quad I_1 \quad \mathbf{7/9\ A}; \; I_{BF} \quad I_1 \quad I_2 \quad \frac{7}{9} \quad \frac{1}{3} \quad \frac{4}{9}\text{ A}$$

$$I_{BC} \quad I_2 \quad \frac{1}{3}\text{ A}; I_{CE} \quad I_2 \quad I_3 \quad \frac{1}{3} \quad \frac{13}{12} \quad \frac{17}{12}\text{ A}$$

$$I_{DC} \quad I_3 \quad \frac{13}{12}\text{ A}$$

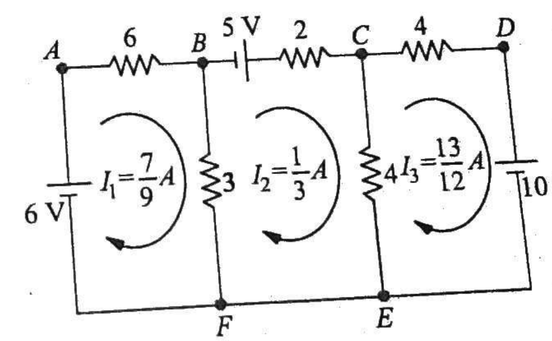


Fig. 2.67

Solution by Using Mesh Resistance Matrix

From inspection of Fig. 2.67, we have

$$R_{11} = 9; \; R_{22} = 9; \; R_{33} = 8$$

$$R_{12} = R_{21} = -3\ \Omega; \; R_{23} = R_{32} = -4\ \Omega; \; R_{13} = R_{31} = 0\ \Omega$$

$$E_1 = 6\text{ V} : E_2 = 5\text{ V}; \; E_3 = -10\text{ V}$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ -10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix} = 9(72-16) + 3(-24) = 432$$

$$\Delta_1 = \begin{vmatrix} 6 & -3 & 0 \\ 5 & 9 & -4 \\ -10 & -4 & 8 \end{vmatrix} = 6(72-16) - 5(-24) - 10(12) = 336$$

$$\Delta_2 = \begin{vmatrix} 9 & 6 & 0 \\ -3 & 5 & -4 \\ 0 & -10 & 8 \end{vmatrix} = 9(40-40) + 3(48) = 144$$

$$\Delta_3 = \begin{vmatrix} 9 & -3 & 6 \\ -3 & 9 & 5 \\ 0 & -4 & -10 \end{vmatrix} = 9(-90+90) - 3(30+24) = -468$$

$$I_1 = \Delta_1/\Delta = 336/432 = 7/9 \text{ A}$$
$$I_2 = \Delta_2/\Delta = 144/432 = 1/3 \text{ A}$$
$$I_3 = \Delta_3/\Delta = -468/432 = -13/12 \text{ A}$$

These are the same values as found above.

2.13. Nodal Analysis with Current Sources

Consider the network of Fig. 2.68 (*a*) which has two current sources and three nodes out of which 1 and 2 are independent ones whereas No. 3 is the reference node.

The given circuit has been redrawn for ease of understanding and is shown in Fig. 2.68 (*b*). The current directions have been taken on the assumption that

1. both V_1 and V_2 are positive with respect to the reference node. That is why their respective curents flow from nodes 1 and 2 to node 3.
2. V_1 is positive with respect to V_2 because current has been shown flowing from node 1 to node 2.

A positive result will confirm out assumption whereas a negative one will indicate that actual direction is opposite to that assumed.

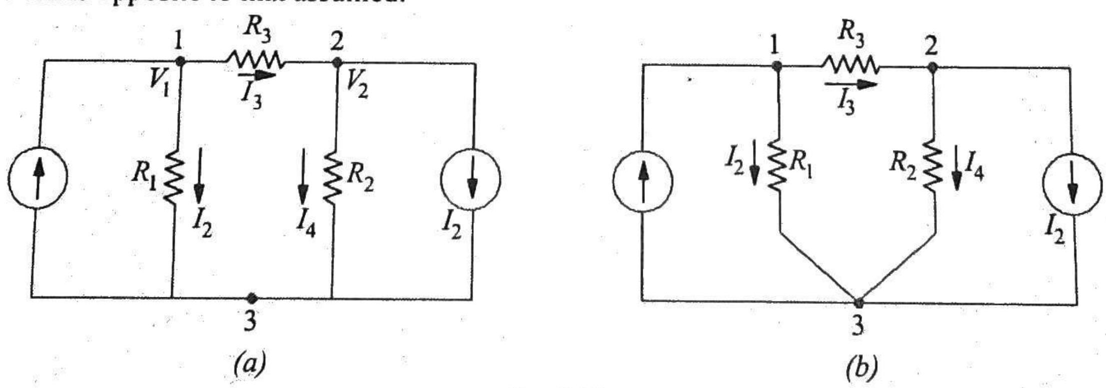


Fig. 2.68

We will now apply *KCL* to each node and use Ohm's law to express branch currents in terms of node voltages and resistances.

Node 1

$$I_1 - I_2 - I_3 = 0 \quad \text{or} \quad I_1 = I_2 + I_3$$

Now

$$I_2 = \frac{V_1}{R_1} \quad \text{and} \quad I_3 = \frac{V_1 - V_2}{R_3}$$

$\therefore$

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} \quad \text{or} \quad V_1\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - \frac{V_2}{R_3} = I_1 \qquad ...(i)$$

Node 2

$$I_3 - I_2 - I_4 = 0 \quad \text{or} \quad I_3 = I_2 + I_4$$

Now,

$$I_4 = \frac{V_2}{R_2} \quad \text{and} \quad I_3 = \frac{V_1 - V_2}{-R_3} \text{ –as before}$$

$\therefore$

$$\frac{V_1 - V_2}{R_3} = I_2 + \frac{V_2}{R_2} \quad \text{or} \quad V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{V_1}{R_3} = -I_1 \qquad ...(ii)$$

The above two equations can also be written by simple inspection. For example, Eq. (*i*) is represented by

1. *product* of potential V_1 and $(1/R_1 + 1/R_3)$ *i.e.* sum of the reciprocals of the branch resistances connected to this node.
2. *minus* the ratio of adjoining potential V_2 and the interconnecting resistance R_3.
3. all the above equated to the current supplied by the current source connected to this node.

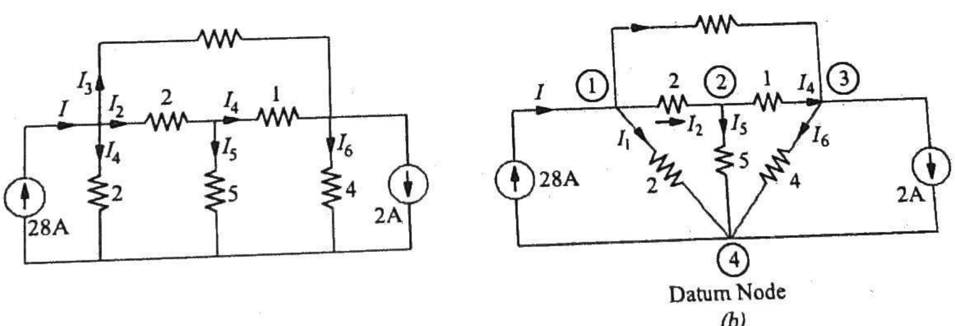
This current is taken *positive* if flowing *into* the node and negative if flowing *out* of it (as per sign convention of Art. 2.3). Same remarks apply to Eq. (*ii*) where I_2 has been taken negative because it flows *away* from node 2.

In terms of branch conductances, the above two equations can be put as

$$V_1 (G_1 + G_3) - V_2G_3 = I_1 \quad \text{and} \quad V_2 (G_2 + G_3) - V_1G_3 = -I_2$$

Example 2.36. *Use nodal analysis method to find currents in the various resistors of the circuit shown in Fig. 2.69* (*a*).

Solution. The given circuit is redrawn in Fig. 2.66 (*b*) with its different nodes marked 1, 2, 3 and 4, the last one being taken as the reference or datum node. The different node-voltage equations are as under :



(*a*)

Fig. 2.69

Node 1
$$V_1\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{10}\right) - \frac{V_2}{2} - \frac{V_3}{10} = 8$$

or
$$11V_1 - 5V_2 - V_3 - 280 = 0 \qquad ...(i)$$

Node 2
$$V_2\left(\frac{1}{2} + \frac{1}{5} + 1\right) - \frac{V_1}{2} - \frac{V_3}{1} = 0$$

or
$$5V_1 - 17\,V_2 + 10\,V_3 = 0 \qquad ...(ii)$$

Node 3
$$V_3\left(\frac{1}{4} + 1 + \frac{1}{10}\right) - \frac{V_2}{1} - \frac{V_1}{10} = -2$$

or $V_1 + 10\,V_2 - 13.5\,V_3 - 20 = 0$ ...(iii)

The matrix form of the above three equations is

$$\begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 280 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{vmatrix} = 1424.5 - 387.5 - 67 = 970$$

$$\Delta_1 = \begin{vmatrix} 280 & -5 & -1 \\ 0 & -17 & 10 \\ 20 & 10 & -13.5 \end{vmatrix} = 34,920, \quad \Delta_2 = \begin{vmatrix} 11 & 280 & -1 \\ 5 & 0 & 10 \\ 1 & 20 & -13.5 \end{vmatrix} = 19,400$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 280 \\ 5 & -17 & 0 \\ 1 & 10 & 20 \end{vmatrix} = 15,520$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{34,920}{970} = 36\text{ V}, \; V_2 = \frac{\Delta_2}{\Delta} = \frac{19,400}{970} = 20\text{ V}, \; V_3 = \frac{\Delta_3}{\Delta} = \frac{15,520}{970} = 16\text{ V}$$

It is obvious that all nodes are at a higher potential with respect to the datum node. The various currents shown in Fig. 2.69 (*b*) can now be found easily.

$$I_1 = V_1/2 = 36/2 = \mathbf{18\ A}$$
$$I_2 = (V_1 - V_2)/2 = (36 - 20)/2 = \mathbf{8\ A}$$
$$I_3 = (V_1 - V_3)/10 = (36 - 16)/10 = \mathbf{2\ A}$$

It is seen that total current, as expected, is $18 + 8 + 2 = \mathbf{28\ A}$

$$I_4 = (V_2 - V_3)/1 = (20 - 16)/1 = \mathbf{4\ A}$$
$$I_5 = V_2/5 = 20/5 = \mathbf{4\ A}, \; I_6 = V_3/4 = 16/4 = \mathbf{4\ A}$$

Example 2.37. *Using nodal analysis, find the different branch currents in the circuit of Fig. 2.70 (a). All branch conductances are in siemens (i.e. mho).*

Solution. Let the various branch currents be as shown in Fig. 2.70 (*b*). Using the procedure detailed in Art. 2.11, we have

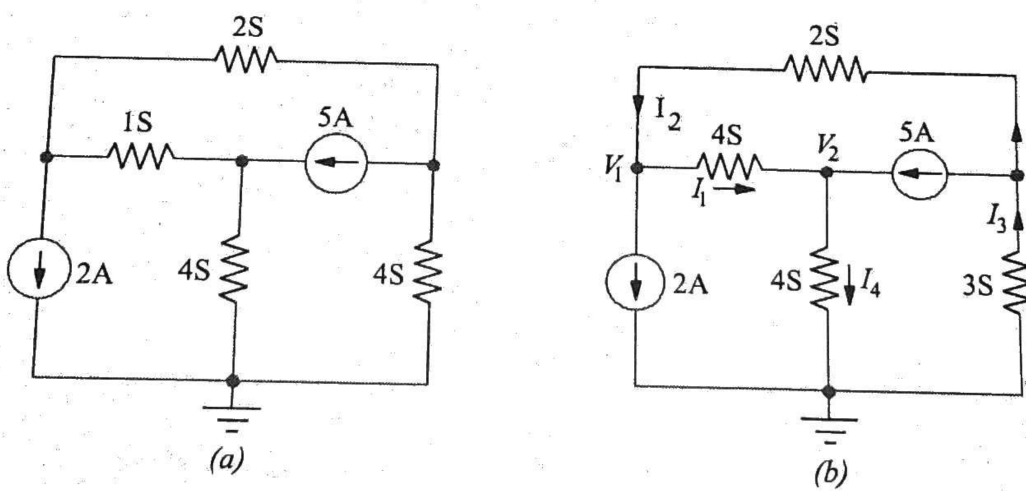


Fig. 2.70

First Node

$$V_1(1 + 2) - V_2 \times 1 - V_3 \times 2 = -2 \quad \text{or} \quad 3V_1 - V_2 - 2V_3 = -2 \qquad ...(i)$$

Second Node

$$V_2(1 + 4) - V_1 \times 1 = 5 \quad \text{or} \quad V_1 - 5V_2 = -5 \qquad ...(ii)$$

Third Node

$$V_3(2 + 3) - V_1 \times 2 = -5 \quad \text{or} \quad 2V_1 - 5V_3 = 5 \qquad ...(iii)$$