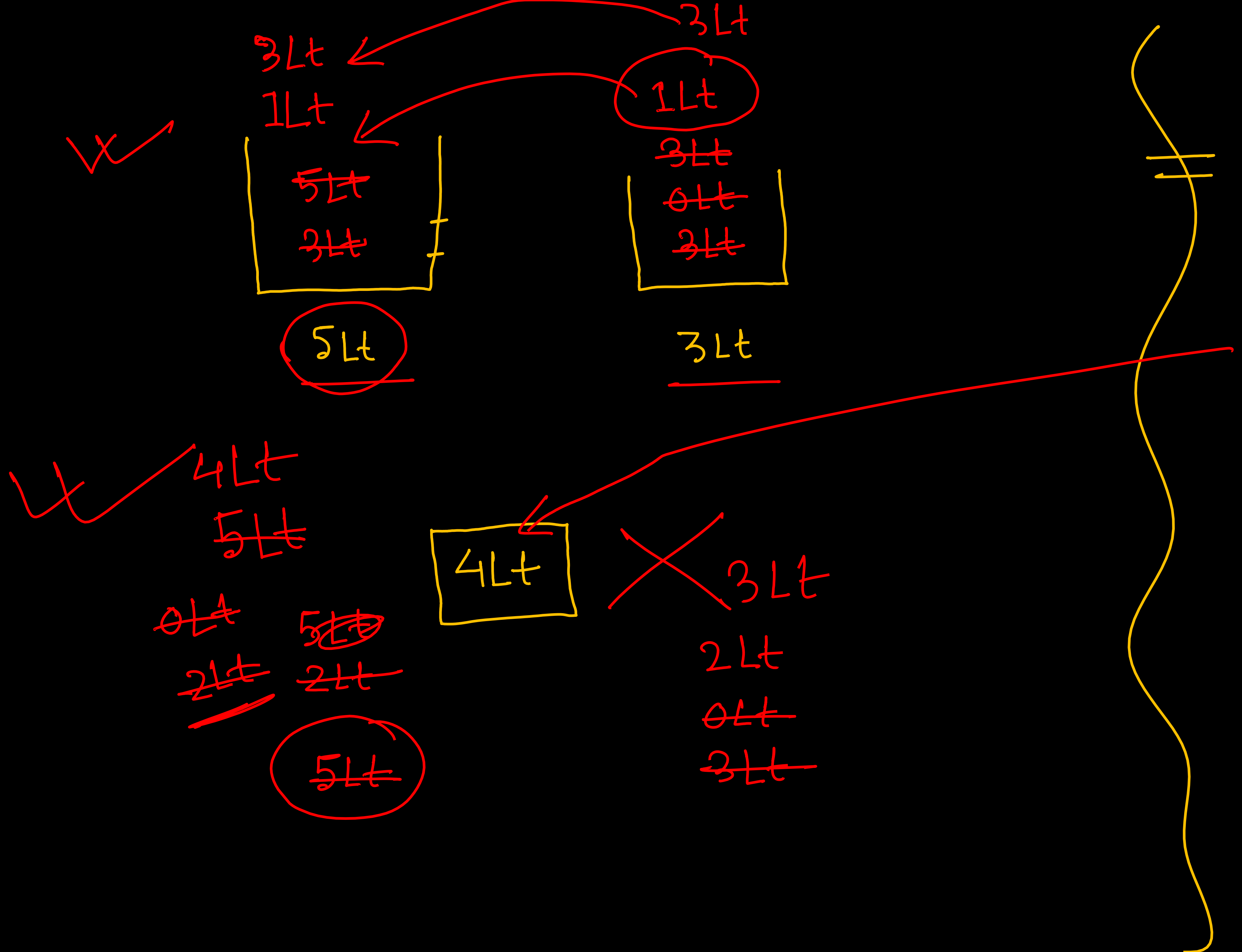
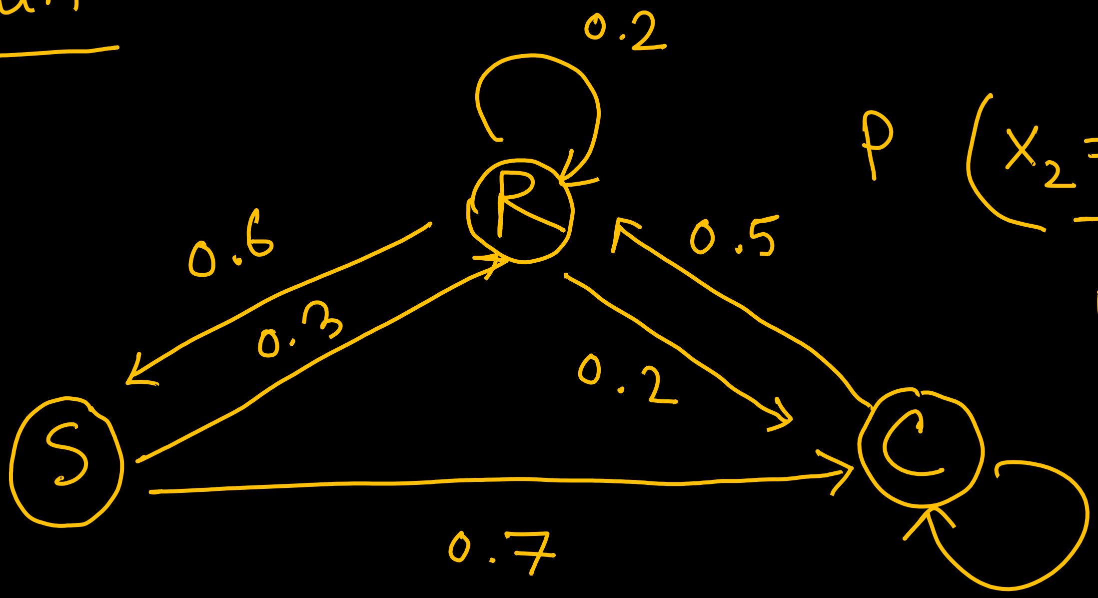

3Lt
1Lt
~~5Lt~~
~~3Lt~~
5Lt
3Lt
1Lt
~~3Lt~~
~~0Lt~~
~~3Lt~~
3Lt
4Lt
~~5Lt~~
~~0Lt~~
~~5Lt~~
~~2Lt~~
~~2Lt~~
~~5Lt~~
4Lt
3Lt
2Lt
~~0Lt~~
~~3Lt~~

Markov Chain



$$P\left(X_2 = \text{"S"} \mid X_1 = \text{"R"}\right) = 0.6$$

$$\boxed{P\left(X_{n+1} = \text{"}x\text{"} \mid X_n = \text{"}x_n\text{"}\right)}$$

Markov chain

" Future state only depends on the current state, not the states before

Naive Bayes

$$P\left(X_{n+1} = \text{"}x\text{"} \mid X_1 = \text{"}x_1\text{"}, X_2 = \text{"}x_2\text{"}, \ldots, X_n = \text{"}x_n\text{"}\right)$$

$$
\begin{array}{c|ccc}
 & R & S & C \\
R & \multicolumn{3}{c}{\multirow{3}{*}{}} \\
\end{array}
$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} R & S & C \end{array} \\
\begin{array}{c} R \\ S \\ C \end{array} & \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}
\end{array} = A
$$

Transition Matrix

$$\pi_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\pi_1 = \pi_0 A = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0 & 0.7 \end{bmatrix}$$

$$\pi_2 = \pi_1 A$$

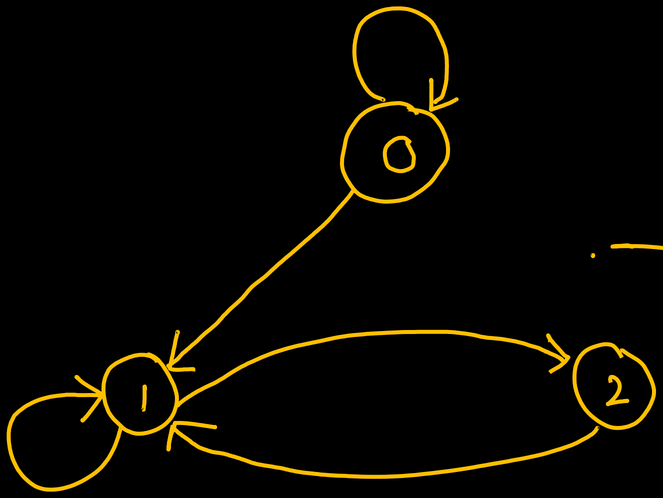
$$= [0.3 \quad 0 \quad 0.7][A]$$

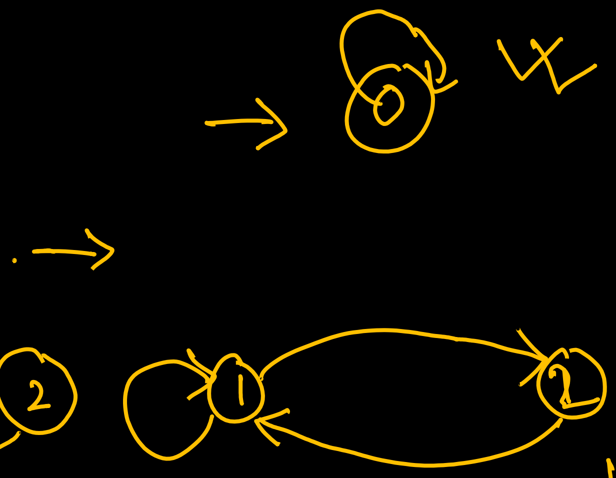
$$\pi_2 = [0.41 \quad 0.18 \quad 0.41]$$

$$\pi_n A = \pi_n \quad [\text{Equilibrium State}]$$

$$\boxed{vA = \lambda v \qquad \lambda = I}$$

$$\pi_n = [0.35211 \quad 0.21127 \quad 0.43662]$$





0 -> 0

0 -> 1

x -> 0

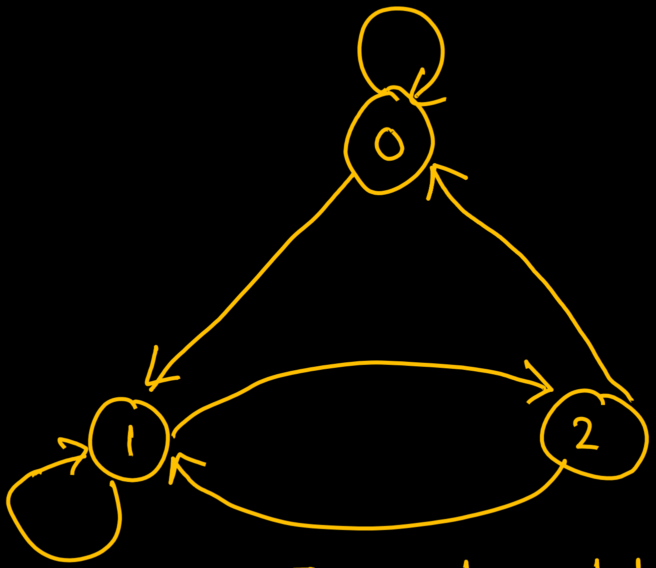
1 -> 2

2 -> 1

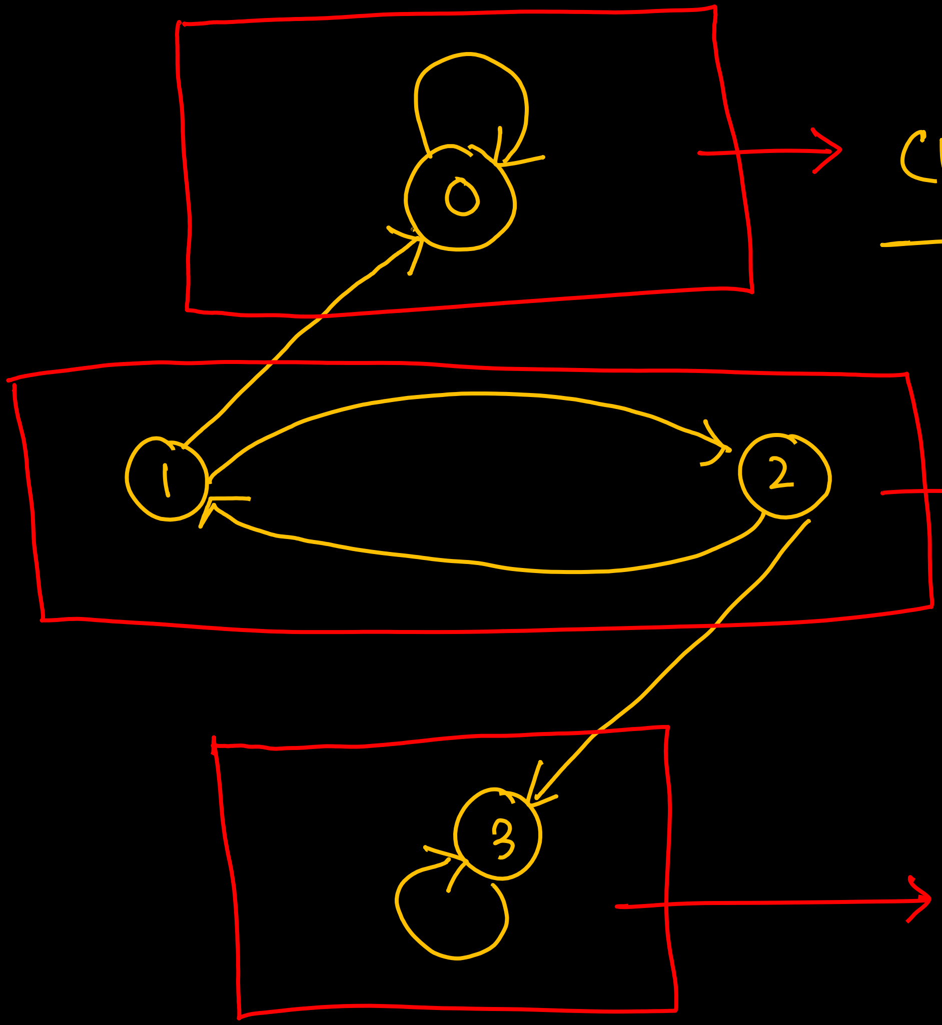
Reducible MC

Transient State -> 0

Recurrent state -> 1, 2



Irreducible MC



Class 0

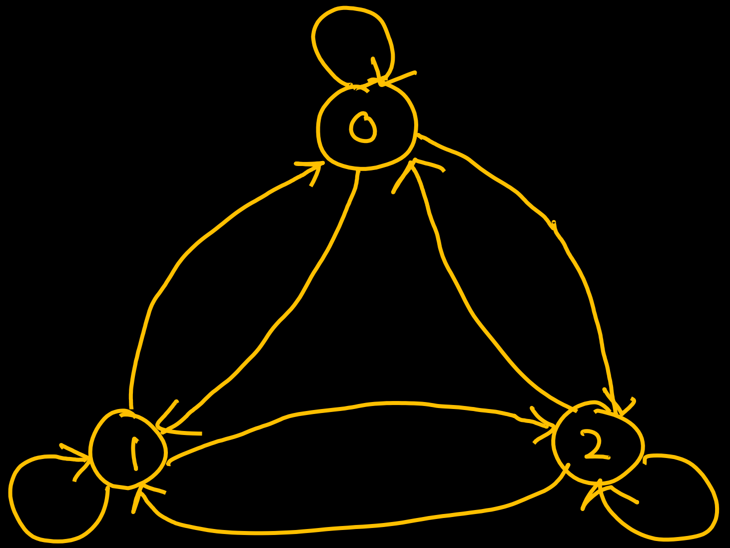
Class 1-2

Class 3

$0 \to 0$

$0 \not\to$

$\boxed{P_{i \to j}(n)}$ ✓



$\left.\begin{array}{l} 0-0 \\ 0-2 \end{array}\right\}$ ✓

$\left.\begin{array}{l} 0-1 \\ 1-2 \end{array}\right\}$ ✓

$\left.\begin{array}{l} 0-2 \\ 2-2 \end{array}\right\}$ ✓

$P_{02}(1) = 0.3 = A_{02}$

$P_{12}(1) = 0.2$

$P_{21}(1) = 0.8$

$$A = \begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 ✓ \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix} \end{array}$$

$$\underline{P_{02}(2)} = A_{00} \times A_{02} + A_{01} \times A_{12} \longrightarrow 1$$

$$+ A_{02} \times A_{22} \longrightarrow 2$$

$$= 0.5 \times 0.3 + 0.2 \times 0.2 + 0.3 \times 0.1 = 0.22$$

$$\underline{P_{10}(2)} = 0.6 \times 0.5 + 0.2 \times 0.6 + 0.2 \times 0.1$$

$$= \boxed{0.44}$$

$$A = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$$

$$P_{ij}(2) = \begin{bmatrix} A_{i0} & A_{i1} & A_{i2} \end{bmatrix} \begin{bmatrix} A_{0j} \\ A_{1j} \\ A_{2j} \end{bmatrix}$$

$$A * A = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix} * \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$$

$$\underset{(3 \times 3)}{A}$$

$$\boxed{P_{ij}(2)}$$

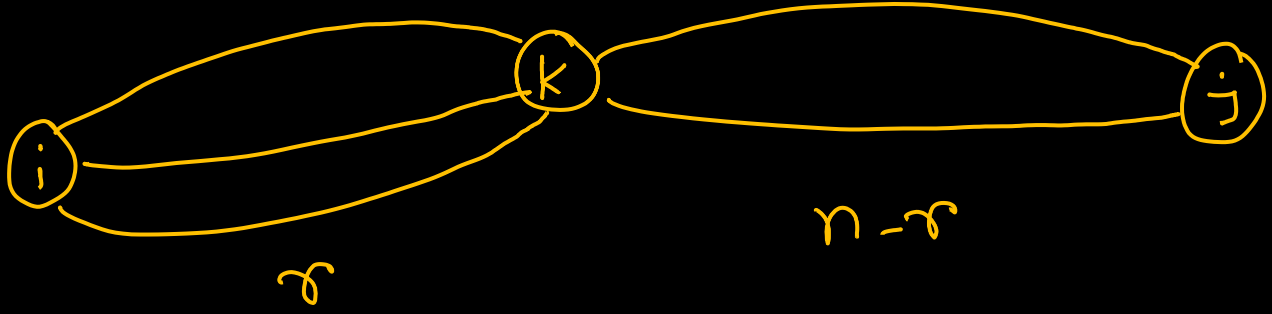
$$\boxed{P_{ij}(2) = A^2_{ij}}$$

$$A^2 = \begin{bmatrix} 0.4 & 0.38 & \underline{0.22} \\ \underline{0.44} & 0.32 & 0.24 \\ 0.54 & 0.26 & 0.20 \end{bmatrix}$$

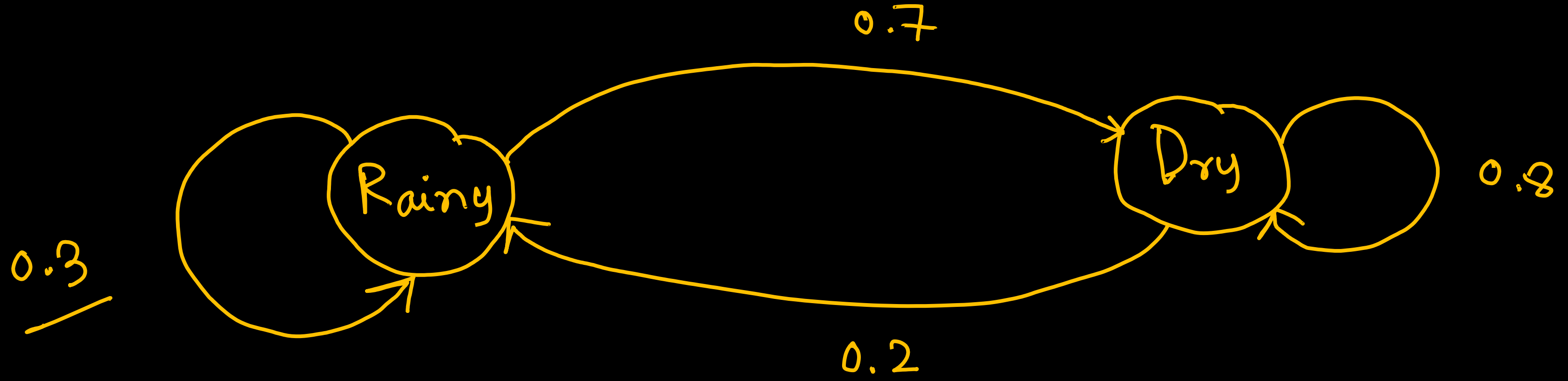
$$\boxed{P_{ij}(n) = A^n_{ij}}$$

* Chapman - Kolmogorov Theorem

$$\boxed{P_{ij}(n) = A^{n}_{ij}}$$



$$\forall \quad \boxed{P_{ij}(n) = \sum_{k} P_{ik}(r) * P_{kj}(n-r)}$$



$P(B|A)$

$P(\text{Rainy}) = 0.4 \qquad P(\text{Dry}) = 0.6$

Rainy → Rainy → Dry → Dry ?

Soln:

$$A = \begin{array}{c c} & \begin{array}{cc} R & D \end{array} \\ \begin{array}{c} R \\ D \end{array} & \begin{bmatrix} 0.3 & 0.7 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

$$\pi = [\,0.4 \quad 0.6\,]$$

$$Q = P(R) * P(R|R) * P(D|R) * P(D|D)$$

$$= 0.4 * 0.3 * 0.7 * 0.8$$

$$= \boxed{} \quad \text{Ans.}$$

$$i \xrightarrow{n \text{ step}} j$$

state i state j

$$r_{ij}(n) = A^{n}_{ij}$$

$$\boxed{r_{ij}(n) = P(X_n = j \mid X_0 = i)}$$

$$i \rightarrow j$$

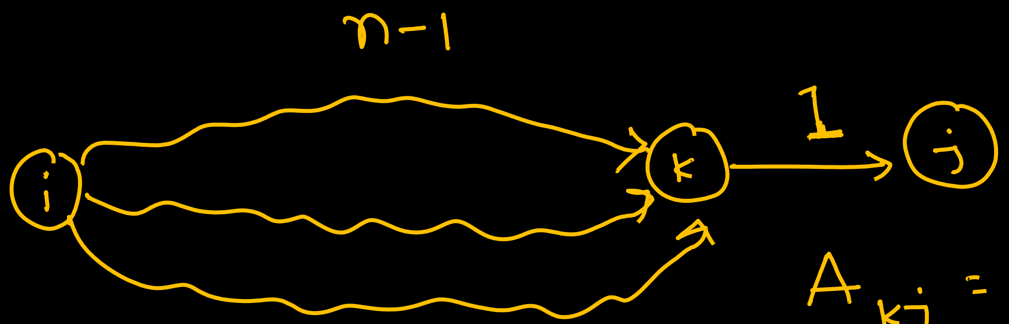
$$i$$

* $$r_{ij}(0) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

* $$r_{ij}(1) = A_{ij} = P_{ij}(X_1 = j \mid X_0 = i)$$

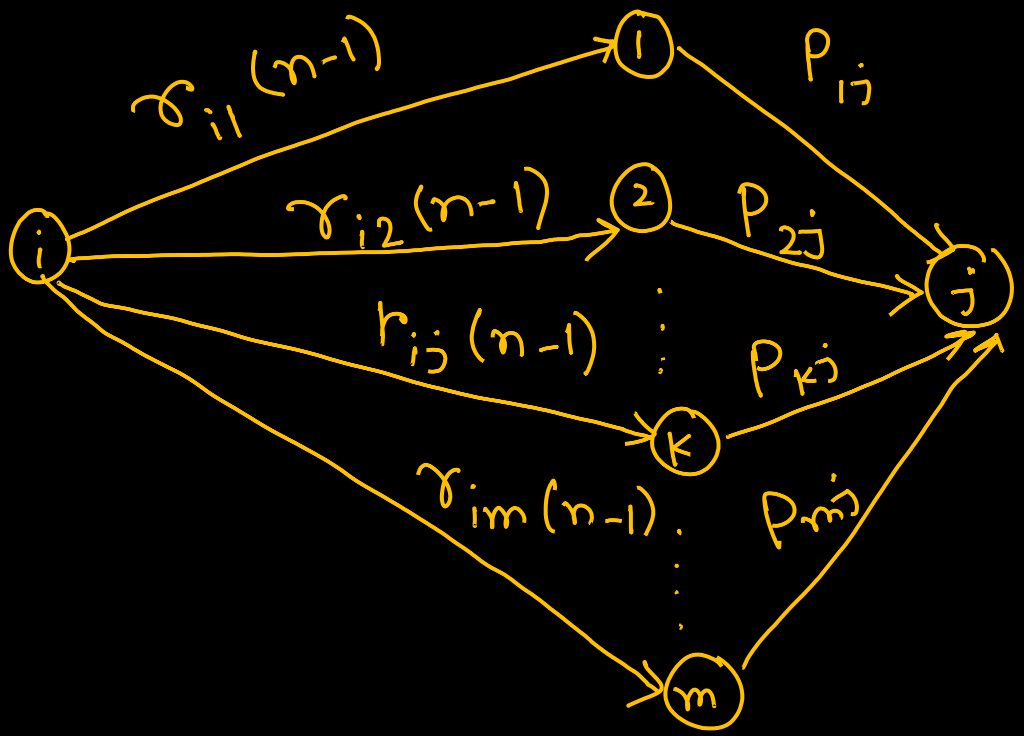
$$\sum r_{ij}(n) = 1$$

$$n \geq 2$$



$$A_{kj} = P_{kj}\left(X_n = j \mid X_{n-1} = k\right)$$

$$\sum_{k} r_{ik}(n-1)$$

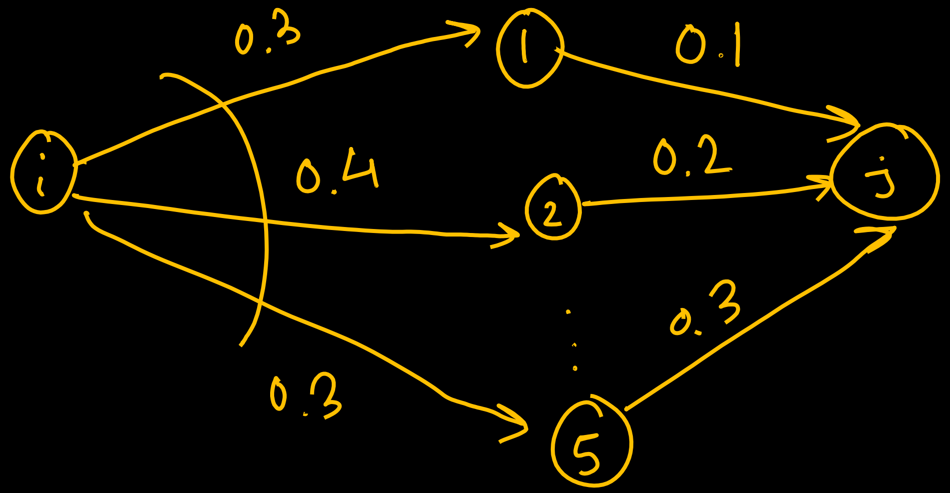


$*$ $$\boxed{r_{ij}(n) = \sum_{k}^{m} \underbrace{r_{ik}(n-1)}_{n-1} * \underbrace{P_{kj}}_{1}}$$

$$i \xrightarrow{1} k \overset{n-1}{\rightsquigarrow} j$$

$$\boxed{r_{ij}(n) = \sum_{k=1}^{m} \underbrace{p_{ik}}_{1} * \underbrace{r_{kj}(n-1)}_{n-1}}$$

$$* \qquad \boxed{r_{ij}(n) = \sum_{k}^{m} r_{ik}(q) * r_{kj}(n-q)}$$



$$A * A = A^2$$

$$A^2 * A = A^3$$

$$A^3 * A = A^4$$

$$P_{ij}(n) = A^n_{ij}$$

Equilibrium state → does not depend on initial state

$$\lim_{n \to \infty} A^\infty$$

* irreducible

$$A^\infty = \begin{bmatrix} 0.54 & 0.45 \\ 0.54 & 0.46 \\ 0.54 & 0.45 \end{bmatrix}$$

$\boxed{i = 0}$

1

2