Integration (Formula):

$\int x^n dx$	$\frac{x^{n+1}}{n+1} + c$		$\int \tan\ x\, dx$	$\ln\lvert\sec x\rvert + c$
$\int dx$	$x + c$		$\int \cot\ x\, dx$	$\ln\lvert\sin x\rvert + c$
$\int \frac{1}{x}\ dx$	$\ln x + c$			
$\int e^x dx$	$e^x + c$		$2 \sin x \cos x$	$\sin 2x$
$\int e^{mx} dx$	$\frac{e^{mx}}{m} + c$		$2\sin^2 x$	$1 - \cos 2x$
			$2\cos^2 x$	$1 + \cos 2x$
$\int \sin x\ dx$	$-\cos x + c$		$\cos 2x$	$\cos^2 x - \sin^2 x$
$\int \sin mx\, dx$	$\frac{-\cos mx}{m} + c$			
$\int \cos x\ dx$	$\sin x + c$		$\sin^2 x + \cos^2 x = 1$	
$\int \cos mx\ dx$	$\frac{\sin mx}{m} + c$		$1 + \tan^2 x = \sec^2 x$	
$\int \sec^2 x\ dx$	$\tan x + c$		$1 + \cot^2 x = \mathrm{cosec}^2 x$	
$\int \sec^2 mx\ dx$	$\frac{\tan mx}{m} + c$			
$\int \mathrm{cosec}^2 x\ dx$	$-\ \cot x + c$			
$\int \mathrm{cosec}^2 mx\ dx$	$\frac{-\cot mx}{m} + c$			
$\int \sec x \tan x\ dx$	$\sec x + c$			
$\int \sec mx \tan mx\ dx$	$\frac{\sec mx}{m} + c$			

1) Integrate: $\int \left(5x^3 + \frac{6}{x^3}\right) dx$

$= 5\frac{x^4}{4} + 6\frac{x^{-2}}{-2} + c$

$= \frac{5}{4}x^4 - \frac{3}{x^2} + c$

2) Integrate: $\int \frac{1 - x^3}{1 - x} dx$

$= \int \frac{(1-x)(1+x+x^2)}{1-x}\ dx$

$= \int (1 + x + x^2)\ dx$

$= x + \frac{x^2}{2} + \frac{x^3}{3} + c$

3) Integrate: $\int \frac{\tan x}{\cot x} dx$

$= \int \tan^2 x\, dx$

$= \int (\sec^2 x - 1) dx = \tan x - x + c$

4) Integrate: $\int \frac{dx}{1 + \sin x}$

$= \int \frac{1 - \sin x}{1 - \sin^2 x} dx$

$= \int \frac{1 - \sin x}{\cos^2 x} dx$

$= \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx$

$$= \int (\sec^2 x - \sec x \tan x) dx$$

$$= \tan x - \sec x + c$$

5) Integrate: $\displaystyle \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$

$$= \int \frac{\sin x + \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x}} dx$$

$$= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int dx = x + c$$

6) Integrate: $\displaystyle \int \sin^3 x \, dx$

$$= \frac{1}{4} \int 4\sin^3 x \, dx$$

$$= \frac{1}{4} \int (3 \sin x - \sin 3x) dx$$

$$= -\frac{3}{4} \cos x + \frac{\cos 3x}{12} + c$$

H.W:

1) $\int \cos^4 x \, dx$	6) $\int \sin^2 x \cos 2x \, dx$
$= \frac{1}{4} \int (2\cos^2 x)^2 \, dx$	$= \int \frac{1}{2} (2\sin^2 x) \cos 2x \, dx$
$= \frac{1}{4} \int (1 + \cos 2x)^2 \, dx$	$= \frac{1}{2} \int (1 - \cos 2x) \cos 2x \, dx$
$= \frac{1}{4} \int (1 + 2 \cos 2x + \cos^2 2x) \, dx$	$= \frac{1}{2} \int (\cos 2x - \cos^2 2x) \, dx$
$= \frac{1}{4} \int \left[1 + 2 \cos 2x + \frac{1}{2} (2\cos^2 2x)\right] dx$	$= \frac{1}{2} \int \left[\cos 2x - \frac{1}{2} (2\cos^2 2x)\right] dx$
$= \frac{1}{4} \int \left[1 + 2 \cos 2x + \frac{1}{2} (1 + \cos 4x)\right] dx$	$= \frac{1}{2} \int \left[\cos 2x - \frac{1}{2} (1 + \cos 4x)\right] dx$
$= \frac{1}{4} \left(x + 2 \frac{\sin 2x}{2} + \frac{1}{2} \left(x + \frac{\sin 4x}{4}\right)\right) + c$	$= \frac{1}{2} \left(\frac{\sin 2x}{2} - \frac{1}{2} \left(x + \frac{\sin 4x}{4}\right)\right) + c$

2) $\int \frac{e^{5\ln x} - e^{4\ln x}}{e^{3\ln x} - e^{2\ln x}}\,dx$ $= \int \left(\frac{e^{\ln x^5} - e^{\ln x^4}}{e^{\ln x^3} - e^{\ln x^2}}\right) dx$ $= \int \left(\frac{x^5 - x^4}{x^3 - x^2}\right) dx$ $= \int x^2 \left(\frac{x^3 - x^2}{x^3 - x^2}\right) dx$ $= \int x^2\,dx$ $= \frac{x^3}{3} + c$	7) $\int \frac{\cos^3 x + \sin^3 x}{\cos x + \sin x}\,dx$ $= \int \frac{(\cos x + \sin x)(\cos^2 x - \cos x \sin x + \sin^2 x)}{\cos x + \sin x}$ $= \int (\cos^2 x + \sin^2 x - \cos x \sin x)\,dx$ $= \int \left(1 - \frac{1}{2}\sin 2x\right) dx$ $= x - \frac{1}{2}\left(\frac{-\cos 2x}{2}\right) + c$ $= x + \frac{\cos 2x}{4} + c$
3) $\int \sin\frac{x}{2}\,\cos\frac{x}{2}\,dx$ $= \frac{1}{2}\int \left(2\sin\frac{x}{2}\,\cos\frac{x}{2}\right) dx$ $= \frac{1}{2}\int \sin x\,dx$ $= \frac{-\cos x}{2} + c$	8) $\int \frac{1 - \sin x}{1 + \sin x}\,dx$ $= \int \frac{(1 - \sin x)^2}{1 - \sin^2 x}\,dx$ $= \int \frac{1 - 2\sin x + \sin^2 x}{\cos^2 x}\,dx$ $= \int \left(\frac{1}{\cos^2 x} - 2\frac{\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\right) dx$ $= \int (\sec^2 x - 2\sec x \tan x + \tan^2 x)\,dx$ $= \int \left(\sec^2 x - 2\sec x \tan x + (\sec^2 x - 1)\right) dx$ $= \tan x - 2\sec x + \tan x - x + c$ $= 2\tan x - 2\sec x - x + c$
4) $\int \frac{\cos 2x}{\cos^2 x \sin^2 x}\,dx$ $= \int \left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x}\right) dx$ $= \int \left(\frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x}\right) dx$ $= \int (\mathrm{cosec}^2 x - \sec^2 x)\,dx$ $= -\cot x - \tan x + c$	9) $\int \frac{1}{\sqrt{x} + \sqrt{x+1}}\,dx$ $= \int \frac{\sqrt{x} - \sqrt{x+1}}{(\sqrt{x})^2 - (\sqrt{x+1})^2}\,dx$ $= -\int (\sqrt{x} - \sqrt{x+1})\,dx$ $= -\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right) + c$

5) $\int \sqrt{1+\sin x}\, dx$ $= \int \sqrt{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\; dx$ $= \int \sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}\, dx$ $= \int \left(\sin\frac{x}{2}+\cos\frac{x}{2}\right) dx$ $= -\frac{\cos\frac{x}{2}}{\frac{1}{2}}+\frac{\sin\frac{x}{2}}{\frac{1}{2}}+c$	10) $\int \frac{2-\sin 2x}{1-\cos 2x}\, dx$ $= \int \left(\frac{2-2\sin x\cos x}{2\sin^2 x}\right) dx$ $= \int \left(\frac{2}{2\sin^2 x}-\frac{2\sin x\cos x}{2\sin^2 x}\right) dx$ $= \int \left(\frac{1}{\sin^2 x}-\frac{\cos x}{\sin x}\right) dx$ $= \int (\mathrm{cosec}^2 x-\cot x)\; dx$ $= -\cot x-\ln\lvert\sin x\rvert+c$