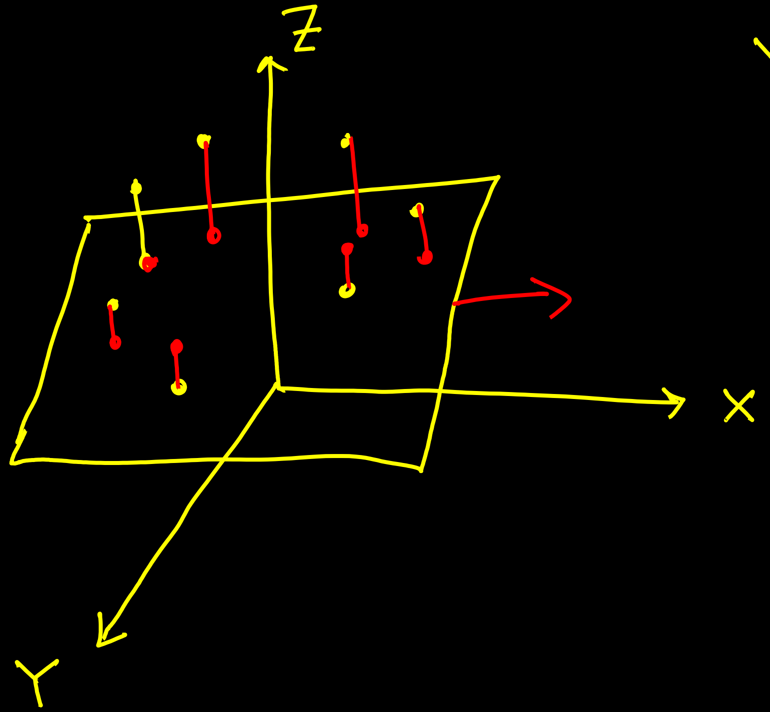
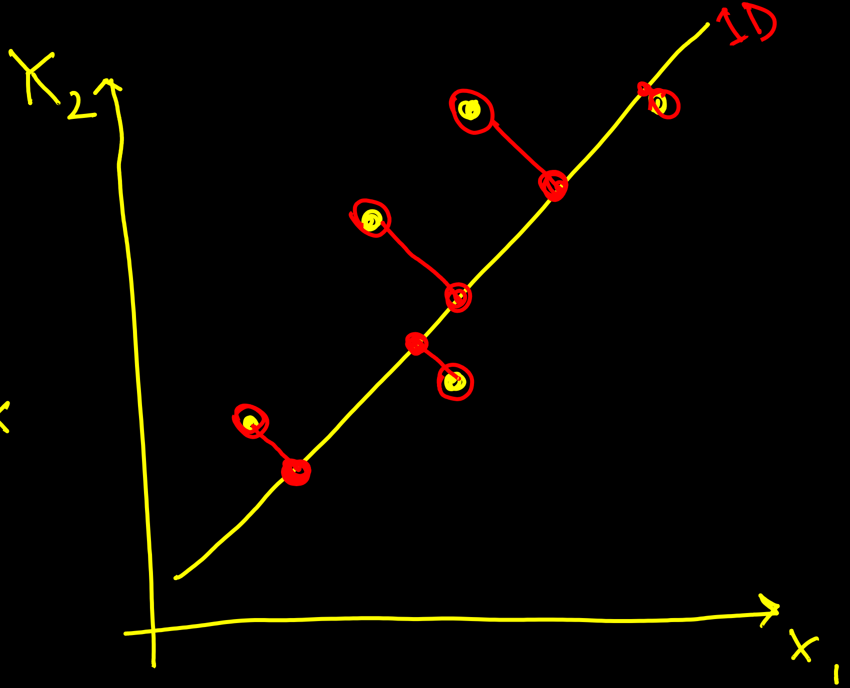
Dimension Reduction





PCA

Principle
Component
Analysis

① Dataset

X_1	4	8	13	7
X_2	11	4	5	14

(ii) Calculating Mean from the features

$$\bar{X}_1 = \frac{1}{4}(4+8+13+7) = 8$$

$$\bar{X}_2 = \frac{1}{4}(11+4+5+14) = 8.5$$

$$\boxed{X_1, X_2}, X_3$$

(iii) Covariance Matrix

$$Cov(x_1, x_1) = \frac{1}{N-1}\sum_{k=1}^{N}(X_{1k} - \bar{X}_1)^2$$

$$= \frac{1}{3}\left\{(4-8)^2 + (8-8)^2 + (13-8)^2 + (7-8)^2\right\}$$

$$= \frac{1}{3}\left\{16 + 0 + 25 + 1\right\} = 14$$

$$C = [2 \times 2] = \begin{bmatrix} \overset{14}{Cov(x_1, x_1)} & \overset{-11}{Cov(x_1, x_2)} \\ \overset{-11}{Cov(x_2, x_1)} & \overset{23}{Cov(x_2, x_2)} \end{bmatrix}$$

$$C = [3 \times 3]$$

$$C = \begin{bmatrix} x_1x_1 & x_1x_2 & x_1x_3 \\ x_2x_1 & x_2x_2 & x_2x_3 \\ x_3x_1 & x_3x_2 & x_3x_3 \end{bmatrix}$$

$$\mathrm{Cov}(x_1, x_2) = \frac{1}{N-1} \sum_{k=1}^{N} (X_{1k} - \bar{X}_1)(X_{2k} - \bar{X}_2)$$

$$= \frac{1}{3} \left\{ (4-8)(11-8.5) + (8-8)(4-8.5) + (13-8)(5-8.5) + (7-8)(14-8.5) \right\}$$

$$= -11$$

$$\mathrm{Cov}(x_2, x_1) = \frac{1}{N-1} \sum_{k=1}^{N} (X_{2k} - \bar{X}_2)(X_{1k} - \bar{X}_1)$$

$$= -11$$

$$\mathrm{Cov}(x_2, x_2) = \frac{1}{N-1} \sum_{k=1}^{N} (X_{2k} - \bar{X}_2)^2$$

$$= \frac{1}{3} \left\{ (11-8.5)^2 + (4-8.5)^2 + (5-8.5)^2 + (14-8.5)^2 \right\} = 23$$

(iv) Calculating eigenvalues

$$\underline{\det}\,(\underline{S - \lambda I}) = 0$$

λ → eigen values

S → Covariance Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} &ax^2 + bx + c = 0 \\ &x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}}$$

$$\Rightarrow \begin{vmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (14-\lambda)(23-\lambda) - (-11)(-11) = 0$$

$$\Rightarrow \lambda^2 - 37\lambda + 201 = 0$$

$$\lambda = 30.38 \,,\ 6.61$$

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 30.38 \\ 6.61 \end{bmatrix}$$

(v) Calculating eigen vectors

$$(S - \lambda_1 I)U = 0$$

$$\Rightarrow \begin{bmatrix} 14-\lambda_1 & -11 \\ -11 & 23-\lambda_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (14-\lambda_1)u_1 - 11u_2 \\ -11u_1 + (23-\lambda_1)u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\|\bar{U}\| = \sqrt{(11)^2 + (14-\lambda_1)^2} = \boxed{19.73}$$

$$(14-\lambda_1)u_1 - 11u_2 = 0$$

$$\Rightarrow (14-\lambda_1)u_1 = 11u_2$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{11}{14-\lambda_1}$$

30.38

$$e_1 = \begin{bmatrix} u_1 / \|\bar{U}\| \\ U_2 / \|\bar{U}\| \end{bmatrix} = \begin{bmatrix} 11/19.73 \\ (14-30.38)/19.73 \end{bmatrix} = \begin{bmatrix} 0.55 \\ -0.83 \end{bmatrix}$$

$$(S - \lambda_2 I) V = 0$$

$$\Rightarrow \begin{bmatrix} 14-\lambda_2 & -11 \\ -11 & 23-\lambda_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(2×2) (2×1)

$$\Rightarrow \begin{bmatrix} (14-\lambda_2)v_1 - 11v_2 \\ -11v_1 + (23-\lambda_2)v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow \frac{V_1}{V_2} = \frac{11}{14-\lambda_2}$$

$\lambda_2 \downarrow 6.61$

(2×1)

$$\|\bar{V}\| = \sqrt{(11)^2 + (14-6.61)^2} = \boxed{13.25}$$

$$e_2 = \begin{bmatrix} v_1 / \|\bar{v}\| \\ v_2 / \|\bar{v}\| \end{bmatrix} = \begin{bmatrix} 11 / 13.25 \\ (14-6.61) / 13.25 \end{bmatrix} = \begin{bmatrix} 0.83 \\ 0.55 \end{bmatrix}$$

(vi) Calculating Principle Component

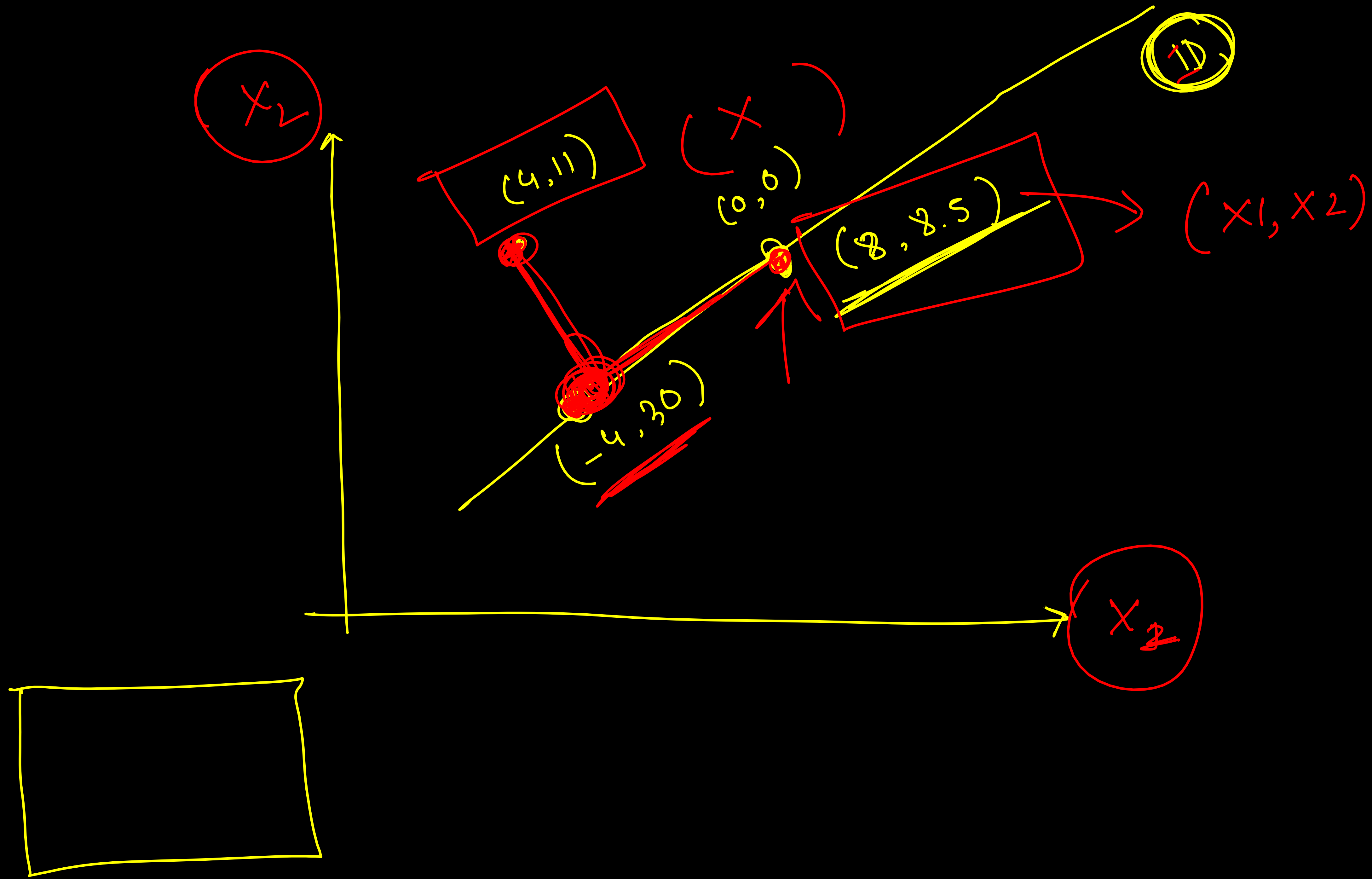
$$e_1 = \begin{bmatrix} 0.55 \\ -0.83 \end{bmatrix}$$

$$e_1^T = \begin{bmatrix} 0.55 & -0.83 \end{bmatrix}$$

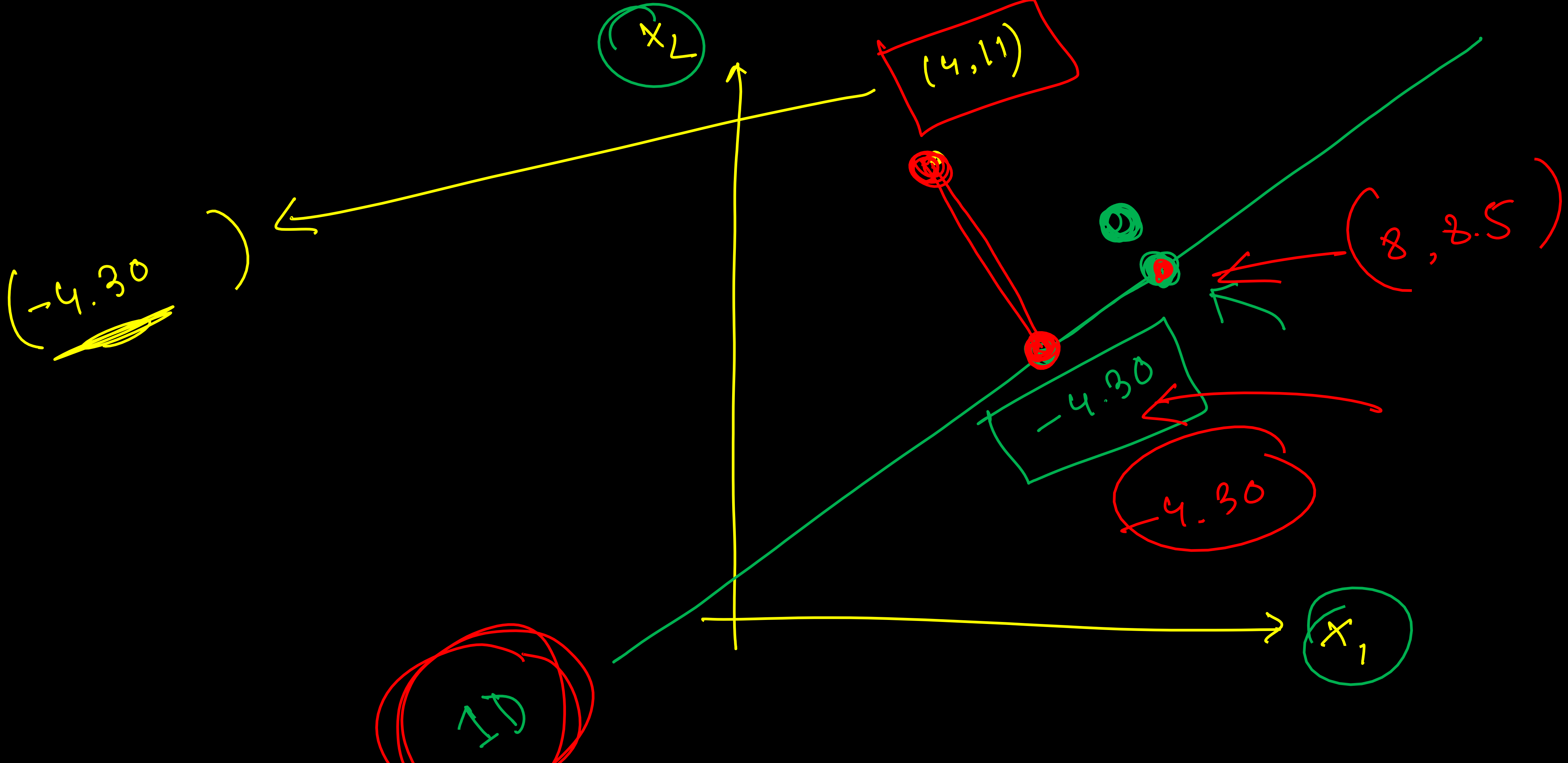
$$e_1^T \begin{bmatrix} \boxed{4} - \bar{x}_1 \\ \boxed{11} - \bar{x}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.55 & -0.83 \end{bmatrix} \begin{bmatrix} 4-8 \\ 11-8.5 \end{bmatrix}$$

$$= \begin{bmatrix} -4.30 \end{bmatrix}$$



X2
(4,11)
(X)
(0,0)
(8,8.5)
(x1,x2)
1D
(-4,30)
X2



$$(\bar{x}_1, \bar{x}_2) = (8, 8.5)$$