07

10. Use Superposition principle to find current in the 10-Ω resistor of Fig. 2.125. All resistances are in ohms. **[1 A]**

11. State and explain Superposition theorem. For the circuit of Fig. 2.126.
    (*a*) determine currents I_1, I_2 and I_3 when switch *S* is in position *b*.
    (*b*) using the results of part (*a*) and the principle of superposition, determine the same currents with switch *S* in position *a*.

[(*a*) 15 A, 10 A, 25 A (*b*) 11 A , 16 A, 27 A] (*Elect. Technology Vikram Univ.*)

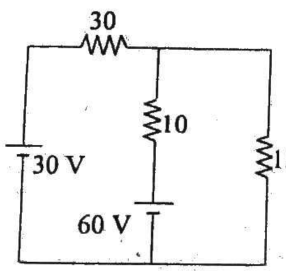


Fig. 2.124

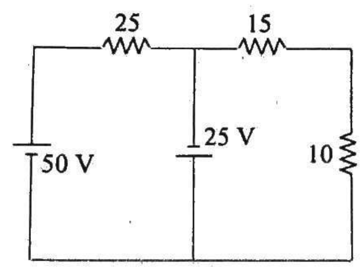


Fig. 2.125

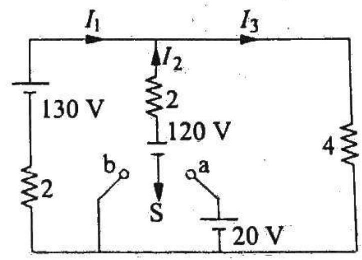
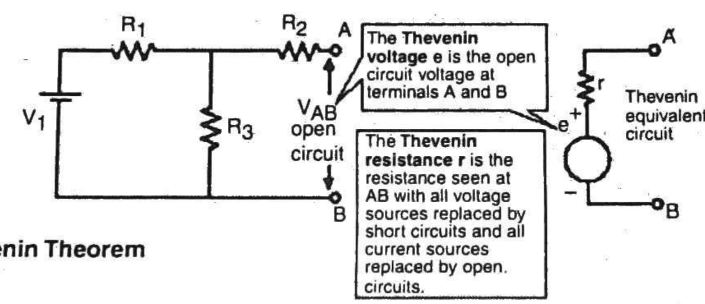


Fig. 2.126

2.18. Thevenin Theorem



Thevenin Theorem

It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by ***a single voltage source with a series resistance***. It makes the solution of complicated networks (particularly, electronic networks) quite quick and easy. The application of this extremely useful theorem will be explained with the help of the following simple example.

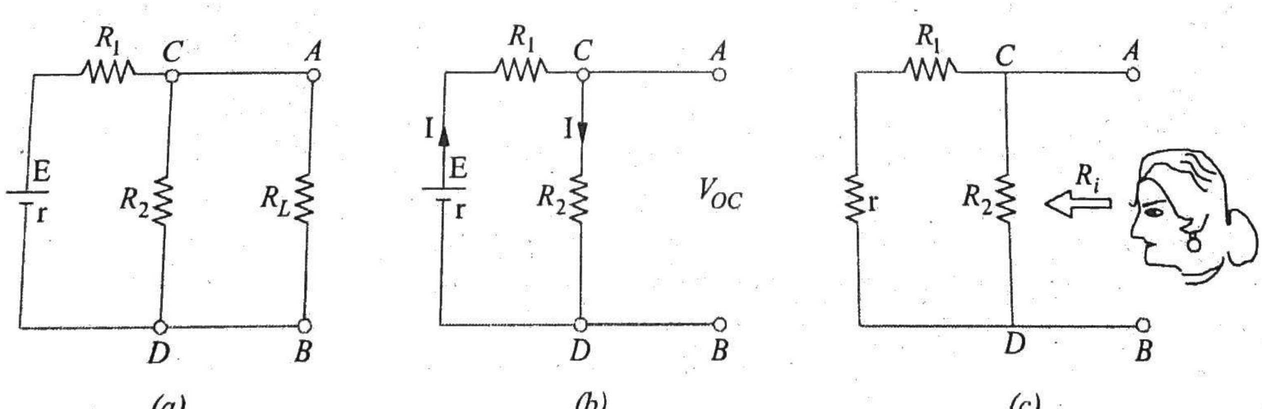


Fig. 2.127

M. L. Thevenin

Suppose, it is required to find current flowing through load resistance R_L, as shown in Fig. 2.127 (*a*). We will proceed as under :

1. Remove R_L from the circuit terminals A and B and redraw the circuit as shown in Fig. 2.127 (*b*). Obviously, the terminals have become open-circuited.
2. Calculate the open-circuit voltage V_{oc} which appears across terminals A and B when they are open *i.e.* when R_L is removed.
   As seen, V_{oc} = drop across $R_2 = IR_2$ where I is the circuit current when A and B are open.
   $I = \dfrac{E}{R_1 + R_2 + r}$ $\quad \therefore V_{oc} = IR_2 = \dfrac{ER_2}{R_1 + R_2 + r}$ [r is the internal resistance of battery]
   It is also called 'Thevenin voltage' V_{th}.
3. Now, imagine the battery to be removed from the circuit, leaving its internal resistance r behind and redraw the circuit, as shown in Fig. 2.127 (*c*). When viewed ***inwards*** from terminals A and B, the circuit consists of two parallel paths : one containing R_2 and the other containing $(R_1 + r)$. The equivalent resistance of the network, as viewed from these terminals is given as

$$R = R_2 \,||\, (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is also called,* Thevenin resistance R_{sh} (though, it is also sometimes written as R_i or R_0).

Consequently, as viewed from terminals A and B, the whole network (excluding R_1) can be reduced to a single source (called Thevenin's source) whose e.m.f. equals $V_{\propto}$ (or V_{sh}) and whose internal resistance equals R_{sh} (or R_i) as shown in Fig. 2.128.

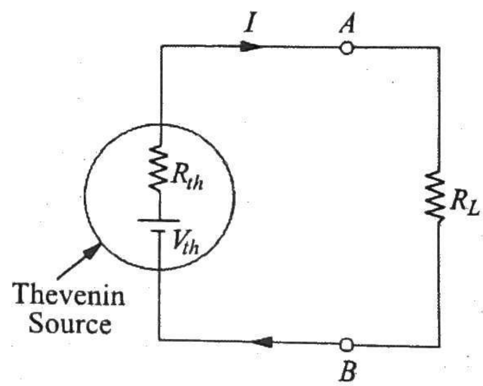


Fig. 2.128

4. R_L is now connected back across terminals A and B from where it was temporarily removed earlier. Current flowing through R_L is given by

$$I = \frac{V_{th}}{R_{th} + R_L}$$

It is clear from above that any network of resistors and voltage sources (and current sources as well) when viewed from any points A and B in the network, can be replaced by a single voltage source and a single resistance** in series with the voltage source.

After this replacement of the network by a single voltage source with a series resistance has been accomplished, it is easy to find current in any load resistance joined across terminals A and B. This theorem is valid even for those linear networks which have a nonlinear load.

Hence, Thevenin's theorem, as applied to d.c. circuits, may be stated as under :

The current flowing through a load resistance R_L connected across any two terminals A and B of a linear, active bilateral network is given by $V_{oc} \,||\, (R_i + R_L)$ where V_{oc} is the open-circuit voltage (i.e. voltage across the two terminals when R_L is removed) and R_i is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.

* After the French engineer M.L. Thevenin (1857-1926) who while working in Telegraphic Department published a statement of the theorem in 1893.

** Or impedance in the case of a.c. circuits.

2.19. How to Thevenize a Given Circuit ?

1. Temporarily remove the resistance (called load resistance R_L) whose current is required.
2. Find the open-circuit voltage V_{oc} which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage V_{th}.
3. Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit *i.e.* infinite resistance. It is also called Thevenin resistance R_{th} or T_i.
4. Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{oc} and whose internal resistance is R_{th} or R_i.
5. Connect R_L back to its terminals from where it was previously removed.
6. Finally, calculate the current flowing through R_L by using the equation,

$$I = V_{th}/(R_{th} + R_L) \quad \text{or} \quad I = V_{oc}/(R_i + R_L)$$

Example 2.59. *Convert the circuit shown in Fig. 2.129 (a), to a single voltage source in series with a single resistor.* **(AMIE Sec. B, Network Analysis Summer 1992)**

Solution. Obviously, we have to find equivalent Thevenin circuit. For this purpose, we have to calculate (*i*) V_{th} or V_{AB} and (*ii*) R_{th} or R_{AB}.

With terminals A and B open, the two voltage sources are connected in subtractive series because they oppose each other. Net voltage around the circuit is $(15-10) = 5$ V and total resistance is $(8 + 4) = 12\ \Omega$. Hence circuit current is = 5/12 A. Drop across 4 Ω resistor = $4 \times 5/12 = 5/3$ V with the polarity as shown in Fig. 2.129 (*a*).

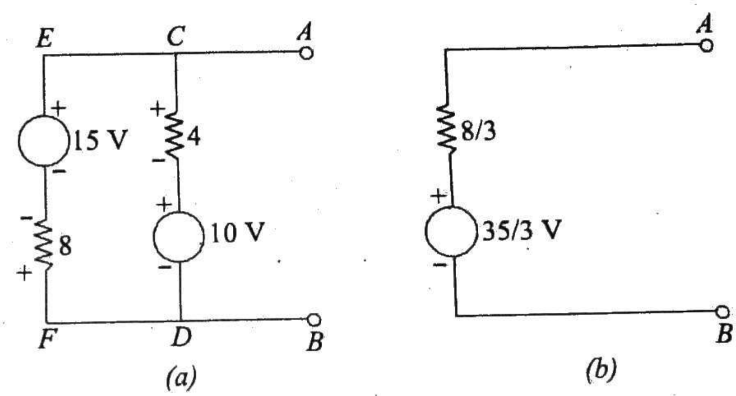


Fig. 2.129

$$\therefore \qquad V_{AB} = V_{th} = +10 + 5/3 = 35/3 \text{ V.}$$

Incidently, we could also find V_{AB} while going along the parallel route $BFEA$.

Drop across 8 Ω resistor = $8 \times 5/12 = 10/3$ V. V_{AB} equal the algebraic sum of voltages met on the way from B to A. Hence, $V_{AB} = (-10/3) + 15 = 35/3$ V.

As shown in Fig. 2.129 (*b*), the single voltage source has a voltage of 35/3 V.

For finding R_{th}, we will replace the two voltage sources by short-circuits. In that case, $R_{th} = R_{AB} = 4 \parallel 8 = 8/3\ \Omega$

Example 2.60. *State Thevenin's theorem and give a proof. Apply this theorem to calculate the current through the 4 Ω resistor of the circuit of Fig. 2.130 (a).* **(A.M.I.E. Sec. B Network Analysis W.)**

Solution. As shown in Fig. 2.130 (*b*), 4 Ω resistance has been removed thereby open-circuiting the terminals A and B. We will now find V_{AB} and R_{AB} which will give us V_{th} and R_{th} respectively. The potential drop across 5 Ω resistor can be found with the help of voltage-divider rule. Its value is $= 15 \times 5/(5 + 10) = 5$ V.

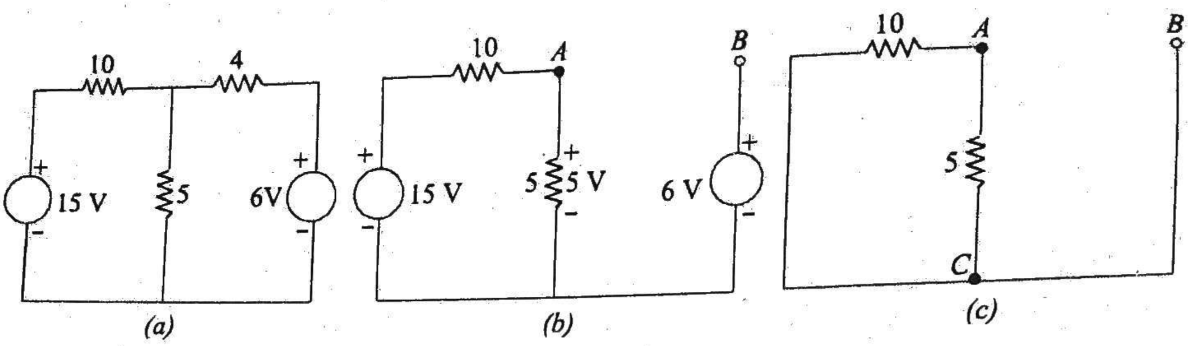


Fig. 2.130

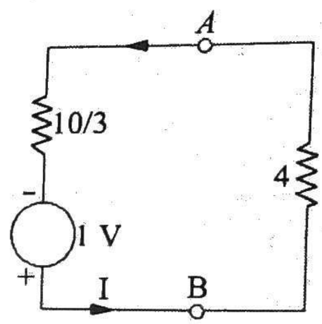


Fig. 2.131

For finding V_{AB}, we will go from point B to point A in the clockwise direction and find the algebraic sum of the voltages met on the way.

$$\therefore \quad V_{AB} = -6 + 5 = -1 \text{ V.}$$

It means that point A is negative with respect to point E, or point B is at a higher potential than point A by one volt.

In Fig. 2.130 (*c*), the two voltage source have been short-circuited. The resistance of the network as viewed from points A and B is the same as viewed from points A and C.

$$\therefore \quad R_{AB} = R_{AC} = 5 \parallel 10 = 10/3\ \Omega$$

Thevenin's equivalent source is shown in Fig. 2.131 in which 4 Ω resistor has been joined back across terminals A and B. Polarity of the voltage source is worth nothing.

$$\therefore \quad I = \frac{1}{(10/3)+4} = \frac{3}{22} = 0.136 \text{ A} \qquad \text{From } E \text{ to } A$$

Example 2.61. *With reference to the network of Fig. 2.132 (a), by applying Thevenin's theorem find the following :*

(i) the equivalent e.m.f. of the network when viewed from terminals A and B.

(ii) the equivalent resistance of the network when looked into from terminals A and B.

(iii) current in the load resistance R_L of 15 Ω **(Basic Circuit Analysis, Nagpur Univ. 1993)**

Solution. (*i*) Current in the network before load resistance is connected [Fig. 2.132 (*a*)]

$$= 24/(12 + 3 + 1) = 1.5 \text{ A}$$

$\therefore$ voltage across terminals $AB = V_{oc} = V_{th} = 12 \times 1.5 = \mathbf{18\ V}$

Hence, so far as terminals A and B are concerned, the network has an e.m.f. of 18 volt (and not 24 V).

(*ii*) There are two parallel paths between points A and B. Imagine that battery of 24 V is removed but not its internal resistance. Then, resistance of the circuit as looked into from point A and B is [Fig. 2.132 (*c*)]

$$R_i = R_{th} = 12 \times 4/(12 + 4) = 3\ \Omega$$

(*iii*) When load resistance of 15 Ω is connected across the terminals, the network is reduced to the structure shown in Fig. 2.132 (*d*).

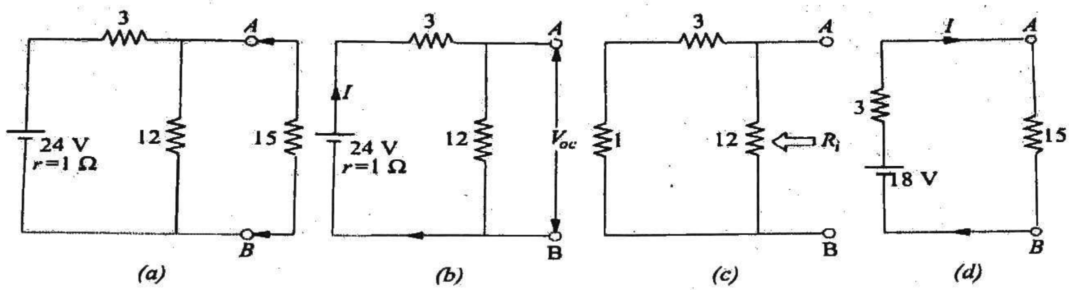


Fig. 2.132

$$I = V_{th}/(R_{th} + R_L) = 18/(15 + 3) = \mathbf{1\ A}$$

Example 2.62. *Using Thevenin theorem, calculate the current flowing through the 4 Ω resistor of Fig. 2.133 (a).*

Solution. (i) Finding V_{th}

If we remove the 4-Ω resistor, the circuit becomes as shown in Fig. 2.133 (*b*). Since full 10 A current passes through 2 Ω resistor, drop across it is 10 × 2 = 20 V. Hence, V_B = 20 V with respect to the common ground. The two resistors of 3 Ω and 6 Ω are connected in series across the 12 V battery. Hence, drop across 6 Ω resistor = 12 × 6/(3 + 6) = **8 V**.

∴ V_A = 8 V with respect to the common ground*

∴ $V_{th} = V_{BA} = V_B - V_A = 20 - 8 = 12$ V—with *B* at a higher potential

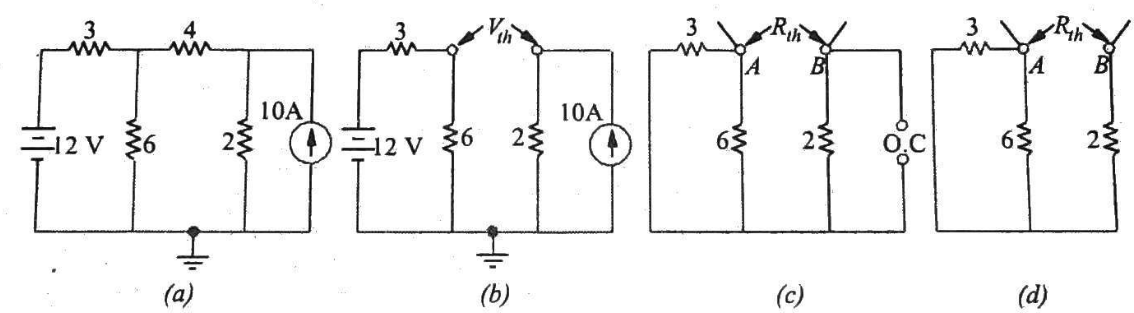


Fig. 2.133

(ii) Finding R_{th}

Now, we will find R_{th} *i.e.* equivalent resistance of the network as looked back into the open-circuited terminals *A* and *B*. For this purpose, we will replace both the voltage and current sources. Since voltage source has no internal resistance, it would be replaced by a short circuit *i.e.* zero resistance. However, current source would be removed and replaced by an 'open' *i.e.* infinite resistance (Art. 1.18). In that case, the circuit becomes as shown in Fig. 2.133 (*c*). As seen from Fig. 2.133 (*d*), $F_{th} = 6 \| 3 + 2 = 4\ \Omega$. Hence, Thevenin's equivalent circuit consists of a voltage source of 12 V and a series resistance of 4 Ω as shown in Fig. 2.134 (*a*). When 4 Ω resistor is connected across terminals *A* and *B*, as shown in Fig. 2.134 (*b*).

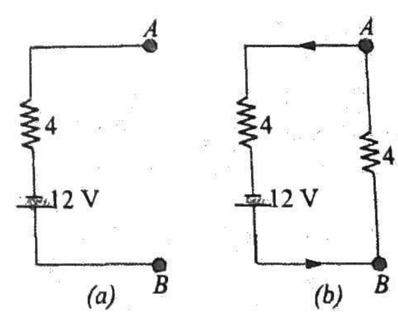


Fig. 2.134

I = 12/(4 + 4) = **1.5 A—from *B* to *A***

* Also, V_A = 12 −drop across 3-Ω resistor = 12 −12 × 3/(6 + 3) = 12− 4 = 8 V

Example 2.63. *For the circuit shown in Fig. 2.135 (a), calculate the current in the 10 ohm resistance. Use Thevenin's theorem only.*

(Elect. Science-I Allahabad Univ. 1992)

Solution. When the 10 Ω resistance is removed, the circuit becomes as shown in Fig. 2.135 (*b*).

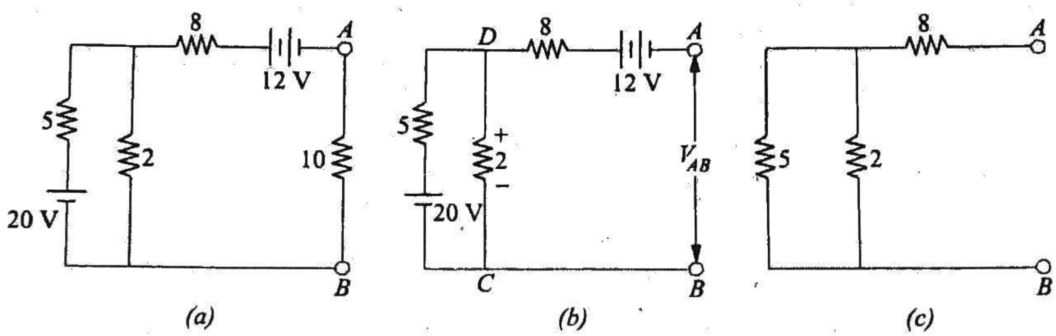


Fig. 2.135

Now, we will find the open-circuit voltage $V_{AB} = V_{th}$. For this purpose, we will go from point B to point A and find the algebraic sum of the voltages met on the way. It should be noted that with terminals A and B open, there is no voltage drop on the 8 Ω resistance. However the two resistances of 5 Ω and 2 Ω are connected in series across the 20-V battery. As per voltage-divider rule, drop on 2 Ω resistance = 20 × 2/(2 + 5) = 5.71 V with the polarity as shown in figure. As per the sign convention of Art.

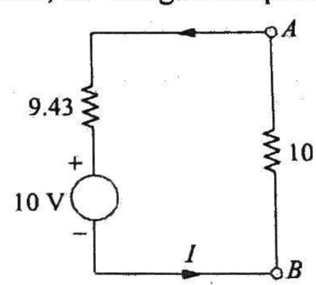


Fig. 2.136 (a)

$$V_{AB} = V_{th} = +5.71 - 12 = -6.29 \text{ V}$$

The negative sign shows that point A is negative with respect to point B or which is the same thing, point B is positive with respect to point A.

For finding $R_{AB} = R_{th}$, we replace the batteries by short-circuits as shown in Fig. 2.128 (*c*).

$$\therefore \qquad R_{AB} = R_{th} = 8 + 2 \parallel 5 = 9.43 \ \Omega$$

Hence, the equivalent Thevenin's source with respect to terminals A and B is as shown in Fig. 2.136. When 10 Ω resistance is reconnected across A and B, current through it is $I = 6.24/(9.43 + 10)$ = **0.32 A.**

Example 2.64. *Using Thevenin's theorem, calculate the p.d. across terminals A and B in Fig. 2.137 (a).*

Solution. (*i*) Finding V_{oc}

First step is to remove 7 Ω resistor thereby open-circuiting terminals A and B as shown in Fig. 2.137 (*b*). Obviously, there is no current through the 1 Ω resistor and hence no drop across it. Therefore $V_{AB} = V_{oc} = V_{CD}$. As seen, current I flows due to the combined action of the two batteries. Net voltage in the $CDFE$ circuit = 18 −6= 12 V. Total resistance = 6 + 3 = 9 Ω. Hence, I = 12/9 = 4/3 A

$$V_{CD} = 6 \text{ V} + \text{drop across } 3\ \Omega \text{ resistor} = 6 + (4/3) \times 3 = 10 \text{ V}^*$$

$$\therefore \qquad V_{oc} = V_{th} = 10 \text{ V.}$$

(*ii*) Finding R_i or R_{th}

As shown in Fig. 2.137 (*c*), the two batteries have been replaced by short-circuits (SC) since their internal resistances are zero. As seen, $R_i = R_{th} = 1 + 3 \parallel 6 = 3\ \Omega$. The Thevenin's equivalent circuit is as shown in Fig. 2.137 (*d*) where the 7 Ω resistance has been reconnected across terminals A and B.

* Also, V_{CD} = 18-drop across 6 Ω resistor = 18 −(4/3) × 6 = 10 V

The p.d. across this resistor can be found with the help of Voltage Divider Rule (Art. 1.15).

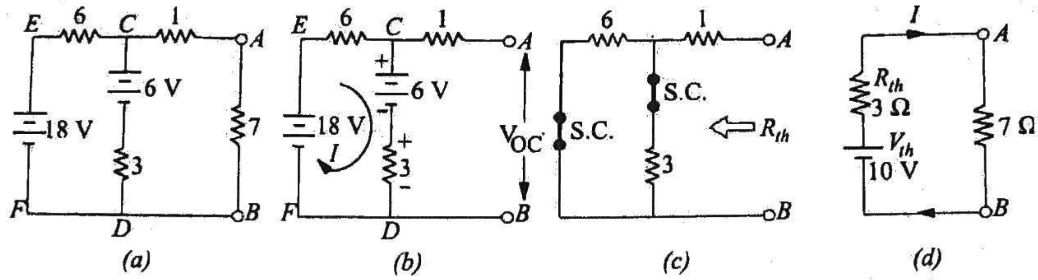


Fig. 2.137

Example 2.65. *Use Thevenin's theorem to find the current in a resistance load connected between the terminals A and B of the network shown in Fig. 2.138 (a) if the load is (a) 2 Ω (b) 1 Ω.*

(Elect. Technology, Gwalior Univ.)

Solution. For finding open-circuit voltage V_{oc} or V_{th} across terminals A and B, we must first find current I_2 flowing through branch CD. Using Maxwell's loop current method (Art. 2.11), we have from Fig. 2.131 (a).

$$-2\,I_1 - 4\,(I_1 - I_2) + 8 = 0 \quad \text{or} \quad 3\,I_1 - 2\,I_2 = 4$$

Also $$-2\,I_2 - 2\,I_2 - 4 - 4\,(I_2 - I_1) = 0 \quad \text{or} \quad I_1 - 2\,I_2 = 1$$

From these two equations, we get $I_2 = 0.25$ A

As we go from point D to C, voltage rise $= 4 + 2 \times 0.25 = 4.5$ V

Hence, $V_{CD} = 4.5$ or $V_{AB} = V_{th} = 4.5$ V. Also, it may be noted that point A is positive with respect to point B.

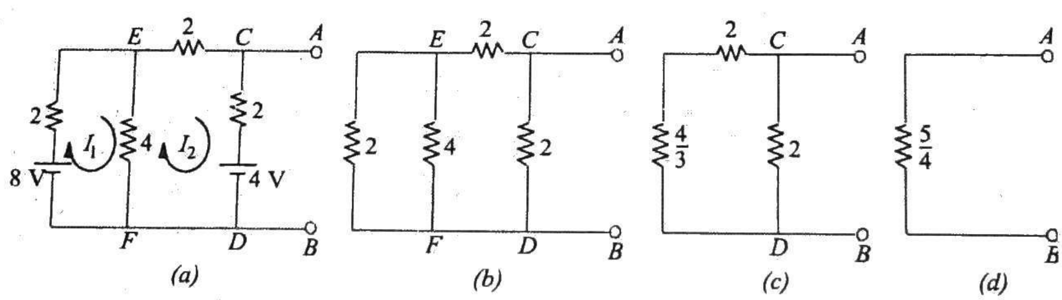


Fig. 2.138

In Fig. 2.138 (b), both batteries have been removed. By applying laws of series and parallel combination of resistances, we get $R_i = R_{th} = 5/4\ \Omega = 1.25\ \Omega$

(i) When $R_L = 2\ \Omega$; $\quad I = 4.5/(2 + 1.25) = $ **1.38 A**

(ii) When $R_L = 1\ \Omega$; $\quad I = 4.5\ (1 + 1.25) = $ **2.0 A**

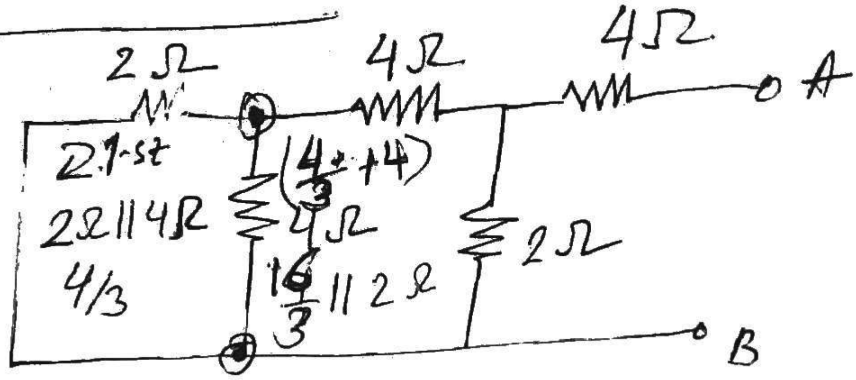
Note. We could also find V_{oc} and R_i by first Thevenining part of the circuit across terminals E and F and then across A and B (Ex. 2.62).

Example 2.66. *The four arms of a Wheatstone bridge have the following resistances :*

AB = 100, BC = 10, CD = 4, DA = 50 Ω. A galvanometer of 20 Ω resistance is connected across BD. Use Thevenin's theorem to compute the current through the galvanometer when a p.d. of 10 V is maintained across AC.

(Elect. Technology, Vikram Univ. of Ujjain)

* To find R_{th}



For 1st — $2\,\Omega \parallel 4\,\Omega$ and it convert like

$$\frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{2 \cdot 4}{2+4} = \frac{4}{3}\,\Omega$$

For 2-nd

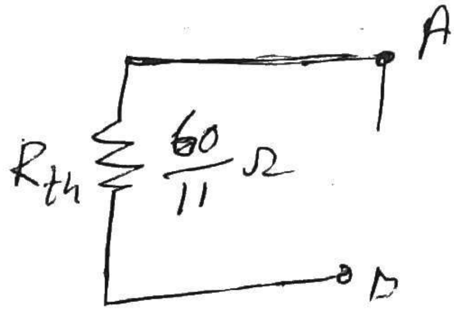
$\frac{4}{3}$ series $4\,\Omega$ and both are parallel to $2\,\Omega$

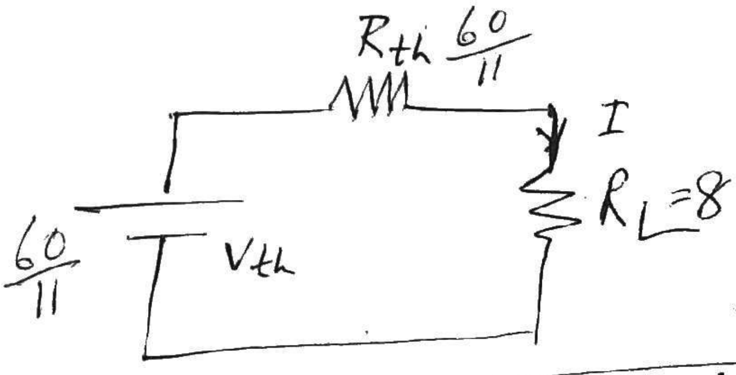
$$\left(\frac{4}{3} + 4\right)\Omega \parallel 2\,\Omega$$

$$\Rightarrow \frac{16}{3}\,\Omega \parallel 2\,\Omega \Rightarrow \frac{32/3}{22/3} = \frac{16}{11}$$

For 3rd

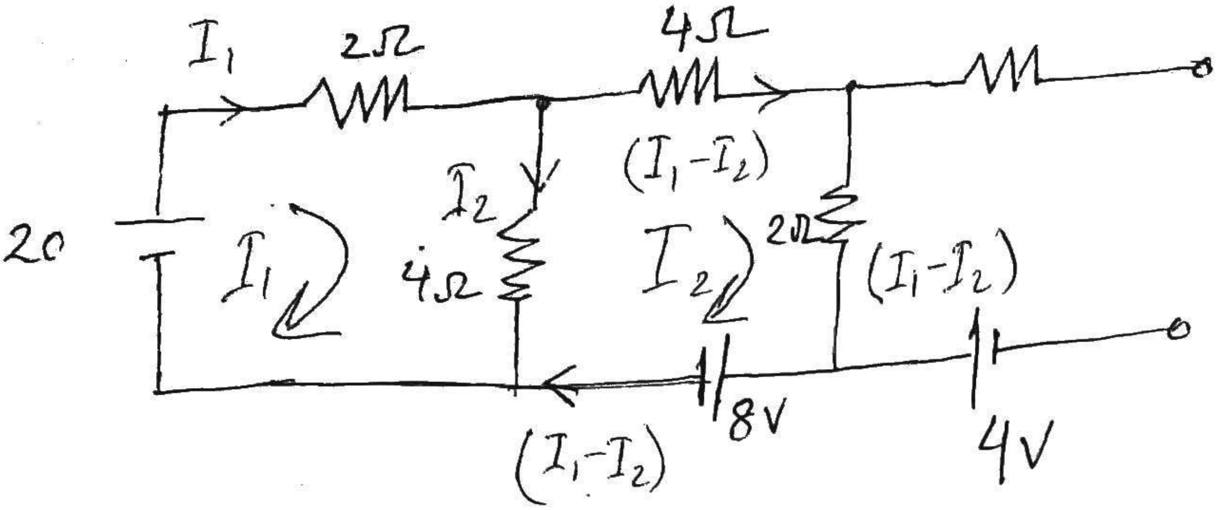
$$\frac{16}{11} + 4 = \frac{60}{11}\,\Omega$$





$$\boxed{I = \frac{V_{th}}{R_{th} + R_L} = \frac{\frac{66}{11}}{\frac{60}{11} + 8} = \frac{60}{148} = \boxed{\frac{15}{37}}}$$

* distribute current Kirchhoff's law



For Loop I_1 (KVL)

$$-2I_1 - 4I_2 + 20V = 0, \text{ ie. } I_1 + 2I_2 = 10 \quad \text{--- Eq}^n (1)$$

For Loop I_2

$$-4(I_1 - I_2) - 2(I_1 - I_2) - 8 + 4I_2 = 0$$

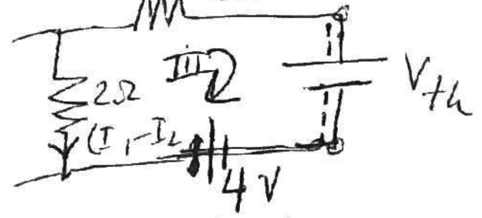
$$-6I_1 + 10I_2 = 8$$

$$-3I_1 + 5I_2 = 4 \quad \text{------ Eq}^n (2)$$

As (1) and (2)

$$I_1 = \frac{42}{11} A, \quad I_2 = \frac{34}{11} A$$

* To find "V_{th}"



V_{th} - এর যেকোন direction এ ধরতে পারে

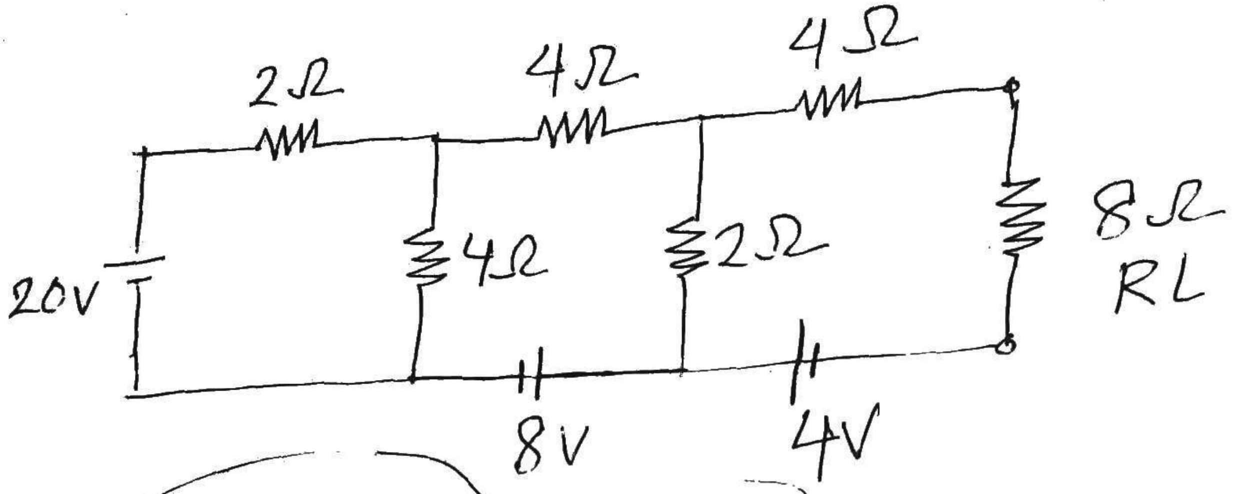
$$-V_{th} + (I_1 - I_2)2 + 4 = 0$$

$$V_{th} = 4 + 2\left[\frac{42}{11} - \frac{34}{11}\right]$$

$$= 4 + 2\left[\frac{8}{11}\right]$$

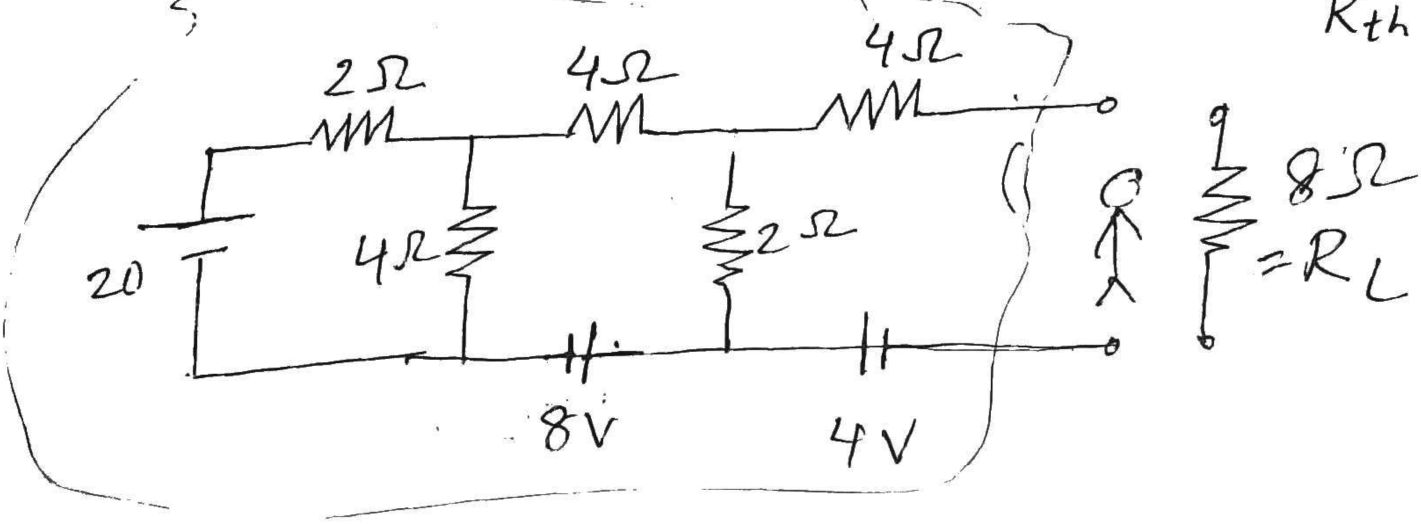
$$= \frac{60}{11} V$$

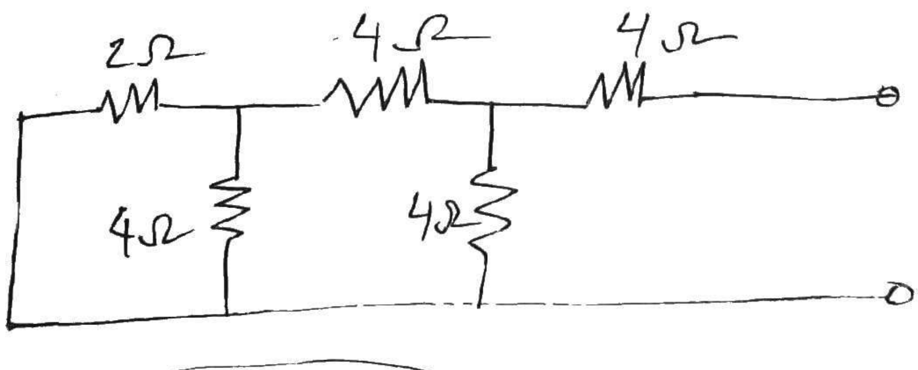
Find the current in 8Ω resistor using Thevenin's Theorem.

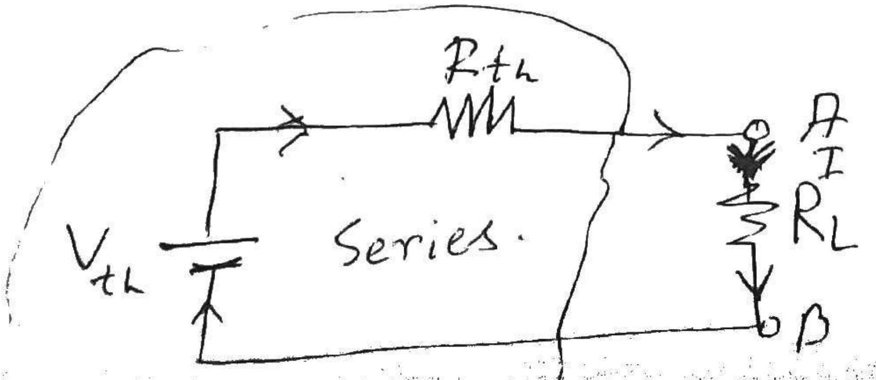


V_{th}

R_{th}







$$I = \frac{V_{th}}{R_{th} + R_L}$$