21. Find the value of the voltage v in the network of Fig. 2.47. [36 V]
22. Determine the current i for the network shown in Fig. 2.48. [–40 A]
23. State and explain Kirchhoff's current law. Determine the value of R_S and R_P in the network of Fig. 2.49 if $V_2 = V_1/2$ and the equivalent resistance of the network between the terminals A and B is 100 Ω

[R_S = 100/3 Ω R_P = 400/3 Ω] (*Elect. Engg. I, Bombay Univ.*)
24. Four resistance each of R ohms and two resistances each of S ohms are connected (as shown in Fig. 2.50) to four terminasl AB and CD. A p.d. of V volts is applied across the terminals AB and a resistance of Z ohm is connected across the terminals CD. Find the value of Z in terms of S and R in order that the current at AB may be V/Z.

Find also the relationship that must hold between R and S in order that the p.d. at the points EF be $V/2$.

[$Z = \sqrt{R(R + 2S)}$; $S = 4R$]

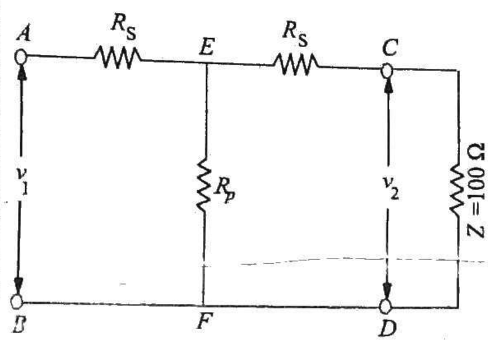

Fig. 2.49

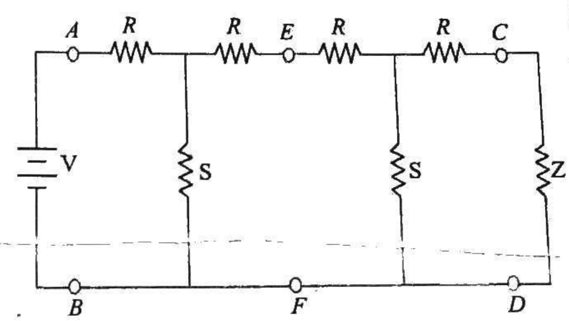

Fig. 2.50

2.10. Maxwell's Loop Curent Method

This method which is particularly well-suited to coupled circuit solutions employs a system of *loop* or *mesh* currents instead of *branch* currents (as in Kirchhoff's laws). Here, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. This method eliminates a great deal of tedious work involved in the branch-current method and is best suited when energy sources are voltage sources rather than current sources. Basically, this method consists of writing loop voltage equations by Kirchhoff's voltage law in terms of unknown loop currents. As will be seen later, the number of independent equations to be solved reduces from b by Kirchhoff's laws to $b - (j - 1)$ for the loop current method where b is the number of branches and j is the number of junctions in a given network.

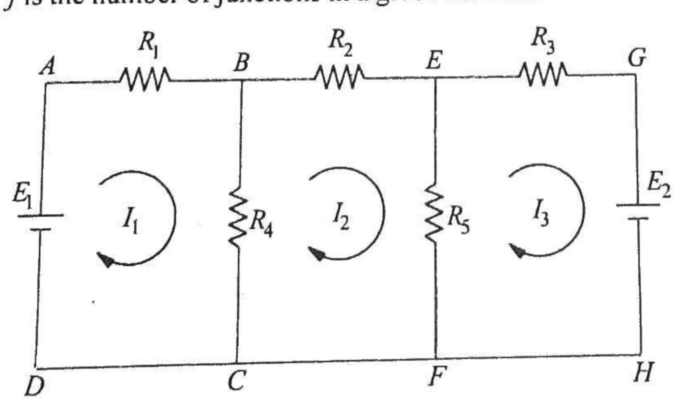

Fig. 2.51

Fig. 2.51 shows two batteries E_1 and E_2 connected in a network consisting of five resistors. Let the loop currents for the three meshes be I_1, I_2 and I_3. It is obvious that current through R_4 (when considered as a part of the first loop) is $(I_1 - I_2)$ and that through R_5 is $(I_2 - I_3)$. However, when R_4 is considered part of the second loop, current through it is $(I_2 - I_1)$. Similarly, when R_5 is considered part of the third loop, current through it is $(I_3 - I_2)$. Applying Kirchhoff's voltage law to the three loops, we get,

$$E_1 - I_1R_1 - R_4(I_1 - I_2) = 0 \quad \text{or} \quad I_1(R_1 + R_4) - I_2R_4 - E_1 = 0 \qquad \text{...loop 1}$$

Similarly, $-I_2R_2 - R_5(I_2 - I_3) - R_4(I_2 - I_1) = 0$

or $I_2R_4 - I_2(R_2 + R_4 + R_5) + I_3R_5 = 0$ ...loop 2

Also $-I_3R_3 - E_2 - R_5(I_3 - I_2) = 0$ or $I_2R_5 - I_3(R_3 + R_5) - E_2 = 0$ ...loop 3

The above three equations can be solved not only to find loop currents but branch currents as well.

2.11. Mesh Analysis Using Matrix Form

Consider the network of Fig. 2.52, which contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be designated as I_1, I_2 and I_3 and all the three may be assumed to flow in the clockwise direction for obtaining symmetry in mesh equations.

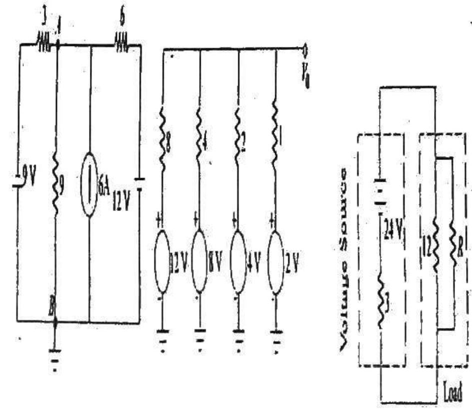

Fig. 2.52

Applying KVL to mesh (*i*), we have

$$E_1 - I_1R_1 - R_3(I_1 - I_3) - R_2(I_1 - I_2) = 0$$

or $(R_1 + R_2 + R_3)I_1 - R_2I_2 - R_3I_3 = E_1$ ...(*i*)

Similarly, from mesh (*ii*), we have

$$E_2 - R_2(I_2 - I_1) - R_5(I_2 - I_3) - I_2R_4 = 0$$

or $-R_2I_1 + (R_2 + R_4 + R_5)I_2 - R_5I_3 = E_2$ ...(*ii*)

Applying KVL to mesh (*iii*), we have

$$E_3 - I_3R_7 - R_5(I_3 - I_2) - R_3(I_3 - I_1) - I_3R_6 = 0$$

or $-R_3I_1 - R_5I_2 + (R_3 + R_5 + R_6 + R_7)I_3 = E_3$ ...(*iii*)

It should be noted that signs of different items in the above three equations have been so changed as to make the items containing self resistances positive (please see further).

The matrix equivalent of the above three equations is

$$\begin{bmatrix} +(R_1 + R_2 + R_3) & -R_2 & -R_3 \\ -R_2 & +(R_2 + R_4 + R_5) & -R_5 \\ -R_3 & -R_5 & +(R_3 + R_5 + R_6 + R_7) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It would be seen that the first item is the first row *i.e.* $(R_1 + R_2 + R_3)$ represents the self resistance of mesh (*i*) which equals the sum of all resistance in mesh (*i*). Similarly, the second item in the first row represents the mutual resistance between meshes (*i*) and (*ii*) *i.e.* the sum of the resistances common to mesh (*i*) and (*ii*). Similarly, the third item in the first row represents the mutual-resistance of the mesh (*i*) and mesh (*ii*).

The item E_1, in general, represents the algebraic sum of the voltages of all the voltage sources acting around mesh (*i*). Similar is the case with E_2 and E_3. The sign of the *e.m.f*'s is the same as discussed in Art. 2.3 *i.e.* while going along the current, if we pass from negative to the positive terminal of a battery, then its *e.m.f.* is taken positive. If it is the other way around, then battery *e.m.f.* is taken negative.

In general, let

R_{11} = self-resistance of mesh (*i*)

R_{22} = self-resistance of mesh (*ii*) *i.e.* sum of all resistances in mesh (*ii*)

R_{33} = Self-resistance of mesh (*iii*) *i.e.* sum of all resistances in mesh (*iii*)

$R_{12} = R_{21}$ = –[Sum of all the resistances common to meshes (*i*) and (*ii*)] *

$R_{23} = R_{32}$ = –[Sum of all the resistances common to meshes (*ii*) and (*iii*)]*

* Although, it is easier to take all loop currents in one direction (Usually clockwise), the choice of direcion for any loop current is arbitrary and may be chosen independently of the direction of the other loop currents.

$R_{31} = R_{13} = -$[Sum of all the resistances common to meshes (*i*) and (*iii*)] *

Using these symbols, the generalized form of the above matrix equivalent can be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

If there are *m* independent meshes in any liner network, then the mesh equations can be written in the matrix form as under :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & \ldots & R_{1m} \\ R_{21} & R_{22} & R_{23} & \ldots & R_{2m} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ R_{31} & R_{32} & R_{33} & \ldots & R_{3m} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \ldots \\ \ldots \\ I_m \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \ldots \\ \ldots \\ E_m \end{bmatrix}$$

The above equations can be written in a more compact form as $[R_m]$ $[I_m] = [E_m]$. It is known as Ohm's law in matrix form.

In the end, it may be pointed out that the directions of mesh currents can be selected arbitrarily. If we assume each mesh current to flow in the clockwise direction, then

(*i*) All self-resistances will always be postive and (*ii*) all mutual resistances will always be negative. We will adapt this sign convention in the solved examples to follow.

The above main advantage of the generalized form of all mesh equations is that they can be easily remembered because of their symmetry. Moreover, for any given network, these can be written by inspection and then solved by the use of determinants. It eliminates the tedium of deriving simultaneous equations.

Example. 2.30. *Write the impedance matrix of the network shown in Fig. 2.53 and find the value of current I_3.* **(Network Analysis A.M.I.E. Sec. B.W. 1980)**

Solution. Different items of the mesh-resistance matrix $[R_m]$ are as under :

$$R_{11} = 1 + 3 + 2 = 6\ \Omega;\ R_{22} = 2 + 1 + 4 = 7\ \Omega;\ R_{33} = 3 + 2 + 1 = 6\ \Omega;$$

$$R_{12} = R_{21} = -2\ \Omega;\ R_{23} = R_{32} = -1\ \Omega;\ R_{13} = R_{31} = -3\ \Omega;$$

$$E_1 = +5\text{ V};\ E_2 = 0;\ E_3 = 0.$$

The mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} = 6(42-1) + 2(-12-3) - 3\,(2+21) = 147$$

$$\Delta_3 = \begin{bmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{bmatrix} = 6 + 2(5) - 3(-35) = 121$$

$$I_3 = \Delta_3/\Delta = \frac{121}{147} = 0.823\text{ A}$$

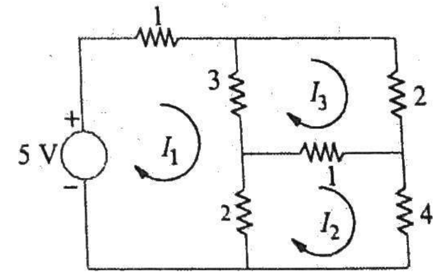

Fig. 2.53

* In general, if the two currents through the common resistance flow in the same direction, then the mutual resistance is taken as negative. One the other hand, if the two currents flow in the same direction, mutual resistance is taken as positive.

Example 2.31. *Determine the current supplied by each battery in the circuit shown in Fig. 2.54.*

(Electrical Engg. Aligarh Univ.)

Solution. Since there are three meshes, let the three loop currents be shown in Fig. 2.51.

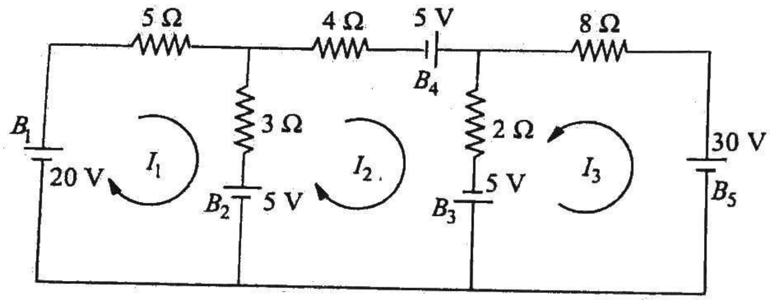

Fig. 2.54

For loop 1 we get

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0 \quad \text{or} \quad 8I_1 - 3I_2 = 15 \qquad ...(i)$$

For loop 2 we have

$$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0 \quad \text{or} \quad 3I_1 - 9I_2 + 2I_3 = -15 \qquad ...(ii)$$

Similarly, for loop 3, we get

$$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0 \quad \text{or} \quad 2I_2 - 10I_3 = 35 \qquad ...(iii)$$

Eliminating I_1 from (*i*) and (*ii*), we get $63I_2 - 16I_3 = 165$ ...(*iv*)

Similarly, for I_2 from (*iii*) and (*iv*), we have $I_2 = 542/299$ A

From (*iv*), $I_3 = -1875/598$ A

Substituting the value of I_2 in (*i*), we get $I_1 = 765/299$ A

Since I_3 turns out to be negative, actual directions of flow of loop currents are as shown in Fig. 2.55.

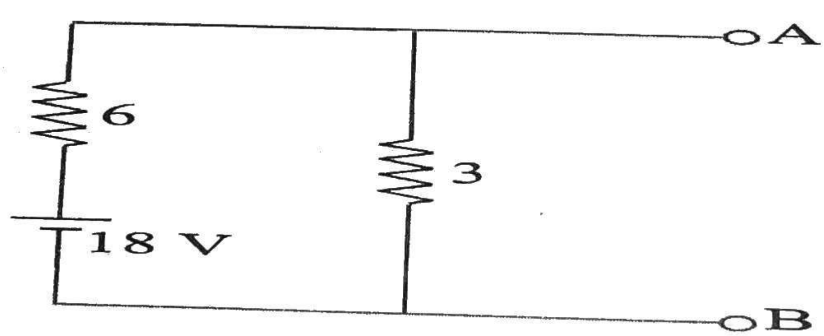

Fig. 2.55

Discharge current of B_1 = **765/299A**

Charging current of $B_2 = I_1 - I_2 =$ **220/299 A**

Discharge current of $B_3 = I_2 + I_3 =$ **2965/598 A**

Discharge current of $B_4 = I_2 =$ **545/299 A**; Discharge current of B_5 = **1875/598 A**

Solution by Using Mesh Resistance Matrix.

The different items of the mesh-resistance matrix $[R_m]$ are as under :

$R_{11} = 5 + 3 = 8\ \Omega$; $R_{22} = 4 + 2 + 3 = 9\ \Omega$; $R_{33} = 8 + 2 = 10\ \Omega$

$R_{12} = R_{21} = -3\ \Omega$; $R_{13} = R_{31} = 0$; $R_{23} = R_{32} = -2\ \Omega$

E_1 = algebraic sum of the voltages around mesh (*i*) = 20 – 5 = 15 V

$E_2 = 5 + 5 + 5 = 15$ V ; $E_3 = -30 - 5 = -35$ V

Hence, the mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} = 8(90-4)+3(-30) = 598$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = 15(90-4)-15(-30)-35\,(6) = 1530$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = 8(150-70)+3(150+0) = 1090$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = 8(-315+30)+3(105+30) = -1875$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1530}{598} = \frac{765}{299}\text{ A} ; I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = \frac{545}{299}\text{ A} ; I_3 = \frac{\Delta_3}{\Delta} = \frac{-1875}{598}\text{ A}$$

Example 2.32. *Determine the current in the 4-Ω branch in the circuit shown in Fig. 2.56.*

(Elect. Technology, Nagpur Univ.)

Solution. The three loop currents are as shown in Fig. 2.53 (*b*).

For loop 1, we have

$-1\,(I_1 - I_2) - 3\,(I_1 - I_3) - 4I_1 + 24 = 0$ or $8I_1 - I_2 - 3I_3 = 24$ ...(*i*)

For loop 2, we have

$12 - 2I_2 - 12\,(I_2 - I_3) - 1\,(I_2 - I_1) = 0$ or $I_1 - 15I_2 + 12I_3 = -12$ ...(*ii*)

Similarly, for loop 3, we get

$-12\,(I_3 - I_2) - 2I_3 - 10 - 3\,(I_3 - I_1) = 0$ or $3I_1 + 12I_2 - 17I_3 = 10$ ...(*iii*)

Eliminating I_2 from Eq. (*i*) and (*ii*) above, we get, $119I_1 - 57I_3 = 372$ ...(*iv*)

Similarly, eliminating I_2 from Eq. (*ii*) and (*iii*), we get, $57I_1 - 111I_3 = 6$ ...(*v*)

From (*iv*) and (*v*) we have,

$$I_1 = 40{,}950/9{,}960 = \mathbf{4.1\ A}$$

Solution by Determinants

The three equations as found above are

$$8I_1 - I_2 - 3I_3 = 24$$
$$I_1 - 15I_2 + 12I_3 = -12$$
$$3I_1 + 12I_2 - 17I_3 = 10$$

Their matrix form is $\begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -12 \\ 10 \end{bmatrix}$

$$\Delta = \begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix} = 664,\ \Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ -12 & -15 & 12 \\ 10 & 12 & -17 \end{bmatrix} = 2730$$

$\therefore\ I_1 = \Delta_1/\Delta = 2730/664 = \mathbf{4.1\ A}$