Integration by Parts

Formula: $\int \mathbf{uv\,dx} = \mathbf{u}\int \mathbf{v\,dx} - \int \left\{\frac{\mathbf{d}}{\mathbf{dx}}(\mathbf{u})\int \mathbf{v\,dx}\right\}\mathbf{dx}$

L	I	A	T	E	C

L = Logarithm (log x)

I = Inverse (x^{-1}, $\sin^{-1} x$)

A = Algeraic Function (x^2)

T = Trigonometric Function (sin x, cos x)

E = Exponential (e^x)

C = Constant (1,2,3, ….)

Ex. 1) $\int \mathbf{x\cos x\,dx}$

$= x\int \cos x\,dx - \int \left\{\frac{d}{dx}(x)\int \cos x\,dx\right\}dx$

$= x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + c$

Ex. 2) $\int \mathbf{\ln x\,dx}$

$= \ln x\int 1\,dx - \int \left\{\frac{d}{dx}(\ln x)\int 1\,dx\right\}dx$

$= \ln x\,.\,x - \int \frac{1}{x}.x\,dx$

$= x\ \ln x - \int 1\,dx$

$= x\ \ln x - x + c$

Ex. 3) $\int \cos^{-1} x \, dx$

$= \cos^{-1} x \int 1 \, dx - \int \left\{ \frac{d}{dx} (\cos^{-1} x) \int 1 \, dx \right\} dx$

$= \cos^{-1} x \, . \, x - \int \frac{-x}{\sqrt{1 - x^2}} dx$ Let, $1 - x^2 = z$

$= x \cos^{-1} x - \int \frac{\frac{dz}{2}}{\sqrt{z}}$ $\Rightarrow -2xdx = dz$

$= x \cos^{-1} x - \frac{1}{2} \frac{\sqrt{z}}{\frac{1}{2}} + c$ $\Rightarrow -xdx = \frac{dz}{2}$

$= x \cos^{-1} x - \frac{1}{2} . 2\sqrt{z} + c$

$= x \cos^{-1} x - \sqrt{1 - x^2} + c$

Ex. 4) $\int \tan^{-1} x \, dx$

$= \tan^{-1} x \int 1 \, dx - \int \left\{ \frac{d}{dx} (\tan^{-1} x) \int 1 \, dx \right\} dx$

$= \tan^{-1} x \, . \, x - \int \frac{1}{1 + x^2} x \, dx$ Let, $1 + x^2 = z$

$= x \tan^{-1} x - \int \frac{\frac{dz}{2}}{z}$ $\Rightarrow 2xdx = dz$

$= x \tan^{-1} x - \frac{1}{2} . \ln z + c$ $\Rightarrow xdx = \frac{dz}{2}$

$= x \tan^{-1} x - \frac{1}{2} . \ln(1 + x^2) + c$

Ex. 5) $\int x^2 e^x \, dx$

$= x^2 \int e^x \, dx - \int \left\{ \frac{d}{dx} (x^2) \int e^x \, dx \right\} dx$

$= x^2 e^x - \int 2x \, e^x \, dx$

$$= x^2e^x - 2\left[x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x\,dx\right\}dx\right]$$

$$= x^2e^x - 2\left[xe^x - \int e^x\,dx\right]$$

$$= x^2e^x - 2(xe^x - e^x) + c$$

$$= x^2e^x - 2xe^x + 2e^x + c$$

Ex. 6) $\int \ln\left(x + \sqrt{x^2 + a^2}\right)dx$

$$= \ln\left(x + \sqrt{x^2 + a^2}\right)\int 1\,dx - \int\left\{\frac{d}{dx}\ln\left(x + \sqrt{x^2 + a^2}\right)\int 1\,dx\right\}dx$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \int \frac{1}{\left(x + \sqrt{x^2 + a^2}\right)}\left(1 + \frac{1}{2\sqrt{x^2 + a^2}}.2x\right)x\,dx$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \int \frac{1}{\left(x + \sqrt{x^2 + a^2}\right)}\left(1 + \frac{x}{\sqrt{x^2 + a^2}}.\right)x\,dx$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \int \frac{1}{\left(x + \sqrt{x^2 + a^2}\right)}\left(\frac{x + \sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2}}.\right)x\,dx$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \int \frac{x}{\sqrt{x^2 + a^2}}\,dx$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \int \frac{x}{\sqrt{x^2 + a^2}}\,dx \qquad \text{Let}, x^2 + a^2 = z$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \int \frac{\frac{dz}{2}}{\sqrt{z}} \qquad \Rightarrow 2xdx = dz$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \frac{1}{2}\frac{\sqrt{z}}{\frac{1}{2}} + c \qquad \Rightarrow xdx = \frac{dz}{2}$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \frac{1}{2}.2\sqrt{z} + c$$

$$= x\ln\left(x + \sqrt{x^2 + a^2}\right) - \sqrt{x^2 + a^2} + c$$

H.W:

1) $\int x \ln x \, dx$	6) $\int \frac{x + \sin x}{1 + \cos x} \, dx$
2) $\int \left(e^{\sqrt{x}} - e^{-\sqrt{x}}\right) dx$	7) $\int \frac{\ln(\ln x)}{x} \, dx$
3) $\int (\sin^{-1} x)^2 \, dx$	8) $\int \frac{\ln(x+1)}{\sqrt{x+1}} \, dx$
4) $\int x^2 \sin^2 x \, dx$	9) $\int \frac{x}{\sec x + 1} \, dx$
5) $\int \frac{x}{1 + \cos x} \, dx$	10) $\int (\ln x)^2 \, dx$