Expected value = weighted sum of probabilities $P(H) = \frac{1}{2}$

$E(H) = 2$

TTH	H	TH	TH	H	TH
3	1	2	2	1	2

$$E(H) = \frac{3+1+2+2+1+2}{6} = \frac{11}{6} \approx 1.8$$

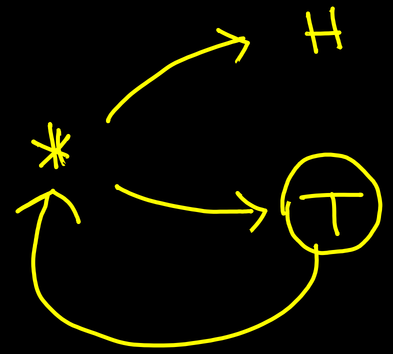
Dice

$$E(D) = \frac{1+2+3+4+5+6}{6} = 3.5$$

$$E(D) = \frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \frac{1}{6} \times 4 + \frac{1}{6} \times 5 + \frac{1}{6} \times 6$$

$$E(D) = \frac{2}{7} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \frac{1}{6} \times 4 + \frac{1}{6} \times 5 + \frac{1}{6} \times 6$$

$$\boxed{E(H) = \underbrace{\frac{1}{2} \times 1}_{H} + \underbrace{\frac{1}{2}\left(E(H) + 1\right)}_{T}}$$



$$\Rightarrow E(H) = \frac{1}{2} + \frac{1}{2} E(H) + \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} E(H) = 1$$

$$\Rightarrow \boxed{E(H) = 2}$$

$$E(HH) = \frac{1}{4}\times 2 + \frac{1}{4}\left(E(HH)+2\right) + \frac{1}{2}\left(E(HH)+1\right)$$

$$HH + HT + T$$

$$\left\{\begin{array}{l} TH \rightarrow \\ HT \rightarrow \\ TT \rightarrow \end{array}\right.$$

$$E(HH) - \frac{1}{4}E(HH) - \frac{1}{2}E(HH) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

$$\Rightarrow \frac{4-1-2}{4}E(HH) = \frac{3}{2}$$

$$\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

$$\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{4}E(HH) = \frac{3}{2}$$

$$\frac{1}{2}$$

$$\boxed{E(HH) = 6}$$

$$E(HHH) = \frac{1}{8} \times 3 + \frac{1}{8}\left(E(HHH)+3\right) + \frac{1}{4}\left(E(HHH)+2\right) + \frac{1}{2}\left(E(HHH)+1\right)$$

HHH HHT HT T

$$E = \left(\frac{1}{2}\right)^{3}$$

W $E(HHH \ldots H) = \boxed{\quad\quad\quad\quad}$?   W W $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

W $E(1 \text{ to } 6) = \boxed{\quad\quad\quad\quad}$?   W W

[2 3 2 4 1 2 4 5 4 3 2 1 2 6]

3	2	-4	5	1	-3	7	-3

k

$$E(M) = \frac{3 + 2 + (E(M)+4) + 5 + 1 + (E(M)+3) + 7 + (E(M)+3)}{8}$$

$$\Rightarrow E(M) = \boxed{} \; ?$$

$$E(HHH\ldots H) = \underbrace{\left(\frac{1}{2}\right)^{n} \times n}_{\boxed{HHH\ldots H}} + \underbrace{\left(\frac{1}{2}\right)^{n-1}\left(E(HHH\ldots H) + n\right)}_{\boxed{\underbrace{HHH\ldots HT}_{(n-1)+1}}}$$

$$+ \underbrace{\left(\frac{1}{2}\right)^{n-2}\left(E(HHH\ldots H) + (n-1)\right)}_{\boxed{HHHH\ldots \underset{\overset{\uparrow}{(n-1)}}{T}\,\square}} + \ldots$$

$$+ \underbrace{\frac{1}{2}\left(E(HHH\ldots H) + 1\right)}_{\boxed{T}}$$

$$E(\underline{HT}) = \underbrace{\frac{1}{4}\times 2}_{HT} + \underbrace{\frac{1}{4}\left(E(HT)+2\right)}_{HH} + \underbrace{\frac{1}{2}\left(E(HT)+1\right)}_{T}$$

$$\begin{cases} E(HTH) = \\ E(TTH) = \end{cases}$$

Ackermann Function

$$A(x,y) = \begin{cases} y+1, & x=0 \\ A(x-1,1), & y=0 \\ A(x-1, A(x, y-1)) & \text{otherwise} \end{cases}$$

$$\underline{A(1,2)} = A(1-1, A(1,2-1))$$
$$= A(0, \underline{A(1,1)})$$
$$= A(0, \underline{A(0,2)})$$
$$= A(0,3)$$
$$= 3+1$$
$$= 4$$

$$\underline{A(1,0)}$$
$$= A(1-1,1)$$
$$= A(0,1)$$
$$= 1+1 = 2$$

$$\underline{A(1,1)}$$
$$= A(1-1, A(1,1-1))$$
$$= A(0, \underline{A(1,0)})$$
$$= \underline{A(0,2)}$$
$$= 2+1 = 3$$

$$A(x,y) = \begin{cases} y+1, & x=0 \\ A(x-1,1), & y=0 \\ A(x-1, A(x,y-1)) & \text{otherwise} \end{cases}$$

$$A(2,1) = A(2-1, A(2, 1-1))$$
$$= A(1, \underline{A(2,0)})$$
$$= A(1,3)$$
$$= A(1-1, A(1, 3-1))$$
$$= A(0, \underline{A(1,2)})$$
$$= A(0,4)$$
$$= 4+1$$
$$= 5$$

$$A(1,0)$$
$$= A(1-1, 1)$$
$$= A(0,1)$$
$$= 2$$

$$\underline{A(2,0)}$$
$$= A(2-1, 1)$$
$$= A(1,1)$$
$$= A(1-1, A(1, 1-1))$$
$$= A(0, \underline{A(1,0)})$$
$$= \underline{A(0,2)}$$
$$= 3$$