VECTOR ALGEBRA

Ex. 1) If $A = 3\mathbf{i} - \mathbf{j} - 4\mathbf{k}$ and $B = -2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, then find

(i) $|2A + B|$ (ii) $|A + B|$ (iii) $|6A - 4B|$

Solution:

(i) $2A + B = 2(3\mathbf{i} - \mathbf{j} - 4\mathbf{k}) + (-2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$

$= 6\mathbf{i} - 2\mathbf{j} - 8\mathbf{k} - 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} = = 4\mathbf{i} + 2\mathbf{j} - 11\mathbf{k}$

$|2A + B| = \sqrt{4^2 + 2^2 + (-11)^2} = \sqrt{16 + 4 + 121} = \sqrt{141}$

Ex. 2) For what value of m, two vectors $A = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ and $B = m\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

will be perpendicular?

Solution: If the vectors A and B are perpendicular then $A.B = 0$

$A.B = (2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}).(m\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}) = 0$

$\Rightarrow 2m + 6 - 24 = 0$

$\Rightarrow 2m = 18$

$\therefore m = 9$

Ex. 3) Find the component of the vector $B = 5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ along the vector $A = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

Solution: The component of the vector B along $A = \frac{A.B}{|A|}\,\widehat{A}$

$A.B = 10 - 3 - 4 = 3$

$|A| = \sqrt{4 + 1 + 4} = 3$

and, $\widehat{A} = \frac{A}{|A|} = \frac{2\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{3}$

The component of the vector B along $A = \frac{3}{3}\,\frac{2\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{3} = \frac{1}{3}(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$

Ex. 4) Find the angle between the vectors $A = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $B = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.

Solution: Let θ be the angle between the two vectors.

Then, $A.B = |A|\,|B| \cos\theta$

$|A| = 3$ and $|B| = \sqrt{35}$

$A.B = 2 - 6 - 5 = -9$

$\therefore -9 = 3\,.\sqrt{35}\cos\theta$

$\therefore \theta = \cos^{-1}\left(-\frac{3}{\sqrt{35}}\right)$

Ex. 5) If $A = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $B = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, then find $A \times B$.

Solution:

$$A \times B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{bmatrix} = \mathbf{i}(6+4) - \mathbf{j}(-4+1) + \mathbf{k}(8+3) = 10\mathbf{i} + 3\mathbf{j} + 11\mathbf{k}$$

Ex. 6) Find the value of **a** such that the three vectors $2\mathbf{i} + \mathbf{j} - \mathbf{k}, 3\mathbf{i} - 2\mathbf{j} + \mathbf{4k}, \mathbf{i} - 3\mathbf{j} + a\mathbf{k}$ will be coplanar.

Solution: Let, $A = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, B = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, C = \mathbf{i} - 3\mathbf{j} + a\mathbf{k}$

When three vectors are in a plane, then $(A \times B).C = 0$

$$A \times B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ 3 & -2 & 4 \end{vmatrix} = \mathbf{i}(4-2) - \mathbf{j}(8+3) + \mathbf{k}(-4-3) = 2\mathbf{i} - 11\mathbf{j} - 7\mathbf{k}$$

Now,

$(A \times B).C = 0$

$\Rightarrow (2\mathbf{i} - 11\mathbf{j} - 7\mathbf{k}).(\mathbf{i} - 3\mathbf{j} + a\mathbf{k}) = 0$

$\Rightarrow 2 + 33 - 7a = 0$

$\therefore a = 5$

Ex.7) Find a unit vector which is perpendicular to the plane of the vector

$A = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, B = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Solution: Given, $A = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, B = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$$A \times B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 2 \\ 2 & -2 & 1 \end{vmatrix} = \mathbf{i}(2+4) - \mathbf{j}(1-4) + \mathbf{k}(-2-4) = 6\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$$

$|A \times B| = 9$

The unit vector which is perpendicular to two vectors $= \dfrac{A \times B}{|A \times B|} = \dfrac{6\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}}{9}$

Ex. 8) Find the value of **a** such that the vectors $P = 2\mathbf{i} + a\mathbf{j} - 3\mathbf{k}, \mathbf{Q} = 6\mathbf{i} - 3\mathbf{j} - 9\mathbf{k}$

will be parallel.

Solution: If P and Q are parallel, then $P \times Q = 0$

$$\Rightarrow \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & a & -3 \\ 6 & -3 & -9 \end{vmatrix} = 0$$

$\Rightarrow \mathbf{i}\,(-9a - 9) - \mathbf{j}(-18 + 18) + \mathbf{k}(-6 - 6a) = 0$

$\Rightarrow a = -1$

H.W:

1) Find the angle between the vectors $A = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $B = \mathbf{i} - \mathbf{j} + \mathbf{k}$

2) Find the angle between the vectors $A = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and $B = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

3) Find the angle between the vectors $A = 3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$ and $B = 4\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

4) If $A = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $B = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, then find $(A + B) \times (A - B)$

5) If $= 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $B = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $C = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ be three vectors,
   (i) Find $A \times (B \times C)$
   (ii) Prove that, $A.(B + C) = A.B + A.C$
   (iii) Find, $|2A - B + C|$

6) Find the value of λ such that the three vectors $\mathbf{i} - \mathbf{j} + \mathbf{k}$, $2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\lambda\mathbf{i} - \mathbf{j} + \lambda\mathbf{k}$ will lie on the same plane.

7) Find a unit vector which is perpendicular to two vector
$\mathrm{A} = 3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}, \mathrm{B} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

8) Find a unit vector which is perpendicular to the plane of the vector
$\mathrm{A} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}, \mathrm{B} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$

9) Find a unit vector which is perpendicular to two vector
$\mathrm{A} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}, \mathrm{B} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

10) Find the value of **m** such that the vectors $\mathrm{P} = 2\mathbf{i} + \mathrm{m}\mathbf{j} - \mathbf{k}$, $\mathbf{Q} = 6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ will be parallel.