Example 6·1. *When a system is taken from the state A to the state B, along the path ACB, 80* joules *of heat flows into the system, and the system does 30* joules *of work (Fig. 6·3).*

(*a*) *How much heat flows into the system along the path ADB, if the work done is 10* joules.

(*b*) *The system is returned from the state B to the state A along the curved path. The work done on the system is 20* joules. *Does the system absorb or liberate heat and how much ?*

(*c*) *If* $U_A = 0$, $U_D = 40$ joules, *find the heat absorbed in the process AD and DB.*

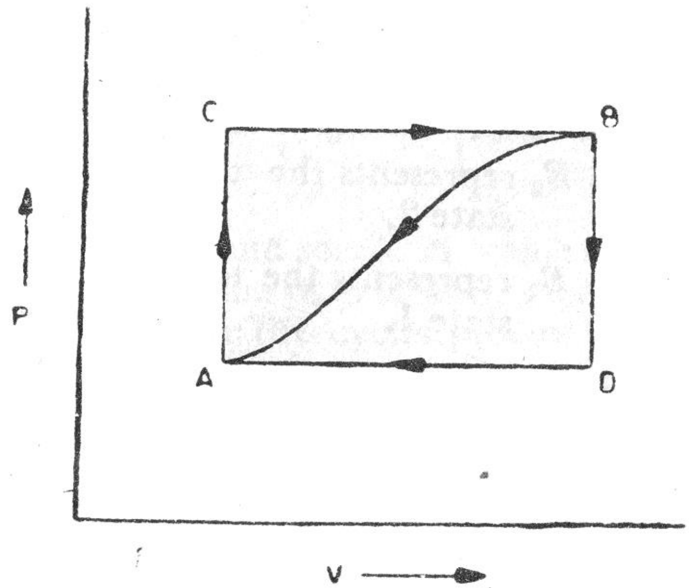

Fig. 6·3

Along the path *ACB*,

$$H_{ACB} = U_B - U_A + W$$

Here $$H = +80 \text{ joules}$$
$$W = +30 \text{ joules}$$

$\therefore$ $$+80 = U_B - U_A + 30$$
$$U_B - U_A = 80 - 30 = \mathbf{50\ joules}$$

(*a*) Along the path *ADB*,

$$W = +10 \text{ joules}$$

$$H_{ADB} = U_B - U_A + W$$
$$H = 50+10 = \mathbf{60\ joules}$$

(*b*) For the curved path from *B* to *A*,

$$W = -20 \text{ joules}$$
$$H = (U_A - U_B) + W$$
$$= -50-20 = \mathbf{-70\ joules}$$

(—ve sign shows that heat is liberated by the system)

(*c*) $U_A = 0, \qquad U_D = 40$ joules

$$U_B - U_A = 50$$

$\therefore$ $U_B = 50$ joules

In the process *ADB*, 10 joules of work is done. **Work done** from *A* to *D* is +10 joules and from *D* to *B* is zero.

For *AD*,

$$H_{AD} = (U_D - U_A) + W$$
$$= 40+10 = 50 \text{ joules}$$

For *DB*

$$H_{DB} = U_C - U_D + W$$
$$= 50-40+0 = \mathbf{10\ joules}$$

Example 6·2. *A motor car tyre has a pressure of 2 atmospheres at the room temperature of 27°C. If the tyre suddenly bursts, find the resulting temperature.*

Here,

$$P_1 = 2 \text{ atmospheres}$$

$$T_1 = 273+27$$

$$= 300 \text{ K}$$

$$P_2 = 1 \text{ atmosphere}$$

$$T_2 = ?$$

$$\gamma = 1{\cdot}4$$

$$\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$$

$$\left(\frac{P_2}{P_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^{\gamma}$$

$$\left(\frac{1}{2}\right)^{0.4} = \left(\frac{T_2}{300}\right)^{1.4}$$

$$0{\cdot}4 \log(0.5) = 1{\cdot}4\,[\log T_2 - \log 300]$$

$$-0{\cdot}1204 = 1{\cdot}4 \log T_2 - 3{\cdot}4680$$

$$1{\cdot}4 \log T_2 = 3{\cdot}4680 - 0{\cdot}1204$$

$$= 3{\cdot}3476$$

$$\log T_2 = \frac{3{\cdot}3476}{1{\cdot}4}$$

$$= 2{\cdot}3911$$

$$T_2 = 246{\cdot}1 \text{ K}$$

$$= -26{\cdot}9°\text{C}$$

Example 6·3. *A quantity of air at 27°C and atmospheric pressure is suddenly compressed to half its original volume. Find the final (i) pressure and (ii) temperature.*

(i) $P_1 = 1$ atmosphere ; $P_2 = ?$, $\gamma = 1{\cdot}4$

$V_1 = V$; $V_2 = \dfrac{V}{2}$

During sudden compression, the process is adiabatic

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_1\left[\frac{V_1}{V_2}\right]^{\gamma}$$

$$= 1[2]^{1.4}$$

$$= 2{\cdot}636 \text{ atmospheres}$$

(ii) $V_1 = V$; $V_2 = \dfrac{V}{2}$

$$T_1 = 300 \text{ K} ; T_2 = ?$$

$$\gamma = 1{\cdot}4$$

$$T_1(V_1)^{\gamma-1} = T_2 (V_2)^{\gamma-1}$$

$$T_2 = T_1[2]^{1.4-1}$$

$$= 300[2]^{0.4}$$

$$= 395{\cdot}9 \text{ K}$$

$$= 122{\cdot}9°\text{C}$$

Example 5·24. *Calculate the mean free path of gas molecules in a chamber of 10^{-6} mm of mercury pressure, assuming the molecular diameter to be 2Å. One gram molecule of the gas occupies 22·4 litres at N.T.P. Take the temperature of the chamber to be 273 K.* (*Agra 1975*)

At 760 mm Hg pressure and 273 K temperature, the number of molecules in 22·4 litres of a gas

$$= 6{\cdot}023 \times 10^{23}$$

Therefore, the number of molecules per cm³ in the chamber at 10^{-6} mm pressure and 273 K temperature

$$n = \frac{6{\cdot}023 \times 10^{23} \times 10^{-6}}{22400 \times 760}$$

$$n = 3{\cdot}538 \times 10^{10} \text{ molecules/cm}^3$$

$$d = 2\text{Å} = 2 \times 10^{-8} \text{ cm}$$

Mean free path,

$$\lambda = \frac{1}{\pi d^2 n}$$

$$= \frac{1}{3{\cdot}14 \times (2 \times 10^{-8})^2 \times 3{\cdot}538 \times 10^{10}} \doteq 2{\cdot}25 \times 10^4 \text{ cm}$$

Example 5·24. *Calculate the Van der Waals constants for dry air, given that*

$T_c = 132$ K, $P_c = 37{\cdot}2$ *atmospheres*,

R per mole $= 82{\cdot}07$ cm^3 *atmos* K^{-1}. (*Delhi 1975*)

Here $P_c = 37{\cdot}2$ atmospheres

$T_c = 132$ K

$R = 82{\cdot}07$ cm^3 atmos K^{-1}

(*i*)
$$a = \frac{27}{64}\,\frac{R^2 T_c^2}{P_c}$$

$$a = \left(\frac{27}{64}\right)\frac{(82{\cdot}07)^2(132)^2}{37{\cdot}2}$$

or
$$a = \mathbf{13{\cdot}31 \times 10^6 \text{ atmos cm}^6}$$

(*ii*)
$$b = \frac{RT_c}{8P_c}$$

$$b = \frac{82{\cdot}07 \times 132}{8 \times 37{\cdot}2}$$

or
$$b = \mathbf{36{\cdot}41 \text{ cm}^3}$$

Example 6·11. *A Carnot's engine is operated between two reservoirs at temperatures of 450* K *and 350* K. *If the engine receives 1000 calories of heat from the source in each cycle, calculate the amount of heat rejected to the sink in each cycle. Calculate the efficiency of the engine and the work done by the engine in each cycle.* (*1* calorie = *4·2* joules).

$$T_1 = 450 \text{ K} ; \qquad T_2 = 350 \text{ K}$$

$$H_1 = 1000 \text{ cal} ; \qquad H_2 = ?$$

$$\frac{H_2}{H_1} = \frac{T_2}{T_1}$$

$$H_2 = H_1 \times \frac{T_2}{T_1}$$

$$= \frac{1000 \times 350}{450} = \mathbf{777{\cdot}77 \ cals}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{350}{450} = \frac{100}{450}$$

$$= 0{\cdot}2222$$

$$\% \text{ efficiency} = \mathbf{22{\cdot}22\%}$$

Work done in each cycle

$$= H_1 - H_2$$

$$= 1000 - 777{\cdot}77$$

$$= 222{\cdot}23 \text{ cal}$$

$$= 222{\cdot}23 \times 4{\cdot}2 \text{ joules}$$

$$= \mathbf{933{\cdot}33 \ joules}$$