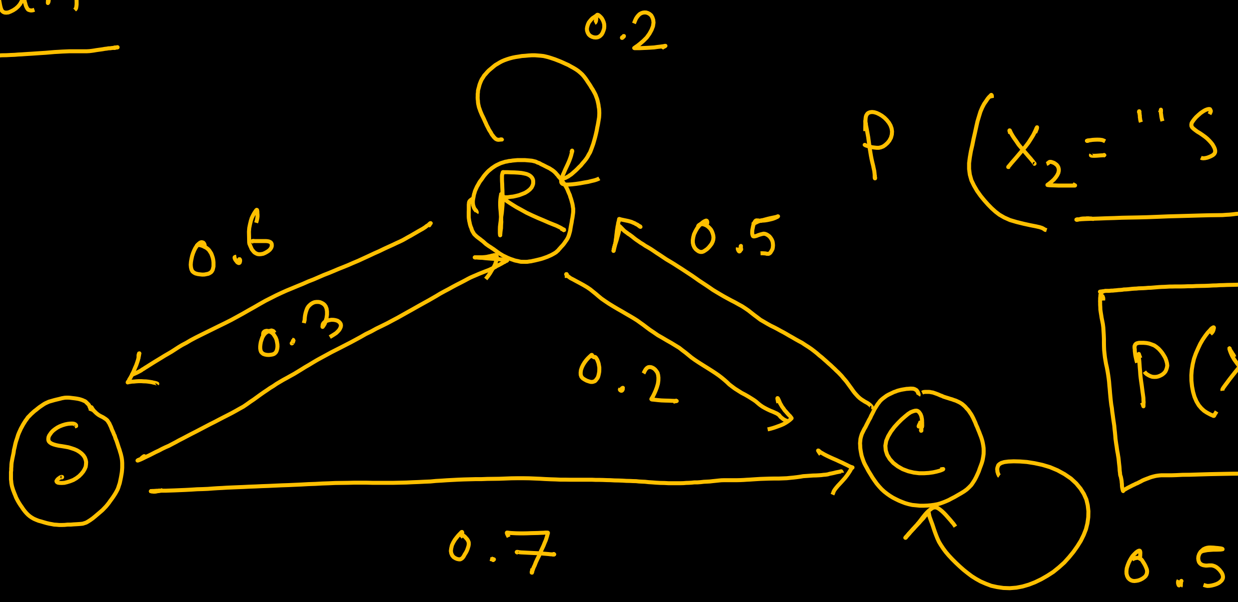
Markov Chain



$$P\left(\underline{X_2 = "S"} \mid \underline{X_1 = "R"}\right) = 0.6$$

$$\boxed{P\left(X_{n+1} = "x" \mid X_n = "x_n"\right)}$$

Markov chain

" Future state only depends on the current state, not the states before

Naive Bayes

$$P\left(X_{n+1} = "x" \mid X_1 = "x_1", X_2 = "x_2", \ldots, X_n = "X_n"\right)$$

$$\begin{array}{c|ccc} & R & S & C \\ \hline R & 0.2 & 0.6 & 0.2 \\ S & 0.3 & 0 & 0.7 \\ C & 0.5 & 0 & 0.5 \end{array} = A$$

Transition Matrix

$$\pi_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\pi_1 = \pi_0 A = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0 & 0.7 \end{bmatrix}$$

$$\pi_2 = \pi_1 A$$

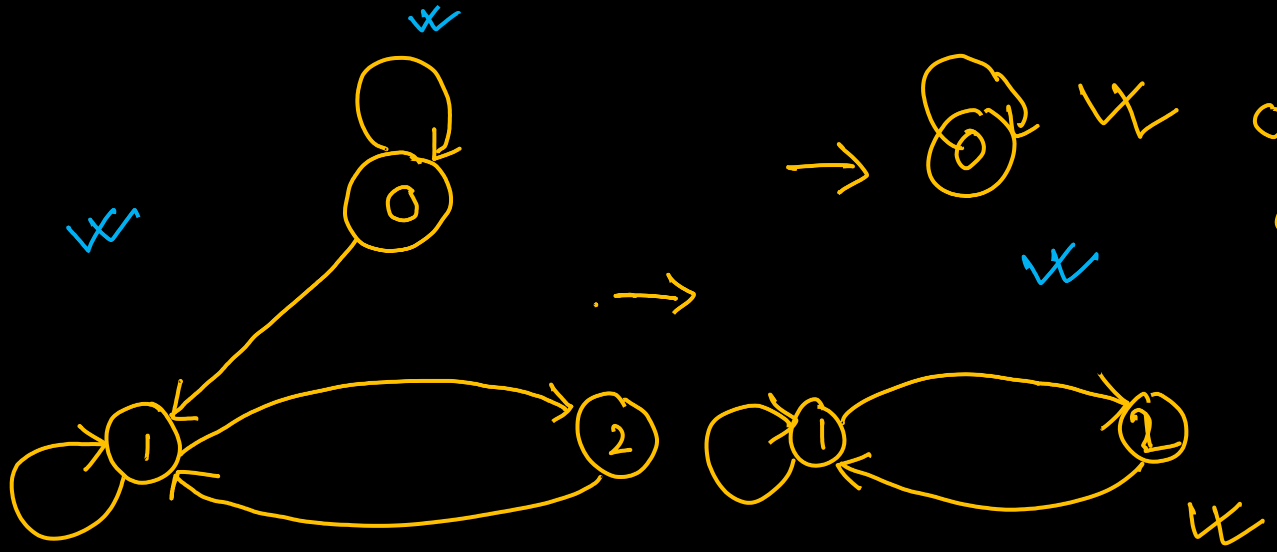
$$= \begin{bmatrix} 0.3 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} A \end{bmatrix}$$

$$\pi_2 = \begin{bmatrix} 0.41 & 0.18 & 0.41 \end{bmatrix}$$

$\pi_n A = \pi_n$ [Equilibrium State]

$$\boxed{vA = \lambda v \qquad \lambda = I}$$

$$\circled{\pi_n} = \underline{\begin{bmatrix} 0.35211 & 0.21127 & 0.43662 \end{bmatrix}}$$

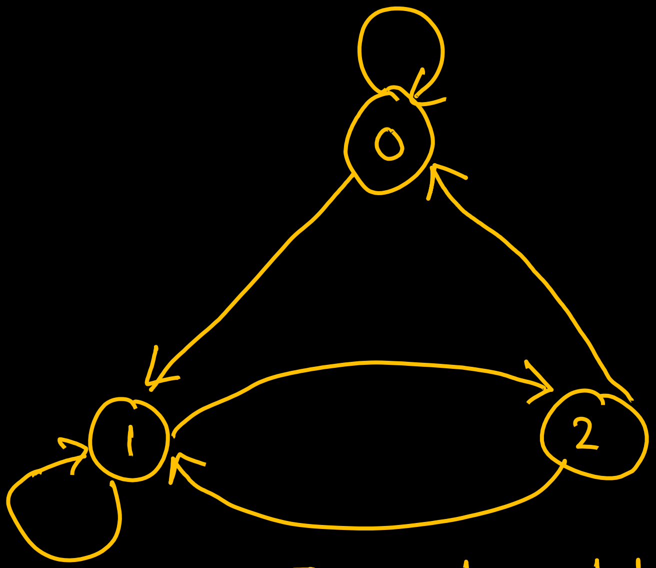


Reducible MC

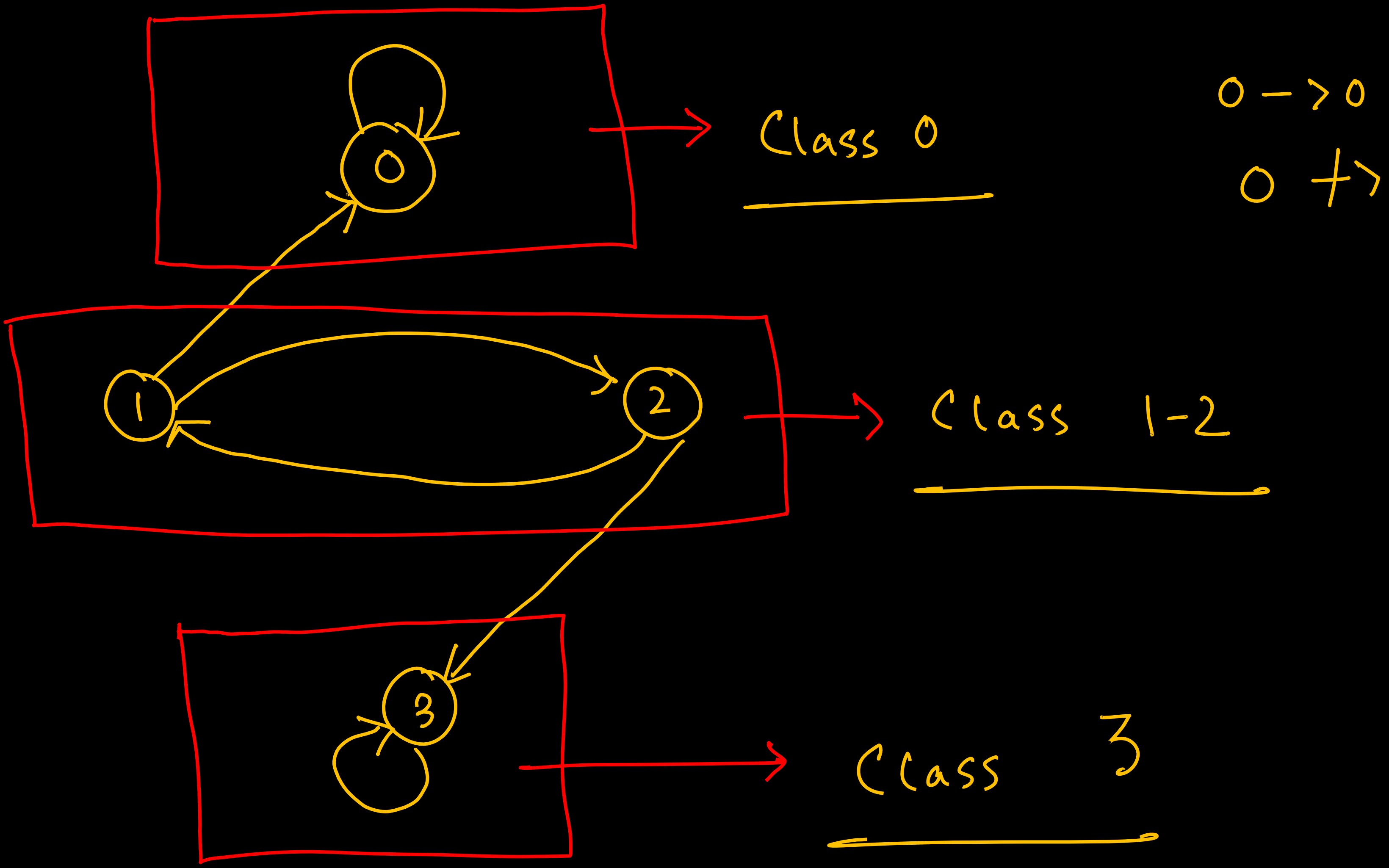
0 -> 0
0 -> 1
x -> 0
1 -> 2
2 -> 1

Transient State -> 0

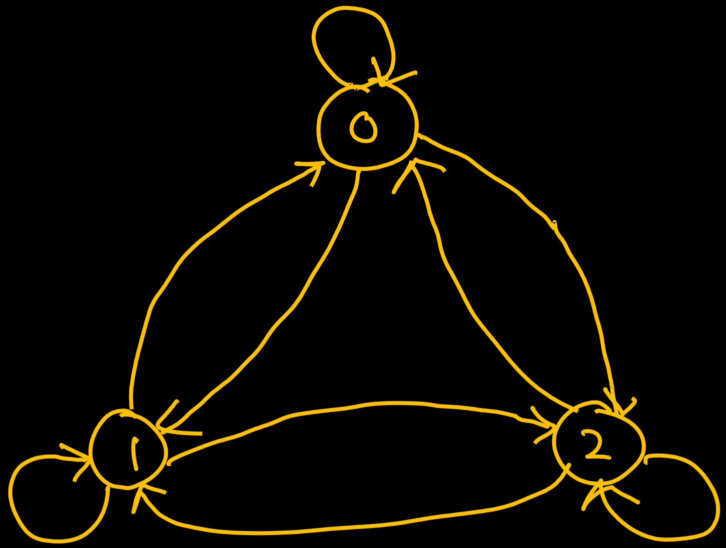
Recurrent state -> 1, 2



Irreducible MC


0
1
2
3
Class 0
Class 1-2
Class 3
0 → 0
0 ↛

$$\boxed{P_{i\to j}(n)}$$ ✓



0 – 0
0 – 2 } ✓

0 – 1
1 – 2 } ✓

0 – 2
2 – 2 } ✓

$$P_{02}(1) = 0.3 = A_{02}$$

$$P_{12}(1) = 0.2$$

$$P_{21}(1) = 0.8$$

$$A = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0\ ✓ & 0.5 & \underline{0.2} & \underline{0.3} \\ 1 & 0.6 & 0.2 & \underline{0.2} \\ 2 & 0.1 & \textcircled{0.8} & 0.1 \end{array}$$

$$\underline{P_{02}(2)} = A_{00}\times A_{02} + A_{01}\times A_{12} \longrightarrow 1$$

$$+ A_{02}\times A_{22} \longrightarrow 2$$

$$= 0.5\times 0.3 + 0.2\times 0.2 + 0.3\times 0.1 = 0.22$$

$$\underline{P_{10}(2)} = 0.6 \times 0.5 + 0.2 \times 0.6 + 0.2 \times 0.1$$

$$= \boxed{0.44}$$

$$A = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$$

$$P_{ij}(2) = \begin{bmatrix} A_{i0} & A_{i1} & A_{i2} \end{bmatrix} \begin{bmatrix} A_{0j} \\ A_{1j} \\ A_{2j} \end{bmatrix}$$

$$A * A = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix} * \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$$

$$\underset{(3 \times 3)}{A}$$

$$\boxed{P_{ij}(2)}$$

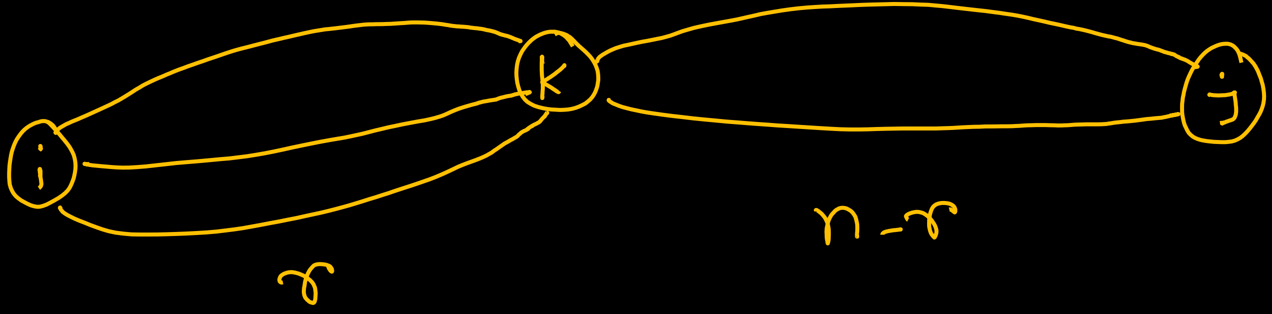
$$A^2 = \begin{bmatrix} 0.4 & 0.38 & \underline{0.22} \\ \underline{0.44} & 0.32 & 0.24 \\ 0.54 & 0.26 & 0.20 \end{bmatrix}$$

$$\boxed{P_{ij}(2) = A^2_{ij}}$$

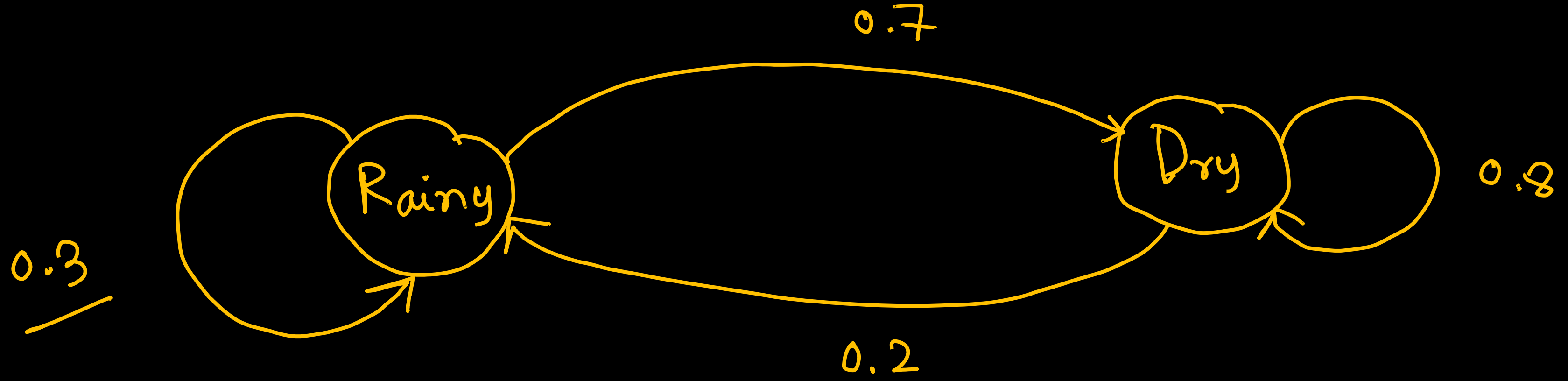
$$\boxed{P_{ij}(n) = A^n_{ij}}$$

* Chapman - Kolmogorov Theorem

$$\boxed{P_{ij}(n) = A^{n}_{ij}}$$



$$\forall \quad \boxed{P_{ij}(n) = \sum_{K} P_{iK}(r) * P_{Kj}(n-r)}$$



$P(B|A)$

$P(\text{Rainy}) = 0.4$ $\quad$ $P(\text{Dry}) = 0.6$

Rainy → Rainy → Dry → Dry ?

Soln:

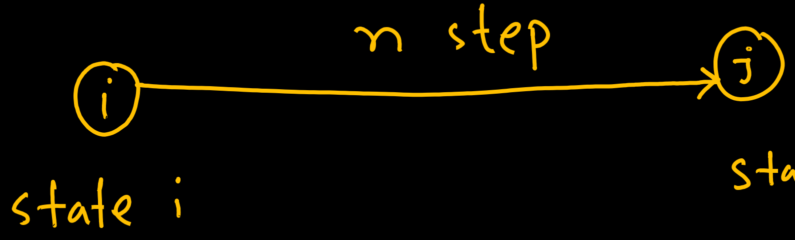
$$A = \begin{array}{c|cc} & R & D \\ \hline R & 0.3 & 0.7 \\ D & 0.2 & 0.8 \end{array}$$

$$\pi = [0.4 \quad 0.6]$$

$$Q = P(R) * P(R|R) * P(D|R) * P(D|D)$$

$$= 0.4 * 0.3 * 0.7 * 0.8$$

$$= \boxed{}$$ Ans.



$$r_{ij}(n) = A^{n}_{ij}$$

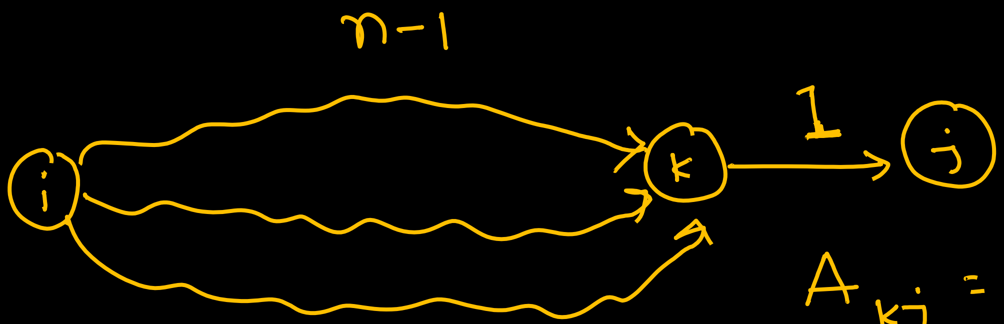
$$\boxed{r_{ij}(n) = P(X_n = j \mid X_0 = i)}$$

⊛ $$r_{ij}(0) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

⊛ $$r_{ij}(1) = A_{ij} = P_{ij}(X_1 = j \mid X_0 = i)$$

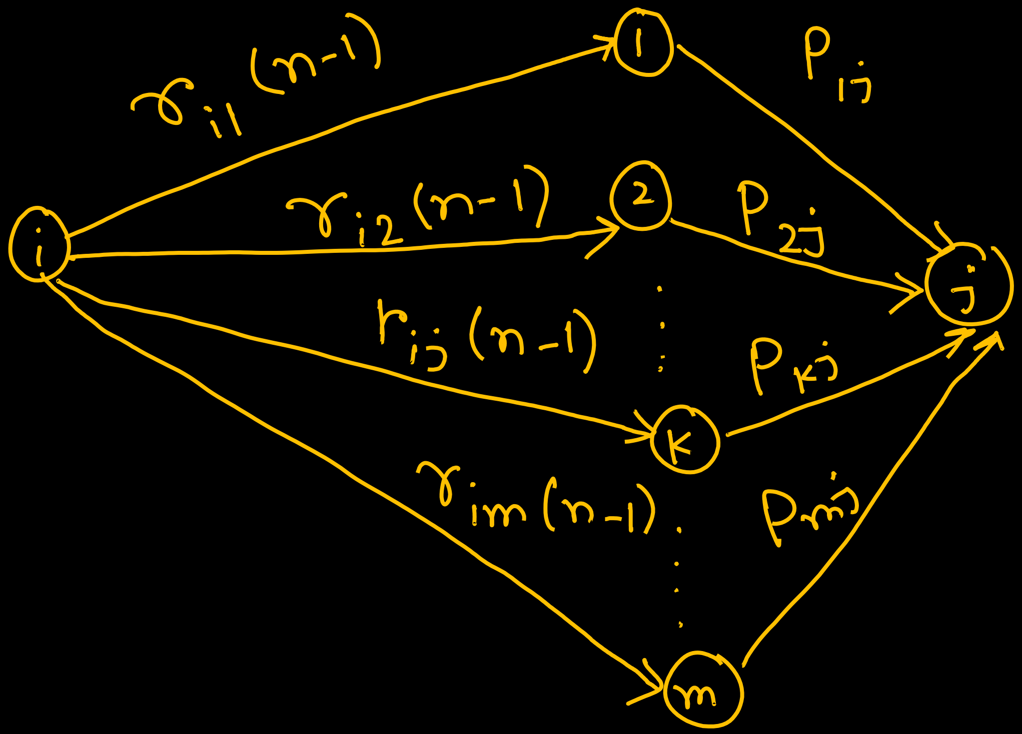
$$\sum r_{ij}(n) = 1$$

$n \geq 2$

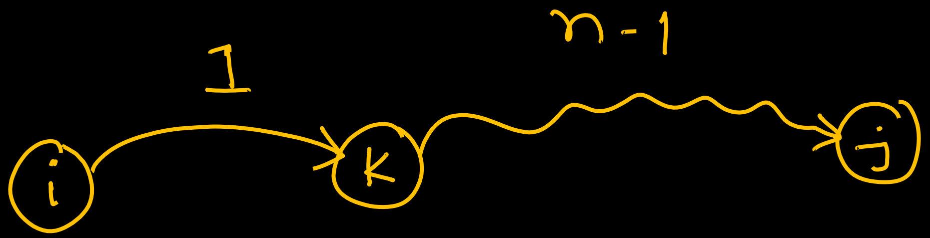


$$A_{kj} = P_{kj}(X_n = j \mid X_{n-1} = k)$$

$$\sum_{k} r_{ik}(n-1)$$

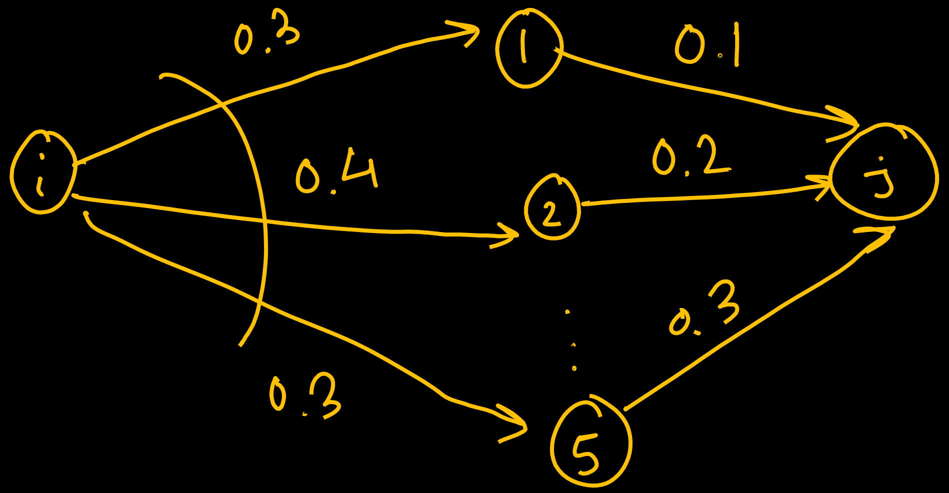


$$\circledast \qquad \boxed{r_{ij}(n) = \underbrace{\sum_{k}^{m} r_{ik}(n-1)}_{n-1} * \underbrace{P_{kj}}_{1}}$$



$$\boxed{r_{ij}(n) = \underbrace{\sum_{k=1}^{m} p_{ik}}_{1} * \underbrace{r_{kj}(n-1)}_{n-1}}$$

⊛ $$\boxed{r_{ij}(n) = \sum_{k}^{m} r_{ik}(q) * r_{kj}(n-q)}$$



$$P_{ij}(n) = A^{n}_{ij}$$

$$A * A = A^2$$

$$A^2 * A = A^3$$

$$A^3 * A = A^4$$

Equilibrium state → does not depends on initial stat

$$\lim_{n\to\infty} A^{\infty}$$

⊛ irreducible

$$A^{\infty} = \begin{bmatrix} 0.54 & 0.45 \\ 0.54 & 0.46 \\ 0.54 & 0.45 \end{bmatrix}$$

$\boxed{i = 0}$

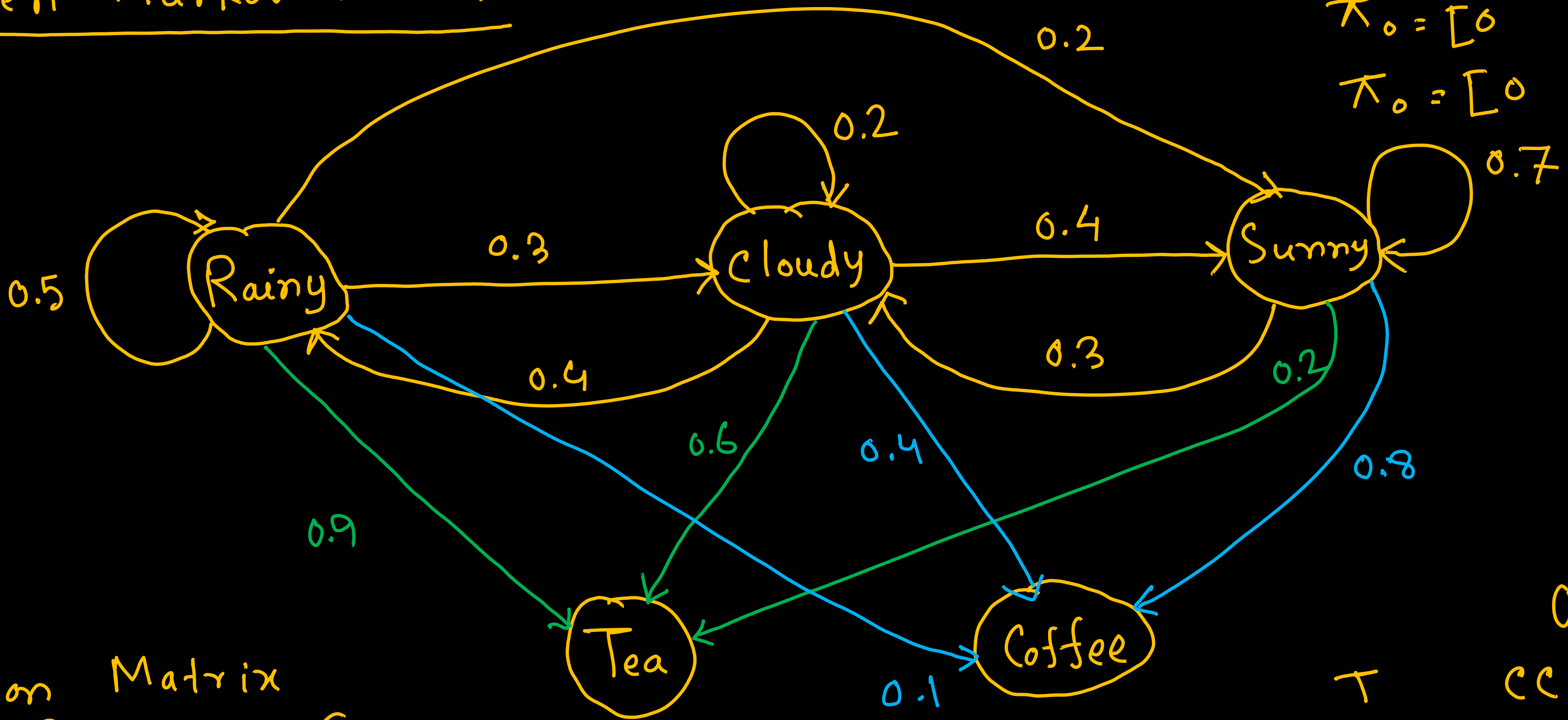
①

②

Hidden Markov Model

$$\left.\begin{array}{l}\pi_0 = [1 \quad 0 \quad 0] \\ \pi_0 = [0 \quad 1 \quad 0] \\ \pi_0 = [0 \quad 0 \quad 1]\end{array}\right\}$$

Hidden states

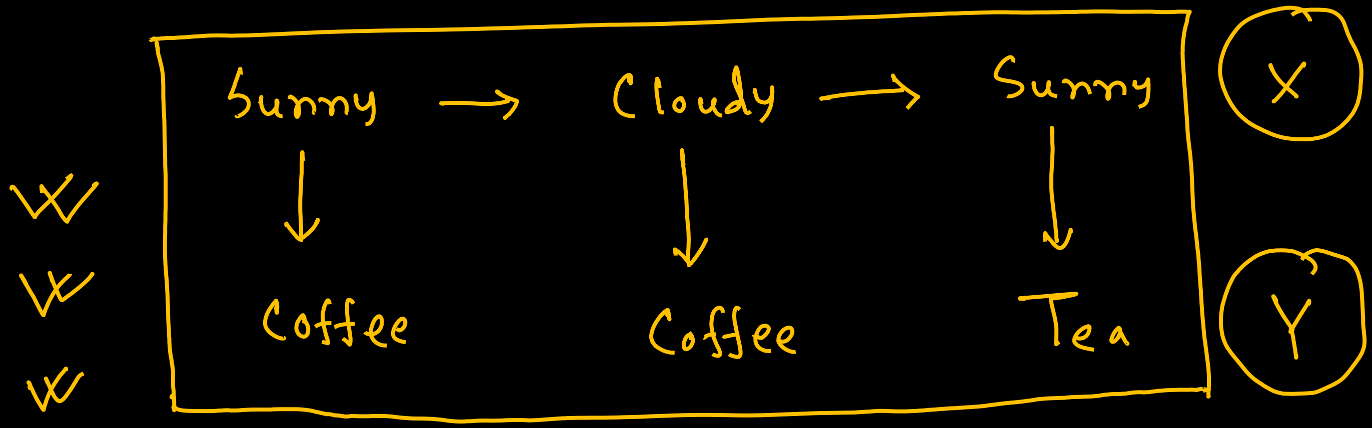


Transition Matrix

$$A = \begin{array}{c|ccc|} & R & C & S \\ R & 0.5 & 0.3 & 0.2 \\ \underline{\underline{C}} & 0.4 & 0.2 & \underline{0.4} \\ S & 0.0 & \underline{0.3} & 0.7 \end{array}$$

Observable states

$$\text{Emission Matrix, } E = \begin{array}{c|cc|} & T & CC \\ R & 0.9 & 0.1 \\ C & 0.6 & \underline{0.4} \\ S & 0.2 & 0.8 \end{array}$$



$$P(\overset{\swarrow}{A} \mid \overset{\swarrow}{B})$$

$$P(Y = CC \rightarrow CC \rightarrow T \,,\, X = S \rightarrow C \rightarrow S)$$

$$\underbrace{P(X_1 = S)}_{0.6} * \underbrace{P(Y_1 = CC \mid X_1 = S)}_{0.8} * \underbrace{P(X_2 = C \mid X_1 = S)}_{0.3} *$$

$$\underbrace{P(Y_2 = CC \mid X_2 = C)}_{0.4} * \underbrace{P(X_3 = S \mid X_2 = C)}_{0.4} * \underbrace{P(Y_3 = T \mid X_3 = S)}_{0.2}$$

$$\rightarrow \pi_0 = \begin{array}{ccc} R & C & S \\ \end{array}$$
$$\left[\begin{array}{ccc} 0 & 0 & 1 \end{array}\right] \checkmark \qquad \pi_1 = \pi_0 A$$

$$\pi_2 = \pi_1 A \qquad \pi_3 = \pi_2 A$$

$$\boxed{\pi_i = \pi_{i-1} A = \pi_i}$$

$$\boxed{\pi A = \pi} \rightarrow \text{Equilibrium state}$$

$$\boxed{\pi = \begin{array}{ccc} R & C & S \\ \left[0.2 \right. & 0.2 & \left. 0.6 \right] \end{array}}$$

$$\boxed{P(A|B) = \frac{P(B|A) * P(A)}{P(B)}}$$

R

$C \rightarrow R \rightarrow C$

$\pi_0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

$\pi_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

$$\operatorname{argmax} \quad P(X = x_1, x_2, x_3, \ldots, x_n \mid Y = y_1, y_2, y_3, \ldots, y_n)$$

$$X = x_1, x_2, x_3 \ldots, x_n$$

$$= \underset{X : x_1, x_2, x_3 \ldots, x_n}{\operatorname{argmax}} \quad \frac{\overset{\propto}{P(Y \mid X)} * \overset{\propto}{P(X)}}{\underbrace{\overset{\propto}{P(Y)}}_{1}} \quad \propto$$

$$P(Y \mid X) = P(y_1 \mid x_1) * P(y_2 \mid x_2) * P(y_3 \mid x_3) * \ldots * P(y_n \mid x_n)$$

$$P(X) = P(x_0) * P(x_1 \mid x_0) * P(x_2 \mid x_1) * \ldots * P(x_i \mid x_{i-1})$$

$$P(Y|X) = \prod_{i=1}^{n} (P(y_i|x_i))$$

$$P(X) = \prod_{i=1}^{n} (P(x_i | x_{i-1}))$$

$$\boxed{\begin{array}{l} \text{argmax} \qquad \prod (P(y_i|x_i) * P(x_i|x_{i-1})) \\ X = x_1, x_2, x_3 \ldots, x_n \end{array}}$$