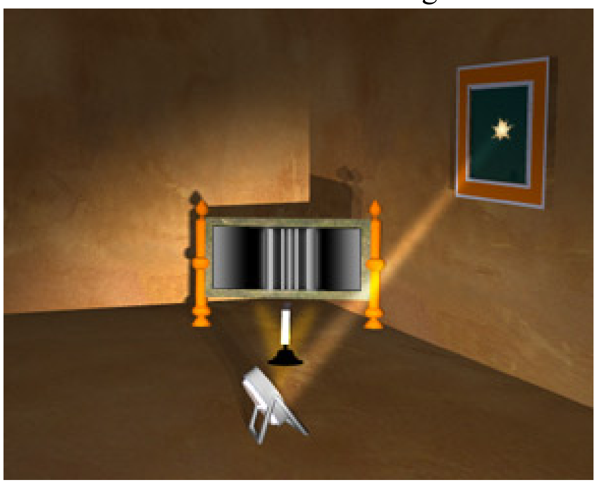
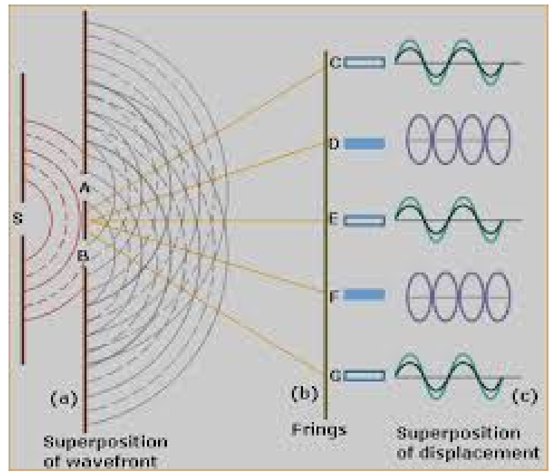
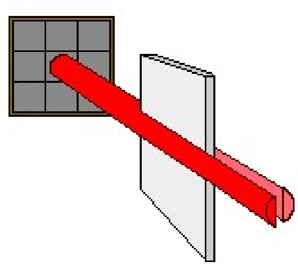
Interference

Definition ***Interference is the addition (superposition) of two or more waves that result in a new wave pattern.*** Constructive interference is a type of interference that occurs at any location along the medium where the two interfering waves have a displacement in the same direction. Destructive interference is a type of interference that occurs at any location where the two interfering waves have a displacement in the opposite direction.





Young's Experiment In 1801, Young devised and performed an experiment to measure the wavelength of light. Thomas Young recognized that if light behaved like a wave, it would be possible to create patterns of constructive and destructive interference using light. He devised an experiment that would force two beams of light to travel different distances before interfering with each other when they reached a screen. To accomplish this, Young set up a mirror to direct a thin beam of sunlight into a darkened room (and an assistant to make sure the mirror aimed the sun's light properly!). Young split the beam in two by placing a very thin card edgewise in the beam, as shown in figure below. Since these two beams emerged from the same source (the sun) they could be considered coming from two coherent sources. Light waves from these two sources (the left side and the right side of the card) would interfere.



A single pinhold beam of sunlight is split into two coherent beams.

Conditions for Constructive and Destructive Interference (why dark and bright fringes occur?)

Consider a monochromatic source of light *S* emitting waves of wavelength λ and two narrow pin holes *A* and *B*. *A* and *B* are equidistant from *S* and act as two virtual sources. Let ***a*** be the amplitude of the waves. The phase difference between the two waves reaching the point *P*, at any instant is δ.

If y_1 and y_2 are the displacements

$$y_1 = a\sin\omega t$$

$$y_2 = a\sin(\omega t + \delta)$$

$$y = y_1 + y_2 = a\sin\omega t + a\sin(\omega t + \delta)$$

$$= a\sin\omega t + a\sin\omega t\cos\delta + a\cos\omega t\sin\delta$$

$$= a\sin\omega t(1+\cos\delta) + a\cos\omega t\sin\delta$$

Taking $a(1+\cos\delta) = R\cos\theta$ ..... (1)

and $a\sin\delta = R\sin\theta$ ..... (2)

We get

$$y = R\sin\omega t\cos\theta + R\cos\omega t\sin\theta$$

$$y = R\sin(\omega t + \theta) \quad \text{..... (3)}$$

Which represents the equation of simple harmonic vibration of amplitude *R*.

Squaring and adding equation (1) and (2)

$$R^2 = a^2\sin^2\delta + a^2(1+\cos\delta)^2$$

$$=2a^2 + 2a^2\cos\delta$$

$$=2a^2(1+\cos\delta)$$

$$= 4\mathrm{a}^2\cos^2\frac{\delta}{2}$$

The intensity at a point is given by the square of the amplitude

$\therefore$ $I = R^2$

or $I = 4a^2\cos^2\dfrac{\delta}{2}$

Special cases:

(i) When the phase difference

$\delta = 0, 2\pi, 2(2\pi), ...n(2\pi)$ *or* the path difference $x = 0, \lambda, 2\lambda..........n\lambda$

$$I = 4a^2$$

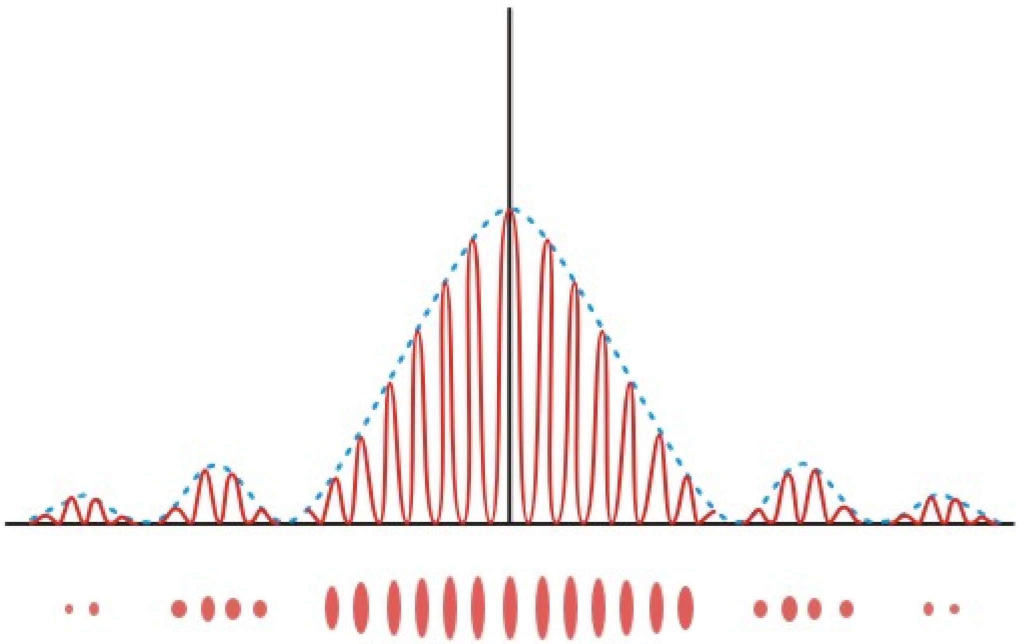
Intensity is maximum when the phase difference is a whole number multiple of 2π or the path difference is a whole number multiple of wavelength.

(ii) When the phase difference

$$\delta = \pi, 3\pi, ...(2n+1)\pi \text{ or the path difference } x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2,} ....(2n+1)\frac{\lambda}{2}$$

$$I = 0$$

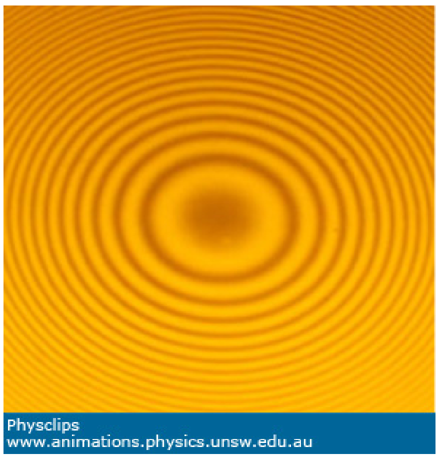
Intensity is minimum when the phase difference is a odd number multiple of half wavelength.

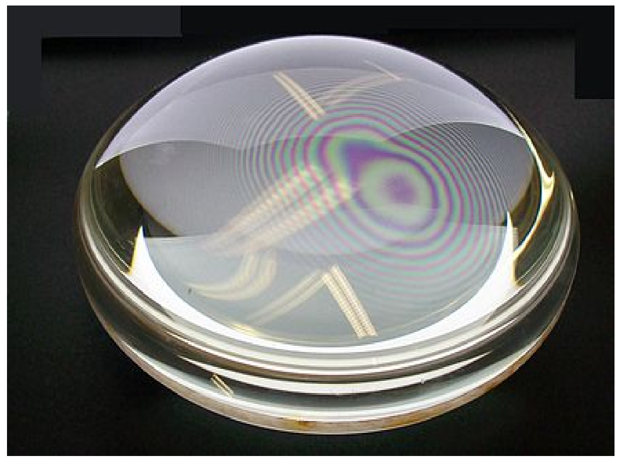


Double Slit Interference Pattern

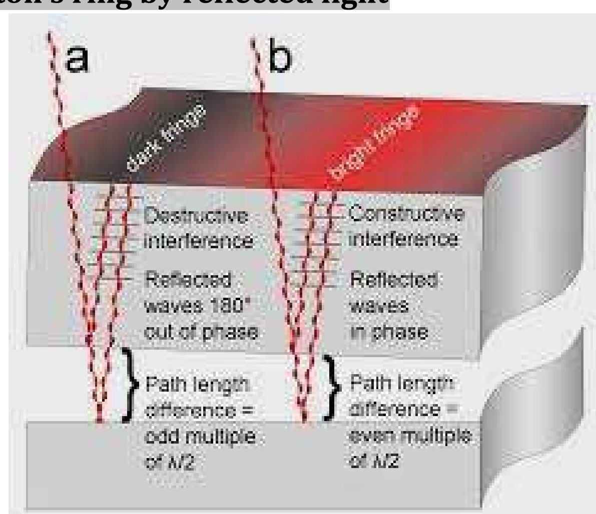
Newton's rings

Definition ***Newton's ring is a noteworthy illustration of the interference of light waves reflected from the opposite surfaces of the thin film of variable thickness.*** Colored rings may also be observed. Newton's rings are caused by the interference effects that occur between light waves reflected at the upper and lower surfaces of the air film separating the lens and the flat surface.





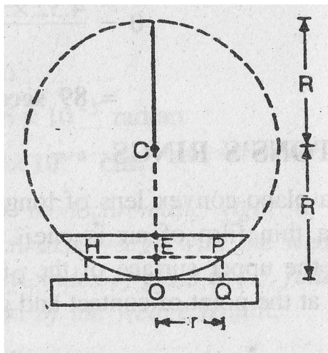
Formation of Newton's ring by reflected light

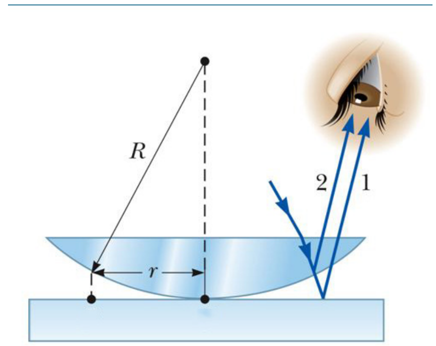


Suppose the radius of curvature of the lens is R and the air film is of thickness t at a distance of $OQ = r$, from the point of contact O. Here interference is due to reflected light. Therefore for bright rings, the optical path difference (*optical path is the product of geometrical distance and refractive index of the medium and in a given time, light travels the same optical path in different media*)

$$2\mu t\cos\theta = (2n-1)\frac{\lambda}{2} \qquad \text{.......... (1)}$$

Where n = 1, 2, 3, ...etc. *[When light is reflected from the surface of an optically denser medium, a phase change π equivalent to a path difference $\frac{\lambda}{2}$ occurs.]*





Here, θ is small, therefore $\cos\theta = 1$ and for air, $\mu = 1$

$$\therefore \quad 2t = (2n-1)\frac{\lambda}{2} \qquad \text{.......... (ii)}$$

For dark rings, $\quad 2\mu t\cos\theta = n\lambda$

Or $\quad 2t = n\lambda$, where n = 0, 1, 2, 3, ... etc. .... (iii)

From the figure, $\quad HE \times EP = OE \times (2R - OE)$

But $\quad HE = EP = r, \quad OE = PQ = t$

$\therefore \; 2R - t = 2R$ (approximately)

$r^2 = 2R.t$

Or $\quad t = \frac{r^2}{2R}$

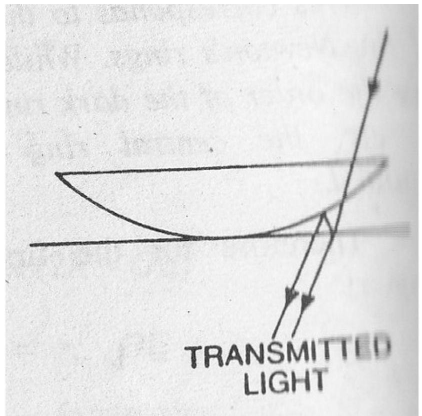
Substituting the value of ***t*** in equation (ii) and (iii),

For bright rings, $\quad r = \sqrt{\frac{(2n-1)\lambda R}{2}}$ .... (iv)

For dark rings, $\quad r = \sqrt{n\lambda R}$ .... (v)

When *n* = *0*, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{-\lambda R}{2}}$ and it is not possible. Therefore, the centre is dark and alternately bright and dark rings are produced.

Formation of Newton's ring by transmitted light



In this case, for bright rings, $2\mu t\cos\theta = n\lambda$ and

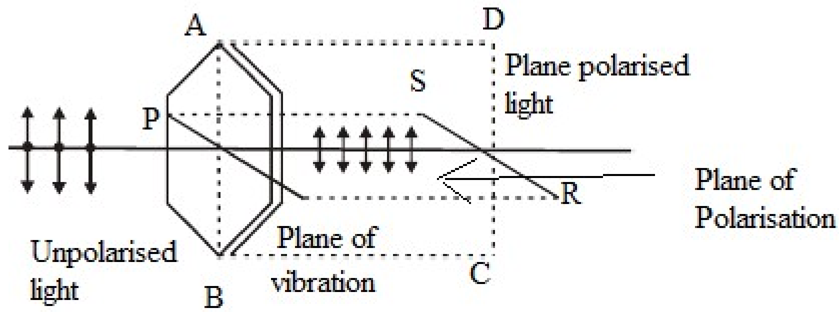
For dark rings, $$2\mu t\cos\theta = (2n-1)\frac{\lambda}{2}$$

Therefore, for bright rings, $r = \sqrt{n\lambda R}$ and for dark rings, $r = \sqrt{\frac{(2n-1)\lambda R}{2}}$. In this case, the central ring is bright.

Polarization

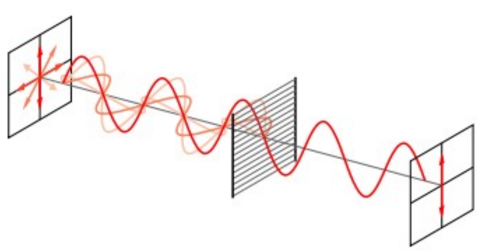
Most sources of electromagnetic radiation contain a large number of atoms or molecules that emit light. The orientation of the electric fields produced by these emitters may not be correlated, in which case the light is said to be *unpolarized.*

Definition ***The process of confining the vibrations of the electric vector of light waves to one direction is called polarization of light.*** The electric field may be oriented in a single direction (linear polarization), or it may rotate as the wave travels (circular or elliptical polarization). Conventionally, when considering polarization, the electric field vector is described and the magnetic field is ignored since it is perpendicular to the electric field.



Absorptive polarizers

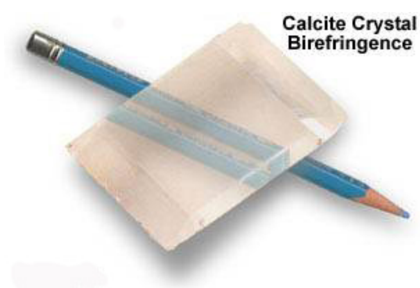
The simplest linear polarizer is the *wire-grid polarizer*, which consists of a regular array of fine parallel metallic wires, placed in a plane perpendicular to the incident beam. Electromagnetic waves which have a component of their electric fields aligned parallel to the wires induce the movement of electrons along the length of the wires. Some energy is lost due to Joule heating in the wires, and the rest of the wave is reflected backwards along the incident beam.



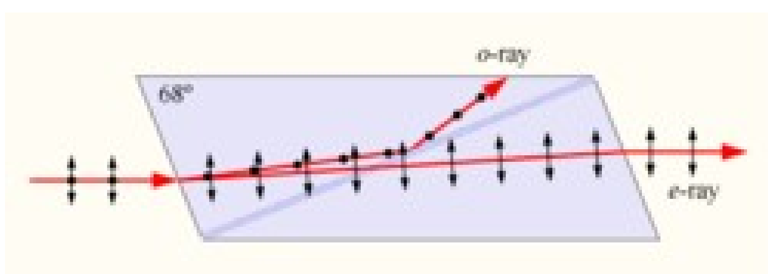
For waves with electric fields perpendicular to the wires, the electrons cannot move very far across the width of each wire; therefore, little energy is lost or reflected, and the incident wave is able to travel through the grid. Since electric field components parallel to the wires are absorbed or reflected, the transmitted wave has an electric field purely in the direction perpendicular to the wires, and is thus linearly polarized.

Double refraction

Definition ***Double refraction is the decomposition of a ray of light into two rays when it passes through certain anisotropic materials, such as calcite crystals.*** The structure of the material with uniaxial anisotropy is such that it has an axis of symmetry which is known as the optical axis (line passing through the optical centre and centre of curvature) of the material and light with linear polarizations parallel and perpendicular to it has unequal indices of refraction, denoted n_e and n_o respectively, where the suffixes stand for **extraordinary** and **ordinary**. If unpolarized light enters the material at a nonzero acute angle to the optical axis, the component (polarized) perpendicular to this axis will be refracted as per the standard law of refraction, while the complementary polarization component will refract at a nonstandard angle determined by the angle of incidence. The light will therefore split into two linearly polarized beams, correspondingly known as ordinary and extraordinary ray. Along the optic axis the *O*-ray and *E*-ray travel with the same speed. Some crystals such as calcite, quartz and tourmaline have only one optic axis, they are uniaxial crystals. Others such as mica and selenite have two optic axis, they are biaxial crystals. The phenomenon is also known as birefringence.



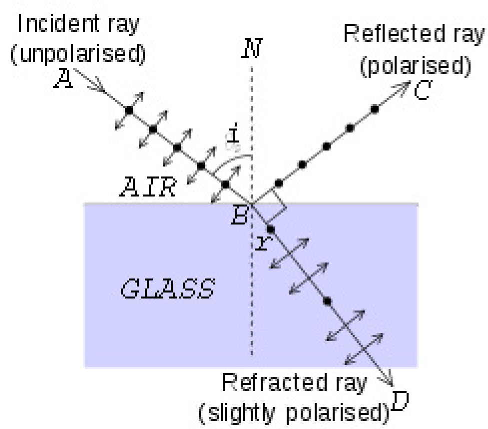
Birefringent polarizers



A Nicol prism was an early type of birefringent polarizer that consists of a crystal of calcite which has been split and rejoined with Canada balsam. Total internal reflection of the *o*-ray occurs at the balsam interface, since it experiences a larger refractive index in calcite than in the balsam, and the ray is deflected to the side of the crystal. The *e*-ray, which sees a smaller refractive index in the calcite, is transmitted through the interface without deflection.

Brewster's Law

In 1811, Brewster found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the *angle of polarization.* He was also able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium. Moreover, the reflected and refracted rays are perpendicular to each other.



Suppose that unpolarized light is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along *BC* and refracted along *BD*.

From Snell's law, $$\mu = \frac{\sin i}{\sin r} \qquad \text{.........} \ (1)$$

From Brewster's law, $$\mu = \tan i = \frac{\sin i}{\cos i} \qquad \text{.........} \ (2)$$

Comparing (1) and (2) $$\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$i = \frac{\pi}{2} - r \qquad or \qquad i + r = \frac{\pi}{2}$$

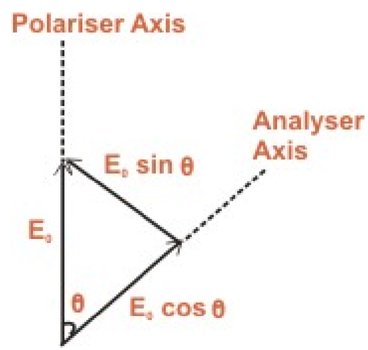
As $i + r = \frac{\pi}{2}$, $\angle CBD$ is also equal to $\frac{\pi}{2}$. Therefore, the reflected and refracted rays are at right angles to each other.

It is clear that the light vibrating in the plane of incidence is not reflected along *BC*. The refracted ray will have both the vibrations *(i)* in the plane of incidence and *(ii)* at right angles to the plane of incidence.

Malus's Law

According to malus, when completely plane polarized light is incident on the analyzer, the intensity *I* of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyzer and the polarizer.

$$i.e.\quad I \propto \cos^2\theta$$



Suppose the angle between the transmission axes of the analyzer and the polarizer is θ. If E_0 is the amplitude of the electric vector transmitted by the polarizer, then intensity I_0 of the light incident on the analyzer is

$$I_0 \propto E_0^2$$

The electric field vector E_0 can be resolved into two rectangular components i.e $E_0 \cos\theta$ and $E_0 \sin\theta$. The analyzer will transmit only the component ($E_0 \cos\theta$) which is parallel to its transmission axis. However, the component $E_0 \sin\theta$ will be absorbed by the analyser. Therefore, the intensity *I* of light transmitted by the analyzer is,

$$I \propto \left(E_0 \cos\theta\right)^2$$

$$\frac{I}{I_0} = \frac{\left(E_0 \cos\theta\right)^2}{E_0^2} = \cos^2\theta$$

$$I = I_0 \cos^2\theta$$

Therefore, $I \propto \cos^2\theta$. This proves law of malus.

When $\theta = 0°$ (or 180°), $I = I_0 \cos^2 0° = I_0$, That is the intensity of light transmitted by the analyzer is maximum when the transmission axes of the analyzer and the polarizer are parallel.

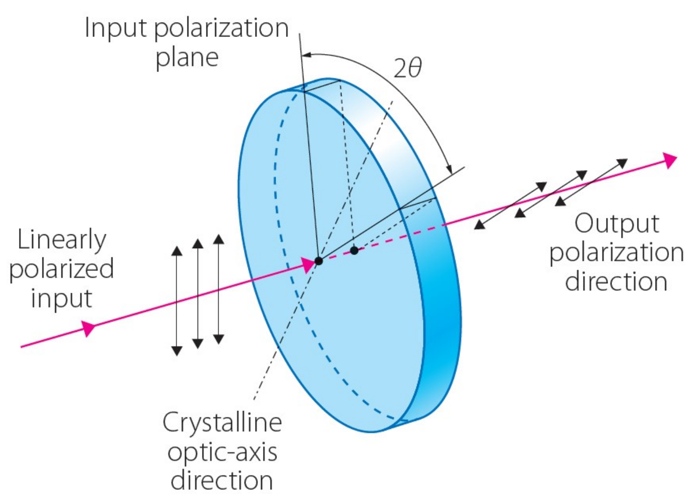
When $\theta = 90°$, $I = I_0 \cos^2 90° = 0$, that is the intensity of light transmitted by the analyzer is minimum when the transmission axes of the analyzer and polarizer are perpendicular to each other.

Waveplates:

Waveplates (retardation plates or phase shifters) are made from materials which exhibit birefringence. The velocities of the extraordinary and ordinary rays through the birefringent materials vary inversely with their refractive indices. The difference in velocities gives rise to a phase difference when the two beams recombine.

Half Waveplate:

The thickness of a half waveplate is such that the phase difference between ordinary and extraordinary ray is *1/2*-wavelength or some multiple of *1/2*-wavelength. A linearly polarized beam incident on a half waveplate emerges as a linearly polarized beam but rotated such that its angle to the optical axis is twice that of the incident beam. Therefore, half-waveplates can be used as polarization rotators.



Performance of half (λ/2) waveplate

If the thickness of the plate is ***t*** and the refractive indices for the ordinary and extraordinary rays are μ_O and μ_E respectively, then

For negative crystals, path difference $= (\mu_O - \mu_E)t$

For positive crystals, path difference $= (\mu_E - \mu_O)t$

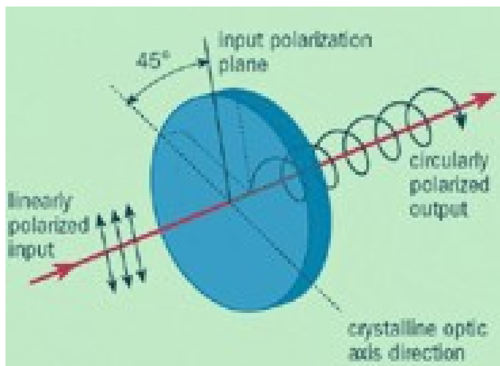
Therefore in calcite, $(\mu_O - \mu_E)t = \dfrac{\lambda}{2}$

Or $$t = \frac{\lambda}{2(\mu_O - \mu_E)}$$

And in quartz, $$t = \frac{\lambda}{2(\mu_E - \mu_O)}$$

Quarter Waveplate:

The thickness of the quarter waveplate is such that the phase difference between ordinary and extraordinary ray is *1/4* wavelength. The emergent beam is circularly polarized.



If the thickness of the plate is ***t*** and the refractive indices for the ordinary and extraordinary rays are μ_O and μ_E respectively, then

For negative crystals, path difference $= (\mu_O - \mu_E)t$

[For ordinary light the velocity is same in all directions and the wavefront is spherical. On the other hand for the extraordinary ray the velocity varies with the direction and the wavefront is ellipsoid of revolution. Along the optic axis both the rays have the same velocity. For negative crystals, $\mu_O \gg \mu_E$*, therefore E-ray travels faster than O-ray at right angles to the direction of the optic axis.]*

For positive crystals, path difference $= (\mu_E - \mu_O)t$

Therefore in calcite, $(\mu_O - \mu_E)t = \frac{\lambda}{4}$

Or $$t = \frac{\lambda}{4(\mu_O - \mu_E)}$$

And in quartz, $$t = \frac{\lambda}{4(\mu_E - \mu_O)}$$