Solution. For silver wire $R_1 = \rho_1 \dfrac{l_1}{A_1}$; For manganin wire, $R_2 = \rho_2 \dfrac{l_2}{A_2}$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \times \frac{l_2}{l_1} \times \frac{A_1}{A_2}$$

Now $\quad A_1 = \pi d_1^2/4$ and $A_2 = \pi d_2^2/4 \qquad \therefore A_1/A_2 = d_1^2/d_2^2$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \times \frac{l_2}{l_1} \times \left(\frac{d_1}{d_2}\right)^2$$

$$R_1 = 1\,\Omega;\ l_2/l_1 = 1/3,\ (d_1/d_2)^2 = (3/1)^2 = 9;\ \rho_2/\rho_1 = 30$$

$$\therefore \quad R_2 = 1 \times 30 \times (1/3) \times 9 = \mathbf{90\ \Omega}$$

Example 1.9. *The resistivity of a ferric-chromium-aluminium alloy is 51×10^{-8} Ω-m. A sheet of the material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine resistance between (a) opposite ends and (b) opposite sides.* **(Electric Circuits, Allahabad Univ. 1983)**

Solution. (*a*) As seen from Fig. 1.5 (*a*) in this case,

$$l = 15 \text{ cm} = 0.15 \text{ m}$$
$$A = 6 \times 0.014 = 0.084 \text{ cm}^2 = 0.084 \times 10^{-4} \text{ m}^2$$

$$R = \rho\frac{l}{A} = \frac{51 \times 10^{-8} \times 0.15}{0.084 \times 10^{-4}} = \mathbf{9.1 \times 10^{-3}\ \Omega}$$

(*b*) As seen from Fig. 1.5(*b*) here

$$l = 0.014 \text{ cm} = 14 \times 10^{-5} \text{ m}$$
$$A = 15 \times 6 = 90 \text{ cm}^2 = 9 \times 10^{-3} \text{ m}^2$$
$$\therefore \quad R = 51 \times 10^{-8} \times 14 \times 10^{-5}/9 \times 10^{-3} = \mathbf{79.3 \times 10^{-10}\ \Omega}$$

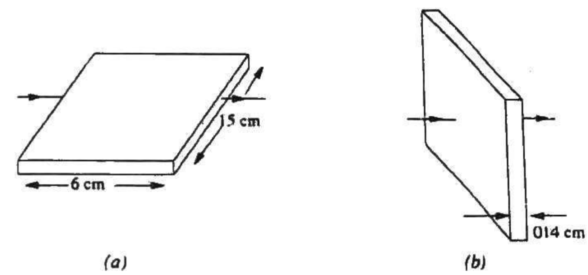


Fig. 1.5

Example 1.10. *The resistance of the wire used for telephone line is 35 Ω per kilometre when the weight of the wire is 5 kg per kilometre. If the specific resistance of the material is 1.95×10^{-8} Ω-m, what is the cross-sectional area of the wire ? What will be the resistance of a loop to a subscriber 8 km from the exchange if wire of the same material but weighing 20 kg per kilometre is used ?*

Solution. Here $R = 35\ \Omega$; $l = 1$ km $= 1000$ m ; $\rho = 1.95 \times 10^{-8}$ Ω-m

Now, $\quad R = \rho\dfrac{l}{A}$ or $A = \dfrac{\rho l}{R} \quad \therefore A = \dfrac{1.95 \times 10^{-8} \times 1000}{35} = \mathbf{55.7 \times 10^{-8}\ m^2}$

In the second case, if the wire is of the same material but weighs 20 kg/km, then its cross-section must be greater than that in the first case.

Cross-section in the second case $= \dfrac{20}{5} \times 55.7 \times 10^{-8} = 222.8 \times 10^{-8}$ m^2

Length of wire $= 2 \times 8 = 16$ km $= 16000$ m $\therefore R = \rho\dfrac{l}{A} = \dfrac{1.95 \times 10^{-8} \times 16000}{222.8 \times 10^{-8}} = \mathbf{140.1\ \Omega}$

Tutorial Problems No. 1.1

1. Calculate the resistance of 100 m length of a wire having a uniform cross-sectional area of 0.1 mm^2 if the wire is made of manganin having a resistivity of 50×10^{-8} Ω-m.

If the wire is drawn out to three times its original length, by how many times would you expect its resistance to be increased ? **[500 Ω ; 9 times]**

In other words, $\frac{V}{I}$ = constant or $\frac{V}{I} = R$

where R is the resistance of the conductor between the two points considered.

Put in another way, it simply means that provided R is kept constant, current is directly proportional to the potential difference across the ends of a conductor. However, this linear relation between V and I does not apply to all non-metallic conductors. For example, for silicon carbide, the relationship is given by $V = KI^m$ where K and m are constants and m is less than unity. It also does not apply to non-linear devices such as Zener diodes and voltage-regulator (VR) tubes.

Example 1.22. *A coil of copper wire has resistance of 90 Ω at 20°C and is connected to a 230-V supply. By how much must the voltage be increased in order to maintain the current constant if the temperature of the coil rises to 60°C ? Take the temperature coefficient of resistance of copper as 0.00428 from 0°C.*

Solution. As seen from Art. 1.10.

$$\frac{R_{60}}{R_{20}} = \frac{1+60\times0.00428}{1+20\times0.00428} \quad \therefore\ R_{60} = 90\times1.2568/1.0856 = 104.2\ \Omega$$

Now, current at 20°C = 230/90 = 23/9 A

Since the wire resistance has become 104.2 Ω at 60°C, the new voltage required for keeping the current constant at its previous value = 104.2 × 23/9 = 266.3 V

∴ increase in voltage required = 266.3 − 230 = **36.3 V**

Example 1.23. *Three resistors are connected in series across a 12-V battery. The first resistor has a value of 1 Ω, second has a voltage drop of 4 V and the third has a power dissipation of 12 W. Calculate the value of the circuit current.*

Solution. Let the two unknown resistors be R_2 and R_3 and I the circuit current.

$$\therefore\quad I^2R_3 = 12 \text{ and } IR_2 = 4 \qquad \therefore\ R_3 = \frac{3}{4}R_2^2.\ \text{Also, } I = \frac{4}{R_2}$$

Now, $I(1+R_2+R_3) = 12$

Substituting the values of I and R_3, we get

$$\frac{4}{R_2}\left(1+R_2+\frac{3}{4}R_2^2\right) = 12 \quad \text{or} \quad 3R_2^2 - 8R_2 + 4 = 0$$

$$\therefore\quad R_2 = \frac{8\pm\sqrt{64-48}}{6}. \quad \therefore\ R_2 = 2\ \Omega \text{ or } \frac{2}{3}\ \Omega$$

$$\therefore\quad R_3 = \frac{3}{4}R_2^2 = \frac{3}{4}\times2^2 = 3\ \Omega \quad \text{or} \quad \frac{3}{4}\left(\frac{2}{3}\right)^2 = \frac{1}{3}\ \Omega$$

$$\therefore\quad I = \frac{12}{1+2+3} = \mathbf{2A} \quad \text{or} \quad I = \frac{12}{1+(2/3)+(1/3)} = \mathbf{6\ A}$$

1.14. Resistance in Series

When some conductors having resistances R_1, R_2 and R_3 etc. are joined end-on-end as in Fig. 1.12, they are said to be connected in series. It can be proved that the equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that (*i*) current is the same through all the three conductors (*ii*) but voltage drop across each is different due to its different resistance and is given by Ohm's Law and (*iii*) sum of the three voltage drops is equal to the voltage applied across the three conductors. There is a progressive fall in potential as we go from point A to D as shown in Fig. 1.13.

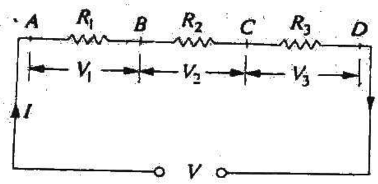


Fig. 1.12

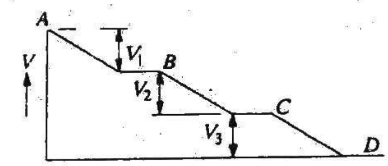


Fig. 1.13

$\therefore$ $$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$ -Ohm's Law

But $$V = IR$$

where R is the equivalent resistance of the series combination.

$\therefore$ $$IR = IR_1 + IR_2 + IR_3 \quad \text{or} \quad R = R_1 + R_2 + R_3$$

Also $$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

As seen from above, the main characteristics of a series circuit are :

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.
5. resistances are additive.
6. powers are additive.

1.15. Voltage Divider Rule

Since in a series circuit, same current flows through each of the given resistors, voltage drop varies directly with its resistance. In Fig. 1.14 is shown a 24-V battery connected across a series combination of three resistors.

Total resistance $R = R_1 + R_2 + R_3 = 12\ \Omega$

According to Voltage Divider Rule, various voltage drops are :

$$V_1 = V \cdot \frac{R_1}{R} = 24 \times \frac{2}{12} = 4\text{ V}$$

$$V_2 = V \cdot \frac{R_2}{R} = 24 \times \frac{4}{12} = 8\text{ V}$$

$$V_3 = V \cdot \frac{R_3}{R} = 24 \times \frac{6}{12} = 12\text{ V}$$

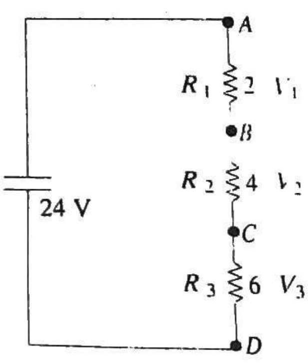


Fig. 1.14

1.16. Resistances in Parallel

Three resistances, as joined in Fig. 1.15 are said to be connected in parallel. In this case (*i*) p.d. across all resistances is the same (*ii*) current in each resistor is different and is given by Ohm's Law and (*iii*) the total current is the sum of the three separate currents.

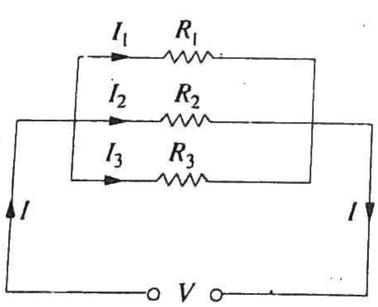


Fig. 1.15

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now, $I = \dfrac{V}{R}$ where V is the applied voltage.

R = equivalent resistance of the parallel combination

$$\therefore \quad \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Also, $G = G_1 + G_2 + G_3$

The main characteristics of a parallel circuit are :

1. same voltage acts across all parts of the circuit.
2. different resistors have their individual current.
3. branch currents are additive.
4. conductances are additive.
5. powers are additive.

Example 1.24. *What is the value of the unknown resistor R in Fig. 1.16 if the voltage drop across the 500 Ω resistor is 2.5 volts ? All resistances are in ohm.*

(Elect. Technology, Indore Univ. April 1990)

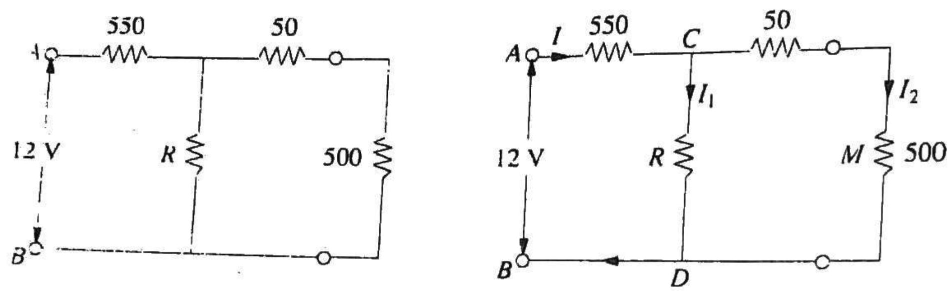


Fig. 1.16

Solution. By direct proportion, drop on 50 Ω resistance

$= 2.5 \times 50/500 = 0.25$ V

Drop across CMD or CD $= 2.5 + 0.25 = 2.75$ V

Drop across 550 Ω resistance $= 12 - 2.75 = 9.25$ V

$I = 9.25/550 = 0.0168$ A, $I_2 = 2.5/500 = 0.005$ A

$I_1 = 0.0168 - 0.005 = 0.0118$ A

$0.0118 = 2.75/R$; **R = 233 Ω**

Example 1.25. *Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B.*

Solution. Resistance between *A* and *C* (Fig. 1.17).

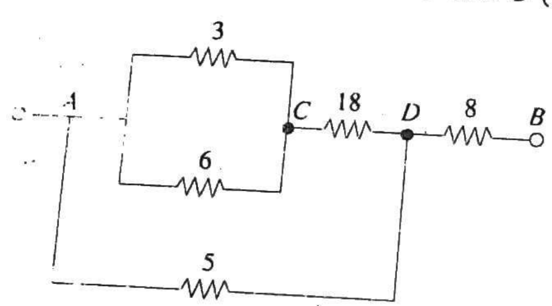


Fig. 1.17

$= 6 \parallel 3 = 2$ Ω

Resistance of branch *ACD*

$= 18 + 2 = 20$ Ω

Now, there are two parallel paths between points *A* and *D* of resistances 20 Ω and 5 Ω. Hence, resistance between *A* and *D*

$= 20 \parallel 5 = 4$ Ω

∴ Resistance between *A* and *B*

$= 4 + 8 = 12$ Ω

Total circuit current

$= 60/12 = 5$ A

Art 1.25

Current through 5 Ω resistance $= 5 \times \frac{20}{25} = 4$ A

Current in branch *ACD* $= 5 \times \frac{5}{25} = 1$ A

∴ P.D. across 3 Ω and 6 Ω resistors $= 1 \times 2 = $ **2 V**

P.D. across 18 Ω resistor $= 1 \times 18 = $ **18 V**

P.D. across 5 Ω resistor $= 4 \times 5 = $ **20 V**

P.D. across 8 Ω resistor $= 5 \times 8 = $ **40 V**

Example 1.26. *A circuit consists of four 100-W lamps connected in parallel across a 230-V supply. Inadvertently, a voltmeter has been connected in series with the lamps. The resistance of the voltmeter is 1500 Ω and that of the lamps under the conditions stated is six times their value when burning normally. What will be the reading of the voltmeter ?*

Solution. The circuit is shown in Fig. 1.18. The wattage of a lamp is given by :

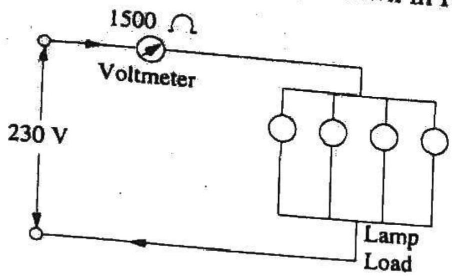


Fig. 1.18

$$W = I^2R = V^2/R$$

$$\therefore \quad 100 = 230^2/R \quad \therefore R = 529\ \Omega$$

Resistance of each lamp under stated condition is

$$= 6 \times 529 = 3174\ \Omega$$

Equivalent resistance of these four lamps connected in parallel = 3174/4 = 793.5 Ω

This resistance is connected in series with the voltmeter of 1500 Ω resistance.

$\therefore$ total circuit resistance = 1500 + 793.5 = 2293.5 Ω

$\therefore$ circuit current = 230/2293.5 A

According to Ohm's law, voltage drop across the voltmeter

$$= 1500 \times 230/2293.5 = \mathbf{150\ V\ (approx)}$$

Example 1.27. *Determine the value of R and current through it in Fig. 1.19, if current through branch AO is zero.*

(Elect. Engg. & Electronics, Bangalore Univ. 1989)

Solution. The given circuit can be redrawn as shown Fig. 1.19 (*b*). As seen, it is nothing else but Wheatstone bridge circuit. As is well-known, when current through branch *AO* becomes zero, the bridge is said to be balanced. In that case, products of the resistances of opposite arms of the bridge become equal.

$$\therefore \quad 4 \times 1.5 = R \times 1\ ;\ R = 6\ \Omega$$

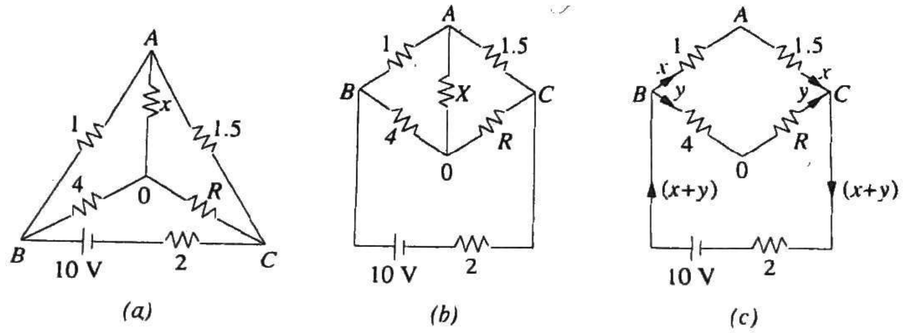


Fig. 1.19

Under condition of balance, it makes no difference if resistance *X* is removed thereby giving us the circuit of Fig. 1.19(*c*). Now, there are two parallel paths between points *B* and *C* of resistances (1 + 1.5) = 2.5 Ω and (4 + 6) = 10 Ω. R_{BC} = 10 || 2.5 = 2 Ω

Total circuit resistance = 2 + 2 = 4 Ω. Total circuit current = 10/4 = 2.5 A.

This current gets divided into two parts at point *B*. Current through *R* is

$$y = 2.5 \times 2.5/12.5 = \mathbf{0.5\ A}$$

Example 1.28. *In the unbalanced bridge circuit of Fig. 1.20(a), find the potential difference that exists across the open switch S. Also, find the current which will flow through the switch when it is closed.*

Solution. With switch open, there are two parallel branches across the 15-V supply. Branch *ABC* has a resistance of (3 + 12) = 15 Ω and branch *ADC* has a resistance of (6 + 4) = 10 Ω. Obviously, each branch has 15 V applied across it.

$$V_B = 12 \times 15/15 = 12\ \text{V};\quad V_D = 4 \times 15/(6+4) = 6\ \text{V}$$

$\therefore$ p.d. across points *B* and *D* = $V_B - V_D = 12 - 6 = 6$ V

When *S* is closed, the circuit becomes as shown in Fig. 1.20(*b*) where points *B* and *D* become electrically connected together.

$$R_{AB} = 3 \,||\, 6 = 2\ \Omega \quad \text{and} \quad R_{BC} = 4 \,||\, 12 = 3\ \Omega$$

$$R_{AC} = 2 + 3 = 5\ \Omega;\quad I = 15/5 = 3\ \text{A}$$

$$V = IR$$

Also $\quad V = I_1 R_1 \quad \therefore \quad IR = I_1 R_1$

or $\quad \dfrac{I}{I_1} = \dfrac{R_1}{R}$ or $I_1 = IR/R_1 \quad \ldots(i)$

Now $\quad \dfrac{I}{R} = \dfrac{I}{R_1} + \dfrac{I}{R_2} + \dfrac{I}{R_3}$

$$R = \frac{R_1R_2R_3}{R_2R_3 + R_3R_1 + R_1R_2}$$

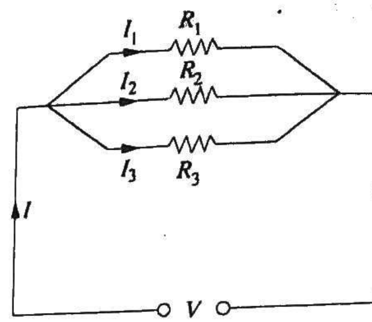


Fig. 1.38

From (i) above, $I_1 = I\left(\dfrac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) = I.\dfrac{G_1}{G_1 + G_2 + G_3}$

Similarly, $\quad I_2 = I\left(\dfrac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) = I.\dfrac{G_2}{G_1 + G_2 + G_3}$

$$I_3 = I\left(\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right) = I.\frac{G_3}{G_1 + G_2 + G_3}$$

Example 1.29. *A resistance of 10 Ω is connected in series with two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied ?*

(Elements of Elect. Engg.-1 ; Bangalore Univ. Jan. 1989)

Solution. The circuit connections are shown in Fig. 1.39.

Drop across 10–Ω resistor = 1.5 × 10 = 15 V

Drop across parallel combination, $V_{AB} = 20 - 15 = 5\text{V}$

Hence, voltage across each parallel resistance is 5V.

$I_1 = 5/15 = 1/3\text{A}$, $I_2 = 5/15 = 1/3$ A

$I_3 = 1.5 - (1/3 + 1/3) = 5/6$ A

$\therefore I_3R = 5$ or $(5/6)\,R = 5$ or $R = \mathbf{6\ \Omega}$

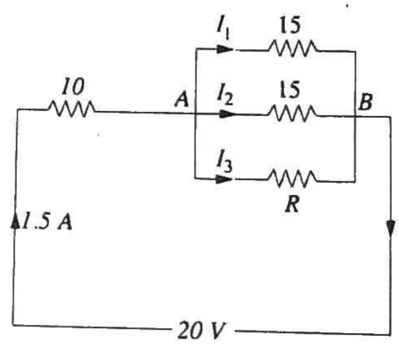


Fig. 1.39

Example 1.30. *If 20 V be applied across AB shown in Fig. 1.40, calculate the total current, the power dissipated in each resistor and the value of the series resistance to halve the total current.*

(Elect. Science-II, Allahabad Univ. 1992)

Solution. As seen from Fig. 1.40. $R_{AB} = 370/199\ \Omega$.

Hence, total current = 20 ÷ 370/199 = **10.76 A.**

$I_1 = 10.76 \times 5(5 + 74/25) = 6.76$ A; $I_2 = 10.76 - 6.76 = 4$ A

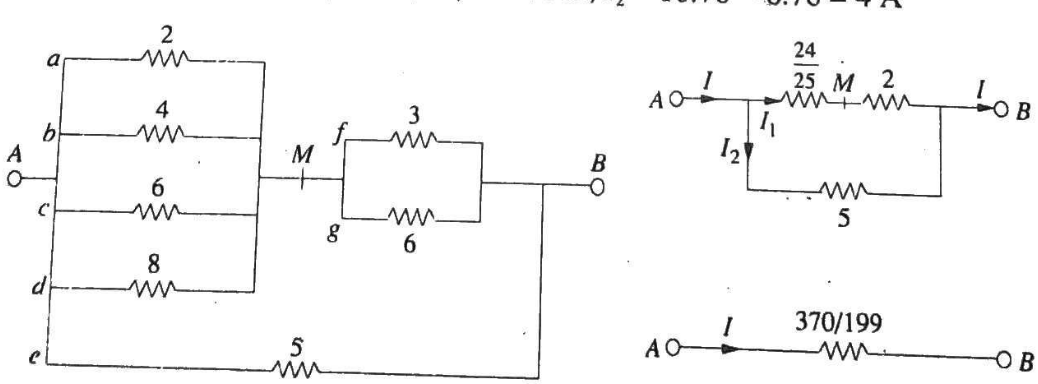


Fig. 1.40

$I_f = 6.76 \times 6/9 = 4.51$ A ; $I_g = 6.76–4.51 = 2.25$ A

Voltage drop across A and M, $V_{AM} = 6.76 \times 24/25 = 6.48$ V

$I_a = V_{AM}/2 = 6.48/2 = 3.24$ A; $I_b = 6.48/4 = 1.62$ A; $I_c = 6.48/6 = 1.08$ A

$I_d = 6.48/8 = 0.81$ A, $I_e = 20/5 = 4$A

Power Dissipation

$P_a = I_a^2 R_a = 3.24^2 \times 2 =$ **21 W**, $P_b = 1.62^2 \times 4 =$ **10.4 W**, $P_c = 1.08^2 \times 6 =$ **7 W**

$P_d = 0.81^2 \times 8 =$ **5.25 W**, $P_e = 4^2 \times 5 =$ **80 W**, $P_f = 4.51^2 \times 3 =$ **61 W**

$P_g = 2.25^2 \times 6 =$ **30.4 W**

The series resistance required is **370/199 Ω**

Incidentally, total power dissipated $= I^2 R_{AB} = 10.76^2 \times 370/199 = 215.3$ W (as a check).

Example 1.31. *Calculate the values of different currents for the circuit shown in Fig. 1.41. What is the total circuit conductance ? and resistance ?*

Solution. As seen, $I = I_1 + I_2 + I_3$. The current division takes place at point B.

As seen from Art. 1.25

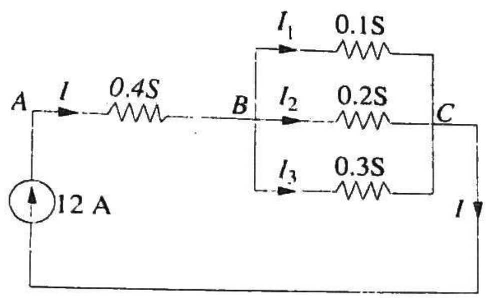


Fig. 1.41

$$I_1 = I \cdot \frac{G_1}{G_1 + G_2 + G_3}$$

$$= 12 \times \frac{0.1}{0.6} = \mathbf{2\ A}$$

$$I_2 = 12 \times 0.2/0.6 = \mathbf{4\ A}$$

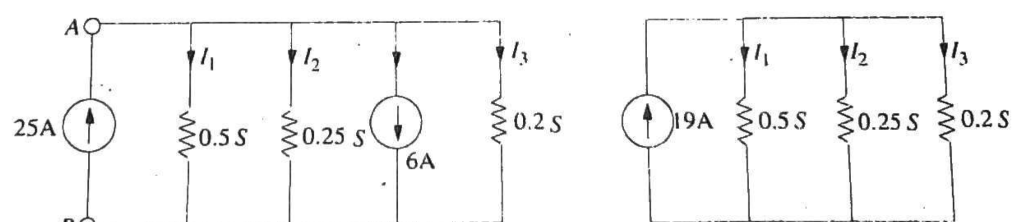
$$I_3 = 12 \times 0.3/0.6 = \mathbf{6\ A}$$

$$G_{BC} = 0.1 + 0.2 + 0.3 = 0.6\text{ S}$$

$$\frac{1}{G_{AC}} = \frac{1}{G_{AB}} + \frac{1}{G_{BC}} = \frac{1}{0.4} + \frac{1}{0.6} = \frac{25}{6}\text{ S}^{-1} \quad \therefore R_{AC} = 1/G_{AC} = \mathbf{25/6\ \Omega}$$

Example 1.32. *Compute the values of three branch currents for the circuit of Fig. 1.42 (a). What is the potential difference between points A and B ?*

Solution. The two given current sources may be combined together as shown in Fig. 1.42 (*b*). Net current = 25 – 6 = 19 A because the two currents flow in opposite directions.



(*a*) (*b*)

Fig. 1.42

Now, $\quad G = 0.5 + 0.25 + 0.2 = 0.95$ S ; $I_1 = I\dfrac{G_1}{G} = 19 \times \dfrac{0.5}{0.95} = \mathbf{10\ A}$

$$I_2 = I\frac{G_2}{G} = 19 \times \frac{0.25}{0.95} = \mathbf{5\ A}\ ;\ I_3 = I\frac{G_3}{G} = 19 \times \frac{0.2}{0.95} = \mathbf{4\ A}$$

$$V_{AB} = I_1 R_1 = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} \quad \therefore V_{AB} = \frac{10}{0.5} = \mathbf{20\ V}$$

The same voltage acts across the three conductances.

We can express the above conclusion thus ; $\Sigma I = 0$ ...at a junction

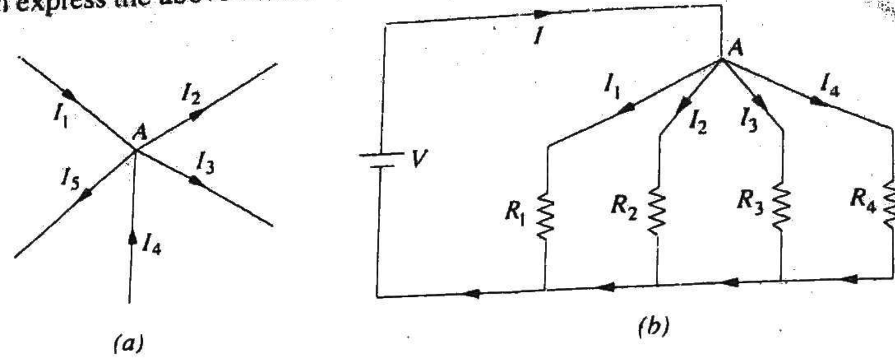


Fig. 2.2

2. Kirchhoff's Mesh Law or Voltage Law (KVL)

It states as follows :

the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.fs. in that path is zero.

In other words, $\Sigma IR + \Sigma$ *e.m.f.* $= 0$ ...round a mesh

It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

The basis of this law is this : If we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started. Hence, it means that all the sources of e.m.f. met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

2.3. Determination of Voltage Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs., otherwise results will come out to be wrong. Following sign convention is suggested :

(a) Sign of Battery E.M.F.

A *rise* in voltage should be given a +ve sign and a *fall* in voltage a −ve sign. Keeping this in mind, it is clear that as we go from the −ve terminal of a battery to its +ve terminal (Fig. 2.3), there is a *rise* in potential, hence this voltage should be given a +ve sign. If, on the other hand, we go from +ve terminal to −ve terminal, then there is a *fall* in potential, hence this voltage should be preceded

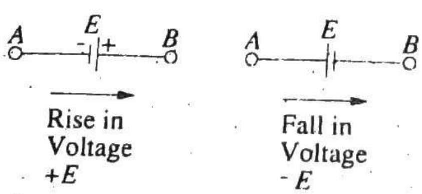


Fig. 2.3

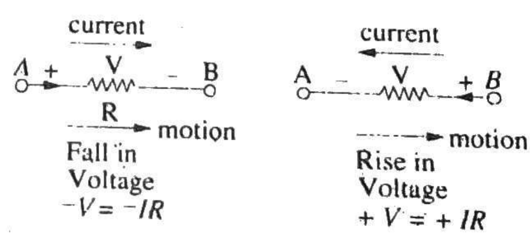


Fig. 2.4

by a −ve sign. *It is important to note that the sign of the battery e.m.f. is independent of the direction of the current through that branch.*

(b) Sign of IR Drop

Now, take the case of a resistor (Fig. 2.4). If we go through a resistor in the *same* direction as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken −ve. However, if we go in a direction opposite to that of the current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit under consideration.

Consider the closed path $ABCDA$ in Fig. 2.5. As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs :

I_1R_1 is $-$ve (fall in potential)
I_2R_2 is $-$ve (" " ")
I_3R_3 is +ve (rise in potential)
I_4R_4 is $-$ve (fall in potential)
E_2 is $-$ve " " ")
E_1 is +ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

or

$$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$

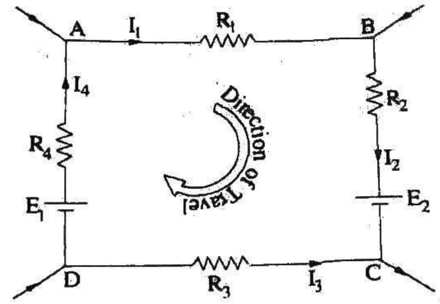


Fig. 2.5

2.4. Assumed Direction of Current

In applying Kirchhoff's laws to electrical networks, the question of assuming proper directior of current usually arises. The direction of current flow may be assumed either clockwise or anti· clockwise. If the assumed direction of current is not the actual direction, then on solving the question this current will be found to have a minus sign. If the answer is positive, then assumed direction i: the same as actual direction (Example 2.10). *However, the important point is that once a particula direction has been assumed, the same should be used throughout the solution of the question.*

Note. It should be noted that Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents However, in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across ; capacitor should be also taken into account (See Example 2.14).

2.5. Solving Simultaneous Equations

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two o three simultaneous equations. These equations can be solved by a systematic elimination of th variables but the procedure is often lengthy and laborious and hence more liable to error. Determinant and Cramer's rule provide a simple and straight method for solving network equations throug manipulation of their coefficients. Of course, if the number of simultaneous equations happens t be very large, use of a digital computer can make the task easy.

2.6. Determinants

The symbol $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is called a determinant of the second order (or 2×2 determinant) becaus it contains two rows (ab and cd) and two columns (ac and bd). The numbers a, b, c and d are calle the elements or constituents of the determinant. Their number in the present case is $= 2^2 = 4$.

The evaluation of such a determinant is accomplished by cross-multiplication as illustrate below :

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The above result for a second order determinant can be remembered as

upper left times lower right **minus** *upper right times lower left*

The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ represents a third-order determinant having $3^2 = 9$ elements. It ma be evaluated (or expanded) as under :

Equivalent resistance of the bridge between points D and B

$$= \frac{\text{p.d. between points } B \text{ and } D}{\text{current between points } B \text{ and } D} = \frac{88/91}{47/91} = \frac{88}{47} = \mathbf{1.87\ \Omega\ (approx)}$$

Solution By Determinants

The matrix form of the three simultaneous equations (1), (2) and (3) is

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} = 1(6+18) - 2(4-8) + 5(18+12) = 182$$

$$\Delta_1 = \begin{bmatrix} 0 & -2 & 4 \\ 0 & -3 & -9 \\ 2 & 2 & -2 \end{bmatrix} = 0(6+18) - 0(4-8) + 2(18+12) = 60$$

$$\Delta_2 = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 0 & -9 \\ 5 & 2 & -2 \end{bmatrix} = 34, \; \Delta_3 = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -3 & 0 \\ 5 & 2 & 2 \end{bmatrix} = 2$$

$$\therefore \quad x = \frac{\Delta_1}{\Delta} = \frac{60}{182} = \frac{\mathbf{30}}{\mathbf{91}}\ \mathbf{A},\; y = \frac{34}{182} = \frac{\mathbf{17}}{\mathbf{91}}\ \mathbf{A},\; z = \frac{2}{182} = \frac{\mathbf{1}}{\mathbf{192}}\ \mathbf{A}$$

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Example 2.9. *Determine the branch currents in the network of Fig. 2.12 when the value of each branch resistance is one ohm.* **(Elect. Technology. Allahabad Univ. 1992)**

Solution. Let the current directions be as shown in Fig. 2.12.

Apply Kirchhoff's Second law to the closed circuit $ABDA$, we get

$$5 - x - z + y = 0 \quad \text{or} \quad x - y + z = 5 \qquad \ldots(i)$$

Similarly, circuit $BCDB$ gives

$$-(x - z) + 5 + (y + z) + z = 0$$

$$\text{or } x - y - 3z = 5 \qquad \ldots(ii)$$

Lastly, from circuit $ADCEA$, we get

$$-y - (y + z) + 10 - (x + y) = 0$$

$$\text{or } x + 3y + z = 10 \qquad \ldots(iii)$$

From Eq. (i) and (ii), we get, $z = 0$

Substituting $z = 0$ either in Eq. (i) or (ii) and in Eq. (iii), we get

$$x - y = 5 \qquad \ldots(iv)$$

$$x + 3y = 10 \qquad \ldots(v)$$

Subtracting Eq. (v) from (iv), we get

$-4y = -5$ or $y = 5/4 = 1.25$ A

Eq. (iv) gives $x = 25/4$ A $= 6.25$ A

Current in branch AB = current in branch BC = **6.25 A**

Current in branch BD = 0; current in branch AD = current in branch DC = **1.25 A**; current in branch CEA = 6.25 + 1.25 = **7.5 A**

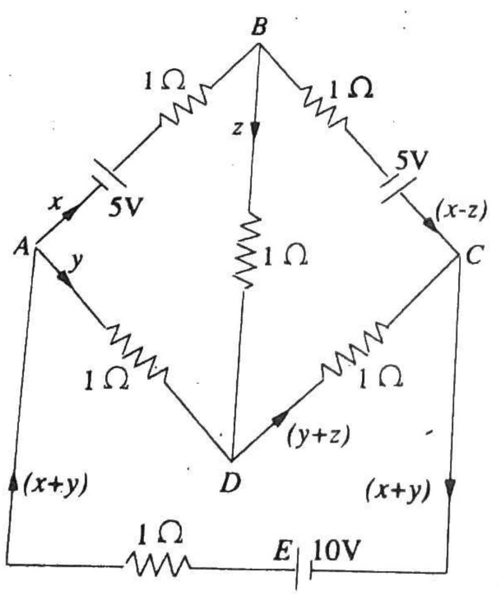

Fig. 2.12

Example 2.10. *State and explain Kirchhoff's laws. Determine the current supplied by the battery in the circuit shown in Fig. 2.12A.*

(Elect. Engg.-I, Bombay Univ. 1987).