L' Hospital's Rule

Ex. 1) $\lim\limits_{x\to 1}\dfrac{x^4-4x^3+6x^2-4x+1}{x^3-3x^2+3x-1}$

Solution: Given,

$$\lim_{x\to 1}\frac{x^4-4x^3+6x^2-4x+1}{x^3-3x^2+3x-1} \quad \left(\frac{0}{0}\,\text{form}\right)$$

$$=\lim_{x\to 1}\frac{4x^3-12x^2+12x-4}{3x^2-6x+3} \quad \left(\frac{0}{0}\,\text{form}\right)$$

$$=\lim_{x\to 1}\frac{12x^2-24x+12}{6x-6} \quad \left(\frac{0}{0}\,\text{form}\right)$$

$$=\lim_{x\to 1}\frac{24x-24}{6}$$

$$=\frac{24-24}{6}=\frac{0}{6}=0$$

Ex. 2) $\lim\limits_{x\to 0} x\ln x$

Solution: Given,

$$\lim_{x\to 0}\ x\ln x \quad (0\times\infty\ \text{form})$$

$$=\lim_{x\to 0}\ \frac{\ln x}{\frac{1}{x}} \quad \left(\frac{0}{0}\,\text{form}\right)$$

$$=\lim_{x\to 0}\ \frac{\frac{1}{x}}{-\frac{1}{x^2}} == \lim_{x\to 0}\ (-x)=0$$

Ex. 3) Proof, $\lim\limits_{x\to 0}\left[\dfrac{1}{x}-\dfrac{1}{x^2}\ \ln(1+x)\right]=\dfrac{1}{2}$

Solution:

$$\text{L.H.S:}\ \lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{x^2}\ \ln(1+x)\right] \quad (\infty\ \text{form})$$

$$= \lim_{x \to 0} \left[\frac{x - \ln(1+x)}{x^2} \right] \qquad \left(\frac{0}{0} \text{form} \right)$$

$$= \lim_{x \to 0} \left[\frac{1 - \frac{1}{1+x}}{2x} \right] \qquad \left(\frac{0}{0} \text{form} \right)$$

$$= \lim_{x \to 0} \left[\frac{\frac{1+x-1}{1+x}}{2x} \right] \qquad \left(\frac{0}{0} \text{form} \right)$$

$$= \lim_{x \to 0} \left[\frac{x}{2x(1+x)} \right] = \lim_{x \to 0} \left[\frac{1}{2(1+x)} \right] = \frac{1}{2}$$

Ex. 4) Show that, $\mathbf{\lim_{x \to 0} \frac{\ln x}{\operatorname{cosec} x} = 0}$

Solution:

L. H. S: $\lim_{x \to 0} \frac{\ln x}{\operatorname{cosec} x} \quad \left(\frac{\infty}{\infty} \text{ form} \right)$

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\operatorname{cosec} x \cot x} \quad \left(\frac{\infty}{\infty} \text{ form} \right)$$

$$= -\lim_{x \to 0} \frac{\sin x \tan x}{x} \qquad \left(\frac{0}{0} \text{form} \right)$$

$$= -\lim_{x \to 0} \frac{\sin x \ \sec^2 x + \tan x \cos x}{1}$$

$$= -(0 + 0) = 0$$

Ex. 5) Show that, $\mathbf{\lim_{x \to \infty} \left[\frac{x^3}{e^{2x}} \right] = 0}$

Solution:

L. H. S: $\lim_{x \to \infty} \left[\frac{x^3}{e^{2x}} \right] \quad \left(\frac{\infty}{\infty} \text{ form} \right)$

$$= \lim_{x \to \infty} \left[\frac{3x^2}{2e^{2x}} \right] \quad \left(\frac{\infty}{\infty} \text{ form} \right)$$

$$= \lim_{x\to\infty}\left[\frac{6x}{4e^{2x}}\right] \quad \left(\frac{\infty}{\infty}\text{ form}\right)$$

$$= \lim_{x\to\infty}\left[\frac{6}{4e^{2x}}\right] \quad \left(\frac{\infty}{\infty}\text{ form}\right)$$

$$= \frac{6}{\infty} = 0$$

<u>H.W:</u>

1) Proof, $\lim_{x\to 0}\left[\frac{2\sin x - \sin 2x}{x - \sin x}\right] = 6$

2) Proof, $\lim_{x\to 0}\left[\frac{1}{x^2} - \frac{1}{x\sin x}\right] = -\frac{1}{6}$

3) $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]$

4) $\lim_{x\to 1}\left[\frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}\right]$

5) $\lim_{x\to\infty}\left[\frac{e^x}{x^2}\right]$

6) $\lim_{x\to 0}\left[\frac{e^x - e^{\sin x}}{x - \sin x}\right]$