SOLUTION

1) $\int \frac{\sqrt{1+\ln x}}{x}\,dx$

$= \int \sqrt{z}\,dz$ Let, $1+\ln x = z$

$= \frac{z^{\frac{3}{2}}}{\frac{3}{2}} + c$ $\Rightarrow \frac{1}{x}\,dx = dz$

$= \frac{2}{3}(1+\ln x)^{\frac{3}{2}} + c$

2) $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx$

$= \int \frac{dz}{\cos^2 z}$ Let, $xe^x = z$

$= \int \sec^2 z\,dz$ $\Rightarrow (xe^x + e^x)dx = dz$

$= \tan z + c$ $\Rightarrow e^x(x+1)dx = dz$

$= \tan(xe^x) + c$

3) $\int \frac{1-\sin x}{x+\cos x}\,dx$

$= \int \frac{dz}{z}$ Let, $x+\cos x = z$

$= \ln z + c$ $\Rightarrow (1-\sin x)dx = dz$

$= \ln(x+\cos x) + c$

4) $\int \frac{e^{\sec^{-1} x}}{x\sqrt{x^2-1}}\,dx$

$= \int e^z\,dz$ Let, $\sec^{-1} x = z$

$= e^z + c$ $\Rightarrow \frac{1}{x\sqrt{x^2-1}}\,dx = dz$

$= e^{\sec^{-1} x} + c$

5) $\int \frac{\sin 2x}{a\sin^2 x + b\cos^2 x}\,dx$

$= \int \frac{\frac{dz}{(a-b)}}{z}$ Let, $a\sin^2 x + b\cos^2 x = z$

$= \frac{1}{a-b}\int \frac{dz}{z}$ $\Rightarrow [a\ 2\sin x\cos x + b\ 2\cos x\ .(-\sin x)]dx = dz$

$= \frac{1}{a-b}\ln z + c$ $\Rightarrow (a\sin 2x - b\sin 2x)dx = dz$

$= \frac{1}{a-b}\ln(a\sin^2 x + b\cos^2 x) + c$ $\Rightarrow \sin 2x\,(a-b)dx = dz$

$\Rightarrow \sin 2x\,dx = \frac{dz}{(a-b)}$

6) $\int \frac{\tan x\sec^2 x}{(a^2 + b^2\tan^2 x)^2}\,dx$

$= \int \frac{\frac{dz}{2b^2}}{z^2}$ Let, $a^2 + b^2\tan^2 x = z$

$= \frac{1}{2b^2}\int \frac{dz}{z^2}$ $\Rightarrow b^2.2\tan x\sec^2 x\,dx = dz$

$= \frac{1}{2b^2}\left(-\frac{1}{z}\right) + c$ $\Rightarrow \tan x\sec^2 x dx = \frac{dz}{2b^2}$

$= \frac{-1}{2b^2}.\left(\frac{1}{a^2 + b^2\tan^2 x}\right) + c$

7) $\int \frac{\sin x}{\sin(x-a)}\,dx$

$= \int \frac{\sin(z+a)}{\sin z}\,dz$ Let, $x - a = z$

$= \int \frac{\sin z \cos a + \cos z \sin a}{\sin z}\,dz$ $\Rightarrow dx = dz$

$= \int \left(\frac{\sin z \cos a}{\sin z} + \frac{\cos z \sin a}{\sin z}\right) dz$ and, $x = z + a$

$= \int (\cos a + \sin a \cot z)\, dz$

$= \cos a \int dz + \sin a \int \cot z\, dz$

$= \cos a \,.\, z + \sin a \ln|\sin z| + c$

$= \cos a\,(x-a) + \sin a \ln|\sin(x-a)| + c$

8) $\int \frac{dx}{x \ln x\ [\ln(\ln x)]}$

$= \int \frac{dz}{z}$ Let, $\ln(\ln x) = z$

$= \ln z + c$ $\Rightarrow \frac{1}{\ln x}.\frac{1}{x}\,dx = dz$

$= \ln(\ln(\ln x)) + c$

9) $\int \frac{\cos 2x}{(\sin x + \cos x)^2}\, dx$

$= \int \frac{\cos 2x}{\sin^2 x + 2 \sin x \cos x + \cos^2 x}\, dx$

$= \int \frac{\cos 2x}{1 + \sin 2x}\, dx$ Let, $1 + \sin 2x = z$

$= \int \frac{\frac{dz}{2}}{z}$ $\Rightarrow 2 \cos 2x\ dx = dz$

$= \frac{1}{2} \ln z + c$ $\Rightarrow \cos 2x\ dx = \frac{dz}{2}$

$= \frac{1}{2} \ln(1 + \sin 2x) + c$

10) $\int \sqrt{\sin x} \cos^3 x\, dx$

$= \int \sqrt{\sin x}\ . \cos^2 x . \cos x\, dx$ Let, $\sin x = z$

$= \int \sqrt{\sin x}\, (1 - \sin^2 x) . \cos x\, dx$ $\Rightarrow \cos x\, dx = dz$

$= \int \sqrt{z}\, (1 - z^2)\, dz$

$= \int \left(z^{\frac{1}{2}} - z^{\frac{5}{2}} \right) dz$

$= \left(\frac{z^{\frac{3}{2}}}{\frac{3}{2}} - \frac{z^{\frac{7}{2}}}{\frac{7}{2}} \right) + c$

$= \frac{2}{3}\, z^{\frac{3}{2}} - \frac{2}{7}\, z^{\frac{7}{2}} + c$

$= \frac{2}{3} (\sin x)^{\frac{3}{2}} - \frac{2}{7} (\sin x)^{\frac{7}{2}} + c$