**Differentiation (Formula):**

| | |
|---|---|
| $\frac{d}{dx}(x^n)$ | $nx^{n-1}$<br>$\frac{d}{dx}(x^4) = 4x^{4-1} = 4x^3$<br>$\frac{d}{dx}(x^{-4}) = -4x^{-4-1} = -4x^{-5}$<br>$\frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{d}{dx}(x^{-4}) = -4x^{-4-1} = -4x^{-5}$<br>$\frac{d}{dx}(\sqrt[4]{x}) = \frac{d}{dx}\left(x^{\frac{1}{4}}\right) = \frac{1}{4}x^{\frac{1}{4}-1} = \frac{1}{4}x^{\frac{-3}{4}}$ |
| $\frac{d}{dx}(c)$ | $0$<br>$\frac{d}{dx}(1) = 0$ |
| $\frac{d}{dx}(\ln x)$ | $\frac{1}{x}$ |
| $\frac{d}{dx}(e^x)$ | $e^x$ |
| $\frac{d}{dx}(\sqrt{x})$ | $\frac{1}{2\sqrt{x}}$<br>$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{\frac{-1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$ |
| | |
| $\frac{d}{dx}(\sin x)$ | $\cos x$ |
| $\frac{d}{dx}(\cos x)$ | $-\sin x$ |
| $\frac{d}{dx}(\tan x)$ | $\sec^2 x$ |
| $\frac{d}{dx}(\cot x)$ | $-\mathrm{cosec}^2 x$ |

| | |
|---|---|
| $\frac{d}{dx}(\sec x)$ | $\sec x \tan x$ |
| $\frac{d}{dx}(\operatorname{cosec} x)$ | $-\operatorname{cosec} x \cot x$ |
| | |
| $\frac{d}{dx}(\sin^{-1} x)$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\frac{d}{dx}(\cos^{-1} x)$ | $-\frac{1}{\sqrt{1-x^2}}$ |
| $\frac{d}{dx}(\tan^{-1} x)$ | $\frac{1}{1+x^2}$ |
| $\frac{d}{dx}(\cot^{-1} x)$ | $-\frac{1}{1+x^2}$ |
| $\frac{d}{dx}(\sec^{-1} x)$ | $\frac{1}{x\sqrt{x^2-1}}$ |
| $\frac{d}{dx}(\operatorname{cosec}^{-1} x)$ | $-\frac{1}{x\sqrt{x^2-1}}$ |

**Ex.1) Find the derivative with respect to x.**

$3 \ln x - 5\sec x + 2 \cot x$

$$= \frac{d}{dx}(3\ln x - 5\sec x + 2 \cot x)$$

$$= 3\frac{d}{dx}(\ln x) - 5\frac{d}{dx}(\sec x) + 2 \frac{d}{dx}(\cot x)$$

$$= 3\frac{1}{x} - 5 \sec x \tan x + 2 (-\operatorname{cosec}^2 x)$$

$$= \frac{3}{x} - 5 \sec x \tan x - 2 \operatorname{cosec}^2 x$$

**Ex.2) Find the derivative with respect to x.**

$$6x^4 - 3x^3 - 4x^{-\frac{1}{2}} + 5$$

$$= \frac{d}{dx}\left(6x^4 - 3x^3 - 4x^{-\frac{1}{2}} + 5\right)$$

$$= 6.4\,x^3 - 3.3\,x^2 - 4.-\frac{1}{2}\,x^{-\frac{3}{2}}$$

$$= 24\,x^3 - 9\,x^2 + 2\,x^{-\frac{3}{2}}$$

**Ex.3) Find the derivative with respect to x.**

$$\sqrt{x} + \sqrt[4]{x} + \frac{3}{x}$$

$$= \frac{d}{dx}\left(\sqrt{x} + \sqrt[4]{x} + \frac{3}{x}\right)$$

$$= \frac{d}{dx}(\sqrt{x}) + \frac{d}{dx}\left(x^{\frac{1}{4}}\right) + 3\frac{d}{dx}(x^{-1})$$

$$= \frac{1}{2\sqrt{x}} + \frac{1}{4}x^{-\frac{3}{4}} + 3(-1)x^{-2}$$

**Ex.4) Find the derivative with respect to x.**

$$\frac{x^3 - 27}{x^2 + 3x + 9}$$

$$= \frac{d}{dx}\left(\frac{x^3 - 27}{x^2 + 3x + 9}\right)$$

$$= \frac{d}{dx}\left(\frac{(x-3)(x^2 + 3x + 9)}{x^2 + 3x + 9}\right)$$

$$= \frac{d}{dx}(x-3) = 1$$

**H.W:**

1) $\frac{d}{dx}\left(7\sin x - 3\cos x - \frac{a}{\sqrt[4]{x}}\right)$

2) $\frac{d}{dx}\left(\frac{x^2 + 5x - 3}{3x^{\frac{1}{2}}}\right)$

3) $\frac{d}{dx}\left(2\sqrt{x} + 9\sqrt[3]{x^7} - \frac{2}{\sqrt[5]{x^2}}\right)$

4) If $s = \sqrt{t} + 7$ find the value of $\frac{ds}{dt}$ when, $t = 9$

| | |
|---|---|
| $\frac{d}{dx}(uv)$ | $u\frac{dv}{dx} + v\frac{du}{dx}$ |
| $\frac{d}{dx}\left(\frac{u}{v}\right)$ | $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ |

**Ex.1) Find the derivative with respect to x.**

(i) $(x^3 + 3)(2x^2 - 1)$

(ii) $x^2\ln x - 8e^x\cos x + 7$

Solution: (i) Given, $(x^3 + 3)(2x^2 - 1)$

$\frac{d}{dx}\left((x^2 + 3)(2x^2 - 1)\right)$

$= (x^2 + 3)\frac{d}{dx}(2x^2 - 1) + (2x^2 - 1)\frac{d}{dx}(x^2 + 3)$

$= (x^2 + 3).4x + (2x^2 - 1).2x$

$= 4x^3 + 12x + 4x^3 - 2x$

$= 8x^3 + 10x$

(ii) Given, $x^2\ln x - 8e^x \cos x + 7$

$$\frac{d}{dx}( x^2\ln x - 8e^x \cos x + 7)$$

$$= \frac{d}{dx}(x^2\ln x) - 8\frac{d}{dx}(e^x \cos x) + \frac{d}{dx}(7)$$

$$= x^2\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x^2) - 8\left[e^x\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(e^x)\right] + 0$$

$$= x^2\frac{1}{x} + \ln x.2x - 8(-e^x \sin x + e^x \cos x)$$

$$= x + 2x\ln x + 8e^x \sin x - 8e^x \cos x$$

**Ex.2) Find the derivative with respect to t.**

$$\frac{\sin t + \cos t}{\sin t - \cos t}$$

$$\frac{d}{dt}\left(\frac{\sin t + \cos t}{\sin t - \cos t}\right)$$

$$= \frac{(\sin t - \cos t)\frac{d}{dt}(\sin t + \cos t) - (\sin t + \cos t)\frac{d}{dt}(\sin t - \cos t)}{(\sin t - \cos t)^2}$$

$$= \frac{(\sin t - \cos t)(\cos t - \sin t) - (\sin t + \cos t)(\cos t + \sin t)}{(\sin t - \cos t)^2}$$

$$= \frac{-(\sin t - \cos t)^2 - (\sin t + \cos t)^2}{(\sin t - \cos t)^2}$$

**<u>H.W:</u>**

1) $\frac{d}{dx}\left(\frac{1 + \sin x}{1 - \sin x}\right)$

2) $\frac{d}{dx}\left(\frac{1 - \tan x}{1 + \tan x}\right)$

3) $\frac{d}{dx}[(6x^3 - x)(10 - 20x)]$

4) $\frac{d}{dx}\left(\sqrt[3]{x^2}(2x - x^2)\right)$

5) $\frac{d}{dx}\left(\frac{4\sqrt{x} - 2x^{-3}}{x^2 - 2x + 2}\right)$

$* \text{ Proof}, \frac{d}{dx}(\sin x) = \cos x$

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\frac{(2x+h)}{2}\ \sin\frac{h}{2}}{h}$$

$$= \lim_{h\to 0} \frac{\cos\frac{(2x+h)}{2}\ \sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \lim_{h\to 0}\left(\cos\frac{2x+h}{2}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$$

$$= \lim_{h\to 0}\left(\cos\frac{2x+h}{2}\right)\ \left[\because \lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) = 1\right]$$

$$= \cos\frac{2x+0}{2} = \cos x$$

$* \text{ Proof of}, \lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) = 1$

By L' Hospitals Rule,

$$\lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) = \lim_{h\to 0}\ \frac{1}{2}\left(\frac{\sin\frac{h}{2}}{h}\right) = \lim_{h\to 0}\ \frac{1}{2}\left(\frac{\left(\cos\frac{h}{2}\right).\frac{1}{2}}{1}\right) = \cos 0 = 1$$