Quiz-3 (Solution)

Integrate the followings:

1) $\displaystyle\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$

$= \displaystyle\int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos \alpha}\, dx$

$= \displaystyle\int \frac{2\cos^2 x - 2\cos^2 \alpha}{\cos x - \cos \alpha}\, dx$

$= 2\displaystyle\int \frac{(\cos x + \cos \alpha)(\cos x - \cos \alpha)}{\cos x - \cos \alpha}\, dx$

$= 2\displaystyle\int (\cos x + \cos \alpha)\, dx$

$= 2(\sin x + x \cos \alpha) + c$

2) $\displaystyle\int \cos^3 x\ \sin^2 x\, dx$

$= \displaystyle\int \sin^2 x . \cos^2 x. \cos x\, dx$ Let, $\sin x = z$

$= \displaystyle\int \sin^2 x\ (1 - \sin^2 x). \cos x\, dx$ $\Rightarrow \cos x\, dx = dz$

$= \displaystyle\int z^2\ (1 - z^2)\, dz$

$= \displaystyle\int (z^2 - z^4)\, dz$

$= \dfrac{z^3}{3} - \dfrac{z^5}{5} + c = \dfrac{(\sin x)^3}{3} - \dfrac{(\sin x)^5}{5} + c$

3) $\displaystyle\int \frac{\sin^5 x}{\cos^7 x}\, dx$

$= \displaystyle\int \frac{\sin^5 x}{\cos^5 x}.\frac{1}{\cos^2 x}\, dx$

$= \displaystyle\int \tan^5 x\ \sec^2 x\, dx$ Let, $\tan x = z$

$$= \int z^5 \, dz \qquad\qquad\qquad \Rightarrow \sec^2 x \, dx = dz$$

$$= \frac{z^6}{6} + c$$

$$= \frac{\tan^6 x}{6} + c$$

4) $\int x\sqrt{x^2 + 1} \, dx$

$$= \sqrt{x^2 + 1} \, x \, dx \qquad\qquad\qquad \text{Let, } x^2 + 1 = z$$

$$= \int \sqrt{z} \frac{dz}{2} \qquad\qquad\qquad \Rightarrow 2x \, dx = dz$$

$$= \frac{1}{2} \frac{z^{\frac{3}{2}}}{\frac{3}{2}} + c \qquad\qquad\qquad \Rightarrow x \, dx = \frac{dz}{2}$$

$$= \frac{1}{2} \cdot \frac{2}{3} (x^2 + 1)^{\frac{3}{2}} + c$$

$$= \frac{1}{3} (x^2 + 1)^{\frac{3}{2}} + c$$

5) $\int \dfrac{4x + 3}{x(2x + 3)(2x + 1)} \, dx$

$$\frac{4x + 3}{x(2x + 3)(2x + 1)} = \frac{A}{x} + \frac{B}{2x + 3} + \frac{C}{2x + 1}$$

$$A = 1, B = -1, C = -1$$

$$\Rightarrow \int \frac{4x + 3}{x(2x + 3)(2x + 1)} \, dx = \int \frac{1}{x} \, dx - \int \frac{1}{2x + 3} \, dx - \int \frac{1}{2x + 1} \, dx$$

$$\Rightarrow \int \frac{4x + 3}{x(2x + 3)(2x + 1)} \, dx = \int \frac{1}{x} \, dx - \frac{1}{2} \int \frac{2}{2x + 3} \, dx - \frac{1}{2} \int \frac{2}{2x + 1} \, dx$$

$$\Rightarrow \int \frac{dx}{(x + 1)(x - 5)} = \ln x - \frac{1}{2} \ln(2x + 3) - \frac{1}{2} \ln(2x + 1) + c$$

6) $\int \dfrac{7x + 4}{x^3 - 4x} dx$

$\dfrac{7x + 4}{x^3 - 4x} = \dfrac{A}{x} + \dfrac{B}{x + 2} + \dfrac{C}{x - 2}$

$A = -1, B = -\dfrac{5}{4}, C = \dfrac{9}{4}$

$\Rightarrow \int \dfrac{7x + 4}{x^3 - 4x} dx = -\int \dfrac{1}{x} dx - \int \dfrac{\frac{5}{4}}{x + 2} dx + \int \dfrac{\frac{9}{4}}{x - 2} dx$

$\Rightarrow \int \dfrac{dx}{(x + 1)(x - 5)} = -\ln x - \dfrac{5}{4} \ln(x + 2) + \dfrac{9}{4} \ln(x - 2) + c$

7) $\int \dfrac{dx}{x^3 - x^2 - 9x + 9}$

$\dfrac{1}{(x - 1)(x + 3)(x - 3)} = \dfrac{A}{x - 1} + \dfrac{B}{x + 3} + \dfrac{C}{x - 3}$

$A = -\dfrac{1}{8}, B = \dfrac{1}{24}, C = \dfrac{1}{12}$

$\Rightarrow \int \dfrac{dx}{x^3 - x^2 - 9x + 9} = -\int \dfrac{\frac{1}{8}}{x - 1} dx + \int \dfrac{\frac{1}{24}}{x + 3} dx + \int \dfrac{\frac{1}{12}}{x - 3} dx$

$\Rightarrow \int \dfrac{dx}{(x + 1)(x - 5)} = -\dfrac{1}{8} \ln(x - 1) + \dfrac{1}{24} \ln(x + 3) + \dfrac{1}{12} \ln(x - 3) + c$