Constraint Optimization Technique

$$\boxed{x^2 + y^2 - 100 = 0}$$

$\rightarrow f(x, y) = 3x + 4y \longrightarrow$ Objective Function

$\rightarrow \underline{x^2 + y^2 = 100} \longrightarrow$ Constraint

$$\boxed{g(x, y) = x^2 + y^2 - 100}$$

Lagrange Multiplier Method

$$\rightarrow \underline{f(x, y)} - \lambda * \underline{g(x, y)} = 0 \quad \text{———} \quad (*)$$

$$\Rightarrow 3x + 4y - \lambda * (x^2 + y^2 - 100) = 0$$

$$\Rightarrow 3x + 4y - \lambda x^2 - \lambda y^2 + 100\lambda = 0$$

$$3x + 4y - \lambda x^2 - \lambda y^2 + 100\lambda = 0$$

Partially derive by 'x', we get,

$$3 + 0 - 2\lambda x - 0 + 0 = 0$$

$$\Rightarrow 3 = 2\lambda x$$

$$\therefore x = \frac{3}{2\lambda}$$

'y', we get

$$0 + 4 - 0 - 2\lambda y + 0 = 0$$

$$\Rightarrow 4 = 2\lambda y$$

$$\therefore y = \frac{4}{2\lambda} = \frac{2}{\lambda}$$

$$x^2+y^2=100$$

$$\Rightarrow \left(\frac{3}{2\lambda}\right)^2+\left(\frac{2}{\lambda}\right)^2=100$$

$$\Rightarrow \frac{9}{4\lambda^2}+\frac{4}{\lambda^2}=100$$

$$\Rightarrow \frac{9+16}{4\lambda^2}=100$$

$$\Rightarrow 25=400\lambda^2$$

$$\Rightarrow \lambda^2=\frac{25}{400}$$

$$\Rightarrow \lambda^2=\frac{1}{16}$$

$$\therefore \lambda=\pm\frac{1}{4}$$

when, $\lambda = \frac{1}{4}$

$$x = \frac{3}{2\lambda}$$

$$= \frac{3}{2 \cdot \frac{1}{4}}$$

$$= 6$$

$$y = \frac{2}{\lambda}$$

$$= 8$$

when, $\lambda = -\frac{1}{4}$

$$\boxed{\begin{aligned} x &= -6 \\ y &= -8 \end{aligned}}$$

$$x^2 + y^2 = 100$$

$$6^2 + 8^2$$

$$= 36 + 64$$

$$= 100$$

$\lambda = \frac{1}{4}$ $\underline{(x, y) = (6, 8)}$ ✓

$\lambda = -\frac{1}{4}$ $\underline{(x, y) = (-6, -8)}$ ✓

$$(-6)^2 + (-8)^2$$

$$= 36 + 64$$

$$= 100$$

$$\boxed{f(x,y) = 3x + 4y} \quad \text{———} \quad \boxed{x^2 + y^2 = 100}$$

$$f(6,8) = 3.6 + 4.8$$

$$= 18 + 32$$

$$= 50 \quad (\text{Max}) \checkmark$$

$$f(-6,-8) = -50 \quad (\text{Min}) \checkmark$$

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$$f(x, y) = y^2 - 4x^2$$ — Objective

$$x^2 + 2y^2 = 4$$ — Constraint

$$g(x, y) = x^2 + 2y^2 - 4$$

$$f(x, y) - \lambda * g(x, y) = 0$$

$$\Rightarrow y^2 - 4x^2 - \lambda * (x^2 + 2y^2 - 4) = 0$$

$$\Rightarrow y^2 - 4x^2 - \lambda x^2 - 2\lambda y^2 + 4\lambda = 0$$

$$y^2 - 4x^2 - \lambda x^2 - 2\lambda y^2 + 4\lambda = 0$$

$$0 - 8x - 2\lambda x - 0 + 0 = 0$$

$$\Rightarrow -8x - 2\lambda x = 0$$

$$\Rightarrow 2\lambda x + 8x = 0$$

$$\Rightarrow \underline{2x}\,\underline{(\lambda + 4)} = 0$$

$2x = 0$ $\quad$ $\lambda + 4 = 0$

$x = 0$ $\quad$ $\boxed{\lambda = -4}$

$$2y - 0 - 0 - 4\lambda y + 0 = 0$$

$$\Rightarrow 2y - 4\lambda y = 0$$

$$\Rightarrow \underline{2y}\,\underline{(1 - 2\lambda)} = 0$$

$y = 0$ $\quad$ $\boxed{\lambda = \frac{1}{2}}$

when $\boxed{\lambda = -4}$

$x = 0$

$x^2 + 2y^2 = 4$

$\Rightarrow 0 + 2y^2 = 4$

$\Rightarrow 2y^2 = 4$

$\Rightarrow y^2 = 2$

$\therefore y = \pm\sqrt{2}$

$$\left.\begin{array}{l} \checkmark\ (x, y) = (0, \sqrt{2}) \\ \checkmark\ (x, y) = (0, -\sqrt{2}) \end{array}\right\}$$

when $\lambda = \frac{1}{2}$

$y = 0$

$x^2 + 2y^2 = 4$

$\Rightarrow x^2 + 0 = 4$

$\Rightarrow x = \pm 2$

$$\left\{\begin{array}{l} (x, y) = (2, 0) \\ (x, y) = (-2, 0) \end{array}\right.$$

$f(x, y) = y^2 - 4x^2$

$(0, \sqrt{2})$

$f(0, \sqrt{2})$

$= (\sqrt{2})^2 - 0$

$= 2$ — (Max)

$(0, -\sqrt{2})$

$f(0, -\sqrt{2})$

$= (-\sqrt{2})^2 - 0$

$= 2$ — (Max)

$(2, 0)$

$f(2, 0) = 0^2 - 4(2)^2$

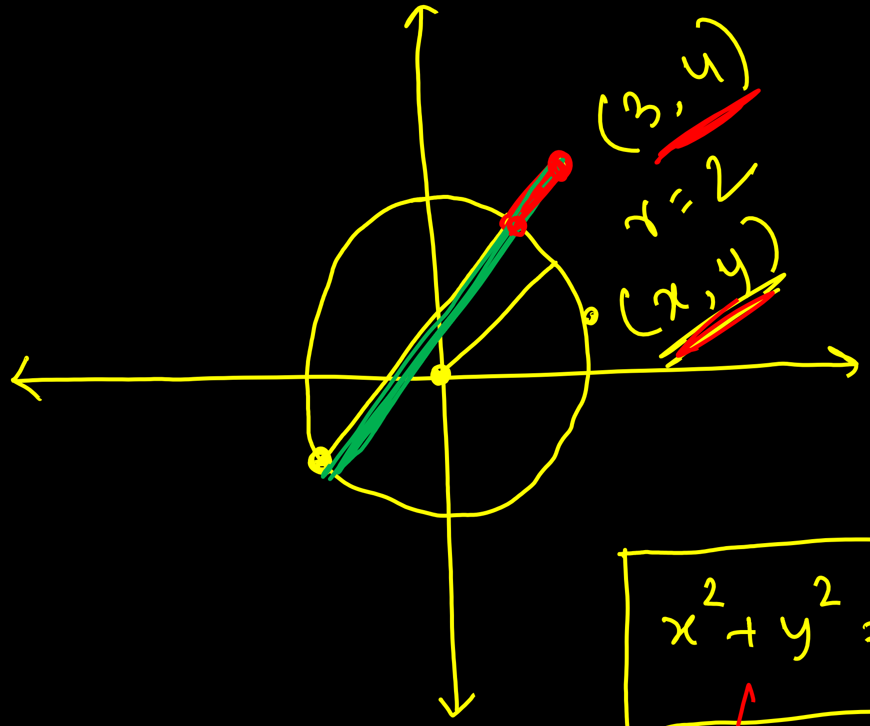
$= 0 - 16$

$= -16$ (Min)

$f(-2, 0) = 0^2 - 4(-2)^2$

$= 0 - 16$

$= -16$ (Min)



objective function $\boxed{x^2 + y^2 = r^2}$

$$\boxed{d = \sqrt{(x-3)^2 + (y-4)^2}}$$

$$\boxed{x^2 + y^2 = 4}$$

$$\boxed{x^2 + y^2 = 4}$$

Constraint