Successive Differentiation

Ex. 1) $\dfrac{d}{dx}\left(\tan\left(\ln \sin e^{\sqrt{x}}\right)\right)$

$= \sec^2\left(\ln \sin e^{\sqrt{x}}\right)\frac{d}{dx}\left(\ln \sin e^{\sqrt{x}}\right)$

$= \sec^2\left(\ln \sin e^{\sqrt{x}}\right)\dfrac{1}{\sin e^{\sqrt{x}}}\dfrac{d}{dx}\left(\sin e^{\sqrt{x}}\right)$

$= \sec^2\left(\ln \sin e^{\sqrt{x}}\right)\dfrac{1}{\sin e^{\sqrt{x}}}\cos e^{\sqrt{x}}\dfrac{d}{dx}\left(e^{\sqrt{x}}\right)$

$= \sec^2\left(\ln \sin e^{\sqrt{x}}\right)\dfrac{1}{\sin e^{\sqrt{x}}}\cos e^{\sqrt{x}}\, e^{\sqrt{x}}\dfrac{d}{dx}\left(\sqrt{x}\right)$

$= \sec^2\left(\ln \sin e^{\sqrt{x}}\right)\dfrac{1}{\sin e^{\sqrt{x}}}\cos e^{\sqrt{x}}\, e^{\sqrt{x}}\dfrac{1}{2\sqrt{x}}$

$= \sec^2\left(\ln \sin e^{\sqrt{x}}\right)\cot e^{\sqrt{x}}\, e^{\sqrt{x}}\dfrac{1}{2\sqrt{x}}$

Ex. 2) $\dfrac{d}{dx}\left\{(x^2+1)\sin^{-1}x + e^{\sqrt{1+x^2}}\right\}$

$= (x^2+1)\dfrac{d}{dx}(\sin^{-1}x) + \sin^{-1}x\dfrac{d}{dx}(x^2+1) + \dfrac{d}{dx}\left(e^{\sqrt{1+x^2}}\right)$

$= \dfrac{(x^2+1)}{\sqrt{1-x^2}} + \sin^{-1}x\,.\,2x + e^{\sqrt{1+x^2}}\dfrac{d}{dx}\left(\sqrt{1+x^2}\right)$

$= \dfrac{(x^2+1)}{\sqrt{1-x^2}} + \sin^{-1}x\,.\,2x + e^{\sqrt{1+x^2}}.\dfrac{1}{2\sqrt{1+x^2}}\dfrac{d}{dx}(1+x^2)$

$= \dfrac{(x^2+1)}{\sqrt{1-x^2}} + \sin^{-1}x\,.\,2x + e^{\sqrt{1+x^2}}.\dfrac{1}{2\sqrt{1+x^2}}.2x$

$= \dfrac{(x^2+1)}{\sqrt{1-x^2}} + \sin^{-1}x\,.\,2x + \dfrac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$

Ex. 3) $\dfrac{\mathbf{d}}{\mathbf{dx}}\left(\sqrt{\mathbf{x}}\,\mathbf{e^x}\,\mathbf{sec\ x}\right)$

$$= \sqrt{x}\frac{d}{dx}(e^x \sec x) + e^x \sec x \frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \sqrt{x}\left\{e^x \frac{d}{dx}(\sec x) + \sec x \frac{d}{dx}(e^x)\right\} + e^x \sec x . \frac{1}{2\sqrt{x}}$$

$$= \sqrt{x}(e^x . \sec x \tan x + \sec x . e^x) + \frac{e^x \sec x}{2\sqrt{x}}$$

$$= \sqrt{x}\, e^x \sec x\,(\tan x + 1) + \frac{e^x \sec x}{2\sqrt{x}}$$

Ex. 4) $\dfrac{\mathbf{d}}{\mathbf{dx}}\left[\mathbf{ln}\left\{\sqrt{\mathbf{1+lnx}} - \mathbf{\ sin\ x}\right\}\right]$

$$= \frac{1}{\left(\sqrt{1+\ln x} - \sin x\right)} \frac{d}{dx}\left\{\sqrt{1+\ln x} - \sin x\right\}$$

$$= \frac{1}{\left(\sqrt{1+\ln x} - \sin x\right)} \left[\frac{1}{2\sqrt{1+\ln x}} \frac{d}{dx}(1+\ln x) - \cos x\right]$$

$$= \frac{1}{\left(\sqrt{1+\ln x} - \sin x\right)} \left[\frac{1}{2\sqrt{1+\ln x}} . \frac{1}{x} - \cos x\right]$$

$$= \frac{1}{\left(\sqrt{1+\ln x} - \sin x\right)} \left[\frac{1}{2x\sqrt{1+\ln x}} - \cos x\right]$$

H.W:

1) $\dfrac{d}{dx}\left(e^{\sin x} \sin(a^x)\right)$

2) $\dfrac{d}{dx}\left[\tan(\ln x^2)\right]$

3) $\dfrac{d}{dx}\left[\tan\left(\ln(\sin e^{x^2})\right)\right]$

Differentiate with respect to x:

1) $\dfrac{\cot x - \tan x}{\cot x + \tan x}$

2) $\dfrac{1}{(3 - x^2)^3}$

3) $\operatorname{cosec} \sqrt{x} + \ln(\sin 2x)$

4) $\sqrt{e^{\sqrt{x}}}$

5) $\sqrt{\sin \sqrt{x}}$

6) $\ln\left(x - \sqrt{x^2 - 1}\right)$

7) $\dfrac{\ln(\cos x)}{x}$

8) $e^{xy} - 4xy$

9) $\sqrt{(x - 3)(x - 4)}$

10) $\dfrac{1}{\sqrt{x + 1} + \sqrt{x + 2}}$