Assignment 1

Instructions:
1. Please read the questions carefully and answer in pen & paper in consecutive order if possible.
2. The assignment consists of 2 questions.
3. Answer all questions.
4. Include a cover page mentioning your ID and Name.
5. Scan your answer scripts and make a single pdf file with a filename of you Reg. ID (e.g. 18101051.pdf)
5. Good luck!

1. Consider the system of linear equations:

$$
\begin{aligned}
x_1 + 5x_2 + 3x_3 &= 4 \\
x_1 - x_2 + 6x_3 &= 16 \\
2x_1 + x_2 \phantom{{}+ 6x_3} &= 5
\end{aligned}
$$

   (a) Write the system as $Ax = b$
   (b) Solve the system using Gaussian Elimination with back-substitution.
   (c) What is the determinant of A?
   (d) Perform one step of the Gauss-Seidel method on this system (put the equations in the most appropriate order), starting with initial guess

   $$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

2. Consider the function $f(x) = 4x - 1 - \sin(x)$ on the interval $0 \le x \le 2$.
   (a) Perform 3 iterations of the Bisection Method on $f(x)$ using the end-points of the interval as initial data. Show the new estimate x_n and $f(x_n)$ at each step.
   (b) Perform 2 iterations of Newton's Method on $f(x)$ with the initial guess $x_0 = 1$. Show the new estimate x_n and $f(x_n)$ at each step.
   (c) What is the absolute difference between the final estimates of the root of $f(x)$ in parts (a) and (b)? If this is the error of the bisection method, how many more iterations of bisection are needed to find the root of $f(x)$ to within an accuracy of 10^{-4}?