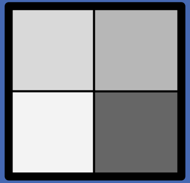
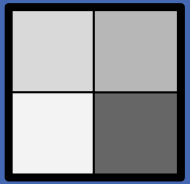
How neural networks work

Brandon Rohrer

A four pixel camera



Categorize images



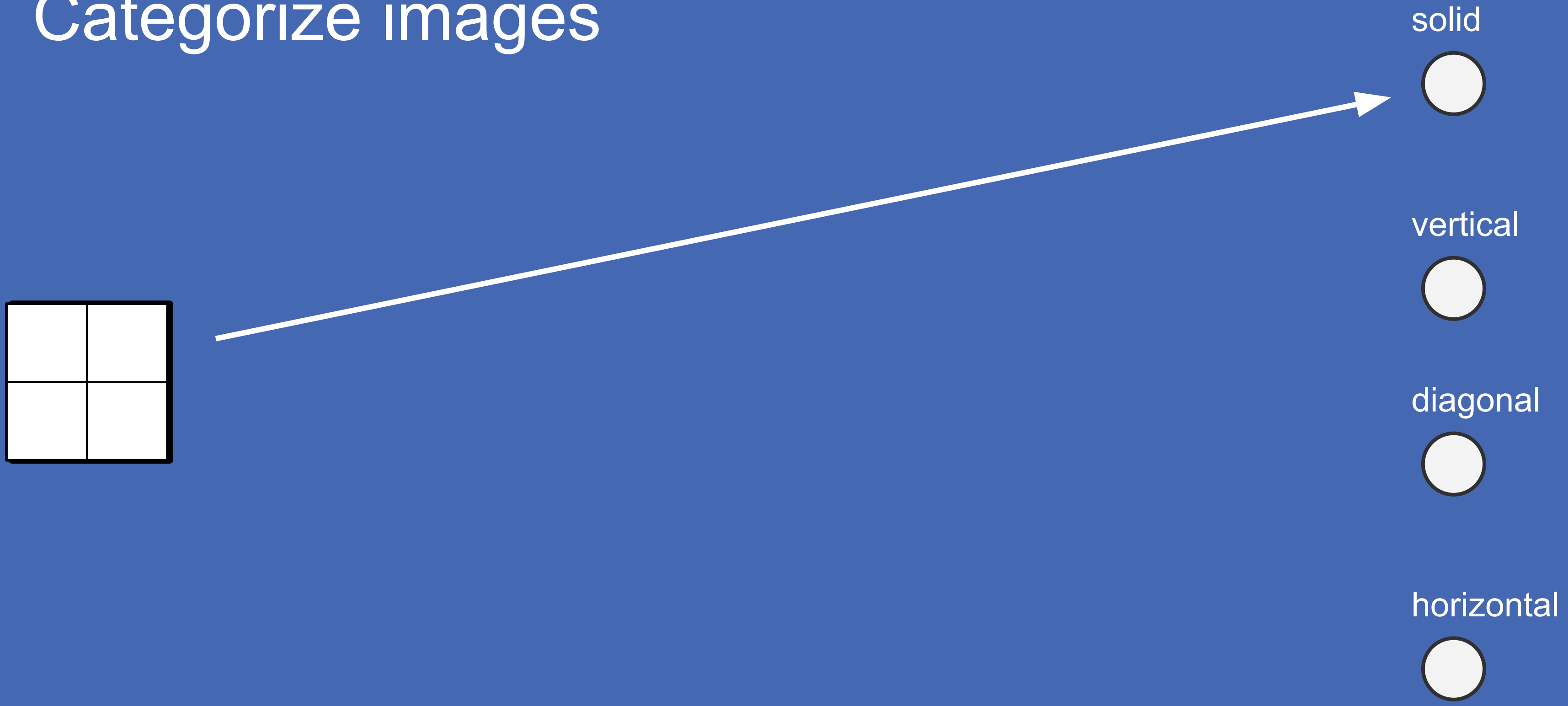
solid

vertical

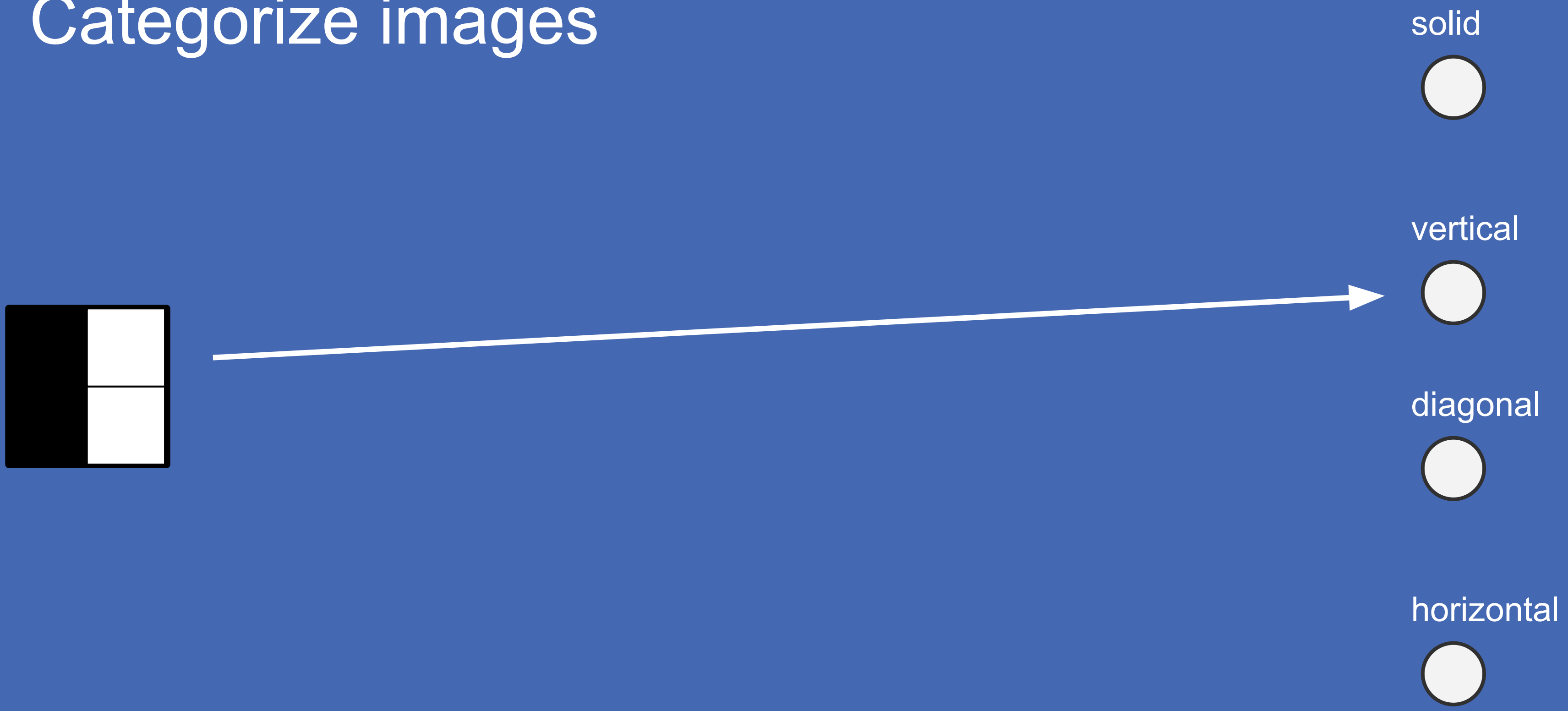
diagonal

horizontal

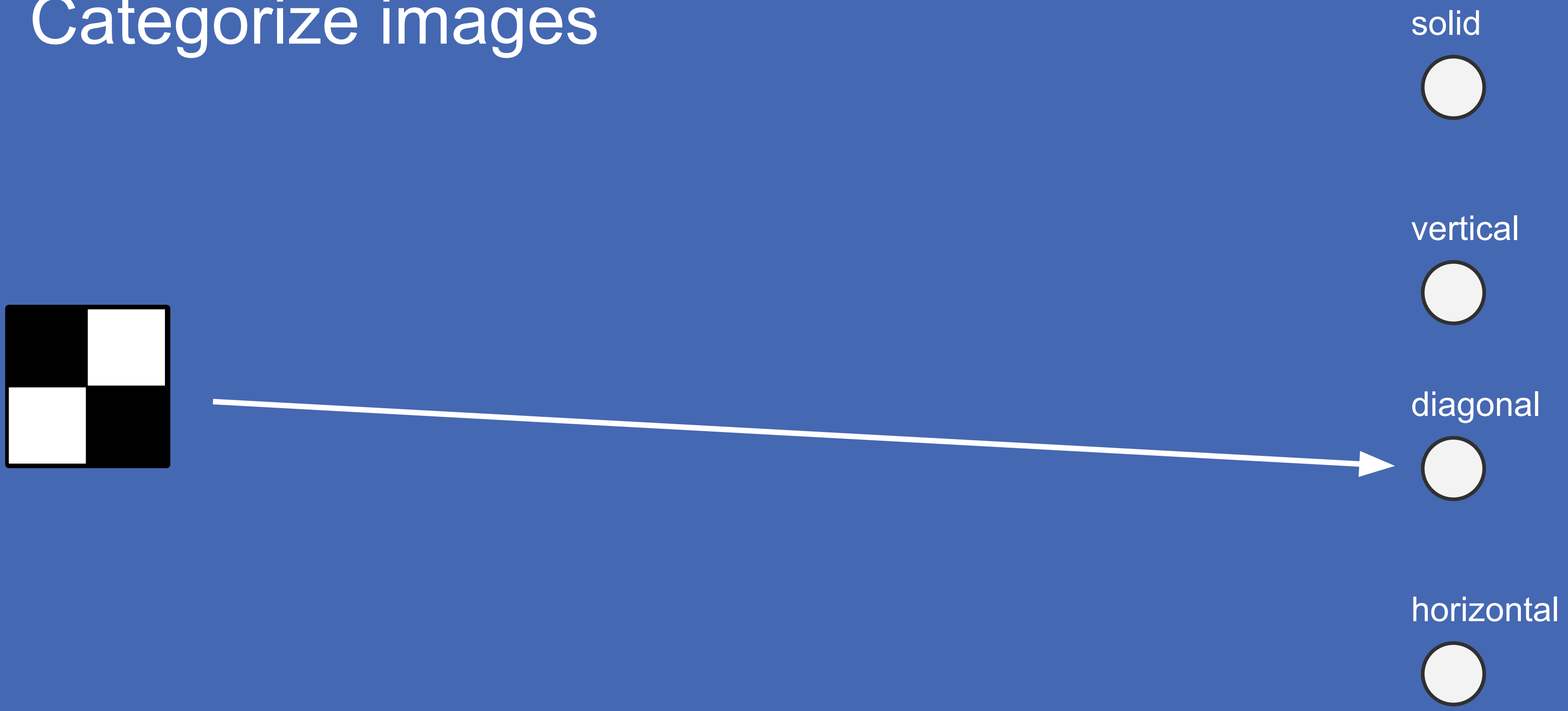
Categorize images



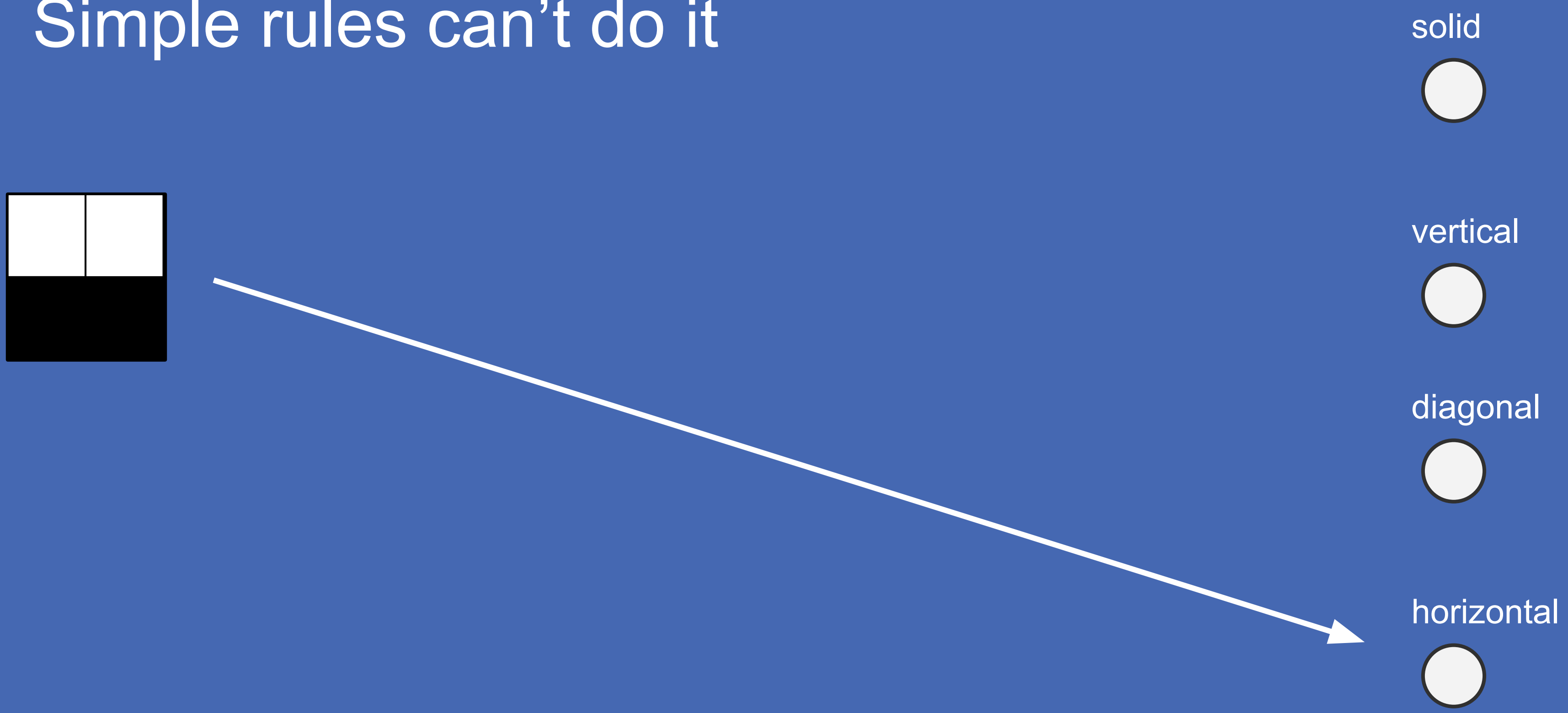
Categorize images



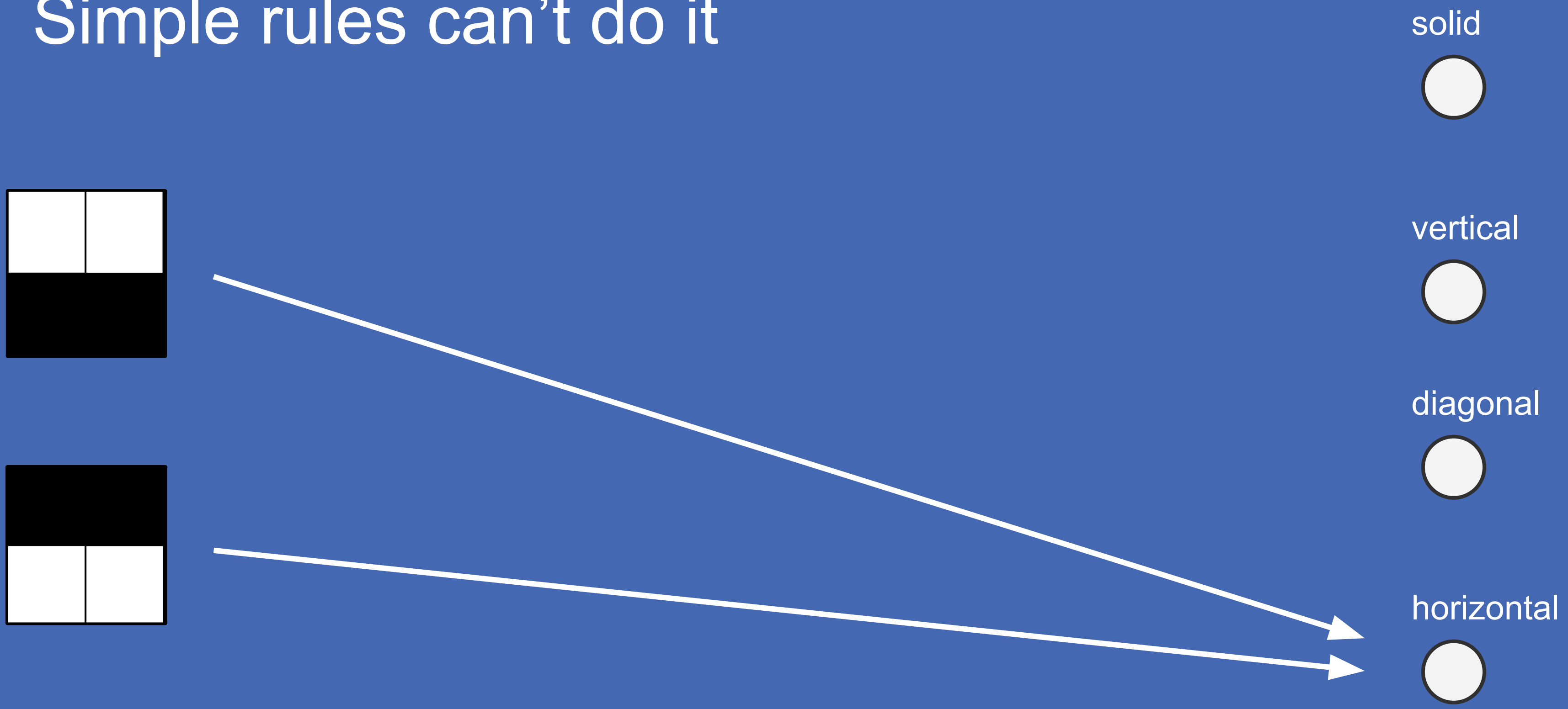
Categorize images



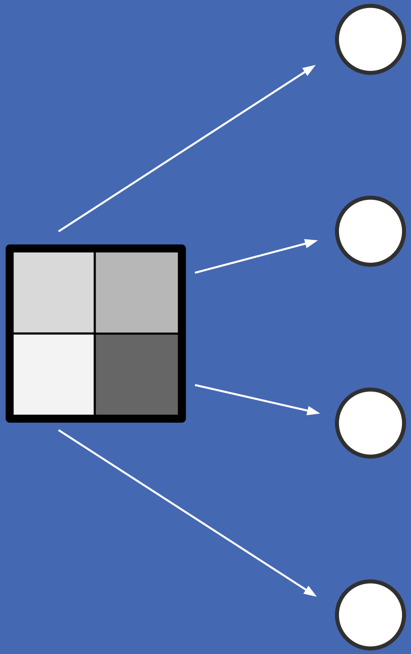
Simple rules can't do it



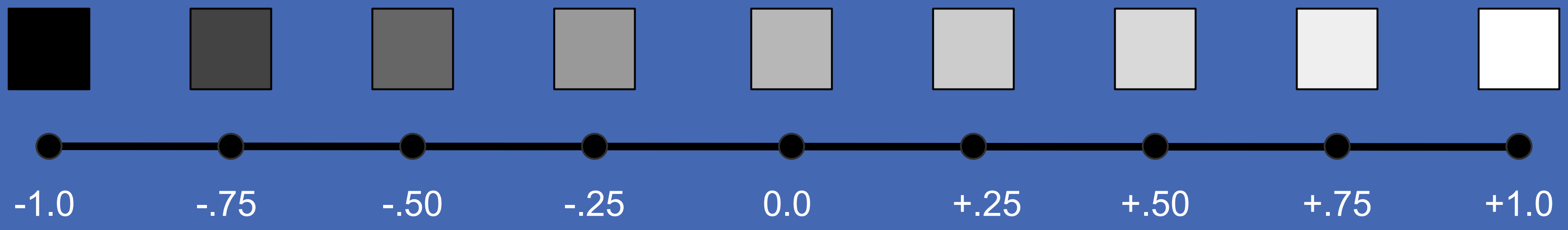
Simple rules can't do it



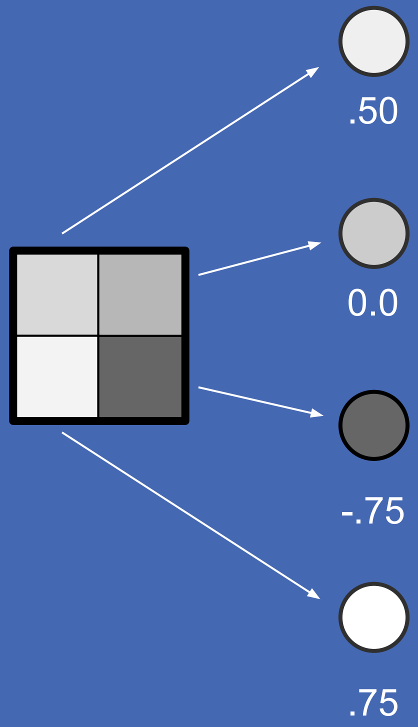
Input neurons



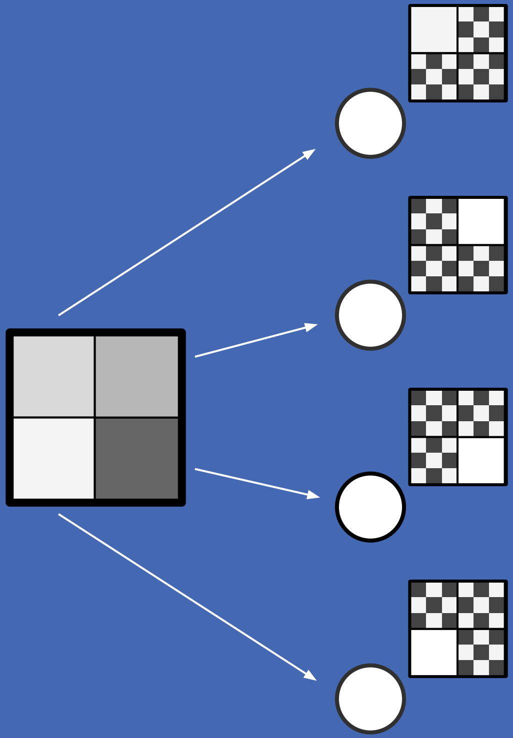
Pixel brightness



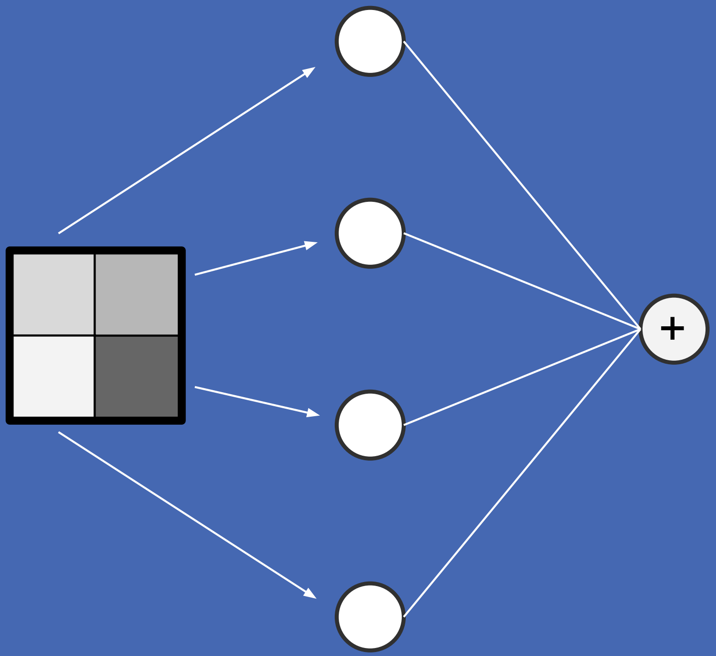
Input vector



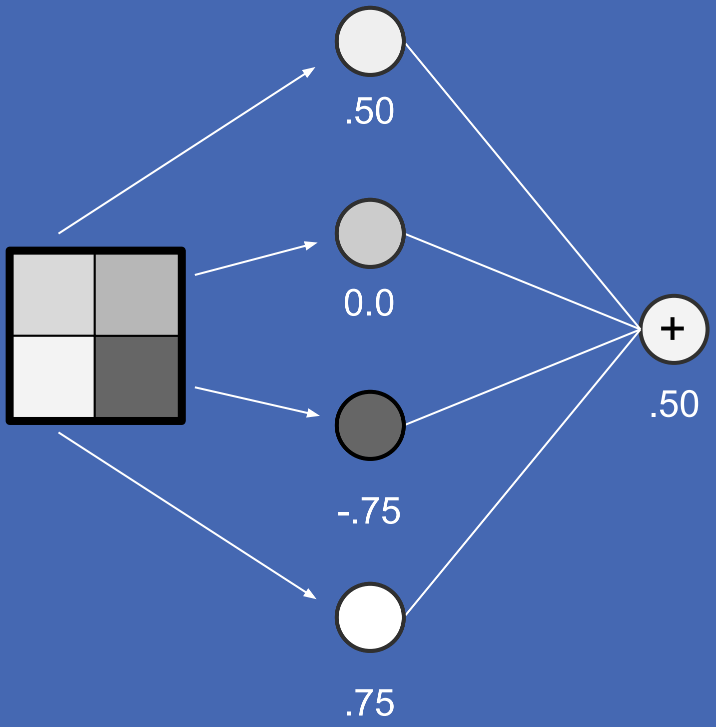
Receptive fields



A neuron

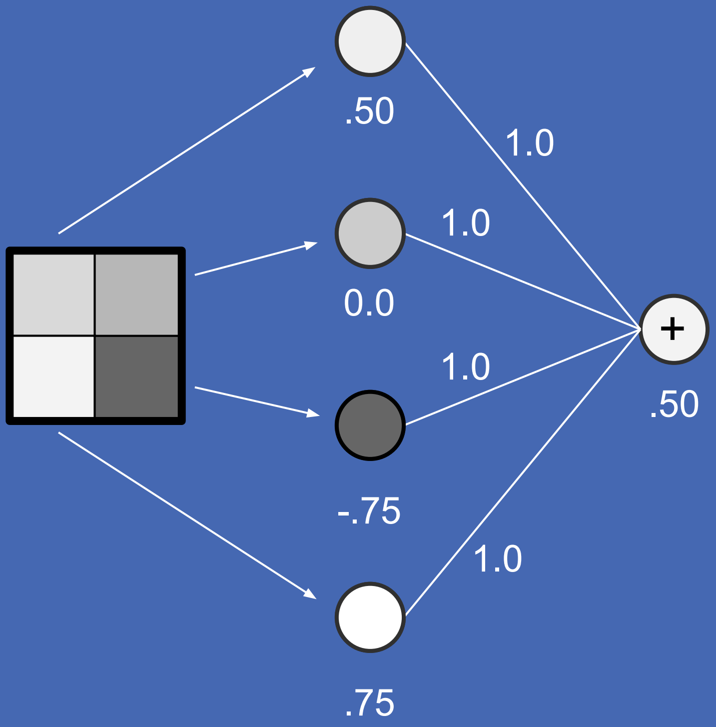


Sum all the inputs



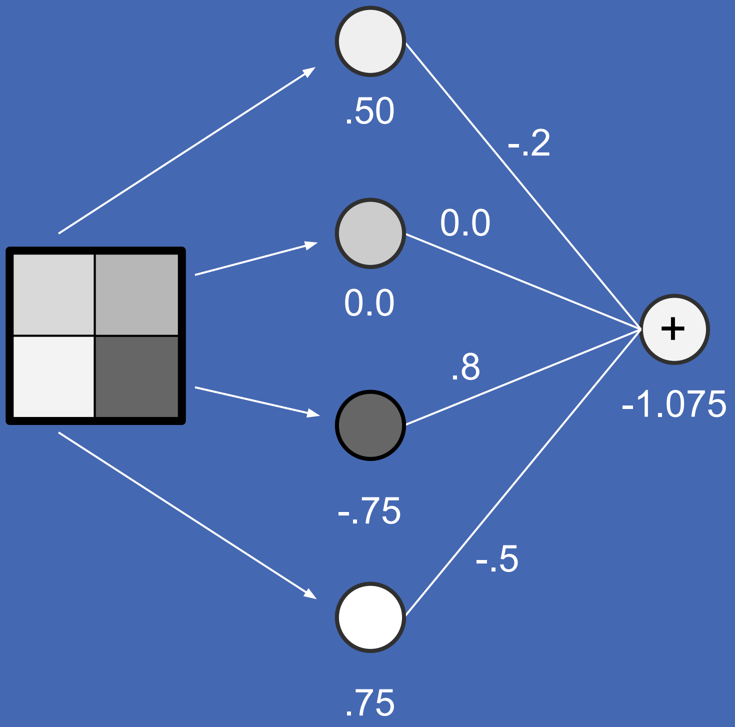
$$
\begin{array}{rr}
 & .50 \\
 & 0.00 \\
 & -.75 \\
+ & .75 \\
\hline
 & .50
\end{array}
$$

Weights



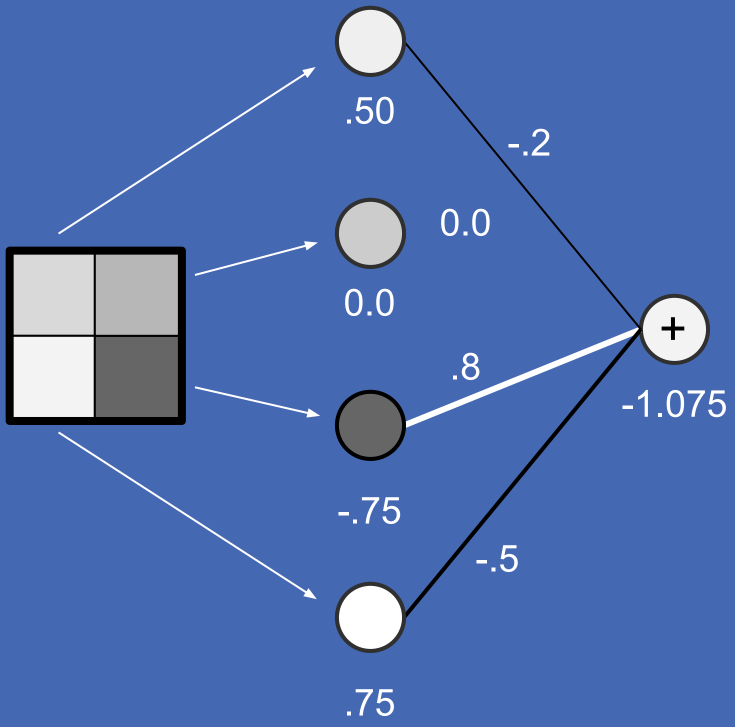
$$
\begin{array}{rrl}
 & .50 & \times\ 1.0 \\
 & 0.00 & \times\ 1.0 \\
 & -.75 & \times\ 1.0 \\
+ & .75 & \times\ 1.0 \\
\hline
 & & .50
\end{array}
$$

Weights



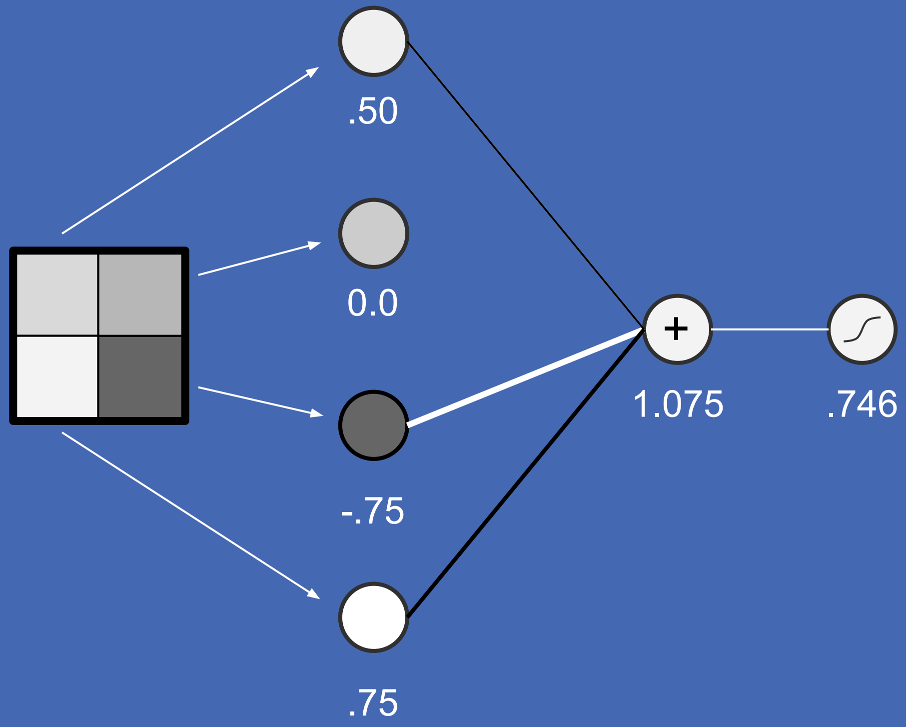
$$
\begin{array}{rrcr}
 & .50 & \times & -.2 \\
 & 0.00 & \times & 0.0 \\
 & -.75 & \times & .8 \\
+ & .75 & \times & -.5 \\
\hline
 & & -1.075 &
\end{array}
$$

Weights

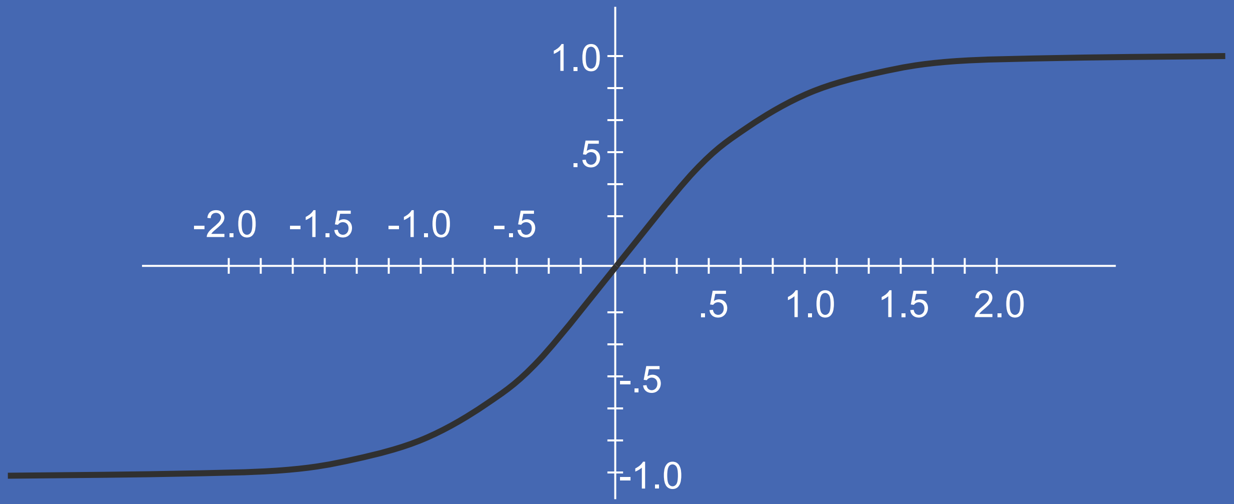


$$
\begin{array}{rrcr}
 & .50 & \times & -.2 \\
 & 0.00 & \times & 0.0 \\
 & -.75 & \times & .8 \\
+ & .75 & \times & -.5 \\
\hline
 & & -1.075 &
\end{array}
$$

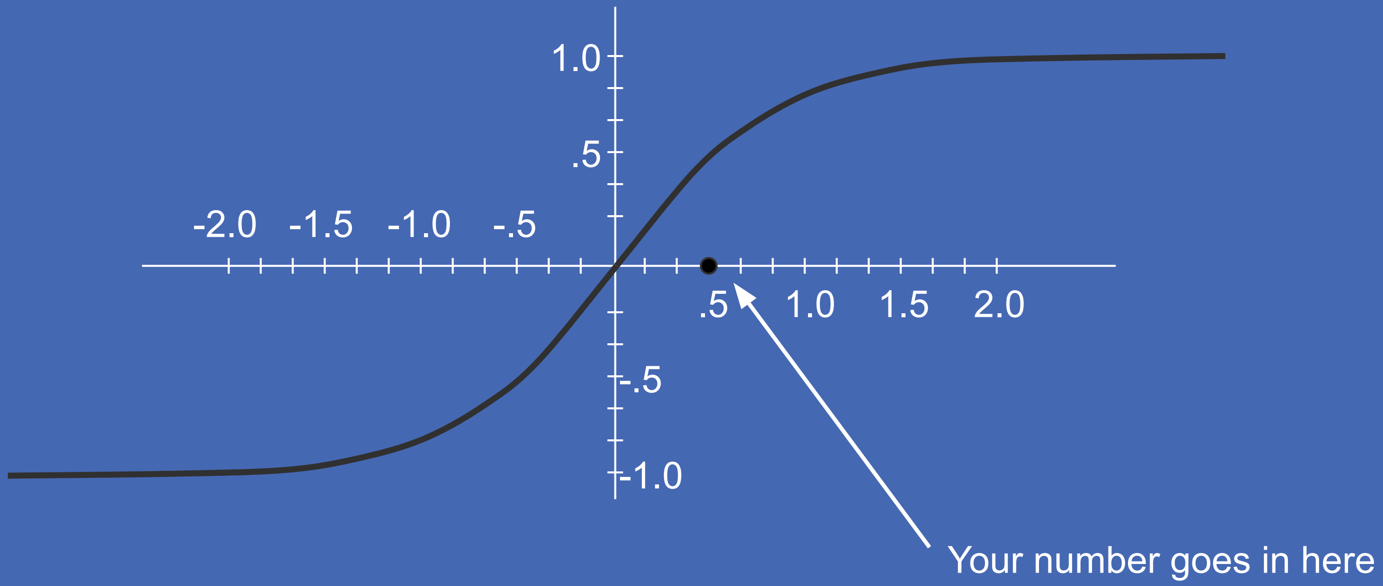
Squash the result



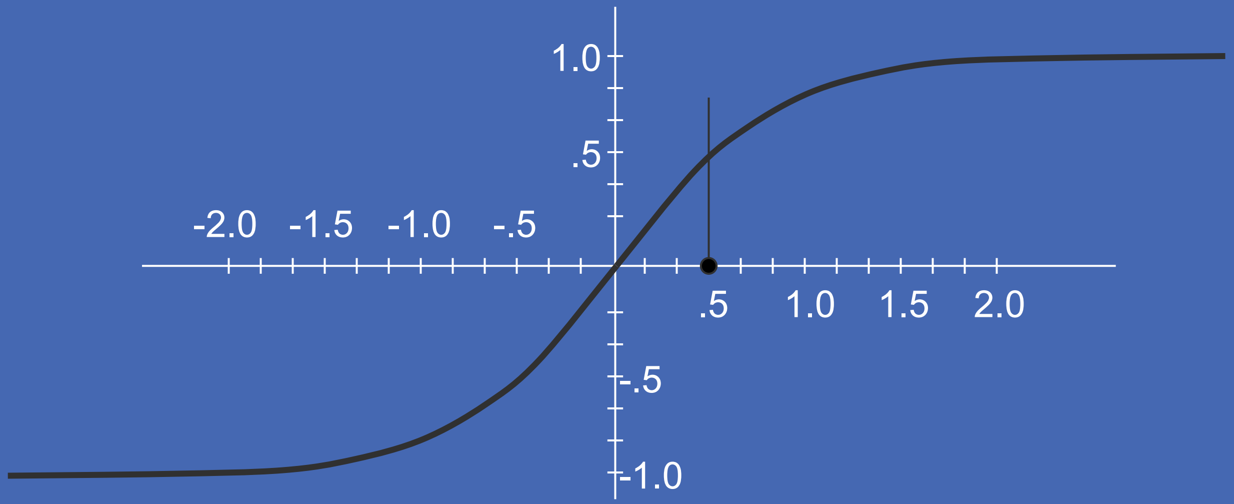
Sigmoid squashing function



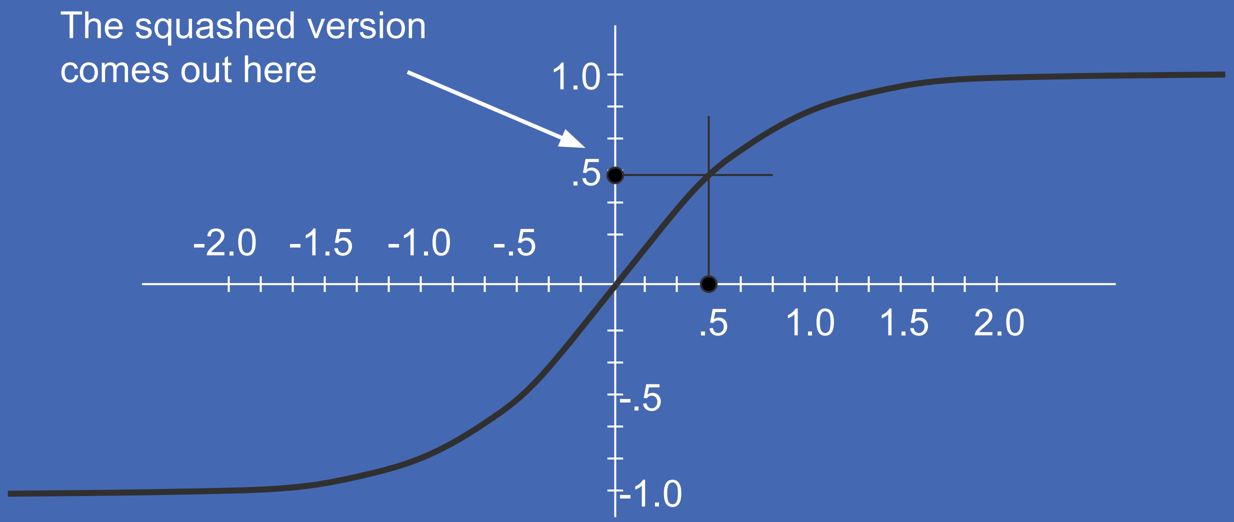
Sigmoid squashing function



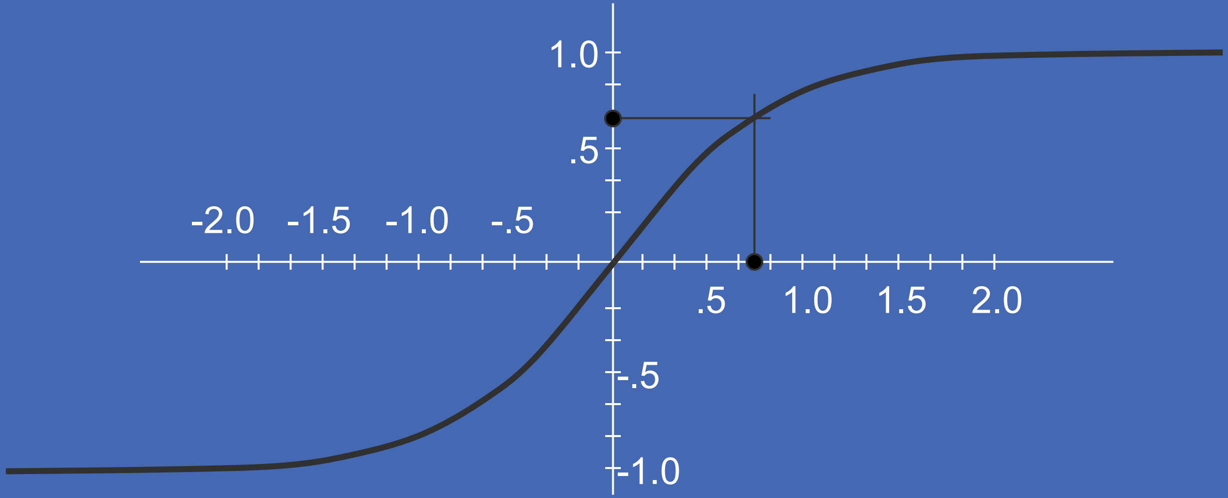
Sigmoid squashing function



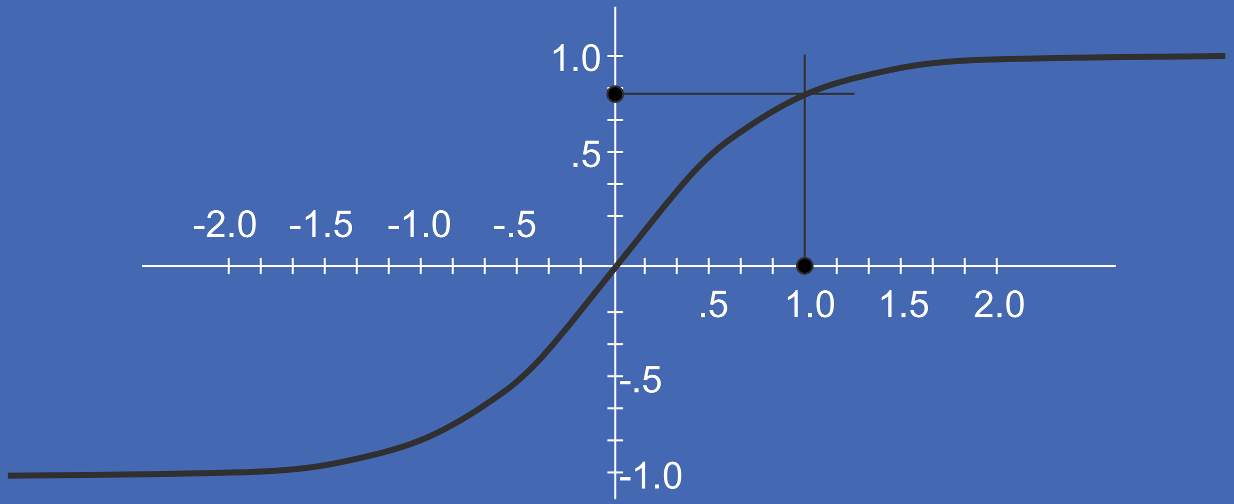
Sigmoid squashing function



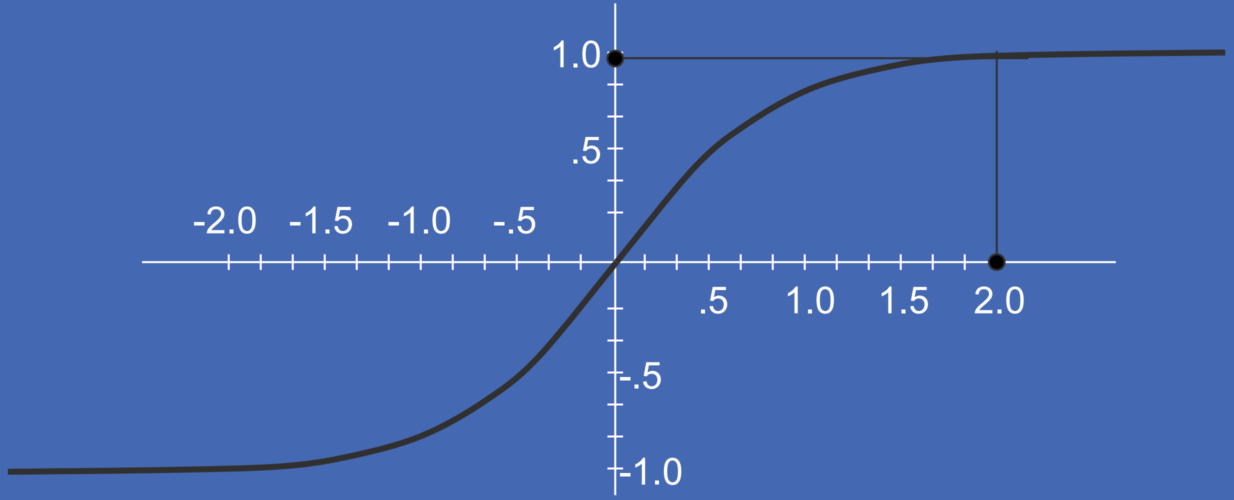
Sigmoid squashing function



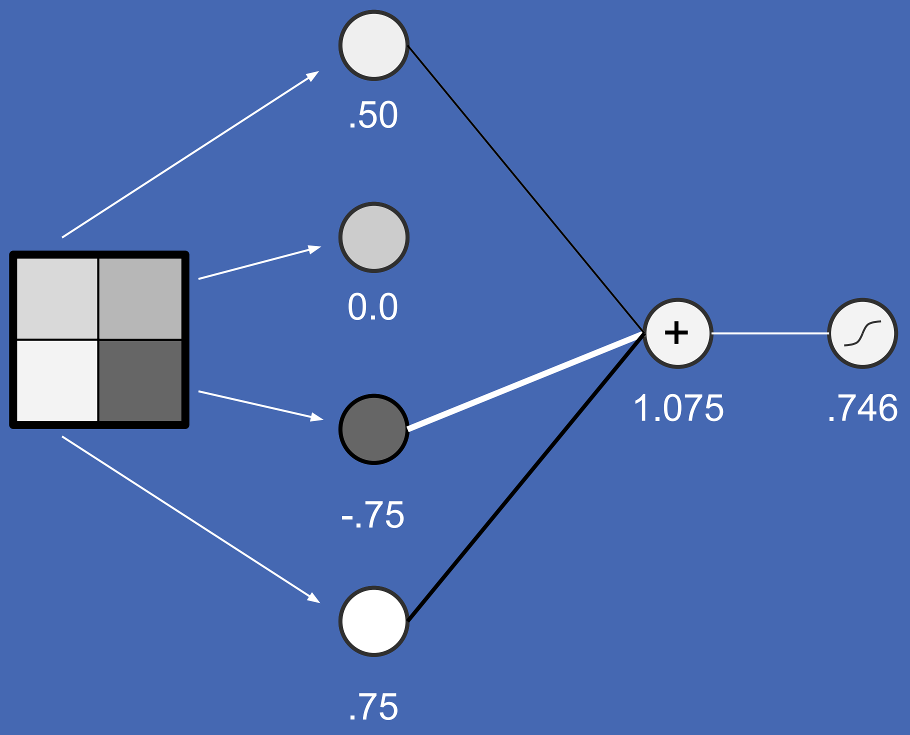
Sigmoid squashing function



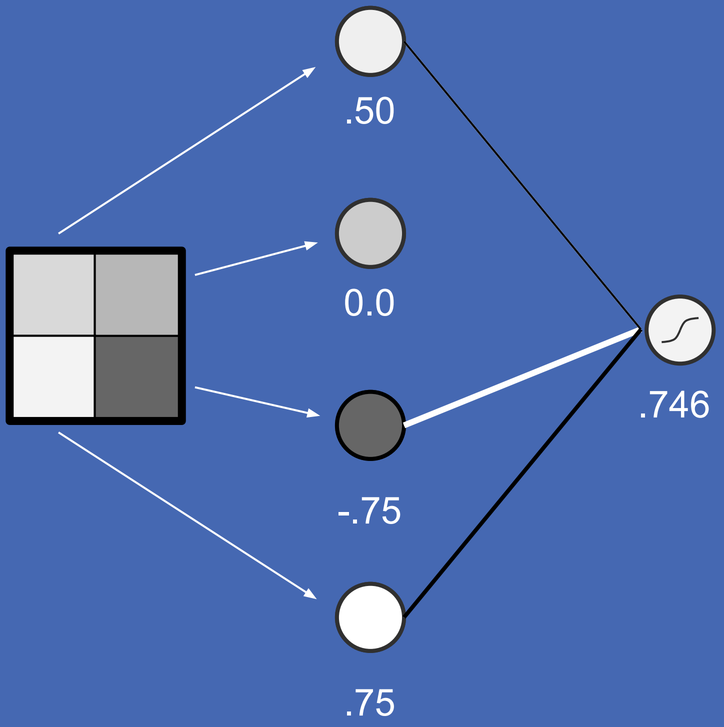
No matter what you start with, the answer stays between -1 and 1.



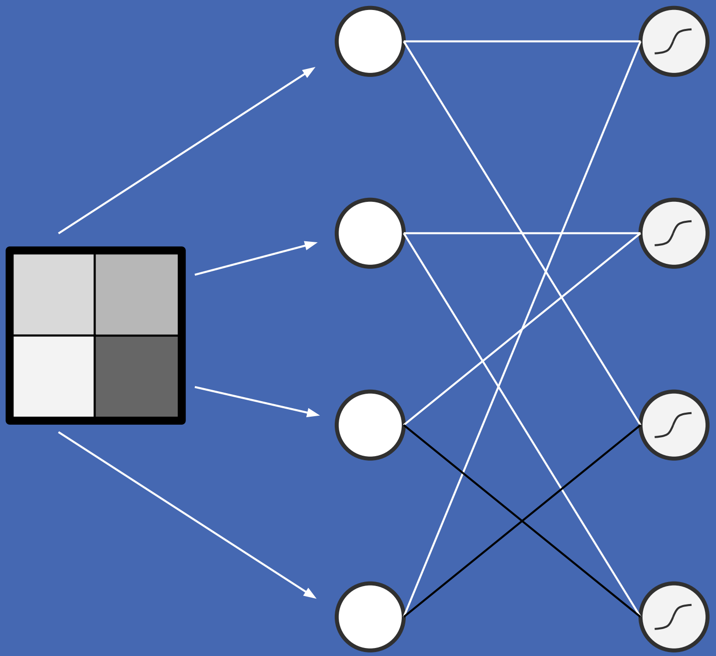
Squash the result



Weighted sum-and-squash neuron

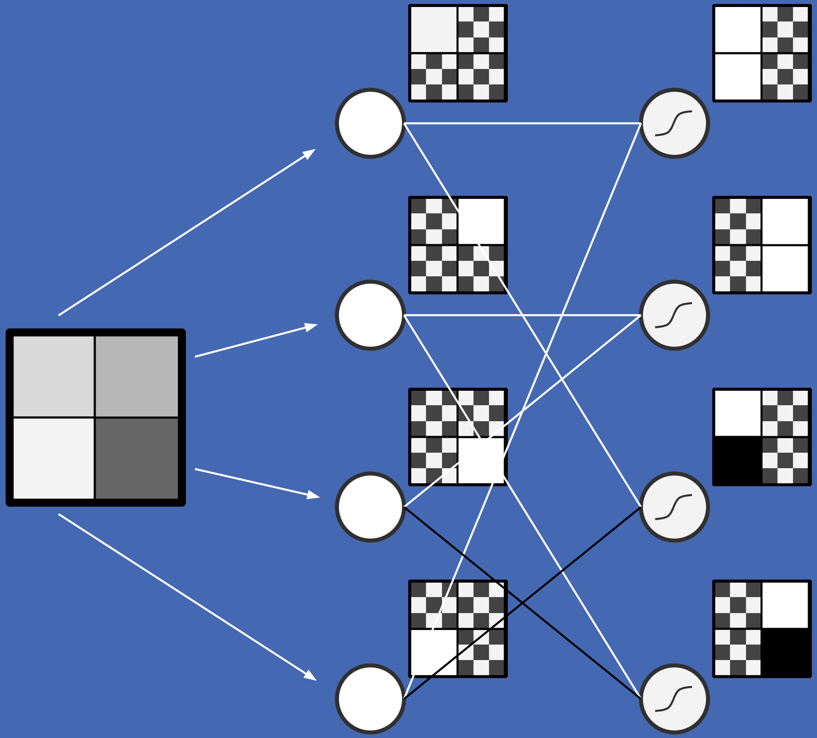


Make lots of neurons, identical except for weights

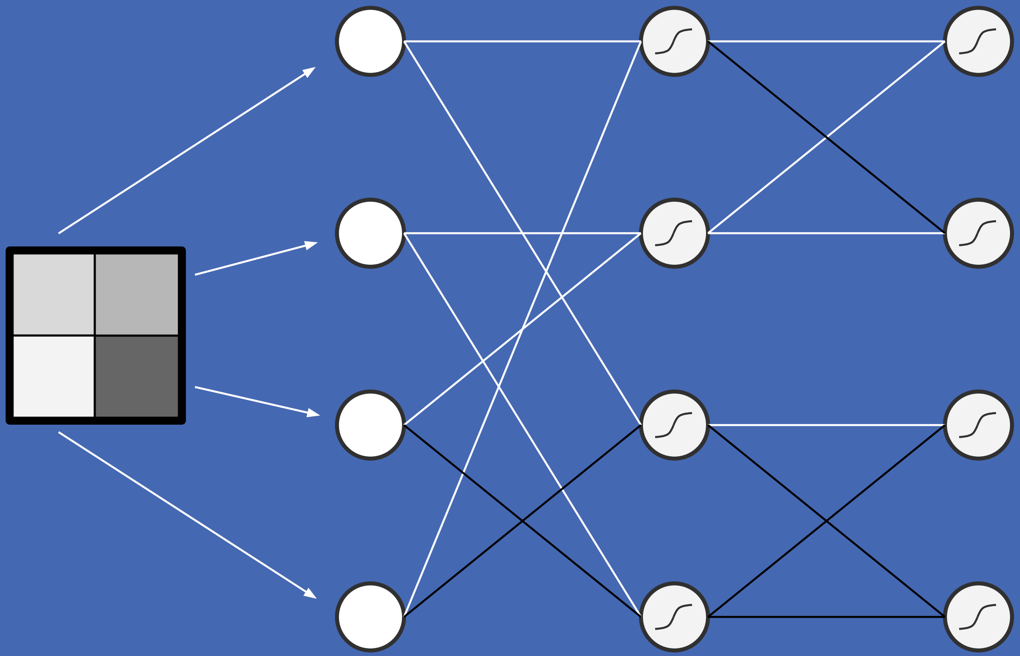


To keep our picture clear,
weights will either be
1.0 (white)
-1.0 (black) or
0.0 (missing)

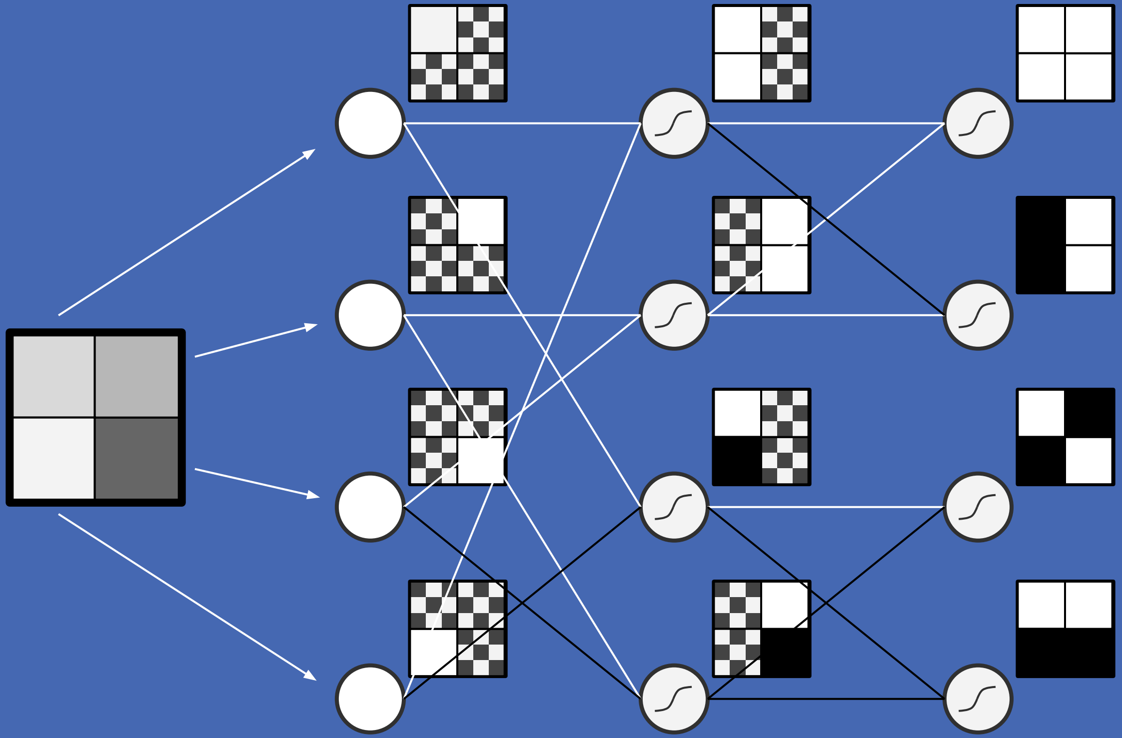
Receptive fields get more complex



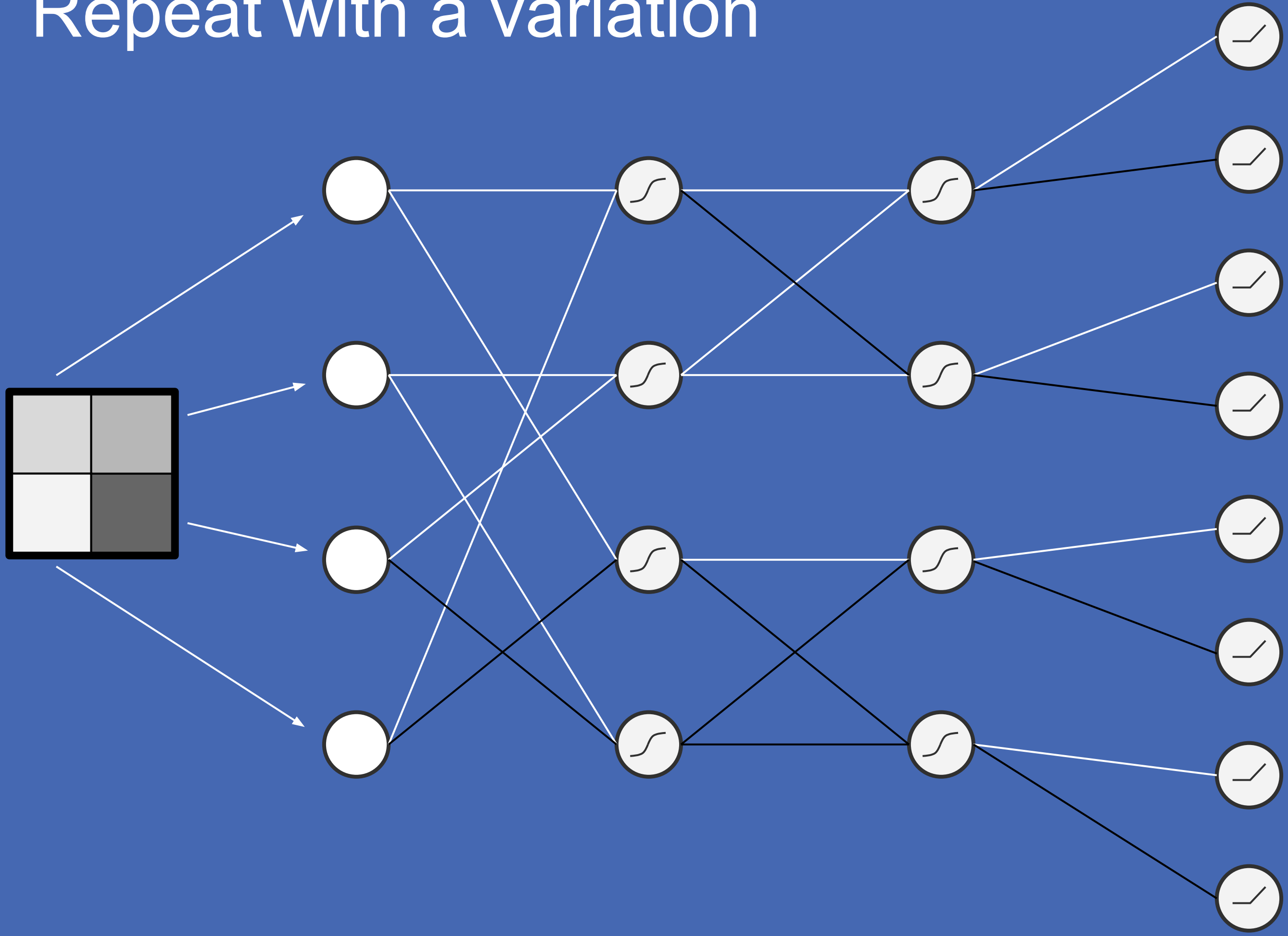
Repeat for additional layers



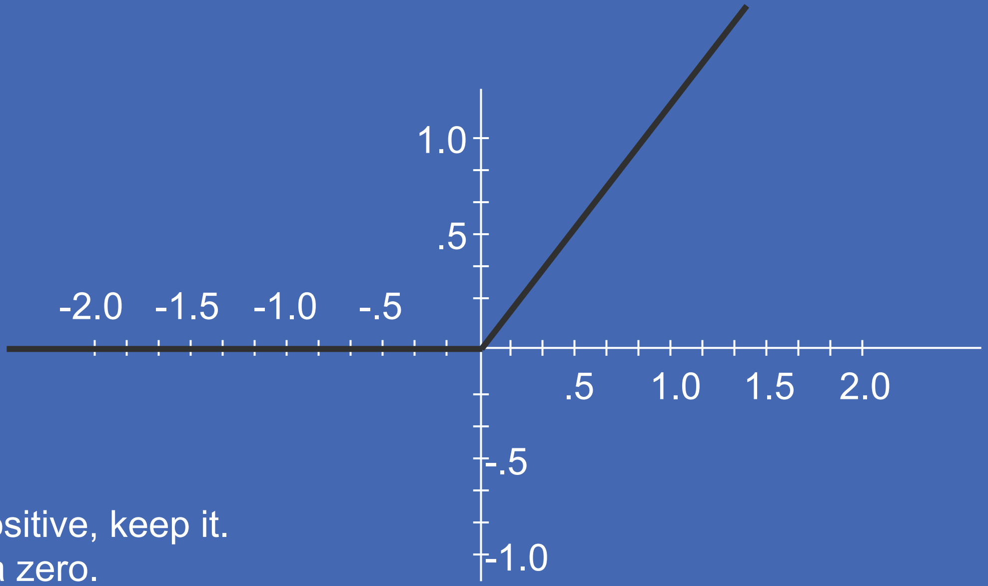
Receptive fields get still more complex



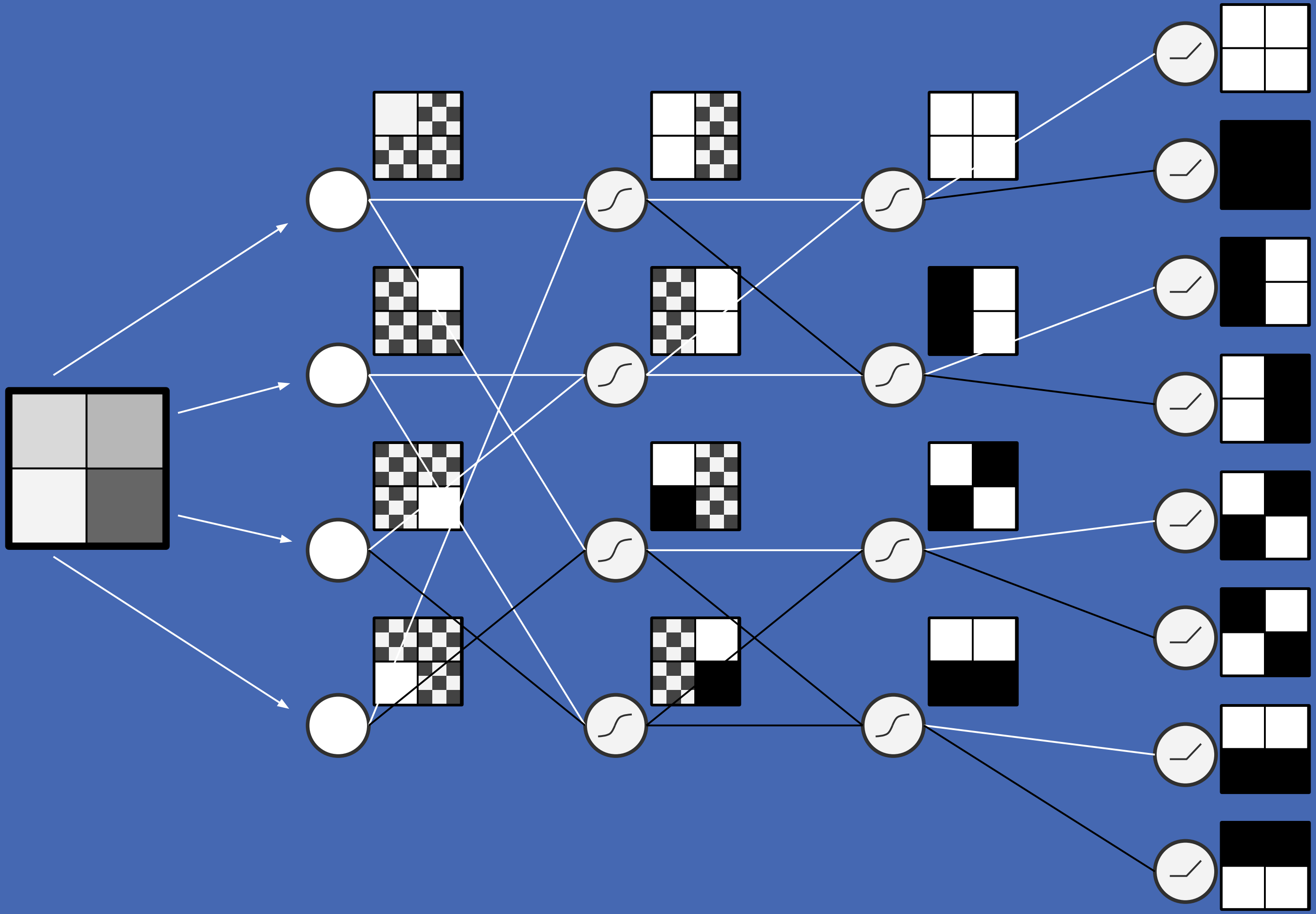
Repeat with a variation



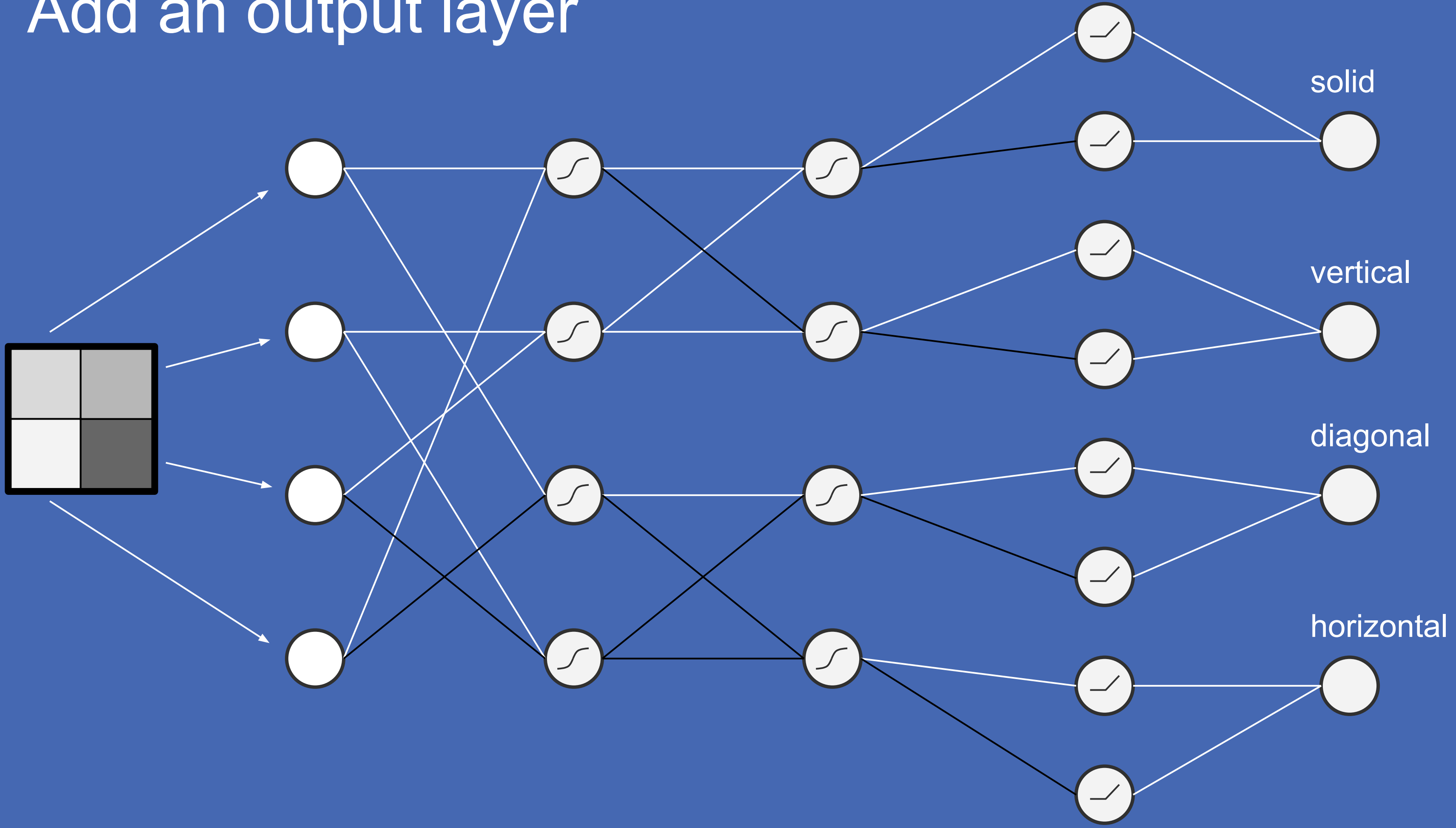
Rectified linear units (ReLUs)

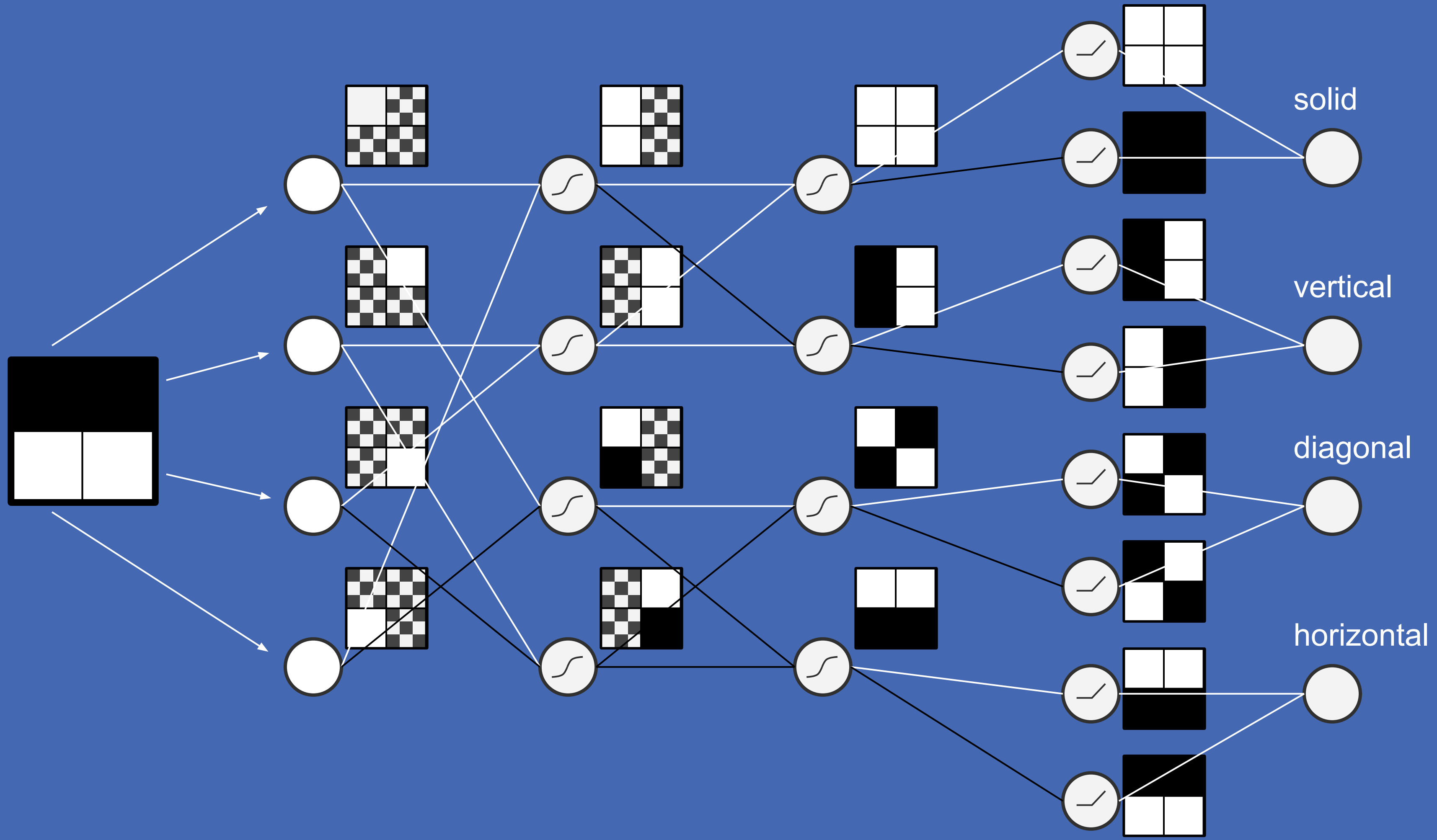


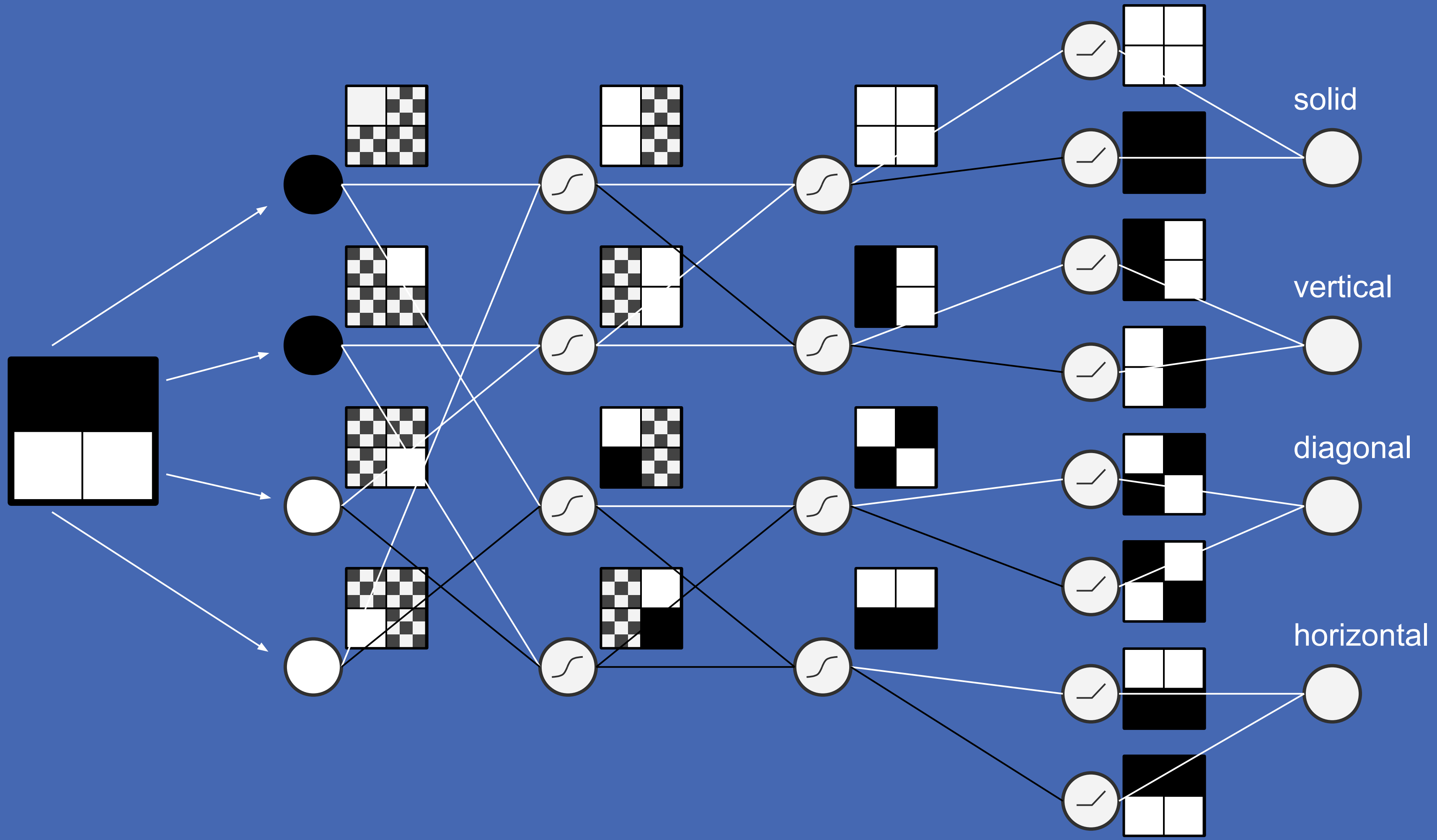
If your number is positive, keep it.
Otherwise you get a zero.

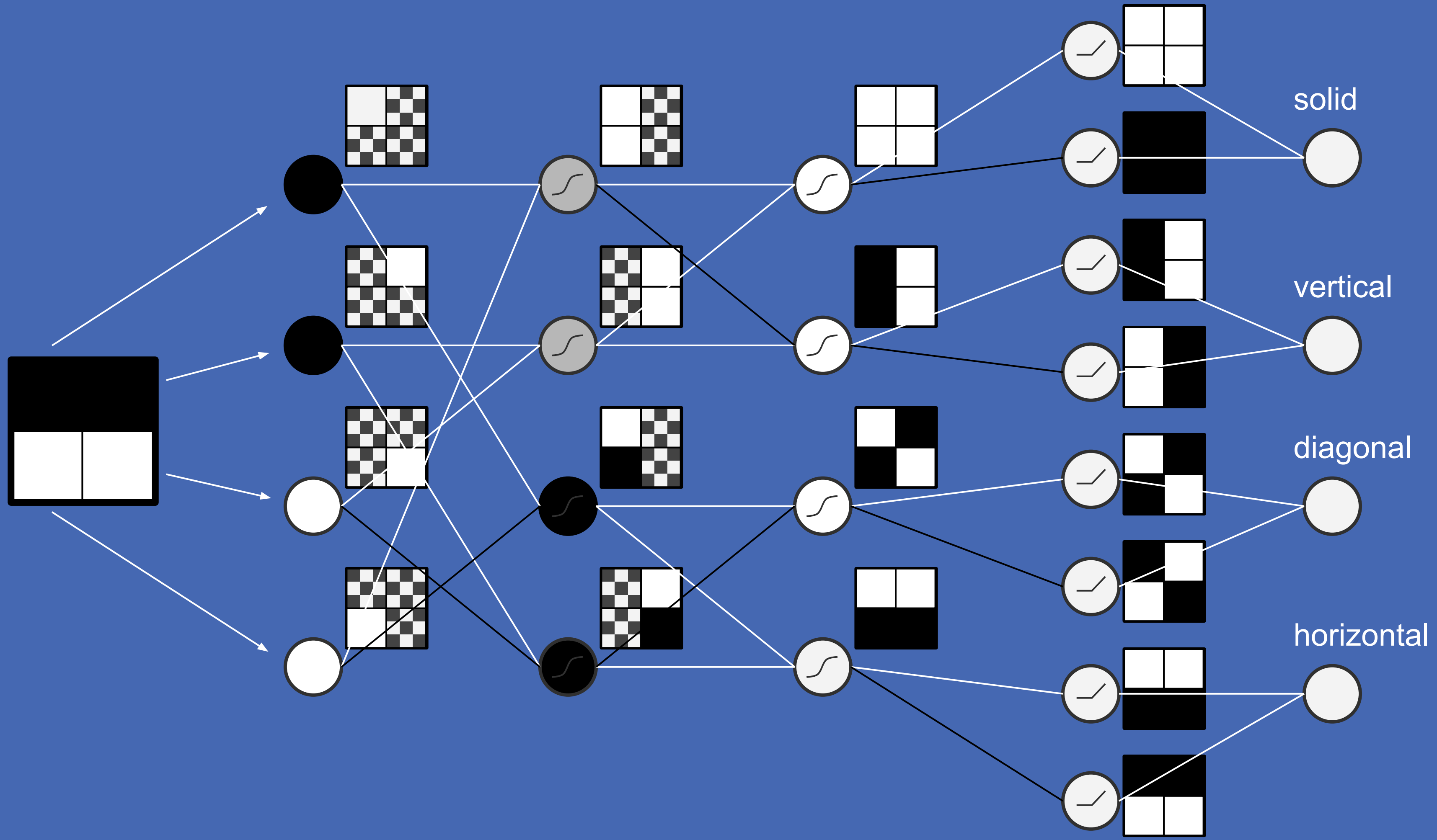


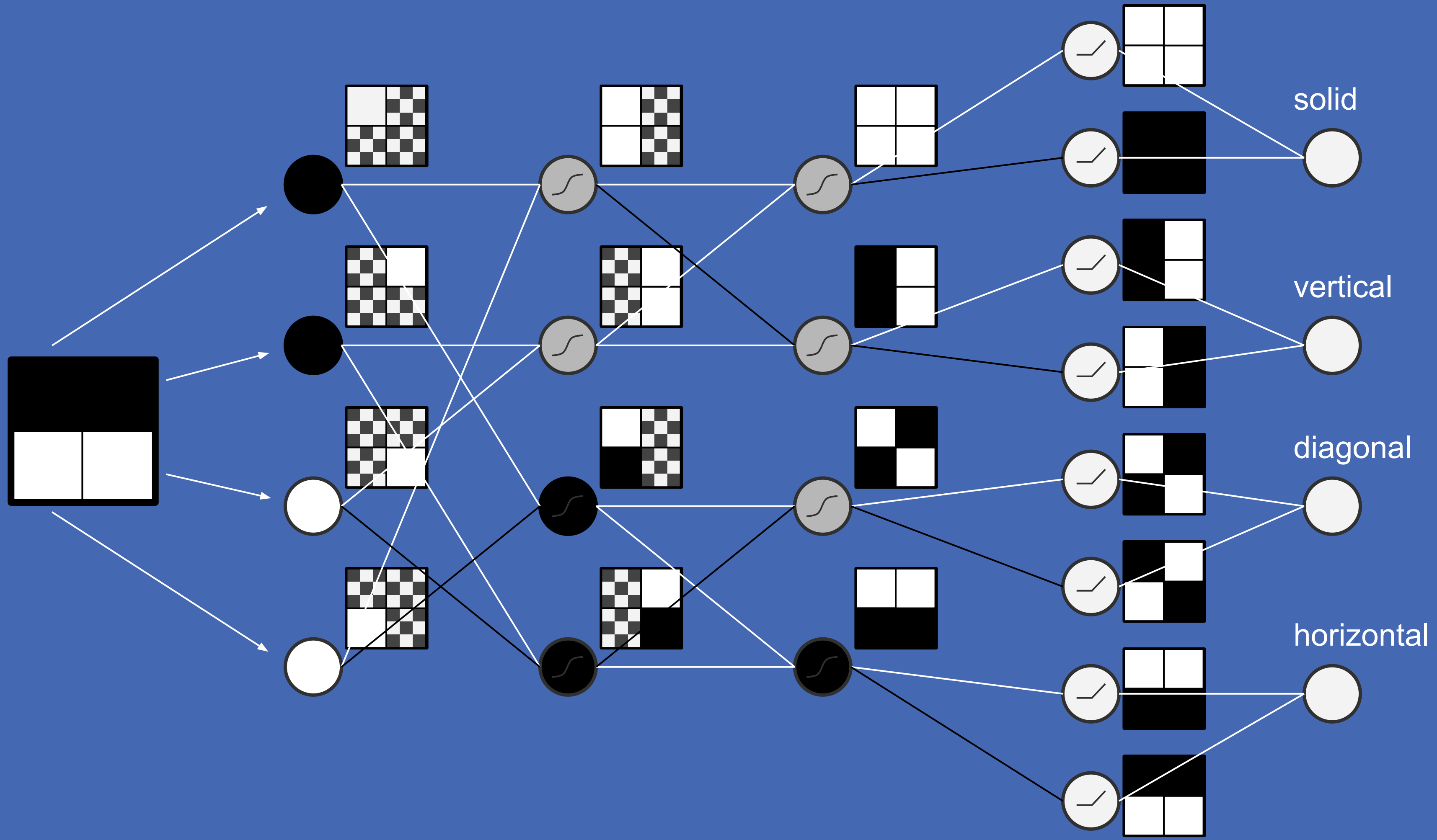
Add an output layer

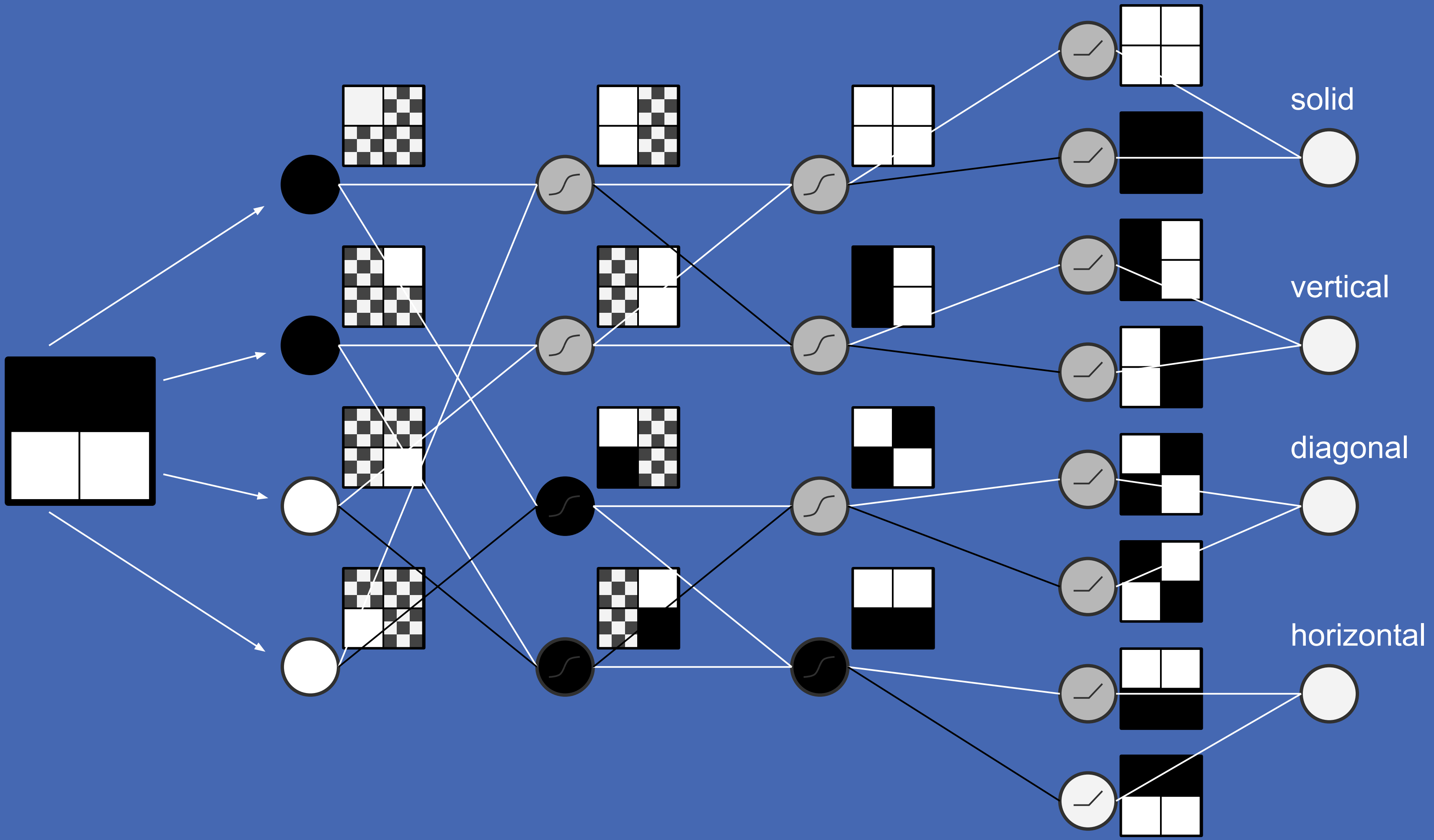


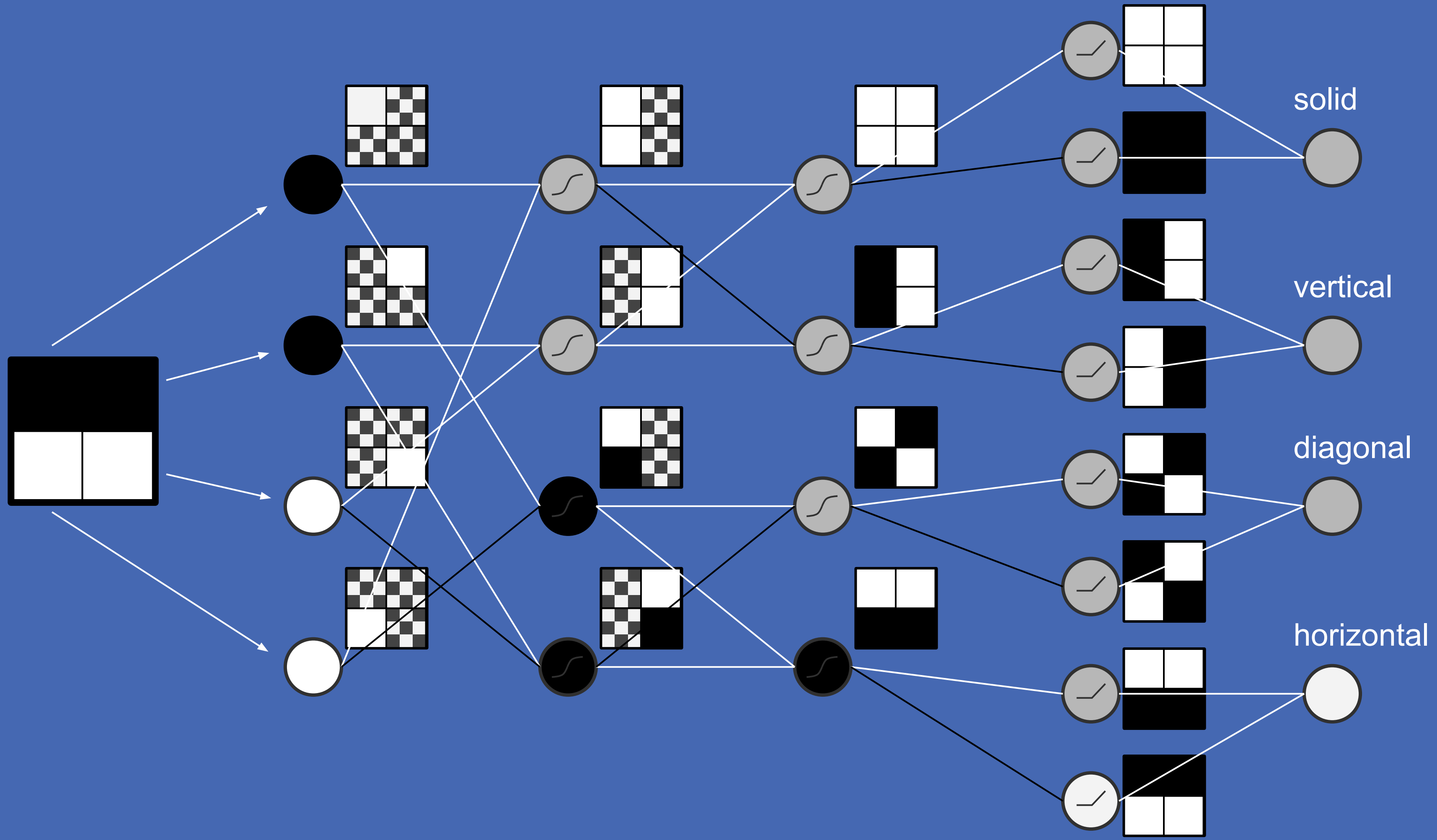

solid
vertical
diagonal
horizontal

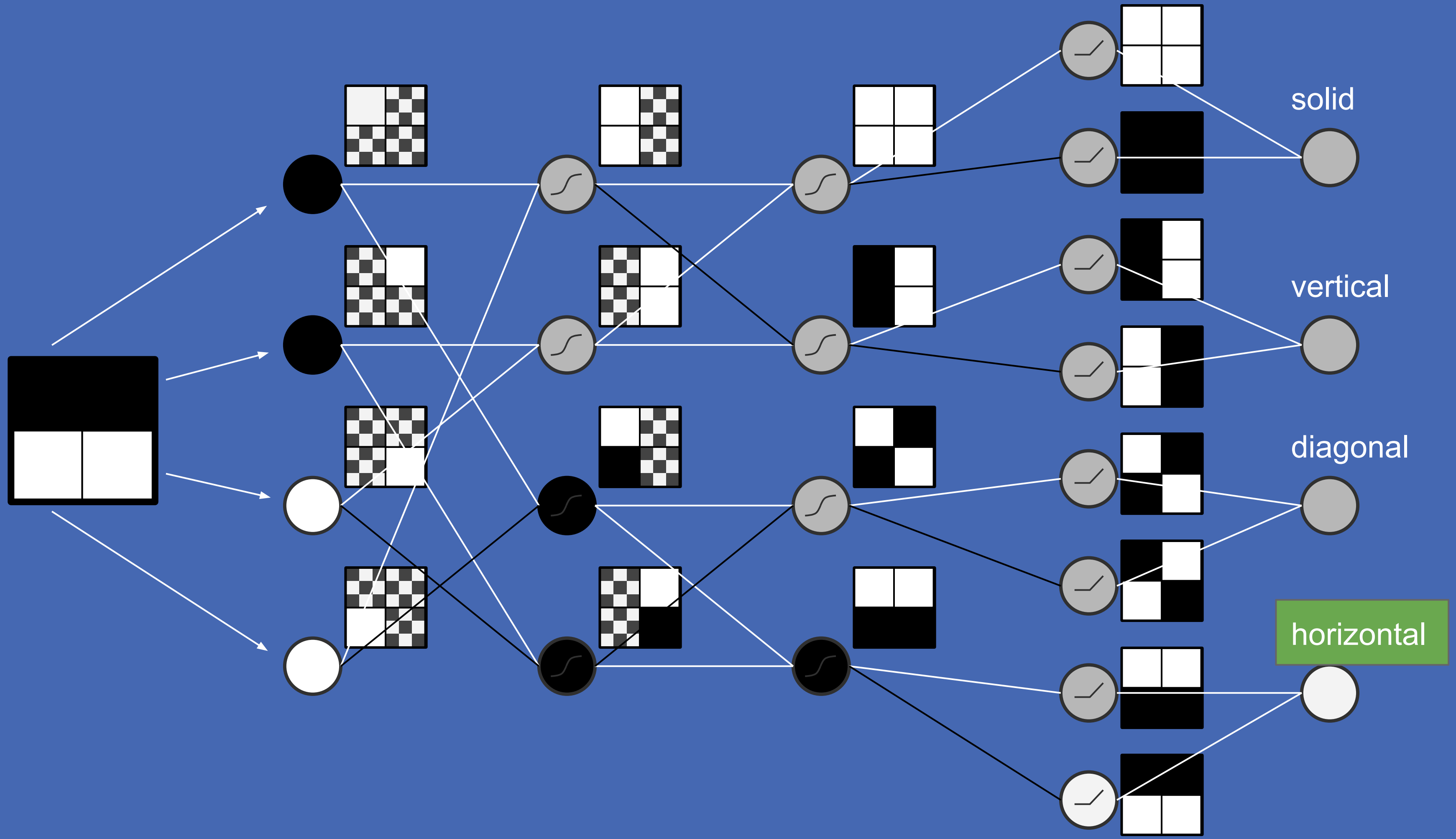

solid
vertical
diagonal
horizontal


solid
vertical
diagonal
horizontal

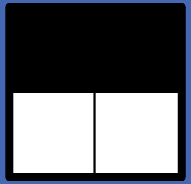

solid
vertical
diagonal
horizontal


solid
vertical
diagonal
horizontal


solid
vertical
diagonal
horizontal

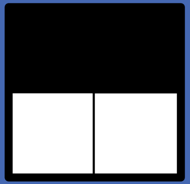

solid
vertical
diagonal
horizontal

Errors



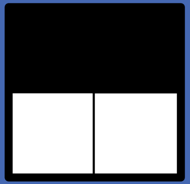
truth	
0.	solid
0.	vertical
0.	diagonal
1.	horizontal

Errors



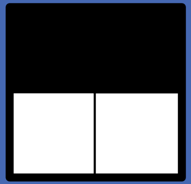
truth	answer	
0.	.5	solid
0.	.75	vertical
0.	-.25	diagonal
1.	-.75	horizontal

Errors



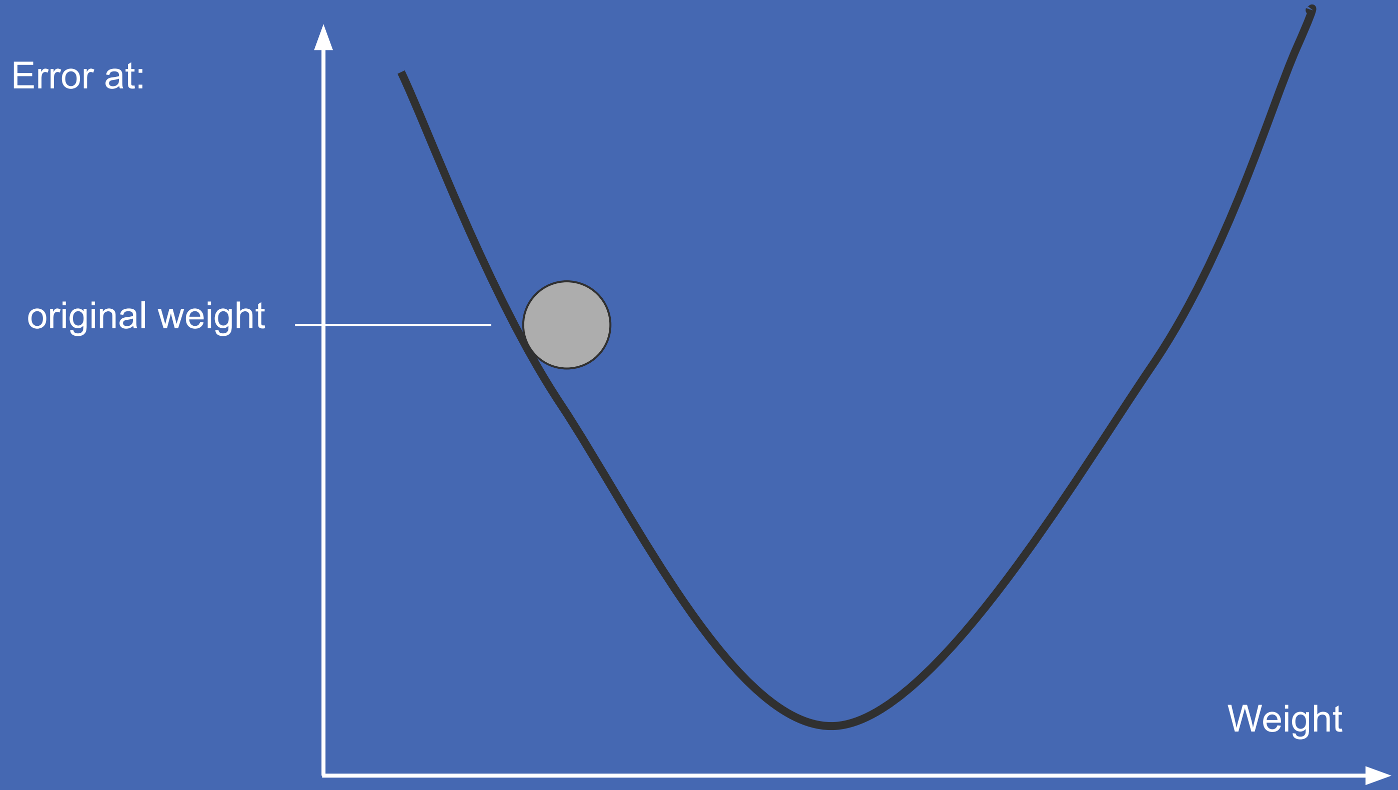
error	truth	answer	
.5	0.	.5	solid
.75	0.	.75	vertical
.25	0.	-.25	diagonal
1.75	1.	-.75	horizontal

Errors

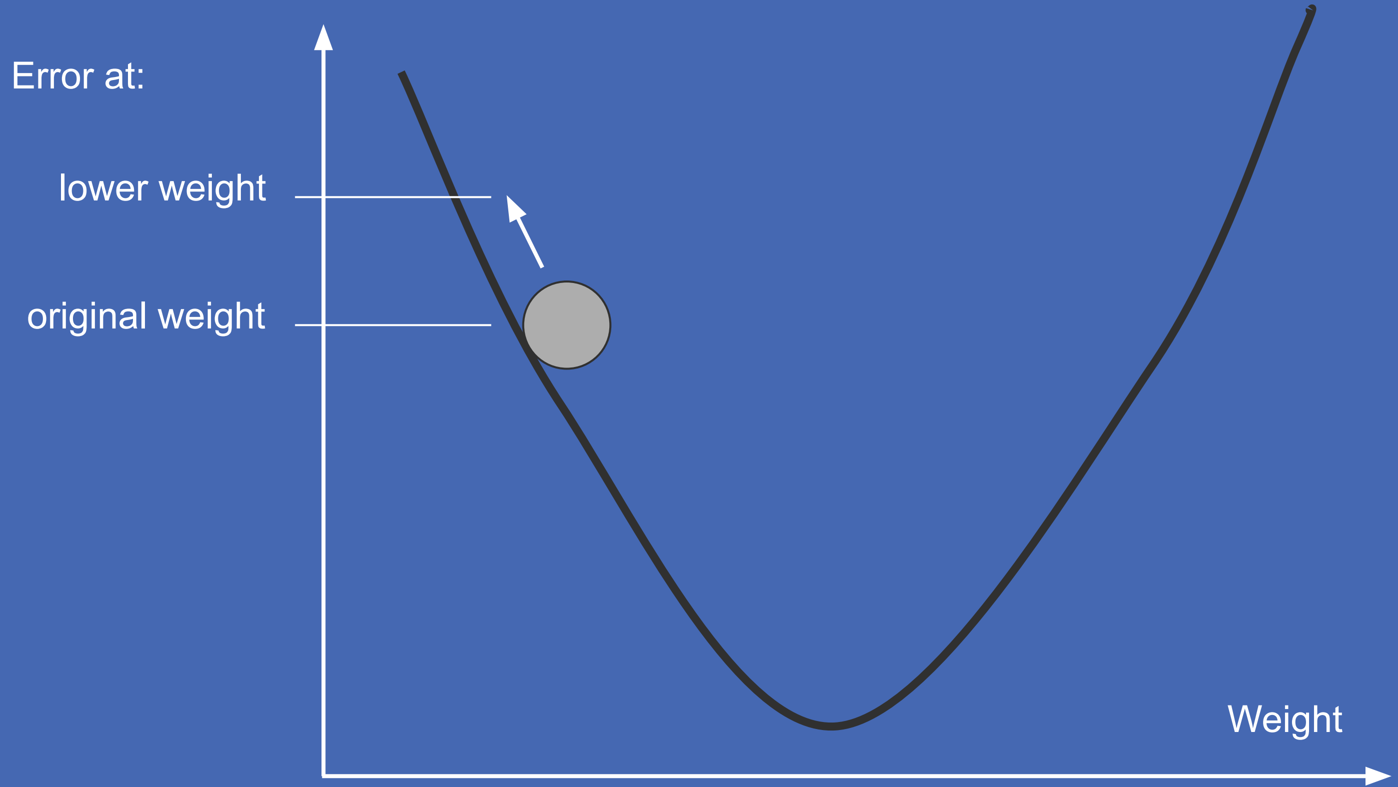


	error	truth	answer	
	.5	0.	.5	solid
	.75	0.	.75	vertical
	.25	0.	-.25	diagonal
	1.75	1.	-.75	horizontal
total	3.25			

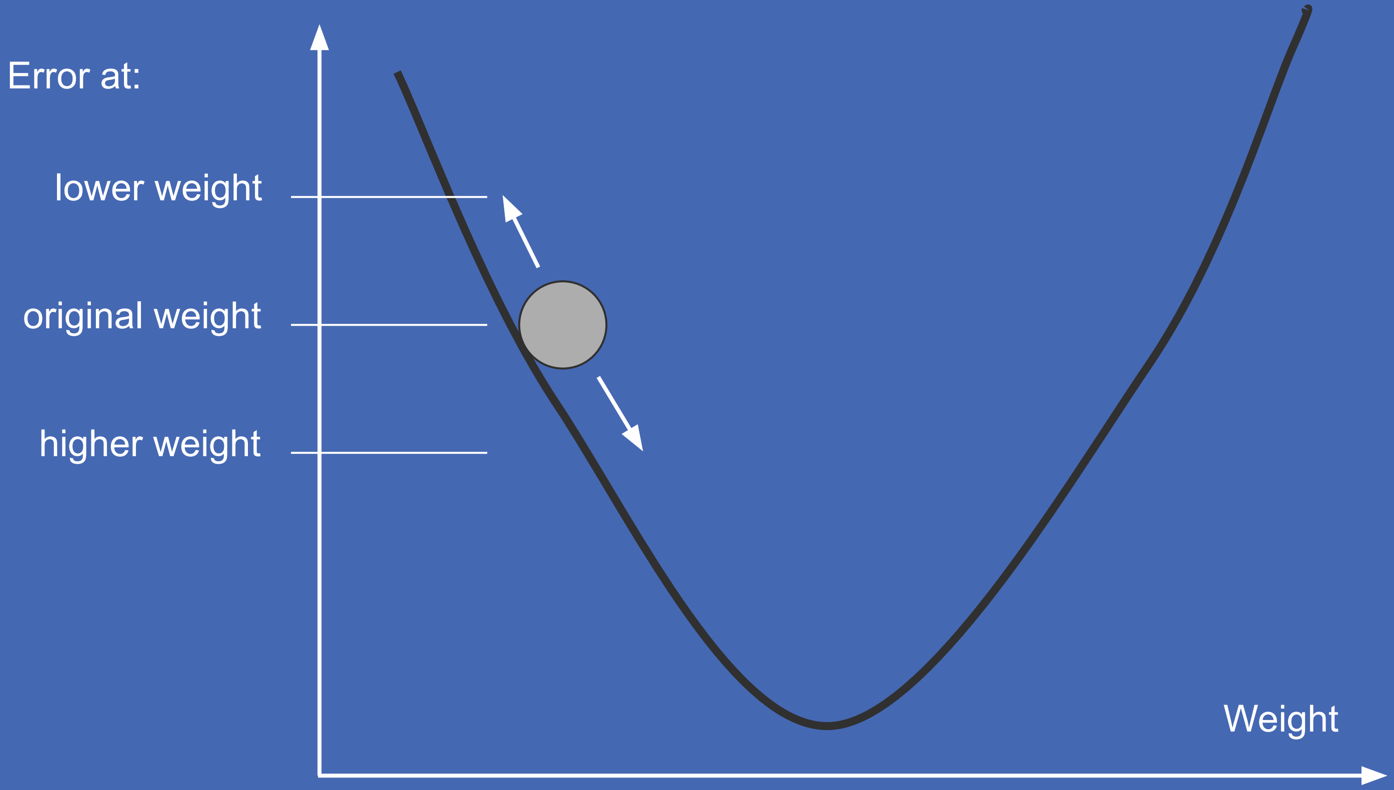
Learn all the weights: Gradient descent



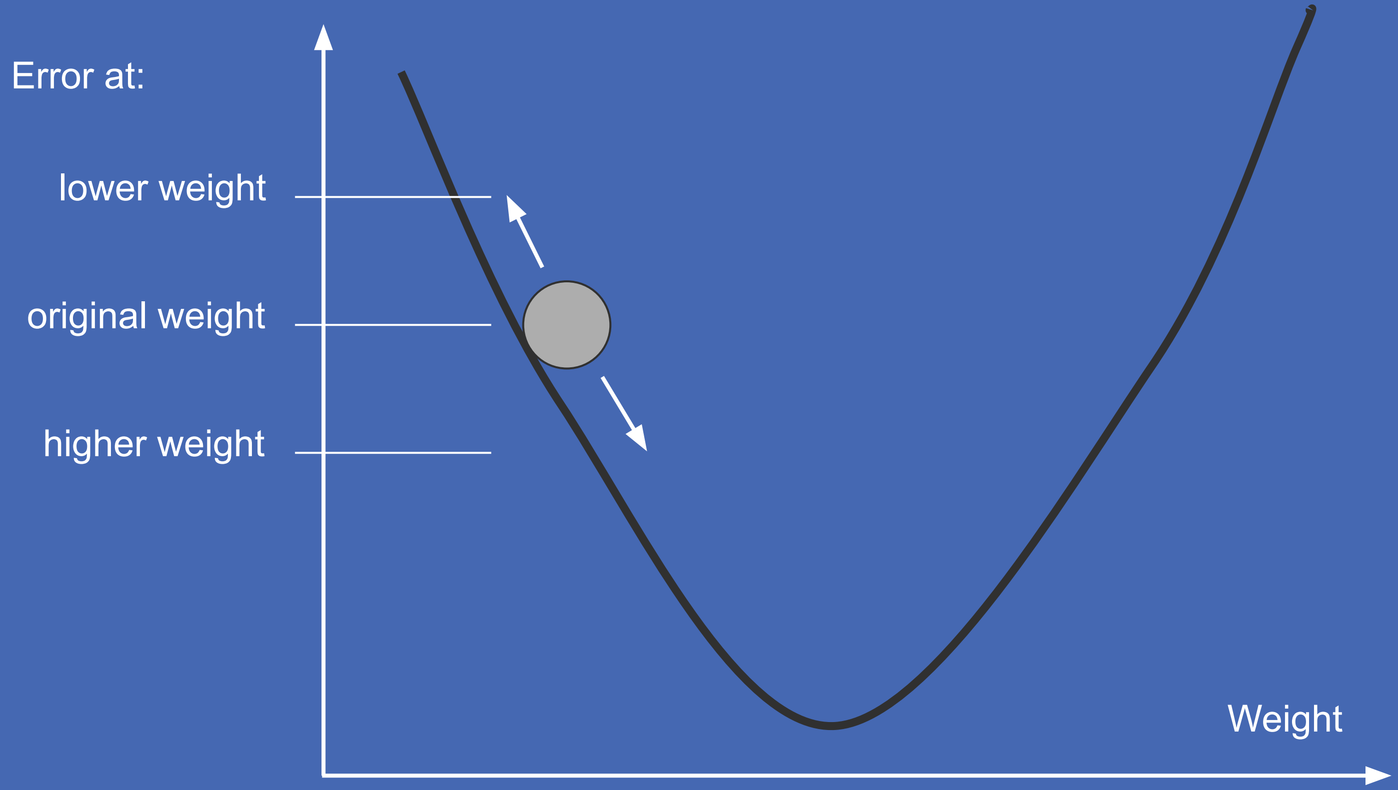
Learn all the weights: Gradient descent



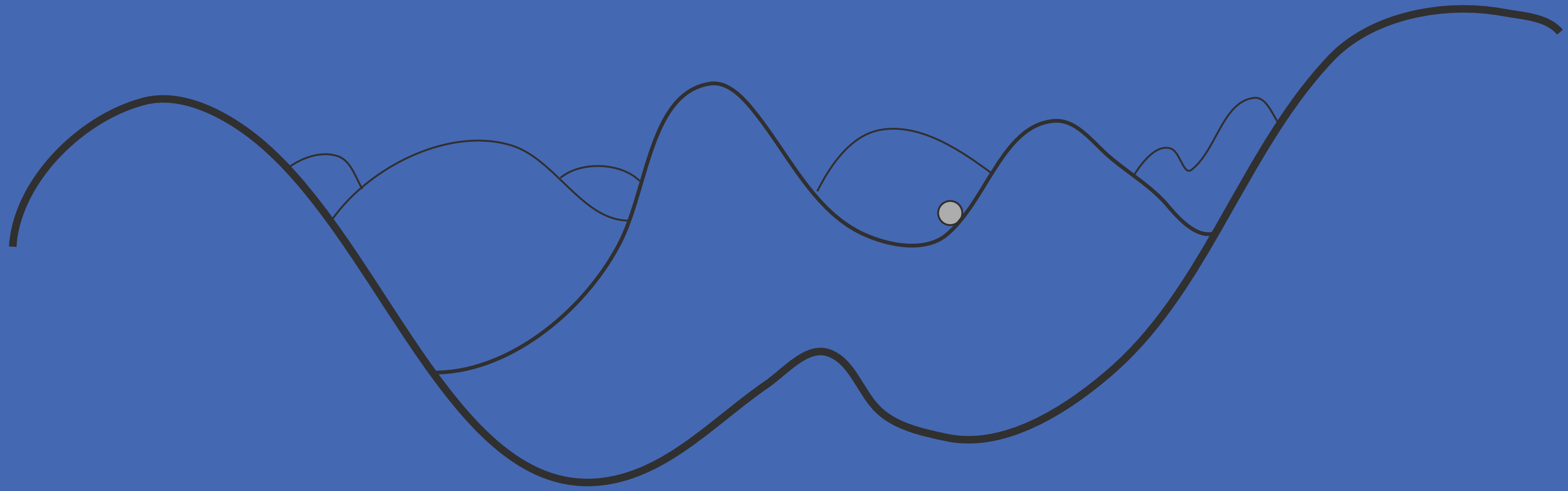
Learn all the weights: Gradient descent



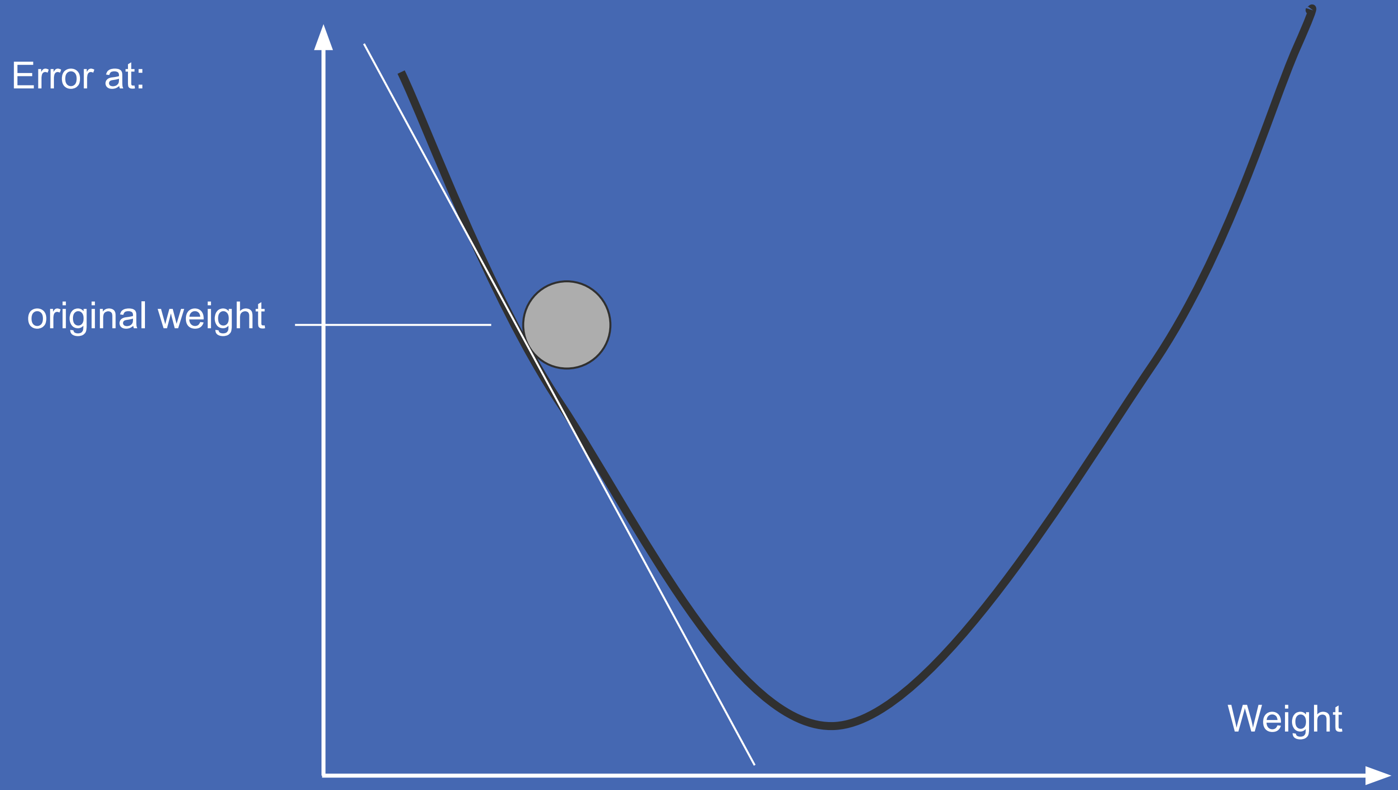
Numerically calculating the gradient is expensive



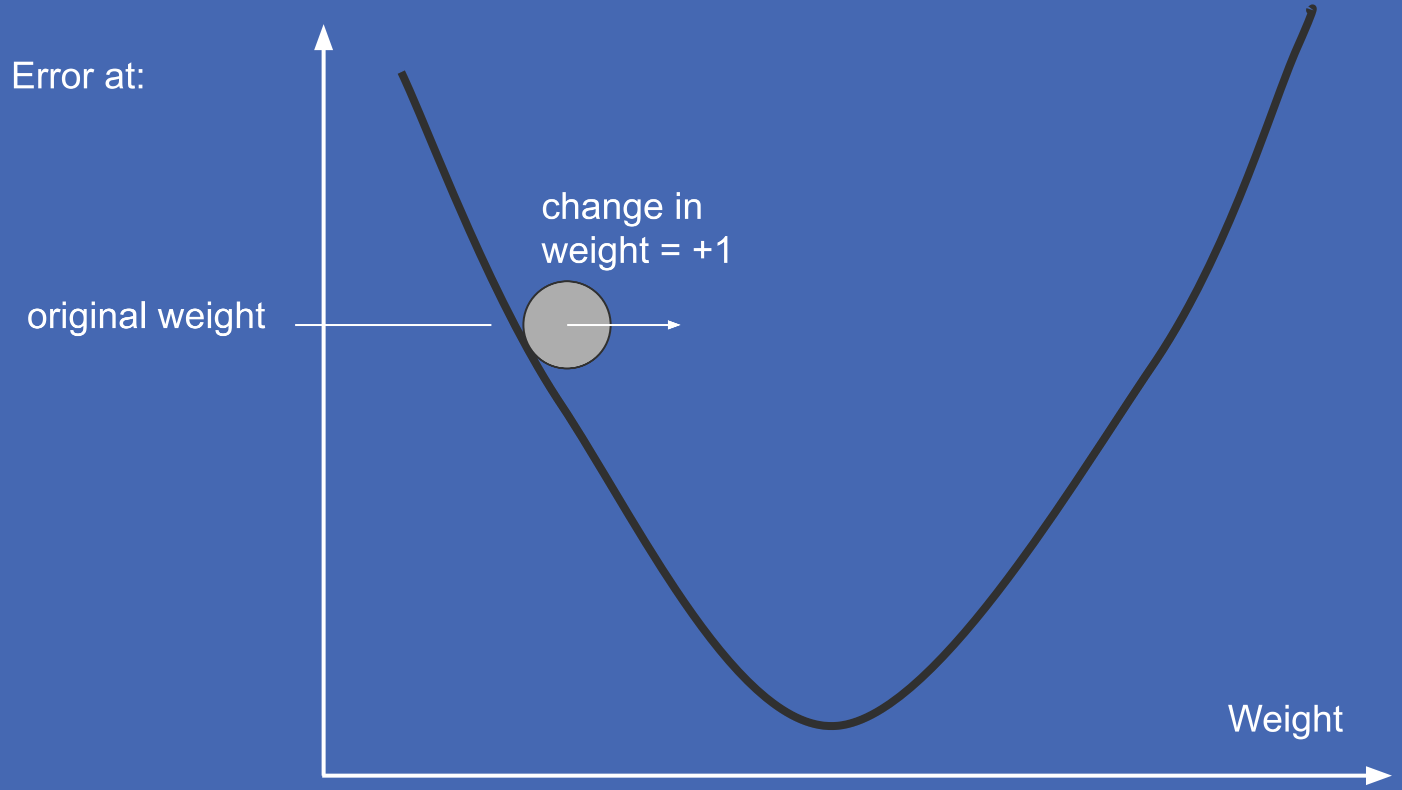
Numerically calculating the gradient is very expensive



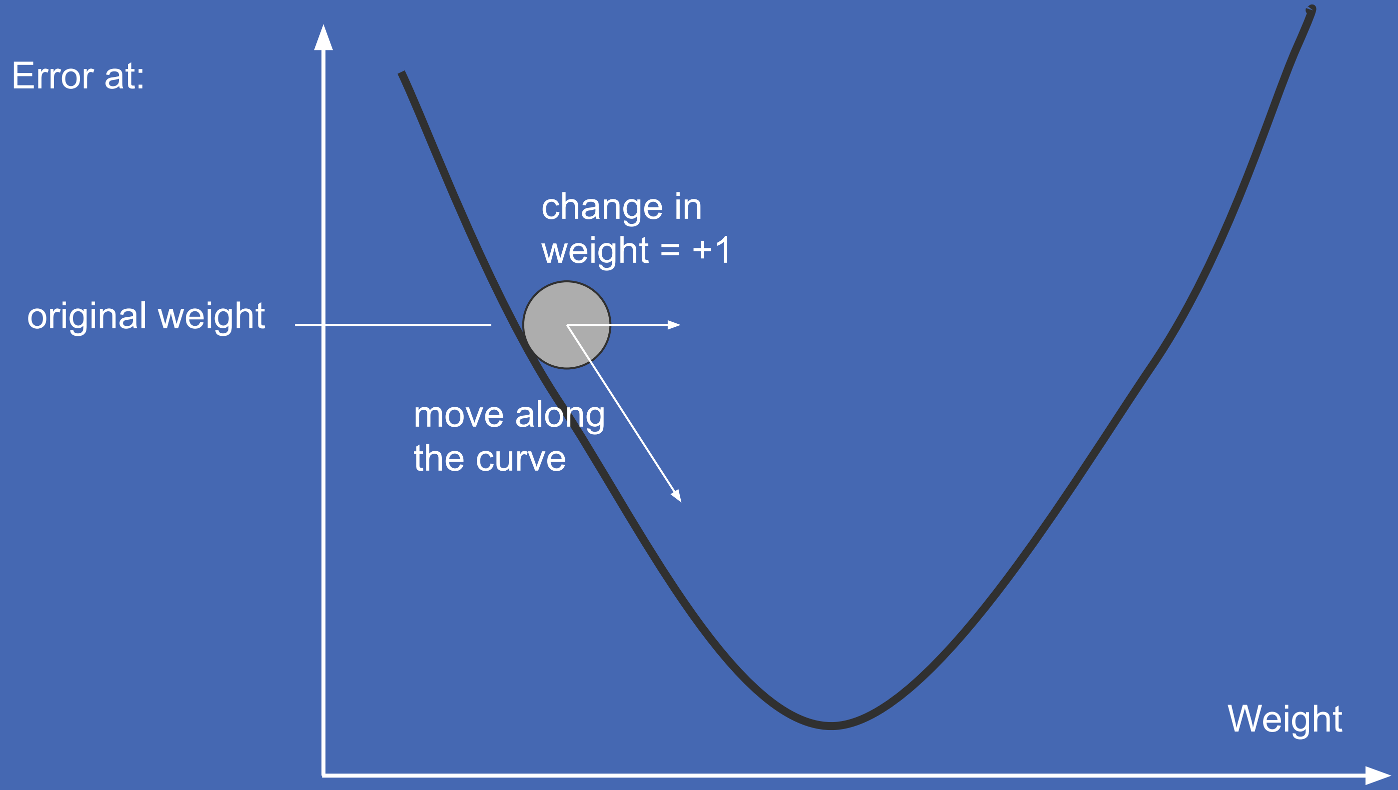
Calculate the gradient (slope) directly



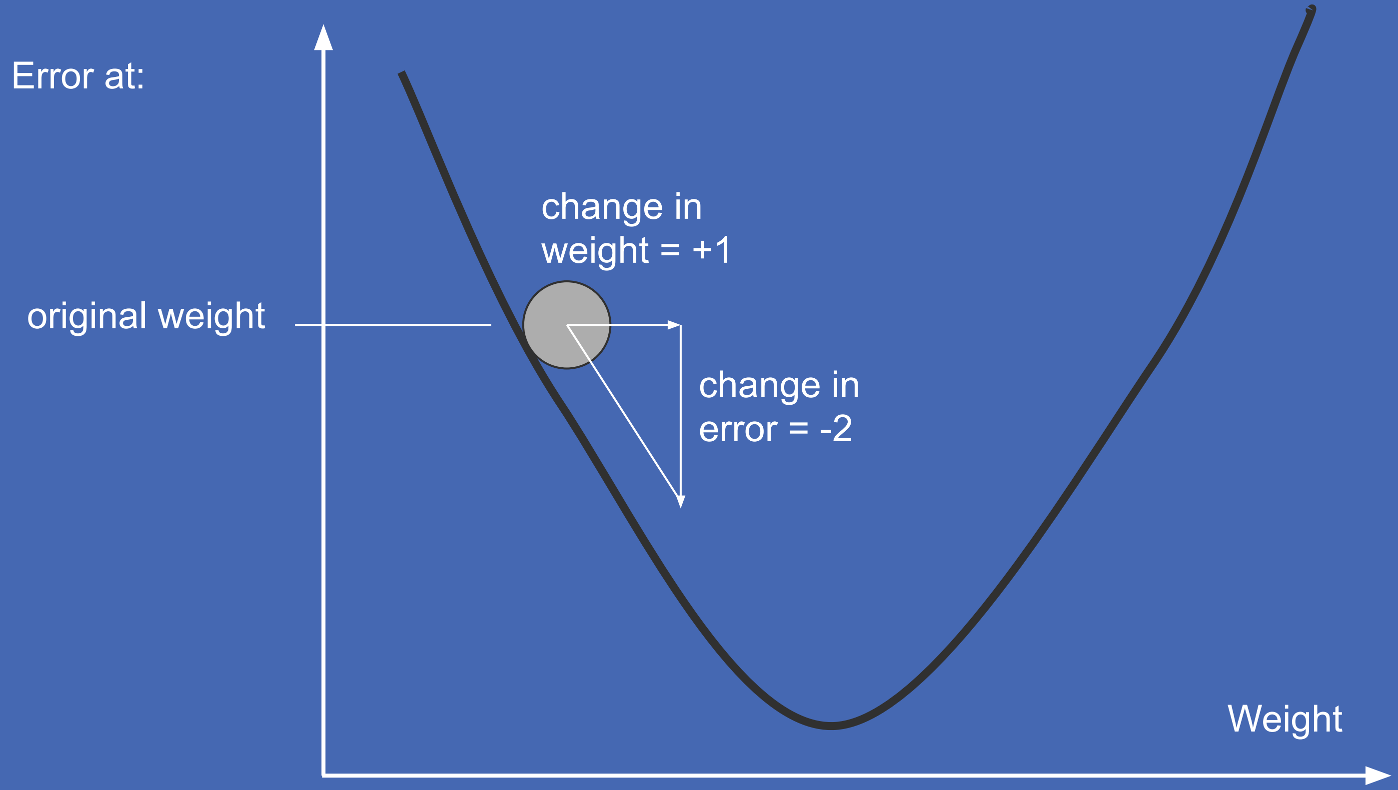
Slope



Slope

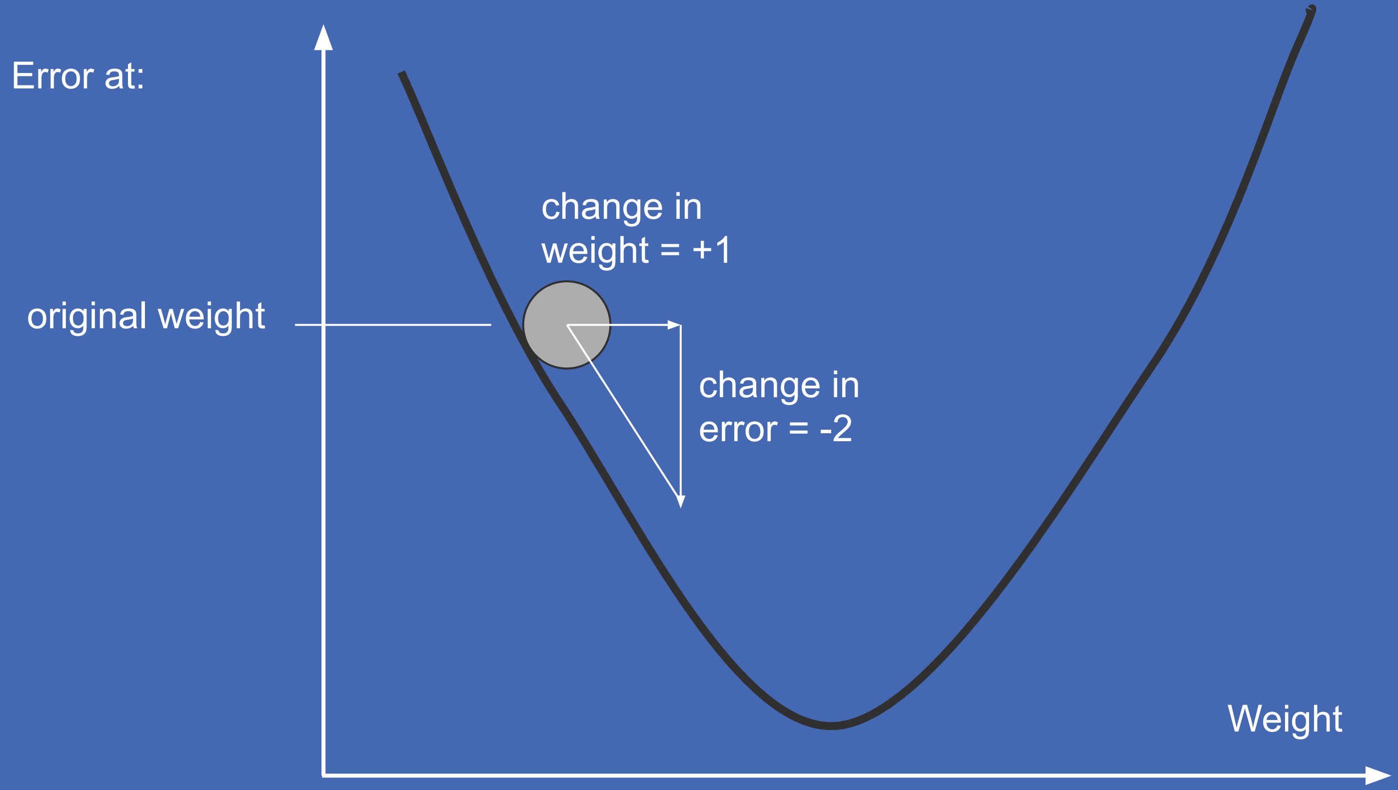

Error at:
change in
weight = +1
original weight
move along
the curve
Weight

Slope


Error at:
change in
weight = +1
original weight
change in
error = -2
Weight

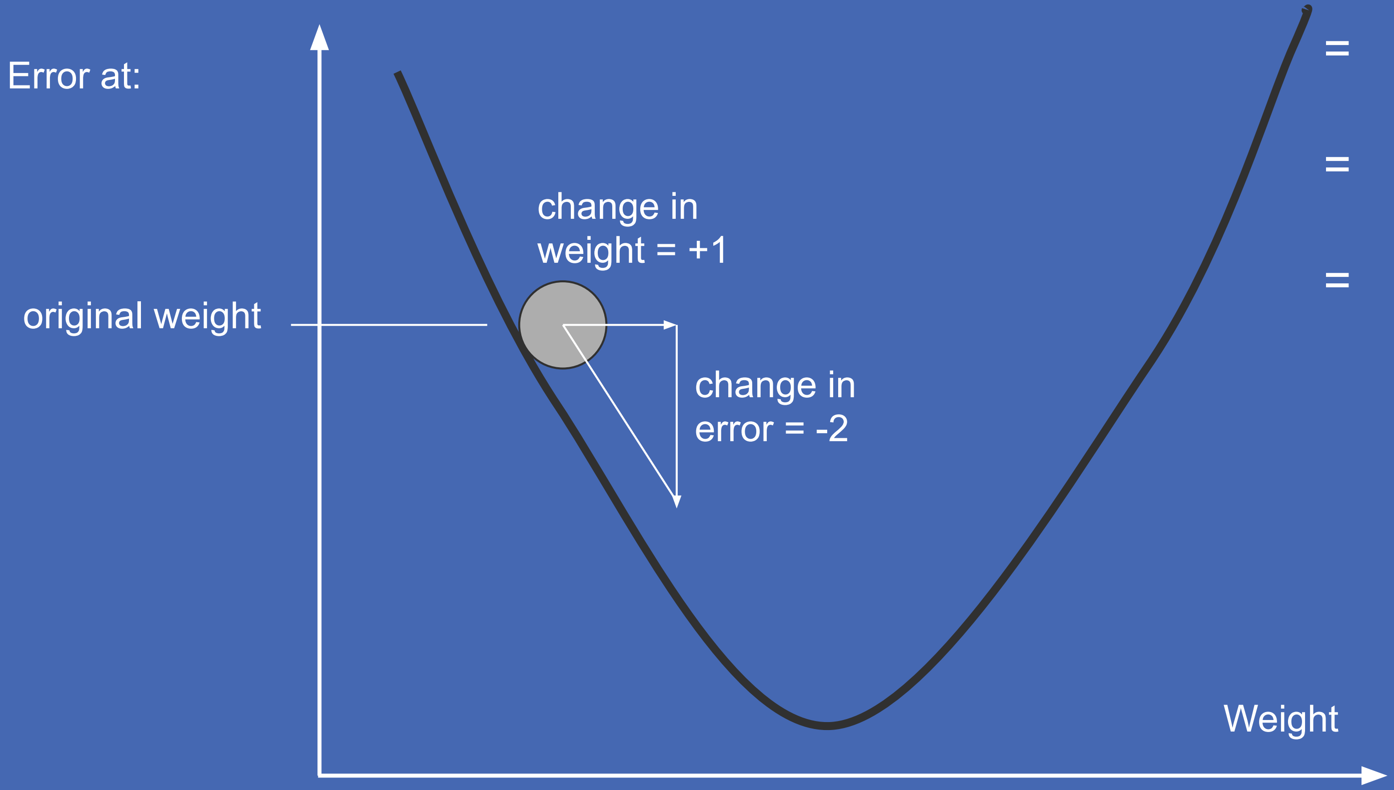
Slope

$$\text{slope} = \frac{\text{change in error}}{\text{change in weight}}$$

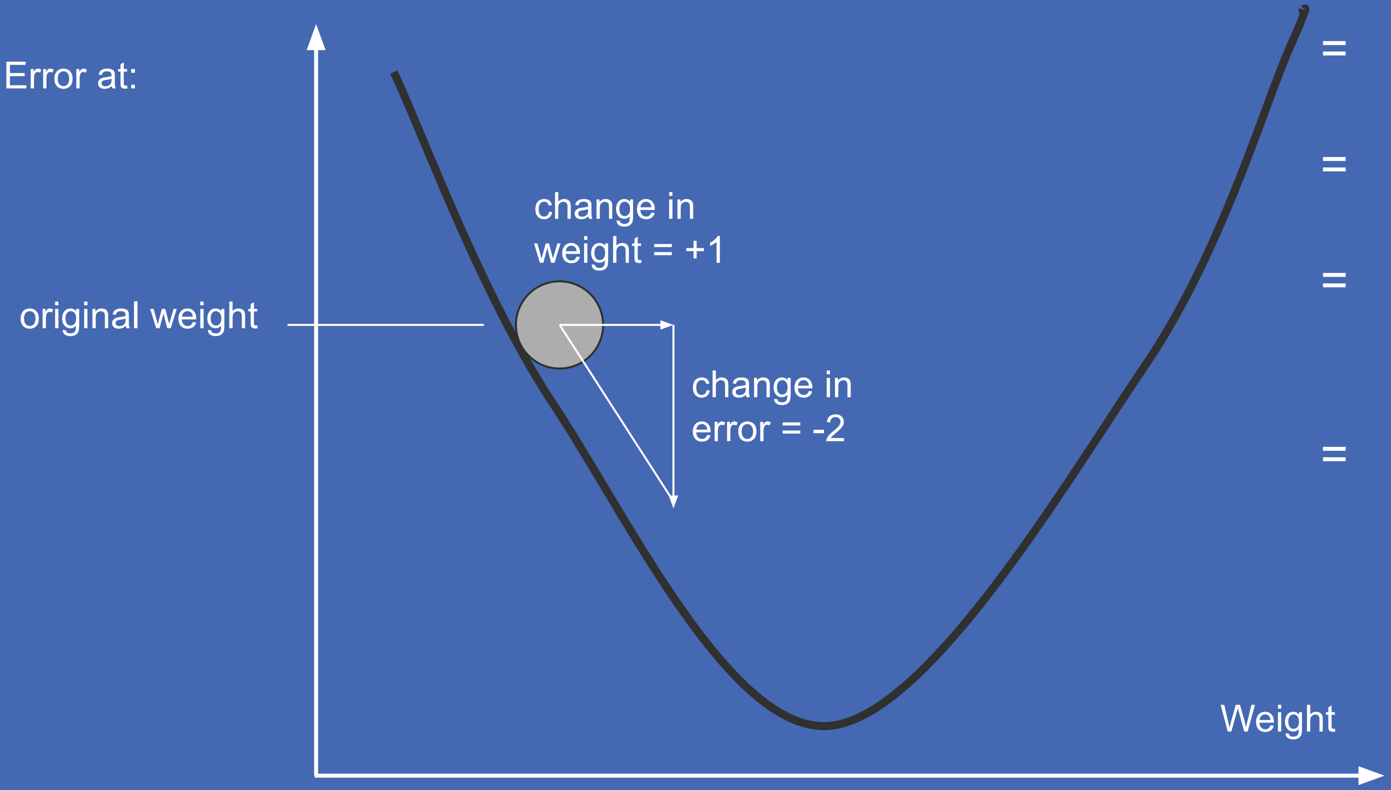


Slope

$$\text{slope} = \frac{\text{change in error}}{\text{change in weight}} = \frac{\Delta\ \text{error}}{\Delta\ \text{weight}} = \frac{d(\text{error})}{d(\text{weight})} = \frac{\partial e}{\partial w}$$



Slope



$$\text{slope} = \frac{\text{change in error}}{\text{change in weight}}$$

$$= \frac{\Delta\ \text{error}}{\Delta\ \text{weight}}$$

$$= \frac{d(\text{error})}{d(\text{weight})}$$

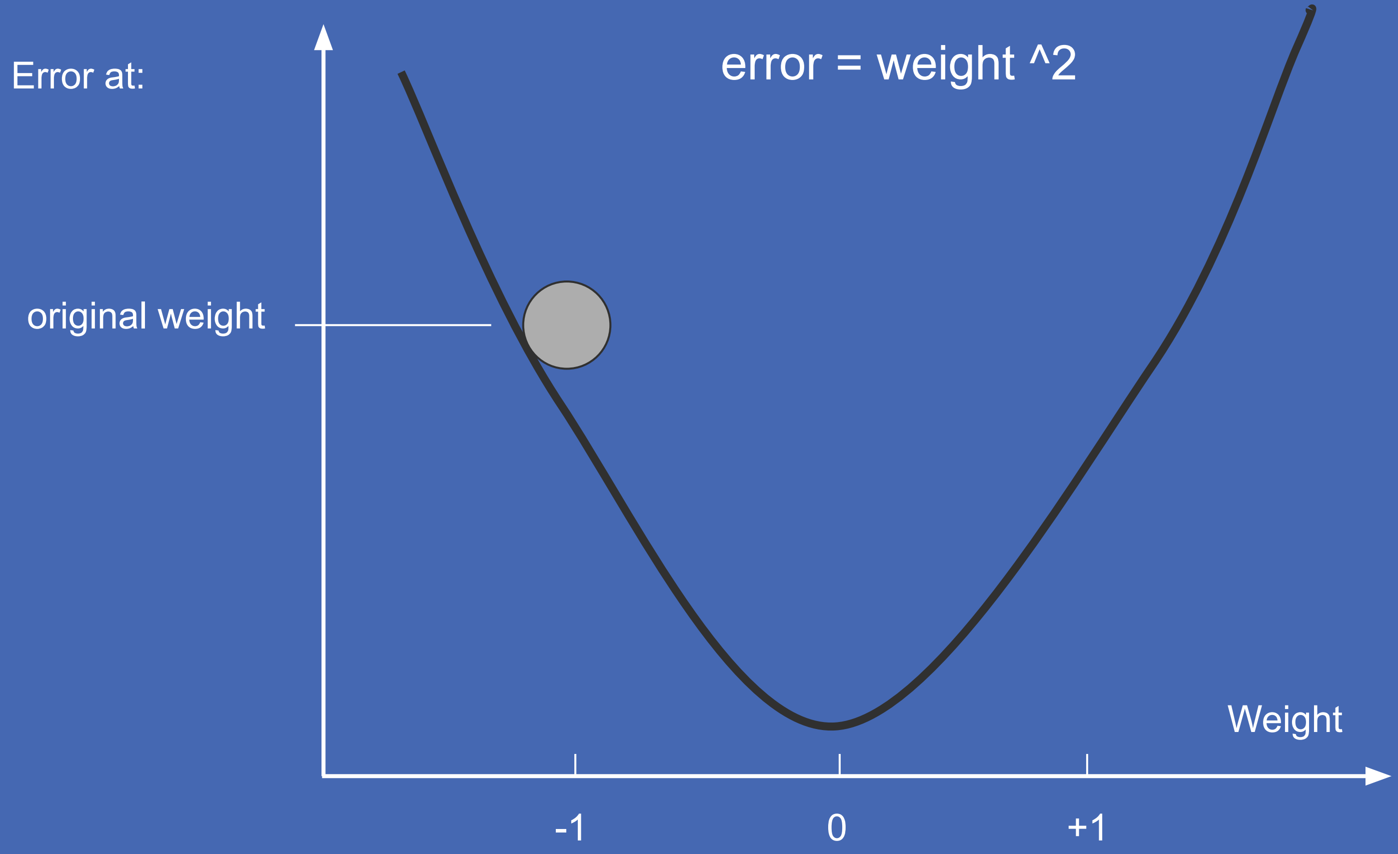
$$= \frac{\partial e}{\partial w}$$

$$= \frac{-2}{+1} = -2$$

Slope

You have to know your error function.
For example:

error = weight ^2

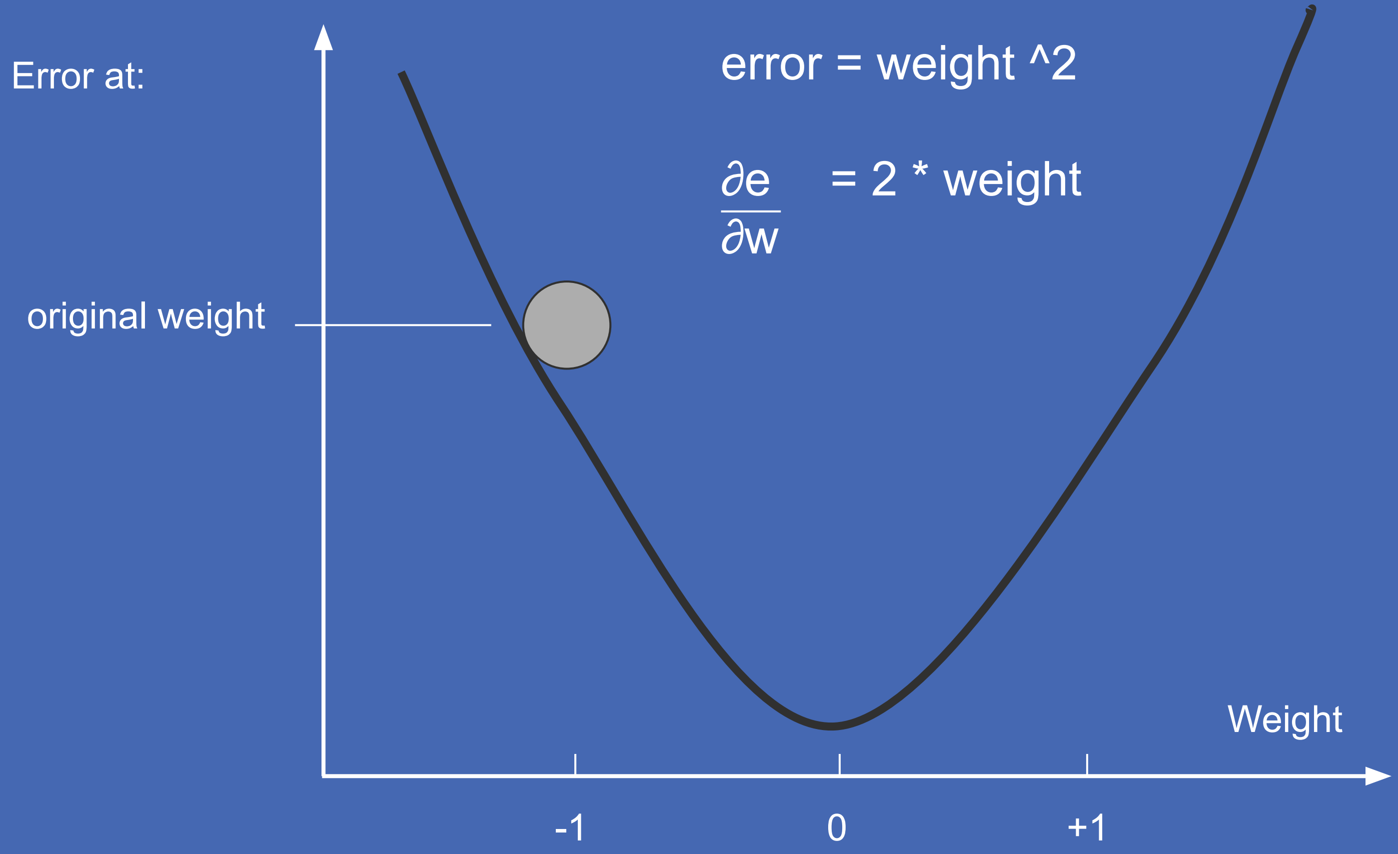


Slope

You have to know your error function.
For example:

error = weight ^2

$$\frac{\partial e}{\partial w} = 2 * \text{weight}$$



Slope

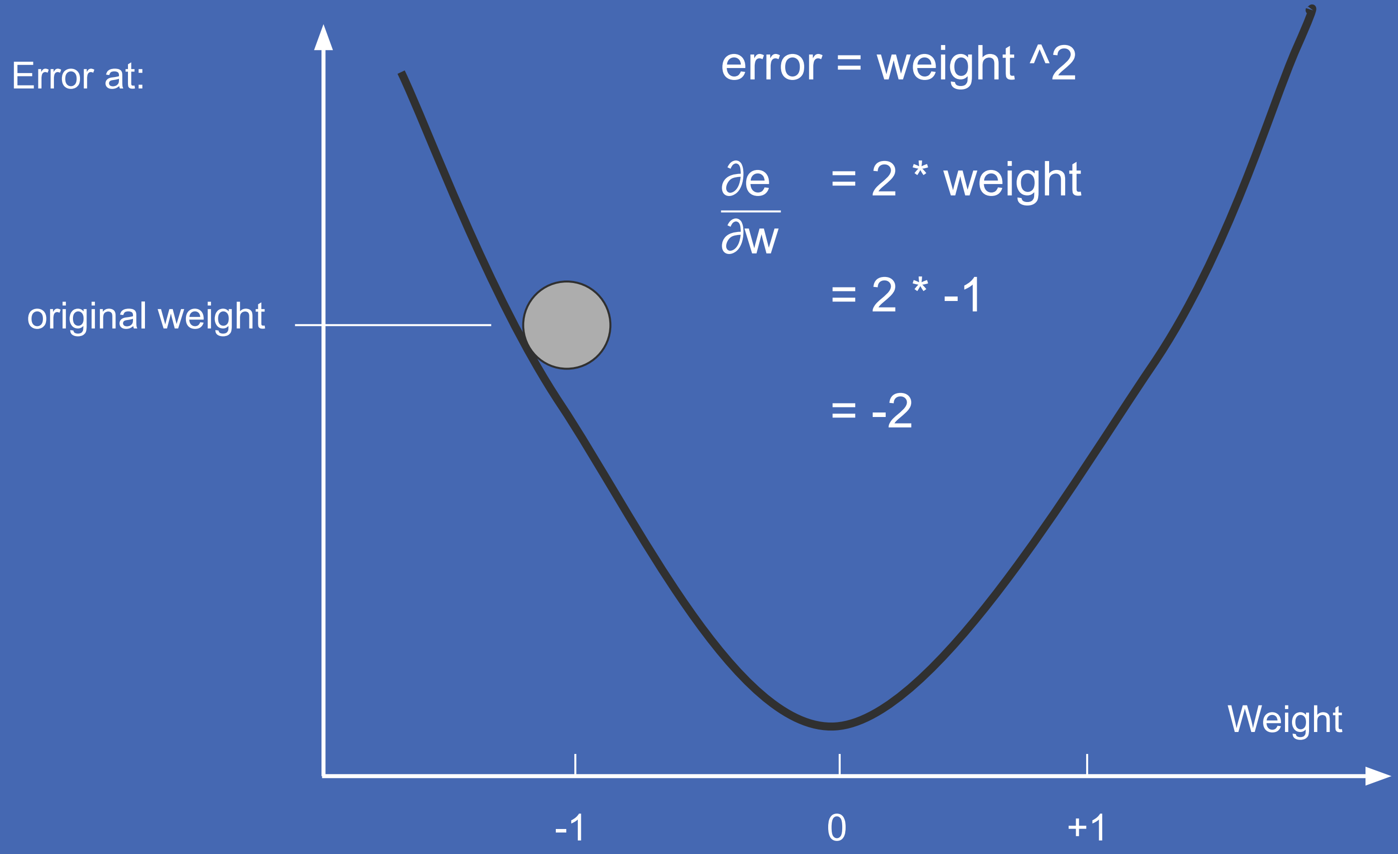
You have to know your error function.
For example:

$$error = weight\ \hat{}2$$

$$\frac{\partial e}{\partial w} = 2 * weight$$

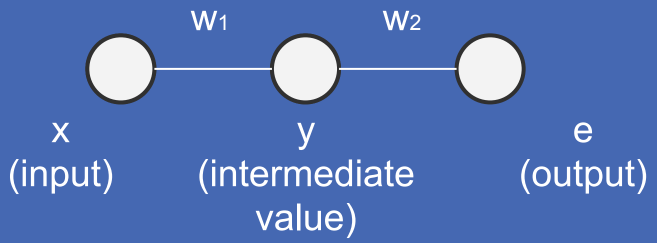
$$= 2 * -1$$

$$= -2$$



Chaining

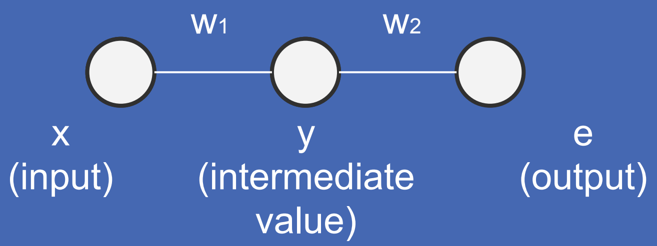
$y = x * w_1$



Chaining

$$y = x * w_1$$

$$\frac{\partial y}{\partial w_1} = x$$



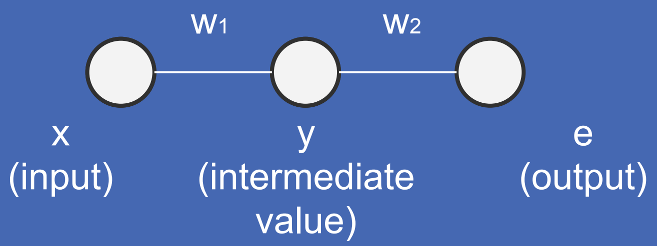
Chaining

$$y = x * w_1$$

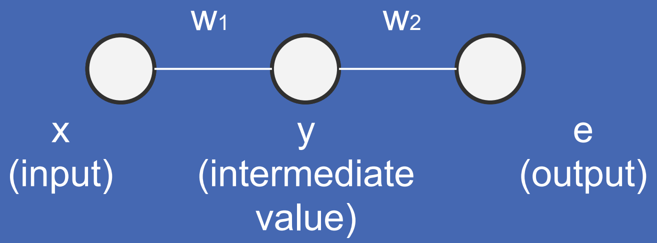
$$\frac{\partial y}{\partial w_1} = x$$

$$e = y * w_2$$

$$\frac{\partial e}{\partial y} = w_2$$



Chaining



$$y = x * w_1$$

$$\frac{\partial y}{\partial w_1} = x$$

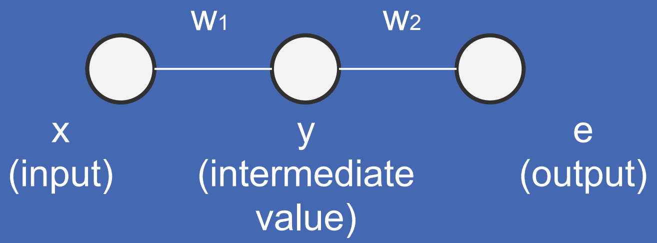
$$e = y * w_2$$

$$\frac{\partial e}{\partial y} = w_2$$

$$e = x * w_1 * w_2$$

$$\frac{\partial e}{\partial w_1} = x * w_2$$

Chaining



$$y = x * w_1$$

$$\frac{\partial y}{\partial w_1} = x$$

$$e = y * w_2$$

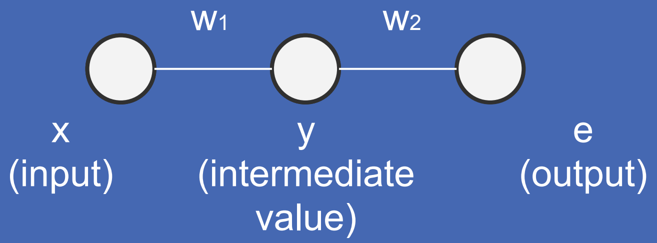
$$\frac{\partial e}{\partial y} = w_2$$

$$e = x * w_1 * w_2$$

$$\frac{\partial e}{\partial w_1} = x * w_2$$

$$\frac{\partial e}{\partial w_1} = \frac{\partial y}{\partial w_1} * \frac{\partial e}{\partial y}$$

Chaining



$$y = x * w_1$$

$$\frac{\partial y}{\partial w_1} = x$$

$$e = y * w_2$$

$$\frac{\partial e}{\partial y} = w_2$$

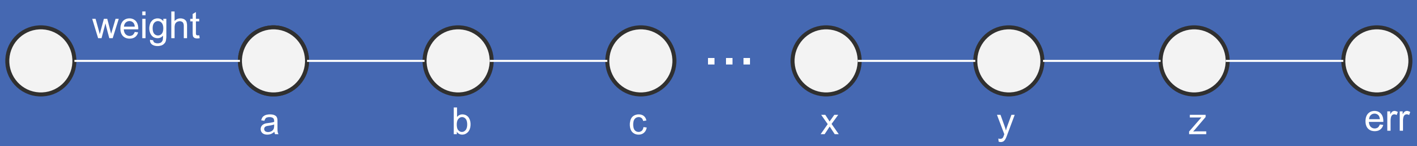
$$e = x * w_1 * w_2$$

$$\frac{\partial e}{\partial w_1} = x * w_2$$

$$\frac{\partial e}{\partial w_1} = \frac{\partial y}{\partial w_1} * \frac{\partial e}{\partial y}$$

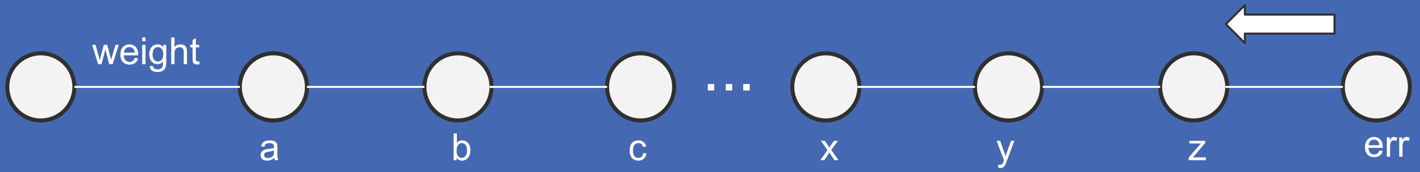
Chaining

$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \ldots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$



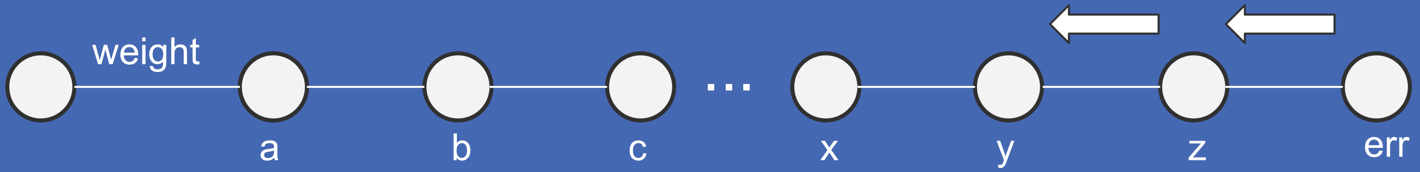
Backpropagation

$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \ldots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$



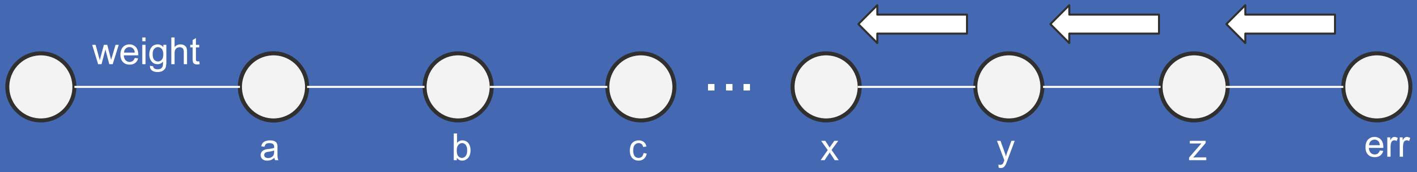
Backpropagation

$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \ldots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$



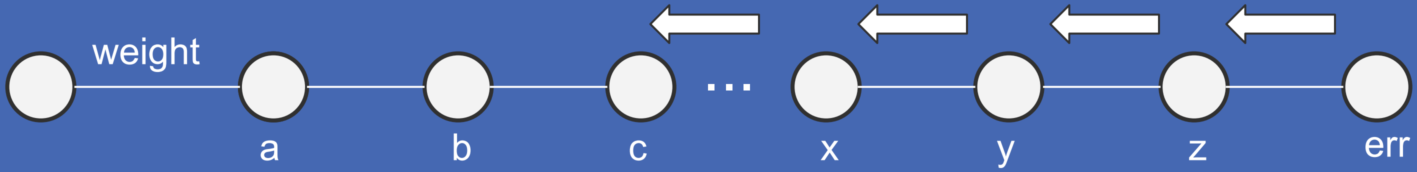
Backpropagation

$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \dots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$



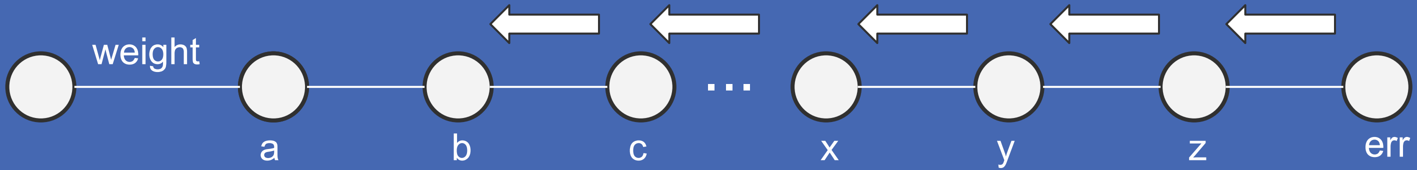
Backpropagation

$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \dots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$



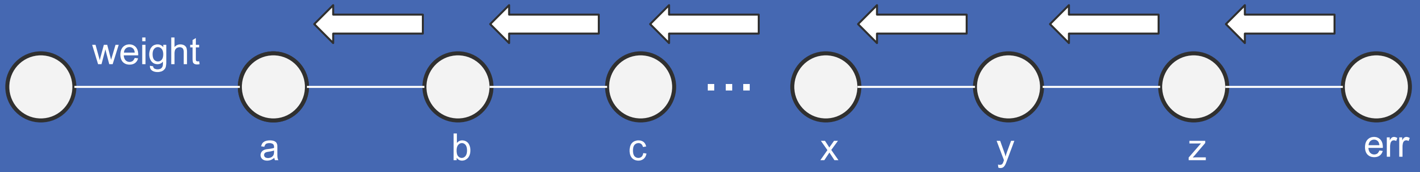
Backpropagation

$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \ldots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$



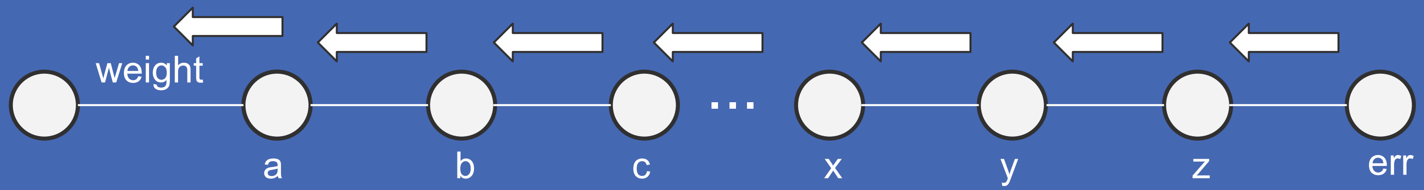
Backpropagation

$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \ldots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$

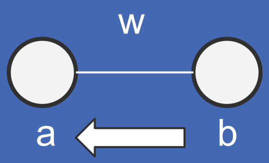


Backpropagation

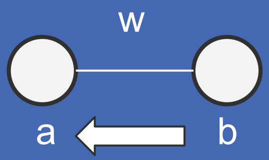
$$\frac{\partial err}{\partial weight} = \frac{\partial a}{\partial weight} * \frac{\partial b}{\partial a} * \frac{\partial c}{\partial b} * \frac{\partial d}{\partial c} * \ldots * \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial err}{\partial z}$$



Backpropagation challenge: weights

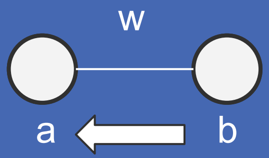


Backpropagation challenge: weights



$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

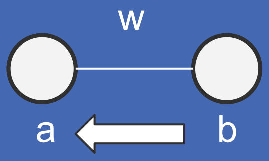
Backpropagation challenge: weights



$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

$$b = wa$$

Backpropagation challenge: weights

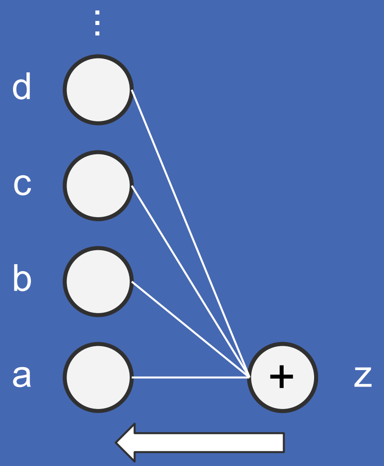


$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

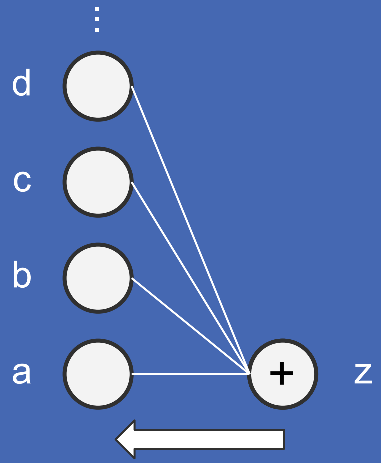
$$b = wa$$

$$\frac{\partial b}{\partial a} = w$$

Backpropagation challenge: sums

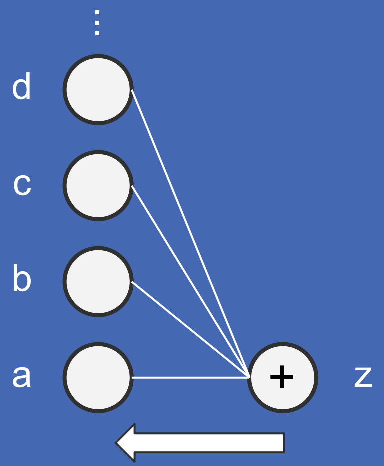


Backpropagation challenge: sums



$$\frac{\partial err}{\partial a} = \frac{\partial z}{\partial a} * \frac{\partial err}{\partial z}$$

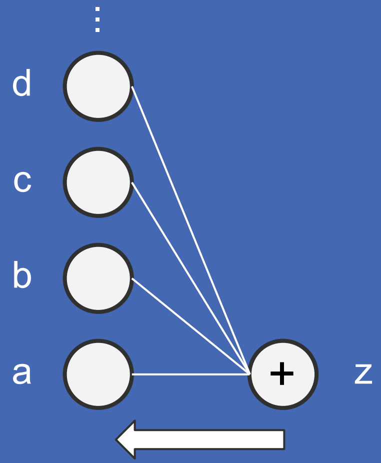
Backpropagation challenge: sums



$$\frac{\partial err}{\partial a} = \frac{\partial z}{\partial a} * \frac{\partial err}{\partial z}$$

$$z = a + b + c + d + \ldots$$

Backpropagation challenge: sums

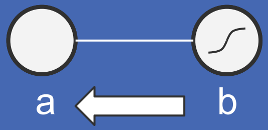


$$\frac{\partial err}{\partial a} = \frac{\partial z}{\partial a} * \frac{\partial err}{\partial z}$$

$$z = a + b + c + d + \ldots$$

$$\frac{\partial z}{\partial a} = 1$$

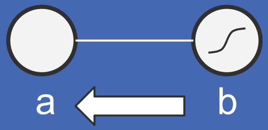
Backpropagation challenge: sigmoid



$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

Backpropagation challenge: sigmoid

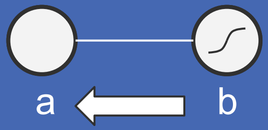
$$b = \frac{1}{1 + e^{-a}}$$



$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

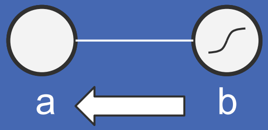
Backpropagation challenge: sigmoid

$$b = \frac{1}{1 + e^{-a}} = \sigma(a)$$



$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

Backpropagation challenge: sigmoid



$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

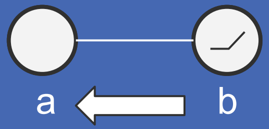
$$b = \frac{1}{1 + e^{-a}}$$

$$= \sigma(a)$$

Because math is beautiful / dumb luck:

$$\frac{\partial b}{\partial a} = \sigma(a) * (1 - \sigma(a))$$

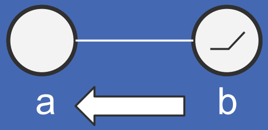
Backpropagation challenge: ReLU



$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

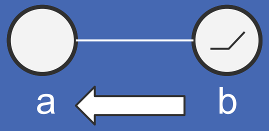
Backpropagation challenge: ReLU

$$
b = \begin{cases} a, & a > 0 \\ 0, & \text{otherwise} \end{cases}
$$



$$
\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}
$$

Backpropagation challenge: ReLU

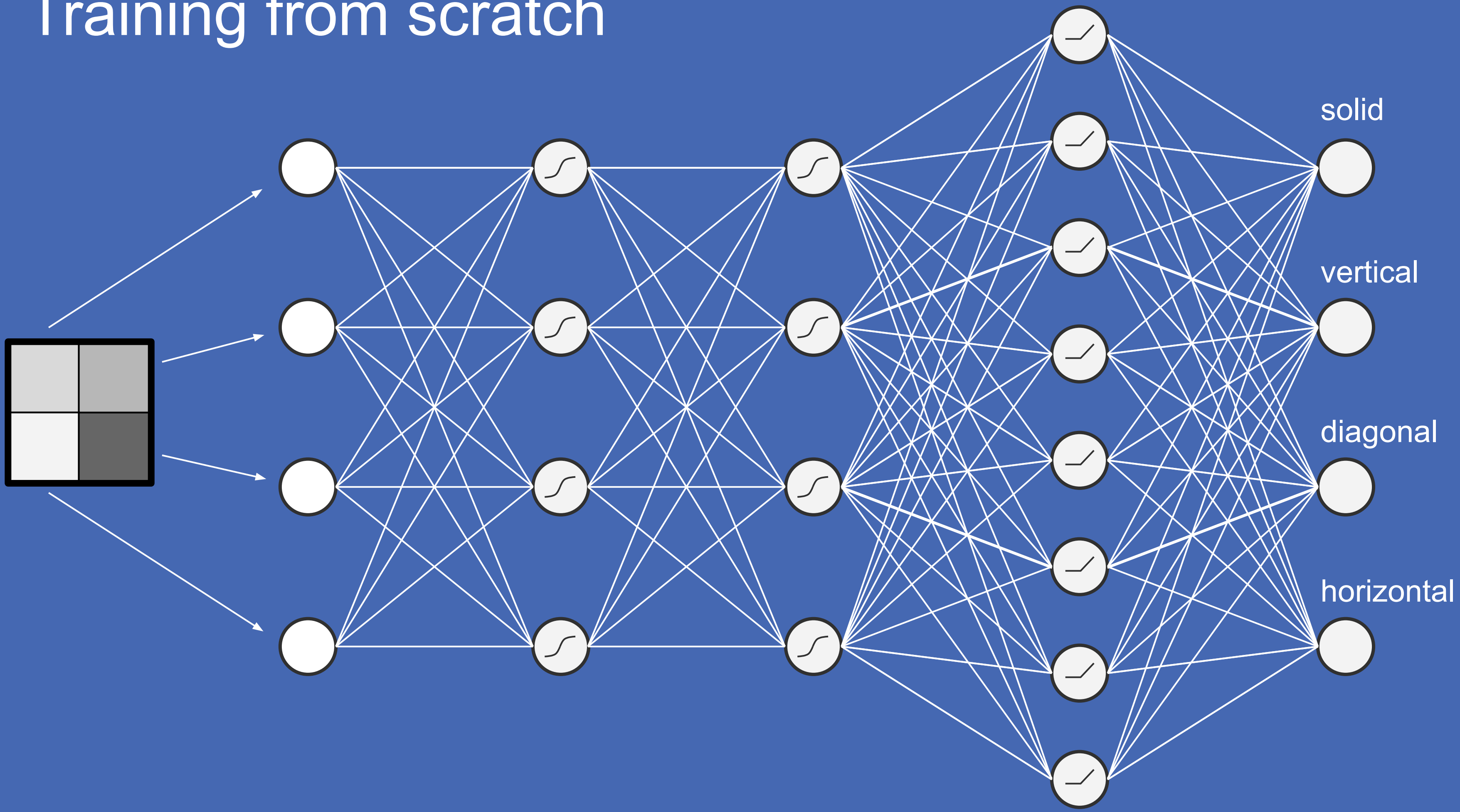


$$\frac{\partial err}{\partial a} = \frac{\partial b}{\partial a} * \frac{\partial err}{\partial b}$$

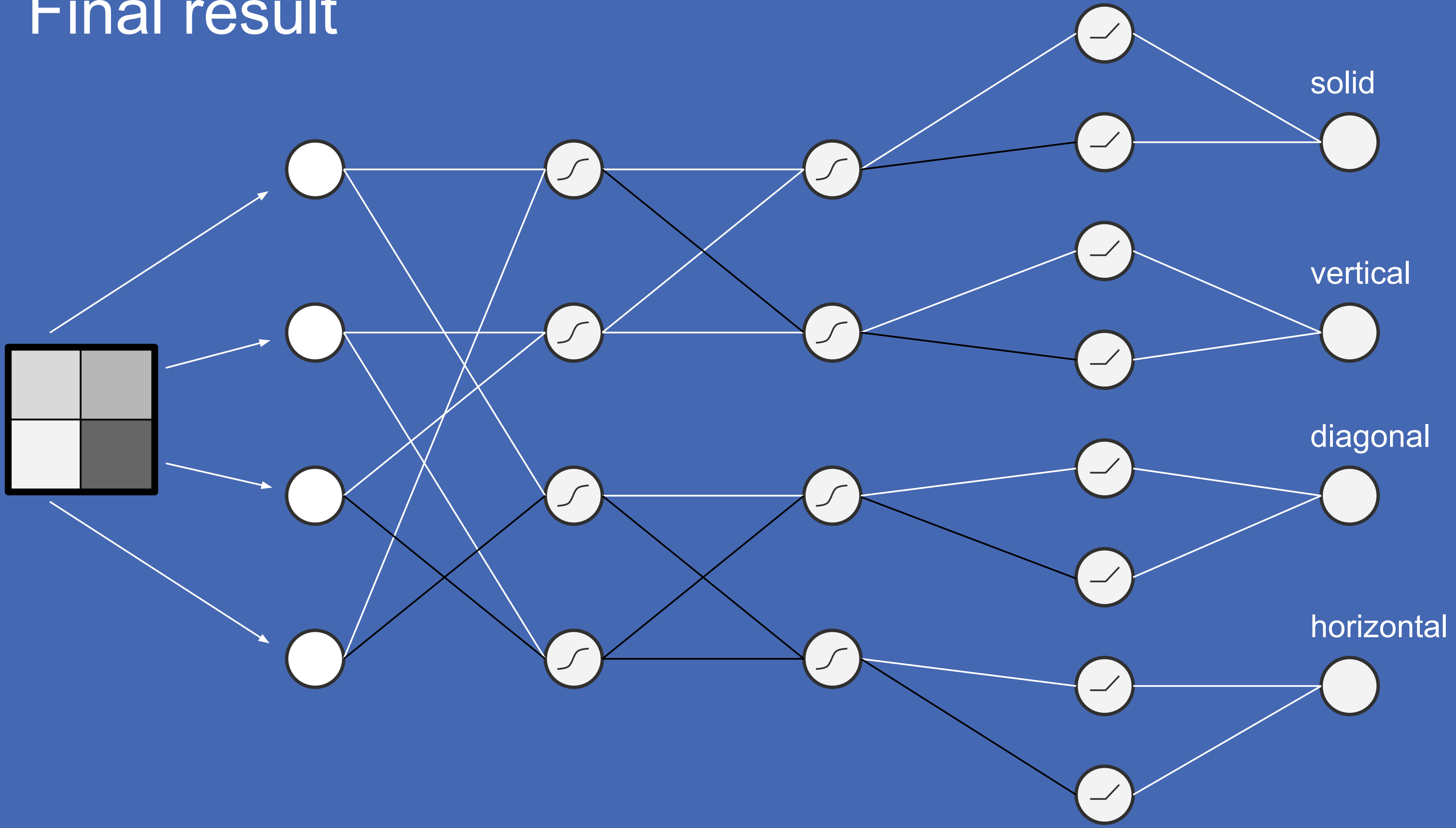
$$b = \begin{cases} a\,, & a > 0 \\ 0\,, & \text{otherwise} \end{cases}$$

$$\frac{\partial b}{\partial a} = \begin{cases} 1\,, & a > 0 \\ 0\,, & \text{otherwise} \end{cases}$$

Training from scratch



Final result



Advanced topics

Bias neurons

Dropout

Backpropagation details

- Andrej Karpathy's [Stanford CS231 lecture]

Backpropagation gotchas

- Andrej Karpathy's article ["Yes you should understand backprop"]

Tips and tricks

- Nikolas Markou's article ["The Black Magic of Deep Learning"]

Data Science and Robots Blog

For more How it Works:

- How Deep Learning works
- How Convolutional Neural Networks work
- How Bayes Law works
- How data science works
- How linear regression works

These slides

https://docs.google.com/presentation/d/1AAEFCgC0Ja7QEl3-wmuvIizbvaE-aQRksc7-W8LR2GY/edit?usp=sharing