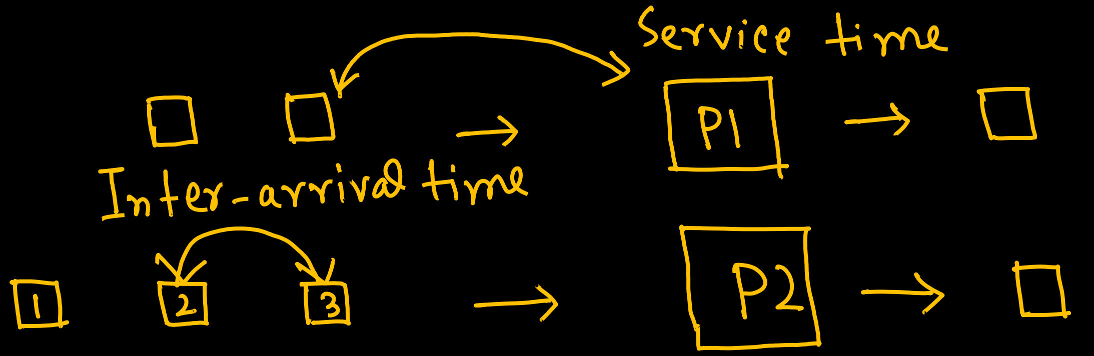
Queue



FIFO - First In, First Out

Queuing Theory — is the measurement of waiting lines which can be used to design services.

Characteristics

1. Input(s) / Arrival time distribution.
2. Output(s) / Departure time distribution.
3. Service channels.
4. Service discipline.
5. Maximum number of customers allowed in a system.
6. Calling source.

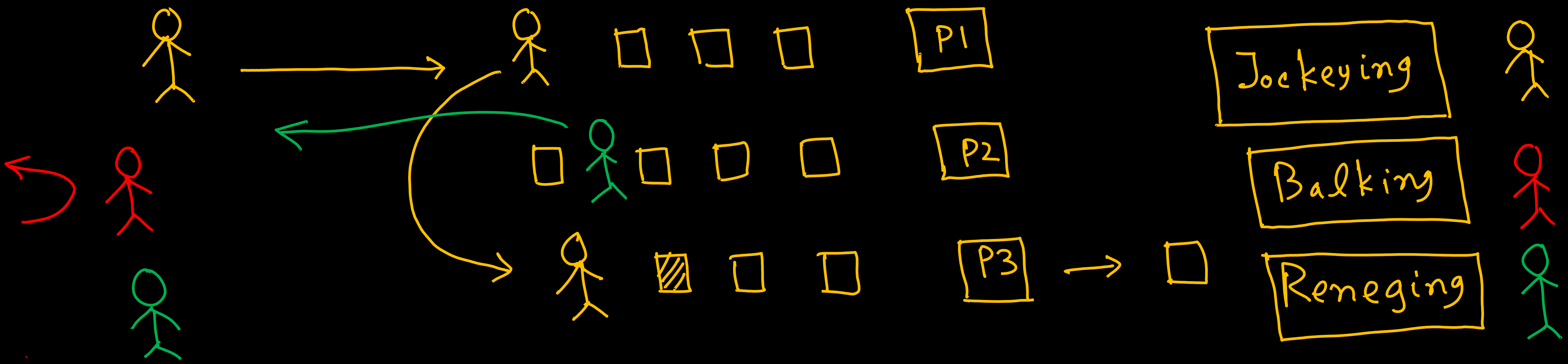
Service Discipline

w 1. First Come, First Serve (FCFS) w

2. Last Come, First Serve (LCFS)

3. Priority based.

4. Service in Random order



Kendall Notation

$\boxed{M/M/1}$

$(a/b/c):(d/e/f)$

a = Input(s) / Arrival time distribution

a/b

M: Poisson arrival distribution. ★

D: Deterministic arrival distribution

E_k: Erlagrian / Gamma inter-arrival distribution

GI: General Independent distribution

G: General distribution

b = Output(s) / Departure time distribution

c = number of service channels.

d = service discipline.

e = Maximum number of customers,

f = Calling source

λ = mean arrival time

$$\overset{7}{①} \to \overset{6}{②} \to \overset{5}{③}$$

$\boxed{Q}$ $\square$ $\square$ $\frac{7+6+5}{3} = 6$

μ = mean service time

$$① \to ② \to ③$$

$\boxed{Q}$ $\frac{4+5+2}{3} = \frac{11}{3}$

ρ = utilization factor

$$= \frac{\lambda}{\mu}$$

$\rho < 1$

$\rho = 1$

$\rho > 1$

n = number of units in the system

$P_n(t)$ = Probability of exactly 'n' units in the system at time 't'.

C = number of parallel servers.

A gas station has one pump which can serve 6 customers per hour. Cars arrive at the station at a rate of 10 per hour which is exponentially distributed.

$\boxed{c = 1}$

10 car / hour

10 car / 60 min

1 car / 6 min

6 car / hour

6 car / 60 min

1 car / 10 min

$\lambda = 6 \text{ min}$

$\mu = 10 \text{ min}$

$\rho = \dfrac{\lambda}{\mu} = \dfrac{6 \text{ min}}{10 \text{ min}} = 0.6$

$(a / b / c) : (d / e / f)$

$(M / M / 1) : (FCFS / \infty / \infty)$

W_s = Expected waiting time per unit in the system.

W_q = " " " " " " queue.

L_s = Expected number of units in the system.

L_q = " " " " " queue.

M/M/1

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{10-6} = \frac{1}{4} = 0.25$$

$$W_q = \rho \cdot W_s = \frac{\lambda}{\mu} \cdot \frac{1}{\mu - \lambda} = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{6}{10(10-6)} = \frac{6}{40}$$

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{6}{10-6} = \frac{6}{4}$$

$$L_q = \rho \cdot L_s = \frac{\lambda}{\mu} * \frac{\lambda}{\mu - \lambda} = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{36}{10(10-6)} = \frac{36}{40}$$

P_0 = Probability of 0 unit in the system

$$= 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{6}{10}$$

$$= 1 - 0.6$$

$$= 0.4$$

$$\boxed{M/M/1}$$

$$P_k = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

$$= \left(\frac{6}{10}\right)^{3+1} = (0.6)^4$$

M/D/1

$$W_q = \frac{\lambda}{2\mu|(\mu-\lambda)|}$$

$$W_s = W_q + \frac{1}{\mu} = \frac{\lambda}{2\mu(\mu-\lambda)} + \frac{1}{\mu} = \frac{\lambda + 2(\mu-\lambda)}{2\mu(\mu-\lambda)}$$

$$= \frac{\lambda + 2\mu - 2\lambda}{2\mu(\mu-\lambda)} = \frac{2\mu-\lambda}{2\mu(\mu-\lambda)}$$

$$L_q = \frac{\lambda^2}{2\mu(\mu-\lambda)}$$

$$L_s = L_q + \frac{\lambda}{\mu} = \frac{\lambda^2}{2\mu(\mu-\lambda)} + \frac{\lambda}{\mu} = \frac{\lambda^2 + 2\lambda(\mu-\lambda)}{2\mu(\mu-\lambda)}$$