Binomial Probability Distribution

- A fixed number of observations (trials), n
 - e.g., 20 tosses of a coin
- Binary random variable
 - e.g., Head or tail in coin toss
 - Often called as success or failure
 - Prob of success is p, and prob of failure is 1-p
- Constant probability for each observation

Binomial example

Take the example of 5 coin tosses

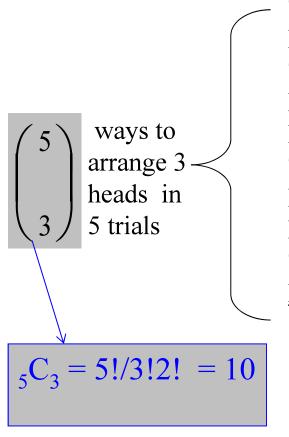
 What's the probability that you flip exactly 3 heads in 5 coin tosses?

Binomial distribution

- Solution:
- One way to get exactly 3 heads: HHHTT
- What's the probability of this exact arrangement?
 - P(heads) x P(heads) x P(heads) x P(tails) x P(tails) = $(1/2)^3$ x $(1/2)^2$
- Another way to get exactly 3 heads: THHHT
 - Probability of this exact outcome = $(1/2) \times (1/2)^3 \times (1/2)$ = $(1/2)^3 \times (1/2)^2$

Binomial distribution

- In fact, $(1/2)^3$ x $(1/2)^2$ is the probability of each unique outcome that has exactly 3 heads and 2 tails
- So, the overall probability of 3 heads and 2 tails is: $(1/2)^3 \times (1/2)^2 + (1/2)^3 \times (1/2)^2 + (1/2)^3 \times (1/2)^2 + \dots$ for as many unique arrangements as there are
- But how many are there??



Outcome	Probability
THHHT	$(1/2)^3 x (1/2)^2$
HHHTT	$(1/2)^3 x (1/2)^2$
TTHHH	$(1/2)^3 x (1/2)^2$
HTTHH	$(1/2)^3 x (1/2)^2$
HHTTH	$(1/2)^3 x (1/2)^2$
THTHH	$(1/2)^3 x (1/2)^2$
HTHTH	$(1/2)^3 x (1/2)^2$
HHTHT	$(1/2)^3 x (1/2)^2$
THHTH	$(1/2)^3 x (1/2)^2$
<u>HTHHT</u>	$(1/2)^3 x (1/2)^2$
10 arrangements $x (1/2)^3 x (1/2)^2$	

The probability of each unique outcome (note: they are all equal)

∴P(3 heads and 2 tails) =
$$\binom{5}{3}$$
 x P(heads)³ x P(tails)²

$$= 10 \times (\frac{1}{2})^{5} = 31.25\%$$

Binomial distribution, generally

Note the general pattern emerging \rightarrow if you have only two possible outcomes (call them 1/0 or yes/no or success/failure) in *n* independent trials, then the probability of exactly *X* "successes"=

