University of Asia Pacific (UAP)

Department of Basic Sciences and Humanities
Course Code: MTH-205
Program: B.Sc. Engineering (CSE)
2nd Year / 2nd Semester
Teacher: Sk. Reza-E-Rabbi (Lecturer)

Topics: Basic Definitions and Solution of Differential Equations (First Order First Degree)

1. Definitions: Differential equation, ODE, PDE, Order and degree of a differential equation, Linear and non-linear differential equation.
2. Form the following differential equations by eliminating arbitrary constants and write down the order and degree of the differential equations obtained. Hence also explain why the respective differential equation is either linear or non-linear.
$$(a)\, y = Ax + A^2,\ (b)\, y = A\cos x + B\sin x,\ (c)\, y^2 = Ax^2 + Bx + C$$
3. Using the method of separation of variables solve the following differential equations.
$$(i)\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y},\ (ii)\, x^4\frac{dy}{dx} + x^3 y = -\sec(xy).$$
4. Define homogeneous differential equation. Solve the following differential equations.
$$(i)\left(2xy + x^2\right)\frac{dy}{dx} = 3y^2 + 2xy,\ (ii)\frac{dy}{dx} = \frac{y}{x} + x\sin\frac{y}{x}.$$
5. What is linear differential equation? Solve the following differential equations.
$$(i)(x+1)\frac{dy}{dx} - y = e^x(x+1)^2,\ (ii)\sin x\frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right).$$
6. Define Bernoulli's equation. Solve the following differential equations.
$$(i)\, x^2 dy + y(x+y)dx = 0,\ (ii)\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y.$$
7. When a differential equation is said to be exact? Solve the following equations.
$$(i)\left\{2xy\cos x^2 - 2xy + 1\right\}dx + \left\{\sin x^2 - x^2 + 3\right\}dy = 0,\ (ii)\left[1 + \log(xy)\right]dx + \left[1 + \frac{x}{y}\right]dy = 0.$$
8. Solve the given equations,
$$(i)\left(2x\log x - xy\right)\mathrm{dy} + 2\,\mathrm{ydx} = 0.$$
$$(ii)\left(y^4 + 2y\right)dx + \left(xy^3 + 2y^4 - 4x\right)dy = 0.$$
$$(iii)\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}.$$

Topics: Equation of First Order and Higher Degree

1. Solve $x^2 = 1 + p^2$.
2. Solve $y = (x - a)p - p^2$.
3. Solve $y = 2px + yp^2$.

Topics: Linear Differential Equations with Constant Coefficients

1. Solve $(i)\ \frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0,\ (ii)\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0.$
2. Solve the following differential equations.

   $(i)\ \frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$, (ii) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2,$

   $(iii)\left(D^2 - 4D + 4\right)y = x^3e^{2x}$, $(iv)\left(D^2 + 6D + 9\right)y = \frac{e^{-3x}}{x^3}.$
3. Solve $\left(D^2 + 4\right)y = \cos 2x.$
4. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x.$
5. Solve the differential equation $\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 10\frac{dy}{dx} = e^{2x}\sin x.$