REVIEW OF DIFFERENTIATION

Rules

1. Constant: $\dfrac{d}{dx} c = 0$

2. Constant Multiple: $\dfrac{d}{dx} cf(x) = c f'(x)$

3. Sum: $\dfrac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

4. Product: $\dfrac{d}{dx} f(x)g(x) = f(x)g'(x) + g(x)f'(x)$

5. Quotient: $\dfrac{d}{dx} \dfrac{f(x)}{g(x)} = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

6. Chain: $\dfrac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

7. Power: $\dfrac{d}{dx} x^n = nx^{n-1}$

8. Power: $\dfrac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} g'(x)$

Functions

Trigonometric:

9. $\dfrac{d}{dx} \sin x = \cos x$

10. $\dfrac{d}{dx} \cos x = -\sin x$

11. $\dfrac{d}{dx} \tan x = \sec^2 x$

12. $\dfrac{d}{dx} \cot x = -\csc^2 x$

13. $\dfrac{d}{dx} \sec x = \sec x \tan x$

14. $\dfrac{d}{dx} \csc x = -\csc x \cot x$

Inverse trigonometric:

15. $\dfrac{d}{dx} \sin^{-1} x = \dfrac{1}{\sqrt{1-x^2}}$

16. $\dfrac{d}{dx} \cos^{-1} x = -\dfrac{1}{\sqrt{1-x^2}}$

17. $\dfrac{d}{dx} \tan^{-1} x = \dfrac{1}{1+x^2}$

18. $\dfrac{d}{dx} \cot^{-1} x = -\dfrac{1}{1+x^2}$

19. $\dfrac{d}{dx} \sec^{-1} x = \dfrac{1}{|x|\sqrt{x^2-1}}$

20. $\dfrac{d}{dx} \csc^{-1} x = -\dfrac{1}{|x|\sqrt{x^2-1}}$

Hyperbolic:

21. $\dfrac{d}{dx} \sinh x = \cosh x$

22. $\dfrac{d}{dx} \cosh x = \sinh x$

23. $\dfrac{d}{dx} \tanh x = \operatorname{sech}^2 x$

24. $\dfrac{d}{dx} \coth x = -\operatorname{csch}^2 x$

25. $\dfrac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$

26. $\dfrac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

Inverse hyperbolic:

27. $\dfrac{d}{dx} \sinh^{-1} x = \dfrac{1}{\sqrt{x^2+1}}$

28. $\dfrac{d}{dx} \cosh^{-1} x = \dfrac{1}{\sqrt{x^2-1}}$

29. $\dfrac{d}{dx} \tanh^{-1} x = \dfrac{1}{1-x^2}$

30. $\dfrac{d}{dx} \coth^{-1} x = \dfrac{1}{1-x^2}$

31. $\dfrac{d}{dx} \operatorname{sech}^{-1} x = -\dfrac{1}{x\sqrt{1-x^2}}$

32. $\dfrac{d}{dx} \operatorname{csch}^{-1} x = -\dfrac{1}{|x|\sqrt{x^2+1}}$

Exponential:

33. $\dfrac{d}{dx} e^x = e^x$

34. $\dfrac{d}{dx} b^x = b^x(\ln b)$

Logarithmic:

35. $\dfrac{d}{dx} \ln|x| = \dfrac{1}{x}$

36. $\dfrac{d}{dx} \log_b x = \dfrac{1}{x(\ln b)}$

BRIEF TABLE OF INTEGRALS

1. $\int u^n \, du = \dfrac{u^{n+1}}{n+1} + C, \ n \neq -1$

2. $\int \dfrac{1}{u} \, du = \ln|u| + C$

3. $\int e^u \, du = e^u + C$

4. $\int a^u \, du = \dfrac{1}{\ln a} a^u + C$

5. $\int \sin u \, du = -\cos u + C$

6. $\int \cos u \, du = \sin u + C$

7. $\int \sec^2 u \, du = \tan u + C$

8. $\int \csc^2 u \, du = -\cot u + C$

9. $\int \sec u \tan u \, du = \sec u + C$

10. $\int \csc u \cot u \, du = -\csc u + C$

11. $\int \tan u \, du = -\ln|\cos u| + C$

12. $\int \cot u \, du = \ln|\sin u| + C$

13. $\int \sec u \, du = \ln|\sec u + \tan u| + C$

14. $\int \csc u \, du = \ln|\csc u - \cot u| + C$

15. $\int u \sin u \, du = \sin u - u \cos u + C$

16. $\int u \cos u \, du = \cos u + u \sin u + C$

17. $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

18. $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

19. $\int \tan^2 u \, du = \tan u - u + C$

20. $\int \cot^2 u \, du = -\cot u - u + C$

21. $\int \sin^3 u \, du = -\frac{1}{3}\left(2 + \sin^2 u\right)\cos u + C$

22. $\int \cos^3 u \, du = \frac{1}{3}\left(2 + \cos^2 u\right)\sin u + C$

23. $\int \tan^3 u \, du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

24. $\int \cot^3 u \, du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$

25. $\int \sec^3 u \, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$

26. $\int \csc^3 u \, du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$

27. $\int \sin au \cos bu \, du = \dfrac{\sin(a-b)u}{2(a-b)} - \dfrac{\sin(a+b)u}{2(a+b)} + C$

28. $\int \cos au \cos bu \, du = \dfrac{\sin(a-b)u}{2(a-b)} + \dfrac{\sin(a+b)u}{2(a+b)} + C$

29. $\int e^{au} \sin bu \, du = \dfrac{e^{au}}{a^2 + b^2}\left(a \sin bu - b \cos bu\right) + C$

30. $\int e^{au} \cos bu \, du = \dfrac{e^{au}}{a^2 + b^2}\left(a \cos bu + b \sin bu\right) + C$

31. $\int \sinh u \, du = \cosh u + C$

32. $\int \cosh u \, du = \sinh u + C$

33. $\int \text{sech}^2 u \, du = \tanh u + C$

34. $\int \text{csch}^2 u \, du = -\coth u + C$

35. $\int \tanh u \, du = \ln(\cosh u) + C$

36. $\int \coth u \, du = \ln|\sinh u| + C$

37. $\int \ln u \, du = u \ln u - u + C$

38. $\int u \ln u \, du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$

39. $\int \dfrac{1}{\sqrt{a^2 - u^2}} \, du = \sin^{-1}\dfrac{u}{a} + C$

40. $\int \dfrac{1}{\sqrt{a^2 + u^2}} \, du = \ln\left|u + \sqrt{a^2 + u^2}\right| + C$

41. $\int \sqrt{a^2 - u^2} \, du = \dfrac{u}{2}\sqrt{a^2 - u^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{u}{a} + C$

42. $\int \sqrt{a^2 + u^2} \, du = \dfrac{u}{2}\sqrt{a^2 + u^2} + \dfrac{a^2}{2}\ln\left|u + \sqrt{a^2 + u^2}\right| + C$

43. $\int \dfrac{1}{a^2 + u^2} \, du = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$

44. $\int \dfrac{1}{a^2 - u^2} \, du = \dfrac{1}{2a}\ln\left|\dfrac{a+u}{a-u}\right| + C$

Note: Some techniques of integration, such as integration by parts and partial fractions, are reviewed in the *Student Resource and Solutions Manual* that accompanies this text.

TABLE OF LAPLACE TRANSFORMS

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. 1	$\dfrac{1}{s}$
2. t	$\dfrac{1}{s^2}$
3. t^n	$\dfrac{n!}{s^{n+1}}$, $\quad n$ a positive integer
4. $t^{-1/2}$	$\sqrt{\dfrac{\pi}{s}}$
5. $t^{1/2}$	$\dfrac{\sqrt{\pi}}{2s^{3/2}}$
6. t^α	$\dfrac{\Gamma(\alpha + 1)}{s^{\alpha+1}}$, $\quad \alpha > -1$
7. $\sin kt$	$\dfrac{k}{s^2 + k^2}$
8. $\cos kt$	$\dfrac{s}{s^2 + k^2}$
9. $\sin^2 kt$	$\dfrac{2k^2}{s(s^2 + 4k^2)}$
10. $\cos^2 kt$	$\dfrac{s^2 + 2k^2}{s(s^2 + 4k^2)}$
11. e^{at}	$\dfrac{1}{s - a}$
12. $\sinh kt$	$\dfrac{k}{s^2 - k^2}$
13. $\cosh kt$	$\dfrac{s}{s^2 - k^2}$
14. $\sinh^2 kt$	$\dfrac{2k^2}{s(s^2 - 4k^2)}$
15. $\cosh^2 kt$	$\dfrac{s^2 - 2k^2}{s(s^2 - 4k^2)}$
16. te^{at}	$\dfrac{1}{(s - a)^2}$
17. $t^n e^{at}$	$\dfrac{n!}{(s - a)^{n+1}}$, $\quad n$ a positive integer
18. $e^{at} \sin kt$	$\dfrac{k}{(s - a)^2 + k^2}$
19. $e^{at} \cos kt$	$\dfrac{s - a}{(s - a)^2 + k^2}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
20. $e^{at} \sinh kt$	$\dfrac{k}{(s - a)^2 - k^2}$
21. $e^{at} \cosh kt$	$\dfrac{s - a}{(s - a)^2 - k^2}$
22. $t \sin kt$	$\dfrac{2ks}{(s^2 + k^2)^2}$
23. $t \cos kt$	$\dfrac{s^2 - k^2}{(s^2 + k^2)^2}$
24. $\sin kt + kt \cos kt$	$\dfrac{2ks^2}{(s^2 + k^2)^2}$
25. $\sin kt - kt \cos kt$	$\dfrac{2k^3}{(s^2 + k^2)^2}$
26. $t \sinh kt$	$\dfrac{2ks}{(s^2 - k^2)^2}$
27. $t \cosh kt$	$\dfrac{s^2 + k^2}{(s^2 - k^2)^2}$
28. $\dfrac{e^{at} - e^{bt}}{a - b}$	$\dfrac{1}{(s - a)(s - b)}$
29. $\dfrac{ae^{at} - be^{bt}}{a - b}$	$\dfrac{s}{(s - a)(s - b)}$
30. $1 - \cos kt$	$\dfrac{k^2}{s(s^2 + k^2)}$
31. $kt - \sin kt$	$\dfrac{k^3}{s^2(s^2 + k^2)}$
32. $\dfrac{a \sin bt - b \sin at}{ab(a^2 - b^2)}$	$\dfrac{1}{(s^2 + a^2)(s^2 + b^2)}$
33. $\dfrac{\cos bt - \cos at}{a^2 - b^2}$	$\dfrac{s}{(s^2 + a^2)(s^2 + b^2)}$
34. $\sin kt \sinh kt$	$\dfrac{2k^2 s}{s^4 + 4k^4}$
35. $\sin kt \cosh kt$	$\dfrac{k(s^2 + 2k^2)}{s^4 + 4k^4}$
36. $\cos kt \sinh kt$	$\dfrac{k(s^2 - 2k^2)}{s^4 + 4k^4}$
37. $\cos kt \cosh kt$	$\dfrac{s^3}{s^4 + 4k^4}$
38. $J_0(kt)$	$\dfrac{1}{\sqrt{s^2 + k^2}}$