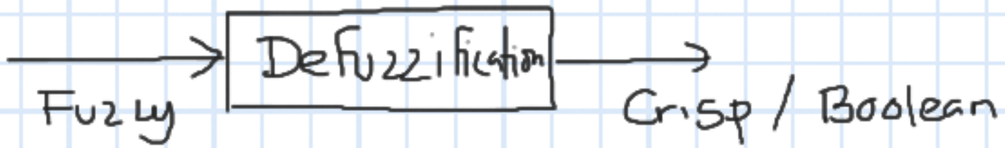
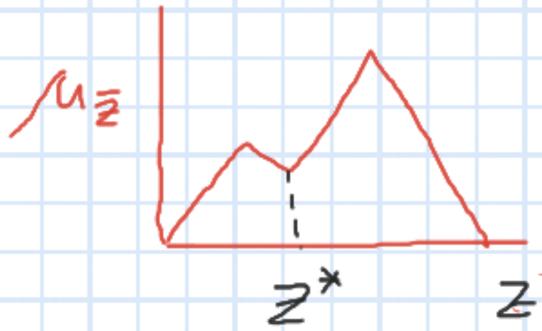
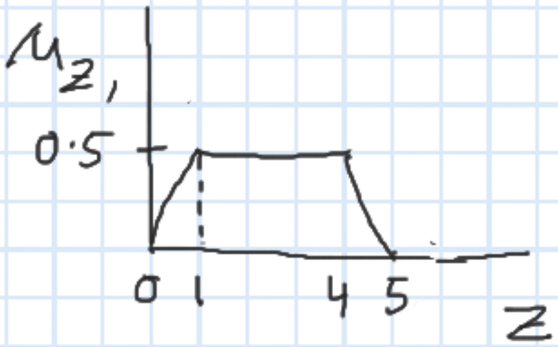
Defuzzification:

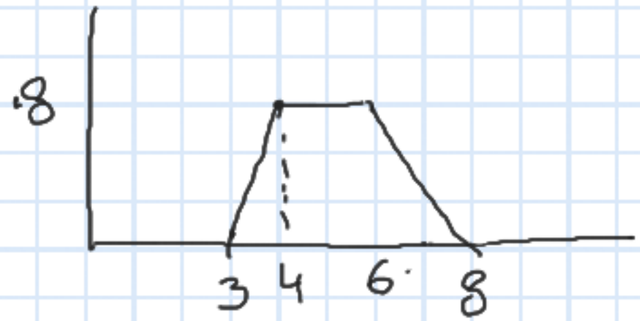


Centroid Method :



$$z^* = \frac{\int \mu_{\bar{z}} \cdot z \, dz}{\int \mu_{\bar{z}} \, dz}$$





Centroid Method ✓

Height " ✓

Weighted Ave " ✓

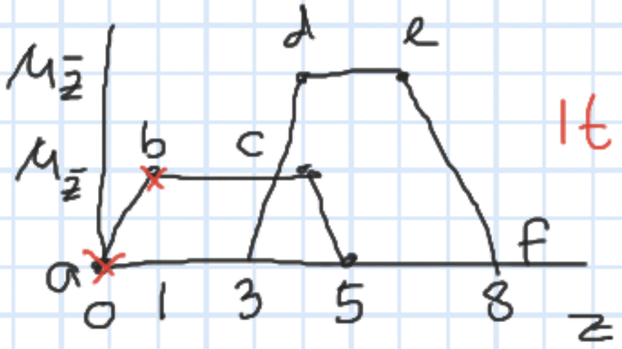
✓ Mean - Max / Middle of max

Centre of Sum ✓

Centre of largest Area

✓ Fast / Last of Maxima

In centroid method, the defuzzified value z^* is given as



It is union of two fuzzy sets

$$z^* = \frac{\int \mu_{\bar{z}} \cdot z \cdot dz}{\int \mu_{\bar{z}} \cdot dz} \quad \text{— (2)}$$

Centroid

ab

$y = mx + c \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{— (1)}$$

$x_1 = 0 \quad y_1 = 0, \quad x_2 = 1 \quad y_2 = 0.5$

$$\frac{y - 0}{x - 0} = \frac{0.5 - 0}{1 - 0} \Rightarrow y = 0.5x \quad \text{for 0 to 1}$$

bc

$y = 0.5$ for 1 to 3.5

cd

$x_1 = 3.5 \quad y_1 = 0.5$

$x_2 = 4 \quad y_2 = 0.8$

$y = \frac{3}{5}x - \frac{8}{5}$ for 3.5 to 4

de

$y = 0.8$ for 4 to 6

ef

$x_1 = 6 \quad y_1 = 0.8$

$x_2 = 8 \quad y_2 = 0$

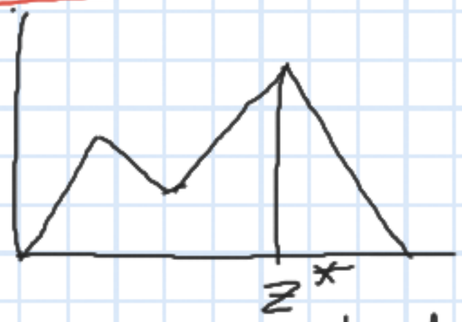
$-.4x + 3.2$

Using eqn (2)

$$z^* = \frac{\int_0^1 0.5z\,z\,dz + \int_1^{3.5} 0.5z\,dz + \int_{3.5}^{4} \left(\frac{3}{5}z - \frac{8}{5}\right)z\,dz + \int_4^6 0.8z\,dz + \int_6^8 (-0.4z + 3.2)z\,dz}{\int_0^1 0.5z\,dz + \int_1^{3.5} 0.5\,dz + \int_{3.5}^{4} \left(\frac{3}{5}z - \frac{8}{5}\right)dz + \int_4^6 .8\,dz + \int_6^8 (-0.4z + 3.2)\,dz} \cong 4.15$$

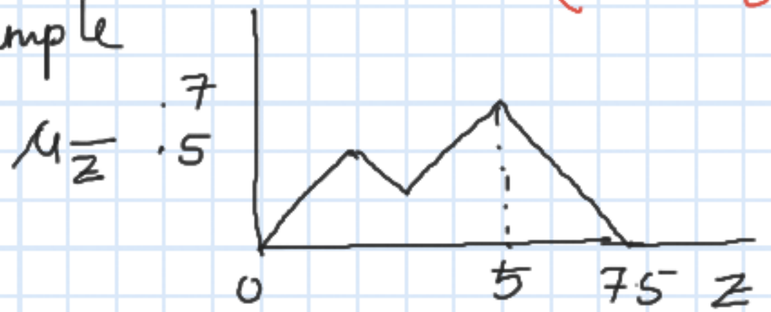
$\boxed{z^* = 4.16}$

HIGHT METHOD / Max Membership Principle



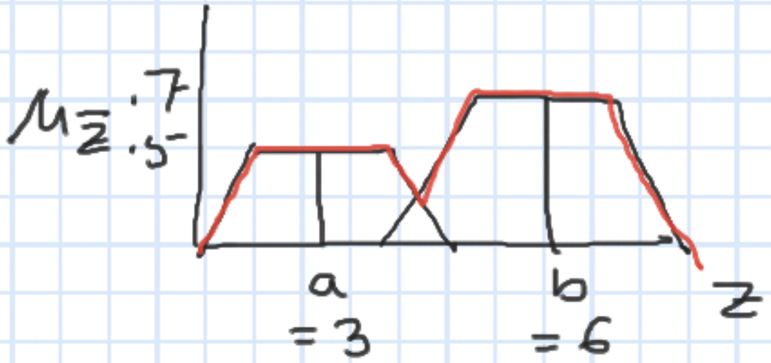
Picked o/p func

Example (Triangular MF)



$z^* = 5$

WEIGHTED AVG METHOD



$$z^* = \frac{\sum \mu \cdot \bar{z}}{\sum \mu}$$

$$= \frac{0.5 \times a + .7 \times b}{0.5 + 0.7}$$

$$= \frac{0.5 \times 3 + 0.7 \times 6}{1.2}$$

$$= ?$$

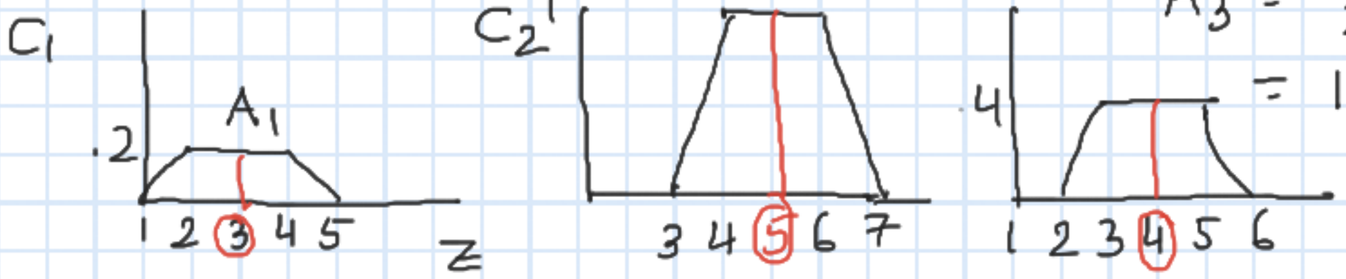
MEAN MAX MEMBERSHIP / MIDDLE OF MAXIMA

$$z^* = \frac{a+b}{2}$$

$$z^* = \frac{5+8}{2}$$

$$= 13/2 = 6.5$$

CENTRE OF SUM



$$A_1 = \frac{1}{2}(b_1 + b_2) \cdot h$$

$$= \frac{1}{2}(2+4)\,0.2$$

$$= 0.6$$

4-2 (5 − 1)

$$A_2 = \frac{1}{2}(4+2) \times 1$$

$$= 3$$

$$A_3 = \frac{1}{2}(4+2) \times 0.4$$

$$= 1.2$$

$$z^* = \frac{\sum z_i A_i}{\sum A_i}$$

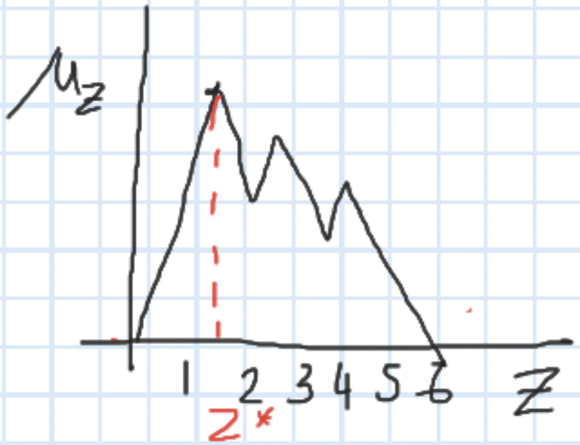
$$= \frac{3 \times .6 + 5 \times 3 + 4 \times 1.2}{.6 + 3 + 1.2}$$

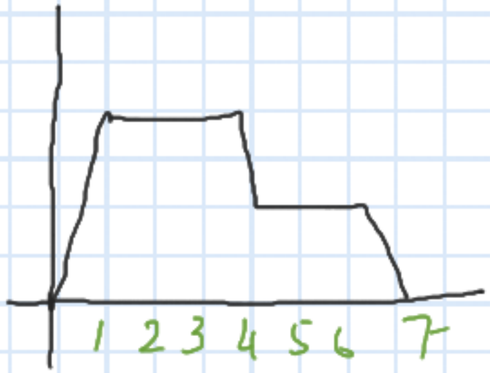
$$= 4.5$$

First of Maxima

$$z^* = \text{1st max height}(\mu_z)$$

$$= 1.5$$





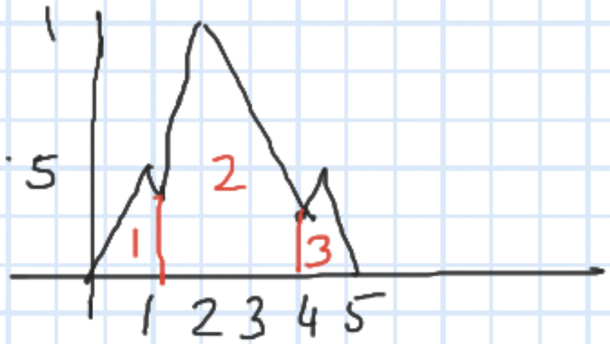
First of max $z^* = 1$

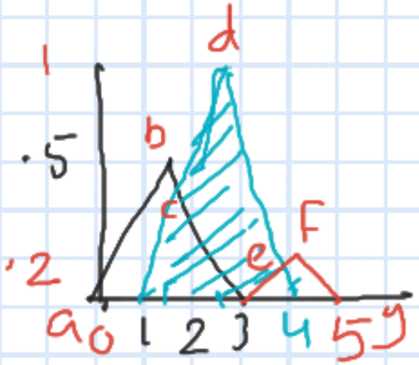
Last of max $z^{\wedge} = 37$

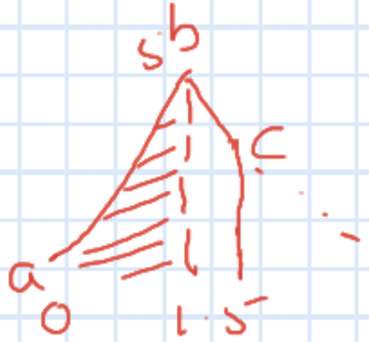
Last of Maxima

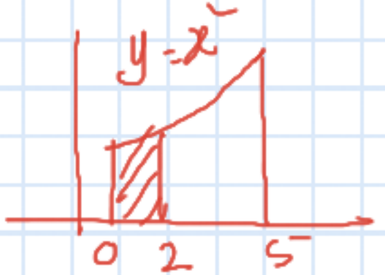
Centre of Largest Area

$$z^* = \frac{\int \mu(z)\, z\, dz}{\int \mu(z)\, dz}$$









$$y = \int_0^{5^2} x^{\frac{1}{2}}\, dx$$

Area of 1 = Area under ab + Area under bc

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-y_1} \;/\; y = mx + c$$

ab $(x_1, y_1) = (0\,0)$ $(x_2, y_2) = (1.5, .5)$

$y = 0.33x \quad [0, 1.5]$

bc

$(x_1, y_1) = (1.5, .5)$ $(x_2, y_2) = (3, 0)$

$y = -.33x + 1 \quad [1.5, 1.7]$

$$\text{Area 1} = \int_0^{1.5} 0.33x\, dx + \int_{1.5}^{1.7} -(.33x + 1)\, dx = 0.44$$

cd

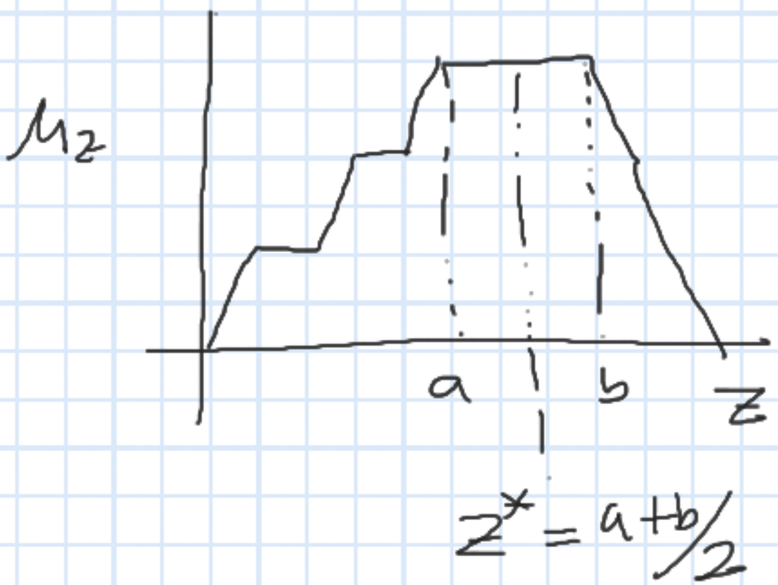
$(x_1, y_1) = (1, 0)$

$(x_2, y_2) = (2.5, 1)$

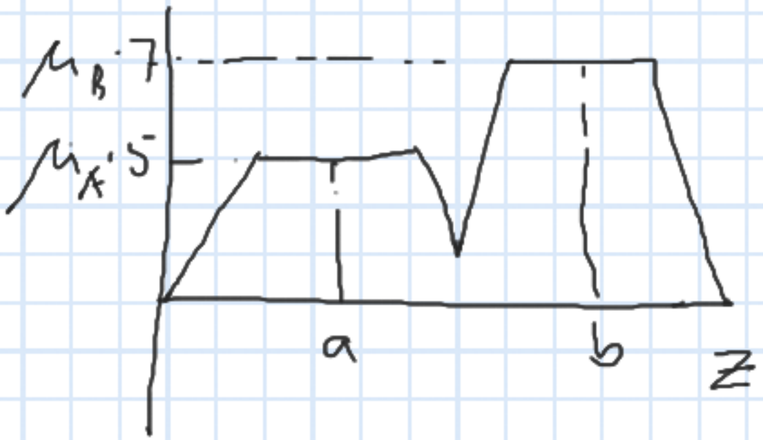
$y = 0.67x - 0.67$

Home Work

Mean Max / Middle of Maxima



Weighted Average Method



$$z^* = \frac{\sum \mu \cdot z}{\sum \mu}$$

$$z^* = \frac{\cdot 7 \times b + \cdot 5 \times a}{\cdot 7 + \cdot 5}$$

$$A^{\alpha} = (x, \mu^{\alpha}(x))$$

$$A = {}^{0.5}/_{1} + {}^{0.7}/_{2} + {}^{1}/_{3}$$

$A^{1/\alpha}$ Small

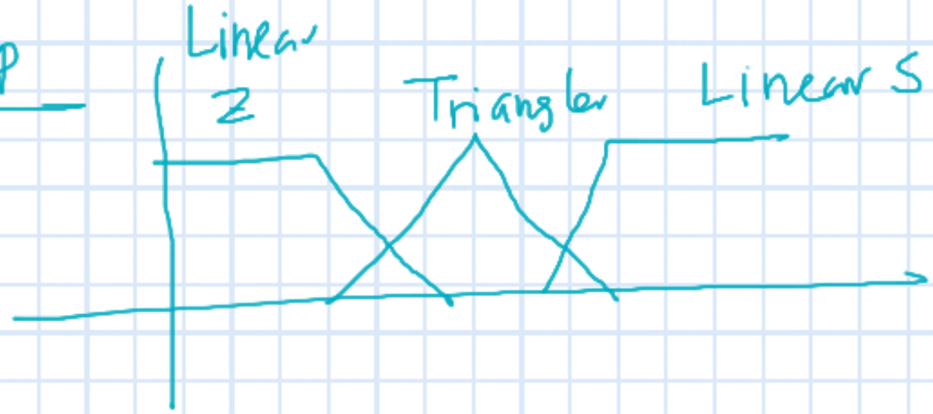
Tall

A^{α} Very Tall

$(A^{\alpha})^{\alpha}$ Very Very Tall

$((A^{\alpha})^{\alpha})^{\alpha}$ Extremely tall

Z / S Membership



Z S

Composition of Fuzy Relations

$$T = R \circ S$$

$3\times3 \quad 3\times2 \quad 2\times3$

1. Max - min Composition

$$\mu_{R\circ S} = \max\left[\min\left(\mu_R(x,y), \mu_S(y,z)\right)\right]$$

2. Max - Product

$$\mu_{R\circ S} = \max\left[\mu_R(x,y) \cdot \mu_S(y,z)\right]$$

(1) Max - Min

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} .5 & .1 \\ .2 & .9 \\ .8 & .6 \end{bmatrix} \end{array}_{3\times2} \qquad S = \begin{array}{c} \\ y_1 \\ y_2 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} .6 & .4 & .7 \\ .5 & .8 & .9 \end{bmatrix} \end{array}_{2\times3}$$

$$\max\left(\min(.5, .6), \min(.1, .5)\right) = .5$$

$$T = R \circ S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} .5 & .4 & .5 \\ .5 & .8 & .7 \\ .6 & .6 & .7 \end{bmatrix} \end{array}$$

(2) Max - Product

$$T = R \circ S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} .3 & .2 & \\ & & \\ & & \end{bmatrix} \end{array}$$

$$\max\left[.5\times.6, \; .1\times.5\right] = \max(.3, 0.05)$$

$$\max\left[.2, 0.08\right] = .2 = 0.3$$