Fuzzy Control

Lect 4 Fuzzy Logic Process

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- Fuzzy Associate Memory(FAM)
- Defuzzification
- Examples

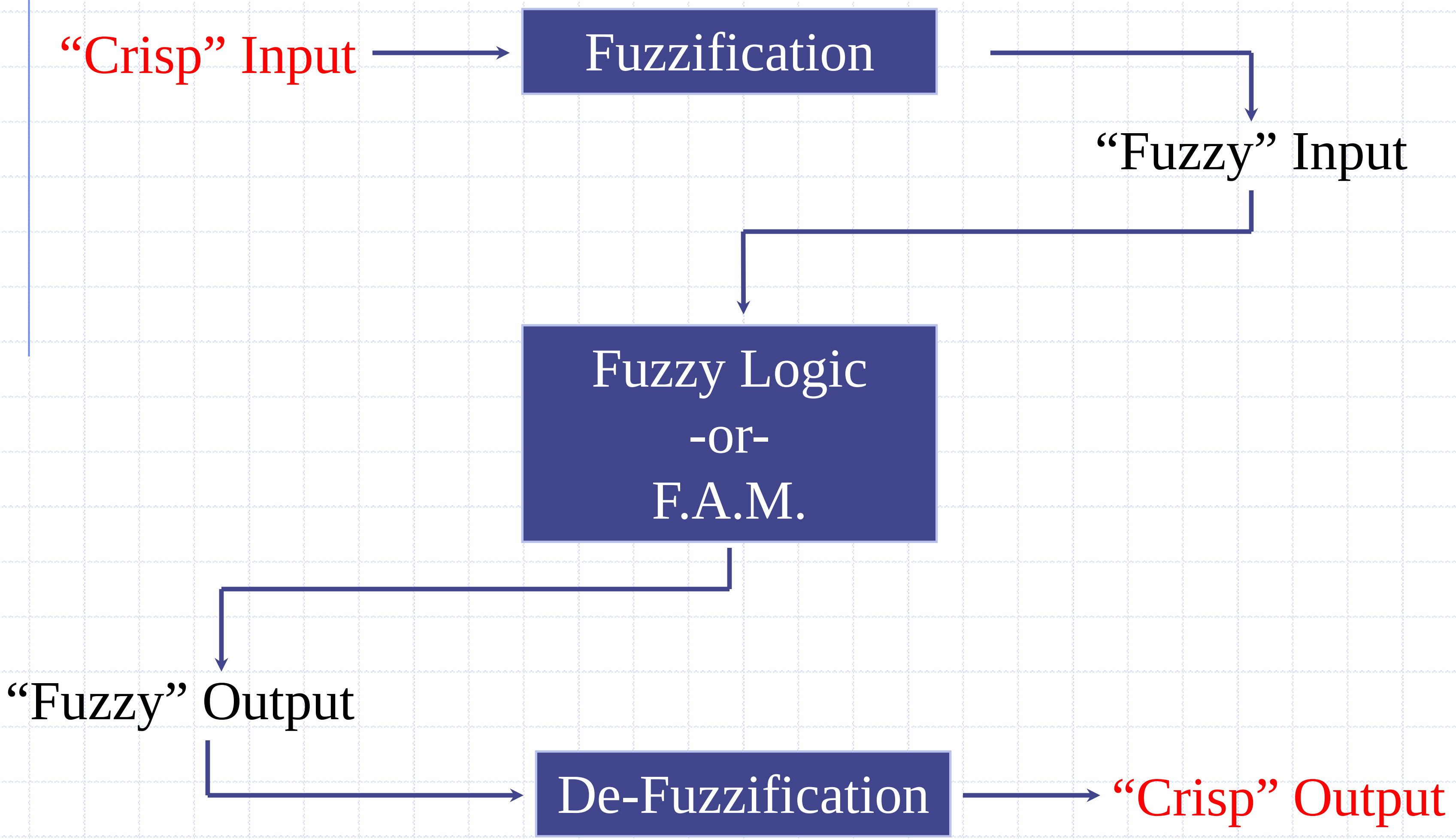
Fuzzy Logic Process

- It is one thing to compute, to reason, and to model with fuzzy information; it is another to apply the fuzzy results to the world around us.
- Despite the fact that the bulk of the information we assimilate every day is fuzzy, most of the actions or decisions implemented by humans or machines are crisp or binary.
- The decisions we make that require an action are binary, the hardware we use is binary, and certainly the computers we use are based on binary digital instructions.

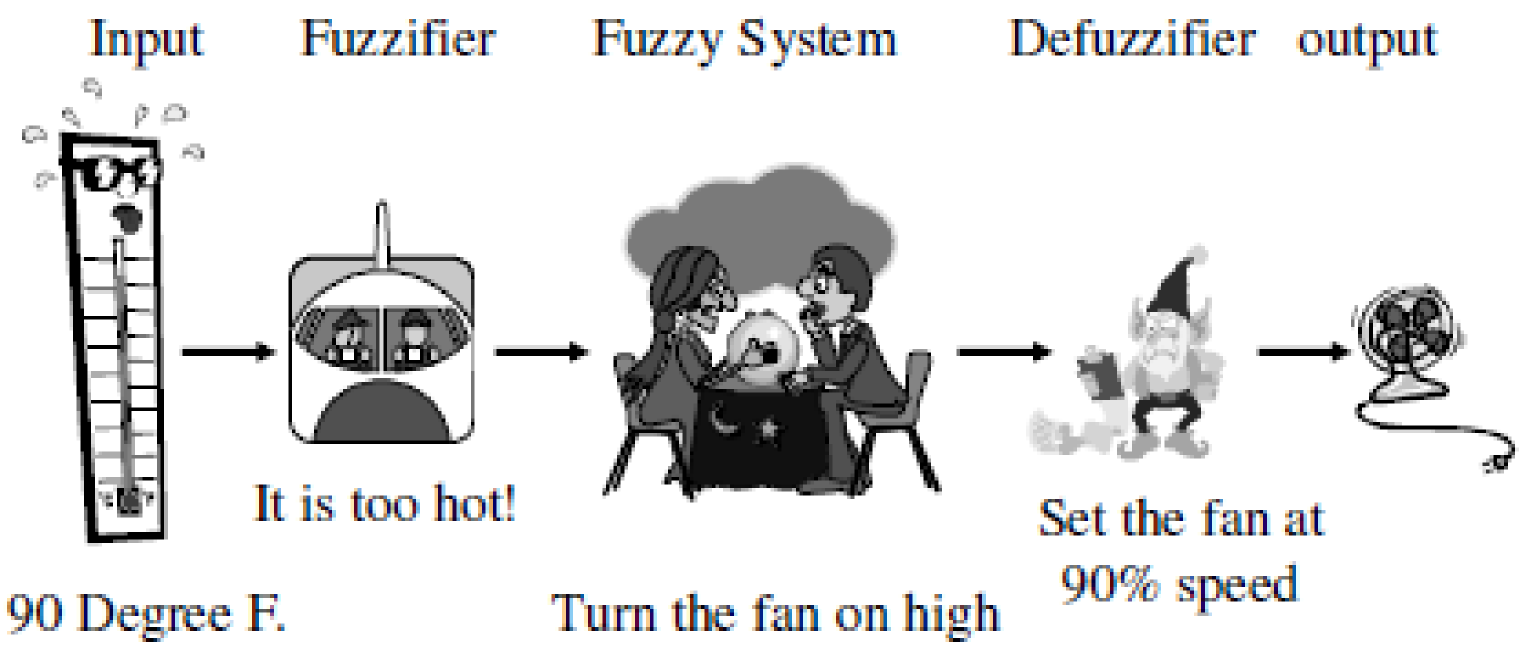
Fuzzy Logic Process

In giving instructions to an aircraft autopilot, it is not possible to turn the plane "slightly to the west"; an autopilot device does not understand the natural language of a human. We have to turn the plane by 15°, for example, a crisp number.

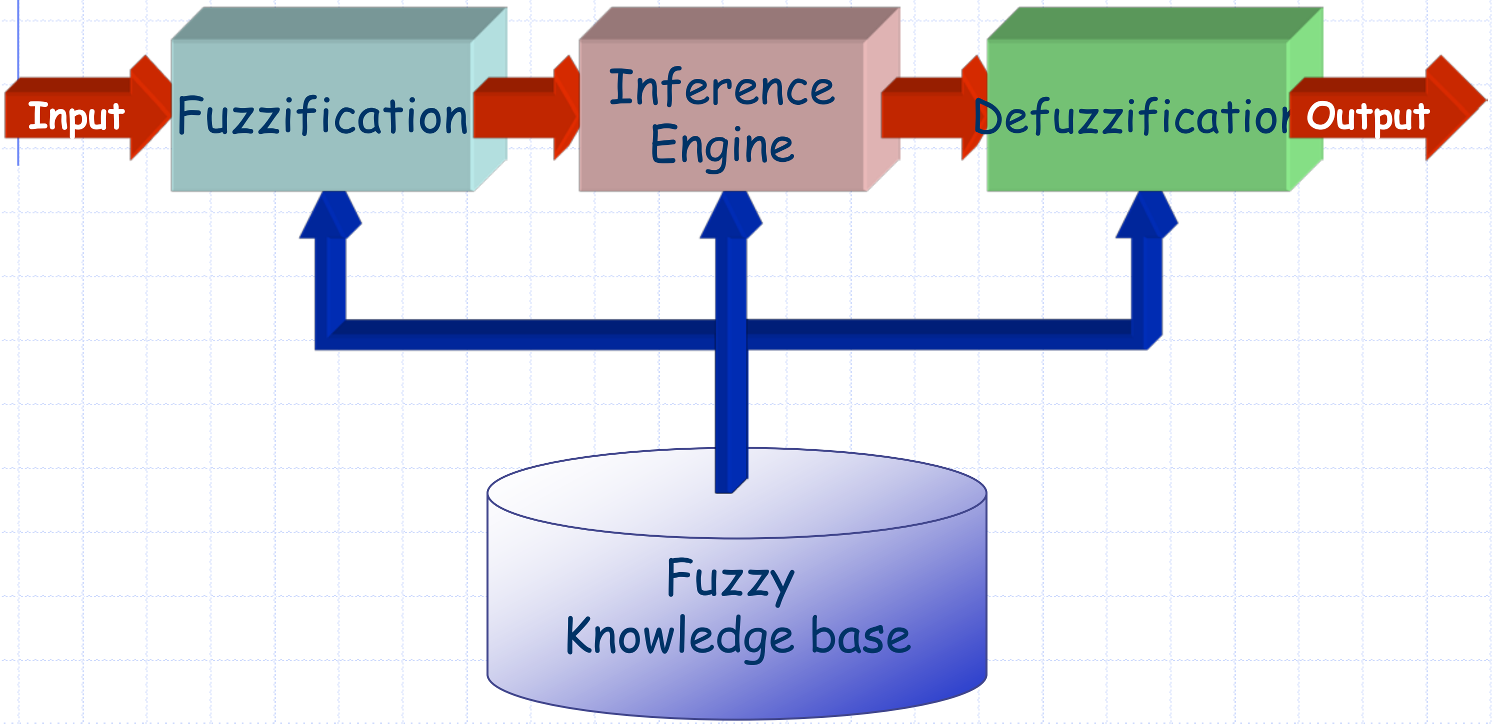
Fuzzy Logic Process



Fuzzy Systems



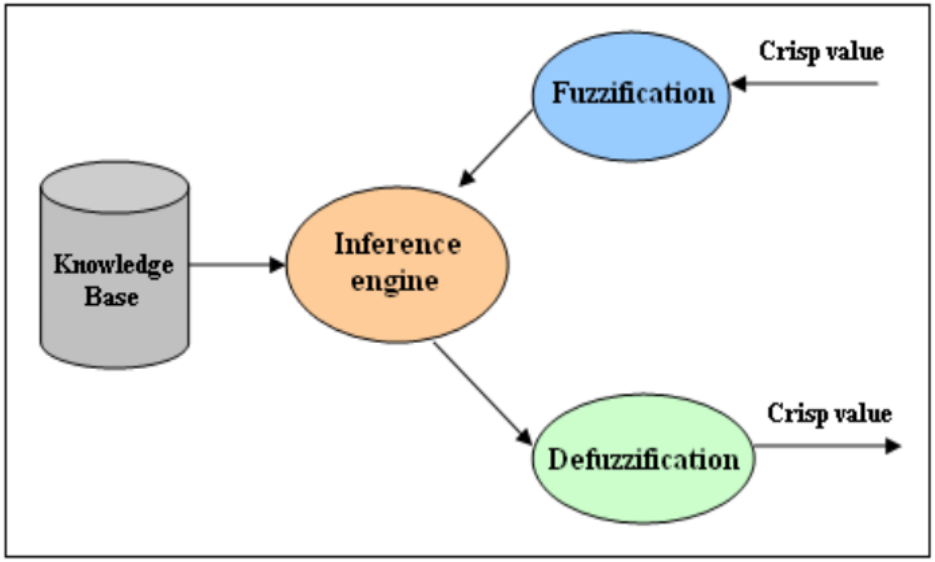
Fuzzy Systems



Fuzzy Systems

In general, a fuzzy inference system consists of four modules:

- Fuzzification module
- Knowledge base
- Inference engine
- Defuzzification module



Why should we use Fuzzy Inference Systems?

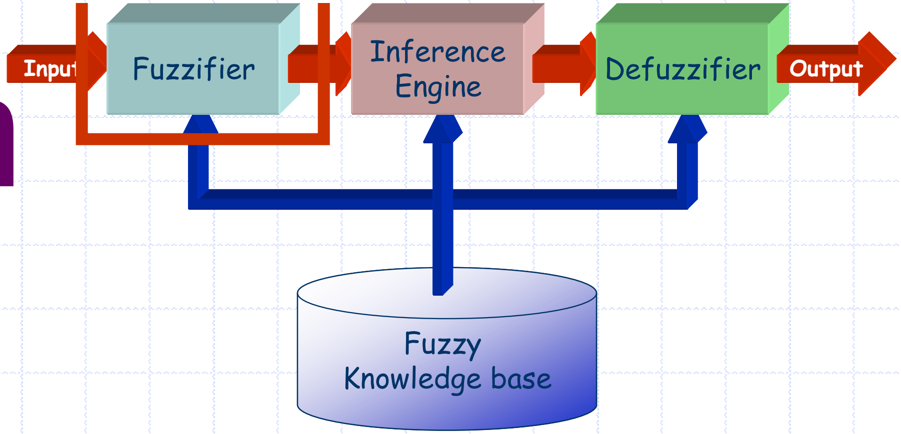
- Fuzzy logic does not solve new problems. It uses new methods to solve everyday problems.
- Mathematical concepts within fuzzy reasoning are very simple.
- Fuzzy logic is flexible: it is easy to modify a FIS just by adding or deleting rules. There is no need to create a new FIS from scratch.
- Fuzzy logic allows imprecise data (it does NOT work with uncertainty): it handles elements in a fuzzy set, i.e. membership values. For instance, fuzzy logic works with 'He is tall to the degree 0.8' instead of 'He is 180cm tall'.

Why should we use Fuzzy Inference Systems?

- Fuzzy logic is built on top of the knowledge of experts: it relies on the know-how of the ones who understand the system.
- Fuzzy logic can be blended with other classic control techniques

Fuzzification

Fuzzification



FUZZIFICATION

- Fuzzification is the process of making a crisp quantity fuzzy.

- In the real world, hardware such as a digital voltmeter generates crisp data, but these data are subject to experimental error.

Fuzzification

- Determine degree of membership of sensor reading

IF Close-by(right-sensor) THEN Left

- Get value of sensor right-sensor
- Find membership value of value in fuzzy set Close-by
- This value determines activation of this rule

Fuzzification

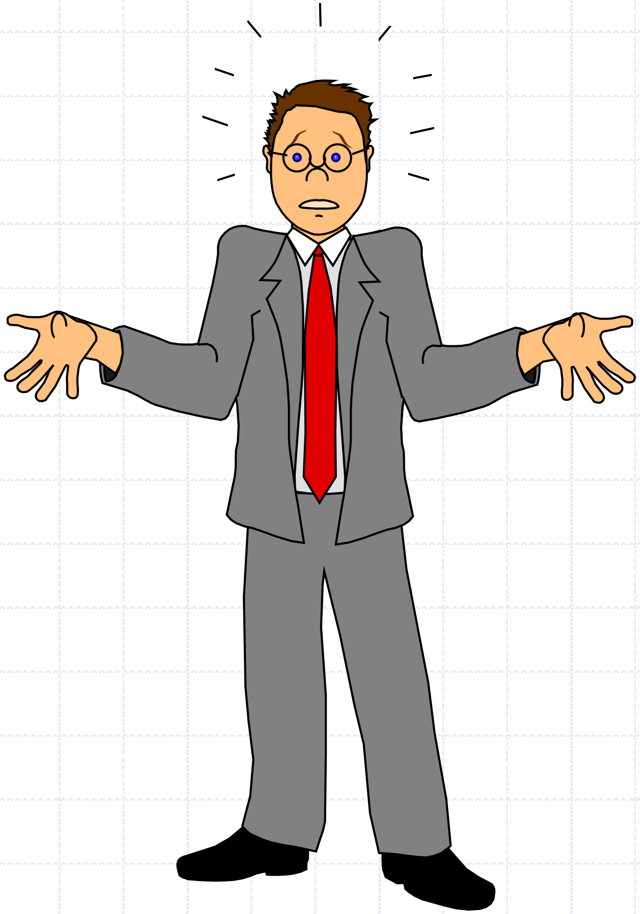
We use Fuzzy Logic to combine predicates

IF Close-by(right-sensor) AND FAR(goal-location)
THEN Left

We do this for all rules.

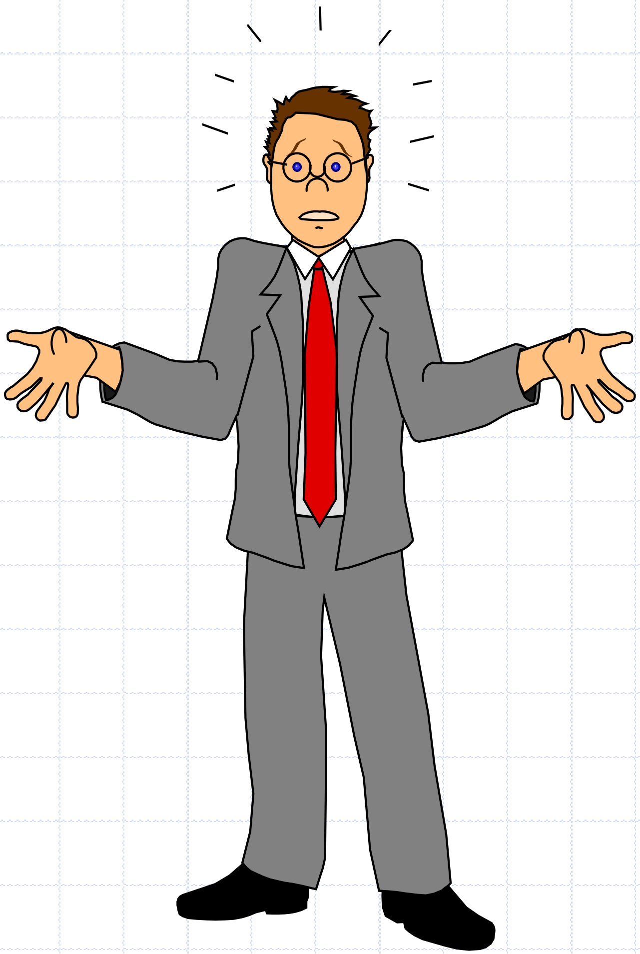
(normally some would get activation 0)

How tall is Kevin?



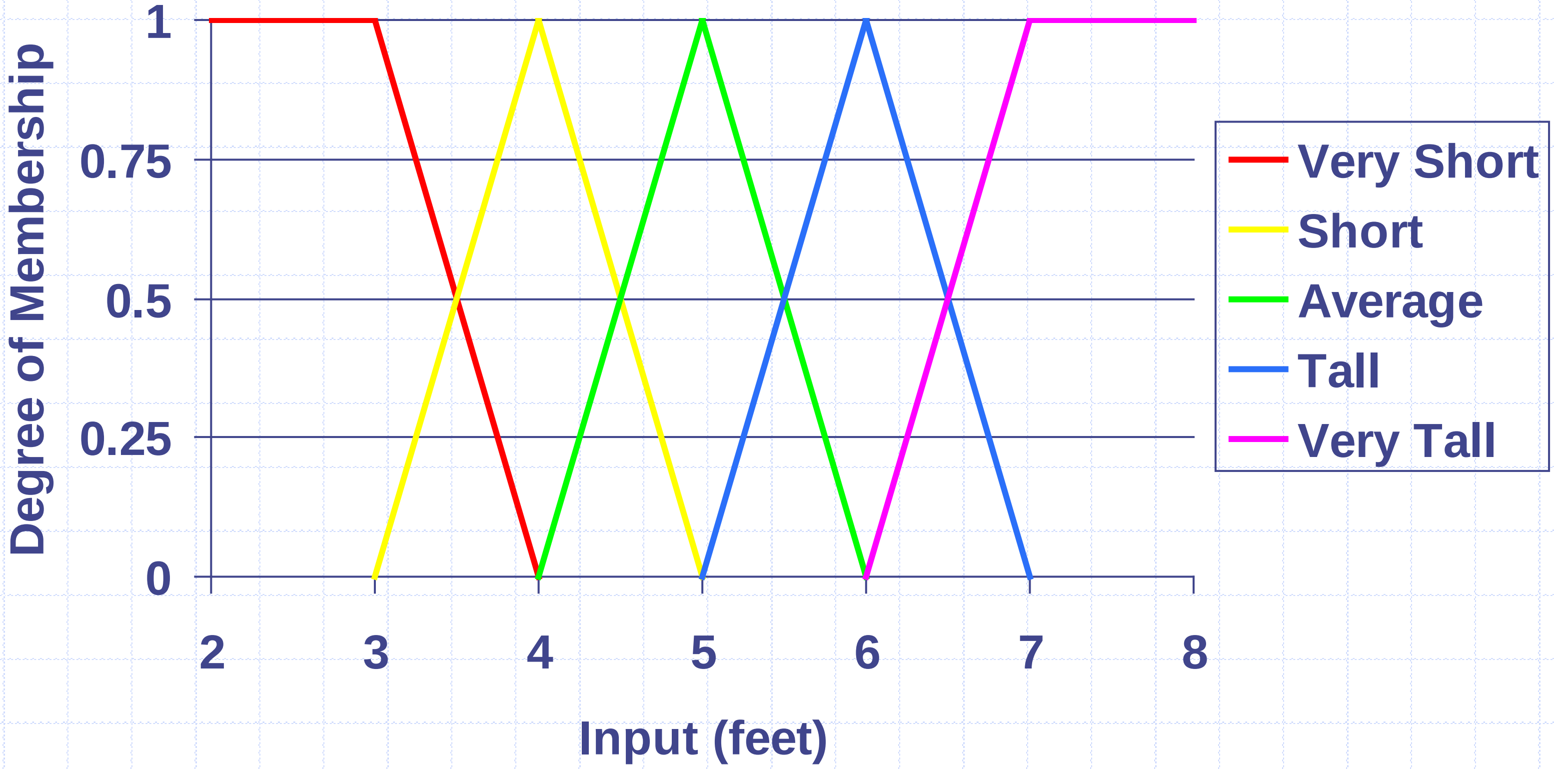
- Very Tall?
- Tall?
- Average?
- Short?
- Very Short?

How tall is Kevin?

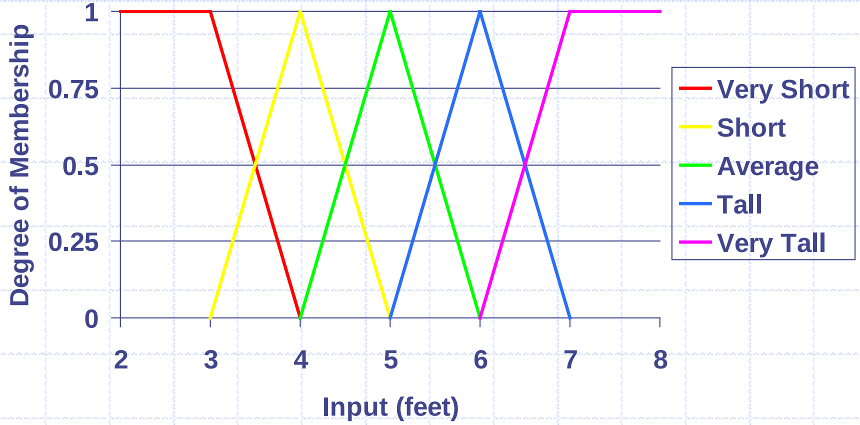


- Very Tall (7 feet)?
- Tall (6 feet)?
- Average (5 feet)?
- Short (4 feet)?
- Very Short (3 feet)?

Fuzzification Rules



Some Examples:



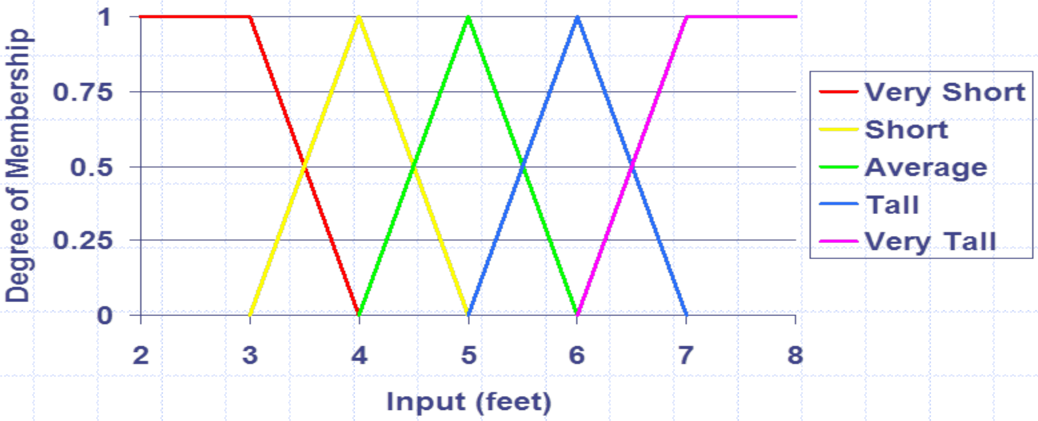
If you are <u>5</u> feet:

- Very tall - 0%
- Tall - 0%
- Average - 100%
- Short - 0%
- Very Short - 0%

Same as Boolean logic (so far...)

- Very Tall (7 feet)?
- Tall (6 feet)?
- Average (5 feet)?
- Short (4 feet)?
- Very Short (3 feet)?

Examples:



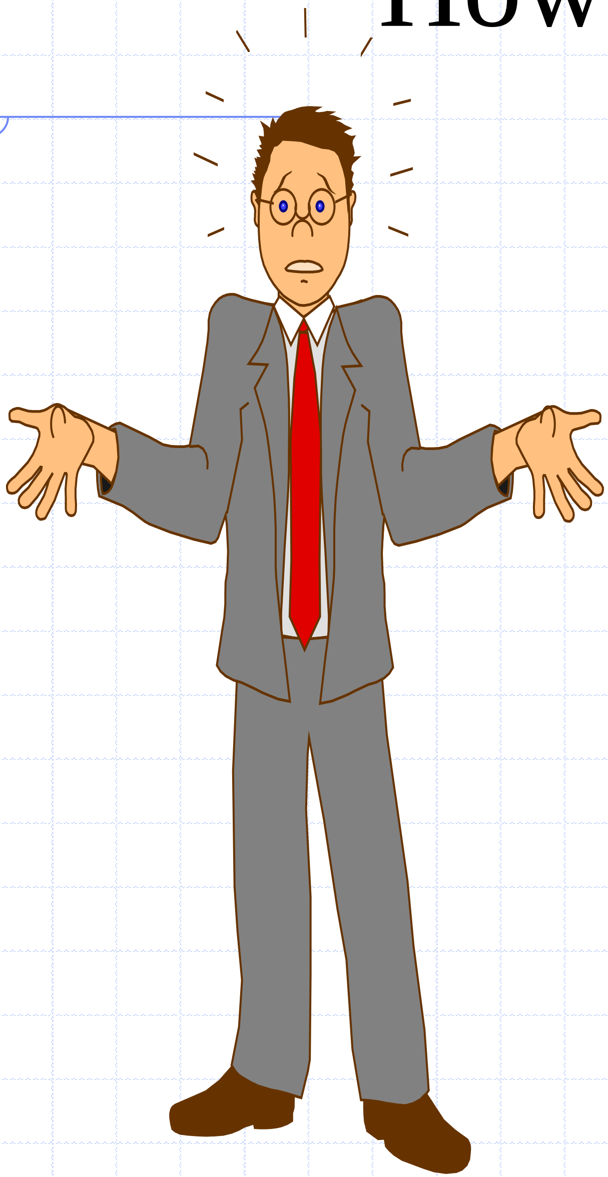
If you are 5½ feet:

- Very tall - 0%
- Tall - **50%**
- Average - **50%**
- Short - 0%
- Very Short - 0%

NOT Boolean logic

- Very Tall (7 feet)?
- Tall (6 feet)?
- Average (5 feet)?
- Short (4 feet)?
- Very Short (3 feet)?

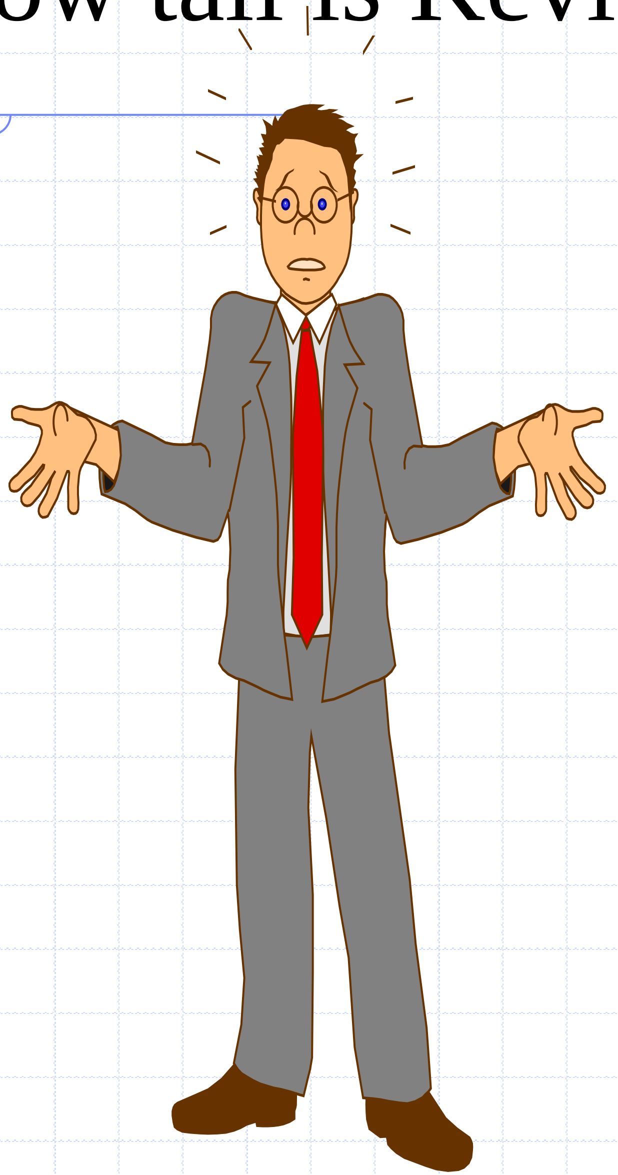
How tall is Kevin?

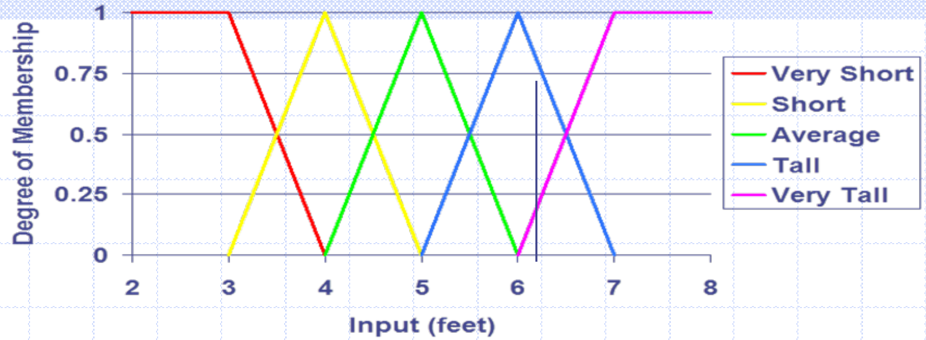


Kevin is 6’ 2”

- Very Tall -
- Tall -
- Average -
- Short -
- Very Short -

How tall is Kevin?





Kevin is 6’ 2”

- Very Tall - 25%
- Tall - 75%
- Average - 0%
- Short - 0%
- Very Short - 0%

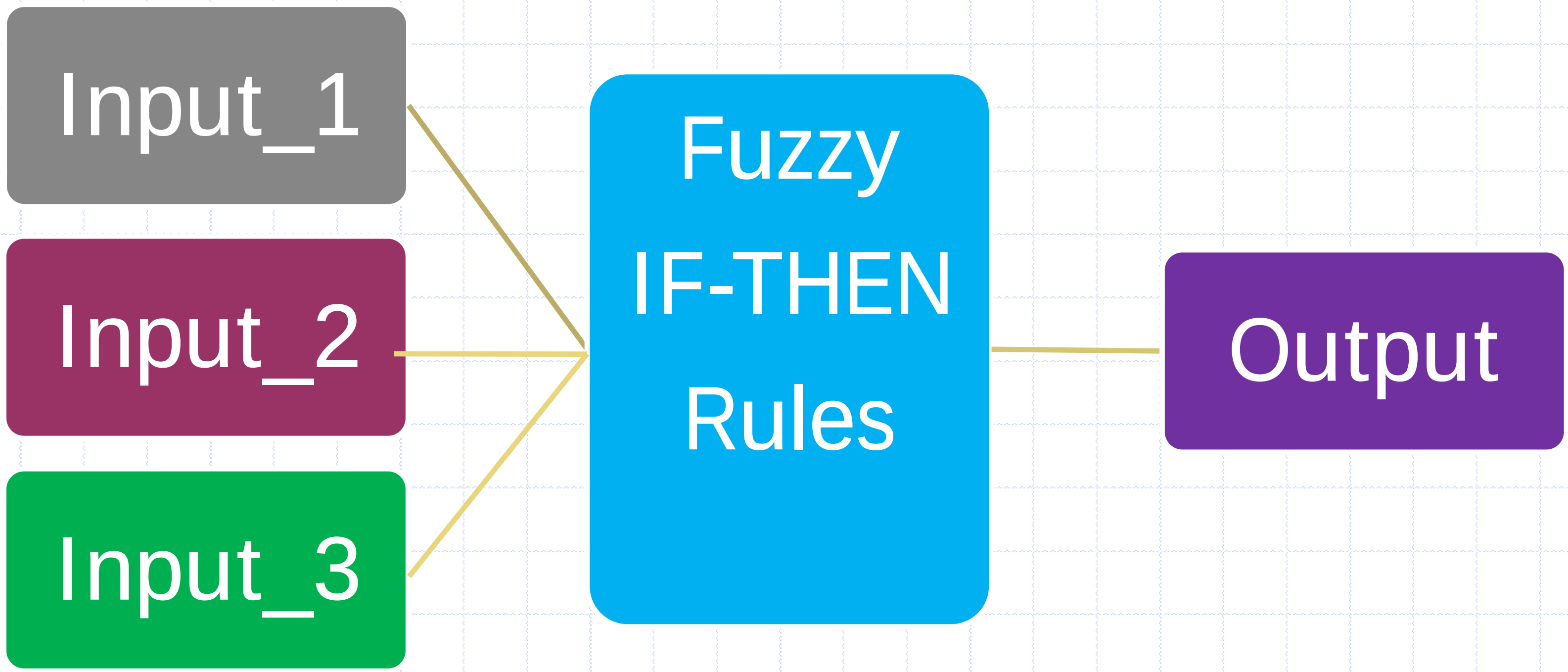
Fuzzification

- Establishes the fact base of the fuzzy system. It identifies the input and output of the system, defines appropriate IF THEN rules, and uses raw data to derive a membership function.
- Consider an air conditioning system that determine the best circulation level by sampling temperature and moisture levels. The inputs are the current temperature and moisture level. The fuzzy system outputs the best air circulation level: “none”, “low”, or “high”. The following fuzzy rules are used:

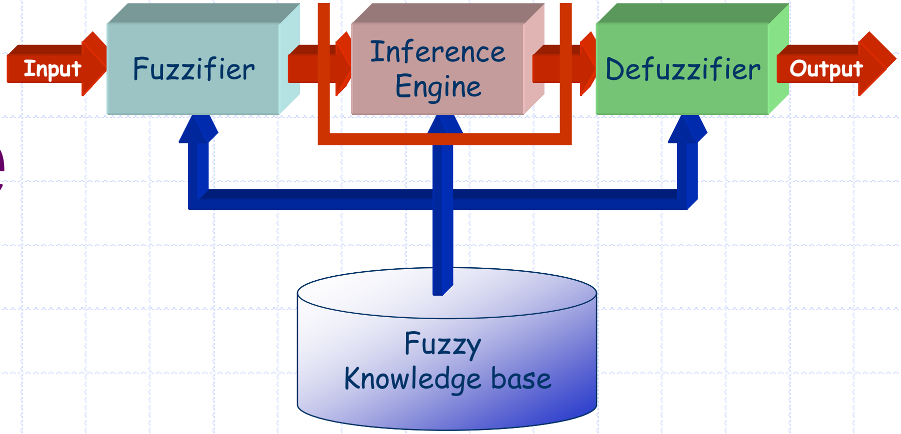
1. If the room is hot, circulate the air a lot.
2. If the room is cool, do not circulate the air.
3. If the room is cool and moist, circulate the air slightly.

- A knowledge engineer determines membership functions that map temperatures to fuzzy values and map moisture measurements to fuzzy values.

Fuzzy Inference (Expert) Systems



Inference Engine



Inference

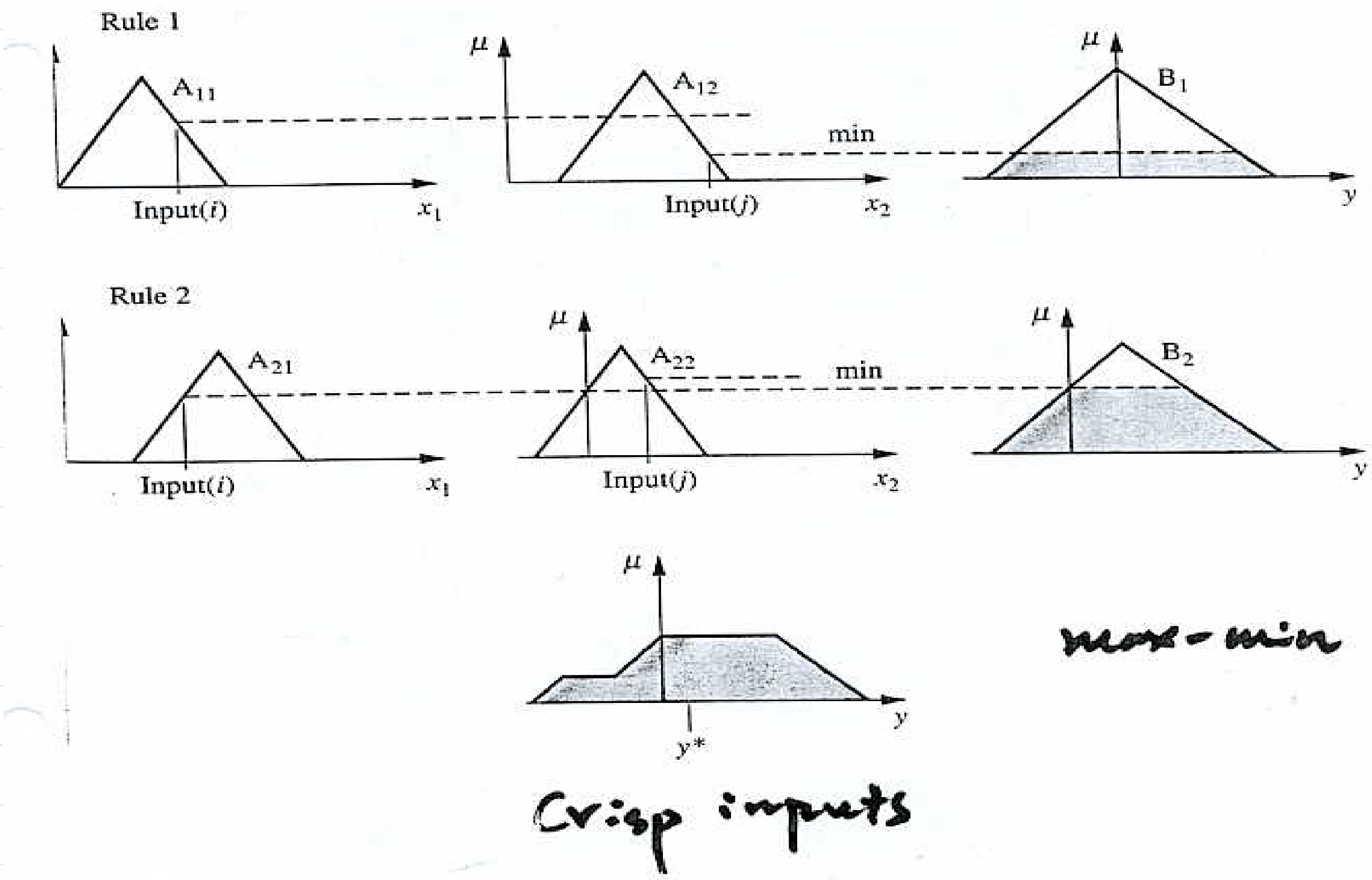
- Evaluates all rules and determines their truth values.
- Continuing the example, suppose that the system has measured temperature and moisture levels and mapped them to the fuzzy values of .7 and .1 respectively. The system now infers the truth of each fuzzy rule. To do this a simple method called MAX-MIN is used. This method sets the fuzzy value of the THEN clause to the fuzzy value of the IF clause. Thus, the method infers fuzzy values of 0.7, and 0.1 for rules 1, 2, and 3 respectively.

Inference Engine

- Fuzzy rules
  - based on fuzzy premises and fuzzy consequences

- e.g.
  - If height is Short and weight is Light then feet are Small
  - Short(height) AND Light(weight) => Small(feet)

Graphical Technique of Inference



Graphical Technique of Inference

Example:

Rule 1: if x1 is $\underset{\sim}{A}_1^1$ and x2 is $\underset{\sim}{A}_2^1$, then y is $\underset{\sim}{B}^1$

Rule 2: if x1 is $\underset{\sim}{A}_1^2$ or x2 is $\underset{\sim}{A}_2^2$, then y is $\underset{\sim}{B}^2$

input(i) = 0.35 input(j) = 55

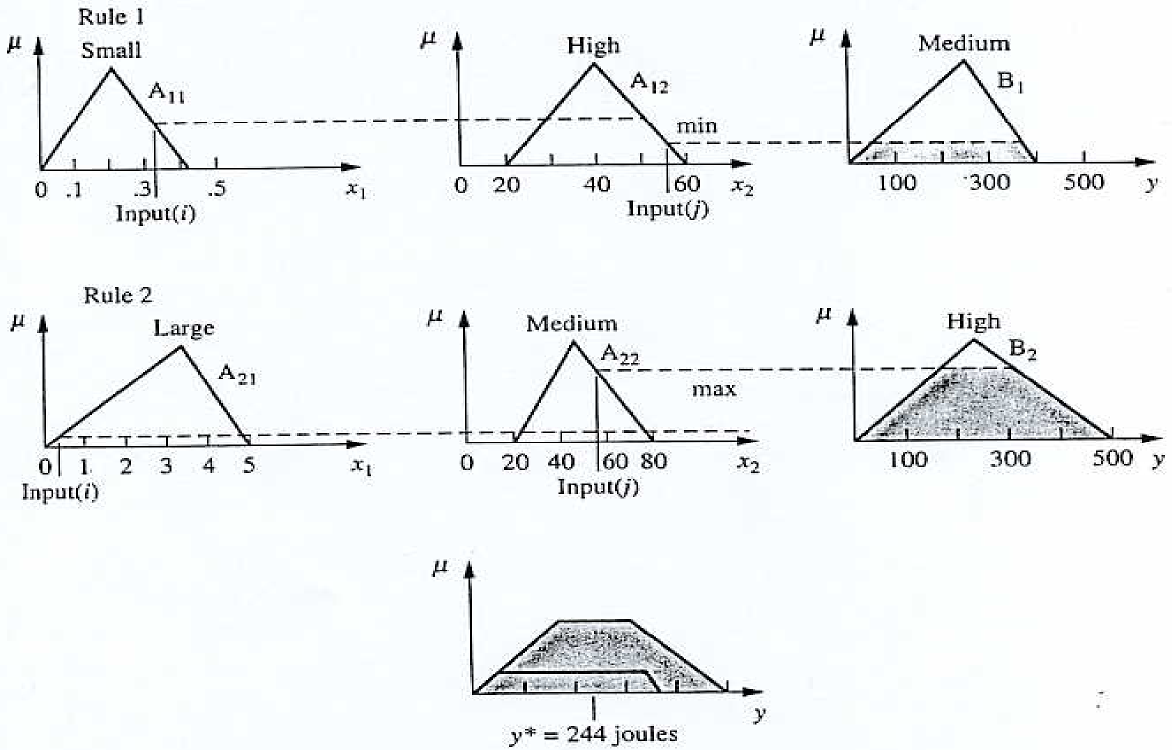


FIGURE 8.11
Fuzzy inference method using the case 1 graphical approach.

Composition

- Combines all fuzzy conclusions obtained by inference into a single conclusion. Since different fuzzy rules might have different conclusions, consider all rules.

- Continuing the example, each inference suggests a different action
  - rule 1 suggests a "high" circulation level
  - rule 2 suggests turning off air circulation
  - rule 3 suggests a "low" circulation level.

- A simple MAX-MIN method of selection is used where the maximum fuzzy value of the inferences is used as the final conclusion. So, composition selects a fuzzy value of 0.7 since this was the highest fuzzy value associated with the inference conclusions.

Fuzzy Logic Rule Base

A **General Fuzzy IF-THEN Rule** has the form

IF a_1 is A_1 AND ... AND a_n is A_n THEN b is B

Using the fuzzy logic **AND** operation, this rule is implemented by

$$\mu_{A_1}(a_1) \wedge \ldots \wedge \mu_{A_n}(a_n) \Rightarrow \mu_B(b)$$

where

$$\mu_{A_1}(a_1) \wedge \ldots \wedge \mu_{A_n}(a_n) = min\{\ \mu_{A_1}(a_1), \ldots, \mu_{A_n}(a_n)\ \}$$

Remarks

(i) There are **no fuzzy logic *OR* operations** in a general fuzzy IF-THEN rule. What should one do if a fuzzy logic implication statement involves the *OR* operation?

(ii) There are **no fuzzy logic *NOT* operations** in a general fuzzy IF-THEN rule. What should one do if a fuzzy logic implication statement involves the *NOT* operation?

(iii) There is only one conclusion after THEN. What if there are more than one conclusion: … **THEN b is B AND** … **AND c is C** ?

Fuzzy Logic Rule Base

(i) Consider the following fuzzy IF-THEN rule with ***OR*** operation:

IF a_1 is A_1 AND a_2 is A_2 OR a_3 is A_3 AND a_4 is A_4 THEN b is B

By convention in logic, this statement is understood as

(IF a_1 is A_1 AND a_2 is A_2) OR (IF a_3 is A_3 AND a_4 is A_4) THEN (b is B)

Hence, it is equivalent to the following two IF-THEN rules:

(1) **IF a_1 is A_1 AND a_2 is A_2 THEN b is B**

(2) **IF a_3 is A_3 AND a_4 is A_4 THEN b is B**

Therefore, the fuzzy logic *OR* operation is not necessary to use

Fuzzy Logic Rule Base

(ii) Consider the fuzzy logic ***NOT*** operation:

"**IF** a **is** **not** A" means "IF $\overline{a}$ is A" or "IF a is $\overline{A}$"

The last two cases "$\overline{a}$ is A" or "a is $\overline{A}$" can be evaluated by

$$\mu_A(\overline{a}) = \mu_{\overline{A}}(a) = 1 - \mu_A(a)$$

Therefore, the fuzzy logic ***NOT*** operation is not necessary to use

Fuzzy Logic Rule Base

(iii) Consider the following case with more than one conclusion:

IF a_1 is A_1 AND a_2 is A_2 THEN b is B AND c is C

This is equivalent to the following two IF-THEN rules:

(1) **IF a_1 is A_1 AND a_2 is A_2 THEN b is B**

(2) **IF a_1 is A_1 AND a_2 is A_2 THEN c is C**

EXAMPLE

Given a fuzzy logic implication statement

IF a_1 is A_1 AND a_2 is <u>not</u> A_2 <u>OR</u> a_3 is <u>not</u> A_3 THEN b is B

How can one rewrite it as a set of equivalent general fuzzy IF-THEN rules in the unified form?

First, **drop** the fuzzy logic ***<u>OR</u>*** operation:

(1) **IF a_1 is A_1 AND a_2 is <u>not</u> A_2 THEN b is B**

(2) **IF a_3 is <u>not</u> A_3 THEN b is B**

EXAMPLE

Then, drop the fuzzy logic ***NOT*** operation by rewriting them as

(1') **IF a_1 is A_1 AND $\overline{a}_2$ is A_2 THEN b is B**

(2') **IF $\overline{a}_3$ is A_3 THEN b is B**

Finally, the above two general fuzzy IF-THEN rules can be **evaluated** as follows:

(1') $\mu_{A_1}(a_1) \wedge \mu_{A_2}(\overline{a}_2) \Rightarrow \mu_B(b)$ where $\mu_{A_2}(\overline{a}_2) = 1 - \mu_{A_2}(a_2)$

(2') $\mu_{A_3}(\overline{a}_3) \Rightarrow \mu_B(b)$ where $\mu_{A_3}(\overline{a}_3) = 1 - \mu_{A_3}(a_3)$

Fuzzy Logic Rule Base

All other fuzzy logic operations can be defined and expressed only by ***AND*** and ***OR*** operations and be evaluated by

$$\mu_{A_1}(a_1) \wedge \mu_{A_2}(a_2) = min\{\ \mu_{A_1}(a_1),\ \mu_{A_2}(a_2)\ \}$$

$$\mu_{A_1}(a_1) \vee \mu_{A_2}(a_2) = max\{\ \mu_{A_1}(a_1),\ \mu_{A_2}(a_2)\ \}$$

$$\mu_A(\ \overline{a}\ ) = \mu_{\overline{A}}\ (a) = 1 - \mu_A(a)$$

$$\mu_A(a \Rightarrow b) = min\{\ 1,\ 1 + \mu_A(b) - \mu_A(a)\ \}$$

$$\mu_A(a \Leftrightarrow b) = 1 - |\ \mu_A(a) - \mu_A(b)\ |$$

Fuzzy Logic Rule Base

Consequently, all finite combinations of fuzzy logic operations can be expressed only by the *AND* operation, so that **IF-AND-THEN** is sufficient for inference.

In summary, a finite fuzzy logic implication statement can always be described by the following **fuzzy logic rule base**:

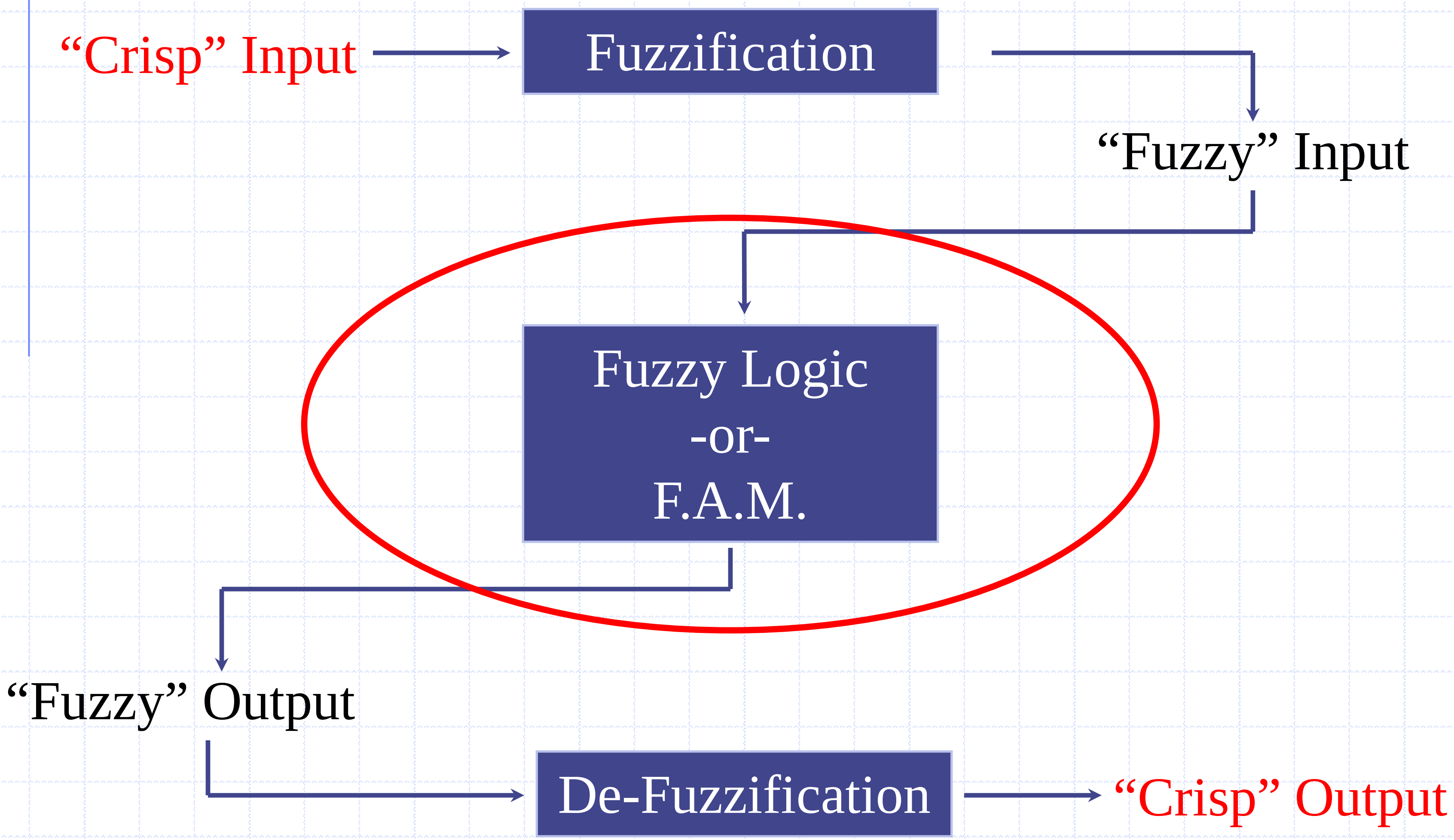
(1) **IF** a_{11} **is** A_{11} **AND ... AND** a_{1n} **is** A_{1n} **THEN** b_1 **is** B_1

(2) **IF** a_{21} **is** A_{21} **AND ... AND** a_{2n} **is** A_{2n} **THEN** b_2 **is** B_2

. . .

(m) **IF** a_{m1} **is** A_{m1} **AND ... AND** a_{mn} **is** A_{mn} **THEN** b_m **is** B_m

Fuzzy Logic Process



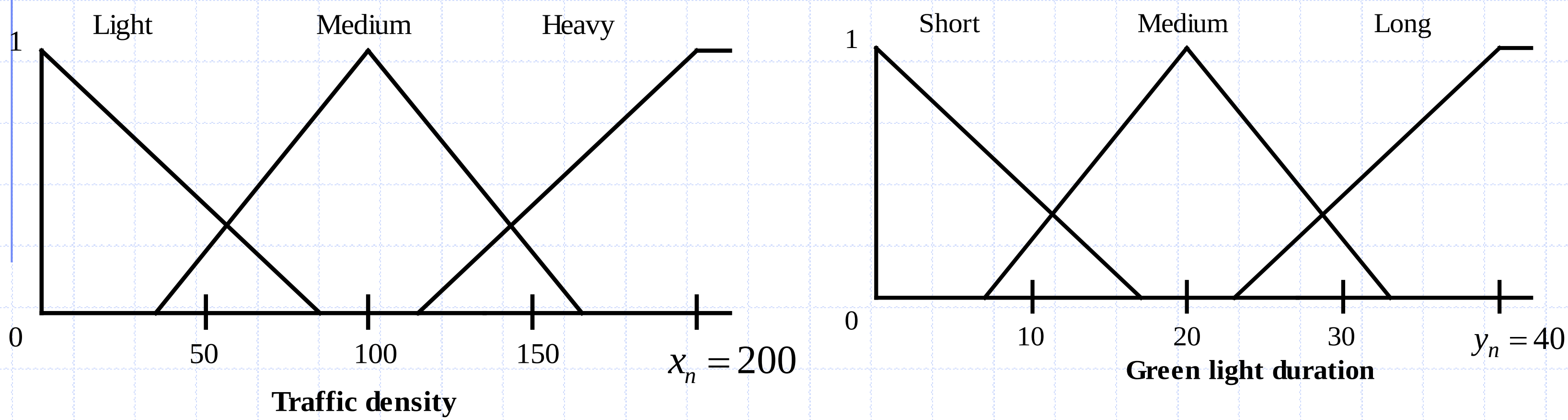
Fuzzy Associative Memory (FAM)

Fuzzy Associative Memory

- It is a Fuzzy Truth Table
- Shows all possible outputs for all possible inputs
- Easy to create!
- Fuzzy associative memories are transformations

FAM map fuzzy sets to fuzzy sets, units cube to units cube.

FAMs as mapping



Three possible fuzzy subsets of traffic-density and green light duration, space X and Y.

Fuzzy Associative Memories (FAMs)

A fuzzy system with n non-interactive inputs and a single output. Each input universe of discourse, x_1, x_2, …, x_n is partitioned into k fuzzy partitions

The total # of possible rules governing this system is given by: $I = k^n$

If x_1 is partitioned into k_1 partitions
x_2 is partitioned into k_2 partitions
:
.
x_n is partitioned into k_n partitions

$$I = k_1 \bullet k_2 \bullet \ldots \bullet k_n$$

Fuzzy Associative Memories (FAMs)

Example: for n = 2

	A1	A2	A3	A4	A5	A6	A7
B1	C1		C4	C4		C3	C3
B2		C1				C2	
B3	C4		C1			C1	C2
B4	C3	C3		C1		C1	C2
B5	C3		C4	C4	C1		C3

A $\rightarrow$ A1 — A7

B $\rightarrow$ B1 — B5

Output: C $\rightarrow$ C1 — C4

$l = k_1 \bullet k_2 = 7*5=35$ rules

Fuzzy Associative Memories (FAMs)

Example: Non-linear membership function: $y = 10 \sin x$

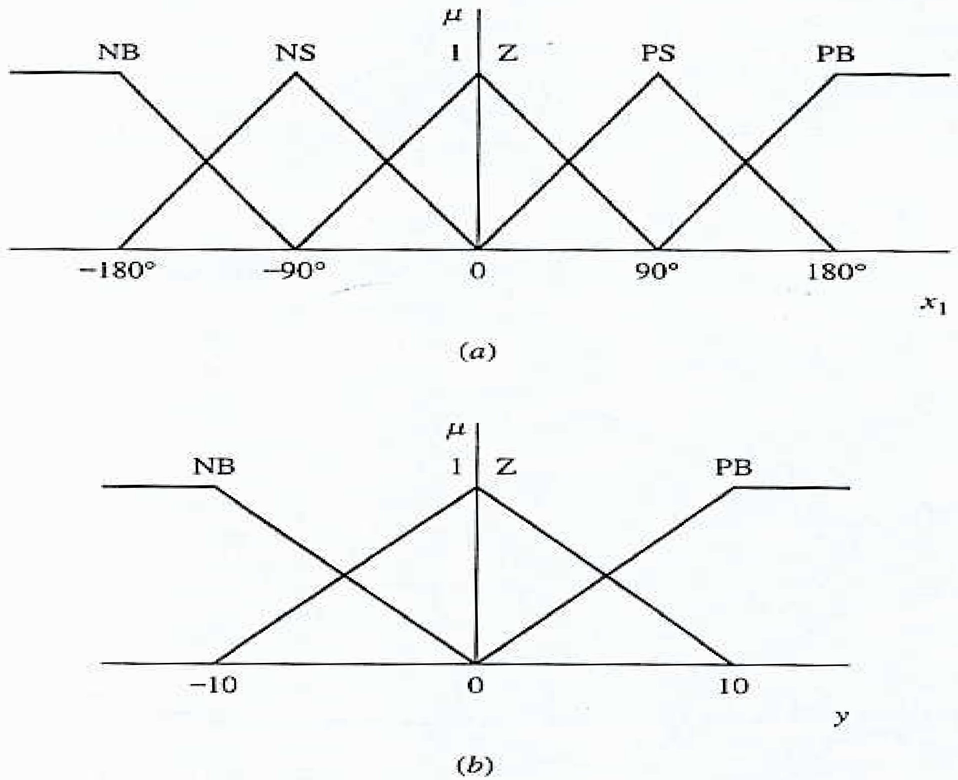
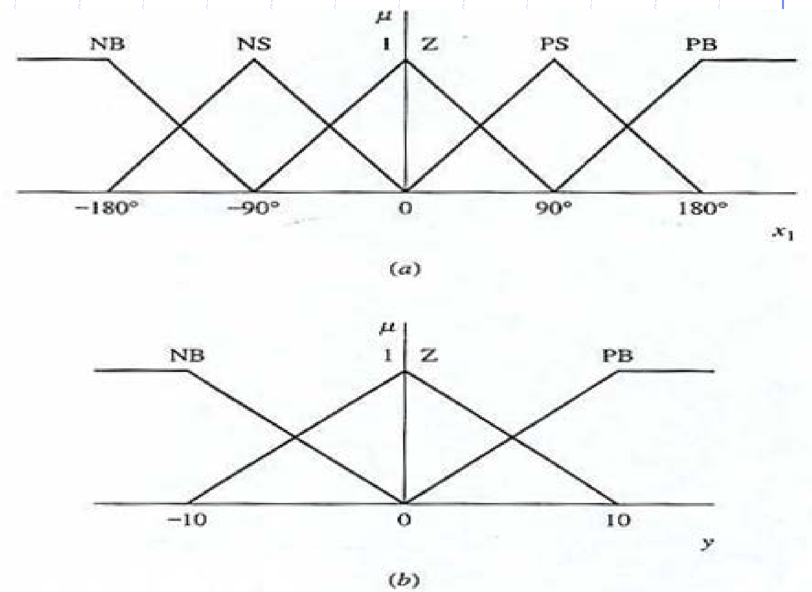


FIGURE 9.13
Fuzzy membership functions for the input and output spaces: (*a*) Five partitions for the input variable, x_1; (*b*) three partitions for the output variable, y.

Fuzzy Associative Memories (FAMs)

Few simple rules for y = 10 sin x

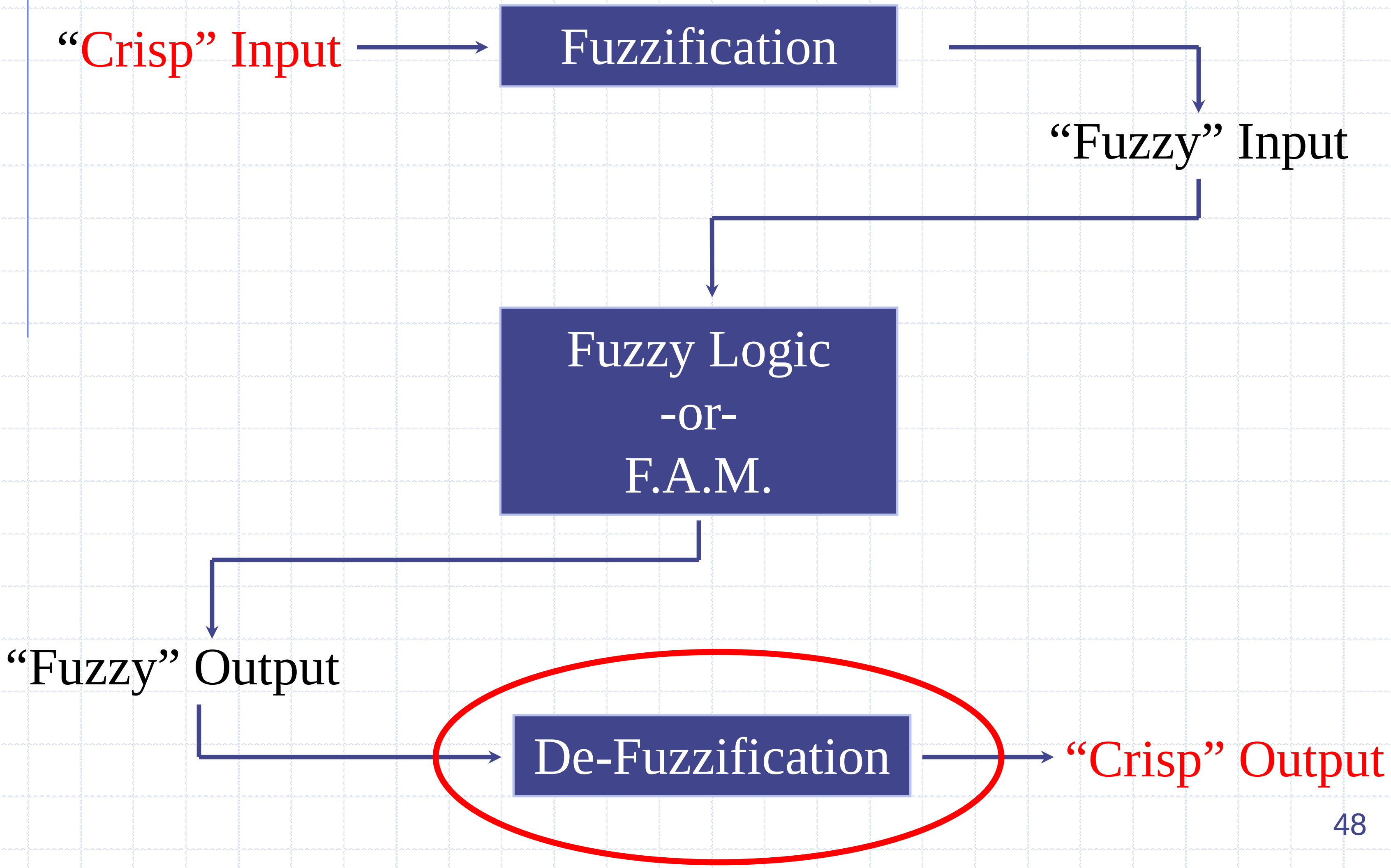
1. IF x1 is Z or P B, THEN y is z.
2. IF x1 is PS, THEN y is PB.
3. IF x1 is z or N B, THEN y is z
4. IF x1 is NS, THEN y is NB



FAM for the four simple rules

x1	N B	N S	z	P S	P B
y	z	N B	z	P B	z

Fuzzy Logic Process



Defuzzification (Fuzzy-To-Crisp conversions)

Using fuzzy to reason, to model
Using crisp to act
Like analog → digital → analog

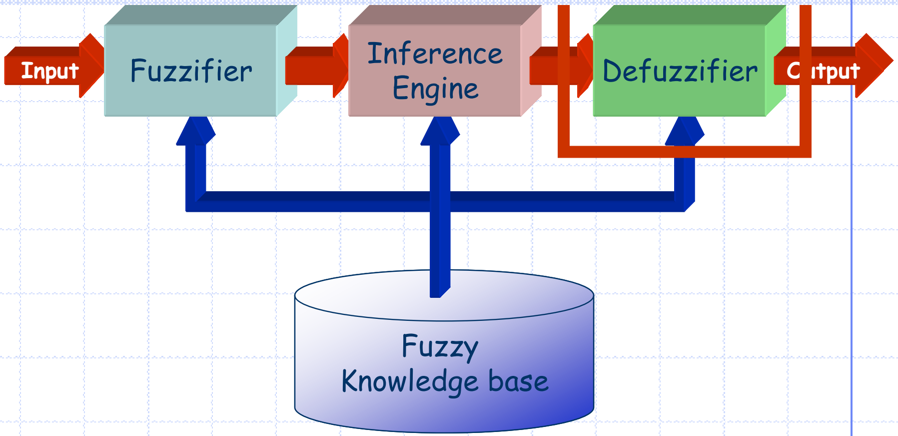
Defuzzification is the process: round it off to the nearest vertex.

Fuzzy set (collection of membership values).

Defuzzification

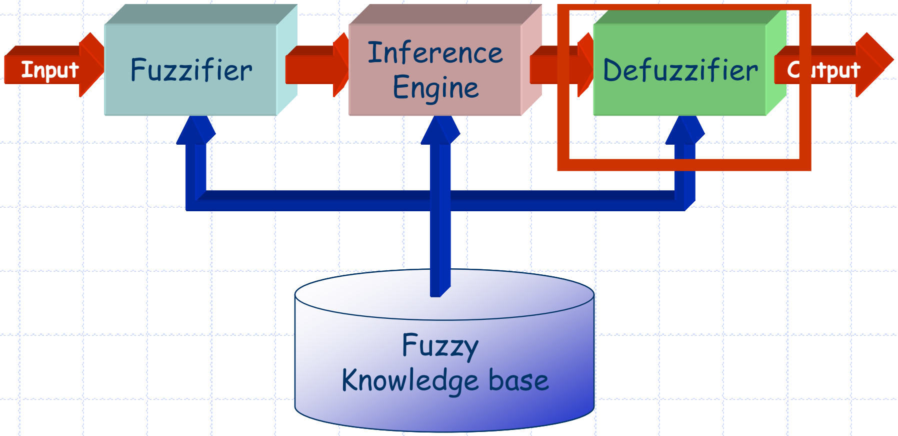
- Convert the fuzzy value obtained from composition into a “crisp” value. This process is often complex since the fuzzy set might not translate directly into a crisp value. Defuzzification is necessary, since controllers of physical systems require discrete signals.
- Defuzzification is the conversion of a fuzzy quantity to a precise quantity, just as fuzzification is the conversion of a precise quantity to a fuzzy quantity.
- The output of a fuzzy process can be the logical union of two or more fuzzy membership functions defined on the universe of discourse of the output variable.

Defuzzification

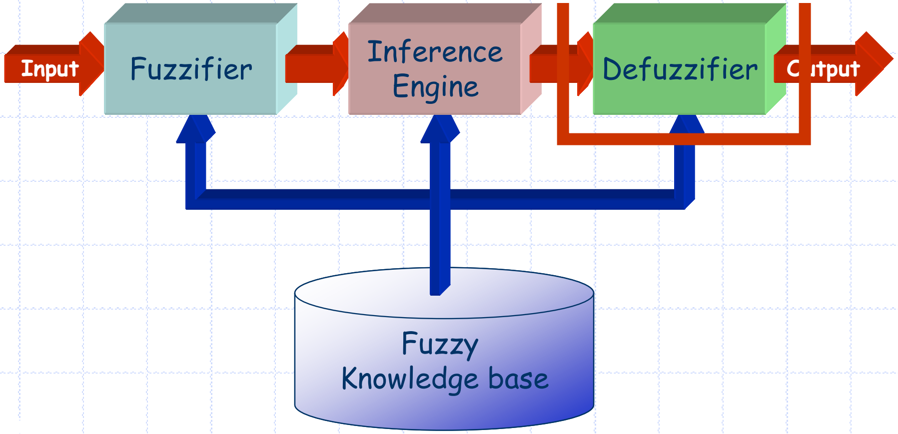


- Converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier.
- Some commonly used defuzzifying methods:
  - Centroid of area (COA)
  - Weighted average method
  - Mean of maximum (MOM)
  - Smallest of maximum (SOM)

Defuzzification



Defuzzification



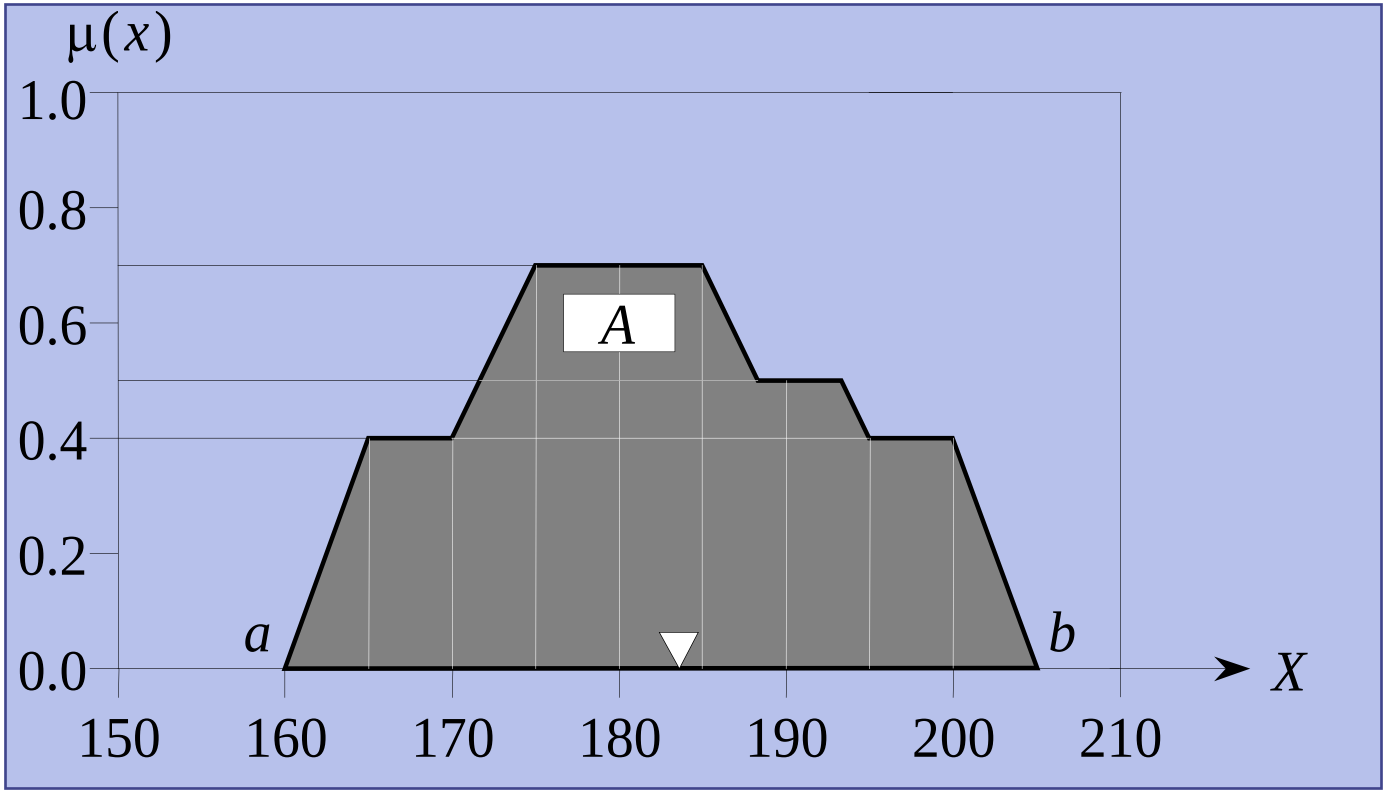
$$z_{COA} = \frac{\int_Z \mu_A(z) z dz}{\int_Z \mu_A(z) dz},$$

Centroid

- It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this centre of gravity (COG) can be expressed as:

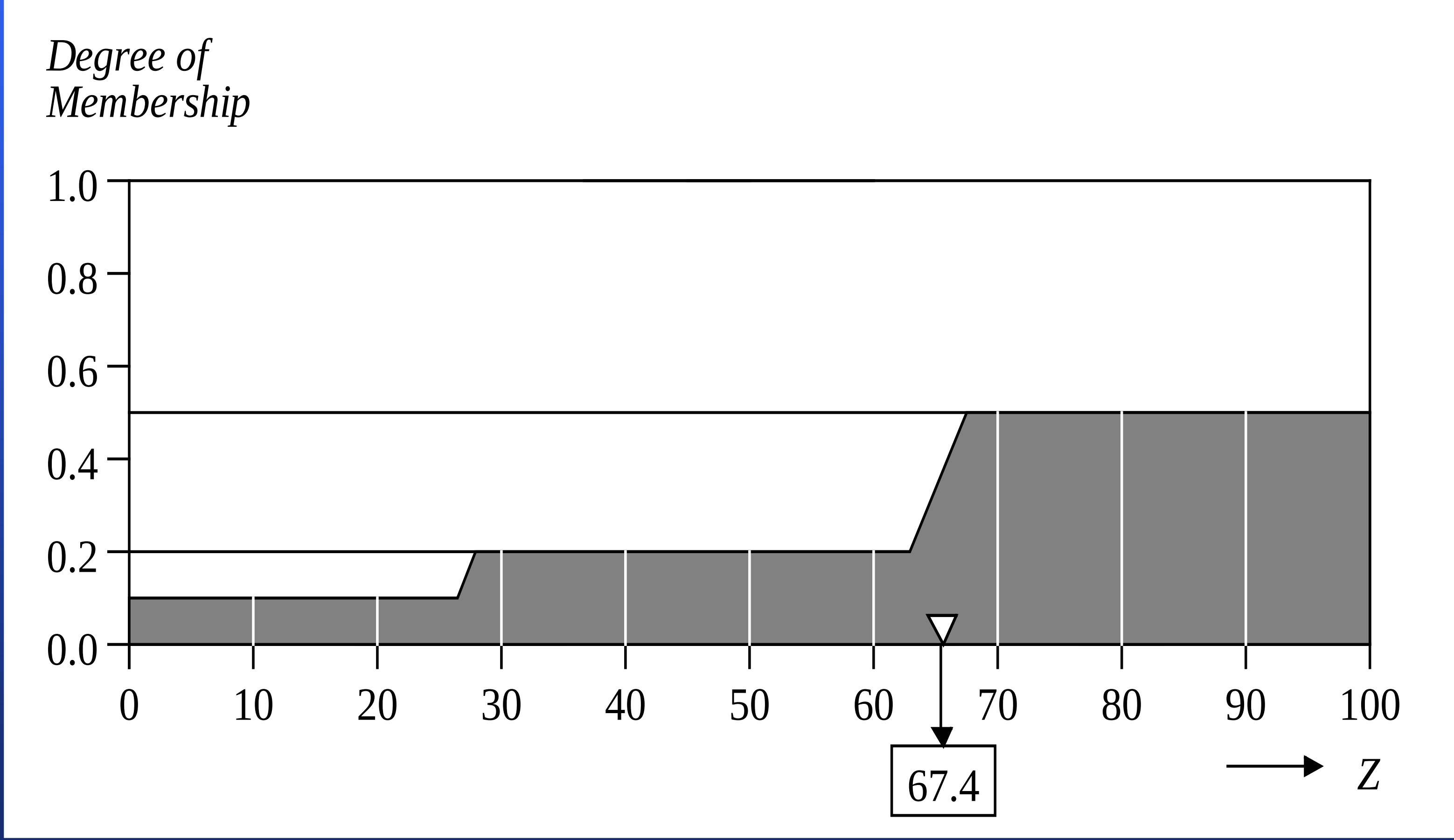
$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

- Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set, *A*, on the interval, *ab*.
- ***A reasonable estimate*** can be obtained by calculating it over a sample of points.

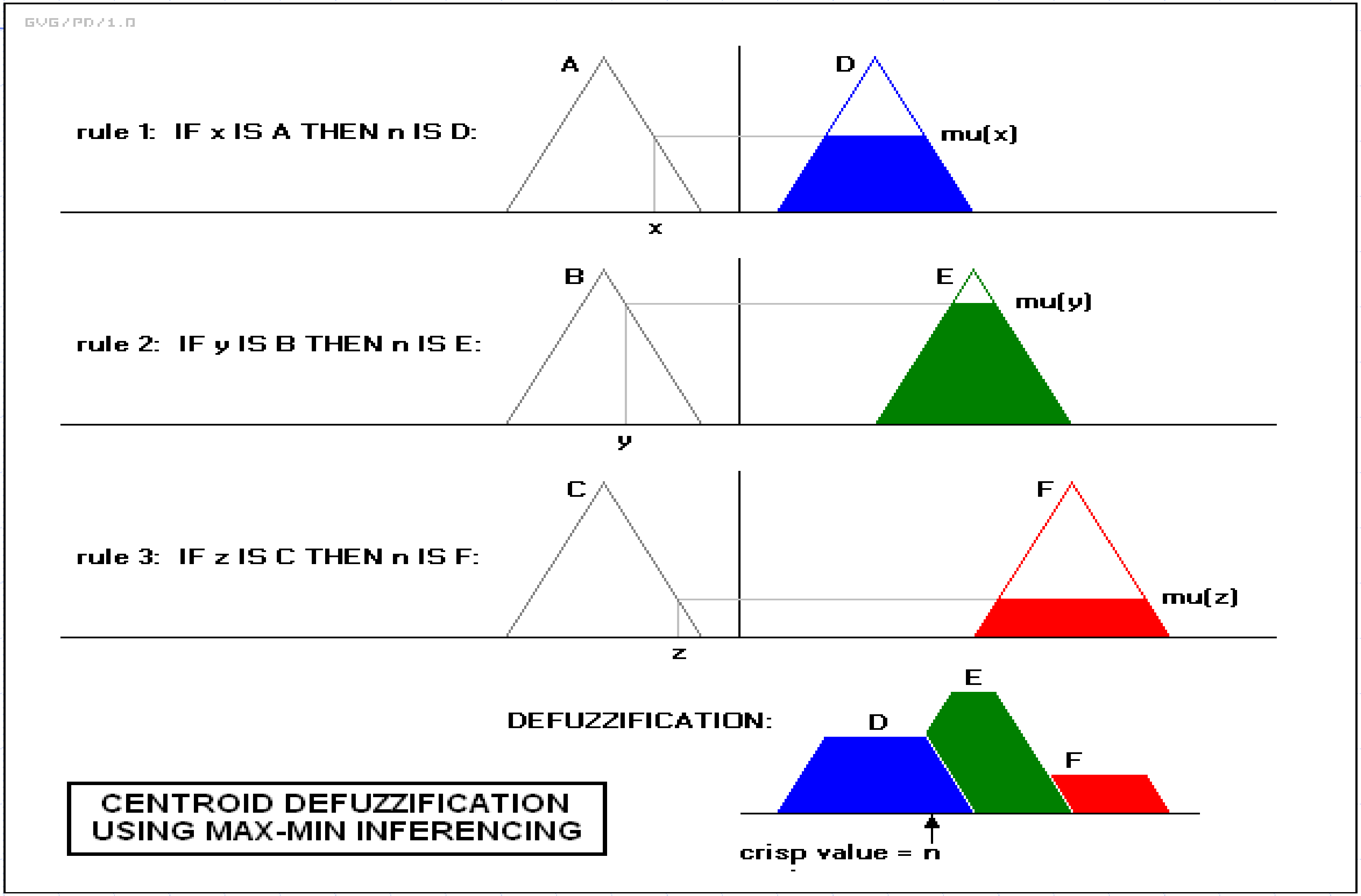


Centre of gravity (COG):

$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



Centroid

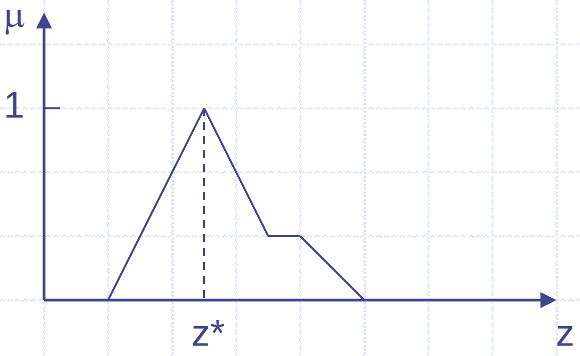


GVG/PD/1.0
rule 1: IF x IS A THEN n IS D:
A
D
mu(x)
x
rule 2: IF y IS B THEN n IS E:
B
E
mu(y)
y
rule 3: IF z IS C THEN n IS F:
C
F
mu(z)
z
DEFUZZIFICATION:
D
E
F
CENTROID DEFUZZIFICATION
USING MAX-MIN INFERENCING
crisp value = n

Defuzzification Methods

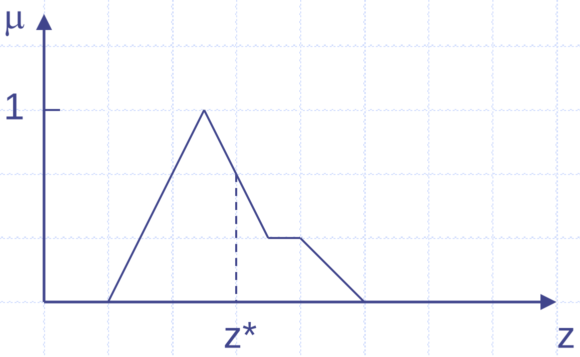
Max-membership principle

$$\mu_c(Z^*) \geq \mu_c(z)\ \forall\ z \in Z$$



Centroid principle

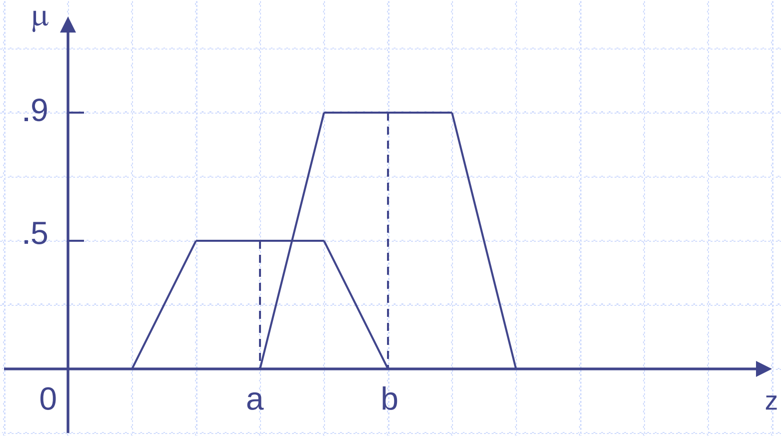
$$z^* = \frac{\int \mu_c(z) \bullet z dz}{\int \mu_c(z) dz}$$



Defuzzification Methods

Weighted average method
(Only valid for symmetrical output membership functions)

$$z^* = \frac{\sum \mu_{\underset{\sim}{c}}(\bar{z}) \bullet \bar{z}}{\sum \mu_{\underset{\sim}{c}}(\bar{z})}$$



$$z^* = \frac{\sum \mu_C(\bar{z}) \cdot \bar{z}}{\sum \mu_C(\bar{z})} \qquad z^* = \frac{a(.5) + b(.9)}{.5 + .9}$$

Formed by weighting each functions in the output by its respective maximum membership value

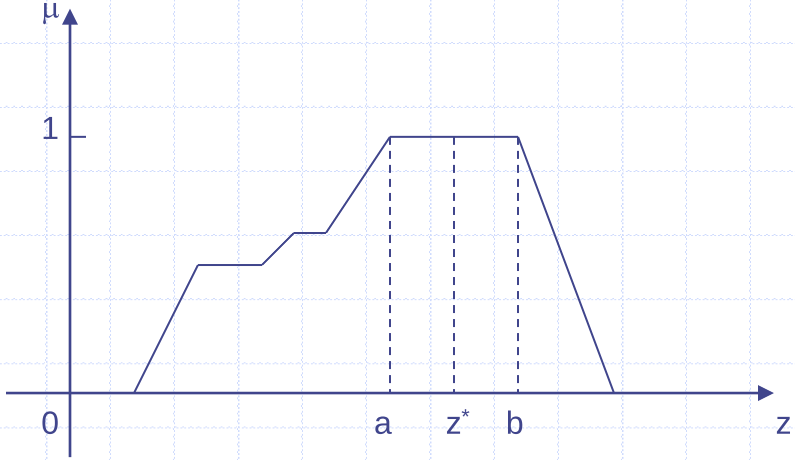
Produces results very close to centroid method

Less computational intensive

Defuzzification Methods

Mean-max membership
(middle-of-maxima method)

$$z^* = (a+b)/2$$



Defuzzification Methods

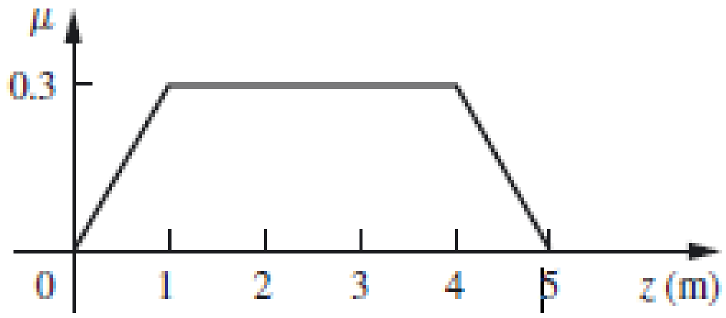
Example 4.3:

A railroad company intends to lay a new rail line in a particular part of a county. The whole area through which the new line is passing must be purchased for right-of-way considerations. *It is surveyed in three stretches*, and the data are collected for analysis. The surveyed data for the road are given by the sets $\underset{\sim}{B}_1, \underset{\sim}{B}_2$ and $\underset{\sim}{B}_3$, where the sets are defined on the universe of right-of-way widths, in meters. For the railroad to purchase the land, it must have an assessment of the amount of land to be bought. The three surveys on the right-of-way width are ambiguous , however, because some of the land along the proposed railway route is already public domain and will not need to be purchased.

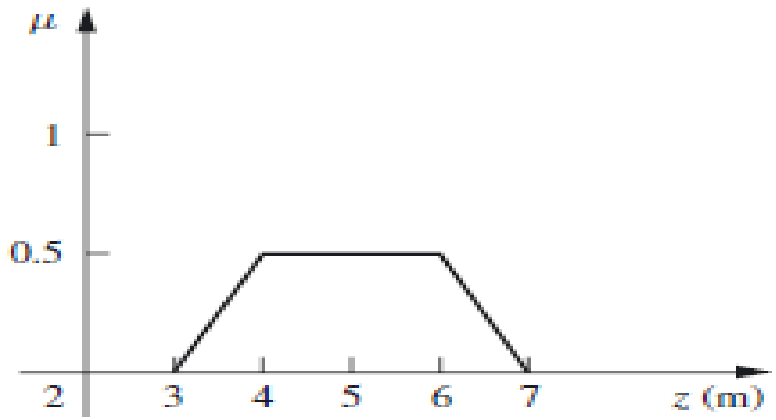
Additionally, the original surveys are so old (circa 1860) that some ambiguity exists on the boundaries and public right-of-way for old utility lines and old roads. The three fuzzy sets $\underset{\sim}{B}_1, \underset{\sim}{B}_2$ and $\underset{\sim}{B}_3$, shown in the figures below, represent the uncertainty in each survey as to the membership of the right-of-way width, in meters, in privately owned land.

We now want to aggregate these three survey results to find the single most nearly representative right-of-way width (z) to allow the railroad to make its initial estimate

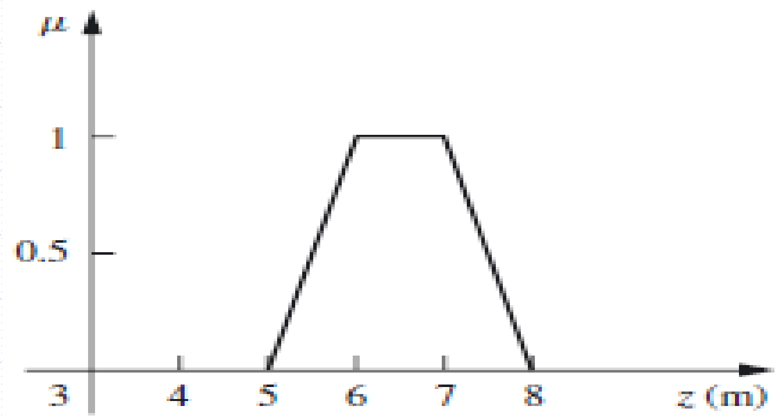
Defuzzification Methods



public right-of-way width (z) for survey 1.



Fuzzy set $\underset{\sim}{B}_2$: public right-of-way width (z) for survey 2.



public right-of-way width (z) for survey 3.

Defuzzification Methods

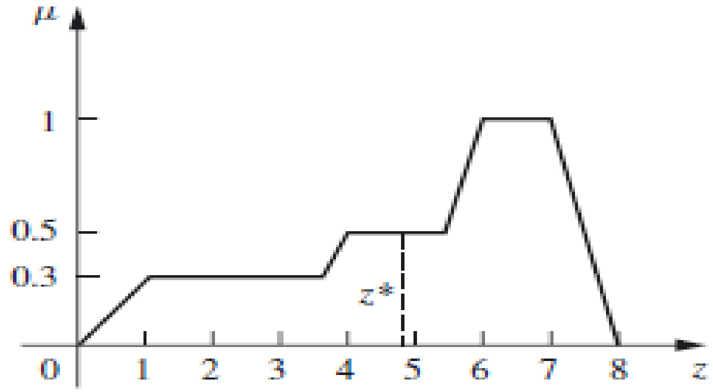
Centroid method:

$$z^* = \frac{\int \mu_{\underset{\sim}{B}}(z) \bullet z dz}{\int \mu_{\underset{\sim}{B}}(z) dz} =$$

$$\left[\int_0^1 (.3z) z dz + \int_1^{3.6} (.3z) dz + \int_{3.6}^4 \left(\frac{z-3}{2} \right) z dz + \int_4^{5.5} (.5) z dz + \int_{5.5}^6 (z-5) z dz + \int_6^7 z dz + \int_7^8 (8-z) z dz \right]$$

$$\div \left[\int_0^1 (.3z) dz + \int_1^{3.6} (.3) dz + \int_{3.6}^4 \left(\frac{z-3}{2} \right) dz + \int_4^{5.5} (.5) dz + \int_{5.5}^6 (z-5) dz + \int_6^7 dz + \int_7^8 (8-z) dz \right]$$

$$= 4.9 meters$$

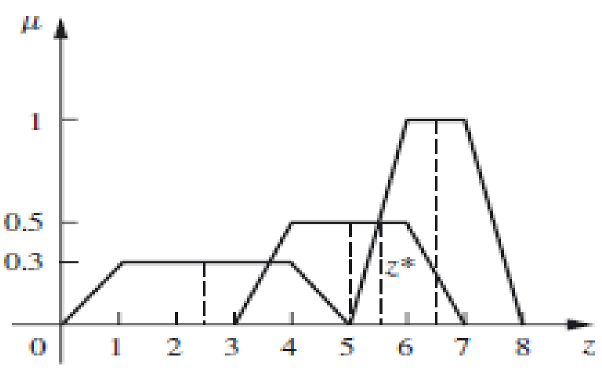


Defuzzification Methods

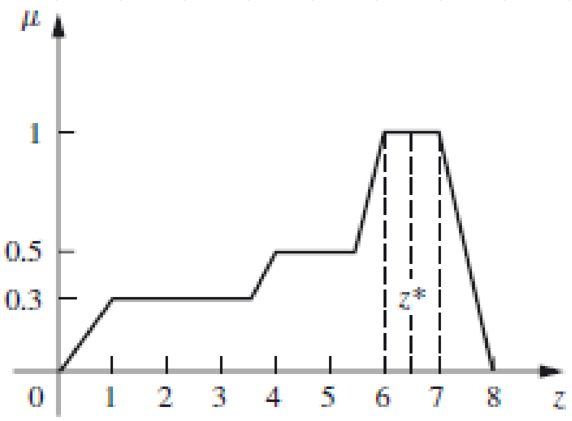
Weighted-Average Method:

$$z^* = \frac{(.3\times 2.5)+(.5\times 5)+(1\times 6.5)}{.3+.5+1} = 5.41 meters$$

Mean-Max Method: $(6+7)/2 = 6.5 meters$



The weighted average method for finding z^*.

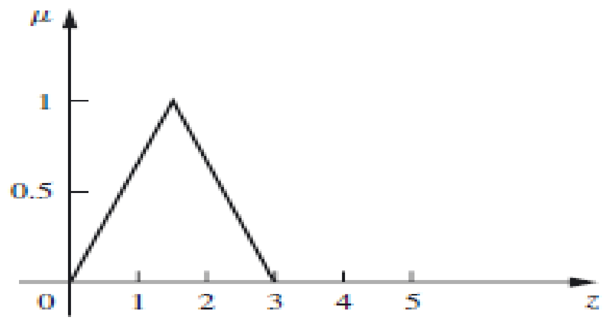


The mean max membership method for finding z^*.

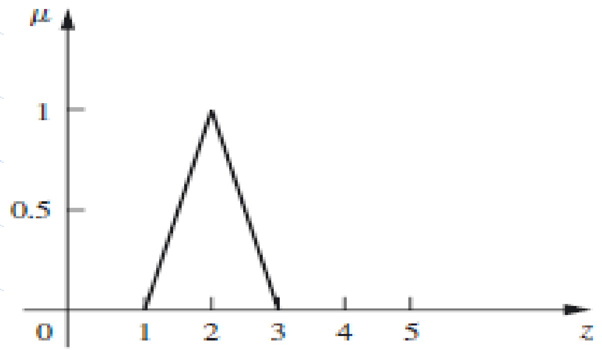
Defuzzification Methods

Example 4.4. Many products, such as tar, petroleum jelly, and petroleum, are extracted from crude oil. In a newly drilled oil well, three sets of oil samples are taken and tested for their viscosity. The results are given in the form of the three fuzzy sets B1, B2, and B3, all defined on a universe of normalized viscosity, as shown in Figures below. we want to find the most nearly representative viscosity value for all three oil samples, and hence find $z*$ for the three fuzzy viscosity sets.

Defuzzification Methods

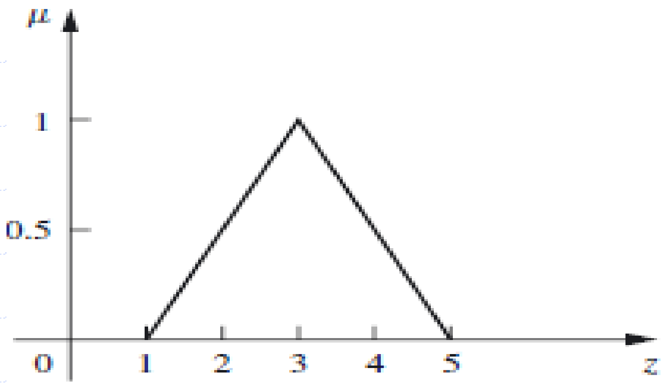


Membership in viscosity of oil sample 1, $\underset{\sim}{B}_1$.

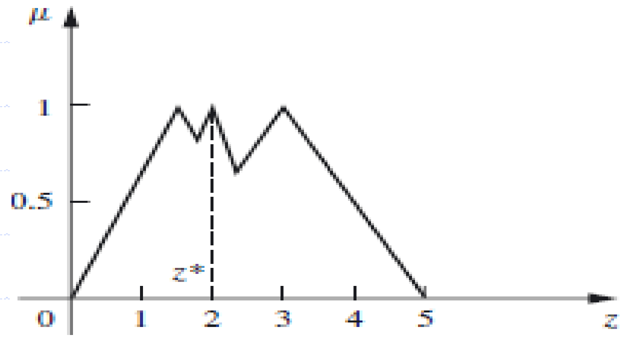


Membership in viscosity of oil sample 2, $\underset{\sim}{B}_2$.

Defuzzification Methods



Membership in viscosity of oil sample 3, $\underset{\sim}{B}_3$.



Logical union of three fuzzy sets $\underset{\sim}{B}_1$, $\underset{\sim}{B}_2$, and $\underset{\sim}{B}_3$.

Defuzzification Methods

According to the centroid method,

$$z^* = \frac{\int \mu_{\underset{\sim}{B}}(z) z\,dz}{\int \mu_{\underset{\sim}{B}}(z)\,dz} =$$

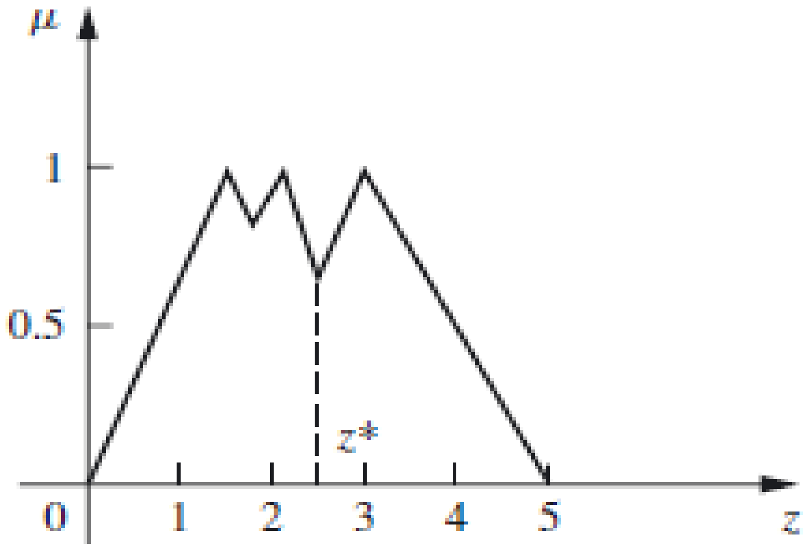
$$\left[\begin{array}{l} \int_0^{1.5} (.67z) z\,dz + \int_{1.5}^{1.8} \left(2 - .67z\right) z\,dz + \int_{1.8}^{2} \left(z - 1\right) z\,dz + \int_{2}^{2.33} \left(3 - z\right) z\,dz \\ + \int_{2.33}^{3} \left(.5z - .5\right) z\,dz + \int_{3}^{5} \left(2.5 - .5z\right) z\,dz \end{array}\right]$$

$$\div \left[\begin{array}{l} \int_0^{1.5} \left(.67z\right) dz + \int_{1.5}^{1.8} \left(2 - .67z\right) dz + \int_{1.8}^{2} \left(z - 1\right) dz + \int_{2}^{2.33} \left(3 - z\right) dz \\ + \int_{2.33}^{3} \left(.5z - .5\right) dz + \int_{3}^{5} \left(2.5 - .5z\right) dz \end{array}\right]$$

$$= 2.5$$

Defuzzification Methods

The centroid value obtained, z^*, is shown in the figure below:

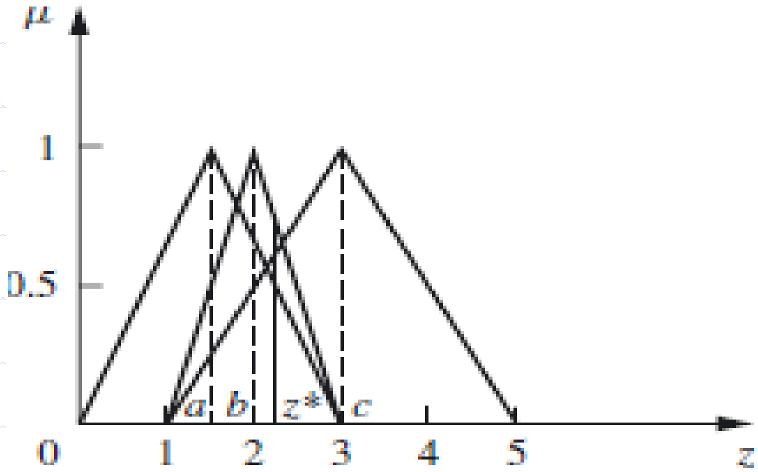


Centroid value z^* for three fuzzy oil samples.

Defuzzification Methods

According to the weighted average method:

$$z^* = \frac{(1\times 1.5)+(1\times 2)+(1\times 3)}{1+1+1} = 2.25$$



Weighted average method for z^*.

Defuzzification Methods

Three other popular methods are available because of their appearance in some applications:(Hellendoorn and Thomas, 1993)

The Center of Sums,
Center of Largest Area,
First of Maxima Methods

Defuzzification Methods

Center of sums Method

- Faster than any defuzzification method
- Involves algebraic sum of individual output fuzzy sets, instead of their union
- Drawback: intersecting areas are added twice.

$$z^* = \frac{\int_z z \sum_{k=1}^{n} \mu_{\underset{\sim}{C}_k}(z)\,dz}{\int_z \sum_{k=1}^{n} \mu_{\underset{\sim}{C}_k}(z)\,dz}$$

It is similar to the weighted average method, but the weights are the areas, instead of individual membership values.

Defuzzification Methods

Center of largest area: If the output fuzzy set has at least two convex sub regions, then the center of gravity (i.e., *z*∗ is calculated using the centroid method) of the convex fuzzy sub region with the largest area is used to obtain the defuzzified value *z*∗ of the output

$$z^* = \frac{\int \mu_{\underset{\sim}{C}_m}(z) z \, dz}{\int \mu_{\underset{\sim}{C}_m}(z) \, dz},$$

Defuzzification Methods

First (or last) of maxima: This method uses the overall output or union of all individual output fuzzy sets C_k to determine the smallest value of the domain with maximized membership degree in C_k.

See Examples 4.5, and 4.6(Page 108)

Picking a Method

Which of these methods is the right one?

There's no simple answer. But if you want to get started quickly, generally the centroid method is good enough. Later you can always change your defuzzification method to see if another method works better.

EXAMPLES

EXAMPLE 1

Suppose that a continuous universe is [-60 +60] and the fuzzy levels are LN [≤ -40], MN [-60 -20], SN [-40 0], ZE [-20 +20], SP [0 +40], MP [+20 +60], and LP [+40 ≥]. The triangular type membership functions are shown in Figure .

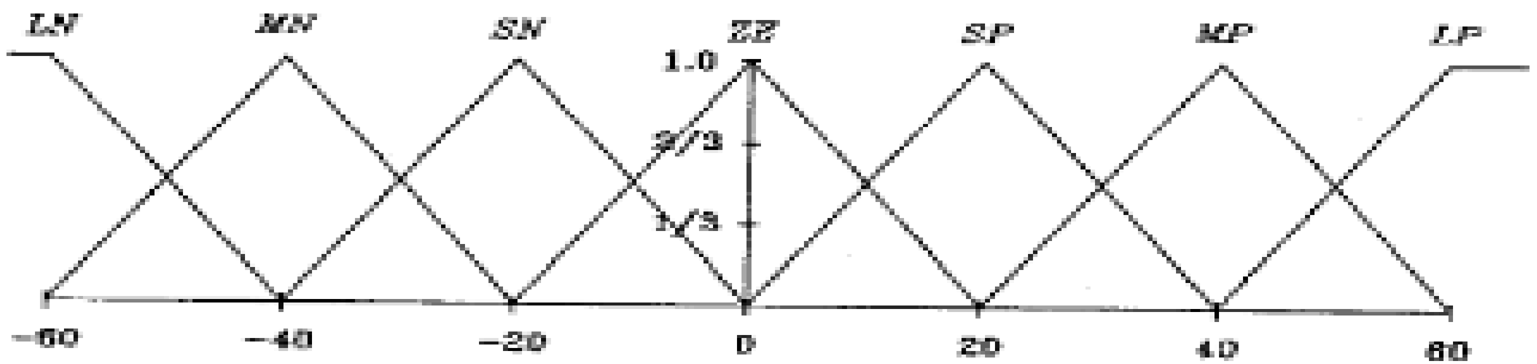


Figure Triangular type membership functions.

EXAMPLE 1

In the FLC, suppose that the FAM rules are the following:

R_1: IF X_1 is SP AND X_2 is MP THEN CA is SP,

R_2: IF X_1 is MP AND X_2 is SP THEN CA is MP.

where X_1 and X_2 represent input signals and CA is an output signal. Further, assume that $X_1 = 35$ and $X_2 = 38$. The defuzzification methods such as the weighted avarage MOM, and Centroid methods are presented in Figure

EXAMPLE 1

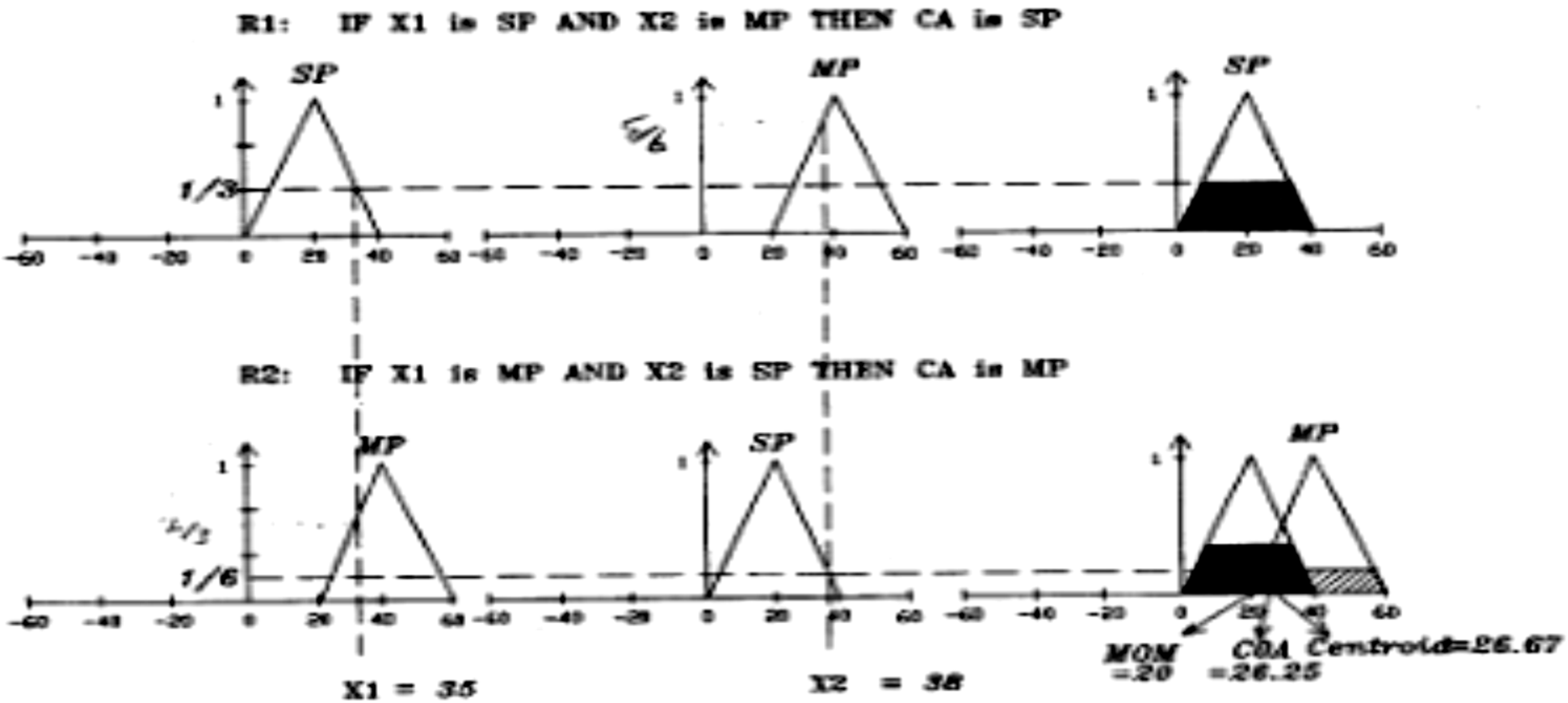


Figure : Defuzzification of the combined FAM rules.

EXAMPLE 1

Then, how are the crisp values of the control actions using the COA, MOM, and Centroid methods calculated? From Figure , in rule 1 R_1, $\mu_{SP}(X_1) = 1/3$ and $\mu_{MP}(X_2) = 5/6$, and the weight of rule 1 is calculated by:

$$w_1 = \min(\mu_{SP}(X_1), \mu_{MP}(X_2)) = \min(1/3, 5/6) = 1/3.$$

and similarly for rule 2:

$$w_2 = \min(\mu_{MP}(X_1), \mu_{SP}(X_2)) = \min(2/3, 1/6) = 1/6.$$

Then, the control actions obtained by combining these FAM rules by these

EXAMPLE 1

Using the COA Method:

$$\text{control action} = \frac{\sum_{i=1}^{q} \mu_C(z_i) z_i}{\sum_{i=1}^{q} \mu_C(z_i)}$$

$= (1/3*10+1/3*20+1/3*30+1/6*40+1/6*50)/(1/3+1/3+1/3+1/6+1/6)$

$=(20+15)/(4/3) = (105)/4 = 26.25.$

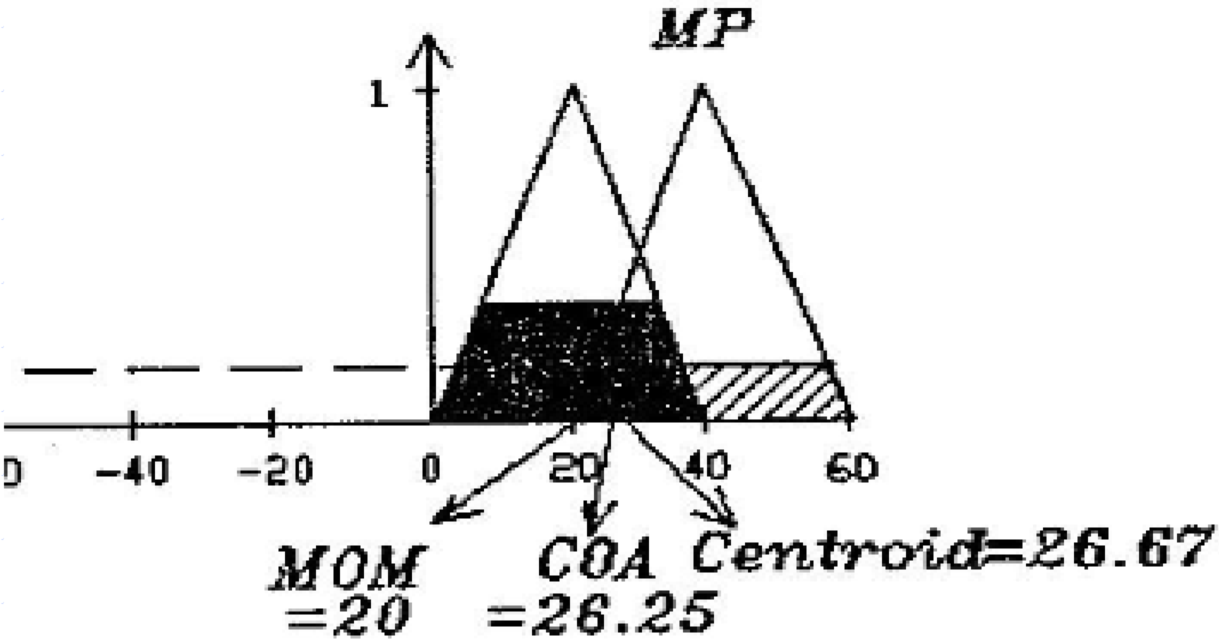
Using the MOM Method:

$$\text{control action} = \sum_{i=1}^{3} \frac{z_i}{3} = (10+20+30)/3 = 20.$$

Using the **weighted average**

$$\text{control action} = \frac{\sum_{i=1}^{2} w_i z_i}{\sum_{i=1}^{2} w_i} = (1/3*20+1/6*40)/(1/3+1/6) = 240/9 = 26.67.$$

EXAMPLE 1



Solar Pool Heater Example

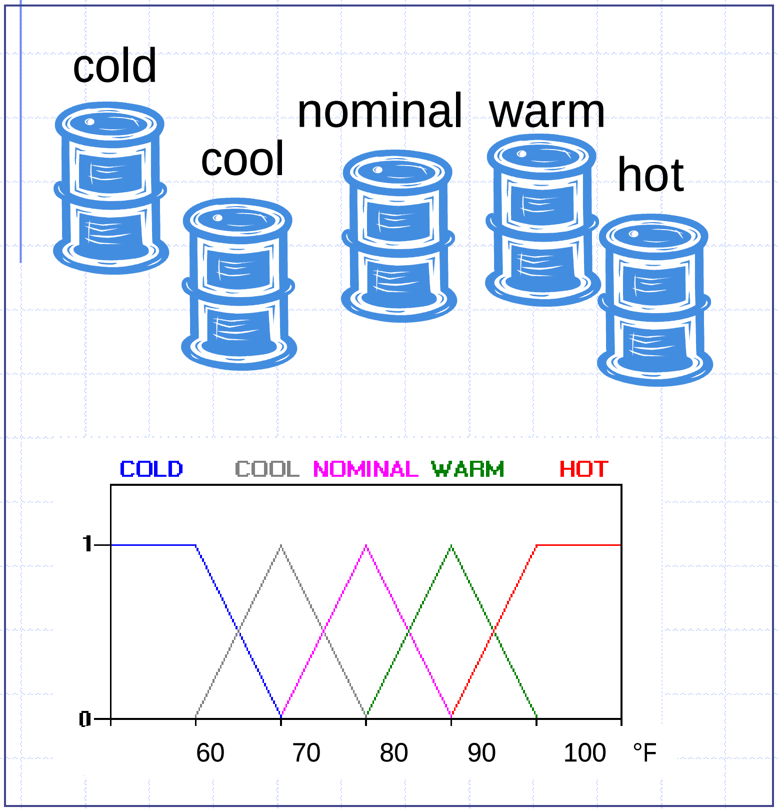
- suppose we measure the pool water temp and the wind speed and we want to adjust the valve that sends water to the solar panels

- we have two input parameters
  temp
  wind_speed

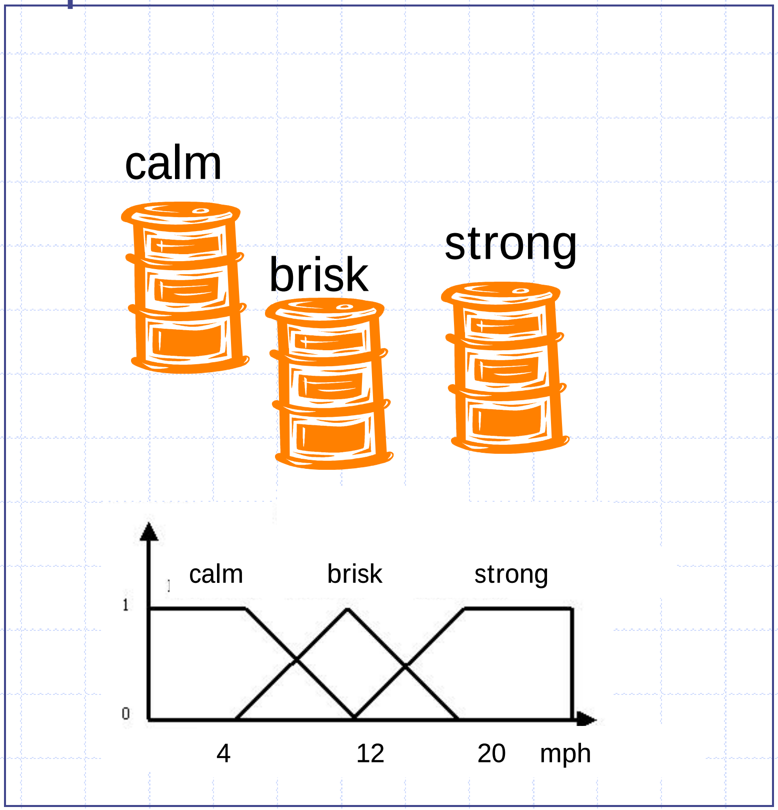
- we have one output parameter
  change_in_valve

Solar Pool Heater Example

- set up membership functions for the inputs
  - for each input, decide on how many categories there will be and decide on their membership functions



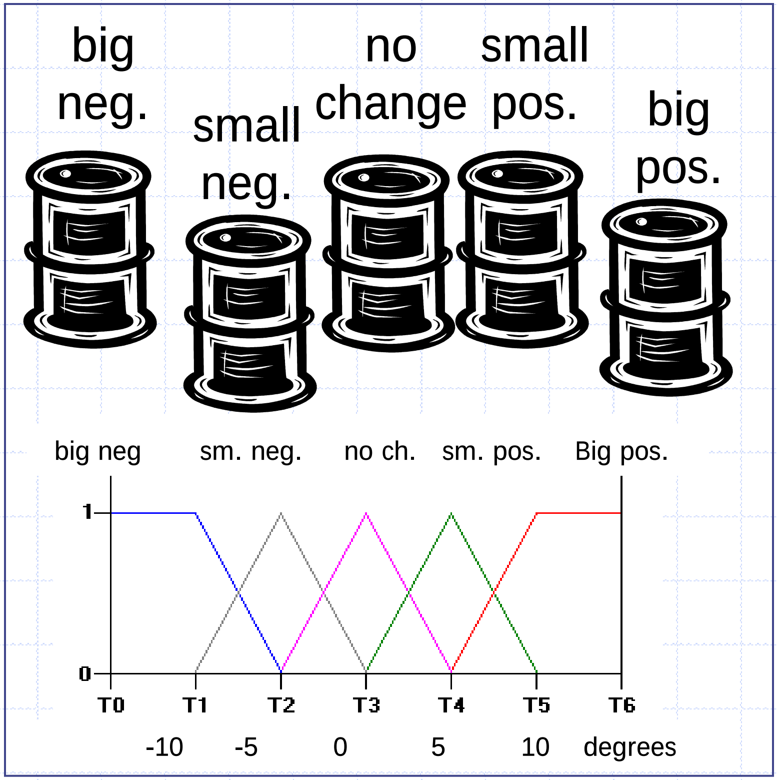
temp



wind_speed

Solar Pool Heater Example

- set up membership functions for the output(s)
  - for each output, decide on how many categories there will be and decide on their membership functions



change_in_valve

Solar Pool Heater Example

set up the rules

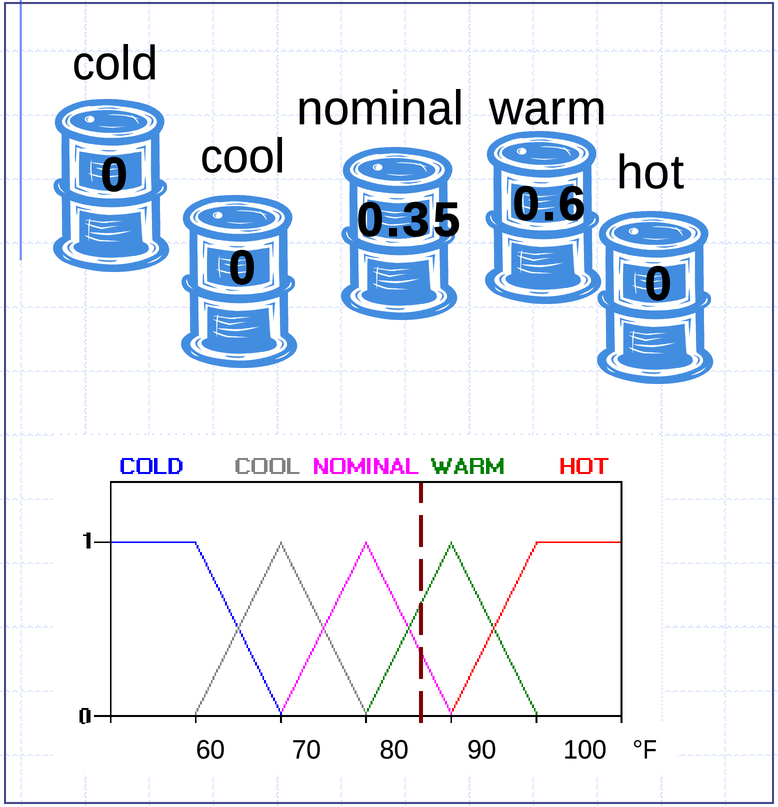
if (**temp** is hot) AND (**wind_speed** is calm)
then (**change_in_valve** is big_negative)

if (**temp** is warm) AND (**wind_speed** is brisk)
then (**change_in_valve** is small_negative)

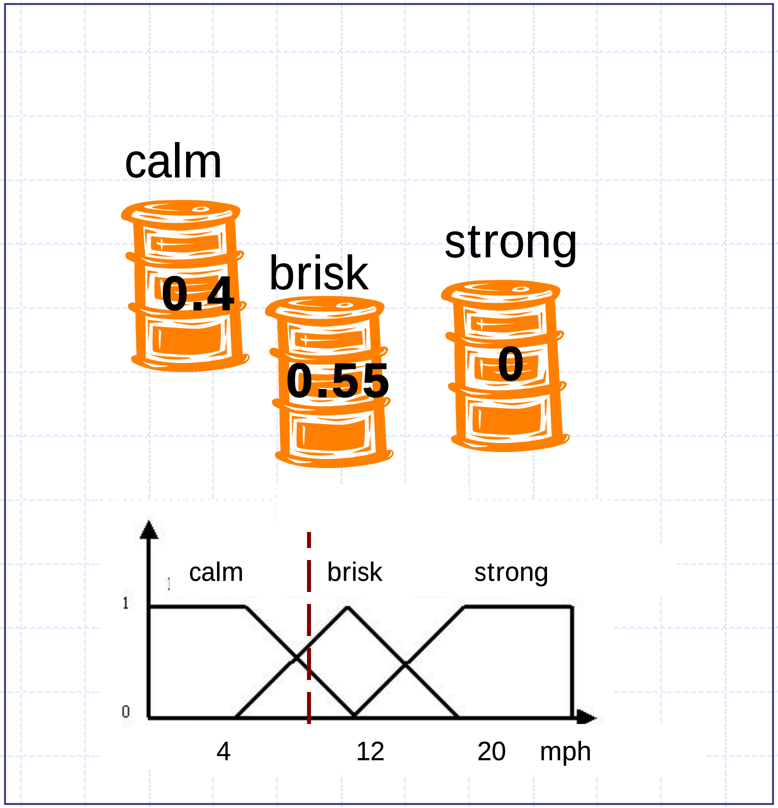
if (**temp** is nominal) OR (**temp** is warm)
then (**change_in_valve** is no_change)

Solar Pool Heater Example

Fuzzify the inputs

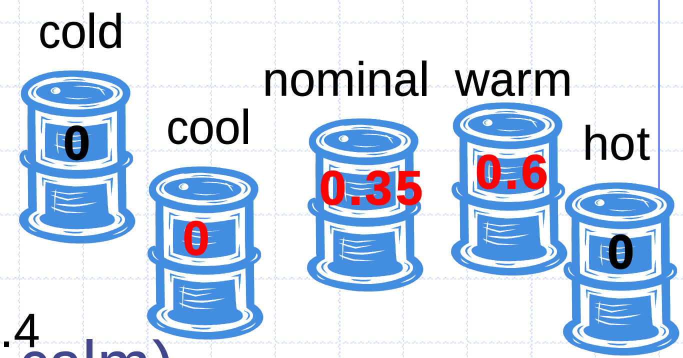


temp = 87°F



wind_speed = 9 mph

Solar Pool Heater Example



- fire the rules

0 0.4

if (**temp** is hot) AND (**wind_speed** is calm)
then (**change_in_valve** is big_negative)

0

0.6 0.55

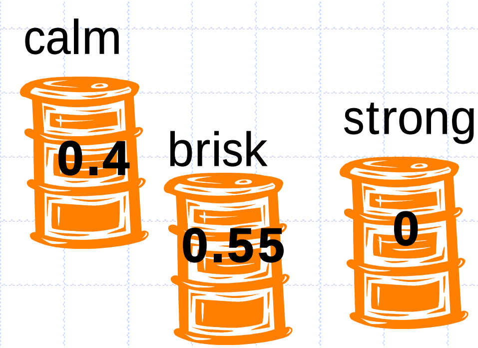
if (**temp** is warm) AND (**wind_speed** is brisk) then (**change_in_valve** is small_negative)

0.55

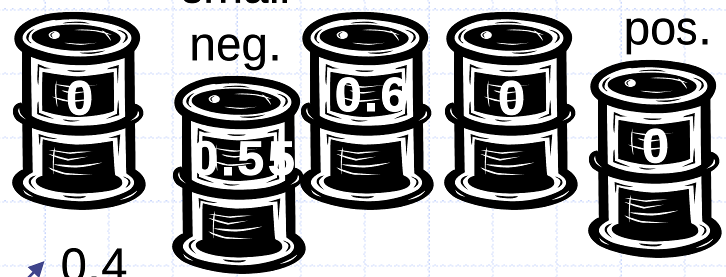
0.35 0.6

if (**temp** is nominal) OR (**temp** is warm)
then (**change_in_valve** is no_change)

0.6



Solar Pool Heater Example



Fire the rules

0 0.4

if (**temp** is hot) AND (**wind_speed** is calm)
then (**change_in_valve** is big_negative)

0

0.6 0.55

if (**temp** is warm) AND (**wind_speed** is brisk)
then (**change_in_valve** is small_negative)

0.55

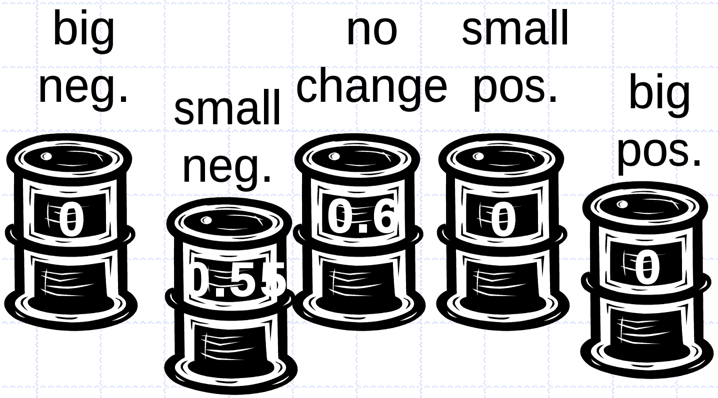
0.35 0.6

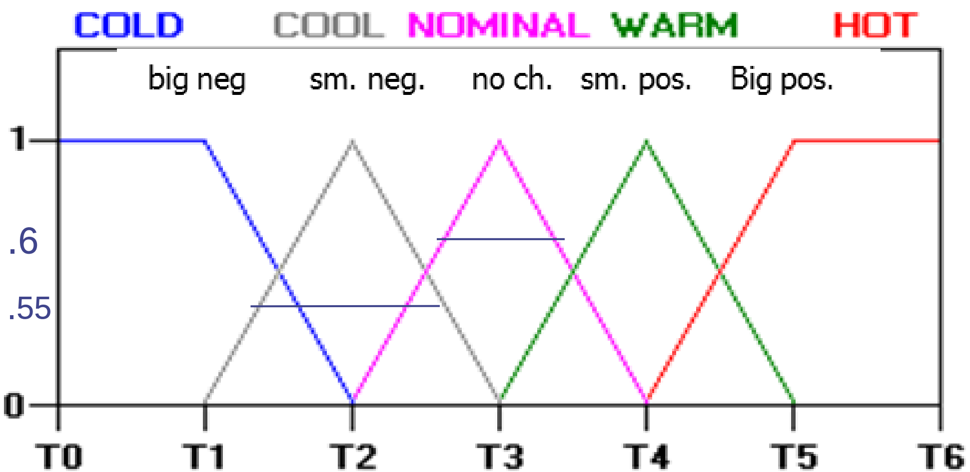
if (**temp** is nominal) OR (**temp** is warm)
then (**change_in_valve** is no_change)

0.6

Solar Pool Heater Example

- Defuzzify the output(s)





HW 2

4.9, 4.10, 5.12, 6.1, 6.4

In Problem 4.9, and 4.10 use any 3 methods for defuzzification.

Due 9/03/2020

Good Luck