

Quantum Computing Workshop

Fundamentals of Quantum Algorithms

CREO Research Center

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Today's Mission:

Discover how quantum algorithms work and run hands-on experiments to experience how quantum computing transforms problem-solving.



Have ready: Laptop with Python installed

Deutsch's problem

Deutsch's problem is very simple – it's the *Parity* problem for functions of the form $f : \Sigma \rightarrow \Sigma$.

There are four functions of the form $f : \Sigma \rightarrow \Sigma$:

a	$f_1(a)$	a	$f_2(a)$	a	$f_3(a)$	a	$f_4(a)$
0	0	0	0	0	1	0	1
1	0	1	1	1	0	1	1

The functions f_1 and f_4 are *constant* while f_2 and f_3 are *balanced*.

Deutsch's problem

Input: $f : \Sigma \rightarrow \Sigma$

Output: 0 if f is constant, 1 if f is balanced

Deutsch's problem

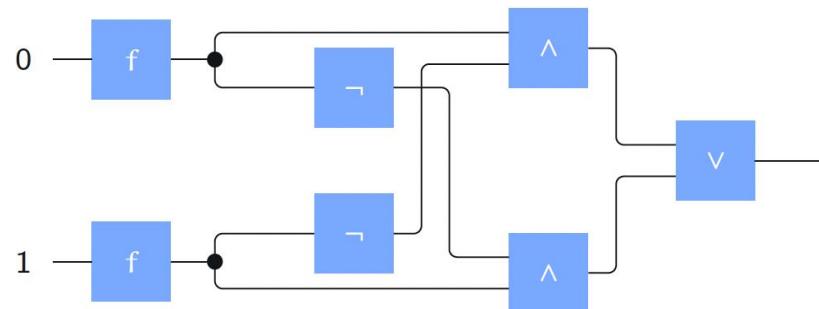
Deutsch's problem

Input: $f : \Sigma \rightarrow \Sigma$

Output: 0 if f is constant, 1 if f is balanced

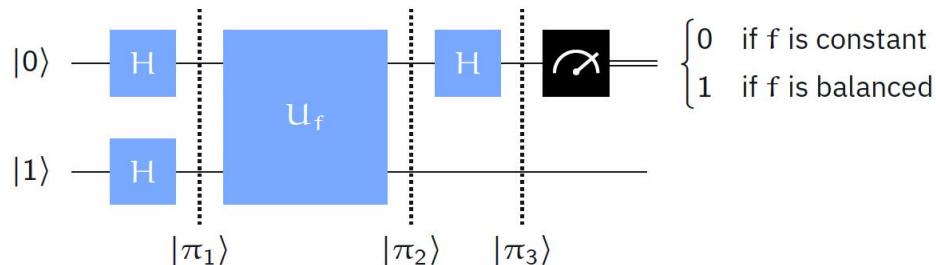
Every *classical* query algorithm must make 2 queries to f to solve this problem – learning just one of two bits provides no information about their parity.

Our query algorithm from earlier is therefore optimal among classical query algorithms for this problem.



Deutsch's algorithm

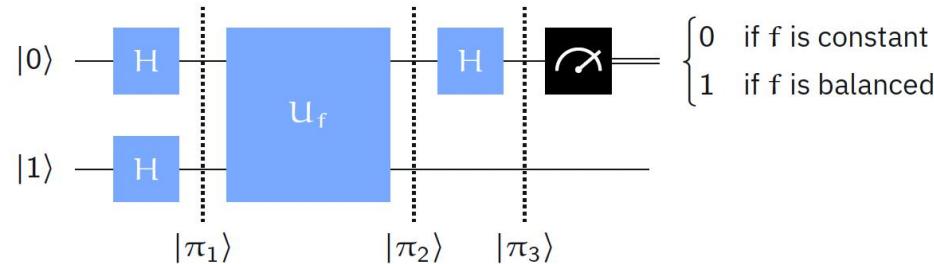
Deutsch's algorithm solves [Deutsch's problem](#) using a single query.



$$\begin{aligned}
 |\pi_1\rangle &= |-\rangle|+\rangle = \frac{1}{2}(|0\rangle - |1\rangle)|0\rangle + \frac{1}{2}(|0\rangle - |1\rangle)|1\rangle \\
 |\pi_2\rangle &= \frac{1}{2}(|0 \oplus f(0)\rangle - |1 \oplus f(0)\rangle)|0\rangle + \frac{1}{2}(|0 \oplus f(1)\rangle - |1 \oplus f(1)\rangle)|1\rangle \\
 &= \frac{1}{2}(-1)^{f(0)}(|0\rangle - |1\rangle)|0\rangle + \frac{1}{2}(-1)^{f(1)}(|0\rangle - |1\rangle)|1\rangle \\
 &= |-\rangle\left(\frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}\right)
 \end{aligned}$$

Deutsch's algorithm

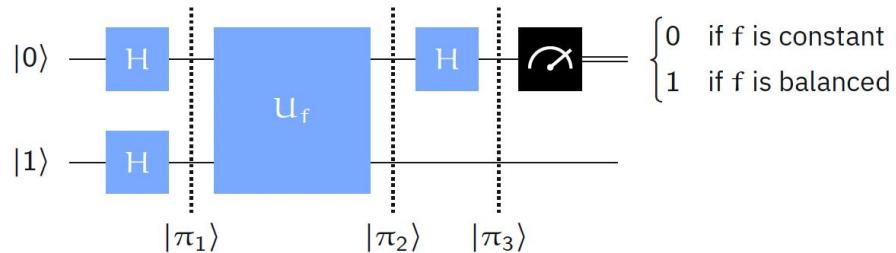
Deutsch's algorithm solves [Deutsch's problem](#) using a single query.



$$\begin{aligned}
 |\pi_2\rangle &= |- \rangle \left(\frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}} \right) \\
 &= (-1)^{f(0)}|- \rangle \left(\frac{|0\rangle + (-1)^{f(0) \oplus f(1)}|1\rangle}{\sqrt{2}} \right) \\
 &= \begin{cases} (-1)^{f(0)}|- \rangle |+\rangle & f(0) \oplus f(1) = 0 \\ (-1)^{f(0)}|- \rangle |-\rangle & f(0) \oplus f(1) = 1 \end{cases}
 \end{aligned}$$

Deutsch's algorithm

Deutsch's algorithm solves [Deutsch's problem](#) using a single query.



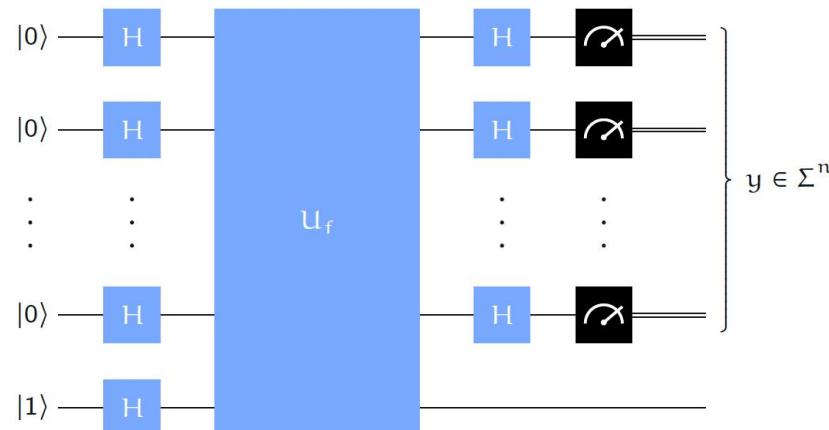
$$|\pi_2\rangle = \begin{cases} (-1)^{f(0)}|-\rangle|+\rangle & f(0) \oplus f(1) = 0 \\ (-1)^{f(0)}|-\rangle|-\rangle & f(0) \oplus f(1) = 1 \end{cases}$$

$$\begin{aligned} |\pi_3\rangle &= \begin{cases} (-1)^{f(0)}|-\rangle|0\rangle & f(0) \oplus f(1) = 0 \\ (-1)^{f(0)}|-\rangle|1\rangle & f(0) \oplus f(1) = 1 \end{cases} \\ &= (-1)^{f(0)}|-\rangle|f(0) \oplus f(1)\rangle \end{aligned}$$

The Deutsch-Jozsa circuit

The Deutsch-Jozsa algorithm extends Deutsch's algorithm to input functions of the form $f : \Sigma^n \rightarrow \Sigma$ for any $n \geq 1$.

The quantum circuit for the Deutsch-Jozsa algorithm looks like this:



We can, in fact, use this circuit to solve multiple problems.

The Deutsch-Jozsa problem

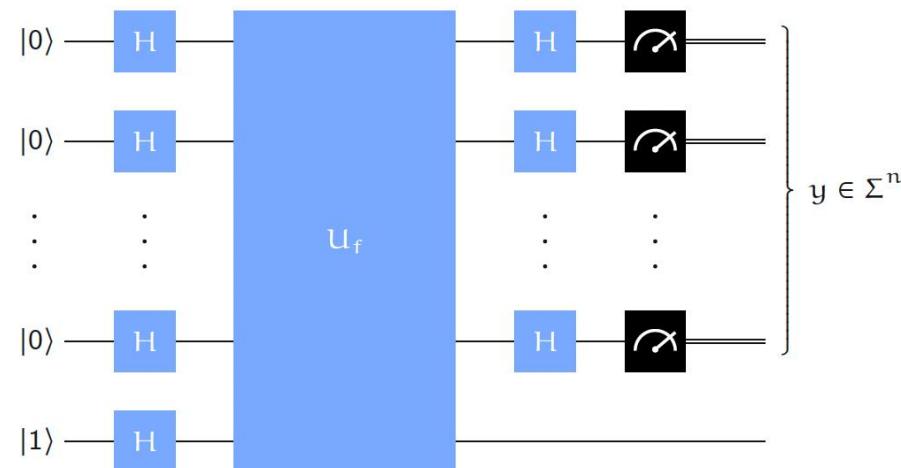
The Deutsch-Jozsa problem generalizes Deutsch's problem: for an input function $f : \Sigma^n \rightarrow \Sigma$, the task is to output 0 if f is constant and 1 if f is balanced.

Deutsch-Jozsa problem

Input: $f : \Sigma^n \rightarrow \Sigma$

Promise: f is either constant or balanced

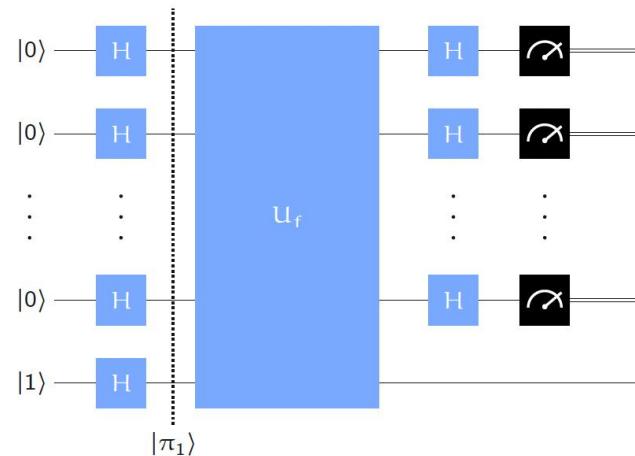
Output: 0 if f is constant, 1 if f is balanced



Output: 0 if $y = 0^n$ and 1 otherwise.

Deutsch-Jozsa analysis

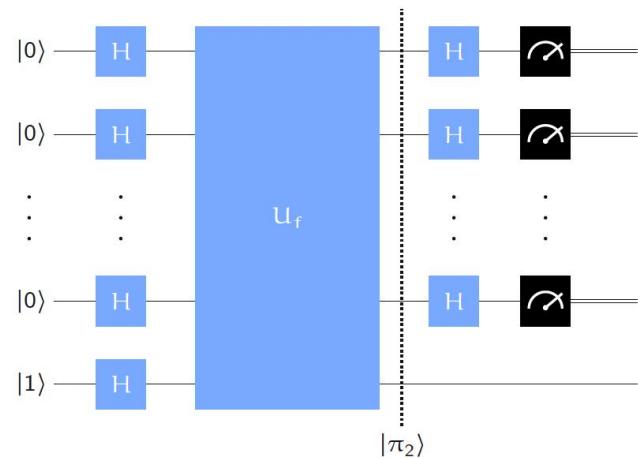
$$H^{\otimes n} |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \Sigma^n} (-1)^{x \cdot y} |y\rangle$$



$$|\pi_1\rangle = |- \rangle \otimes \frac{1}{\sqrt{2^n}} \sum_{x \in \Sigma^n} |x\rangle$$

Deutsch-Jozsa analysis

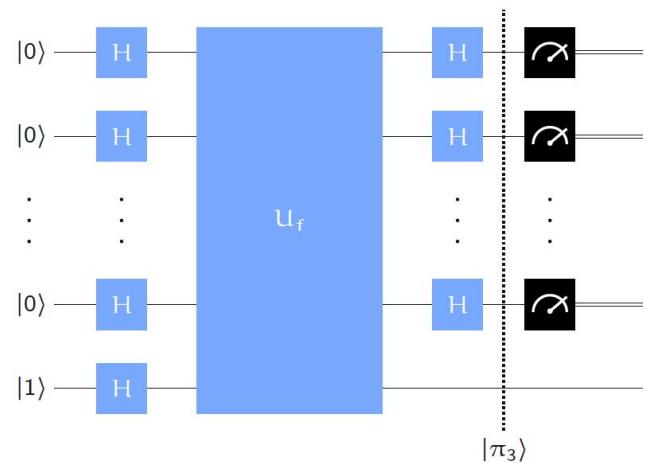
$$H^{\otimes n} |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \Sigma^n} (-1)^{x \cdot y} |y\rangle$$



$$|\pi_2\rangle = |- \rangle \otimes \frac{1}{\sqrt{2^n}} \sum_{x \in \Sigma^n} (-1)^{f(x)} |x\rangle$$

Deutsch-Jozsa analysis

$$H^{\otimes n} |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \Sigma^n} (-1)^{x \cdot y} |y\rangle$$



$$|\pi_3\rangle = |- \rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{f(x) + x \cdot y} |y\rangle$$

Deutsch-Jozsa analysis

The probability for the measurements to give $y = 0^n$ is

$$p(0^n) = \left| \frac{1}{2^n} \sum_{x \in \Sigma^n} (-1)^{f(x)} \right|^2 = \begin{cases} 1 & \text{if } f \text{ is constant} \\ 0 & \text{if } f \text{ is balanced} \end{cases}$$

The Deutsch-Jozsa algorithm therefore solves the Deutsch-Jozsa problem without error with a single query.

Any *deterministic* algorithm for the Deutsch-Jozsa problem must at least $2^{n-1} + 1$ queries.

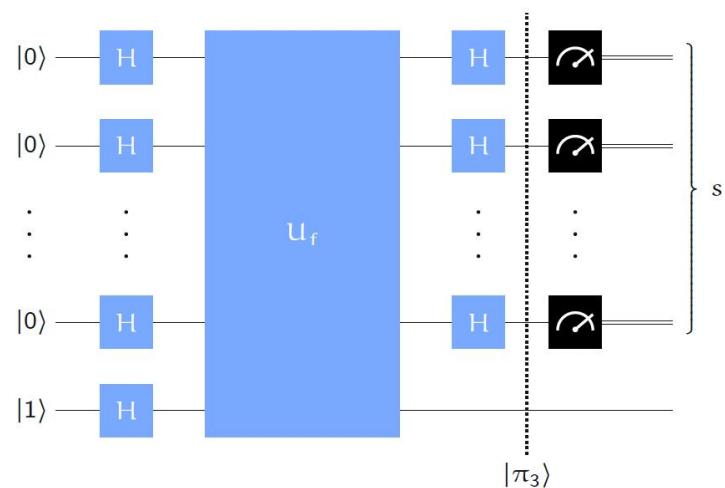
The Bernstein-Vazirani problem

Bernstein-Vazirani problem

Input: $f : \Sigma^n \rightarrow \Sigma$

Promise: there exists a binary string $s = s_{n-1} \dots s_0$ for which
 $f(x) = s \cdot x$ for all $x \in \Sigma^n$

Output: the string s



The Bernstein-Vazirani problem

$$\begin{aligned} |\pi_3\rangle &= |- \rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{f(x)+x \cdot y} |y\rangle \\ &= |- \rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{s \cdot x + y \cdot x} |y\rangle \\ &= |- \rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{(s \oplus y) \cdot x} |y\rangle \\ &= |- \rangle \otimes |s\rangle \end{aligned}$$

The Deutsch-Jozsa circuit therefore solves the Bernstein-Vazirani problem with a single query.

Any probabilistic algorithm must make at least n queries to find s .

<https://learning.quantum.ibm.com/course/fundamentals-of-quantum-algorithms>

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<https://github.com/Qiskit/textbook/blob/main/notebooks/quantum-machine-learning/supervised.ipynb>



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