



# Quantum Computing Workshop

Fundamentals of Quantum Algorithms

CREO Research Center

Presenter: Chibuike Okekeogbu



## Today's Mission:

Discover how quantum algorithms work and run hands-on experiments to experience how quantum computing transforms problem-solving.



Have ready: Laptop with Python installed

# Deutsch's problem

Deutsch's problem is very simple — it's the **Parity** problem for functions of the form  $f : \Sigma \rightarrow \Sigma$ .

There are four functions of the form  $f : \Sigma \rightarrow \Sigma$ :

$a$	$f_1(a)$	$a$	$f_2(a)$	$a$	$f_3(a)$	$a$	$f_4(a)$
0	0	0	0	0	1	0	1
1	0	1	1	1	0	1	1

The functions  $f_1$  and  $f_4$  are **constant** while  $f_2$  and  $f_3$  are **balanced**.

## Deutsch's problem

Input:  $f : \Sigma \rightarrow \Sigma$

Output: 0 if  $f$  is constant, 1 if  $f$  is balanced

# Deutsch's problem

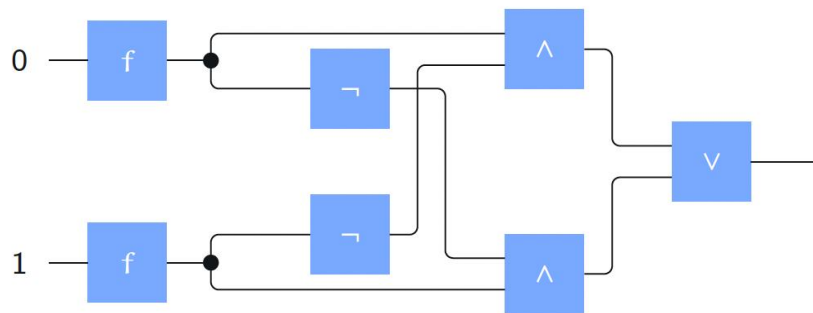
## Deutsch's problem

Input:  $f : \Sigma \rightarrow \Sigma$

Output: 0 if  $f$  is constant, 1 if  $f$  is balanced

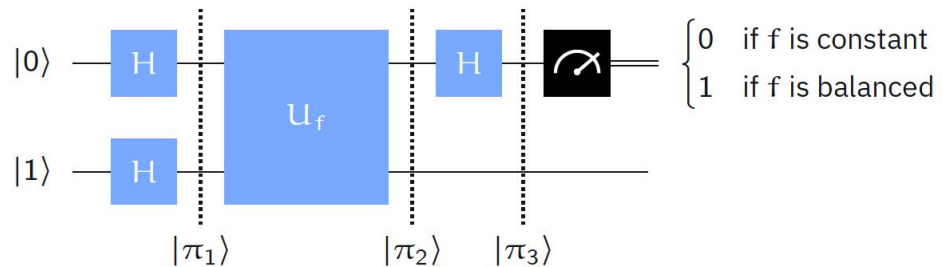
Every *classical* query algorithm must make 2 queries to  $f$  to solve this problem — learning just one of two bits provides no information about their parity.

Our query algorithm from earlier is therefore optimal among classical query algorithms for this problem.



# Deutsch's algorithm

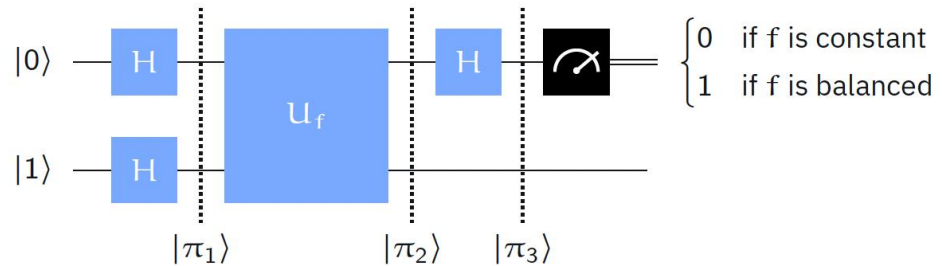
Deutsch's algorithm solves [Deutsch's problem](#) using a single query.



$$\begin{aligned}
 |\pi_1\rangle &= |-\rangle|+\rangle = \frac{1}{2}(|0\rangle - |1\rangle)(|0\rangle + |1\rangle) + \frac{1}{2}(|0\rangle - |1\rangle)(|1\rangle) \\
 |\pi_2\rangle &= \frac{1}{2}(|0 \oplus f(0)\rangle - |1 \oplus f(0)\rangle)|0\rangle + \frac{1}{2}(|0 \oplus f(1)\rangle - |1 \oplus f(1)\rangle)|1\rangle \\
 &= \frac{1}{2}(-1)^{f(0)}(|0\rangle - |1\rangle)|0\rangle + \frac{1}{2}(-1)^{f(1)}(|0\rangle - |1\rangle)|1\rangle \\
 &= |-\rangle \left( \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}} \right)
 \end{aligned}$$

# Deutsch's algorithm

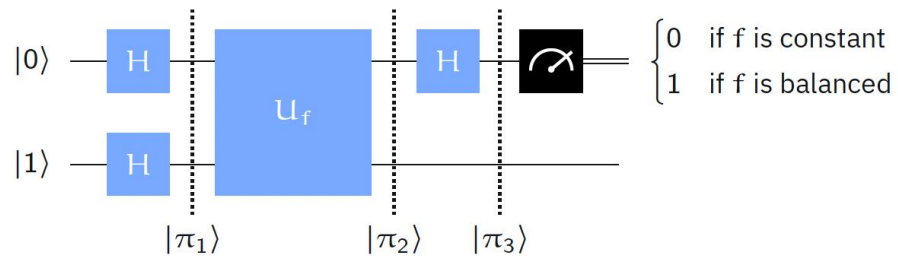
Deutsch's algorithm solves [Deutsch's problem](#) using a single query.



$$\begin{aligned}
 |\pi_2\rangle &= |-\rangle \left( \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}} \right) \\
 &= (-1)^{f(0)} |-\rangle \left( \frac{|0\rangle + (-1)^{f(0) \oplus f(1)}|1\rangle}{\sqrt{2}} \right) \\
 &= \begin{cases} (-1)^{f(0)} |-\rangle |+\rangle & f(0) \oplus f(1) = 0 \\ (-1)^{f(0)} |-\rangle |-\rangle & f(0) \oplus f(1) = 1 \end{cases}
 \end{aligned}$$

# Deutsch's algorithm

Deutsch's algorithm solves *Deutsch's problem* using a single query.



$$|\pi_2\rangle = \begin{cases} (-1)^{f(0)} |-\rangle|+\rangle & f(0) \oplus f(1) = 0 \\ (-1)^{f(0)} |-\rangle|-\rangle & f(0) \oplus f(1) = 1 \end{cases}$$

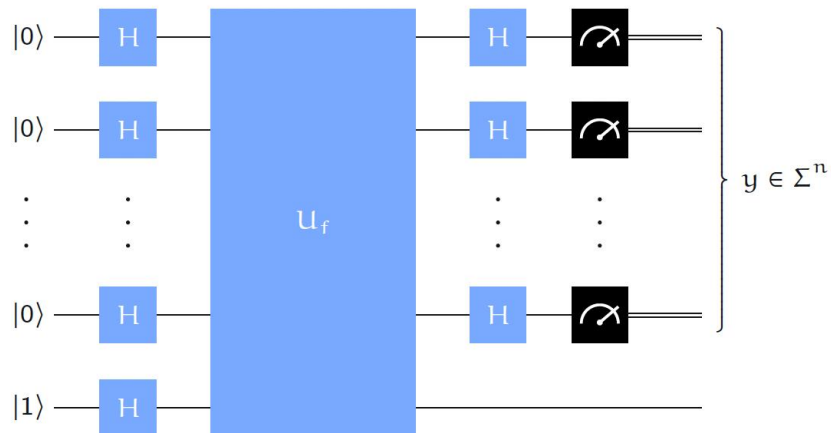
$$|\pi_3\rangle = \begin{cases} (-1)^{f(0)} |-\rangle|0\rangle & f(0) \oplus f(1) = 0 \\ (-1)^{f(0)} |-\rangle|1\rangle & f(0) \oplus f(1) = 1 \end{cases}$$

$$= (-1)^{f(0)} |-\rangle |f(0) \oplus f(1)\rangle$$

# The Deutsch-Jozsa circuit

The Deutsch-Jozsa algorithm extends Deutsch's algorithm to input functions of the form  $f : \Sigma^n \rightarrow \Sigma$  for any  $n \geq 1$ .

The quantum circuit for the Deutsch-Jozsa algorithm looks like this:



We can, in fact, use this circuit to solve multiple problems.

# The Deutsch-Jozsa problem

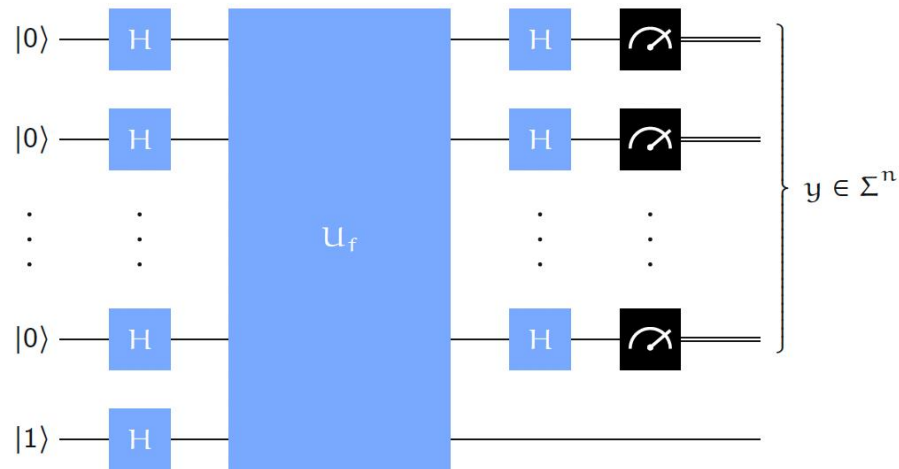
The Deutsch-Jozsa problem generalizes Deutsch's problem: for an input function  $f : \Sigma^n \rightarrow \Sigma$ , the task is to output 0 if  $f$  is constant and 1 if  $f$  is balanced.

## Deutsch-Jozsa problem

Input:  $f : \Sigma^n \rightarrow \Sigma$

Promise:  $f$  is either constant or balanced

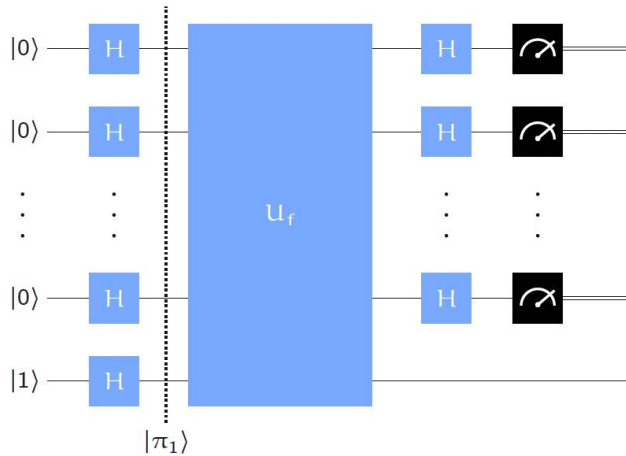
Output: 0 if  $f$  is constant, 1 if  $f$  is balanced



Output: 0 if  $y = 0^n$  and 1 otherwise.

# Deutsch-Jozsa analysis

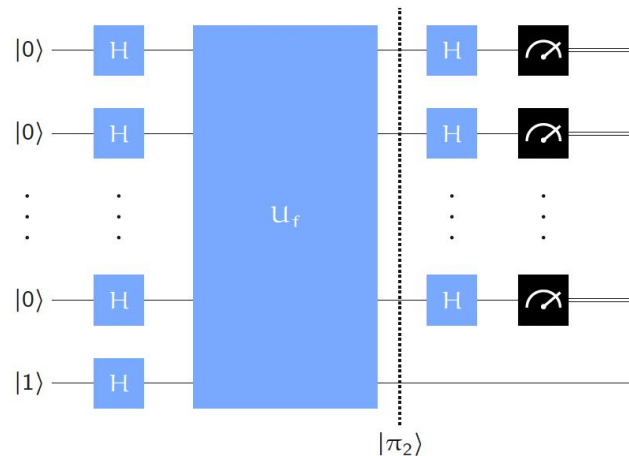
$$H^{\otimes n} |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \Sigma^n} (-1)^{x \cdot y} |y\rangle$$



$$|\pi_1\rangle = |-\rangle \otimes \frac{1}{\sqrt{2^n}} \sum_{x \in \Sigma^n} |x\rangle$$

# Deutsch-Jozsa analysis

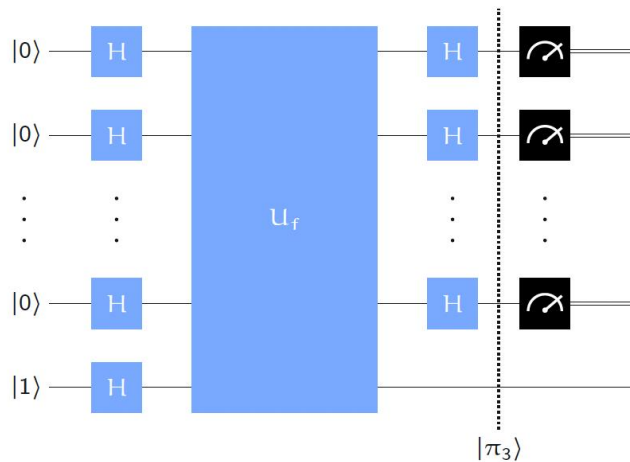
$$H^{\otimes n} |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \Sigma^n} (-1)^{x \cdot y} |y\rangle$$



$$|\pi_2\rangle = |-\rangle \otimes \frac{1}{\sqrt{2^n}} \sum_{x \in \Sigma^n} (-1)^{f(x)} |x\rangle$$

# Deutsch-Jozsa analysis

$$H^{\otimes n} |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \Sigma^n} (-1)^{x \cdot y} |y\rangle$$



$$|\pi_3\rangle = |-\rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{f(x) + x \cdot y} |y\rangle$$



# Deutsch-Jozsa analysis

The probability for the measurements to give  $y = 0^n$  is

$$p(0^n) = \left| \frac{1}{2^n} \sum_{x \in \Sigma^n} (-1)^{f(x)} \right|^2 = \begin{cases} 1 & \text{if } f \text{ is constant} \\ 0 & \text{if } f \text{ is balanced} \end{cases}$$

The Deutsch-Jozsa algorithm therefore solves the Deutsch-Jozsa problem without error with a single query.

Any **deterministic** algorithm for the Deutsch-Jozsa problem must at least  $2^{n-1} + 1$  queries.

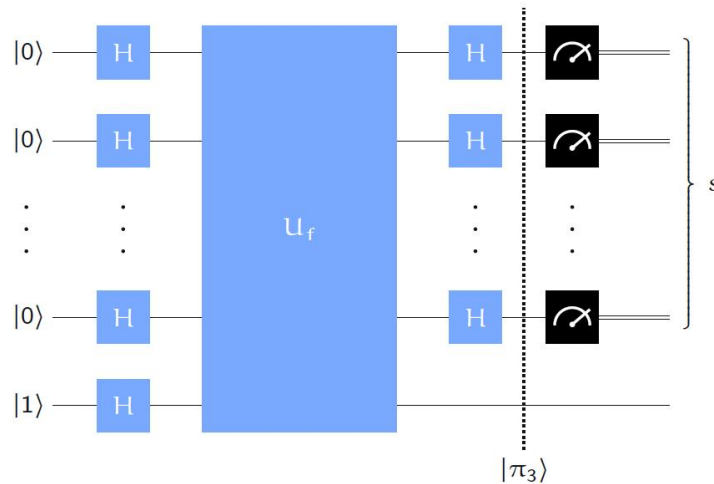
# The Bernstein-Vazirani problem

## Bernstein-Vazirani problem

Input:  $f : \Sigma^n \rightarrow \Sigma$

Promise: there exists a binary string  $s = s_{n-1} \cdots s_0$  for which  
 $f(x) = s \cdot x$  for all  $x \in \Sigma^n$

Output: the string  $s$





## The Bernstein-Vazirani problem

$$\begin{aligned} |\pi_3\rangle &= |-\rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{f(x)+x \cdot y} |y\rangle \\ &= |-\rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{s \cdot x + y \cdot x} |y\rangle \\ &= |-\rangle \otimes \frac{1}{2^n} \sum_{y \in \Sigma^n} \sum_{x \in \Sigma^n} (-1)^{(s \oplus y) \cdot x} |y\rangle \\ &= |-\rangle \otimes |s\rangle \end{aligned}$$

The Deutsch-Jozsa circuit therefore solves the Bernstein-Vazirani problem with a single query.

Any probabilistic algorithm must make at least  $n$  queries to find  $s$ .



<https://learning.quantum.ibm.com/course/fundamentals-of-quantum-algorithms>

<https://www.ibm.com/quantum/qiskit>

<https://quantum.ibm.com>

[https://learning.edx.org/course/course-v1:PurdueX+ECE\\_69501.3+1T2024/home](https://learning.edx.org/course/course-v1:PurdueX+ECE_69501.3+1T2024/home)

[https://qiskit-community.github.io/qiskit-machine-learning/tutorials/02a\\_training\\_a\\_quantum\\_model\\_on\\_a\\_real\\_dataset.html#](https://qiskit-community.github.io/qiskit-machine-learning/tutorials/02a_training_a_quantum_model_on_a_real_dataset.html#)

<https://github.com/Qiskit/textbook/blob/main/notebooks/quantum-machine-learning/supervised.ipynb>



