Life Insurance: Premium Payment Model

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Introduction

Life insurance is a vital financial instrument that provides protection and financial security to families in the event of an unexpected death. In this payment model, I incorporate mortality probabilities, interest rates, and benefit amounts to calculate the actuarial present value (APV) of the life insurance policy. By equating the APV to the present value of annuity payments, I can determine the fair annuity amount that a policyholder should pay.

Data Collection and Research

Term Life Insurance

- Provides coverage for a specific period (10 to 30 years)

-If the policyholder dies within the term, the death benefit is paid to the beneficiaries.

-No cash value component

Permanent Life Insurance

-Life long coverage

-Combines death benefit with cash value component

-Many types: Whole life insurance, Universal life insurance, Variable life insurance

Actuarial Table

Period Life Table, 2020, as used in the 2023 Trustees Report

Exact age	Male			Female		
	Death probability [a]	Number of lives [b]	Life expectancy	Death probability [a]	Number of lives [b]	Life expectancy
0	0.005837	100,000	74.12	0.004907	100,000	79.78
1	0.000410	99,416	73.55	0.000316	99,509	79.17
2	0.000254	99,376	72.58	0.000196	99,478	78.19
3	0.000207	99,350	71.60	0.000160	99,458	77.21
4	0.000167	99,330	70.62	0.000129	99,442	76.22
5	0.000141	99,313	69.63	0.000109	99,430	75.23
6	0.000123	99,299	68.64	0.000100	99,419	74.24
7	0.000113	99,287	67.65	0.000096	99,409	73.25
8	0.000108	99,276	66.65	0.000092	99,399	72.25
9	0.000114	99,265	65.66	0.000089	99,390	71.26

Force of Mortality

u(x) = A + B * e^(C * x)

Gompertz-Makeham distribution

Model

Assumptions

1. Homogenous population: death probability applied equally to any individual
2. Stable interest rates: might change as the economy entering recessions
3. Static mortality rates: life tables data might change as technology advances
4. Independent events: death probability is independent at each age

P(a): the probability of death at age a, derived from a life table.

T: be the term of the insurance policy in years.

A: be the policyholder's current age.

B: be the benefit amount paid to the beneficiary upon the insured's death.

i: the annual interest rate, which is used to discount future cash flows.

APV: the actuarial present value of the term life insurance policy, which is the expected present value of future cash flows (i.e., the benefit paid upon death) within the term.

Annuity term: the annual premium the insured pays to maintain the term life insurance policy for the specified term.

$$\mathrm{APV}_{\mathrm{term}} = B \cdot \sum_{a} [P(a) \cdot v^{(a+1)}]$$

a in range(A, A+T-1)

$$\mathrm{Annuity}_{\mathrm{term}} \cdot \sum_{a} [p(a) \cdot v^{(a)}]$$

v = 1/(i+1)

Results

```python
from tabulate import tabulate

def annuity_payment_results(start_age, end_age, T, B, i):
    results = []
    for age in range(start_age, end_age, 5):
        annuity_term_male = annuity_payment(age, T, B, i, 'male')
        annuity_term_female = annuity_payment(age, T, B, i, 'female')
        results.append((age, annuity_term_male, annuity_term_female))
    return results

# Constants
start_age = 20          # Starting age
end_age = 75            # Ending age
T = 20                  # Term of the insurance policy in years
B = 300000              # Benefit amount paid to the beneficiary upon the insured's death
i = 0.05                # Annual interest rate

# Calculate annuity payments for all males and females from 20 to 60 years old
results = annuity_payment_results(start_age, end_age, T, B, i)
headers = ["Age", "Annuity Amount (Male)", "Annuity Amount (Female)"]
print(tabulate(results, headers=headers, floatfmt=".2f"))
```

```
  Age    Annuity Amount (Male)    Annuity Amount (Female)
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   20                   589.40                     249.11
   25                   724.05                     338.80
   30                   898.11                     461.46
   35                  1167.20                     640.94
   40                  1620.97                     923.82
   45                  2357.66                    1366.07
   50                  3435.81                    2013.13
   55                  4952.54                    2993.07
   60                  7314.69                    4631.29
   65                 11234.18                    7541.84
   70                 18362.36                   13093.76
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Strengths

Simplicity

Well-established principles

Adaptive

Interest rate considerations

Limitations

Simplicity

No considerations of expenses

Non-updated life table

No individual risk assessment

Alternative scenarios

Dynamic interest rates

Policy expenses

Individual risk factors

Policyholder behaviour

Conclusion

This project highlighted the importance of mathematical modeling in the insurance industry and demonstrated the potential for continued innovation and refinement in actuarial science. By building upon our current model and incorporating new insights and variables, we can continue to improve our understanding of the risks and rewards associated with life insurance, ultimately benefiting both insurers and policyholders alike.

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