Institute of Petroleum Engineering

Pore Architecture Reconstruction (PAR) of Heterogeneous Structure from 2D Images

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Introduction

Accurate characterization of porous media is a crucial step in pore-scale modelling of flow and transport properties. However, 3D micro CT is constrained by the trade-off between resolution and sample size, which means that only a limited range of pore sizes can be included in a single sample. As an alternative, one could reconstruct 3D pore-solid models based on high resolution 2D training images (TI). An optimization based algorithm is adapted for this task, by iteratively minimizing the difference between a local pattern in resultant model and its nearest pattern in TI.

Key points of methodology

I. Enrich pattern information – distance map transformation

Pattern based reconstruction techniques rely on the similarity, measured by some distance functions, of certain patterns. The limitation of pattern size (due to memory usage, computation complexity and pattern variety) often leads to a failure to capture heterogeneity and complex shapes of larger regions, resulting in the overestimation of large pore (or solid) structures and absence of complex pore connections in the reconstruction (Figure 1a). Therefore, a distance map transformation was applied to both pore and solid phase. The complexity of TI is better captured, thus improving the heterogeneous pattern reproduction significantly. (Figure 1b)

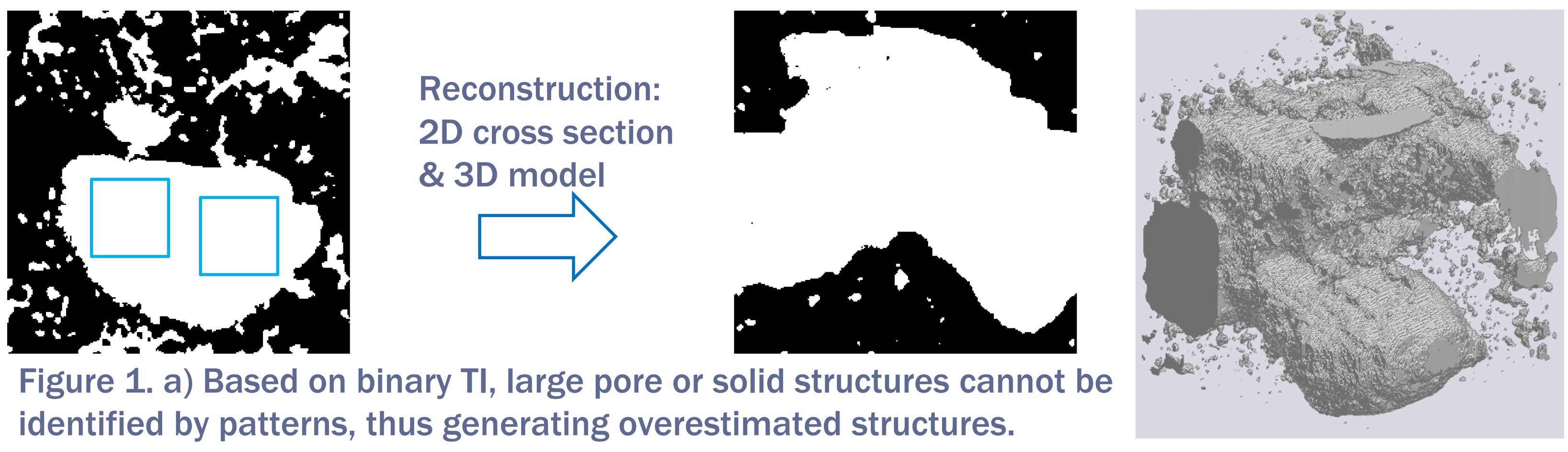


Figure 1. a) Based on binary TI, large pore or solid structures cannot be identified by patterns, thus generating overestimated structures.

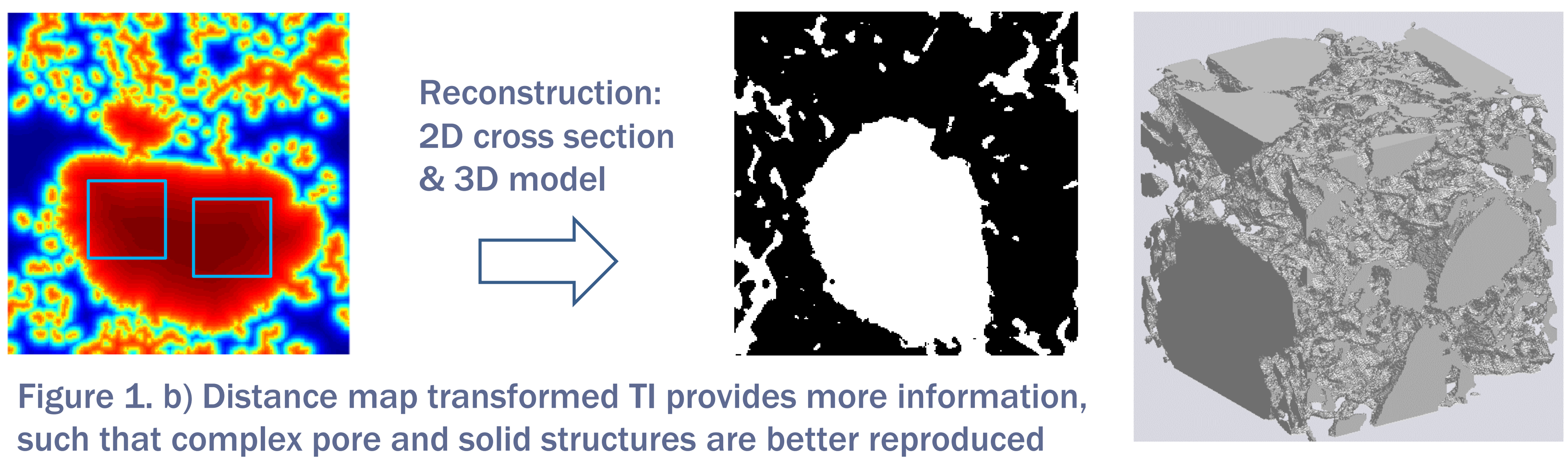


Figure 1. b) Distance map transformed TI provides more information, such that complex pore and solid structures are better reproduced

II. Minimize 2D to 3D intersection conflict – pixel based optimization

Pattern based 2D to 3D reconstruction algorithms suffer from the conflict of 2D orthogonal patterns intersecting in 3D. It is very difficult to find matching patterns (that are consistent with existing reconstruction voxels) from TI in X,Y and Z directions simultaneously – this happens to every voxel. This often leads to discontinuities (see Figure 2). To alleviate the inconsistency, we use the iterative Expectation-Maximization optimization process. (Kopf, 2007) that minimizes the mismatch between the orthogonal patterns in the reconstruction and their nearest (most similar) pattern found in the TI (see below).

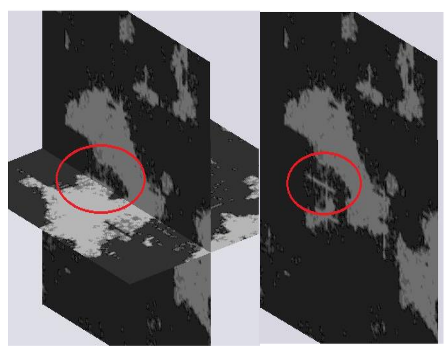


Figure 2. Mismatched intersection patch generate discontinuity.

TI e ⟶ **Nearest pattern** $e(s)$ ⟷ **Model pattern** s

$$E(s;e(s)) = \sum_{s} \|s - e(s)\|^2$$

E-step:
Minimize $E(s;e(s))$ w.r.t. s by least square solver

$$s_v = \frac{\sum_{p\in\mathcal{N}(v)} e(s)_p}{\mathcal{N}}$$

M-step:
Find nearest pattern $e(s)$ from TI for current s

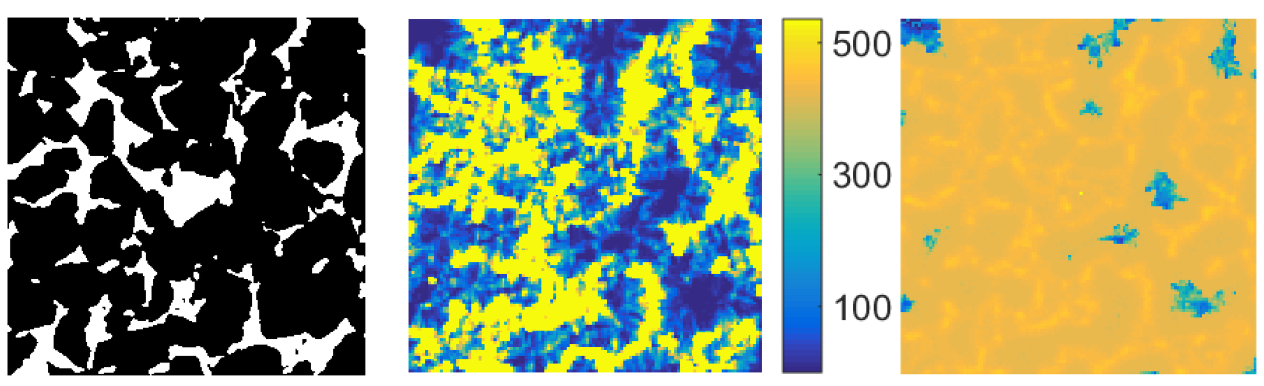
$$\operatorname*{argmin}_{e(s)} \|s - e(s)\|^2$$

III. Avoid poor local minima– position histogram matching

The energy minimization process corresponding to the above E-M optimization often converges to an incorrect local minimum, as it only accounts for the similarity of local patterns. In many cases with all local patterns fitting well with corresponding regions in TI, the overall result may be quite different, i.e. the global statistics and structure are not preserved. Figure 3 illustrates a local minima situation: the energy is minimized, but the usage of patterns (see position histogram H in Figure3b) in pore regions is much higher than that in solid regions, resulting in inaccurate porosity (+68%). To address this problem, a reweighting scheme (see below) is employed in both E-step and M-step in the optimization algorithm (Chen, 2010). The aim is to ensure that most local information in TI is uniformly utilized (Figure3c), such that the global statistics and structure can be reproduced.

E-step: $s_v = \frac{\sum_{p\in\mathcal{N}(v)} w_p e(s)_p}{\sum_{p\in\mathcal{N}(v)} w_p}$ $\quad w_p = \frac{w_p}{1+ max[0,H_{pos}(e(s)_p)-\theta_{pos}]}$

M-step: $d = w_i \cdot \|s - e(s)\|^2$ $\quad w_i = 1 + max[0, H_{idx}(e(s)_i) - \theta_{idx}]$



a) TI: ϕ= 20.6%

b) No histogram control ϕ= 34.7% (+68%)

c) With histogram control ϕ= 21.1% (+2.4%)

Figure 3. a) Berea TI b) Position histogram without control c) Position histogram with control

Results

Using identical or three different orthogonal 2D training images (TI), we are able to reconstruct high quality 3D structures of various types, including (see Figure 4):

- Uniform (a & b)
- Long range connection (c & d)
- Heterogeneous (d & f)
- Anisotropic (e)
- Bimodal pore size distribution (f)

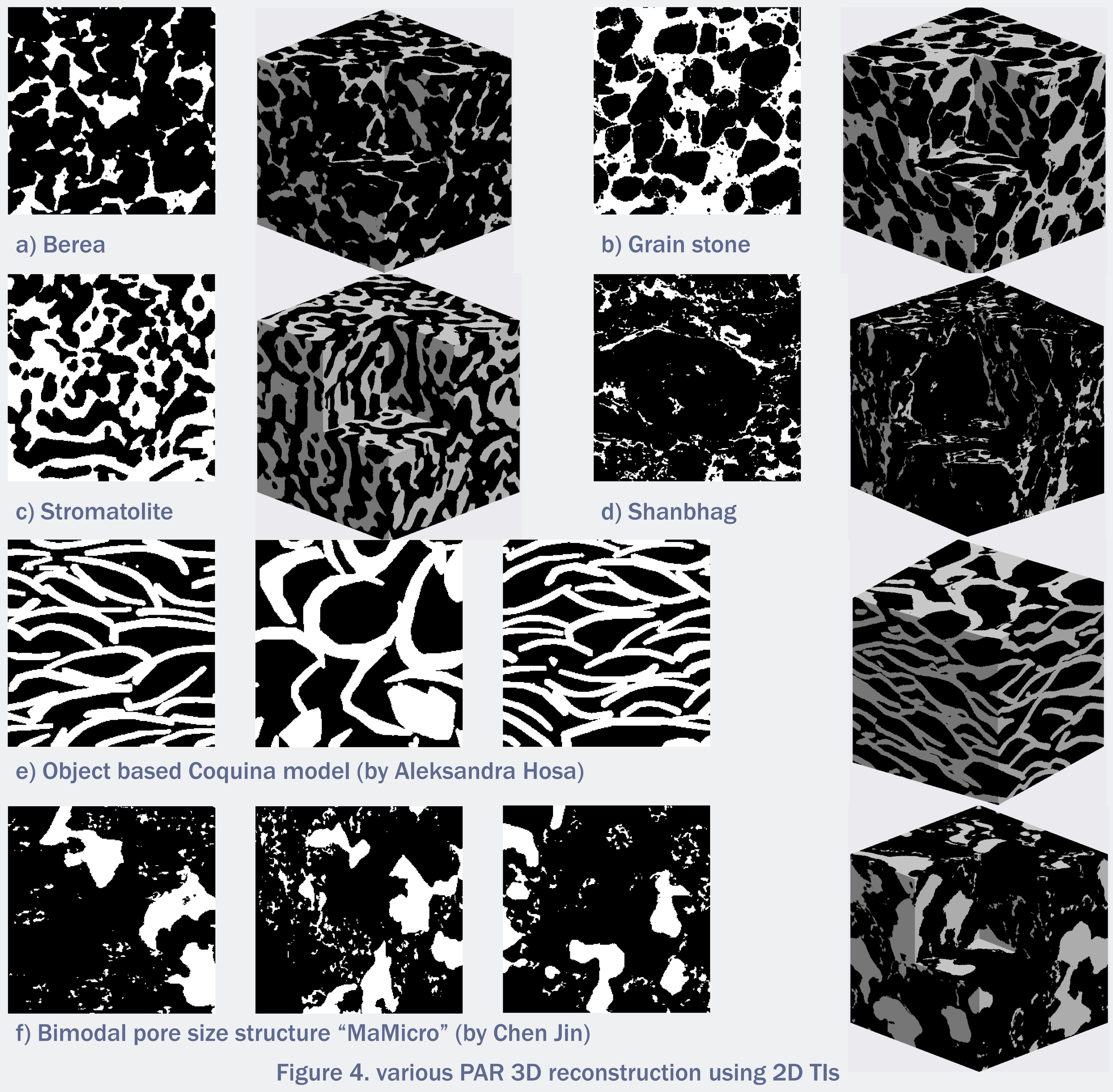


Figure 4. various PAR 3D reconstruction using 2D TIs

Evaluation

Besides visual similarity, good agreement was observed between reconstruction and 3D data in terms of both morphological and petro-physical properties (computed by pore network model). Typical statistics are compared and illustrated, including (see Figure 5):

Geometric properties:
- Two-point connectivity
- Multiple-point connectivity

Pore network statistics:
- Pore size distribution
- Euler connectivity function

Network predicted flow properties:
- Relative permeability during drainage
- Capillary pressure

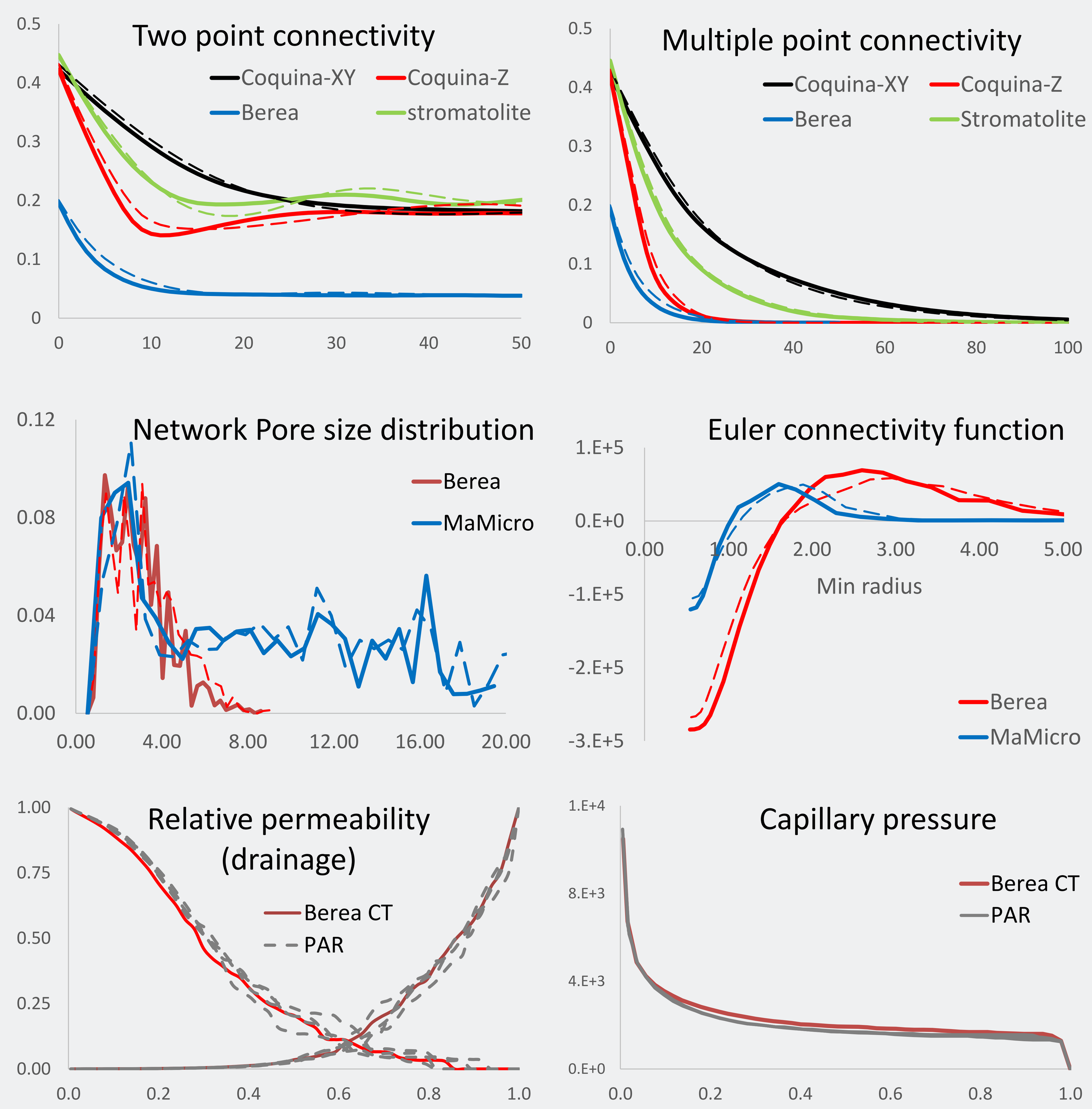


Figure 5. Good agreements in terms of morphological and petro-physical properties

Conclusion

An optimization based algorithm for reconstructing 3D pore structure using 2D images is presented. The information of binary 2D TI is enriched by distance map transformation, enabling accurate reproduction of varying pore/solid sizes. The 3D consistency is significantly improved by minimization of mismatches iteratively. The global statistics is further honored using position histogram matching. The effectiveness of our algorithm is demonstrated by reconstructing various heterogeneous structures and preserving good agreements of both morphological and petro-physical properties.

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References:
- Kopf, J., Fu, C.-W., Cohen-Or, D., Deussen, O., Lischinski, D., & Wong, T.-T. (2007). Solid texture synthesis from 2D exemplars. ACM Transactions on Graphics, 26(3), 2.
- Chen, J., & Wang, B. (2010). High quality solid texture synthesis using position and index histogram matching. Visual Computer, 26(4), 253–262.