Decision Making and Reinforcement Learning

Module 6: Monte Carlo Methods

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Topics

- Value estimation using sample averaging
- First-visit Monte Carlo prediction

- State-action values
- ε–greedy on-policy MC control

- Importance sampling
- Off-policy MC control

Learning Objectives

- **Estimate** state values using MC prediction
- **Estimate** state-action values using ε−greedy MC control and use them to learn or improve policies
- **Use** importance sampling to estimate values of different policies
- **Implement** off-policy MC control to learn optimal policies

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- Examples from real life
- How do humans learn? Other biological organisms?

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- Idea: Approximate the expectation by taking *averages* of sample reward sequences over multiple *episodes*

MC: State Value Estimation

- Idea: $V(s)$ can be estimated by averaging utilities observed *after* visiting s
- Think of each sample as a path from a given node to a leaf node in search tree

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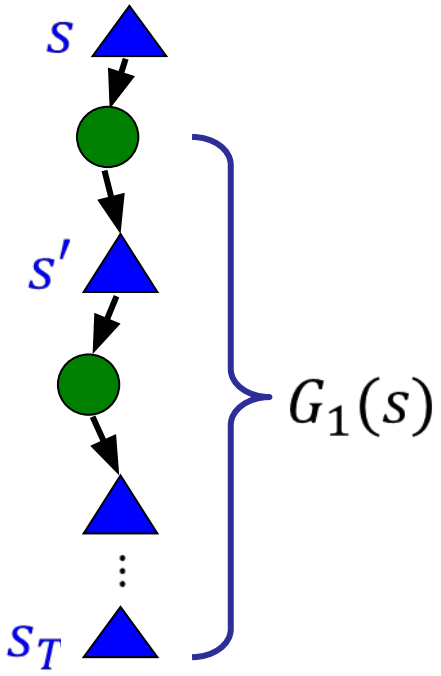

Episode 1

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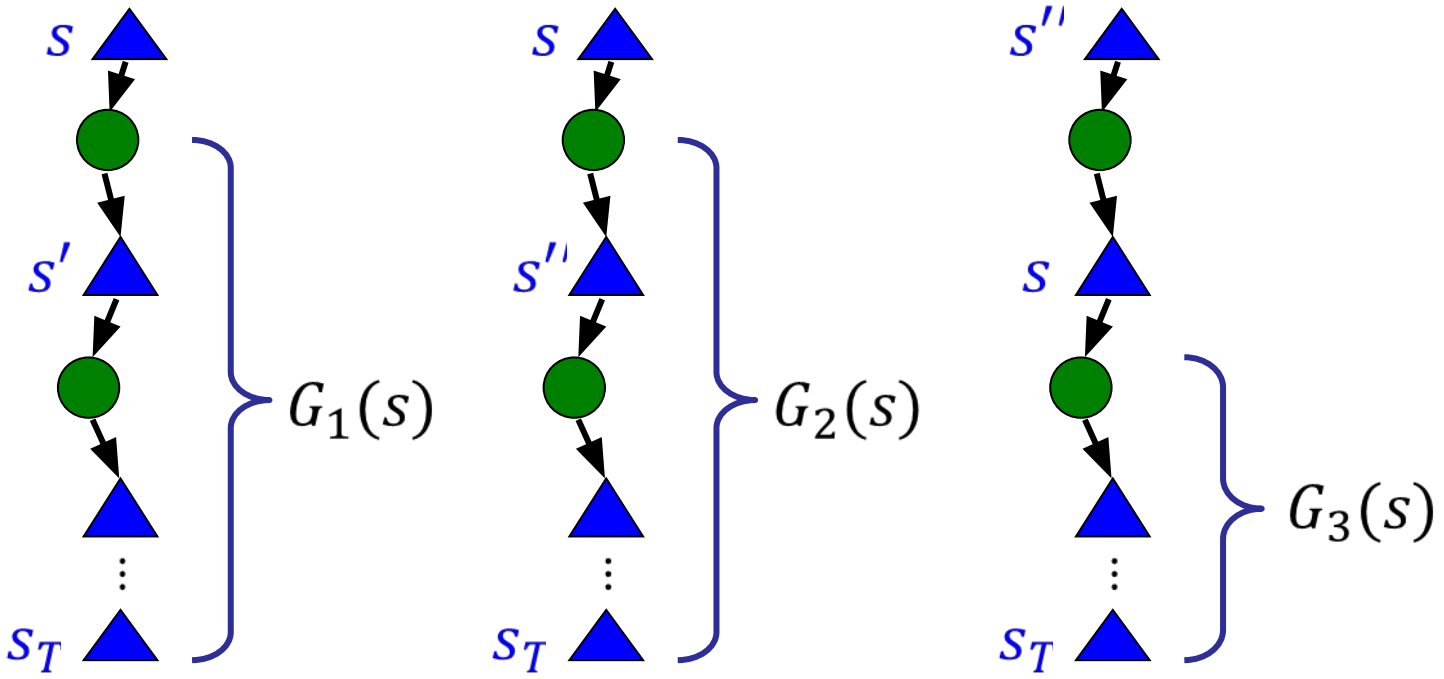

Episode 1 Episode 2 Episode 3

$$\cdots \qquad V(s) = \frac{1}{N}\sum_{i=1}^{N} G_i(s)$$

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Example: Mini-Gridworld

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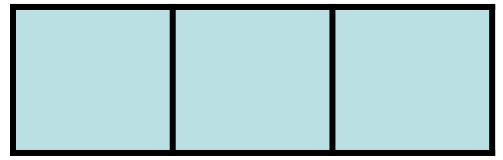

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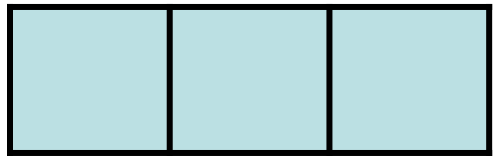
- Each episode ends after 5 actions (finite-horizon)



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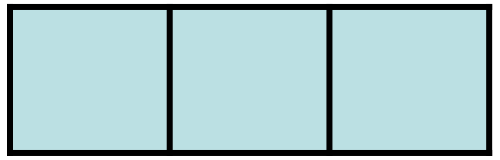

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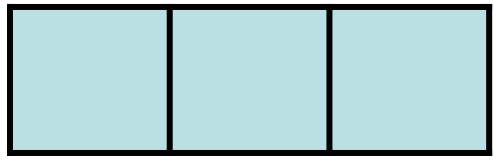

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$$V^{\pi}(A) = 3 + \gamma(-2) + \gamma^2(1) + \gamma^3(-2) + \gamma^4(3) = 2.1875$$

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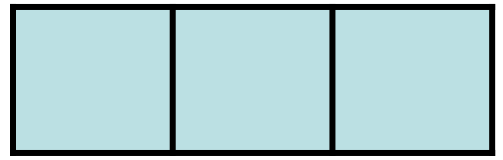

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$$V^{\pi}(B) = 1 + \gamma(-2) + \gamma^2(3) = 0.75$$

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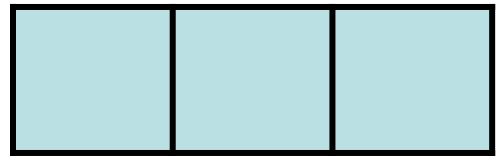

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$$V^{\pi}(C) = -2 + \gamma(3) = -0.5$$

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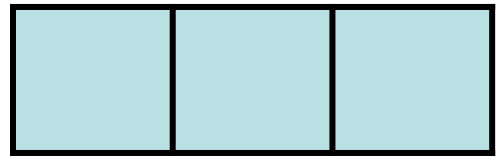

- Episode 2: $(A, -2, B, +3, A, -2, B, +1, C, -2)$

$\gamma = 0.5$

$$G(A) = -1$$

$$G(B) = 2$$

$$G(C) = -2$$

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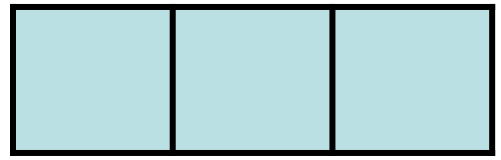

- Episode 2: $(A, -2, B, +3, A, -2, B, +1, C, -2)$ $\gamma = 0.5$

$$G(A) = -1 \qquad V^{\pi}(A) \leftarrow \frac{1}{2}\left(V^{\pi}(A) + G(A)\right) = 0.59375$$

$$G(B) = 2 \qquad V^{\pi}(B) \leftarrow \frac{1}{2}\left(V^{\pi}(B) + G(B)\right) = 1.375$$

$$G(C) = -2 \qquad V^{\pi}(C) \leftarrow \frac{1}{2}\left(V^{\pi}(C) + G(C)\right) = -1.25$$

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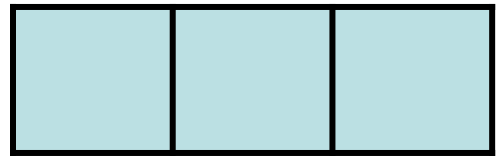

- Episode 3: $(C, +1, C, -2, B, +3, A, -2, B, +3)$

$\gamma = 0.5$

$$G(A) = -0.5 \qquad V^{\pi}(A) \leftarrow \frac{1}{3}\left(2V^{\pi}(A) + G(A)\right) = 0.229$$

$$G(B) = 2.75 \qquad V^{\pi}(B) \leftarrow \frac{1}{3}\left(2V^{\pi}(B) + G(B)\right) = 1.833$$

$$G(C) = 0.6875 \qquad V^{\pi}(C) \leftarrow \frac{1}{3}\left(2V^{\pi}(C) + G(C)\right) = -0.604$$

Finer Points

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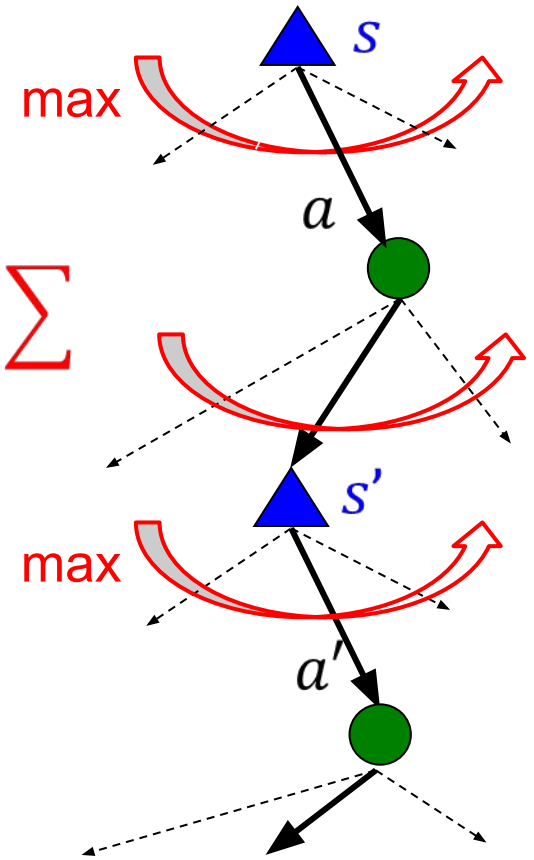
- Result: Computational complexity of estimating specific state values is independent of state space size!
- Can choose to focus on certain states and ignore others

State-Action Values

- Suppose we are given a value function V^{π}
- How do we compute the associated policy π?

$$\pi(s) \leftarrow \operatorname{argmax}_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^{\pi}(s')]$$

don't have these!



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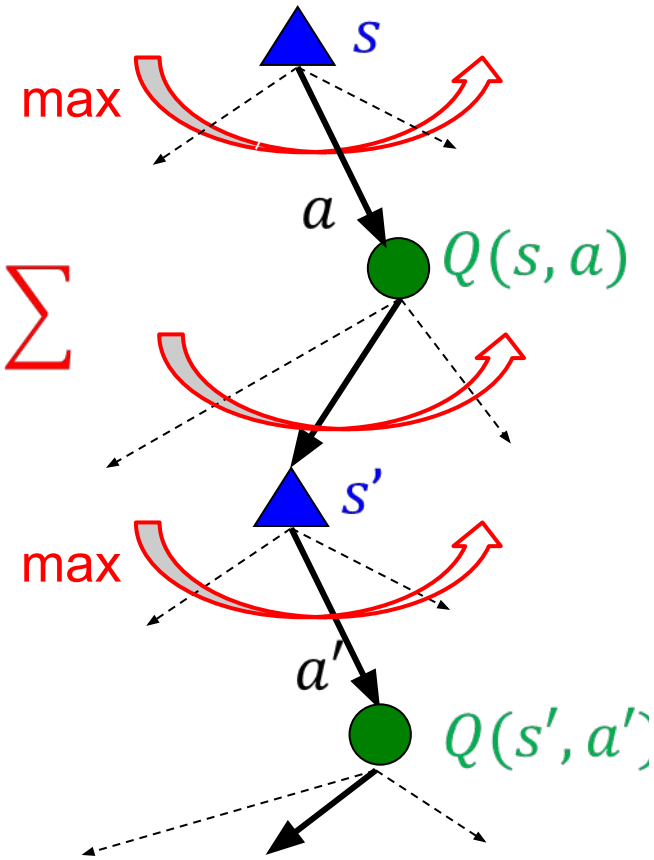

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- Idea: *Learn* **state-action values** values at the chance nodes

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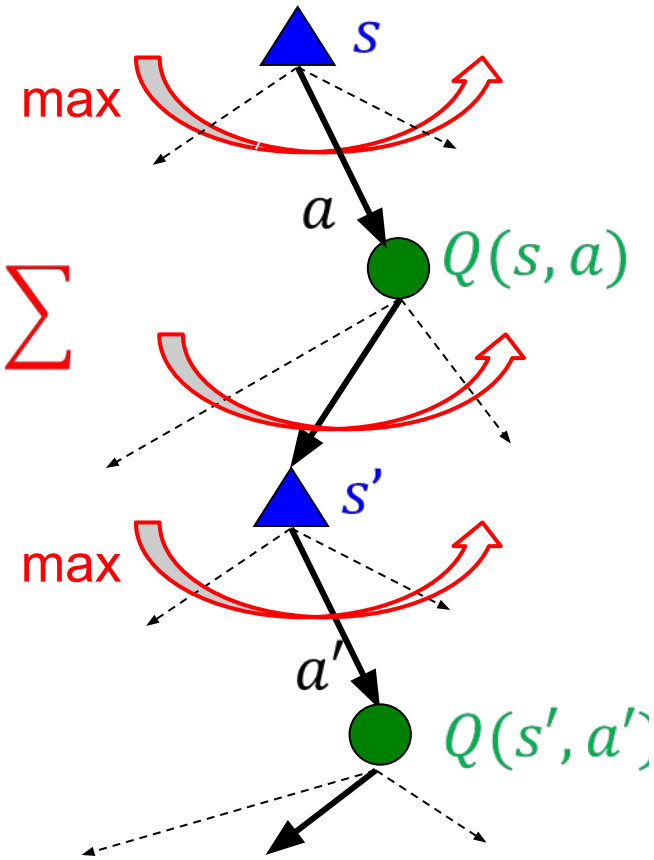
- $Q(s,a)$ is the *expected* utility of s after taking action a and then following π thereafter



Q Values, State Values, and Policy

- **Q values** can be defined as functions of V^* or recursively as functions of themselves

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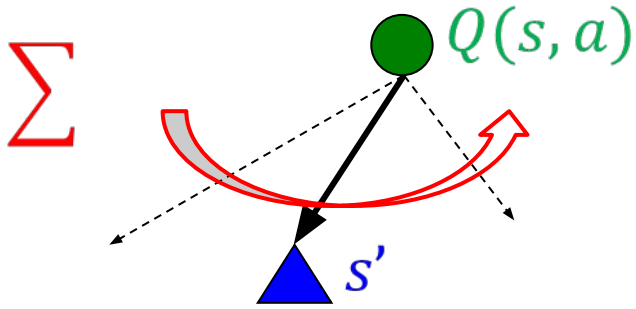

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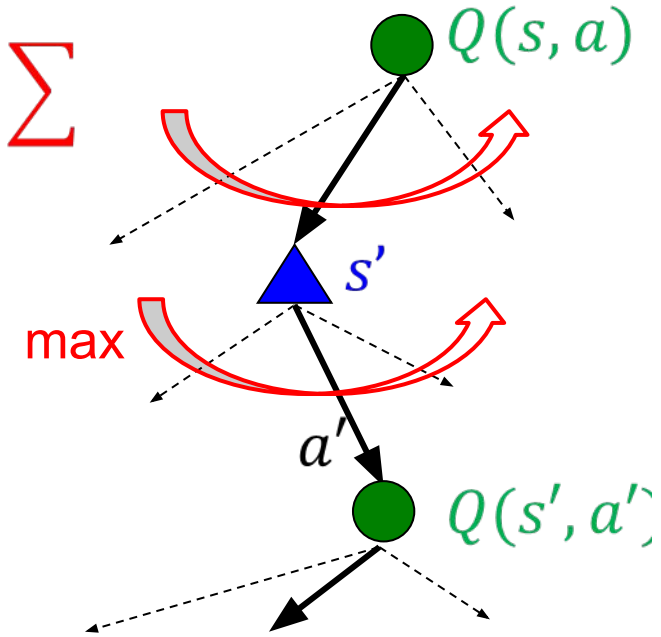

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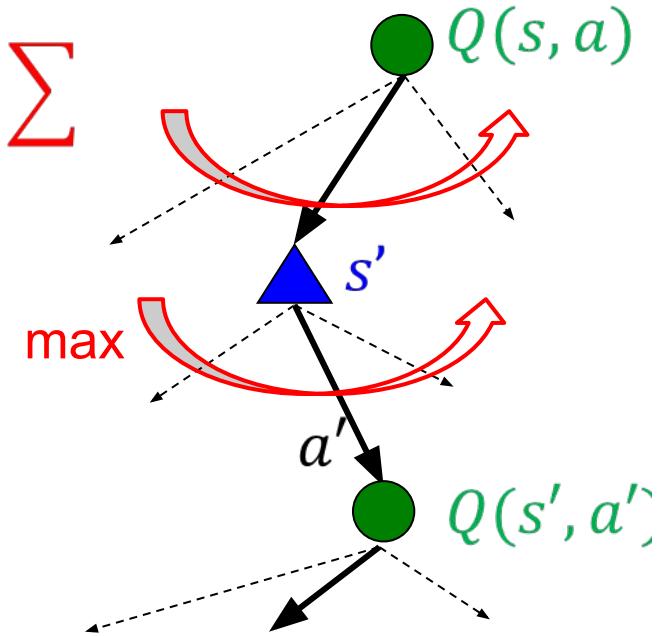


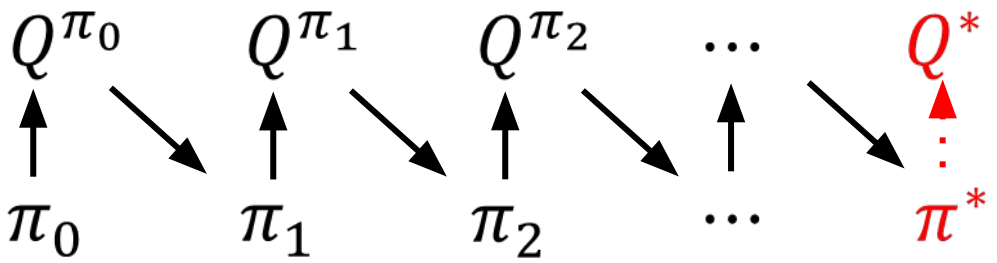
- To learn policies using RL, we will first learn Q values
- Then use these to compute V^* or π^*

$$V^*(s) = \max_a Q(s,a)$$

$$\pi^*(s) = \operatorname{argmax}_a Q(s,a)$$

Monte Carlo Control

- Goal: Learn optimal (or close to optimal) policy via Monte Carlo RL
- Idea from policy iteration: Alternate between evaluation and improvement



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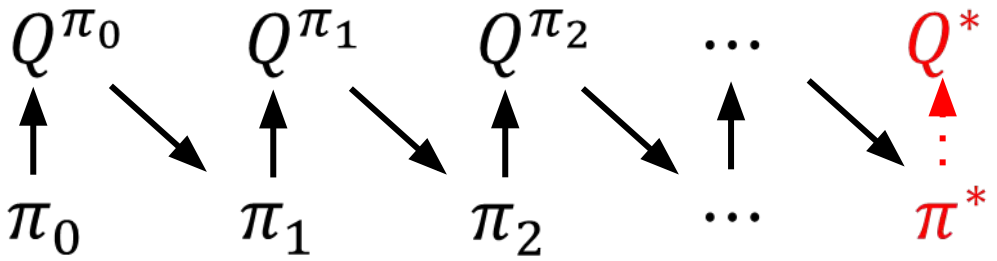

- Evaluation can be done using MC prediction with one or more episodes
- Improvement: Select new actions greedily

$$\pi_{i+1}(s) = \underset{a}{\operatorname{argmax}}\, Q^{\pi_i}(s, a)$$

Convergence and Exploration

- Problem: Policy evaluation requires many episodes
- Instead of waiting for current policy values to converge, we can do one evaluation and one improvement step per episode

- As in generalized policy iteration, this will converge to optimal values and policy
- Argmax operation can only improve current policy

- When learning a new policy we need to experience all relevant state-action pairs
- **Exploring starts**: Choose a random state-action to start in each episode
- Theoretical convergence after infinite episodes

ε-Greedy Policies

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- **ε-greedy policy**: Choose best action most of the time, but with small probability ε, execute random action instead

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Stochastic policy:

$$\pi(a|s) = \begin{cases} 1 - \varepsilon & \text{for } a = \underset{a'}{\operatorname{argmax}}\, Q(s, a') \\ \dfrac{\varepsilon}{|A(s)| - 1} & \text{for all other actions } a \end{cases}$$

ε–Greedy MC Control

- **Initialize** $Q^{\pi}(s,a) \leftarrow 0, Count(s,a) \leftarrow 0, \forall s \in S, \forall a \in A$
- **Initialize** $\varepsilon \leftarrow$ small number, $\pi \leftarrow$ an ε-greedy policy

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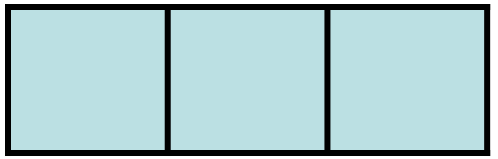
    - $Q^{\pi}(s_t, a_t) \leftarrow (Count(s_t, a_t) \times Q^{\pi}(s_t, a_t) + G)/(Count(s_t, a_t) + 1)$
    - $Count(s_t, a_t) \leftarrow Count(s_t, a_t) + 1$

Example: Mini-Gridworld

- States: A, B, C; actions: L, R
- All Q values initialized to 0; $\gamma = 0.8$



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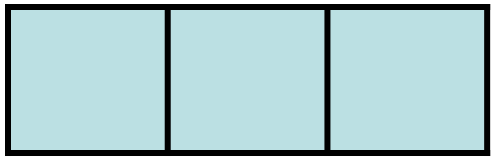

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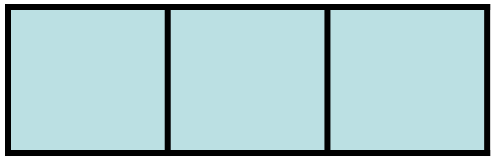

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- Following Q values change:

$$Q(A, L) = 3 + \gamma(-2) + \gamma^2(1) + \gamma^3(-2) = 1.016$$

$$Q(B, R) = 1 + \gamma(-2) = -0.6$$

$$Q(C, L) = -2$$

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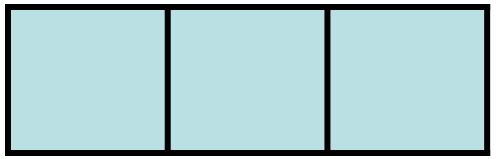

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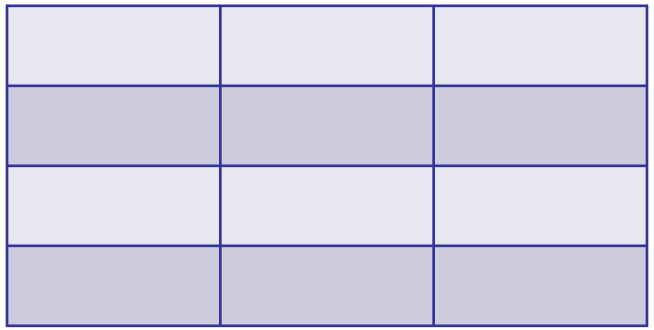

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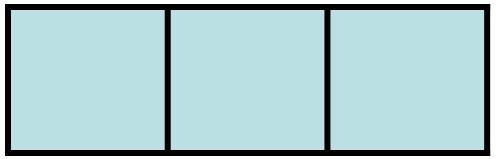

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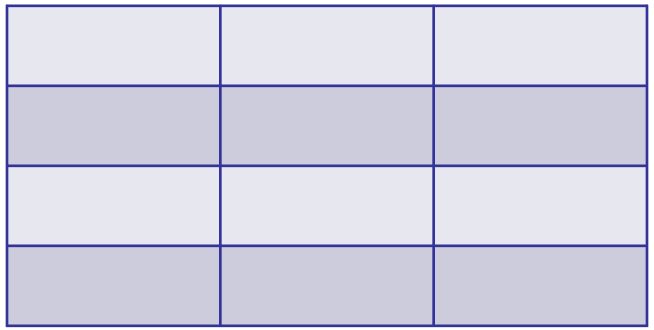

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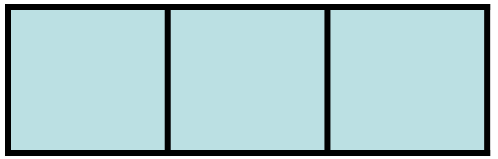

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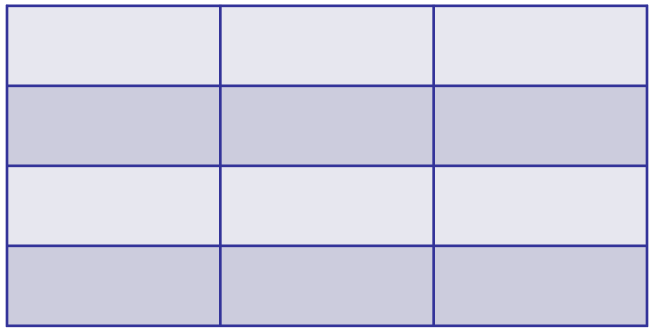

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- **On-policy** learning: Learn the values for the policy that we are following, or the *behavior policy*
- If we wanted the optimal policy, we would have to decrease ε over time

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- Off-policy *prediction*: Estimate values for π using episodes from b
- We require that all state-actions allowed by π also allowed by b: $\pi(a_j|s_j) > 0 \Rightarrow b(a_j|s_j) > 0$

Importance Sampling

- Suppose we observe an episode from behavior policy b:
  $s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1}, r_{t+2}, \dots, s_{T-1}, a_{T-1}, r_T$

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- We can this weight to transform the value of a state-action pair under b into one as if generated by π

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- Recall that computation of G_t, the state-action utility of (s_t, a_t), comes from the rewards seen *after* seeing (s_t, a_t)

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- When updating $Q(s_t, a_t)$ with estimates from multiple episodes, we compute a *weighted average* by dividing by the sum of all weights

Off-Policy MC Control with Weighted IS

- **Initialize** $Q^{\pi}(s,a) \leftarrow 0, W(s,a) \leftarrow 0, \forall s \in S, \forall a \in A$
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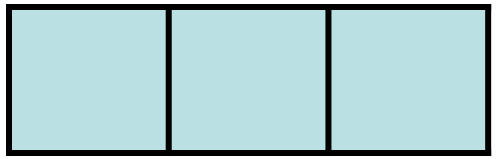

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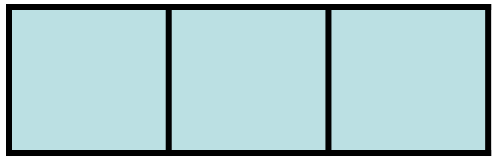

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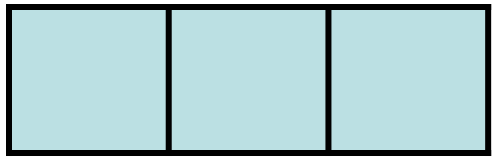

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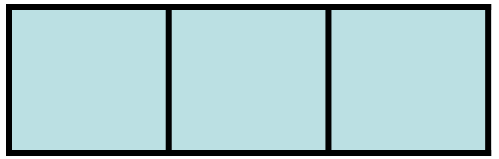

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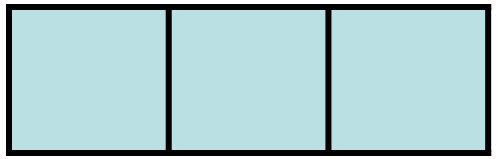

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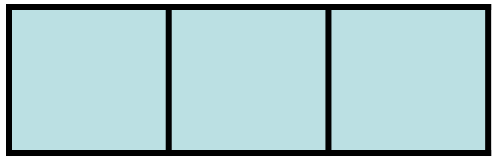

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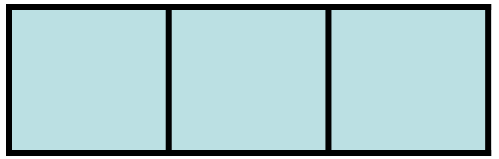

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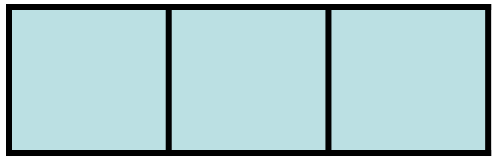

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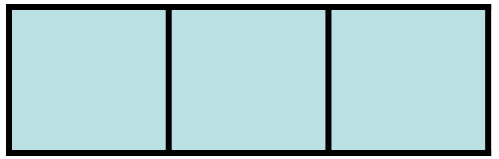

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- Convergence to optimal values will occur in the limit
- Can be slow if exploration is too common
- Experience from beginning of episodes is often wasted

Summary

- MC methods learn value functions and policies from experience
- Useful when model is unknown or difficult to compute
- Useful for focusing on subset of state space

- MC prediction averages utilities from episode sequences to evaluate policies
- Exploration is required for MC control and learning new policies

- *On-policy* methods such as ε-greedy optimize policies that maintain exploration
- *Off-policy* methods use importance sampling to transform utilities generated by a *behavior policy* to estimate those of a *target policy*