2.2. Inferences for Two Means (Two-Sample Cases)

Example of the Pooled t-test (a Two-Mean t-test for Independent Samples)

Step 1: Pooled-sample t-test is selected since it fits the research desgin (no pairing) (all assumptions are met as stated)

Step 2:
H_0 : $\mu_1 = \mu_2$
H_a : $\mu_1 \neq \mu_2$
Parameter: $\mu_1 - \mu_2 = \mu_{Mbudya} - \mu_{Bongoyo}$

Step 3:
Estimate of the difference between means:
$\bar{y}_1 - \bar{y}_2 = 67.4444 - 71.9375 = -4.4931$

Estimate of s_p:
$$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(18-1)10.573^2+(16-1)10.382^2}{18+16-2}} = 10.484$$

$$SE(\mu_{Mbudya} - \mu_{Bongoyo}) = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 10.484\sqrt{1/18 + 1/16} = 3.60228$$

$$t = \frac{(\bar{y}_1-\bar{y}_2)-\delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(\bar{y}_1-\bar{y}_2)-\delta_0}{SE(\bar{y}_1-\bar{y}_2)} = \frac{-4.4431}{3.60228} = -1.247$$

Step 4: df = 18 + 16 - 2 = 32

P-value: (0.10 < P < 0.15) x 2 (two-tailed) or 0.20 < P < 0.30

This is weak evidence against H_0.

$P > \alpha$ (0.05), do not reject H_0.

Step 5: At the 5% signifcance level, the data do not provide sufficient evidence to conclude that there is a difference in the mean heights of colonies at Mbudya and Bongoyo islands.

Example of Calculating a Pooled t-interval

> Using the data and information given for the pooled t-test above, calculate a 95% confidence interval for the difference between mean heights of colonies of Acropora formosa found on the reef crests at Mbudya Island and Bongoyo Island (Dar es Salaam).

For 95% CI, $\alpha = 1 - 0.95 = 0.05$

$t_{\alpha/2} = t_{0.05/2} = t_0.025 = 2.042$

Estimate = $\bar{y}_1 - \bar{y}_2 = -4.4931$

$SE(\bar{y}_1 - \bar{y}_2) = 3.60228$

Confidence Interval:
$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2} \times SE(\bar{y}_1 - \bar{y}_2) = -4.4931 \pm 7.3559 = (-11.85, 2.86)\ cm$

Conclusion: We can be 95% confident that the difference between the mean heights of colonies at Mbudya and Bongoyo Islands is between -11.85 and 2.86 cm.

Hypothesis Test and Confidence Interval: Hypothesis test has P-value which tells you how confident your conclusion is, while confidence interval does not.

Example of Relating Hypothesis Test and Confidence Interval for the Difference Between Two Means

Results from the hypothesis test showed that, at a significance level 5% (α = 0.05), there was no difference in the mean heights of Acropora formosa colonies at Mbudya and Bongoyo Islands.

Results from calculating the confidence interval showed that the (1 – α)% or 95% confidence interval for the difference between the two means (-11.85, 2.86 cm) includes 0. Therefore, we can be 95% confident that the difference between the two means is 0 (not significantly different from 0).

Therefore, the two types of inferential statistics give the same conclusion.

2.2.2. Inferences for Two Population Means: Using Two Paired Samples