6.3. Non-Additive (With Interaction) Model in Comparison with Additive (Without Interaction) Model in Two-Factor ANOVA

Previous Example on Golf Balls (Comparing Non-Additive and Additive Models):

(a) At the 5% significance level, based on the perform a test to determine whether the overall additive Model (Model 1) is significant.

H_0 : The overall additive model is not significant.
H_a: The overall additive model is significant.

$\text{Overall SS (Corrected SS)} = SSA + SSB = 32085.778 + 801.348 = 32888.127$
$\text{Total sample size} = 2 \text{ club types} \times 4 \text{ ball brands} \times 4 \text{ replicates per combination} = 32$

$$F \text{ (Overall model)} = \frac{CorrectSS/(a-1+b-1)}{SSE/(n-1-(a-1)-(b-1))} = \frac{CorrectedMS}{MSE} = \frac{328888.126/(2-1+4-1)}{1584.816/(32-1-(2-1)-(4-1))} = 140.076$$

$df = (4, 27)$
$P < 0.001$, there is extremely strong evidence against H_0.
$P < \alpha(0.05)$, therefore reject H_0.

At the 5% significance level, there is sufficient evidence that the overall additive model is significant.

(b) At the 5% significance level, perform the most appropriate test to determine whether there is a main effect of club type on mean distance.

F (main effect of Factor A (club type))

$$F_A = \frac{SSA/(a-1)}{SSE/(n-1-(a-1)-(b-1))} = \frac{MSA}{MSE} = \frac{32086.778/(2-1)}{1584.816/(32-1-(2-1)-(4-1))} = 546.652$$

$df = (1, 27)$
$P < 0.001$, there is extremely strong evidence against H_0.
$P < \alpha(0.05)$, reject H_0.

At the 5% significance level, there is sufficient evidence that there is a signifificant main effect of club type.

Extra Sum-of-Squares F-test for Testing AB Interaction by Comparing Additive and Non-additive Models:

(a) By comparing the additive model and the non-additive models presented in the tables above, at the 5% significance level, perform the most appropriate test to determine whether the effect of club type depends on ball brand. In other words, test whether there is an interaction effect between club type and ball brand, after accounting for club type and ball brand (as main effects)

$H_0: \mu(Distance|Club, Ball) = \beta_0 + Club + Ball$ (No interaction - Additive model = Reduced model (Model 1))
$H_a: \mu(Distance|Club, Ball) = \beta_0 + Club + Ball + Club \times Ball$ (With interaction - Nonadditive model = Full model (Model 2))

$df_E(reduced) = (n-1) - (a-1) - (b_1) = 32 - 1 - (2-1) - (4-1) = 27$
$df_E(full) = n - ab = 32 - 2 \times 4 = 24$

$F = \frac{[SS_E(reduced) - SS_E(full)]/[df_E(reduced) - df_E(full)]}{SS_E(full)/df_E(full)} = \frac{(1584.816-820.113)/(27-24)}{820.113/24} = 7.459$

$df = (3, 24)$
$0.001 < P < 0.005$, there is very strong evidence against H_0
$P < \alpha(0.05)$, therefore reject H_0.

At the 5% significance level, there is sufficient evidence to conclude that there is a significant interaction effect of club type times ball brand, after accounting for club type and ball brand.