# 2.3 Parametric Methods, Transformations, and Nonparametric Methods

## Example on Transformation & Inferences After Transformations:

> Students were randomly allocated to two groups, one group to a new program and the other group to a standard program. At the end of the experiment, their test scores were recorded (scale = 0 – 700). Since the test scores are not normally distributed and the standard deviations are very different, the log (natural) transformed data are also shown as well as summary statistics. At the 10% significance level, test whether there is a difference in the test scores of students undertaking the two programs and calculate 90% confidence limits.

**Conclusions based on the log transformed data:**

Hypothesis test:

> At the 10% significance level, there is moderate evidence that there is a difference in the means of the logged test scores between the new program and the standard program (Two-sample pooled t-test: t = 1.72, df = 26, P = 0.096).

Confidence interval:

> The estimate of the difference between the means of the logged test scores of the new program and the standard program is 0.982 and the 90% confidence interval for the additive effect of the new program on the test scores is between 0.011 and 1.954. [Also, we can be 90% confident that there is a difference between the means of new and standard programs because 0 is not inside this confidence interval.]

**Back Transformation of the Estimate and Confidence Int. and Interpretation on the original scale:**

Estimate of the difference: $e^{0.982} = 2.670$

Lower endpoint of CI: $e^{0.011} = 1.011$

Upper endpoint of CI: $e^{0.1954} = 7.057$

*Conclusions on the original scale (indicating the multiplicative effet of the treatment):*

Conclusion of the hypothesis test: At the 10% significance level, the median test score of those who took the new program is estimated to be 2.670 times the median test score of theose in the standard program.

We use medians in non-normal distributions.

Conclusion of the confidence interval: A 90% CI for the ratio of the medians in the original scale $\left[\frac{Med(new)}{Med(standard)}\right]$ is $(e^{0.011}, e^{1.954}) = (1.011, 7.057)$
Or, it is estimated with 90% Confidence that the median test score for the new program is between 1.011 and 7.057 times the median test score for the standard program.

**NOTE**: Also, this means that we can be 90% confident that there is a difference between the medians of new and standard programs because 1 (NOT 0) is not inside this confidence interval (1.011, 7.057). This is because ln 1 = 0 and the antilog of 0 = 1(that is, $e^0 = 1$)

Furthermore:

$e^{\overline{LnY_1} - \overline{LnY_2}} = e^{4.257-3.275} = e^{0.982} - 2.670$ estimates $\left[\frac{Median(Y_1)}{Median(Y_2)}\right]$ (population prameters), estimated by $\frac{Median(sample1)}{Median(sample2)} = \frac{86.5}{32.5} = 2.662$

### Back Transformation in Reverse

[Reverse means: Subtracting Standard minus New, instead of New minus Standard (as above)]
Back Transformation of the estimate and confidence interval to the original data:

Estimate of the difference = $e^{-0.982} = 0.3746$
Lower endpoint of CI = $e^{-1.954} = 0.1417$
Upper endpoint of CI = $e^{-0.011} = 0.9891$

Conclusion of the hypothesis test on the original scale:
At the 10% significance level, the median tests score of thoes who took the standard program is estimated to be 0.3746 times the median test score of the new program.

Conclusion of the confidence interval on the original scale: A 90% CI for the ratio of the medians in the original scale $\left[\frac{Med(standard)}{Med(new)}\right]$ is $(e^{-1.954}, e^{-0.011}) = (0.1417, 0.9891)$

Again, since 1 is not inside this confidence interval, we can be 90% confident that there is a difference between the medians of new and standard program.

**Comparing the two results:**

Estimate of the difference for New vs. Standard = 2.670
Estimate of the difference for Standard vs. New = 0.3746
2.670 is the inverse of 0.3746

# 2.2.2 Inferences for Two Population Means

(back to previous example)

### Additional Example

> A fuel manufacturer wanted to test the effectiveness of a new gasoline additive. A random sample of 6 cars were driven one week without the additive and one week with the additive

(a) The confidence interval for the difference in mileage without and with the additive is (3.22094, 0.21906). Determine the confidence level at which this interval was calculated.

Margin of error = $\frac{\text{Upper endpoint - Lower endpoint}}{2} = \frac{3.22094-0.21906}{2} = 1.50094$

df = n - 1 = 5

$\mathrm{ME} = t_{\alpha/2} \times \frac{S_d}{\sqrt{n}} \rightarrow t_{\alpha/2} = \mathrm{ME} \times \frac{\sqrt{n}}{S_d} = 1.50094 \times \frac{\sqrt{6}}{1.43} = 2.571 = t_{0.025}$

$\alpha/2 = 0.025 \rightarrow \alpha = 0.05$

Confidence level = 1 - 0.05 = 95%

(b) Calculate a 99% confidence interval for the difference in mileage without and with the additive.

$\bar{d} = \frac{\text{lower endpoint + upper endpoint}}{2} = -1.72$

At df = 5, $t_{\alpha/2} = t_{0.005} = 4.032$
$\bar{d} \pm t_{\alpha/2} \times \frac{S_d}{\sqrt{n}} = -1.72 \pm 4.032 \times \frac{1.43}{\sqrt{6}} = -1.72 \pm 2.3539 = (-4.074, 0.634)$

It is estimated with 99% confidence that the difference in mileage without and with the additive is in the interval.

(c) Compare the confidence interval given in part (a) and the confidence interval you calculated in part (b). Based on each of these confidence intervals, is there a difference in mileage without and with the additive. Explain why you either got the same conclusion or different conclusions from the two intervals.

> The confidence interval given in part (a), (3.22094, 0.21906) − is shorter and more precise than the confidence interval calculated in part (b), (4.074, 0.634). Based on the 95% confidence interval given in part (a), we would conclude that there is a difference in mileage without and with the additive, it does not contain 0.
> However, based on the 99% confidence interval calculated in part (b), we would conclude that there is no difference, because it does contain 0.

## Research Problem on Exercise Program to Reduce Weight

> It is claimed that a certain exercise program will reduce body weight by more than 20 kg within 6 months in seriously overweight people. The table shows the body weights of a random sample of 15 people before and after undertaking this program. At the 1% significance level, test whether this claim is true.

Parameter = $\mu_d = \mu_{before} - \mu_{after}$
$H_0 : \mu_{before} - \mu_{after} = 20\mathrm{kg}$

$H_a : \mu_{before} - \mu_{after} > 20\text{kg}$
Hypothesized difference = $\Delta_0 = 20$

$$t = \frac{\bar{d} - \Delta_0}{S_d/\sqrt{n}} = \frac{\bar{d} - \Delta_0}{SE(\bar{d})} = \frac{21.8 - 20}{3.167/\sqrt{15}} = 2.200$$

df = n - 1 = 15 - 1 = 14
P-value : 0.02 < P < 0.025, there is strong evidence against $H_0$.
$P > \alpha(0.01)$, therefore do not reject $H_0$.

<u>Conclusion</u>: At the 1% significance level, the data do not provide enough evidence to prove the claim that the exercise program reduces body weight by more than 20kg in seriously overweight people.