3.3.2 Bonferroni's Method of Multiple Comparisons

RETURN to the Research Problem on pages 9-12: Do exercise level and sodium intake level (explanatory variables) have an effect on blood pressure (response variable)?

Number of multiple comparisons: $m = \frac{k(k-1)}{2} = \frac{6(6-1)}{2} = 15$
Individual error rate: $\alpha_I = \frac{\alpha_F}{m} = 0.03/15 = 0.002$

At $df = n - k = 48 - 6 = 42, t_{\alpha/2} = t_{0.002/2} = t_{0.001} = 3.307$

$$\text{ME}_{ij} = t_{n-k,\,\alpha/2} \times \sqrt{\text{MSE}}\sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

$$\text{MSE} = \frac{\text{SSE}}{n-k} = \frac{3342.815}{48-6} = 79.616071$$

$$\text{ME}_{ij} = 3.307 \times \sqrt{79.616071}\sqrt{1/8 + 1/8} = 3.307 \times 8.922784 \times 0.5 = 14.754$$

$\mu_i \neq \mu_2$, if $|\bar{y}_i - \bar{y}_j| \geq ME_{ij}$

>>>>>>>>>> **Matrix**

	NEx-TypNa	MEx-TypNa	VEx-TypNa	NEx-RedNa	MEx-RedNa	VEx-RedNa
NEx-TypNa						
MEx-TypNa	5.50 < 14.75					
VEx-TypNa	14.50 < 14.75	9.00 < 14.75				
NEx-RedNa	9.00 < 14.75	3.50 < 14.75	5.50 < 14.75			
MEx-ReNa	16.50 > 14.75*	11.00 < 14.75	2.00 < 14.75	7.50 < 14.75		
VEx-RedNa	22.63 > 14.75*	17.13 > 14.75*	8.13 < 14.75	13.63 < 14.75	6.13 < 14.75	

* Indicates pairwise comparisons that can be declared different

Means Comparisons Diagram

VigEx-RedNa	ModEx-RedNa	VigEx-TypNa	NoEx-RedNa	ModEx-TypNa	NoEx-TypNa
131.75	137.88	139.88	145.38	148.88	153.38

Conclusion in words: It is estimated with 97% confidence that there is insufficient evidence of a difference in mean SBP between VigEx-RedNa, ModEx-RedNa, VigEx-TypNa and NoEx-RedNa, nor between ModEx-RedNa, VigEx-TypNa, NoEx-RedNa, and ModEx-TypNa, nor between Vig.Ex-TypNa, NoEx-RedNa, ModEx-TypNa, and NoEx-TypNa. All other pairs can be declared different.

Note: Bonferroni controls overall error rate by calculating $\alpha_I = \frac{\alpha_F}{m}$:

1. Higher critical value
2. Wider confidence interval
3. More conservative than Tukey's.

3.4 Linear Combinations (Contrasts) (=Planned Comparisons)

Applications of Linear Contrasts in a Rice Experiment

> Research Problem:
> Determine whether there are differences in effectiveness between the following using linear contrasts:
>
> 1. Control (by itself) versus the biofertilizers (combined)
> 2. The biofertilizers (combined) versus the chemical nitrogen fertilizers (combined)

(a) Develop a linear contrast and perform a hypothesis test, at the 5% significance level, to determine whether there is a difference in mean yield of rice between the control group (C) (by itself) and the two biofertilizers (B) (combined).

Develop the linear combinatio of state parameter:
$\gamma_{C-B} = \frac{\mu_c}{1} - \frac{\mu_u + \mu_A}{2}$

Parameter:
$\gamma_{C-B} = \mu_c - \frac{1}{2}\mu_U - \frac{1}{2}\mu_A$
$H_0 : \gamma = 0$
$H_a : \gamma \neq 0$

Estimate:
$\hat{\gamma}_{C-B} = \hat{\gamma_C} - 1/2\bar{\mu_0} - 1/2\bar{\mu_A} = 2.557 - 1/2 \times 4.333 - 1/2 \times 5.050$

Standard dev of estimate: $S_p = \sqrt{\text{MSE}} = \sqrt{0.22285} = 0.4721$

$SE(\hat{\gamma}) = S_p\sqrt{\frac{C_1^2}{n_1} + \frac{C_2^2}{n_2} + ... + \frac{C_k^2}{n_k}} = 0.4721\sqrt{\frac{1^2}{7} + \frac{(-1/2)^2}{6} + ... + \frac{(-1/2)^2}{6}} = 0.4721 \times 0.4756 = 0.2245$

$t = \frac{\hat{\gamma} - 0}{SE(\hat{\gamma})} = \frac{-1.1345}{0.2245} = -5.053$

$df = n - k = 33 - 5 = 28$
P-value: $(P < 0.0005) \times 2 \Rightarrow P < 0.001$

There is extremely strong evidence against H_0.
$P < \alpha\ (0.05)$, therefore reject H_0.

Conclusion: At the 5% significance level, the data provide sufficient evidence to conclude that there is a difference in mean yield of rice between the control group (C) (by itself) and the two biofertilizers (B) (combined).

(b) Calculate a 95% confidence interval for the linear contrast in part (a).

For 95% CI, $\alpha = 0.05$
At $df = n - k = 33 - 5 = 28,\ t_{\alpha/2} = t_{0.025} = 2.048$

$\hat{\gamma}\ \pm\ \text{Crit. value} \times SE(\hat{\gamma}) = -1.1345 \pm 2.048 \times 0.2245 = -1.1345 \pm 0.4598 = (-1.59, -0.67)$

Conclusion: it is estimated with 95% confidence that the difference in mean yield of rice between the control group C and the two fertilizers is between 01.59 and -0.67.
(Note: Since 0 is not in the interval, it confirms that there is a difference)

(c) Develop a linear contrast and perform a hypothesis test, at the 5% significance level, to determine whether there is a difference in mean yield of rice between the biofertilizers (combined) versus the chemical nitrogen fertilizers (combined).

$\gamma_{B-N} = \frac{\mu_U + \mu_A}{2} - \frac{\mu_{N_1} + \mu_{N_2}}{2}$

Parameter: $\gamma_{B-N} = 1/2\mu_U + 1/2\mu_A - 1/2\mu_{N_1} - 1/2\mu_{N_2}$

$$H_0 : \gamma = 0 \quad \text{versus } H_a : \gamma \neq 0$$

Estimate

$$\hat{\gamma}_{B-N} = \frac{1}{2}\bar{y}_U + \frac{1}{2}\bar{y}_A - \frac{1}{2}\bar{y}_{N1} - \frac{1}{2}\bar{y}_{N2}$$
$$= \frac{1}{2}(4.333) + \frac{1}{2}(5.050) - \frac{1}{2}(4.457) - \frac{1}{2}(4.957) = -0.0155$$

Standard error

$$SE(\hat{\gamma}) = 0.4721\sqrt{\frac{(1/2)^2}{6} + \frac{(1/2)^2}{6} + \frac{(-1/2)^2}{7} + \frac{(-1/2)^2}{7}}$$
$$= (0.4721)(0.3934) = 0.1857$$

t-statistic:

$$t = \frac{\hat{\gamma} - 0}{SE(\hat{\gamma})} = \frac{-0.0155 - 0}{0.1857} = -0.083$$

$df = n - k = 33 - 5 = 28$

P-value: (P < 0.25) x 2 = P < 0.50. There is weak evidence against H_0.
Since P > α (0.05), do NOT reject H_0.

Conclusion: At the 5% significance level, there is insufficient evidence to conclude that there is a difference in mean yield of rice between the biofertilizers (combined) versus the chemical nitrogen fertilizers (combined).

Research Conclusion: Biofertilizers *Azolla-Anabaena* and *Utricularia*-Cyanophyta can be applied on rice to increase crop yield with effects comparable to the application of chemical nitrogen fertilizers. At the same time, these biofertilizers save costs and are an "environmentally-friendly" alternative. Also, the biofertilizers help to control weeds and mosquitoes.

Post Hoc Tests (Tukey's and Bonferroni's Methods) on Rice Experiment

Tukey:
Means Comparisons Diagram:

Control	Ultricularia	N1	N2	Azolla
3.557	4.333	4.457	4.957	5.050
|_____|	|_____|			

Conlcusion: We can be 95% confident that the mean yield of the control group is different from that of the other groups, but no other means are significantly different.

Bonferroni:
Means Comparison Diagram:

Control	Ultricularia	N1	N2	Azolla
3.557	4.333	4.457	4.957	5.050
|_____|				
	|_____|			

Conclusion: We can be 95% confident that the mean yiled of the control group is different than that of all other groups except Ultricularia. No other means are significantly different.