6. TWO-FACTOR ANOVA

6.2. Two-Factor ANOVA with Interaction (with Replication, Balanced Data)

Research Problem Involving Two-Factor ANOVA, Followed by Multiple Comparisons Tests Example on Golf Balls:

Checking Assumptions:

- Randomness and Independence: "Completely randomized design" covers both of these assumptions.
- Normality: The Q-Q plot shows that the points are reasonably close to a straight line. Thus, the distance variable is approximately normally distributed.
- Equal standard deviations: Levene's test gives P = 0.307. Since this is a large P value, H_0 is not rejected. Thus the standard deviation for all the combinations of the two-factors are not significantly different.

(a) At the 5% significance level, perform the most appropriate test to determine whether the overall model is significant.

H_0 : All treatment combinations (driver/ball) have equal means (the overall model is not significant)
H_a : At least two treatment combinations have different means (the overall model is significant)

$\text{Overall model SS (Corrected SS)} = SSA + SSB + SSAB = 32086.778 + 801.348 + 764.703 = 33652.829$
OR
$\text{Overall SS} = \text{Corrected Total SS} - \text{Error SS} = 34472.942 - 820.113 = 33652.829$

[Note: Corrected Total SS = Total SS - Intercept SS]

$\text{Overall model df (Corrected df)} = df(A) + df(B) + df(AB) = (a-1) + (b-1) + (a-1)(b-1) = (2-1) + (4-1) + (2-1)(4-1) = 7$
OR
$\text{Overall df} = ab - 1 = 7$

$n = a \times b \times \text{number of replicates per column} = 2 \times 4 \times 4 = 32$

$\text{Error df} = n - ab = 32 - 2 \times 4 = 24$

$F(Overall) = F(Corrected) = \frac{CorrectedSS/(ab-1)}{ErrorSS/(n-ab)} = \frac{CorrectedMS}{MSE} = \frac{33652.829/7}{820.113/24} = \frac{4807.547}{14.17138} = 140.689$

$df = (7, 24)$. $P < 0.001$. There is extremely strong evidence against H_0.

$P < \alpha(0.05)$. Therefore reject H_0.

At the 5% significance level, the data provide sufficient evidence that the treatment means for distance are not all the same for all club/ball combinations; therefore, the overall model is significant.

(b) At the 5% significance level, perform the most appropriate test to determine whether there is a main effect of club type on mean distance.

H_0: There is no main effect of club type. (The means for all club types are equal)
H_a: There is a main effect of club type. (At least two means for club types are different)

F (Main effect of club type):
$$F_A = \frac{SSA/(a-1)}{SSE/(n-ab)} = \frac{MSA}{MSE} = \frac{32086.778/(2-1)}{820.113/24} = 938.996$$

$df = (1, 24), P < 0.01$. There is extremely strong evidence against H_0.
$P < \alpha(0.05)$. Therefore reject H_0.

At the 5% significance level, the data provide sufficient evidence that there is a siginificant main effect of club type, that is, the mean distance are not all the same for the different club types, averaging over all brands of balls.

(c) At the 5% significance level, perform the most appropriate test to determine whether there is a main effect of ball brand on mean distance.

H_0: There is no main effect of ball brand. (The means for all ball brands are equal)
H_a: There is a main effect of ball brand. (At least two means for ball brands are different)

F (Main effect of club type):
$$F_A = \frac{SSB/(b-1)}{SSE/(n-ab)} = \frac{MSB}{MSE} = \frac{801.348/(4-1)}{820.113/24} = 7.8169$$

$df = (3, 24)$. $P < 0.01$. There is extremely strong evidence against H_0.
$P < \alpha(0.05)$. Therefore reject H_0.

At the 5% significance level, the data provide sufficient evidence that there is a siginificant main effect of ball type on mean distance.

(d) At the 5% significance level, perform the most appropriate test to determine whether the effect of club type on mean distance depends on ball brand. (In other words, test whether there is an interaction effect between club type and ball brand.)

H_0 : There is no interaction effect between club type and ball brand. (All interaction terms equal 0)
H_a : There is an interaction effect between club type and ball brand. (At least one interaction term is not 0)

F (interaction):
$$F_A = \frac{SSAB/(a-1)(b-1)}{SSE/(n-ab)} = MSAB = \frac{764.70/(2-1)(4-1)}{820.113/24} = 7.459$$

$df = (3, 24)$. $0.001 < P < 0.005$, very strong evidence against H_0.
$P < \alpha$ (0.05), reject H_0.

At the 5% significance level, the data provide sufficient evidence that there is significant interaction effect of club type times ball brand, that is, at least one interaction term is not 0.