2.1.1. Research Problem

One-Mean t-test

Step 1: The one-sample t-test is selected
Purpose of the research problem: To test the differnece between on epopulation mean(based on the sample mean) and the hypothesized mean of 75cm.
Assumptions:

1. Random sample (stated in the question)
2. Normal population. Since the P-value for the Shapiro-WIlk Test (P==0.775) is greater than $\alpha = 0.05$ this distribution is not signifcantly diffrent from a normal distribution. Therefore, even a small sample size is adequate.
3. σ is not known (therefore t-test must be applied using the sample standard deviation).

Step 2: State the nul and alternative hypotheses
$H_0 : \mu = 75$ (mean height of the colones found on this reef crest is not shorter than the countrywide mean height of 75cm).
$H_a : \mu < 75$ (mean height shorter than 75cm).
Parameter μ = mean height of A. formosa colonies on the reef crest of Mbudya Island.

Step 3: Obtain the calculated value (or observed value) of the test statistic:
Estimate of the population = $\bar{y} = 67.4444cm$
Standard error(SE) of the estimate of the sample mean
$SE(\bar{y}) = \frac{S}{\sqrt{n}} = \frac{10.5731}{\sqrt{18}} = 2.4921$
$t = \frac{\bar{y}-\mu_0}{\frac{S}{\sqrt{n}}} = \frac{\bar{y}-\mu_0}{SE(\bar{y})} = \frac{67.4444-75}{2.4921} = -3.032$
df = 18 - 1 = 17
Since the t-table is one-tailed and the test is one-tailed, we do not double the P-value
P-value = 0.005 > P > 0.0025
or 0.0025 < P < 0.005
(exact P-value = 0.004)
$P < \alpha$, therefore reject H_0

Two-tailed test

Step 1: Same as above
Step 2:
$H_0 : \mu = 75$ (mean height of the colones found on this reef crest is not shorter than the countrywide mean height of 75cm).
$H_a : \mu \neq 75$ (mean height different than 75cm).
Parameter μ = mean height of A. formosa colonies on the reef crest of Mbudya Island.
Step 3: t = -3.032

Step 4: 0.005 < P < 0.01
Exact P-value = 0.008 (Very strong evidence)
$P < \alpha$, therefore reject H_0.

Note: In the above example, the evidence against H0 is stronger when doing the one-tailed test as opposed to doing the two-tailed hypothesis. The lower P-value in the one-tailed test indicates stronger evidence. However, both cases are and considered as having very strong evidence (0.001 < P ≤ 0.01).

Examples of One-Mean Confidence Interval:

a. Calculate a 95% confidence interval (using the same sample size n = 18) for the mean height of colonies of Acropora formosa found on the reef crest of Mbudya Island.

df = n - 1 = 17
For 95% CI, $\alpha = 1 - 0.95 = 0.05$
$t_{\alpha/2} = t_{0.05/2} = 2.110$
SE of the estimate: same as above = 2.4921
$\bar{y} \pm t_{\alpha/2} \times SE(\bar{y}) = 67.4444 \pm 2.110 \times 2.4921 = (62.19, 72.70)$
Conclusion: We can be 95% confident that the mean height of colonies is between 62.19 cm and 72.70cm

b. Calculate a 99% confidence interval (using the same sample size n = 18)

For 99% CI, $\alpha = 1 - 0.99 = 0.01$
$$t_{\alpha/2} = t_{0.01/2} = t_{0.005} = 2.898$$
$$\bar{y} \pm t_{\alpha/2} \times SE(\bar{y}) = 67.4444 \pm 2.898 \times 2.4921 = (60.22, 74.67)$$

c. Suppose you increase the same size to n = 50 and you still get the same sample mean and sample standard deviation, calculate a 95% confidence interval.

df = 50 - 1 = 49
For 95% CI, $\alpha = 1 - 0.95 = 0.05$
$$t_{\alpha/2} = t_{0.05/2} = t_{0.025} = 2.021$$
$$SE(\bar{y}) = \frac{S}{\sqrt{n}} = \frac{10.5731}{\sqrt{50}} = 1.4953$$
$$\bar{y} \pm t_{\alpha/2} \times SE(\bar{y}) = 67.4444 \pm 2.021 \times 1.4953 = (64.42, 70.47)$$