4.4. Testing the Significance of the Model using the Regression ANOVA Test

> At the 5% significance level, test for a relationship between Effort Index and performance in Chemistry. [In other words, test whether the slope of the regression line is significant.] The regression line has been shown to be linear. Assume that all other assumptions are met.

$H_0 : \beta_1 = 0$ (There is no relationship between Effort index and Chemistry marks)
$H_a : \beta_1 \neq 0$ (There is a relationship)

$SS_{Total} = S_{yy} = 1470$
$SS_{REGR} = \frac{(S_{xy}^2)}{S_{xx}} = \frac{140.5^2}{14.8333} = 1330.8064$
$SS_{Err} = SS_{Total} - SS_{REGR} = 1470 - 1330.8064 = 139.1936$

$$F = \frac{SS_{REGR}/1}{SS_{Err}/(n-2)} = \frac{MS_{REGR}}{MS_{Err}} = \frac{1330.8064/1}{139.1936/(6-2)} = \frac{1330.8064}{34.7984} = 38.242$$

$df = (1, n-2) = (1, 4)$
P-value: $0.001 < P < 0.005$, this is very strong evidence against H_0.
$P < 0.05$, therefore reject H_0.

Conclusion: At the 5% significance level, the data provide sufficient evidence to conclude that there is a relationship between performance in Chemistry and Effort index.

4.5. Inferences for the Slope of the Population Regression Line (Regression t-test)

Testing the Significance of the Slope using the Regression t-test

> At the 5% significance level, test for a relationship between Effort Index and performance in Chemistry. [In other words, test whether the slope of the regression line is significant.] The regression line has been shown to be linear. Assume that all other assumptions are met.

Step 1: Regression t-test is selected because the purpose is to test if the slope is significantly different from 0.

Step 2: H_0: $\beta_1 = 0$ (There is no relationship between performance in Chemistry and Effort index.)
H_a: $\beta_1 \neq 0$ (There is a relationship between performance in Chemistry and Effort index.)

Step 3: Compute the three sums of squares

$$SS_{TOTAL} = S_{yy} = \sum (y_i - \bar{y})^2 = 1,470$$

$$SS_{REGR} = \frac{(S_{xy})^2}{S_{xx}} = \frac{(140.5)^2}{14.8333} = 1330.8064$$

$$SS_{ERROR} = SS_{TOTAL} - SS_{REGR} = 1470 - 1330.8064 = 139.1936$$

Standard error of the model: $\hat{\sigma} = \sqrt{\frac{SS_{Err}}{n-2}} = \sqrt{\frac{139.1936}{6-2}} = 5.8990$

Standard error of the slope: $SE(\hat{\beta}_1) = \frac{\hat{\sigma}}{\sqrt{S_{xx}}} = \frac{5.8990}{\sqrt{14.8333}} = 1.5316$

$t = \frac{\hat{\beta}_1}{\hat{\sigma}/\sqrt{S_{xx}}} = \frac{\hat{\beta}_1}{SE(\hat{\beta}_1)} = \frac{9.4719}{1.5316} = 6.184$

$df = n - 2 = 6 - 2 = 4$
P-value: $(0.001 < P < 0.0025) \times 2 \Rightarrow (0.002 < P < 0.005)$, this is very strong evidence against H_0.
$P < \alpha$ (0.05), therefore reject H_0.

At the 5% significance level, the data provide sufficient evidence to conclude that there is a relationship between performance in Chemistry and Effort index.

One-tailed test: Since it would be safe to predict that Effort index could increase performance, we could do a right-tailed test

$H_0 : \beta_1 = 0$
$H_a : \beta_1 \neq 0$

Again, t = 6.184.
P-value is: 0.001 < P < 0.0025. Exact P-value (from computer output): $P = 0.003475/2 = 0.0017375$. Computer output is always two-tailed.

At the 5% significance level, there is sufficient evidence that there is a positive relationship between performance and Effort index.