

Multiple regression equation:

5. Multiple Regression

Multiple Regression : $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \epsilon$

Independent Variable

Example: Annual sales depend upon 2 or more factors
Profit, Cost & Total sales are independent variables, dependent variable is Annual sales.

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \epsilon$

β_0 = Intercept value of dependent variable
i.e. y intercept value of y when all other parameters are set to zero.

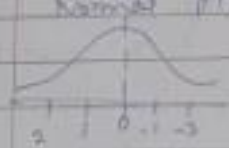
$\beta_1, \beta_2, \dots, \beta_k$ = Slope coefficients

β_1 = Regression Coefficient β_1 of the independent variable x_1

ϵ = Random error (has much variation there in our estimate of y)

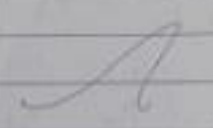
Ques Difference b/w Simple Linear & Multiple Linear Regression?

1. Normal Distribution: Mean = Mode = Median.
bell shape curve. (Symmetrical graph)



1. symmetric 2. peaked

3. Unsymmetrical graph
skewed graph



↑ positive skew
↓ negative skew

Formula

$$a = \bar{Y} - b_1 \bar{X}_1 - b_2 \bar{X}_2$$

$$b_1 = \frac{(\sum x_2^2)(\sum x_1 y) - (\sum x_1 x_2)(\sum x_2 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$

$$b_2 = \frac{(\sum x_1^2)(\sum x_2 y) - (\sum x_1 x_2)(\sum x_1 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$

$$\sum x_1^2 = \sum X_1 X_1 - \frac{(\sum X_1)(\sum X_1)}{N}$$

$$\sum x_2^2 = \sum X_2 X_2 - \frac{(\sum X_2)(\sum X_2)}{N}$$

$$\sum x_1 y = \sum X_1 Y - \frac{(\sum X_1)(\sum Y)}{N}$$

$$\sum x_2 y = \sum X_2 Y - \frac{(\sum X_2)(\sum Y)}{N}$$

$$\sum x_1 x_2 = \sum X_1 X_2 - \frac{(\sum X_1)(\sum X_2)}{N}$$

$$\sum X_1 = 20$$

$$\sum X_2 = 24$$

$$\sum X_1 X_1 = 90$$

$$\sum X_2 X_2 = 148$$

$$\sum X_1 Y = 95.8$$

$$\sum X_2 Y = 45.6$$

$$X_1 X_2 = 99$$

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#	Y	X ₁	X ₂	X ₁ X ₁	X ₂ X ₂	X ₁ X ₂	X ₁ Y	X ₂ Y
	-3.7	3	8	9	64	24	-11.1	-29.6
	3.5	4	5	16	25	20	14	17.5
	2.5	5	7	25	49	35	12.5	17.5
	11.5	6	3	36	9	18	69	69 34.5
	5.7	2	1	4	1	2	11.4	5.7
X	\bar{Y}	\bar{X}_1	\bar{X}_2					
	19.5	20	24	90	148	99	95.8	45.6

$$Y = b_0 + b_1 X_1 + b_2 X_2$$

$$b_0 = \bar{Y} - b_1 \bar{X}_1 - b_2 \bar{X}_2$$

$$b_1 = \frac{(\sum X_2^2)(\sum X_1 Y) - (\sum X_1 X_2)(\sum X_2 Y)}{(\sum X_1^2)(\sum X_2^2) - (\sum X_1 X_2)^2}$$

$$b_2 = \frac{(\sum X_1^2)(\sum X_2 Y) - (\sum X_1 X_2)(\sum X_1 Y)}{(\sum X_1^2)(\sum X_2^2) - (\sum X_1 X_2)^2}$$

$$\sum x_1^2 = \sum x_1 x_1 - \frac{(\sum x_1)(\sum x_1)}{N}$$

$$\sum x_2^2 = \sum x_2 x_2 - \frac{(\sum x_2)(\sum x_2)}{N}$$

$$\sum x_1 y = \sum x_1 y - \frac{(\sum x_1)(\sum y)}{N}$$

$$\sum x_2 y = \sum x_2 y - \frac{(\sum x_2)(\sum y)}{N}$$

$$\sum x_1 x_2 = \sum x_1 x_2 - \frac{(\sum x_1)(\sum x_2)}{N}$$

$$\bar{x}_1 = 5$$

$$\bar{x}_2 = 4.8$$

$$\bar{y} = 19.5$$

$$\sum x_1^2 = 90 - \frac{(20)(20)}{5}$$

$$= 90 - \frac{400}{5}$$

$$= 90 - 80 = 10$$

$$\sum x_2^2 = 148 - \frac{(24)(24)}{5}$$

$$= 148 - \frac{504}{5}$$

$$= 148 - 100.8$$

$$= 47.2$$

$$\sum x_1 y = 95.8 - \frac{(20)(19.5)}{5}$$

$$= 95.8 - \frac{390}{5}$$

$$= 95.8 - 78$$

$$= 17.8$$

$$\sum x_2 y = 45.6 - \frac{(24)(19.5)}{5}$$

$$= 45.6 - \frac{468}{5} = 45.6 - 93.6$$

$$= -48$$

$$\begin{aligned} \sum x_1 x_2 &= 99 - \frac{(20)(24)}{5} \\ &= 99 - 96 \\ &= 3 \end{aligned}$$

$$\begin{aligned} b_1 &= \frac{(47.2)(17.8) - (3)(-48)}{(10)(47.2) - (3)^2} \\ &= \frac{840.16 + 144}{472 - 9} = 2.125 \end{aligned}$$

$$\begin{aligned}
 b_2 &= \frac{(10)(-48) - 3(17.8)}{(10)(41.2) - (3)^2} \\
 &= \frac{-480 - 53.4}{412 - 9} = \frac{-533.4}{403} \\
 &= -1.323
 \end{aligned}$$

$$\begin{aligned}
 b_0 &= (3.9) - (2.125)(4) - (-1.323)(4.8) \\
 &= 3.9 - 8.5 + 6.3504 \\
 &= 1.7504
 \end{aligned}$$

$$\begin{aligned}
 y &= b_0 + b_1x_1 + b_2x_2 \\
 &= (1.7504) + (2.125)(3) + (-1.323)(2) \\
 &= 1.7504 + 6.375 - 2.646 \\
 &= 5.4794
 \end{aligned}$$