

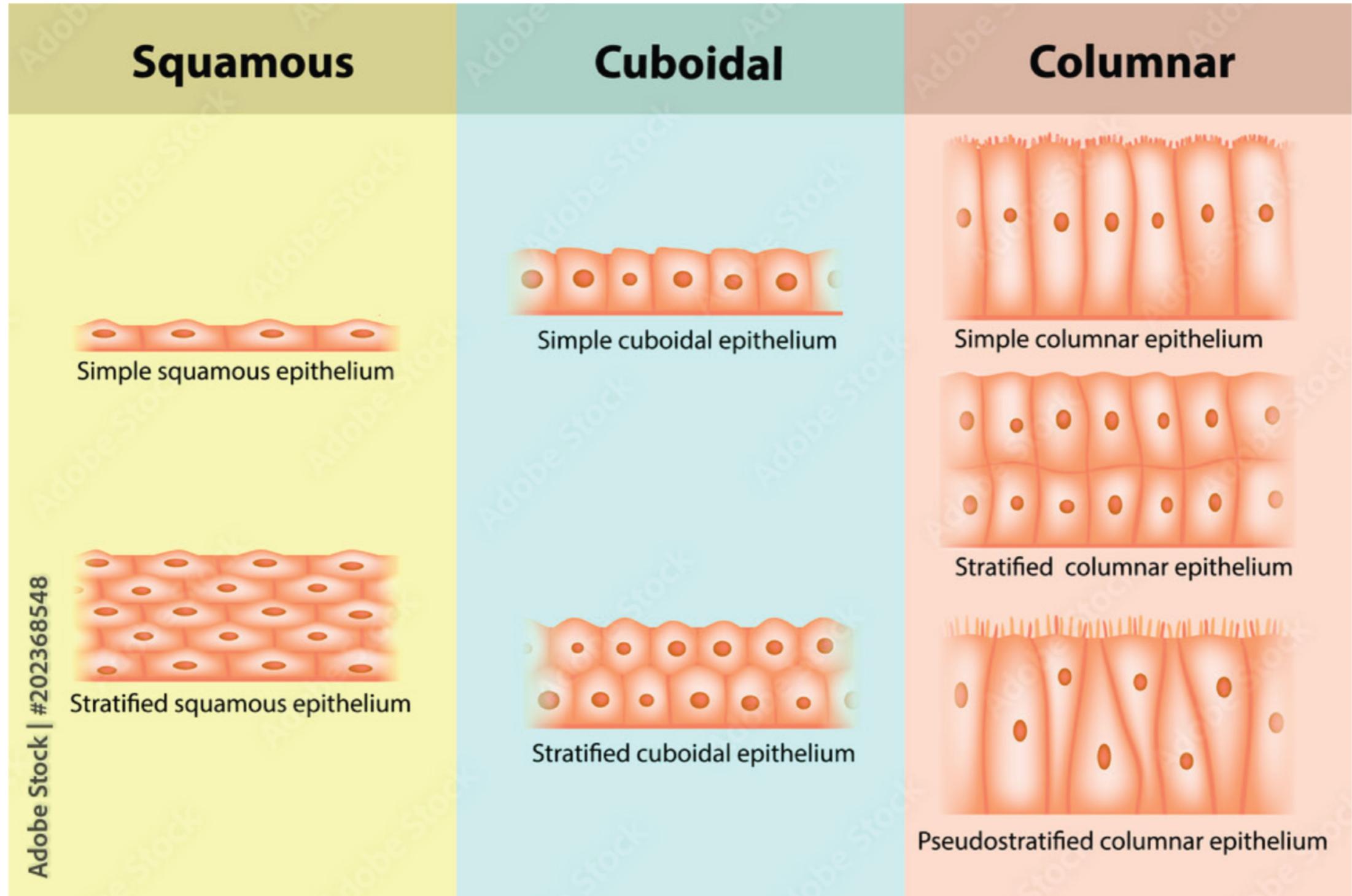
Master 2 ICFP - Physics of Multicellular Systems

Lesson 6

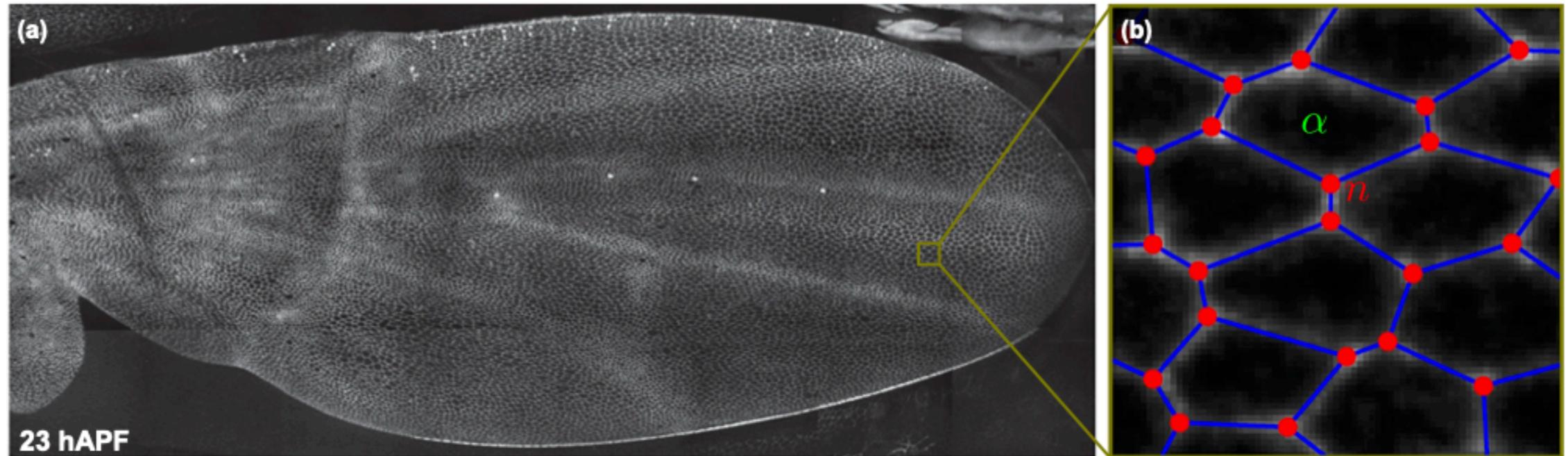
Vertex models of tissues

Hervé Turlier - March 6th 2022

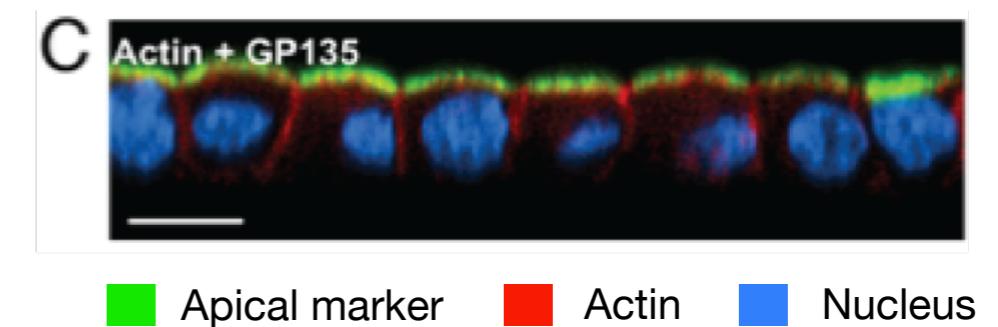
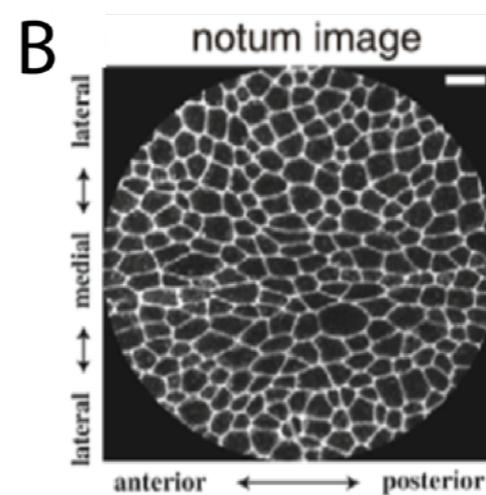
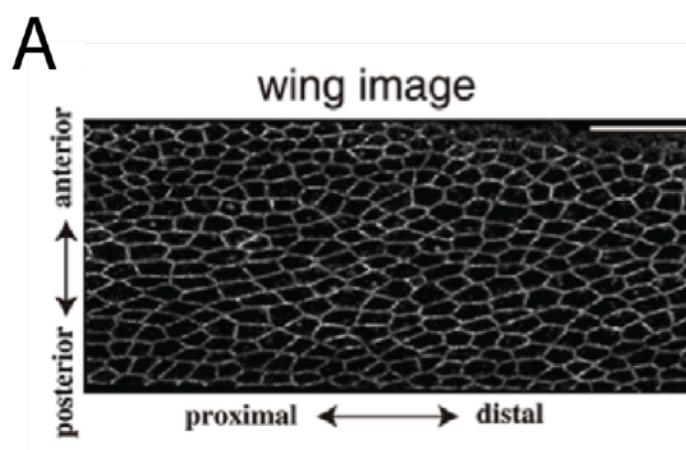
Epithelial tissues



Epithelial monolayers



Merkel et al. *PRE* 2017

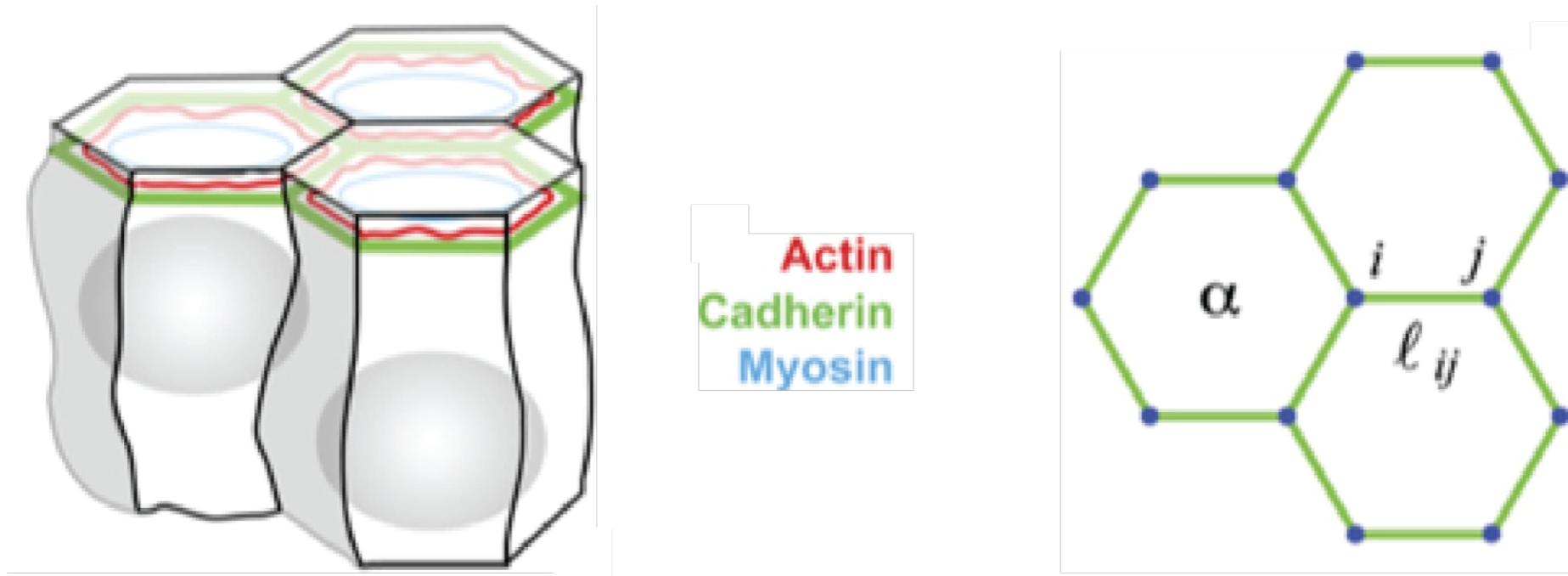


Ishihara et al. *EPJE* 2013

Harris et al. *PNAS* 2012

2D vertex model

Farhadifar et al. *Curr Biol* 2007



$$\mathcal{E} = \sum_{\alpha} \frac{\kappa_{\alpha}}{2} (A_{\alpha} - A_{\alpha}^0)^2 + \sum_{\langle i,j \rangle} \Lambda_{ij} \ell_{ij} + \frac{1}{2} \sum_{\alpha} \Gamma_{\alpha} P_{\alpha}^2$$

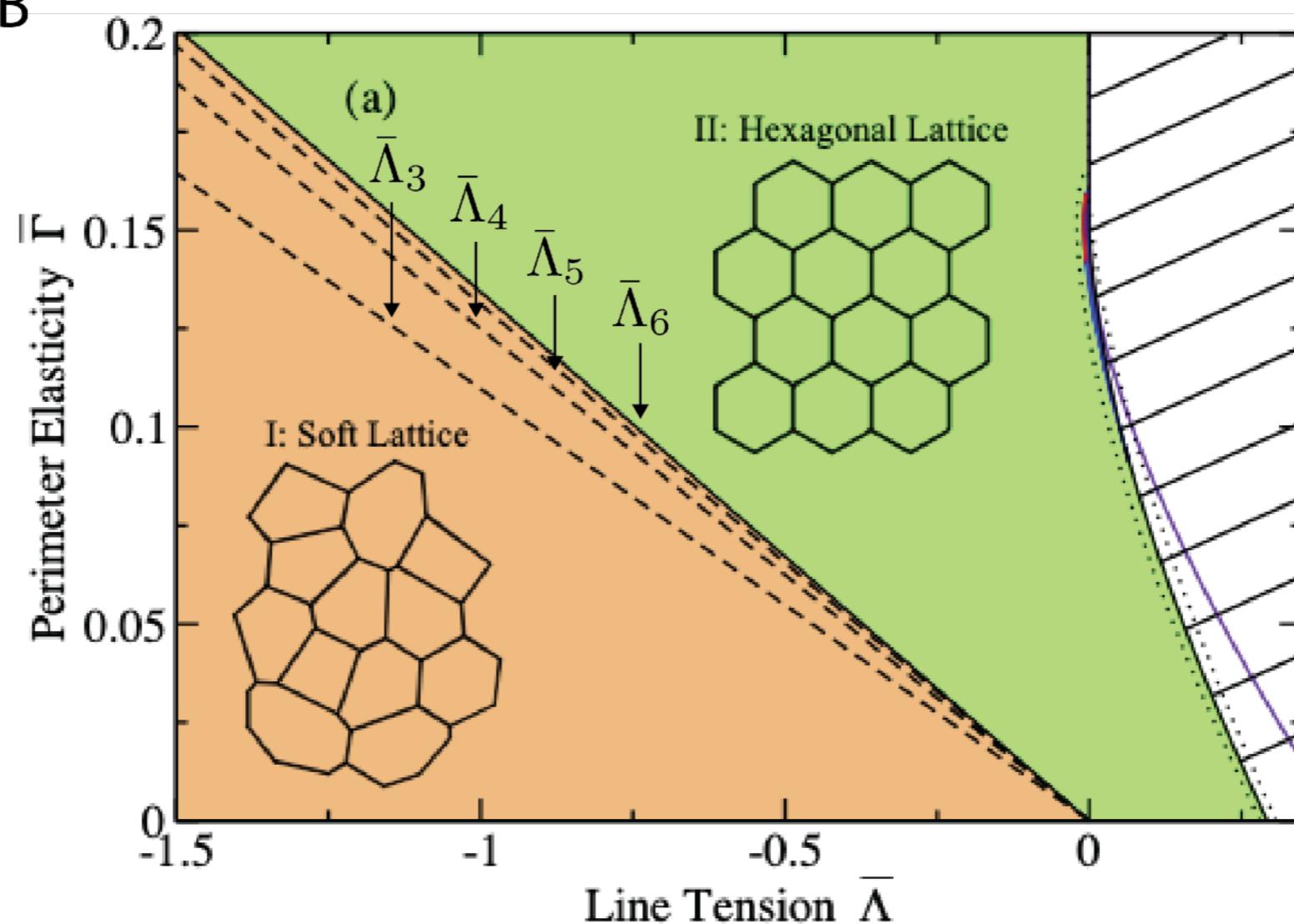
2D vertex model

Fundamental energy states - Staple et al. *EPJE* 2010

A

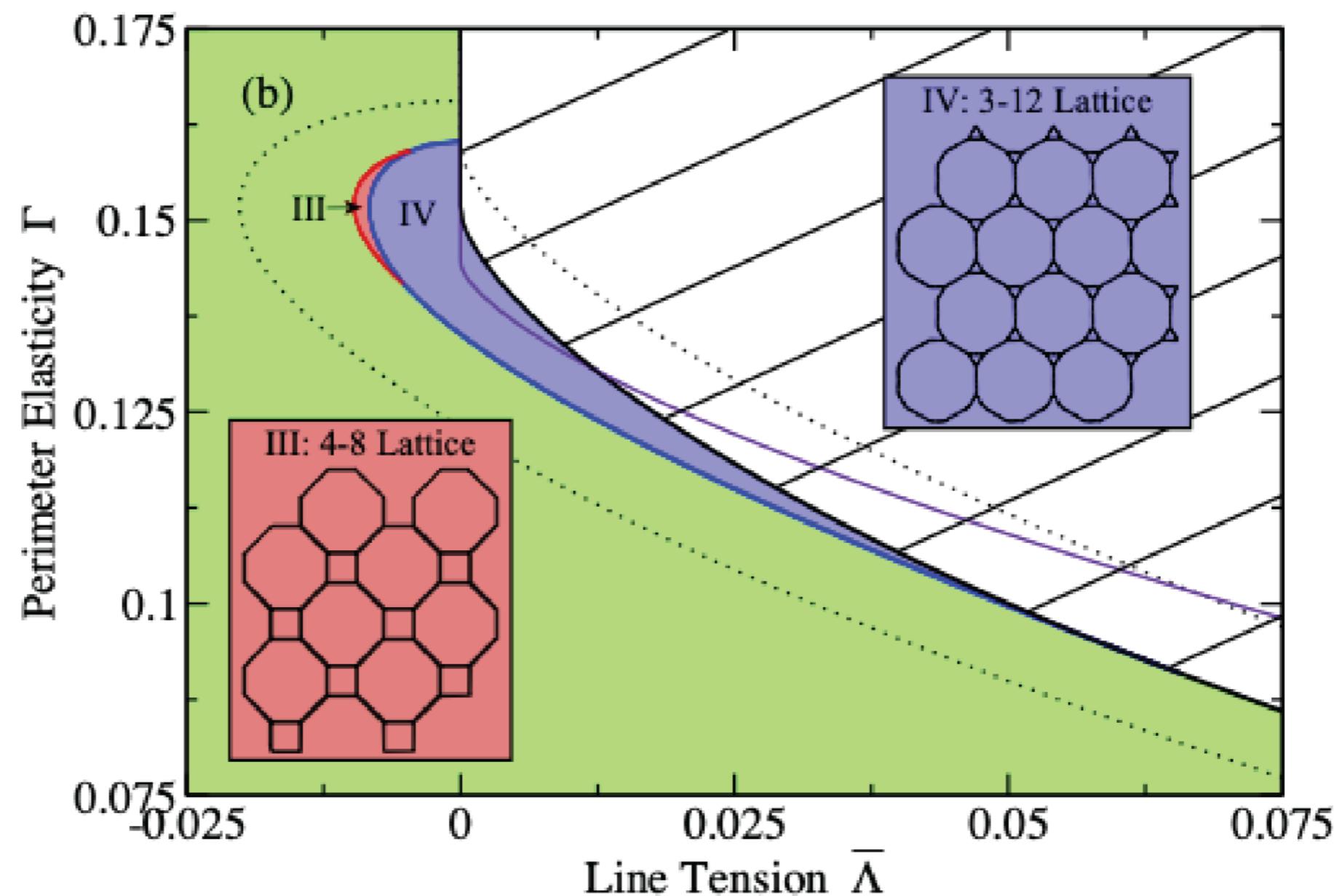
Parameter values		Ground state (identical cells)
$\bar{\Lambda} < -2^{5/2} 3^{1/4} \bar{\Gamma}$	-	irregular polygons ($a = 1, p = p_0$)
$-2^{5/2} 3^{1/4} \bar{\Gamma} \leq \bar{\Lambda} < 2 \cdot 3^{-5/2} (\sqrt{3} - 12\bar{\Gamma})^{3/2}$	$\bar{\Gamma} < \sqrt{3}/12$	$\left. \begin{array}{l} \\ \end{array} \right\} \text{hexagonal lattice}$
$-2^{5/2} 3^{1/4} \bar{\Gamma} \leq \bar{\Lambda} < 0$	$\bar{\Gamma} \geq \sqrt{3}/12$	
$\bar{\Lambda} \geq 2 \cdot 3^{-5/2} (\sqrt{3} - 12\bar{\Gamma})^{3/2}$	$\bar{\Gamma} < \sqrt{3}/12$	$\left. \begin{array}{l} \\ \end{array} \right\} \text{collapsed lattice } (a = 0, p = 0)$
$\bar{\Lambda} \geq 0$	$\bar{\Gamma} \geq \sqrt{3}/12$	

B



2D vertex model

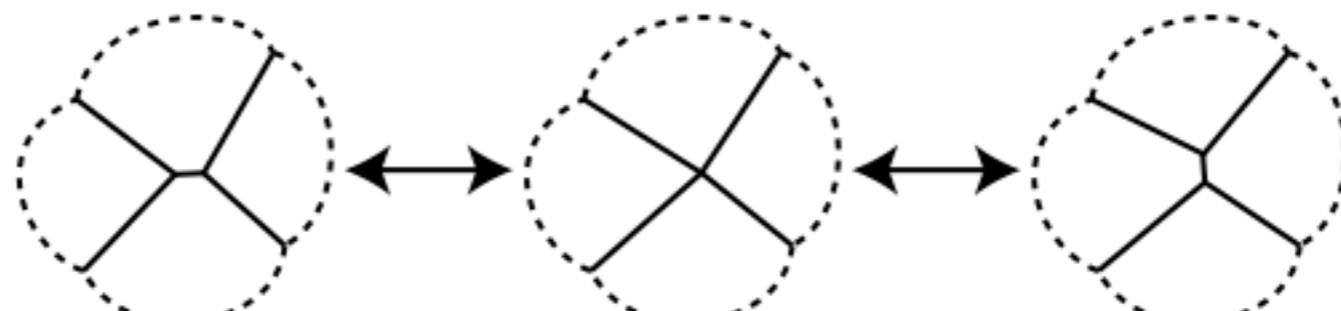
Fundamental energy states - Staple et al. *EPJE* 2010



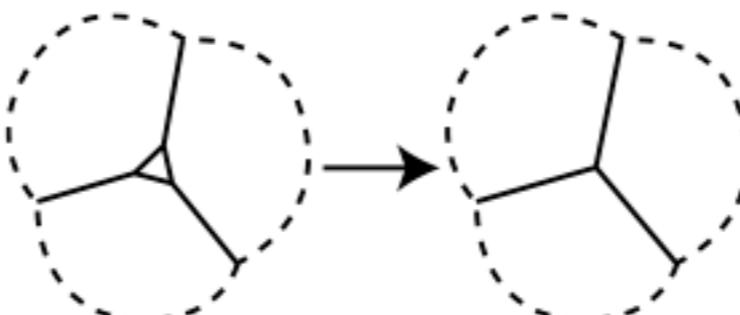
2D vertex model

Farhadifar et al. *Curr Biol* 2007

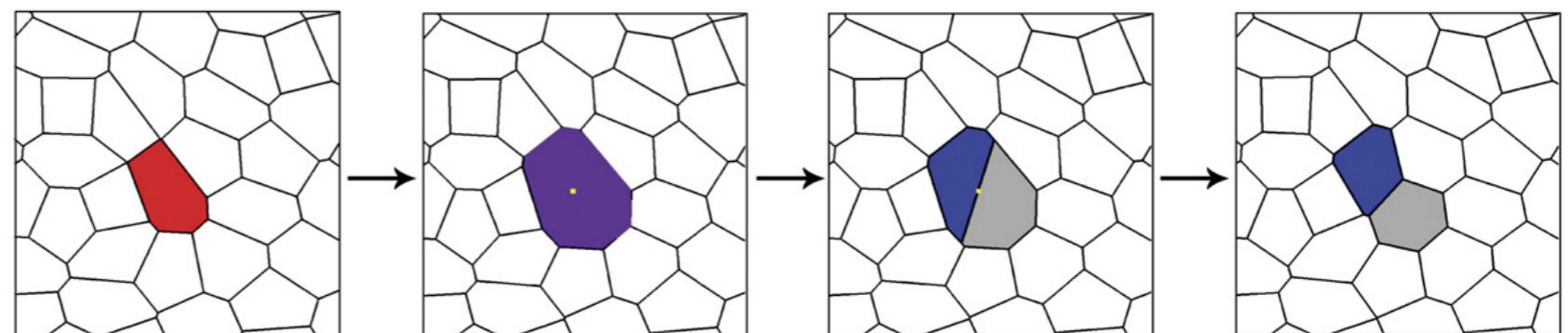
T1 transition



T2 transition

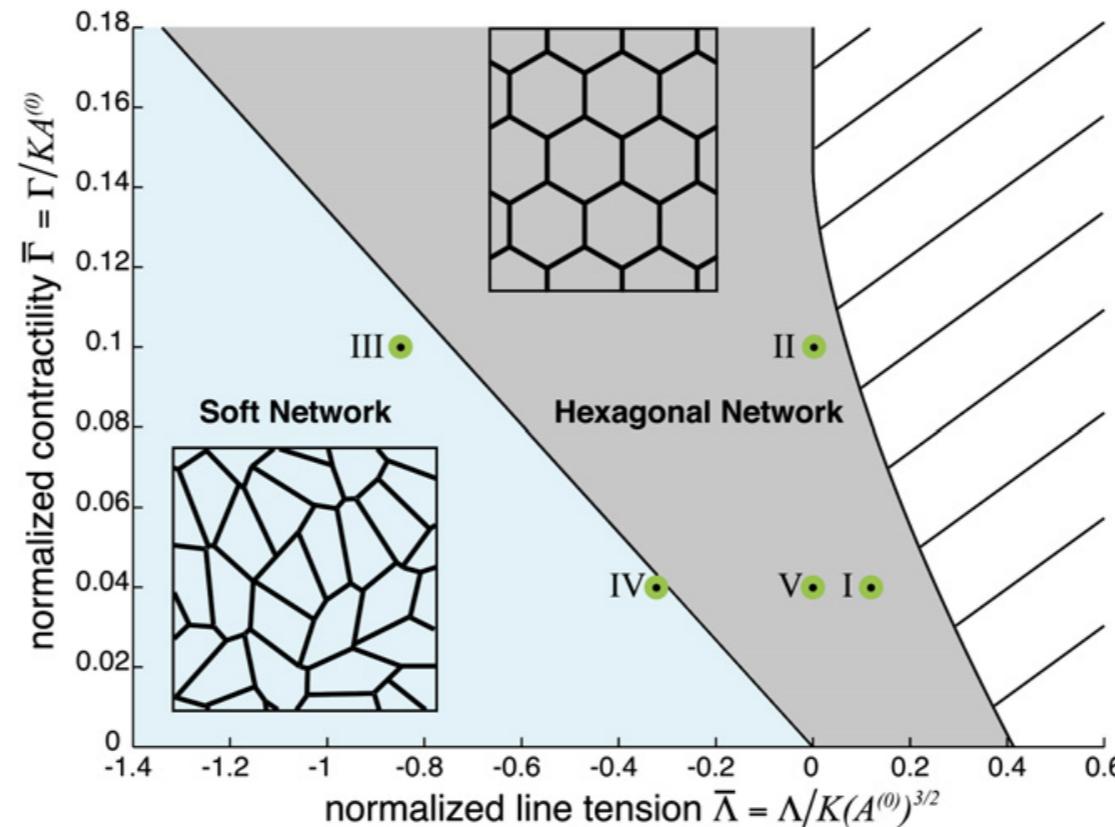


Cell division

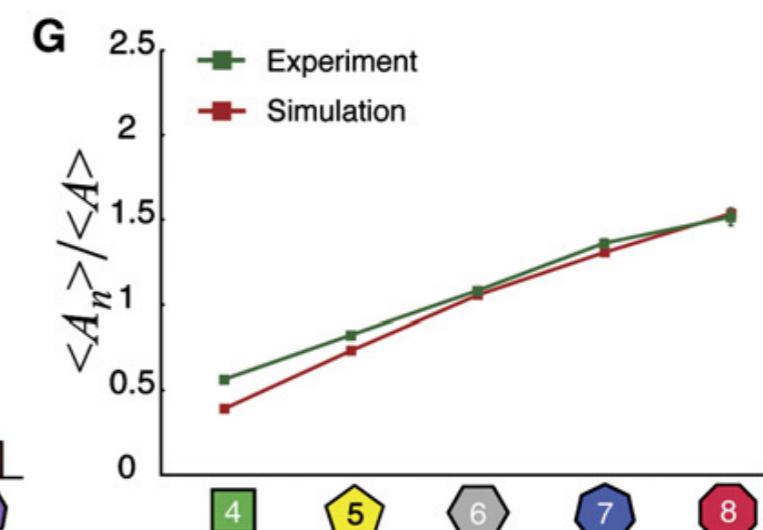
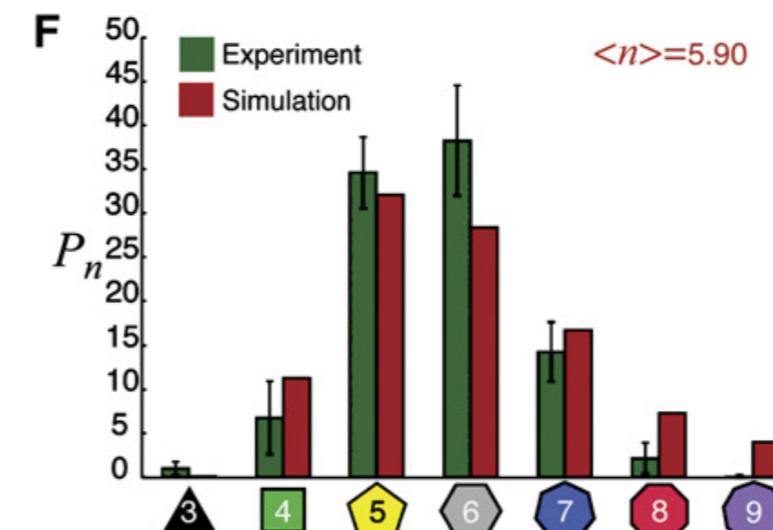
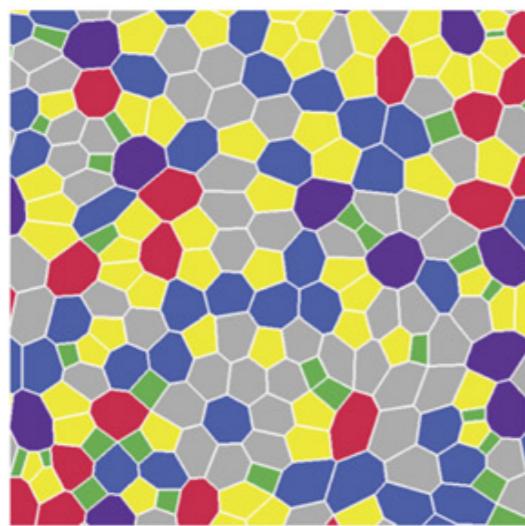


2D vertex model

Farhadifar et al. *Curr Biol* 2007

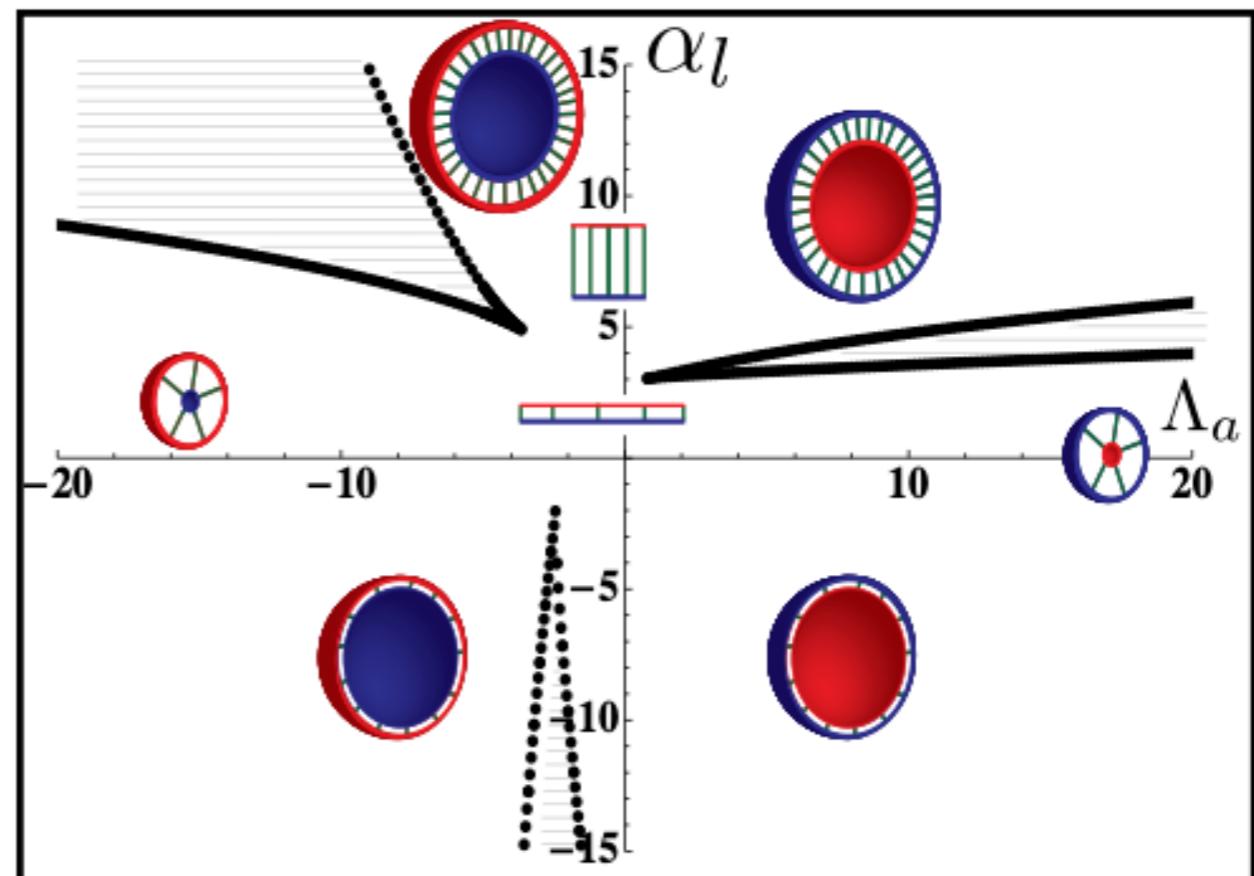
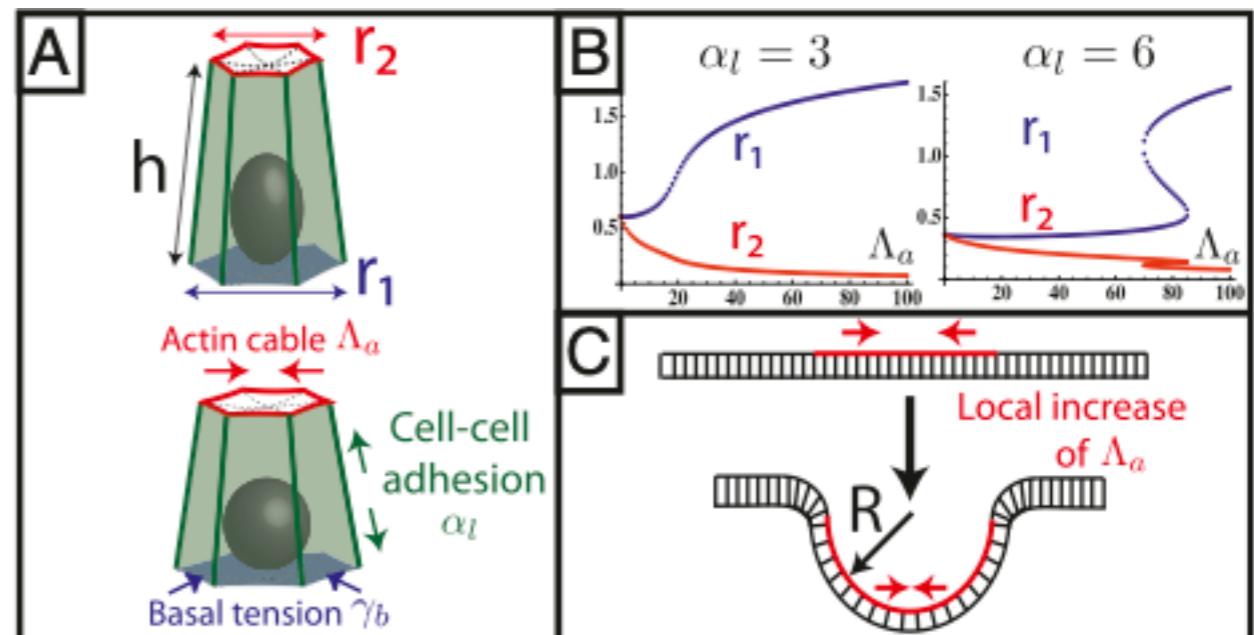
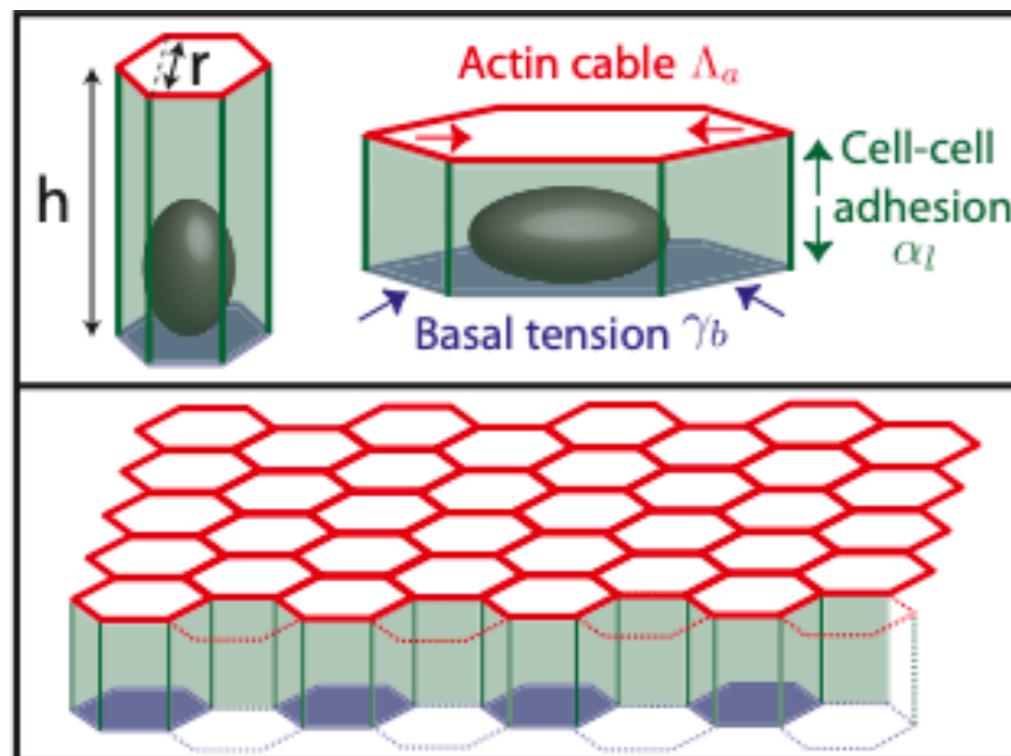


E
case I



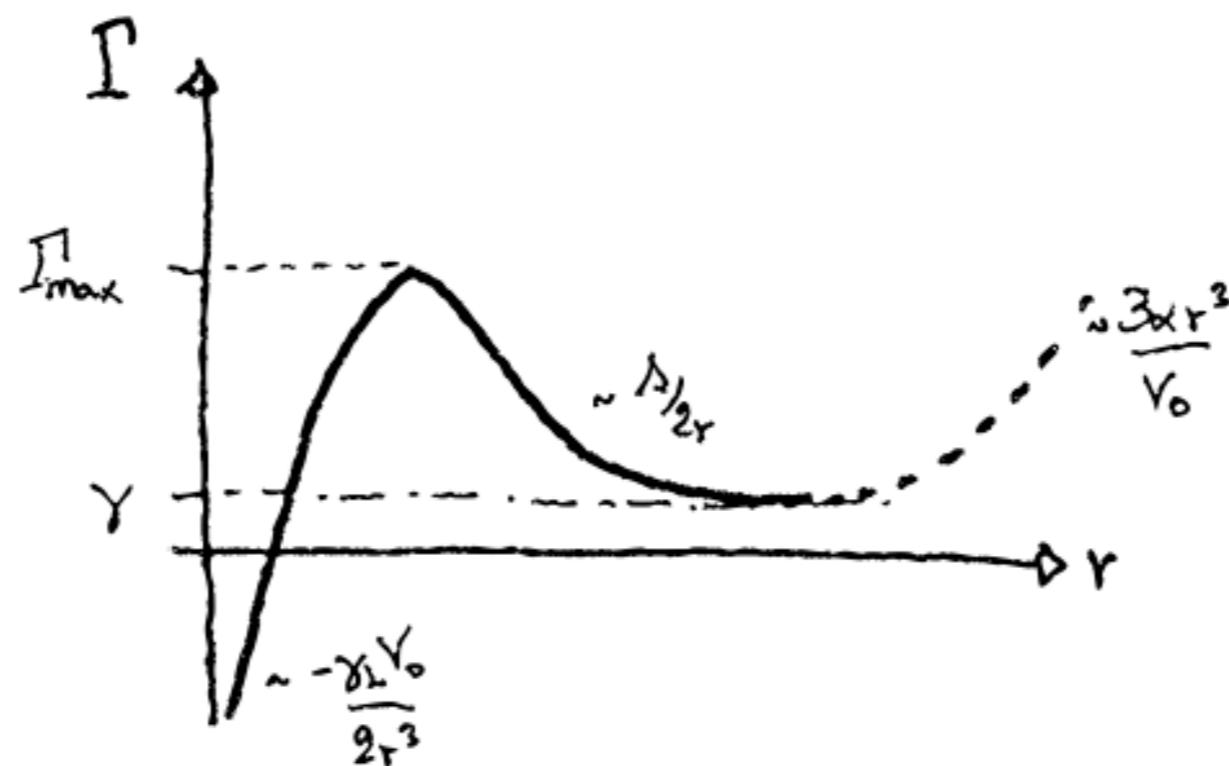
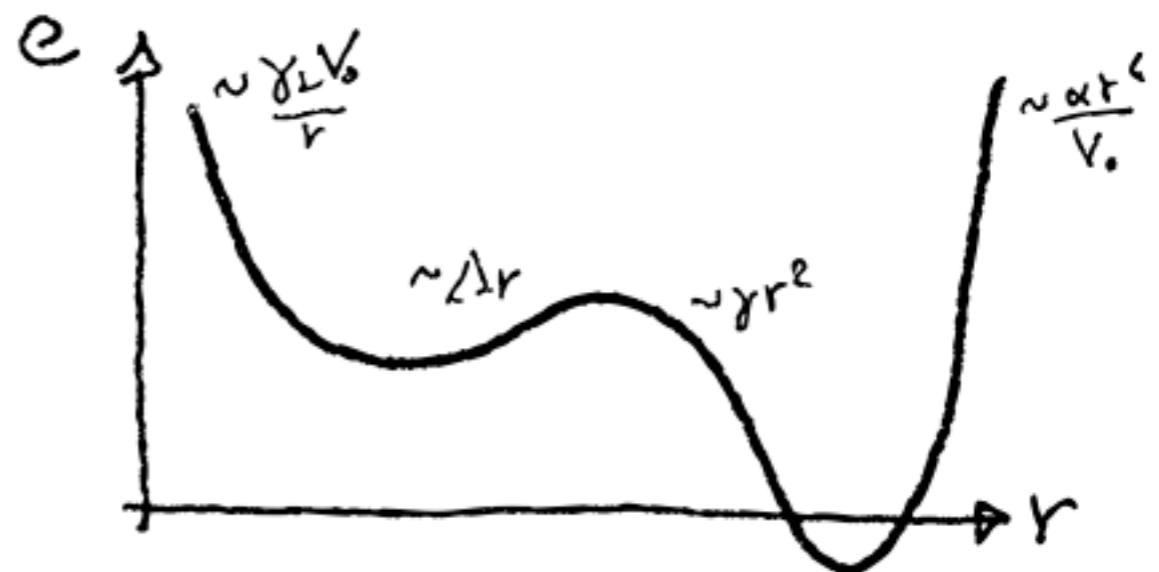
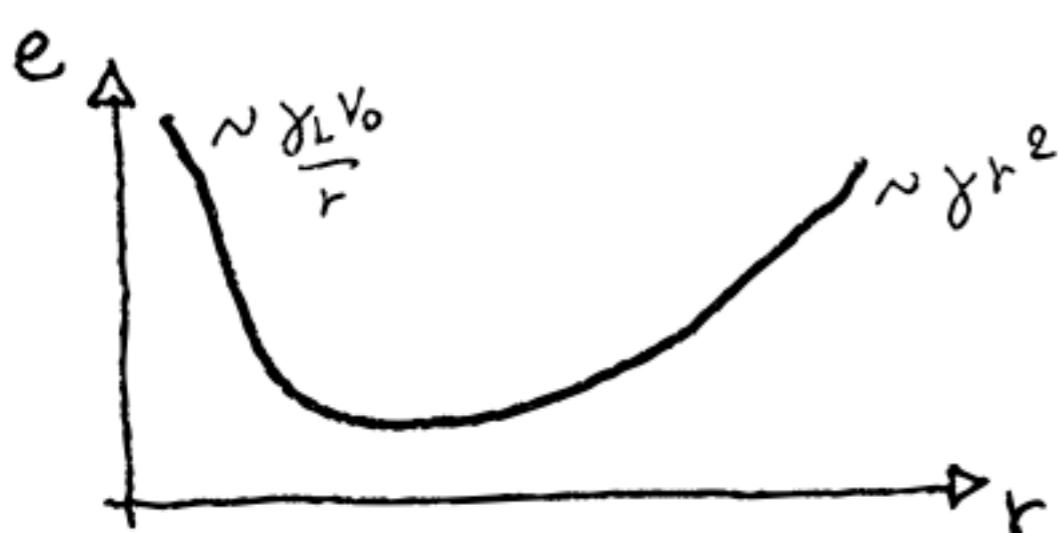
3D vertex models

Analytical model - Hannezo et al. PNAS 2014



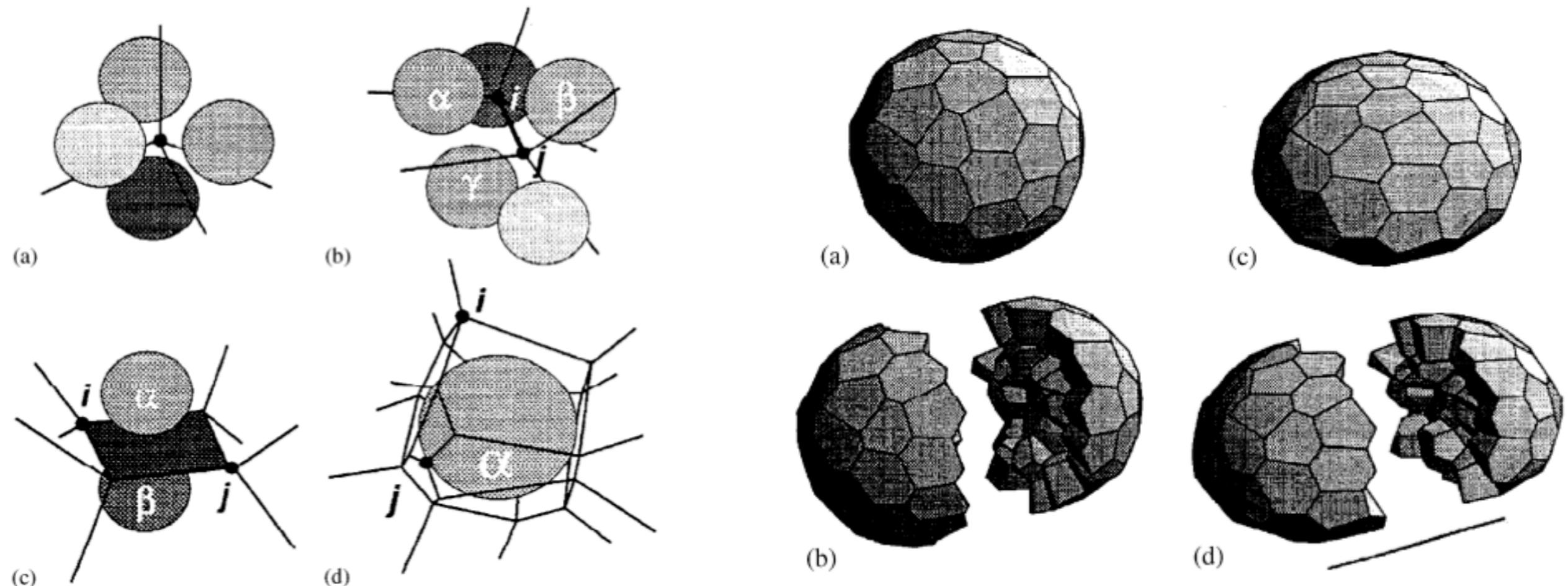
3D vertex models

Analytical model - Hannezo et al. PNAS 2014



3D vertex models

Numerical models - Honda et al. *J Theo Biol* 2004

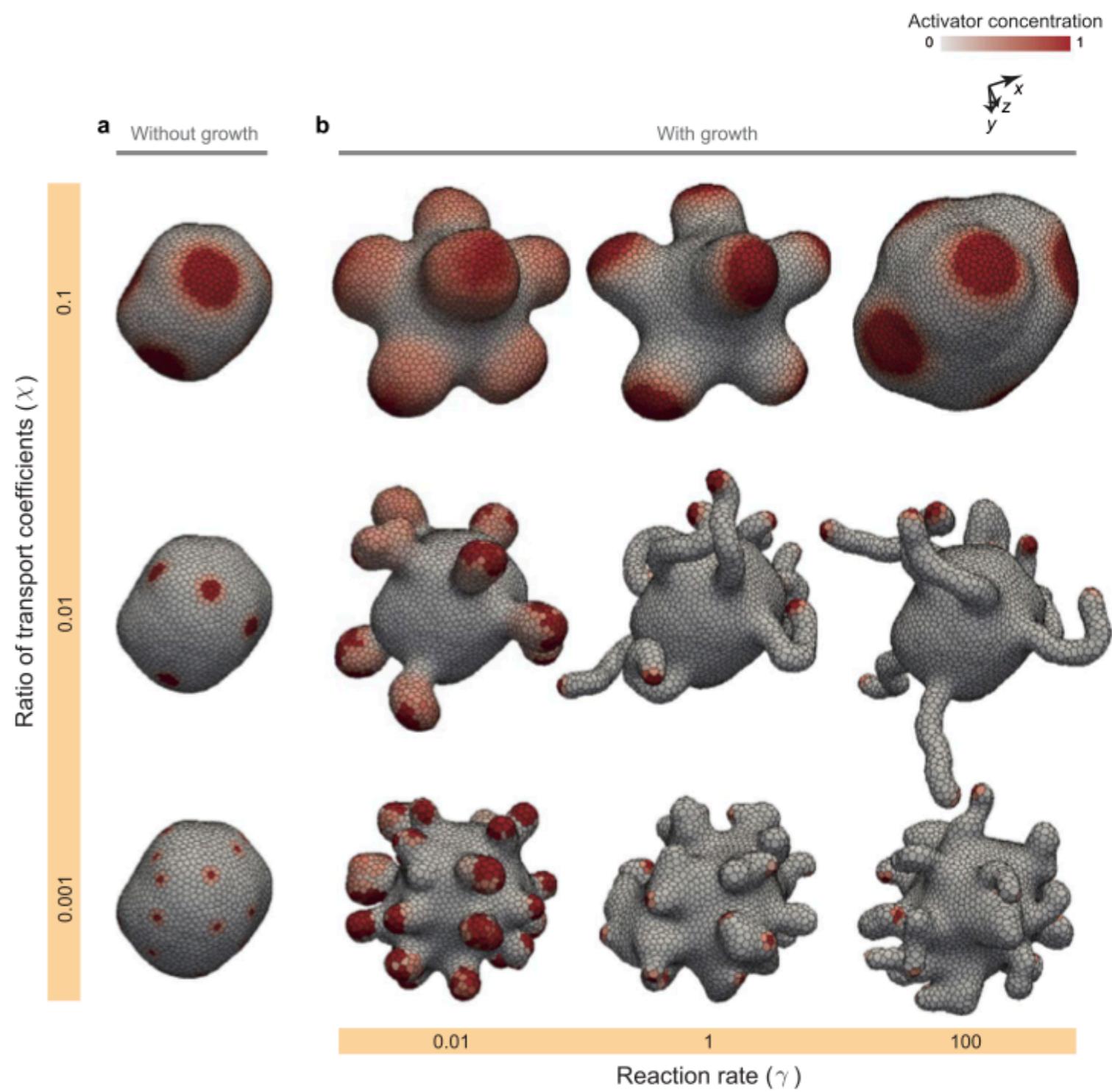
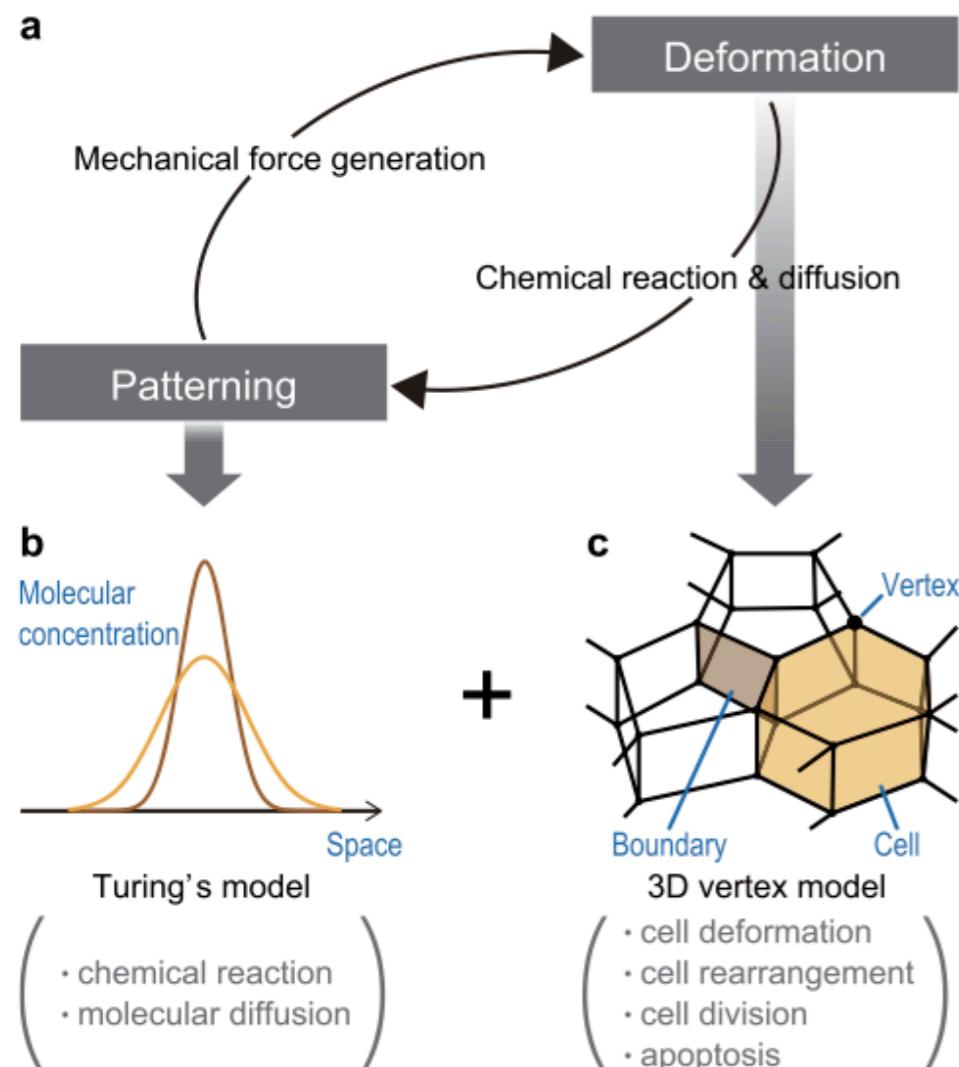


$$\eta \frac{d\mathbf{r}_i}{dt} = -\nabla_i \mathcal{E}$$

$$\mathcal{E} = \sigma \sum_{<\alpha, \beta>} A_{\alpha\beta} + \sigma_0 \sum_{<\alpha, 0>} A_{\alpha 0} + \kappa \sum_{\alpha} (V_{\alpha} - V_0)^2 + \rho \sum_{\alpha} z_{\alpha} V_{\alpha} + w_{\text{floor}} \sum_{\alpha} \frac{1}{1 + e^{az_{\alpha}}}$$

3D vertex models

Numerical models - Okuda et al. *Sci Reports* 2018



3D vertex models

Numerical models - Maître, Turlier et al. *Nature* 2016

