1 Diversity combining

In order to investigate the implementation of combining within the comm toolbox, the first step is to look at the theory. The channel of the transmission can be defined as

$$r_l = \alpha_l \cdot s + n \tag{1.1}$$

where r_l is used for the receive signal of a single aperture l, s for the transmit signal and n for the additive white noise. The complex channel coefficient for channel l is described here as α_l. In a first investigation it shall be assumed that an ideal phase compensation takes place within the coherent transmission system, therefore α_l is considered as a purely real factor. Thus, it is a pure amplitude scaling. As usual with the AWGN channel, we assume a standard normal distribution ($\mu = 0, \sigma_N^2 = 1$) for the noise process n. Knowing that the linear SNR can be defined as the square amplitude by variance with $SNR_{lin} = \frac{A^2}{\sigma_N^2}$ of the noise process, assuming an amplitude of the transmit signal s as 1, the channel coefficient can be defined from this using

$$\alpha_l = \sqrt{SNR_{lin,l}} \tag{1.2}$$

in relation to the given channel SNR. In order to obtain a combined signal, equal-gain combining (EGC) and maximal-ratio combining (MRC) will be investigated as combining methods in the following. These are defined as

$$r_{MRC} = \sum_{l=1}^{L} \alpha_l \cdot r_l \tag{1.3, [1], p.44}$$

$$r_{EGC} = \sum_{l=1}^{L} r_l \tag{1.4, [1], p.45}$$

where r_{MRC} describes the combined signal in the case of MRC combining and r_{EGC} respectively in the case of EGC. L describes the overall amount of diversity channels. As will be clear, the MRC method uses the channel coefficients to weight the individual channels, while EGC uses no weighting (or weighting with 1 per channel). The difficulty here is to predict the expected results of these combinations. While in the case of equal mean SNRs per diversity channel MRC and EGC perform equally well, in the case of different SNRs per diversity channel the resulting SNR of the MRC method outperforms the EGC method. The larger the difference within the channel coefficients of the individual channels, the stronger the difference between MRC and EGC. The theory of the resulting SNRs of the combined signals are given by

$$\gamma_{R,MRC} = \sum_{l=1}^{L} \gamma_k \tag{1.5, [1], p.44}$$

$$\gamma_{R,EGC} = \frac{1}{L} \cdot \left(\sum_{l=1}^{L} |\alpha_l| \right)^2 \quad \text{(1.6, [2], p.165)}$$

where γ_R describes the resulting linear SNR. As can be seen from the formulas, this resulting SNR is only dependent on the SNRs of the individual channels and can therefore be used without dependence on any distributions.

The direct diversity gain of both methods, i.e., the improvement of the combined SNR over the mean SNR of the individual channels, can also be easily determined for MRC by

$$\bar{\gamma}_{R,MRC} = L \quad \text{(1.7, [1], p.44)}$$

where $\bar{\gamma}_R$ denotes the SNR gain. This equotation is also independent of the predominant distribution within the channel coefficients.

For EGC, this is more difficult and needs to be investigated in more depth. To provide a framework for comparison, the theory for the mobile radio channel is used and Rayleigh-distributed amplitude values are assumed for it. In this case, the diversity gain can then be achieved by means of

$$\bar{\gamma}_{R,EGC} = \left[1 + (L-1)\frac{\pi}{4} \right] \quad \text{(1.8, [1], p.45)}$$

where $\pi/4$ appears as a Rayleigh-specific component. Deeper explanations of this fact can be found in [3].

2 Notes

As you can see, the theory of EGC gain is depending on the the given distribution of the channel coefficients, so far the SNR of the single channels. Thats why someone have to discuss how to reproduce this specific distribution in the given context. Assuming that a Rayleigh channel will be characterized by Rayleigh distributed amplitudes, the distribution of the SNR must be adjusted accordingly for tractability. Since the relationship between the amplitudes of the signal and the resulting SNR is quadratic with

$$A = \sqrt{SNR} \quad \text{(1.9)}$$

it can be assumed that the relation between the Rayleigh random variable R and the SNR should also be quadratic. It is desired to determine an average value for amplitudes from a Rayleigh distribution using the given SNR. With

$$E(R) = A = \sqrt{SNR} \;\rightarrow\; E(R^2) = SNR \quad \text{(2.0)}$$

these relation can be shown in a more mathematically way. Using Mathematica, it was determined that the expected value $E()$ of an R^2 type distribution can be described by $E(R^2) = 2\sigma^2$. Thus, to variably set the mean SNR, a Rayleigh distribution with a mode σ of $\sigma = \sqrt{SNR_{lin}/2}$ must be generated as α_l channel coefficients.

3 List of Symbols

l	diversity channel
L	amount of diversity channel
r_l	receive signal of diversity channel l
α_l	complex channel coefficient of diversity channel l
r_{MRC}	signal after MRC combining
r_{EGC}	signal after EGC combining
$\gamma_{R,MRC}$	SNR after MRC combining
$\gamma_{R,EGC}$	SNR after EGC combining
$\bar{\gamma}_{R,MRC}$	SNR gain after MRC combining
$\bar{\gamma}_{R,EGC}$	SNR gain after EGC combining

References

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[2] H. Nuszkowski, *Digitale Signalübertragung im Mobilfunk*. Vogt, 2010.

[3] D. Brennan, "Linear diversity combining techniques," *Proceedings of the IRE*, vol. 47, pp. 1075–1102, jun 1959.