United in Booms, Divided in Busts: Regional House Price Cycles and Monetary Policy

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Abstract

Regions within an economically integrated area, whether a sovereign state or a monetary union, exhibit divergence in house prices during economic busts and convergence during booms. This paper documents two novel facts that jointly explain this phenomenon. First, house price expectations are formed by extrapolating current house prices into the future, a process that intensifies during busts compared to booms. Second, regional heterogeneities in house prices and economic activity arise due to differences in housing supply constraints. We construct a two-region currency union model incorporating a housing sector and extrapolative belief updating regarding house prices. To solve the model we propose a novel solution method. Our model successfully replicates the observed empirical facts. Moreover, we demonstrate that an increased focus on house prices by the monetary authority reduces variations in output and house prices, as well as cross-regional disparities, albeit at the cost of a higher variance of aggregate inflation.

JEL Codes: E31, E32, E52, F45

Keywords: Monetary Policy, Currency Area, Structural Heterogeneity, Subjective House Price Expectations, Housing Booms, Asset Price Learning

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I. Introduction

Most developed countries repeatedly experience episodes of booms and busts in housing prices. Given that housing constitutes the most important, by financial value, asset for the majority of households, the bust periods have often coincided with severe recessions and substantial economic costs. Consequently, house price cycles have become a primary concern not only for central banks but also for policymakers at large. Moreover, house prices do not exhibit uniform trends across different regions within an economically integrated area, be it a sovereign state or a currency union. Notably, during bust periods, there is a pronounced divergence in house prices across regions, with some regions experiencing a moderate bust and others suffering a more severe downturn. This phenomenon raises several important and unresolved questions: What underlying factors contribute to this pattern? What are the appropriate policy responses under these circumstances?

Our study contributes to the existing literature by addressing the aforementioned questions. We start by documenting two novel facts. First, households form their house price expectations by extrapolating observed house price growth into the future. During bust periods, we observe stronger updating of expectations based on realized house prices, making house prices themselves more volatile. Second, we find house prices and economic activity to be more responsive to a common exogenous monetary policy shock in regions where housing supply is more constrained. Combining both facts we find that cross-regional house price variation is increasing in busts. The underlying intuition is that in response to a common shock house prices are more responsive in more supply constrained regions. The intensified expectation updating during busts amplifies the differential response in house prices across regions, leading to increased regional disparities. In light of these findings, we examine the implications of a central bank that targets house prices in addition to inflation. Our analysis reveals that a stronger focus on house prices results in a reduction of aggregate volatility in output and house prices, as well as a decrease in cross-regional variation in inflation, output, and house prices. However, this comes at the cost of higher aggregate volatility in inflation.

We begin by empirically documenting our two facts. For the first fact, we focus on the US at a federal level. As a starting point we document, that house prices are much more responsive to a monetary policy shock in times of a house price bust. We then move on to house price expectations and demonstrate that house price expectations violate the rational expectations hypothesis. Our analysis reveals that subjective beliefs Bayesian updating models, as proposed by Adam et al. (2017) and Adam et al. (2022), can approximate the formation of house price beliefs. Furthermore, we present evidence suggesting that households tend to update their expectations more strongly during bust periods. This implies a heightened responsiveness of house prices during busts, consistent with our initial finding. To our knowledge, we are the first to document this phenomenon. Turning to the second fact, we establish that house prices and economic activity exhibit greater responsiveness in regions with more supply constraints within an economically integrated area. This observation is valid for both the United States and the Euro Area. While a similar observation regarding house prices has been made by Aastveit and Anundsen (2022), our findings extend beyond house prices to demonstrate that this pattern also holds for economic activity more broadly. Additionally, we highlight that this pattern is observable within the Euro Area as well. It is crucial to note that the positive correlation between regional house prices and economic activity can be attributed to extrapolative updating of house prices. An increase in house prices in supply constrained regions leads to higher expected house price growth, which subsequently results in increased housing investment and economic activity. Finally, we investigate the interaction between these two empirical facts. Cross-regional heterogeneity in supply constraints results in differential house price responses. Due to asymmetric boom/bust updating behavior, we observe an amplification of differential house price responses during busts, while these responses are muted during booms. Our data corroborate this, showing that cross-regional standard deviations are larger during busts compared to booms. This finding is consistent across both the United States and the Euro Area.

To provide economic intuition for our empirical findings and to study policy counterfactuals, we construct a two-region New Keynesian model. The model builds on Benigno (2004) and incorporates two regions under a single monetary policy authority. We extend the model by including imperfect cross-regional bond markets, a housing sector, and subjective beliefs about house prices, in the spirit of Adam et al. (2022). To solve the model to first-order, we introduce a novel solution method to solve general equilibrium models with extrapolative asset price beliefs. Our solution method has the advantage of confining subjective beliefs to asset prices, while the rational expectations hypothesises holds for all other variables. Additionally, the first-order approximation renders our model compatible with standard solution methods used in the DSGE literature, such as native Dynare routines for solving and estimating models, making it easy to handle and scale, and amenable to Ramsey-type analyses of optimal policy.[1] Our model quantitatively matches the peak response of a monetary policy shock observed in the data. This is true for both boom and bust episodes and unconditionally. An equivalent rational expectations model fails to match these quantitative responses, yielding peak responses significantly lower than those observed in the data. Furthermore, our model qualitatively matches the cross-regional heterogeneities observed empirically. Regions with more constrained housing supply tend to experience larger responses in house

[1]Our solution method contrasts with that introduced by Winkler (2020), which, although based on first-order approximation, does not confine subjective beliefs to specified asset prices and results in belief spill-overs to all other prices. Conversely, Adam et al. (2022) solve a similar model using non-linear solution techniques; however, their approach does not explicitly characterize the entire sequence of variables households need to form subjective expectations about.

prices and economic activity following a monetary policy shock. Finally, the model generates larger cross-regional house price variations during busts compared to booms, thus aligning with all the empirical facts documented above. Importantly, the equivalent rational expectations model fails to match any of these facts. Finally, we examine the implications of assigning a larger weight to house prices in the Taylor rule on the volatility and cross-regional variation of inflation, output, and house prices. Our findings indicate that an increased emphasis on house prices within the Taylor rule results in greater inflation variability but reduced variability in both output and house prices. Additionally, we observe a decline in cross-regional variation across all variables. We conclude that targeting house prices within the Taylor rule effectively reduces aggregate volatility in output and house prices, as well as cross-regional variation, albeit at the cost of an increased aggregate inflation volatility.

Related literature. We connect to the literature that investigates asset pricing with capital gains extrapolation. Notably, Adam et al. (2017) and Winkler (2020) examine asset price learning in the context of stock markets, whereas Adam et al. (2012), Caines and Winkler (2021), and Adam et al. (2022) focus on house prices. Our contribution to this body of literature lies in emphasizing the asymmetric evolution of asset price beliefs during boom and bust episodes. Additionally, we introduce a novel solution method for models incorporating capital gains extrapolation. Kaplan et al. (2020) demonstrate that changes in agents' house price expectations are crucial for explaining house price volatility during the Great Recession. However, their model does not utilize extrapolative belief updating; instead, changes in agents' beliefs are modeled exogenously. Our findings align with their analysis but extend it by providing empirical evidence through the estimation of a subjective beliefs model. Moreover, our analysis examines the role of monetary policy across different boom and bust episodes and regions, whereas Kaplan et al. (2020) focus solely on the Great Recession at a federal level, without incorporating monetary policy considerations. We also contribute to the literature examining regional housing supply variations in the United States. Studies by Mian et al. (2013), Mian and Sufi (2014), and Guren et al. (2021) leverage housing supply elasticities to explore the housing wealth effect. Our work is also closely related to the studies by Aastveit and Anundsen (2022) and Aastveit et al. (2023), which demonstrate that house prices in U.S. metropolitan areas with more inelastic housing supply exhibit greater responsiveness to monetary policy shocks. We focus on a state level instead of metropolitan areas which allows us to extend our analysis to economic activity more broadly. Further, we also show that the same patterns can be observed in the Euro Area. While the aforementioned studies are primarily empirical, our contribution lies in providing theoretical insights to interpret our empirical findings. Finally, we focus on a setup involving multiple regions governed by a single monetary authority, thereby connecting to the literature on cross-regional heterogeneities in currency unions. Studies by

Benigno (2004), Galí and Monacelli (2008), and Kekre (2022) study optimal policy in a currency union setting. Additionally, Calza et al. (2013), Slacalek et al. (2020),Bletzinger and von Thadden (2021), Pica (2021), Almgren et al. (2022), and Corsetti et al. (2022) examine various sources of heterogeneity within currency unions and their effects on economic activity. In this strand of literature, cross-region heterogeneities include variations in price and wage setting, the share of hand-to-mouth consumers, and mortgage market dynamics. To our knowledge, we are the first to study supply-side housing constraints and connect these to the formation of subjective house price beliefs.

The rest of this paper is organized as follows. In Section II, we describe the empirical facts mentioned above; Section III holds an outline of our theoretical model together with an explanation of our solution method and some analytical insights on how the expectations formation mechanism shapes house prices in general equilibrium; Section IV presents the results of our simulation exercises, and Section V contains the policy exercise. Section VI concludes.

II. Empirical Evidence

This section presents our two key empirical facts. First, we show that house price beliefs are not formed according to rational expectations and that the belief updating process varies between booms and busts. Second, we provide evidence that regional differences in house price responses to a common monetary policy shock are driven by cross-regional heterogeneity in housing supply elasticities. Finally, we analyze the interaction between boom-bust asymmetries and cross-region heterogeneities.[2] Due to data availability we will focus in the first section only on the US. In the following sections, we extend our analysis to the Euro Area.

II.A House price beliefs and boom-bust dynamics

We start by studying the response of aggregate house prices to a monetary policy shock using local projections. To understand how house prices behave in booms and busts, we condition on whether house prices have been increasing or decreasing in the past. The estimation Equation is given by:

$$y_{t+h} = \alpha^h + \mathbb{1}(\Delta q_{t-1} > 0) \times \beta_1 \epsilon_t^{MP} + \mathbb{1}(\Delta q_{t-1} < 0) \times \beta_2 \epsilon_t^{MP} + x_t + u_{t+h} \tag{1}$$

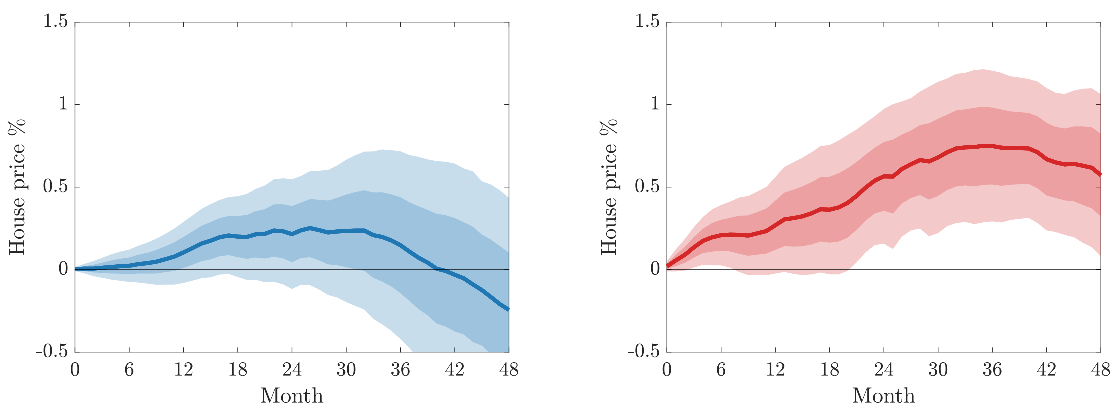
Δq_{t-1} denotes the month-on-month percentage change of house prices. The monetary policy shock, ϵ_t^{MP}, is the high frequency identified and orthogonalized shock from Bauer and Swanson

[2]All data sources are explicitly stated in the Appendix (A.1).

(2023). The left-hand side variable, y_{t+h}, is the log house prices. The sample runs from 1990-2019 and is in monthly frequency. The controls, x_t, contain 12 lags of the left-hand side variable, log of industrial production, the log of CPI, the FFR, and the shocks. We will focus on expansionary monetary policy shocks throughout the whole empirical analysis. Figure (1) plots the results. The blue line shows the response if house prices were increasing in the past, and the red line if they were decreasing. We find that house prices are notably more responsive to monetary policy shocks in times when they have been decreasing. The peak response of the point estimates almost doubles. Also, for the most part of the dynamic response, the boom and bust confidence intervals measured at one standard deviation, do not overlap. This highlights that the difference is statistically significant. In summary, house prices tend to respond stronger in busts than in booms. A potential explanation for this finding could be that house price expectations evolve differently in booms relative to busts. Stronger reaction of expectations in busts relative to booms would lead to a stronger response in housing demand, which would in turn lead house prices to react more strongly. We will now explicitly focus on this channel.

Figure 1: House price response to monetary policy shock, boom-bust

(a) $\Delta q_{t-1} > 0$ (b) $\Delta q_{t-1} < 0$



Notes: Responses to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% (Newey-West).

Moving to house price expectations, we are first interested in whether house price expectations are formed according to rational expectations. To answer this question, we study the response of forecast errors to a monetary policy shock. Forecast errors are interesting for two reasons. First, they show whether house price expectations follow rational expectations. Under rational expectations forecast errors should not respond to the shock. This becomes obvious in our local projections setup. We define forecast errors as as follows:

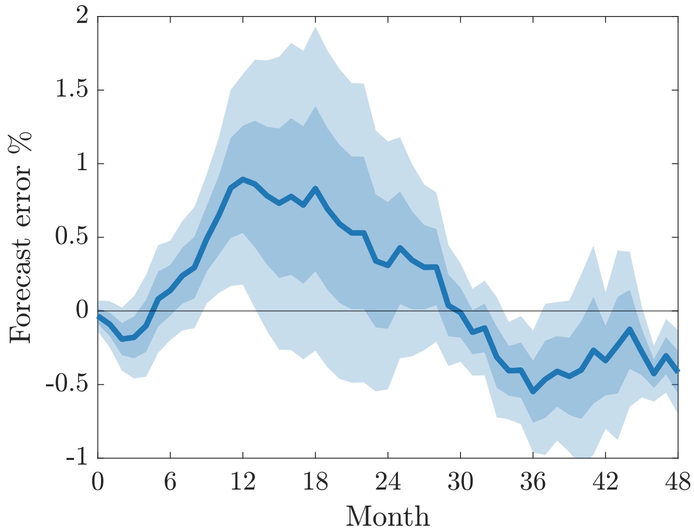
$$fe_t = \Delta q^y_{t+12} - \mathbb{E}^{\mathcal{P}}_t \Delta q^y_{t+12}$$

where Δq^y_{t+12} is the year-on-year percentage change in house prices. $\mathbb{E}^{\mathcal{P}}_t \Delta q^y_{t+12}$ is the expected year-on-year percentage change in 12 month formed today. The local projections we estimate are given by Equation (2). At the time the shock hits, in period t, the forecast errors seen on the left-hand side already contain the forecast formed today for 12 month ahead. In other words, the shock is contained in the information set of the agent and should be taken into account. Under rational expectations agents will perfectly use this information and forecast errors should not respond. The second insight we may gain form this exercise is on how house price expectations are formed. Given the response of the forecast errors is non-zero, the dynamics reveal how households' expectations evolve over time. This will enable us to zoom in on a specific belief updating process that explains our observations. We use the expectations data from the Michigan Survey on Consumer Sentiment. The frequency is monthly and the sample runs from 2007 to 2019. As controls, we include 6 lags of the forecast error and the monetary policy shock. We estimate the following equation:

$$fe_{t+h} = \alpha^h + \beta^h \epsilon_t^{MP} + x_t + u_{t+h} \qquad h = 0, 1, ..., H \tag{2}$$

Figure (2) shows the results. The impulse response reveals a non-zero and highly significant response in forecast errors. We can therefore reject rational expectations. On the dynamic behaviour we find that for the first 30 months the forecast error is positive, indicating over-pessimism on the side of the agents. After 30 months the forecast error turns negative, indicating over-optimism. For both cases, over pessimism and over optimism, the IRF is significant at a 95% confidence level. Further, the hump-shaped dynamics in the forecast errors indicate sluggish updating in the expectation formation process. These findings are in line with Adam et al. (2022).

Figure 2: Forecast error response to a monetary policy shock, US



Notes: Response to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% Newey-West).

To further investigate the belief formation, we will now introduce a specific belief updating process and evaluate its' empirical performance. We follow Adam et al. (2017) and Adam et al. (2022) and choose a process with a prior belief and an updating component. Specifically, this formulation follows a Bayesian belief updating model. For a formal derivation, the reader is referred to the model section. Importantly, this type of belief formation leads to sluggish belief updating, and in response to an expansionary shock, we would observe first over-pessimism followed by over-optimism. This matches our empirical findings from above. We test two specifications of belief updating processes, a linear one, and a specification that allows for regime switches. We include the regime switching model because it has the potential to explain the asymmetric boom-bust responses observed in house prices. The linear process reads:

$$\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1} = \varrho \bar{m}(1-\varrho) + (\varrho - g) \times \mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t + \varrho g \times \Delta q_{t-1} \tag{3}$$

The parameter ϱ in Equation (3) captures the persistence in the updating process on prior beliefs $\mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t$. The Kalman gain, g, governs the speed of the updating with respect to observed house price changes Δq_{t-1}. $\bar{m}$ are long-run house prices growth expectations. The belief updating model is in monthly frequency, and the sample is 2007 to 2022.[3] We obtain the month-on-month percentage change in house price expectations, $\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1}$, by dividing the year-on-year percentage changes

[3]We can expand the sample for this exercise as we are not constraint by the length of the monetary policy shocks, which stop in 2019.

from the Michigan Survey by 12. To estimate the belief updating process we feed in realized past house price growth data, Δq_{t-1}. For a given tuple $(\varrho, g, \bar{m})$ and by recursively updating $\mathbb{E}^{\mathcal{P}}_{t-1}\Delta q_t$, we obtain a sequence of house price growth expectations. We then use a solver choosing $(\varrho, g, \bar{m})$ such that the MSE between the data and the fitted values is minimized.[4] For the estimation procedure we also impose that $\varrho, g \in (0, 1)$.

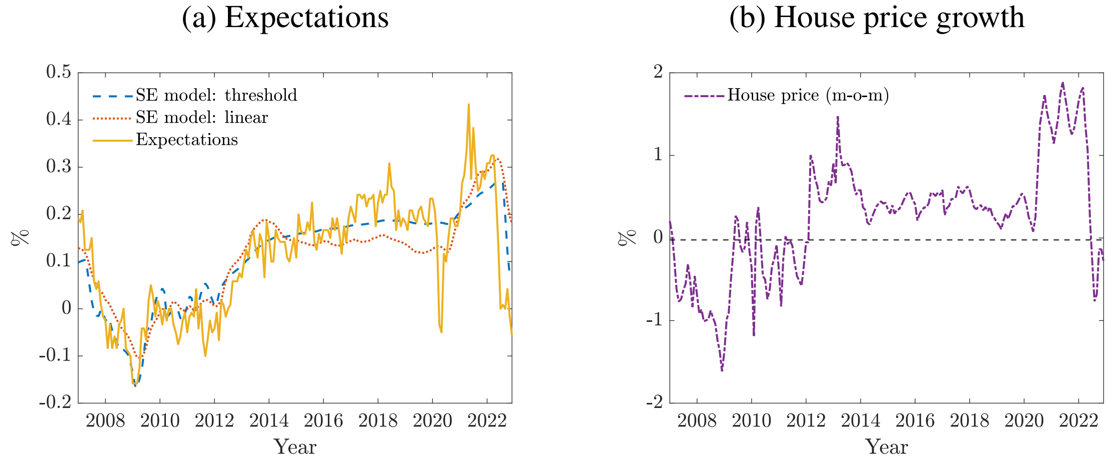
Our second specification introduces regime changes to the updating process. Equation (4) allows for heterogeneity in ϱ and g. The estimation procedure is equivalent to the linear model, only now we choose $(\varrho^h, \varrho^l, g^h, g^l, \omega, \bar{m})$ to minimize the MSE between fitted values and the data. Notice that the threshold is also estimated and that the regime depends on past observed house price growth, Δq_{t-1}.

$$\begin{aligned}\mathbb{E}^{\mathcal{P}}_t \Delta q_{t+1} = \mathbb{1}(\Delta q_{t-1} > \omega)\left(\varrho^h \bar{m}(1-\varrho^h) + (\varrho^h - g^h) \times \mathbb{E}^{\mathcal{P}}_{t-1}\Delta q_t + \varrho^h g^h \times \Delta q_{t-1}\right) + \\ \mathbb{1}(\Delta q_{t-1} < \omega)\left(\varrho^l \bar{m}(1-\varrho^l) + (\varrho^l - g^l) \times \mathbb{E}^{\mathcal{P}}_{t-1}\Delta q_t + \varrho^l g^l \times \Delta q_{t-1}\right)\end{aligned} \quad (4)$$

The estimation results are presented in Figure (3). The solid yellow line in panel (a) plots the expectations data from the Michigan survey, the blue dashed line the threshold model estimates, and the dotted red line the linear model. The dashed-dotted purple line in panel (b) depicts the month-on-month house price growth, and the dashed black line is the threshold for the threshold model. Turning to the linear model first, we find that the model does a reasonable job of explaining the expectation formation process. However, it is not able to capture the full extent of the bust in 2009 and lags behind in the bust episodes around the Great Recession and at the end of 2022. Further, in the recovery after the Recession, it underestimates expectations for house price growth. In comparison, the threshold model improves on exactly these dimensions. It is able to capture the bust in its' full extent without an obvious lag. It also performs much better in the recovery episodes. The estimated threshold for this model is close to zero, which is consistent with the threshold chosen in the local projections exercise above (see Equation (1)). This suggests that the updating behaviour differs in times when house prices rise compared to when they fall. Going forward, we will refer to booms as times when house prices have been increasing $(\Delta q_{t-1} > 0)$, and busts when they have been decreasing $(\Delta q_{t-1} < 0)$.

[4] This procedure demands a starting value for $\mathbb{E}^{\mathcal{P}}_{t-1}\Delta q_t$ which can affect the outcomes. To minimize the effect of the starting values on the estimated parameters, we start in 1987 to feed in monthly house price growth data. Thereby obtaining initial values for 2007M1. Changing the starting value 1987 has insignificant effects on our results.

Figure 3: House price belief model, US



Notes: SE model, threshold (blue, dashed): fitted values of Equation (4); SE model, linear (red,dotted): fitted values of Equation (3); Expectations (yellow, solid): mean expectations data from the Michigan Survey; House price (m-o-m) (purple, dashed-dotted): Month on month percentage change in house price.

The estimated parameters from the models are shown in Table (1). We find a ϱ of 0.91 and a Kalman gain of 0.02 for the linear model. These parameters are roughly in line with the literature. For the threshold model, we find a ϱ^h of 0.97, and a Kalman gain, g^h, 0.005 in booms. In busts, ϱ^l decreases to 0.77, while g^l increases to 0.076. The differences in the parameters across regimes transparently show why the threshold model can fit the data better than the linear model: In busts when house prices and beliefs fall drastically in a short period, a higher Kalman gain and a lower persistence parameter enable faster pass-through from observed house prices to beliefs. In booms, when house prices and beliefs recover steadily over a longer period, expectations are better matched by a slow-moving process with high persistence. This is achieved by a higher degree of persistence ϱ^h and a lower degree of updating g^h. Intuitively, in a boom agents believe that house prices will continue growing steadily and pay little attention to changes in house price growth. In busts the opposite is the case, agents observe dramatic changes in asset prices and pay close attention to their recent development, leading to a higher degree of belief updating. Finally, the MSE is reduced by 25% in the threshold model relative to the linear model. We also perform several robustness exercises. First, we impose the estimated parameters from above and below the threshold, subsequently on the linear model. We find a significant increase in the MSE, implying that the differences in the estimated parameters of the threshold model matter for the model performance. Second, we impose that ϱ (g) are linear across regimes while the respective other parameter can vary. We re-estimate the model for this exercise. We find that the MSE decreases by more relative to the linear model if the Kalman gain is allowed to vary across regimes. This shows that differences in the Kalman gain are more important in matching the data.

Table 1: House price belief model: parameters

$Specification$	ϱ^{lin}	ϱ^{h}	ϱ^{l}	g^{lin}	g^{h}	g^{l}	$Threshold$	MSE^{i}/MSE^{lin}
$Baseline:$								
$linear$	0.91			0.020				1.0
$threshold$		0.97	0.77		0.005	0.076	−0.023	0.75
$Robustness:$								
$high\ regime\ beliefs$	0.97			0.005				2.10
$low\ regime\ beliefs$	0.77			0.076				1.37
$\varrho\ linear$		0.94	0.94		0.003	0.028	−0.118	0.81
$g\ linear$		0.99	0.79		0.010	0.010	0.008	0.94

Notes: linear: estimated parameters from Equation (3); threshold: estimated parameters from (4); Robustness exercises are explained in the main text.

One striking result from our threshold model is that g^h is fairly low, while g^l is fairly high. To enhance our understanding of what might be driving these results, we cut the sample before COVID, in 2019, and re-estimate the model. The intuition is the following. During and after COVID, house prices and beliefs were very volatile, potentially affecting our results given our small sample. The results are depicted in Appendix (A.2). The difference in the Kalman gains is significantly reduced and the estimated parameters seem more reasonable. On the one hand, this uncovers small sample issues. On the other hand, it also emphasizes how large bust episodes can affect the belief formation process. It is therefore important to pay attention to differences in the belief formation in boom-bust episodes. To test whether our estimated belief process inherits the desired features, namely a sluggish response in booms and a swift response in busts, we backcast expectations data until 1990 using the threshold model. We then run local projections on the estimated expectations data using again the monetary policy shock. Appendix (A.2) plots the results. We find a much more sluggish response if house prices are increasing, while expectations react faster if they are decreasing. Running the same exercise on the actual data reveals the same pattern, although the sample of the data is much shorter and the data contain a lot of noise. Finally, we also find that forecast errors respond differently in booms compared to busts.

II.B Cross-regional heterogeneity

We will now move on to study cross-regional differences in house prices, and other variables of interest, to a common shock. To study the reaction of these various regional variables to a common monetary policy shock, we estimate panel local projections to an externally high-frequency identified monetary policy shock. Equation (5) represents the empirical specification:

$$y_{n,t+h} = \alpha_n^h + \beta^h \epsilon_t^{MP} + \gamma^h \epsilon_t^{MP} \times z_n + x_{n,t} + u_{n,t+h} \qquad h = 0, 1, \ldots, H \tag{5}$$

For the left-hand side variable, y_n, we choose house prices, variables relating to construction sector activity, and general economic activity in a given region. ϵ_t^{MP} denotes the monetary policy shock. We further interact this shock with a regional-specific variable capturing supply-side heterogeneities, which we denote as z_n. $x_{n,t}$ is a vector of aggregate and regional controls. A time-fixed effect and a region-fixed effect are also included. We use this empirical specification for the US, as well as for the Euro Area. For consistency, we study a one standard deviation monetary policy shock and standardize the interaction coefficients throughout all exercises.

Evidence from the US. For the US we focus on state-level data in the cross-section.[5] The left-hand side in the local projection is the following. First, we study the nominal house prices. Second, on the construction sector level, we focus on the number employed in the construction sector and the number of building permits issued. Third, to capture general economic activity, we use the unemployment rate and the number employed in the retail sector. Employment in the retail sector is generally used as a proxy for household consumption on the regional level (Mian and Sufi, 2014; Guren et al., 2021). With exception of the unemployment rate, we take the log of each of the left-hand side variables. The monetary policy shock is the same as before and taken from Bauer and Swanson (2023). The interaction term is the house price sensitivity indicator from Guren et al. (2021). It measures the responsiveness of metropolitan area house prices to an increase in house prices at the Census region level controlling for a broad range of local economic conditions. It aims to capture housing supply side heterogeneities .[6] In our context, we will think about this indicator as capturing housing supply side constraints. Specifically, the time it takes to construct houses. We show that the housing sensitivity indicator is correlated with a time-to-build measures on a Census division level in Appendix (A.4). This interpretation also renders a direct comparability from the US supply side constraints to the Euro Area supply side constraints. As Guren et al. (2021) refer to their sensitivity measure as capturing housing supply elasticities, we will henceforth refer to regions with less or more supply constraints as regions with lower or higher elasticity, respectively. To obtain state level housing sensitivity measures, we aggregate up the metropolitan level data by weighing them according to population size. The vector of controls includes respectively 8 lags

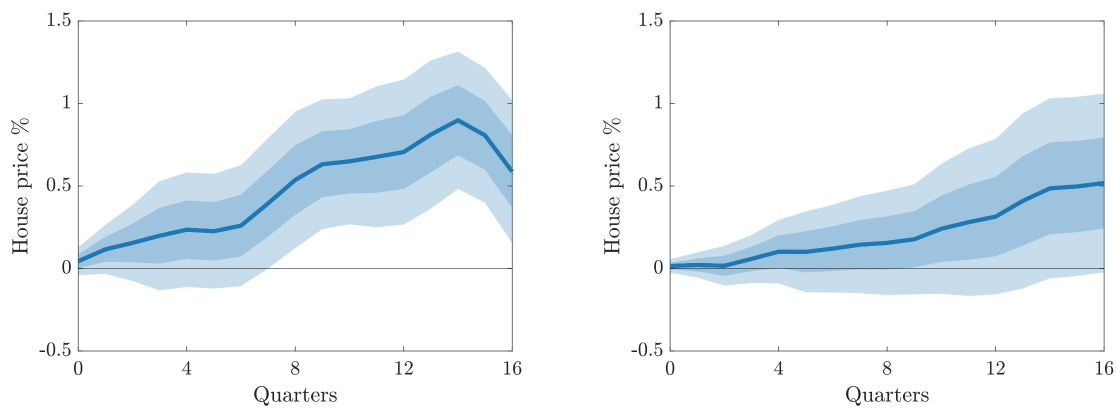
[5]Hawaii and Alaska are not included in the sample due to insufficient data coverage.

[6]This indicator can be understood as a proxy for supply-side elasticities in the spirit of Saiz (2010). Contrary to the supply side elasticities estimated by Saiz (2010), the house price sensitivity indicator is uncorrelated with demand-side characteristics.

of the left-hand side variable, the log of US GDP, the log the GDP deflator, the FFR, the shock, and the interacted shock term. The sample runs from 1990 to 2019 and is in quarterly frequency. Figure (4) plots the response of nominal house prices to an expansionary monetary policy shock. We find a sizeable and persistent increase in the mean coefficient, as seen in panel (a). Further, the interaction coefficient, shown in panel (b), is positive and also persistent. Both IRFs are significant at least at a 90% confidence interval. Our findings indicate that house prices are increasing in response to an expansionary monetary policy shock and they do so more in states where supply is more constrained. These findings are in line with Aastveit and Anundsen (2022).

Figure 4: House price in response to monetary policy shock, US

(a) Nominal house price, mean β^h (b) Nominal house price, interaction γ^h

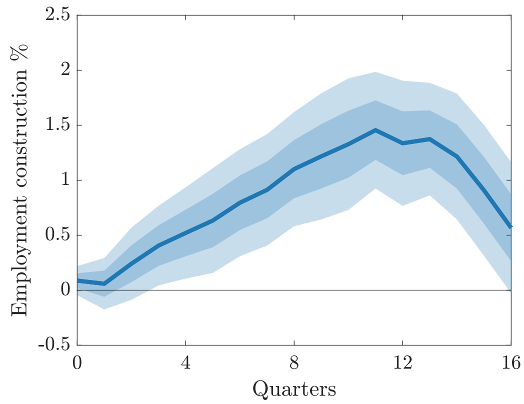


Notes: Responses to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

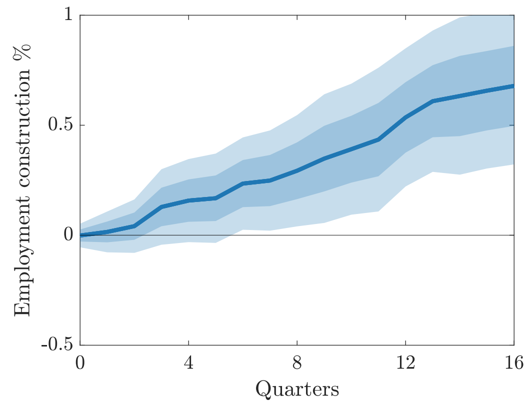
Next, we focus on how heterogeneity of housing supply elasticities translates into real economic activity in response to a monetary policy shock. Figure (5) plots the responses of employment in the construction sector and building permits. We find that employment in construction and building permits increase in response to the shock (panels (a) and (c)). We further observe that they do increase more in regions where housing supply is more constrained (panels (b) and (d)). All IRFs are sizeable and significant at a 95% level.

Figure 5: Economic activity in the construction sectorin responses to monetary policy shock, US

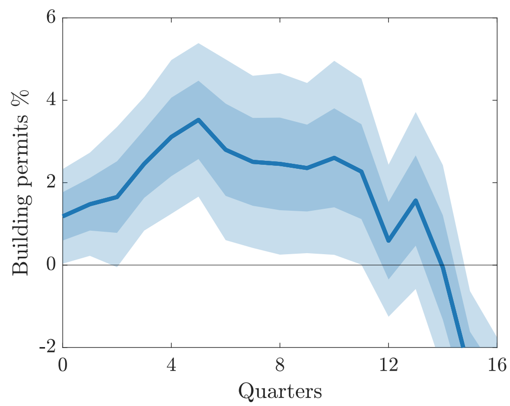
(a) Employment in construction, mean β^h



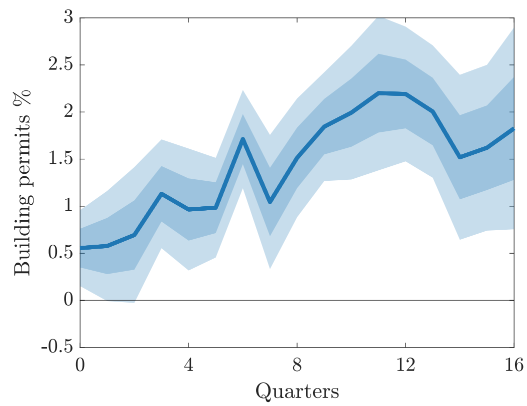
(b) Employment in construction, interaction γ^h



(c) Building permits, mean β^h



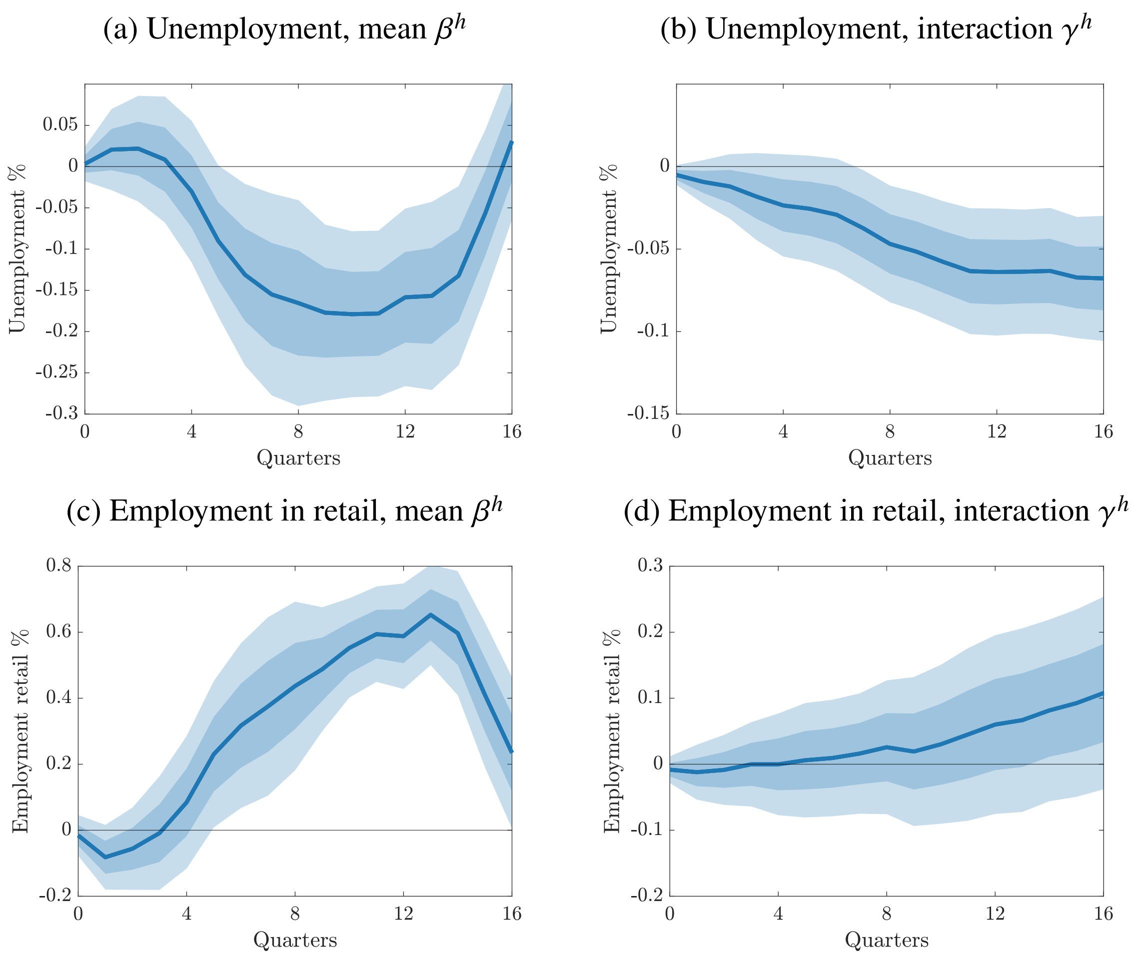
(d) Building permits, interaction γ^h



Notes: Responses to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

These results carry over to the general economy. Panel (a) and (c) in Figure (6) show that the unemployment rate decreases in response to the shock, while employment in retail increases. Again this pattern is more pronounced in more supply constrained states as shown in panels (b) and (d). Apart from the interaction coefficient of employment in the retail sector, all IRFs are highly significant. To summarize, we find that not only house prices respond stronger in supply-inelastic regions in response to monetary policy shocks, but also the same regions experience an economic expansion relative to more supply-elastic states. In other words, supply inelastic states are booming relative to more supply elastic states. This finding is ex-ante non-trivial. Why should economic activity be more responsive in regions where house prices are increasing more strongly? Extrapolative house price belief updating provides an answer to this question. If agents in an inelastic supply region observe a large increase in house prices, they will extrapolate observed house price growth into the future and expect further increases. This will lead to an increase in investment into the housing sector and eventually larger aggregate activity. In more elastic supply regions this dynamic will be muted as the increase in the house price is not as large and therefore belief extrapolation is less pronounced. In summary, extrapolative belief updating leads to regional, demand-driven booms that are larger in more inelastic supply regions, explaining the empirically observed patterns.

Figure 6: General economic activity in responses to monetary policy shock, US



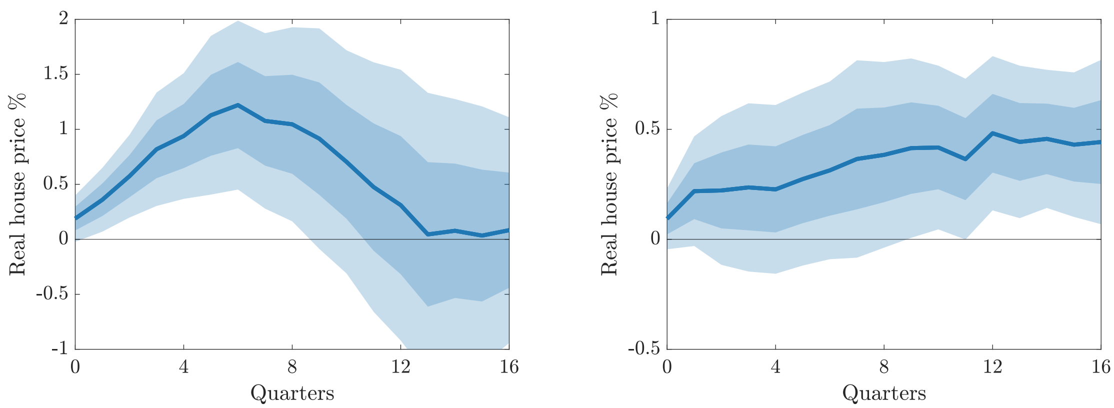
Notes: Responses to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

Evidence from the Euro Area. The general setup is the same as for the US: We estimate Equation (5) for house prices, variables measuring activity in the housing sector, and variables measuring general economic activity. These variables consist of real house prices, housing investment, building permits, real GDP, and unemployment. Again, we take the log for all of these variables but the unemployment rate. The cross-section is taken at a country level. The monetary policy shock is high-frequency identified from overnight interest swaps at a one-year horizon and taken from

Altavilla et al. (2019). As an interaction term, we use the days it takes to obtain a building permit in a given country, provided by the World Bank database. The sample runs from 2000 to 2019 and is in quarterly frequency. The countries contained in the sample are Austria, Germany, Spain, Finland, France, Ireland, Italy, the Netherlands, and Portugal. The vector of controls consists of 6 lags of the following variables: The left-hand-side variable, log GDP, log HICP, the EONIA, the shock, and the shock interacted with the interaction term.[7] Figure (7) plots the response of real property prices to an expansionary monetary policy shock. We find that house prices increase (panel (a)) and that they increase more in countries where supply is more inelastic (panel (b)). Both IRFs are significant at a 95% confidence level.

Figure 7: House price in response to monetary policy shock, EA

(a) Real property price, mean β^h (b) Real property price, interaction γ^h

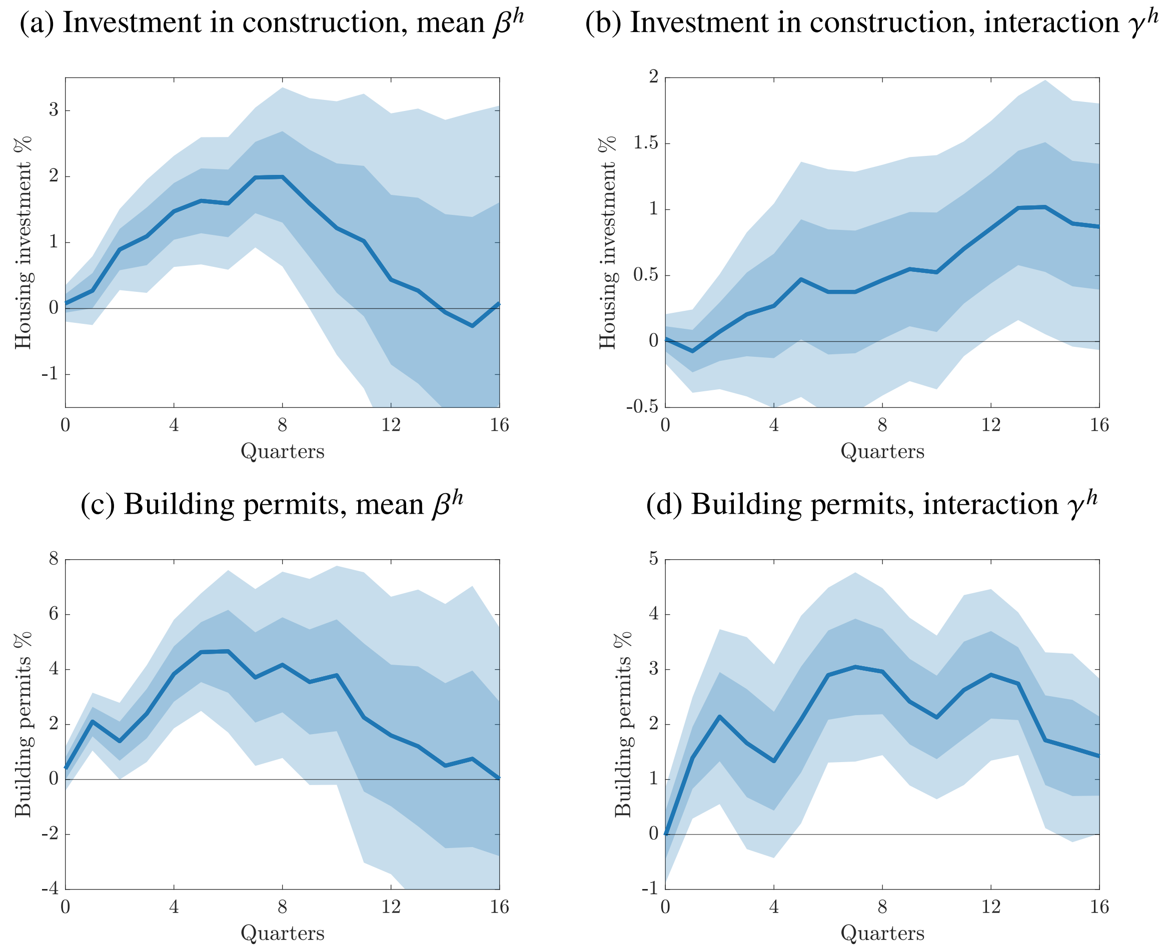


Notes: Responses to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

Figure (8) shows the responses of the log of residential housing investment and the log building permits to an expansionary monetary policy shock. Focusing on the construction sector first, we find that housing investment and building permits increase in response to the shock (panels (a) and (c)), and they do so more in more supply inelastic countries (panels (b) and (d)). All responses are significant at a 95% confidence level.

[7]For housing investment we drop log GDP as a control as it strongly correlates with the left-hand-side variable.

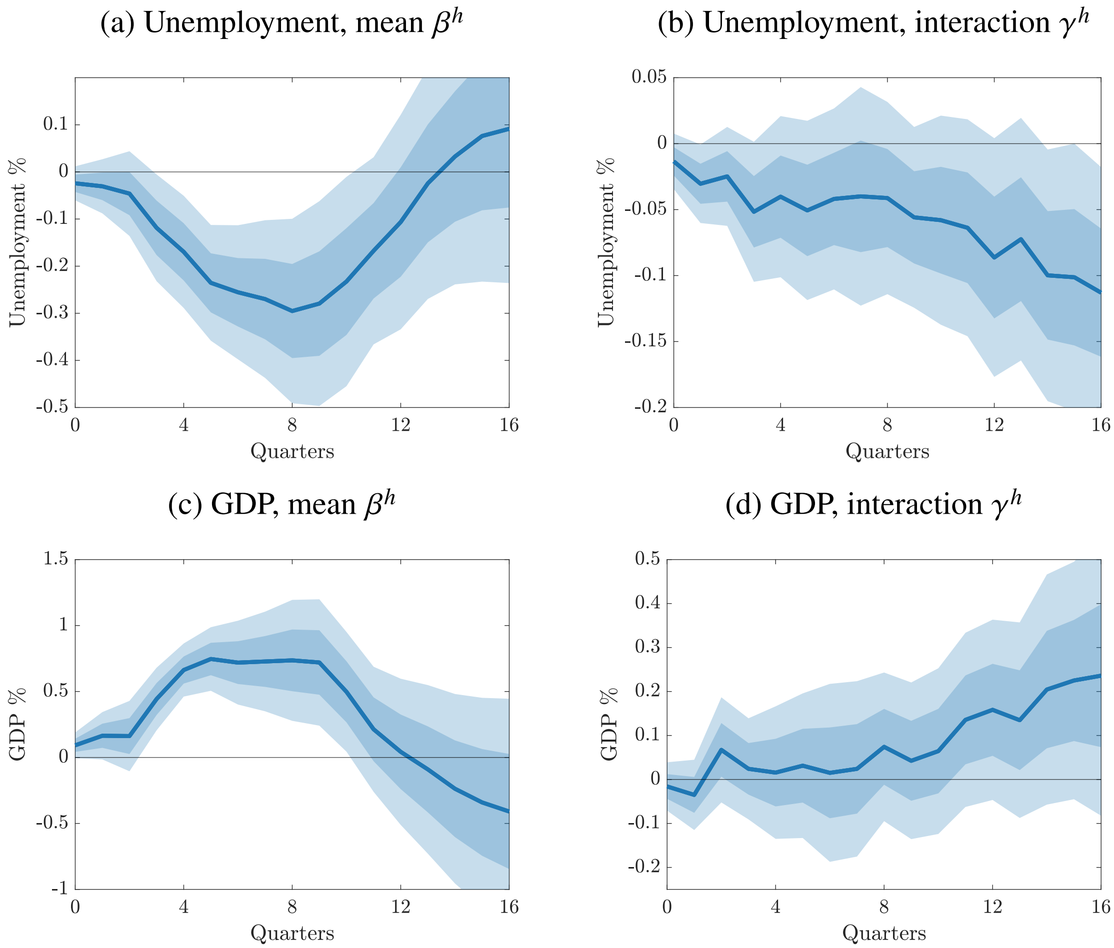
Figure 8: Economic activity the construction sector in responses to monetary policy shock, EA



Notes: Responses to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

Turning to the general economy, shown in Figure (9), we find that the unemployment rate decreases and real GDP increases, as shown in panels (a) and (c). Panels (b) and (d) show that the unemployment rate falls more, and real GDP increases more, in more supply-constrained countries. Apart from the interaction term of real GDP, all IRFs are significant at a 95% confidence level. Our findings from the Euro Area mirror the ones from the US. Not only do house prices increase more in supply-inelastic regions, but economic activity also expands more. This suggests that these dynamics are not limited to the US, but may be understood as a general feature of countries or currency unions facing regional heterogeneity on the housing supply side.

Figure 9: General economic activity in responses to monetary policy shock, EA



Notes: Responses to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

II.C Cross-region heterogeneity in booms and busts

We have demonstrated our two key findings: first, that house price beliefs are not formed according to rational expectations and are updated differently during booms compared to busts; second, that house prices respond differently across regions due to heterogeneity on the supply side. In this section, we will explore how these two findings interact. We have shown that house prices are more responsive when beliefs are updated more strongly. Consequently, significant belief updating can amplify regional house price differences, as initially small disparities are extrapolated into the future to a greater degree. If house price beliefs are updated more strongly during bust episodes, we should expect to observe higher cross-regional variation in house prices within a country or currency union during these periods. In more technical terms, cross-regional heterogeneity on the supply side leads to differential house price responses, and stronger belief updating amplifies these differential regional responses.

A straightforward way to test this hypothesis is to study cross-regional variation in house prices in boom and bust episodes. For each point in time we compute the cross-regional standard deviation of house price growth rates in the United States, respectively the Euro Area. We then split the sample into a boom and a bust sub-sample, and compute means and medians of the cross-regional standard deviations in each sub-sample. For the sample split, we use two procedures. First, we consider a period a boom period if the aggregate house prices on the country or currency union level are increasing. Second, we label the period from 2007-2012 as a bust episode, which roughly matches the low-regime episode from our house price belief model. We do this exercise using the quarterly state and country-level house price data from our panel local projections exercises, for the US and the Euro Area respectively. The results are shown in Table (2). We find that house price variation across regions is larger in bust than in boom episodes. This holds for the US, as well as for the Euro Area. It also holds across both boom-bust specifications. In relative terms, busts produce a 9% – 30% higher cross-regional standard deviation than booms, depending on the measurement and the geographic location.

Table 2: Cross-regional house price variation in booms and busts

	US		*EA*	
	mean	*median*	*mean*	*median*
Full Sample	0.95	0.83	1.72	1.61
	(0.06)	(0.06)	(0.23)	(0.23)
$Boom:\ \Delta q_t^{agg} > 0$	0.91	0.81	1.57	1.59
	(0.06)	(0.06)	(0.21)	(0.21)
$Bust:\ \Delta q_t^{agg} < 0$	1.12	1.02	2.04	2.02
	(0.06)	(0.06)	(0.24)	(0.24)
$\frac{Bust}{Boom}$	1.23	1.26	1.30	1.27
$Boom:\ No\ Recession$	0.92	0.83	1.58	1.57
	(0.06)	(0.06)	(0.21)	(0.21)
$Bust:$ 2007 – 2012	1.08	0.90	2.02	1.97
	(0.06)	(0.06)	(0.25)	(0.25)
$\frac{Bust}{Boom}$	1.17	1.09	1.27	1.26
Sample period	1990 – 2019		2000 – 2019	
Number regions	51		8	

Notes: The Table shows the mean and median estimates of cross-sectional variances of house prices within currency unions. The number in brackets below the point estimates denotes standard errors.

III. Model

In this section, we describe our two-region currency union model, inspired by Benigno (2004). A notable difference in our model is the incorporation of incomplete bond markets across regions. Additionally, we include a housing sector and allow for subjective beliefs in the formation of house price expectations. We differentiate between a "home" region and a "foreign" region, with all foreign region variables denoted by an asterisk. The primary distinction between regions is the time required to construct houses, referred to as "time to build." This parameter enables us to model supply-side heterogeneities in a tractable manner.[8]

III.A The Economy

Households. A representative domestic household derives utility from consuming domestic and foreign varieties, leisure, and housing. The preferences are as follows:

$$\mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left(\frac{\xi_{c,t}\left(c_t - \bar{h} c_{t-1}^a\right)^{1-\sigma}}{1-\sigma} + \frac{\xi_{h,t} h_t^{1-\nu}}{1-\nu} - \frac{\chi n_t^{1+\varphi}}{1+\varphi} \right)$$

$$c_t = \left[\lambda^{\varsigma} c_{H,t}^{1-\varsigma} + (1-\lambda)^{\varsigma} c_{F,t}^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}}$$

$$c_{H,t} = \gamma \left[\int c_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}, \quad c_{F,t} = (1-\gamma)\left[\int c_{F,t}(j^*)^{\frac{\epsilon-1}{\epsilon}} dj^*\right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\mathbb{E}_0^{\mathcal{P}}$ denotes the subjective expectations operator discussed in Section III.A. ξ_j, $j \in \{c, h, a, x, i\}$, denote model-exogenous shock terms, h_t and n_t denote housing and hours worked respectively. $\bar{h}$ denotes habit formation on last periods aggregate consumption c_{t-1}^a, while $\gamma \in (0,1)$ is the measure of households in the home economy. Following Benigno (2004), γ is simultaneously the economic size of the home region, i.e. the mass of variety-producing firms. c_t denotes consumption of the domestic basket that is assembled from the home-good and the foreign-good which in turn are CES-aggregates of two groups of varieties. Consumers in F also consume the home- and foreign-good, albeit with different weights: $c_t^* = [(1-\lambda^*)^{\varsigma} c_{H,t}^{1-\varsigma} + (\lambda^*)^{\varsigma} c_{F,t}^{1-\varsigma}]^{\frac{1}{1-\varsigma}}$. A preference bias for goods produced in the respective region of residence ("home bias") arises if $\lambda, 1-\lambda^* \neq \gamma$ and throughout the paper we maintain the assumptions that (i) the degree of home bias is symmetric:

[8] Arguably, this formulation is closer to the empirical evidence of the Euro Area presented above. Although a time-to-build measure at the state level is not available for the United States, it is possible to use U.S. Census data at the census division level to compute such a measure. Aggregating house price sensitivity indicators to the census division level reveals a positive correlation between these indicators and the time-to-build measure (see Appendix A.4). Thus, employing time to build as a proxy for supply-side differences in the U.S. is a reasonable approximation.

$\gamma(1-\lambda) = (1-\gamma)(1-\lambda^*)$, and (ii) the bias is such that households favor domestically produced products, $\lambda \geq \gamma$. Standard algebra on CES-aggregation allows to aggregate the market prices for consumption goods into price indices

$$\gamma \int P_{H,t}(j) c_{H,t}(j) dj = c_{H,t} P_{H,t}, \quad P_{H,t} = \left[\int P_{H,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}},$$
$$c_{H,t} P_{H,t} + c_{F,t} P_{F,t} = P_t c_t, \quad P_t = \left[\lambda P_{H,t}^{1-\frac{1}{\varsigma}} + (1-\lambda) P_{F,t}^{1-\frac{1}{\varsigma}} \right]^{\frac{1}{1-\frac{1}{\varsigma}}},$$

The household budget constraint is then given by:

$$c_t + q_t(h_t - (1-\delta)h_{t-1}) + b_{t+1} + \frac{P_{H,t}}{P_t} x_t = w_t n_t + (1+r_t) b_t + q_t \cdot H(x_{t-\tau}, \xi_{x,t}) - T_t + \Sigma_t + \mathrm{b}_t.$$

The budget constraint is expressed in units of the region-H final consumption basket, c. Σ_t are profits from all domestic firms, which are owned evenly by all domestic households, w_t is the real wage, T_t are government lump-sum taxes, and b_t is a one-period nominal zero-coupon bond that is traded union-wide. q_t denotes the real house price and x_t is the number of domestic consumption units dedicated to the production of new housing units. To invest into housing production, households need to purchase domestically produced goods and transform them into housing investment units. Housing production is defined as $H(x_{t-\tau}, \xi_{x,t}) = \xi_{x,t} \frac{x_t^\eta}{\eta}, \eta \in (0,1)$. τ denotes the number of quarters it takes to construct new housing units. Housing investment done in period t will therefore lead to a return in period $t+\tau$. We assume that τ is heterogeneous across countries and that $\tau^* > \tau$. In the steady state we will calibrate ξ_h and ξ_h^* s.t. $b_{ss} = b_{ss}^* = 0$. Hence, in steady state there are no net debtor and net creditor regions. This ensures that the only form of regional heterogeneity is situated on the housing supply side.

Housing units can be retained to enjoy housing services, or sold on the housing market. It is convenient to express the bond holdings in units of region H's final basket. The real interest rate r_t is taken as given by households and is determined in equilibrium by the following Fisher-type equation: The value of bond holdings in units of numéraire is $B_t = P_t \cdot b_t$ and the nominal bond pays $i_{t-1} - \psi b_t$ units of currency as interest.[9] The real interest rate is thus given by

$$1 + r_t = \frac{1 + i_{t-1} - \psi b_t}{1 + \pi_t}$$

[9]The nominal interest rate is elastic in the aggregate holdings of bonds by domestic households. We follow Schmitt-Grohé and Uribe (2003) to ensure stationarity of the first-order dynamics. In Appendix B we provide a simple micro-foundation for debt-elastic interest rates.

where $\pi_t := P_t/P_{t-1} - 1$. Finally, $\mathrm{b}_t := (\beta^{-1} - 1)\big(\gamma + (1-\gamma)\frac{P^*_{t-1}}{P_{t-1}}\big)(1+\pi_t)^{-1}\bar{b}$, taken as exogenous by the household, captures payment streams between H and F that guarantee that households are content with holding no bonds in the non-stochastic steady state with zero inflation and real exchange rate parity.[10]

Subjective House Price Expectations. As is standard in the literature on capital gain extrapolation (e.g. Adam and Marcet, 2011; Adam et al., 2017), households are endowed with a set of beliefs in the form of a probability measure over the full sequence of variables that they take as given, henceforth external variables: $(\xi_t, r_t, w_t, \Sigma_t, T_t, \mathrm{b}_t, \pi_t, (P_{H,t}/P_{F,t}), q_t)_{t\geq 0}$. This measure we denote as $\mathcal{P}$. Rational expectations are a special case of this setup in the form that households' beliefs agree with the objective, or equivalently "true" or "equilibrium-implied", distribution of external variables, $\mathcal{P} = \mathbb{P}$. Although households may hold expectations that are generally inconsistent with the equilibrium-implied (conditional) distribution of external variables, it is worth emphasizing that first they have a time-consistent set of beliefs, and second they behave optimally given their beliefs. That is, households are *internally rational* in the sense of Adam and Marcet (2011). Moreover, the fact that all households are identical in beliefs and preferences is not common knowledge among agents so households cannot discover the misspecification of their beliefs, $\mathcal{P} \neq \mathbb{P}$, by eductively reasoning through the structure of the economy. Given the observed path of external variables up to period t, households then use this information and $\mathcal{P}$ to form a conditional expectation over the continuation sequence of external variables, which we denote as $\mathbb{E}^{\mathcal{P}}_t$. We denote the conditional rational expectations operator as usual by $\mathbb{E}_t$.

We assume that agents have rational expectations with respect to all external variables, except for house prices, q_{t+s}.[11] Households entertain the idea that house prices follow a simple state-space

[10] Given that bond holding entails a real cost in equilibrium, see footnote 9, introducing the payments b_t is a way to ensure that there are no bond holding costs in the non-stochastic steady state with zero inflation and real exchange rate parity (i.e. $1 + \pi_t = 1 + \pi^*_t = 1 + \pi_{H,t} = 1 + \pi_{F,t} = \frac{P_{H,t}}{P_{F,t}} = 1$). This ensures that this steady state is efficient, given that fiscal policy undoes the monopolistic competition distortion. b_t may be interpreted as the real interest rate paid by a non-marketable nominal consol, that perpetually pays the nominal rate $(\beta^{-1} - 1)(\gamma + (1-\gamma)P^*_{t-1}/P_{t-1})$ and of which the household is endowed with $\bar{b}$ units. The endowments of these consols ensure that nominal payments balance, i.e. $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$, see Appendix B. Since we will linearize the model around a steady-state with zero bond holding, b_t will be zero in equilibrium.

[11] Formally, $\mathcal{P} := \mathbb{P}_{-q} \otimes \mathcal{P}_q$, where $\mathbb{P}_{-q}$ is the objective measure over sequences of external variables without house prices, $\mathcal{P}_q$ is the measure over sequences of house prices implied by the described perceived model of house prices, and $\otimes$ is the product measure. Since we are interested in a first-order solution to the model, it does not matter what households perceive to be the dependence structure between house prices and the other external variables.

model:

$$\begin{aligned} \ln\frac{q_{t+1}}{q_t} &= \ln m_{t+1} + \ln e_{t+1} \\ \ln m_{t+1} &= \varrho \ln m_t + \ln v_{t+1}, \quad \varrho \in (0,1) \\ \begin{pmatrix}\ln e_t & \ln v_t\end{pmatrix}' &\sim \mathcal{N}\left(\begin{pmatrix}-\frac{\sigma_e^2}{2} & -\frac{\sigma_v^2}{2}\end{pmatrix}, \begin{pmatrix}\sigma_e^2 & 0 \\ 0 & \sigma_v^2\end{pmatrix}\right) \end{aligned} \tag{6}$$

Hence, agents perceive house price growth rates as the sum of a transitory and a persistent component. Crucially, $\ln e_t$ and $\ln v_t$ are not observable to the agents, rendering $\ln m_t$ unobservable. Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system:[12]

$$\begin{aligned} \ln\frac{q_{t+1}}{q_t} &= \varrho \ln \overline{m}_t + \ln \widehat{e}_{t+1} \\ \ln \overline{m}_t &= \varrho \ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2} + g \cdot \left(\ln \widehat{e}_t + \frac{\sigma_e^2 + \sigma_v^2}{2}\right) \end{aligned}$$

where $\ln \overline{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2+\sigma_v^2}{\sigma^2+\sigma_v^2+\sigma_e^2}$ is the steady-state Kalman filter gain, $\sigma^2 = \frac{1}{2}\left[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}\right]$ is the steady-state Kalman filter uncertainty, and $\ln \widehat{e}_t$ is perceived to be a white noise process.

To avoid simultaneity in the house price we modify the belief setup following Adam et al. (2017).[13] We obtain the same observable system but with lagged information being used in the posterior mean updating equation:

$$\ln \overline{m}_t = (\varrho - g)\left(\ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2}\right) + g\left(\ln\frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2}\right). \tag{7}$$

Under this formulation, the posterior mean is pre-determined. We may now derive the posterior mean on the $s > 0$ periods ahead of price:

$$\mathbb{E}_t^{\mathcal{P}} q_{t+s} = q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho\frac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2\left(\varrho\frac{1-\varrho^s}{1-\varrho}\right)^2\right) \cdot \exp(V), \quad V \propto \sigma_v^2 \tag{8}$$

Explicit derivations can be found in Appendix C.1. This arrangement of subjective beliefs over house prices follows closely Winkler (2020) and Caines and Winkler (2021). To build some

[12] We assume agents' prior variance equals the steady-state Kalman variance.

[13] q_t appears twice: in the forecast equation, and in the Kalman-updating Equattion through $\ln \widehat{e}_t$. Since q_t depends on $\overline{m}_t$, but the latter also depends on the former, it is not assured that at any point an equilibrium asset price exists and whether it is unique. See Adam et al. (2017) for the details. The idea of the modification is to alter agents' perceived information setup in that they observe each period one component of the lagged transitory price growth.

intuition we can log-linearize equations (6) and (8) around a steady state. Below we show that this steady-state exists. The house price beliefs are given by:

$$\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} = q_t + \frac{1-\varrho^s}{1-\varrho} \varrho \widehat{\widehat{m}}_t \tag{9}$$

The house price belief updating follows:

$$\widehat{\widehat{m}}_t = (\varrho - g)\widehat{\widehat{m}}_{t-1} + g(\widehat{q}_{t-1} - \widehat{q}_{t-2}) \tag{10}$$

Variables denoted with a "$\widehat{\ }$" express the respective variables in percent deviations from its' steady-state value. Equattion (9) shows that future house price beliefs depend on the current house price and today's beliefs. As $\varrho \in (0,1)$, the weight on the beliefs increases in the forecast horizon. Current house prices translate one-to-one into house price expectations. Hence, today's house price is extrapolated into the future. Turning to the belief updating Equattion (9), we see that beliefs have an autoregressive component and are updated according to past observed house price changes. House price updating is increasing in the Kalman gain, g, and decreasing in the persistence of the beliefs ϱ. Combining Equattion (9) and (10) will give Equattion (3), which we used in the empirical analysis.

In our empirical findings we have shown that the house price belief updating process varies across booms and busts. To study this behavior in our model we specify a threshold model version. The only differences to the linear model lie in the house price belief Equattion and the belief updating equation. The one period ahead forecast is given by:

$$\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+1} = \mathbb{1}\big(\widehat{q}_{t-1} - \widehat{q}_{t-2} > 0\big)\Big[q_t + \varrho^h \widehat{\widehat{m}}_t\Big] + \mathbb{1}\big(\widehat{q}_{t-1} - \widehat{q}_{t-2} < 0\big)\Big[q_t + \varrho^l \widehat{\widehat{m}}_t\Big] \tag{11}$$

And the belief updating Equattion is given by

$$\begin{aligned}\widehat{\widehat{m}}_t = \mathbb{1}\big(\widehat{q}_{t-1} - \widehat{q}_{t-2} > 0\big)&\Big[(\varrho^h - g^h)\widehat{\widehat{m}}_{t-1} + g^h(\widehat{q}_{t-1} - \widehat{q}_{t-2})\Big] \\ &+ \mathbb{1}\big(\widehat{q}_{t-1} - \widehat{q}_{t-2} < 0\big)\Big[(\varrho^l - g^l)\widehat{\widehat{m}}_{t-1} + g^l(\widehat{q}_{t-1} - \widehat{q}_{t-2})\Big]\end{aligned} \tag{12}$$

Combining these equations will give the formulation we used in the empirical exercise. Applying the intuition from above, $\varrho^h > \varrho^l$ and $g^h < g^l$ imply stronger updating in busts compared to booms.

Firms and price setting. We assume a continuum of monopolistically competitive firms that produce intermediate good varieties and have the same beliefs as households. Firm beliefs, however, concern only variables over which households have rational expectations. Therefore, firms are rational. Firm j buys labor $n_t(j)$ from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The variety is bought by households from both regions. The firm sets its retail price $P_{H,t}(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs. Formally the firm solves:

$$\max_{P_{H,t}(j)} \mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{P_t} \left[P_{H,t}(j) y_{H,t}(j) - (1-\tau^\ell) W_t n_t(j) - P_{H,t} \frac{\kappa}{2} \left(\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1 \right)^2 y_{H,t} \right]$$

$$\text{with} \quad y_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} y_{H,t}$$

with $y_{H,t}(j) = \xi_{a,t} n_t(j)$. $\Lambda_t = u'_{c,t}/u'_{c,0}$ denotes the stochastic discount factor and τ^ℓ is a wage subsidy paid by the government. It is selected such that the monopolistic competition distortion is offset in the non-stochastic steady state. The subsidy is financed through a lump-sum tax on the firm. In symmetric equilibrium, all firms choose the same price, $P_{H,t}(j) = P_{H,t} \forall j$ and we receive the New Keynesian Phillips curve:

$$(\Pi_{H,t} - 1)\Pi_{H,t} = \beta \mathbb{E}_t^{\mathcal{P}} \left[\frac{\Lambda_{t+1} y_{H,t+1}}{\Lambda_t y_{H,t} \Pi_{t+1}} (\Pi_{H,t+1} - 1)\Pi_{H,t+1}^2 \right] + \frac{1}{\kappa} \left((1-\epsilon) + \epsilon(1-\tau^\ell) \frac{w_t P_t}{\xi_{a,t} P_{H,t}} \right) \quad (13)$$

The real wage and gross producer price inflation are defined as $w_t = \frac{W_t}{P_t}$ and $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$ respectively. Furthermore, it is convenient to define the terms of trade, i.e. the relative price of the H- and F-good: $s_t := \frac{P_{H,t}}{P_{F,t}}$.

Monetary authority. The monetary authority sets the nominal interest rates according to a standard Taylor rule targeting currency union consumer price inflation:

$$i_t = \frac{1}{\beta} (\Pi_t^{cu})^{\phi_\pi} \xi_{i,t} \quad (14)$$

Currency union inflation is the average of region-level consumer price inflation, weighted by the country size: $\Pi_t^{cu} = (\Pi_t)^\gamma \left(\Pi_t^*\right)^{1-\gamma}$.

Market clearing. To achieve goods market clearing, each goods market for a variety j must clear. For notational convenience, we define $\mathrm{y}_{H,t}(j) := c_{H,t}(j) + x_{H,t}(j) + \Psi_{t,H}(j)$, as the total demand for good (H, j) coming from one typical H-consumer. $\Psi_t := (1+\pi_t)^{-1} \frac{\psi}{2} (b_t)^2$ is the real cost of intermediating the position of an H-citizen in the union-wide bond. This cost, just like consumption and housing investment, gets passed along down to the varieties: $\Psi_{t,H} := (\frac{P_{H,t}}{P_t})^{-\frac{1}{\varsigma}} \lambda \Psi_t$. Goods market clearing across all goods markets requires:

$$y_t := \int y_{H,t}(j)dj = \gamma \int \mathrm{y}_{H,t}(j)dj + (1-\gamma) \int \mathrm{y}^*_{H,t}(j)dj + \int \Phi_t(j)dj$$

where $\Phi_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \Phi_t$ and $\Phi_t = \frac{\kappa}{2}\left(\Pi_{H,t}-1\right)^2 y_{H,t}$ account for the price adjustment costs from the firm side. Since housing investment requires domestically produced goods we have that $x_t = x_{H,t}$ and $x^*_t = x^*_{F,t}$. And therefore $x^*_{H,t} = 0$ and $x_{F,t} = 0$. Aggregation and successive substitution eventually yields the domestic and foreign aggregate good market clearing conditions:

$$\left(1 - \frac{\kappa}{2}(\Pi_{H,t}-1)^2\right) y_t \gamma = \gamma \mathrm{y}_{H,t} + (1-\gamma)\mathrm{y}^*_{H,t}$$
$$\left(1 - \frac{\kappa}{2}(\Pi_{F,t}-1)^2\right) y^*_t (1-\gamma) = \gamma \mathrm{y}_{F,t} + (1-\gamma)\mathrm{y}^*_{F,t}$$

Further, the bond market clearing condition is given by:

$$\gamma P_t b_{t+1} + (1-\gamma) P^*_t b^*_{t+1} = 0.$$

Market clearing in the housing sectors is given by:

$$H(x_{t-\tau}, \xi_t) = (h_t - (1-\delta)h_{t-1}),$$
$$H(x^*_{t-\tau^*}, \xi^*_{x,t}) = (h^*_t - (1-\delta)h^*_{t-1}).$$

Finally, the balance-of-payments Equattion ensures that the household budget constraints hold:

$$\gamma \mathrm{y}_{F,t} P_{F,t} - P_{H,t}(1-\gamma)\mathrm{y}^*_{H,t} + \gamma\left(P_t b_{t+1} - (1+i_{t-1})P_{t-1}b_t - \mathrm{b}_t\right) = 0.$$

Equilibrium. We adopt the equilibrium concept of Internally Rational Expectations Equilibrium, as defined in Adam and Marcet (2011):

Definition 1 (Internally Rational Expectations Equilibrium)**.** *An IREE consists of three bounded stochastic processes: shocks* $(\xi_t)_{t\geq 0}$*, allocations* $([c_{H,t}(j)]_{j\in[0,1]}, [c^*_{F,t}(j^*)]_{j^*\in[0,1]}, b_t, b^*_t, h_t, h^*_t, x_t, x^*_t, n_t, n^*_t)_{t\geq 0}$ *and prices* $(w_t, w^*_t, q_t, q^*_t, i_t, [P_{H,t}(j)]_{j\in[0,1]}, [P_{F,t}(j^*)]_{j^*\in[0,1]})_{t\geq 0}$*, such that in all t*

1. *households choose* $[c_{H,t}(j)]_{j\in[0,1]}, [c^*_{F,t}(j^*)]_{j^*\in[0,1]}, b_t, b^*_t, h_t, h^*_t, x_t, x^*_t, n_t, n^*_t$ *optimally, given their beliefs* $\mathcal{P}$*,*

2. *firms choose* $([P_{H,t}(j)]_{j\in[0,1]}, [P_{F,t}(j^*)]_{j^*\in[0,1]})_{t\geq 0}$ *optimally, given their beliefs* $\mathcal{P}$*,*

3. *the monetary authority acts according to the Taylor rule* (14)*,*

4. *markets for consumption good varieties, hours and housing clear given the prices, and the balance-of-payments Equattion holds.*

Appendix C.2 contains (i) a proof that Walras' law holds for our economy and (ii) the derivation of the Balance-of-Payments condition. Appendix C.3 presents the system of equations that characterizes the IREE.

III.B Solution method

We solve our model to first order around a non-stochastic and efficient steady state. This preserves analytic tractability at many points in the model and allows us to derive results on household behavior under capital gain extrapolation in closed form. The model presented in Section III.A admits a unique non-stochastic steady state with zero net inflation and parity of the terms of trade:

Lemma 1 (Non-stochastic steady state)**.** *Consider the model economy presented in Section III.A. As the variance of shocks (actual or perceived) fades,* $\mathrm{Var}[\|\xi_t\|] \to 0$, $\xi_t = (\xi_{a,t}, \xi_{c,t}, \xi_{h,t}, \xi_{x,t}, \xi_{i,t}, e_t, v_t)^\top$, *there exists one and only one steady state in which net inflation is zero,* $\pi_{H,ss} = \pi_{F,ss} = \pi_{ss} = \pi^*_{ss} = 0$, *and in which the terms of trade are at parity,* $s_{ss} = 1$.

Proof. See Appendix C.4 ■

Linearizing models with capital gain extrapolation is not straightforward. In fact, to the best of our knowledge, we are the first to provide a first-order approximation to a model with capital gain extrapolation under the assumption that agents hold rational expectations outside of the asset pricing block.[14] The entire exposition focuses on the typical linearized household problem in region H with all derivations being analogous for the typical household in region F.[15] For expositional clarity, we omit habit formation in consumption and the specification that housing investment occurs in domestically produced goods. These simplifications do not affect the fundamental logic of our solution method.

Standard first-order solution techniques for models with rational expectations rely on a recursive representation of the equilibrium conditions. For instance, the inter-temporal consumption decision

[14] Winkler (2020) proposes the "conditionally model-consistent expectations" (CMCE) concept as a starting point for linearizing models with capital gain extrapolation. Under CMCE, however, beliefs over all external variables are distorted relative to rational expectations. In our approach, linearized decision rules may be obtained under the assumption that belief distortions apply only to asset prices, allowing to confine the deviations from rational expectations to exactly those variables where survey data allows to discipline the expectations-modeling choices.

[15] For any variable $\mathrm{var}_t \notin \{b_t, b^*_t, \Sigma_t, \Sigma^*_t\}$ define $\widehat{\mathrm{var}_t} := \frac{\mathrm{var}_t - \mathrm{var}_{ss}}{\mathrm{var}_{ss}} \simeq \ln \mathrm{var}_t - \ln \mathrm{var}_{ss}$ to first order. For b_t, Σ_t (analogously for b^*_t, Σ^*_t) define $\widehat{\mathrm{var}_t} := \frac{\mathrm{var}_t - \mathrm{var}_{ss}}{y_{ss}}$, which allows for the case that $\mathrm{var}_{ss} = 0$. (That is, we scale deviations in bond holdings and profits by GDP.) Note furthermore that $\widehat{1 + r_{t+1}} \simeq \ln(1 + r_{t+1}) - \ln(1 + r_{ss}) \simeq r_{t+1} + \ln\beta$. We abuse notation slightly and write $\widehat{r}$ instead of $\widehat{1 + r}$.

is captured by the forward recursion, the Euler equation:

$$\widehat{c}_t = \mathbb{E}_t\widehat{c}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+1}.$$

Under subjective expectations an equivalent formulation exists:

$$\widehat{c}_t = \mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t^{\mathcal{P}}\widehat{r}_{t+1}.$$

At this point, it is important to note that first, we can characterize external variables for which households have distorted expectations. In our case this is only the house price, $\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s}$, for which we have derived a subjective belief model. Second, for all other external variables, for which households have rational expectations, we can formulate all equilibrium conditions recursively in the usual manner. In this case, the equilibrium-implied distribution measure applies and we can drop $\mathcal{P}$ from the expectations operator. The difficulty arises concerning expectations of household choices, in our case $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}$. These variables depend on external variables over which households hold subjective and rational expectations. In this case, we neither have a specified subjective beliefs model, nor we can ignore the subjective probability measure $\mathcal{P}$.[16] Our solution method allows us to identify these variables in closed form. We provide a detailed derivation in Appendix C.5, and concentrate here on conveying the intuition of our solution approach.[17]

In solving for the subjectively optimal plan we exploit two key insights into how households behave to first-order that are valid irrespective of which set of beliefs they hold.[18] First, since there is only one budget constraint, there is only one inter-temporal trade-off, namely in consumption, c. Given a path for consumption, the first-order conditions for housing, hours worked, and housing investment uniquely pin down a mapping from external sequences to decisions for these variables. This insight allows us to concentrate on finding the optimal path for consumption. The second insight is that to first-order, the permanent income hypothesis holds and consumption depends only on the path of real interest rates, an external variable, and the subjectively expected lifetime income.

[16]The reason is that households have distorted expectations over at least one price sequence and therefore will make distorted choices; in particular they plan to make choices in the future that are inconsistent with what these choices will be in equilibrium. If we ignored this, i.e. exchanged $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}$ for $\mathbb{E}_t\widehat{c}_{t+s}$ in the forward iteration above, the computed equilibrium would be different from the IREE in Definition 1.

[17]For tractability, we set habit formation, $\bar{h}$, to zero and we assume that housing investment is made out of the aggregate consumption goods bundle c_t instead of only locally produced goods. Relaxing these assumptions does not affect our methodological approach.

[18]In particular, our solution method can be used to solve RE models.

By iterating forward the Euler Equattion we receive

$$\widehat{c}_t = -\frac{1}{\sigma}\sum_{s\geq 0}\mathbb{E}_t\widehat{r}_{t+s+1} + \lim_{s\to\infty}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s},$$

This shows that the only subjective expectation variable to remain is $\lim_{s\to\infty}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}$. We refer to this variable henceforth as "terminal consumption" and denote, for conciseness, as $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$.[19] Using these insights the household first-order conditions are given by the equations in (15). For expositional brevity we have dropped the shock terms and set time-to-build to zero.

$$\begin{array}{rl}
\text{Rational Expectations} & \text{Subjective Expectations} \\
\varphi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t & \varphi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t \\
\widehat{q}_t = (1-\eta)\widehat{x}_t & \widehat{q}_t = (1-\eta)\widehat{x}_t \\
\widehat{c}_t = -\frac{1}{\sigma}\sum_{s\geq 0}\mathbb{E}_t\widehat{r}_{t+s+1} & \widehat{c}_t = -\frac{1}{\sigma}\sum_{s\geq 0}\mathbb{E}_t\widehat{r}_{t+s+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty \\
\widehat{h}_t = \frac{\sigma}{\nu(1-\bar{\beta})}\left(\widehat{c}_t - \bar{\beta}\mathbb{E}_t\widehat{c}_{t+1}\right) - \frac{1}{\nu}\frac{\widehat{q}_t - \bar{\beta}\mathbb{E}_t\widehat{q}_{t+1}}{1-\bar{\beta}} & \widehat{h}_t = \frac{\sigma}{\nu(1-\bar{\beta})}\left(\widehat{c}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}\right) - \frac{1}{\nu}\frac{\widehat{q}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}}{1-\bar{\beta}}
\end{array} \tag{15}$$

where $\overline{\beta} := \beta(1-\delta)$. From this representation it becomes clear, that the difference between the subjective beliefs and the rational expectations model lies in the measure $\mathcal{P}$ expectations with respect to consumption and house prices, and the existence of the terminal consumption variable, $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$. Subjective beliefs with respect to house prices are unproblematic, as these are pinned down by the subjective beliefs model on house prices. Further note, that subjective beliefs on future consumption can be characterized through the Euler Equattion once we have a representation for terminal consumption. Therefore, in order to solve the subjective expectations model, it suffices to find a representation for terminal consumption. We find this characterization by combining the first-order conditions with the linearized household budget constraint. After iterating over this equation, we can find a closed form expression for terminal consumption.

Proposition 1 (Terminal consumption)**.** *To first-order around the non-stochastic steady-state terminal consumption, $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$, is given by:*

$$\begin{aligned}
\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = {} & \frac{\delta q_{ss}h_{ss}/\nu}{c_{ss} + \sigma/\varphi\cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu}\cdot\left[\widehat{q}_t + \widehat{\widehat{m}}_t\cdot\frac{\varrho}{1-\varrho}\left(1 + \frac{1-\overline{\beta}\varrho}{1-\overline{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho}\frac{1-\beta}{\delta}\right)\right] \\
& + \frac{y_{ss}}{c_{ss} + \sigma/\varphi\cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu}\frac{1-\beta}{\beta}\cdot\left[\sum_{n=1}^{\infty}\beta^n\mathbb{E}_t\{z_{t+n}^*\} + \widehat{b}_{t+1}\right],
\end{aligned} \tag{16}$$

[19] We have directly used the fact that households have rational expectations over external variables other than house prices, $\mathbb{E}_t^{\mathcal{P}}\text{var}_{t+1} = \mathbb{E}_t\text{var}_{t+1}$ for any variable $\text{var}_{t+1} \neq q_{t+1}$.

z^*_{t+n} is a function of external variables the household has rational expectations about and is specifically stated in Appendix C.5.

Proof. See Appendix C.5 ∎

The notation "ss" in the subscript of a variable, denotes its' steady-state values. Proposition (1) shows that terminal consumption consists of three parts. First, expectations about the future evolution of external variables for which the household has subjective beliefs. In our case this is only the house price and it is represented in Equattion (16) through the presence of the current house price, $\widehat{q}_t$, and posterior beliefs about house prices $\widehat{m}_t$. Second, it depends on the expectations of all variables the household has rational expectations about, which are collected in z^*_{t+n}. Third, it depends on today's bond choices $\widehat{b}_{t+1}$. This structure allows us to economically interpret the meaning of the subjective beliefs terminal consumption variable. In subjective beliefs the deviation of consumption from its' steady-state depends on the expected path of external variables and today's savings. It therefore captures a subjective beliefs wealth effect. Naturally the question arises if this wealth effect is also present under rational expectations. For expositional purposes it is informative to consider a transitory shock to the model. In response to this transitory shock, the terminal consumption variable under rational expectations will always be zero. Due to rational expectations households understand that the shock is purely transitory and therefore that there will be no wealth changes in the far future as the economy converges back to its' steady state. The non-zero result under subjective beliefs arises because households extrapolate today's price changes into the future and consequently expect wealth changes.

Discussion. Our method has two important advantages over previous approaches to solving asset price learning models. First, we solve the model using a first-order approximation which makes it fast to solve, easily scalable, and amenable to the analysis of Ramsey-optimal policies. The literature has previously relied on non-linear solution techniques (Adam et al., 2017), or hybrid[20] techniques (Adam et al., 2022) to solve these models. Hence, solution procedures are much more involved and limits to computational capacity may constrain the solution of larger-scale models. Second, our solution method confines subjective expectations to house prices. A previously developed method by Winkler (2020) and Caines and Winkler (2021), which also relies on perturbation, assumes household expectations to conform with the concept of conditionally model-consistent expectations. Under this concept, subjective expectations about one variable lead to spillovers to expectations about other variables. Thus, households will form subjective beliefs across all model variables. In our approach, households only hold subjective expectations with respect to one

[20] Adam et al. (2022) linearize all model equations except the households' demand Equattion for housing.

variable, while they remain rational with respect to all other variables. This method of explicitly characterizing choices in terms of lifetime income is general in the sense that it allows solving for household decisions under any time-consistent set of beliefs.

III.C Rational vs. Subjective Expectations: Analytical Insights

To understand how the expectation formation mechanism shapes the behavior of house prices, and particularly the capacity of the model to generate amplification in the house price response to shocks, we study a strongly simplified model in this section, comparing the effects of the two alternative assumptions: Rational Expectations and Subjective Expectations. Specifically, we consider a one-region, zero liquidity endowment economy with instantaneous housing production, that is we set the time to build to zero.[21] We study the path of house prices within an exercise that, while keeping everything tractable, allows us to understand the effects of expansionary monetary policy: the consumption endowment process is selected such that the real rate drops by ε on impact of the shock and returns to 0 afterwards. This is arguably a parsimonious way to model expansionary monetary policy. Detailed derivations are relegated to Appendix C.7.

Rational Expectations. The first-order dynamics of house prices in response to the endowment shock may be found by solving the housing first order condition:

$$\widehat{q}_t - \bar{\beta}\mathbb{E}_t\widehat{q}_{t+1} = \sigma(\widehat{c}_t - \bar{\beta}\mathbb{E}_t\widehat{c}_{t+1}) - \nu(1-\bar{\beta})\widehat{h}_t.$$

After substituting for $\widehat{h}_t$ using the housing market clearing condition and the housing investment FOC, and after solving the difference equation, we obtain the following Proposition.

Proposition 2 (House prices under Rational Expectations)**.** *In the outlined one-region, zero liquidity endowment economy with instantaneous housing production, and considering a shock to endowments that leads the real rate to drop by ε in $t = 0$ before returning to 0 afterwards, house prices evolve as*

$$\widehat{q}_t = \varepsilon\varpi/\bar{\beta}\cdot\left((\varpi/\beta)^t(1-\varpi(1-\delta)) - \mathbb{1}_{t\geq 1}(\varpi/\beta)^{t-1}(1-\delta)\right),$$

where $\varpi/\beta \in (0,1)$ is defined in Appendix C.7. Using the properties of ϖ established in Appendix C.7, it is easy to show that: $\widehat{q}_0$ is positive but smaller than ε, and $\widehat{q}_t$ is negative for any $t > 0$.

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[21] We refrain from modeling the housing supply as inelastic since, as it will turn out, the expansion in the housing stock that results from the initial house price surge has radically different implications on the behavior of house prices under rational and subjective expectations.

The intuition for this result is as follows: since the real rate drops and income expands, housing demand rises on impact, pushing up prices; this incentivizes the construction of new housing which exerts downward pressure on the house price; once the drop in the real rate is nullified in $t \geq 1$, the only force left that acts on house prices is the increased housing stock, exerting downward pressure on the house price, leading it to be negative. As is typical for rational expectations, agents perfectly anticipate this drop in the house price, leading to an additional downward pressure already on impact of the shock. In sum, the house price thus reacts less than one-for-one to the expansionary shock.

Subjective Expectations. Under subjective expectations, house prices continue to be determined by the housing FOC – but we must now deal with the additional complication of the wealth effect, $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$. We keep the analysis as parsimonious as possible by selecting the sequence of endowments not only such that the real rate behaves as described above, but also such that the wealth effect is muted, $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty = 0$, $\forall t$.[22] Since terminal consumption reacts positively to the current house price and posterior beliefs about future house price growth, this exercise can be considered a lower bound on the potential of subjective expectations to generate amplification in the house price response to expansionary shocks. In the absence of a wealth effect, the housing FOC reads:

$$\widehat{q}_t = \frac{\mathbb{1}_{t=0}}{1-\bar{\beta}} \varepsilon - \nu \widehat{h}_t + \frac{\bar{\beta}}{1-\bar{\beta}} \varrho \widehat{\widehat{m}}_t.$$

In contrast to the case of Rational Expectations, this Equattion is purely backward-looking. On impact of the shock we have $\widehat{\widehat{m}}_0 = 0$ and $\widehat{h}_0 = \frac{\delta \eta}{1-\eta} \widehat{q}_0$. Based on this, we can compute the response of house prices in period 0 and 1 in response to the shock.

Proposition 3 (House prices under Subjective Expectations)**.** *In the outlined one-region, zero liquidity endowment economy with instantaneous housing production, and considering a shock to endowments that (i) leads the real rate to drop by ε in $t = 0$ before returning to 0 afterwards and (ii) mutes the wealth effect, $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty = 0$, $\forall t$, house prices evolve as*

$$\begin{aligned}
\widehat{q}_0 &= \varepsilon / \vartheta, \quad \vartheta := (1-\bar{\beta}) \left(1 + \nu \frac{\delta \eta}{1-\eta}\right), \\
\widehat{q}_1 &= \widehat{q}_0 \left(\bar{\beta} \varrho g - (1-\bar{\beta})(1-\delta) \frac{\delta \nu \eta}{1-\eta}\right) / \vartheta.
\end{aligned}$$

Under the empirically relevant case that $\bar{\beta} > \eta$, it can be shown that a sufficient condition for ϑ to be smaller than unity, and thus for the impact response to be more than one-for-one, is that $\delta \nu < 1$.

[22] To be able to make this simplification, we must show that it is indeed possible to find a sequence of $\widehat{y}_t$ that produces this wealth effect; this is presented in Appendix C.7.

In contrast to Rational Expectations, Subjective Expectations can generate amplification in the house price response to expansionary shocks under reasonable parametrizations. As time progresses, agents update their beliefs about the future path of house prices – depending on the parametrization, the house price under SE may be positive even after the actual real-rate impulse dissipated. This is a feature that is not shared by RE as it is crucial for this result is that under RE, forward-looking agents understand the effects of higher housing supply in the future for house prices today; while under SE backward-looking agents not only do not take into account the downward price pressure of an expanded future housing supply, but also up their willingness to pay for housing in light of an upward past trajectory of house prices.

IV. Results

In this section we will present our findings. We will focus on the model parameterization, and then move on to the results. First, we show that the model is able to replicate aggregate house price responses to a monetary policy shock, conditional and unconditional on booms and busts. We then show that it can also replicate cross-regional differences in house prices and economic activity. Finally, the model can qualitatively capture the documented boom-bust-asymmetry in cross-regional variations.

IV.A Parameterization

Table (3) lists the parameter values, all of which are symmetric across regions. First, focusing on the household sector, the labor disutility shifter, the inverse Frisch elasticity, and the intertemporal elasticity of substitution are set to standard values in line with the literature. The discount factor is set to achieve a 2% steady-state interest rate. Habit formation is chosen according to Smets and Wouters (2007). We further assume the regions are symmetric in size. We set housing depreciation to 2% per quarter. Moving to the production sector, we set the home bias, the elasticity of substitution across regions and goods in accordance with Bletzinger and von Thadden (2021). After linearizing Equation (13), the slope of the Phillips Curve is given by $\frac{\epsilon-1}{\kappa} = 0.02$. This is in line with the recent literature.[23] Finally, for the linear model, not considering boom-bust dynamics, we will always choose the persistence parameter of house price beliefs and the Kalman gain as estimated by the linear model in Table (1). For exercises focusing on boom-bust dynamics, we will

[23] Adam and Billi (2006) set the slope of the Phillips Curve to 0.057. However, it has been argued that the slope of the Phillips Curve has decreased, see i.e Del Negro et al. (2020) and Hazell et al. (2022). We choose a middle ground. This parameterization is also in line with Bilbiie (2018).

choose different parameterizations in the belief updating process. We will be more precise below. The Taylor rule weight on inflation is 1.5, which is standard.

Table 3: Model parameters (symmetric parameters)

Parameter		Value	Description	Source/ Target
Households	χ	1.000	labor disutility shifter	standard
	φ	1.250	inverse Frisch elasticity	standard
	σ	2.000	inverse of intertemporal EOS	standard
	ν	1.500	housing utility elasticity	standard
	δ	0.02	housing depreciation	2% quarterly depreciation
	β	0.995	discount factor	standard for quarterly frequency
	$\bar{h}$	0.700	habit in consumption	Smets and Wouters (2007)
	γ	0.500	relative region size	symmetric regions
Goods aggregation & production	λ	0.800	home bias	
	ς	1.000	EOS across regions	Bletzinger and von Thadden (2021)
	ϵ	6.000	EOS across varieties	
	κ	250	price adjustment costs	slope of 0.02 for the Phillips curve
	η	0.8	elasticity of housing production	Adam et al. (2022)
House price beliefs	ϱ^{lin}	0.910	Persistence in house price beliefs	estimated
	g^{lin}	0.02	Kalman gain	estimated
Policy	ϕ_π	1.500	Taylor coefficient	standard

Notes: All parameters depicted above are equal across countries. One period in the model is one quarter.

Table 4 shows the parameter values for the non-symmetric parameters, as well as the allocation and prices in the non-stochastic steady state. We choose a higher degree of time-to-build in the foreign region. The steady-state value for the housing preference shifter is chosen such that we attain a symmetric steady-state in the allocation variables. This modeling choice ensures that all cross-regional differences result from the structural heterogeneity on the housing supply side. The steady-state value for the house price is the only variable that differs across countries. Symmetric steady-state values for bond levels, which are zero for both countries, also imply that there is no net-borrower or net-saver country in the steady-state. Changes in monetary policy will therefore not lead to Fisherian debt revaluation effects. We parameterize the model such that we match the housing sector in the US economy. The investment to output ratio, $\frac{x_{ss}}{y_{ss}}$, amounts to roughly 4%, while the size housing sector relative to the rest of the economy, $\frac{\delta q_{ss} h_{ss}}{y_{ss}}$, is roughly 5%. The former matches the residential investment to GDP ratio for the US, the latter the size of the residential housing sector relative to GDP.

Table 4: Asymmetric model parameters and steady-state values

Domestic	Value	Foreign	Value	Description
τ	2	τ^*	4	Time-to-build
$\xi_{x,ss}$	0.800	$\xi^*_{x,ss}$	0.800	housing productivity shifter
$\xi_{h,ss}$	0.950	$\xi^*_{h,ss}$	0.960	housing preference shifter
c_{ss}	2.065	c^*_{ss}	2.065	consumption
x_{ss}	0.087	x^*_{ss}	0.087	housing investment
h_{ss}	7.104	h^*_{ss}	7.104	housing
y_{ss}	2.152	y^*_{ss}	2.152	output
b_{ss}	0.000	b^*_{ss}	0.000	bond holdings
q_{ss}	0.783	q^*_{ss}	0.775	house price
w_{ss}	1.000	w^*_{ss}	1.000	wage

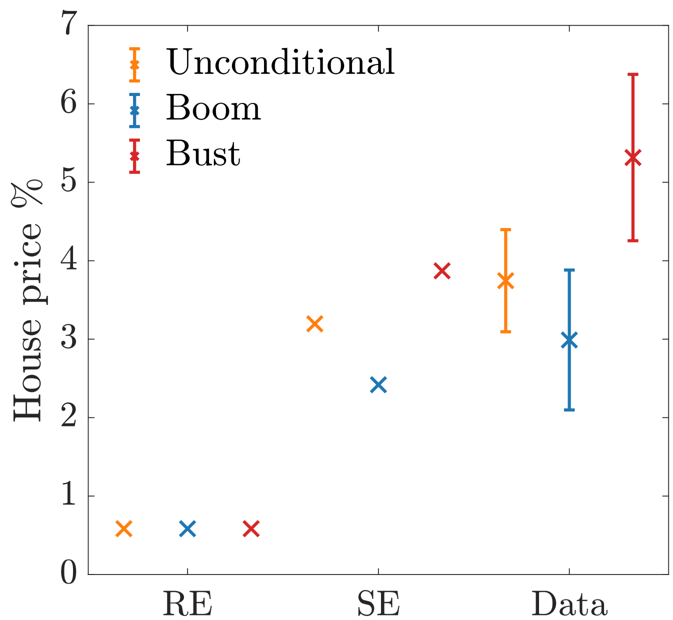
Notes: All steady-states except the house price are symmetric across countries. Time-to-build, and the steady state housing preference shifter are not symmetric across countries. All elements of ξ_{ss} not explicitly mentioned assume the value 1.

IV.B Aggregate house prices in booms and busts

We will start by studying the model performance on an aggregate level. All aggregate variables are a convex combination of the regional variables weighted by their size. Hence, the aggregate house price is given by $\widehat{q}_t^{agg} = \gamma \widehat{q}_t + (1 - \gamma)\widehat{q}_t^*$. We will study the aggregate house price response in the model and the data, unconditionally and conditionally on being in a boom or a bust. For the unconditional response in the model we simply consider the linear model using the parameterization described above. For the unconditional empirical response we estimate Equation (1) and drop the conditionality on the boom-bust regimes with respect to the monetary policy shocks. The conditional empirical response is given by local projections estimated from Equation (1). This set-up estimates house price responses to a monetary policy shock conditional on being in a boom or a bust. An appropriate model counterpart produces the response to a monetary policy shock conditional on being in either a boom or bust regime. To construct this conditionality in the model, we use the linear model but adjust the model parameters such that the model captures either boom or bust dynamics in house price updating. We use the parameters estimated in Table (1). For the boom we have a persistence parameter of $\varrho^h = 0.97$ and a Kalman gain of $g^h = 0.005$. For the bust period we have $\varrho^l = 0.77$ and $g^l = 0.028$. Note that for the bust case the Kalman gain lies below the actually estimated parameter.[24] Importantly, the respective parameters have no effect on the steady-state. Therefore, the shock will hit both models in the same steady-state but the dynamics will be different due to differential updating behaviour. We consider a 25 basis points expansionary monetary policy shock across all models and their empirical counterparts. We will first focus on the magnitudes of the responses in the model and the data, after which we move on to the dynamics. Figure (10) plots the peak responses in the model and the data, unconditional and conditional on booms and busts. First, it is apparent that the rational expectations model is unable to match the peak response observed in the data, unless augmented with additional features. Further, the response is equivalent across boom and bust periods as there is no source of asymmetry in the rational expectations model. Moving to the subjective beliefs model we see a significant improvement. The unconditional response increases relative to the rational expectations model by a factor of 6. In the model the peak is roughly at 3.2, while in the data we find a peak response of 3.7. Taking estimation uncertainty into account, the subjective beliefs model is well in range of what we observe in the data. Moving to the house price response in a boom, we find a peak response of 2.4 in the model, and 2.9 in the data. Again the subjective belief model lies within one standard deviation of the empirical estimate. With respect to the bust, we find in the model and the data a significantly stronger response to the shock. However, the subjective beliefs model under-preforms to a small degree and the peak response lies slightly outside the one standard deviation confidence band. This is most likely due to the downward adjustment in the Kalman gain used in the model.

[24] A high Kalman gain introduces a strong amplification force into the model – so strong that particularly high gain values render the law of motion for the models' state variables unstable. We choose the highest Kalman gain that permits existence of a stable solution to the model.

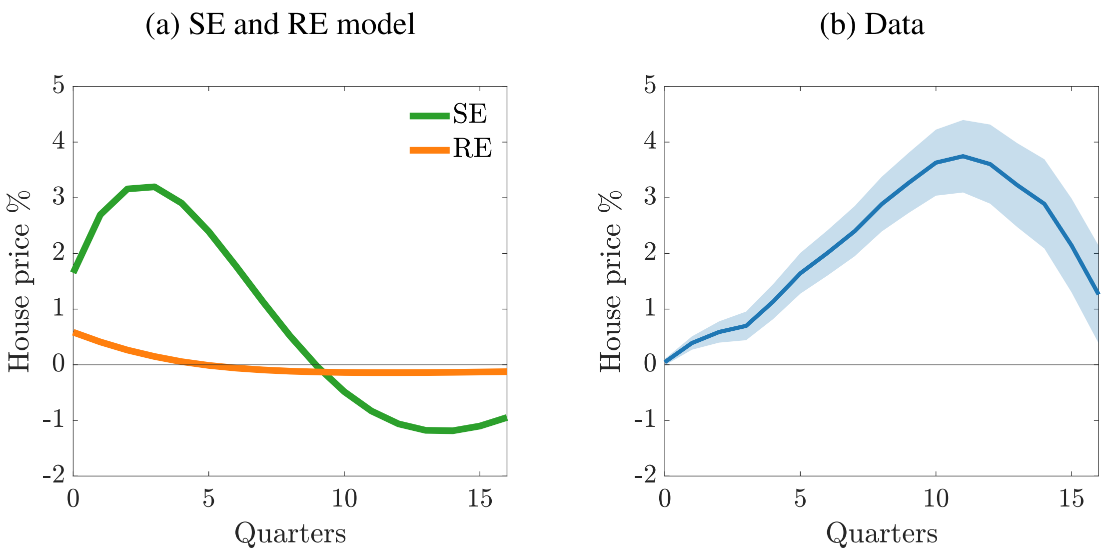
Figure 10: Peak response of house prices to a monetary policy shock: model vs. data



Notes: Response to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newy-West); Paramterization: $\varrho^h = 0.97$, $g^h = 0.005$ (boom), $\varrho^l = 0.97$, $g^l = 0.028$ (bust).

We will now turn to the dynamic responses in the model and the data. Figure (11) shows the response of house prices to a 25 basis point expansionary monetary policy shock in the model and in the data unconditional on booms and busts. The empirical response is depicted in panel (b). The model counterparts, in panel (a), show the response of the linear model under subjective beliefs and rational expectations. In terms of dynamics we observe a very sluggish response in the data, the peak response is only reached after 10 quarters. In the rational expectations model, we observe no hump shaped pattern at all. House prices peak on impact and return to the steady-state thereafter. In the subjective beliefs model we do observe some sluggishness in the house price dynamics. House prices respond on impact, increase for the first four quarters, after which the model converges back to its' steady-state. The dynamic can be explained through the extrapolative belief structure. After the initial shock, agents believe that house prices will increase further and invest into housing. They will continue to do so until their beliefs are not met, after which they will adjust their behaviour and the model returns to it's steady-state. Quantitatively, it turns out that this behaviour is too short-lived to explain the sluggishness observed in the data. The inability of DSGE models to match the persistence in the data is well known. Therefore, medium sized DSGE models such as Smets and Wouters (2007) tend to add backward looking components, for instance habit formation, to match these dynamics. While the extrapolative beliefs do improve the model performance to a certain degree, they cannot capture the full persistence of the response. This indicates that some sort of sluggish adjustment behaviour in housing demand, for instance habit formation on housing or housing search frictions, are needed to exactly match the data.

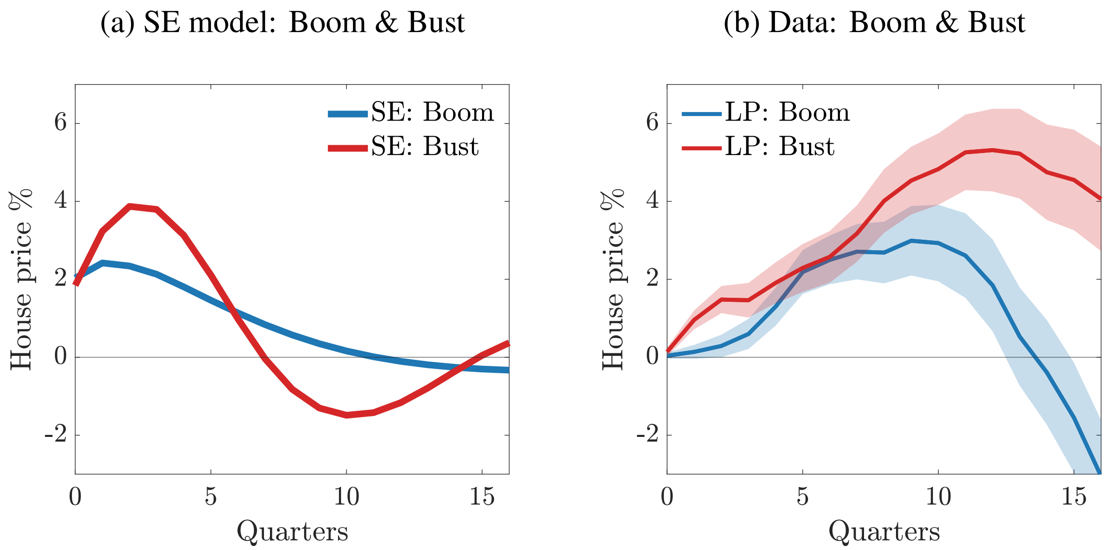
Figure 11: House price response to monetary policy shock, model vs. data



Notes: Responses to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newy-West).

We will now turn to house prices responses in booms and busts. Panel (b) in Figure (12) plots the empirically estimated impulse responses from the local projections exercise. The dynamics are equivalent to the ones depicted in Figure (1), but scaled to match a 25 basis point increase. We only plot 68% confidence bands. Panel (a) shows the subjective beliefs model responses for the boom and bust parameterization discussed above. In terms of dynamics we observe the same short-comings as in the unconditional case. The model is unable to capture the full persistence observed in the data. We however do observe a less persistent response in the boom relative to the bust. The model is able to capture this pattern qualitatively.

Figure 12: House price response to monetary policy shock in booms and busts, model vs. data



Notes: Responses to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newy-West); Paramterization: $\varrho^h = 0.97$, $g^h = 0.005$ (boom), $\varrho^l = 0.97$, $g^l = 0.028$ (bust).

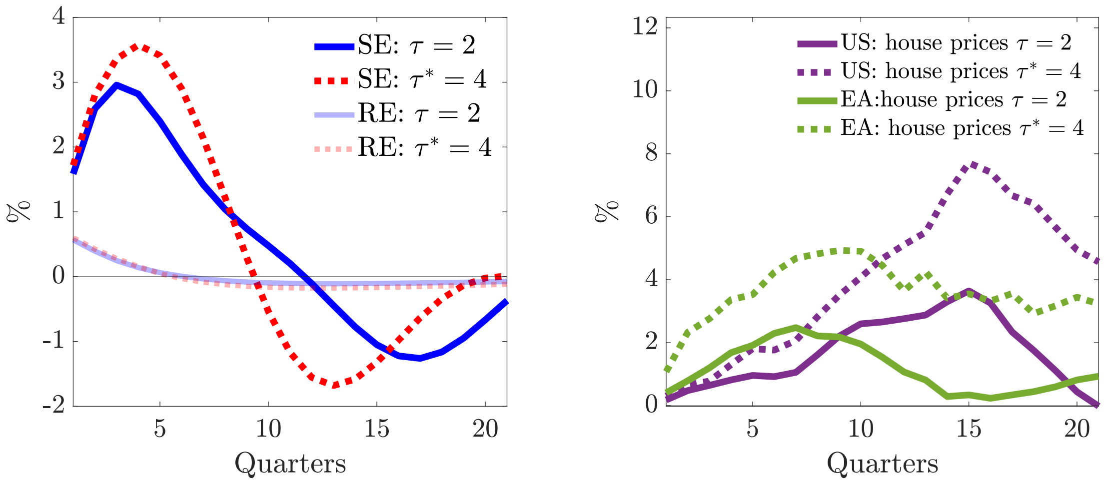
IV.C Cross-regional heterogeneity

Unconditional cross-regional heterogeneity. We will start by evaluating the model performance with respect to cross-regional differences without conditioning on booms and busts. To do so we compute the linear model with time-to-build $\tau = 2$ and $\tau^* = 4$. We use the local projections from section (II.B) to compute empirical counterparts. For the Euro Area, we can directly use the interaction term to compute empirical counterparts. For the US we use the correlation of the house price sensitivity indicators and time-to-build on a census division level (Appendix (A.4)) to compute the appropriate empirical counterparts. Figure (13) plots the response of house prices to a 25 basis points expansionary monetary policy shock. In panel (a) we show the responses of the subjective beliefs and the rational expectations model for both regions. Panel (b) plots the local projections for time-to-build 2 and 4, both for the US and the Euro Area. First, one can note that the rational expectations model is unable to generate sizeable cross-regional differences. The subjective beliefs model on the other hand does create sizeable differences between regions. The more supply-constrained region is more responsive to the shock. This is qualitatively in line with what we find in the US and the Euro Area. In terms of quantitative differences, the model however seems to under-predict cross-regional differences, as for instance the differences between regions at the peaks are more pronounced in the data than in the model. Similarly, the dynamics in terms of persistence can not be matched as we discussed already above.

Figure 13: House price response to monetary policy shock, model vs. data

(a) SE and RE model

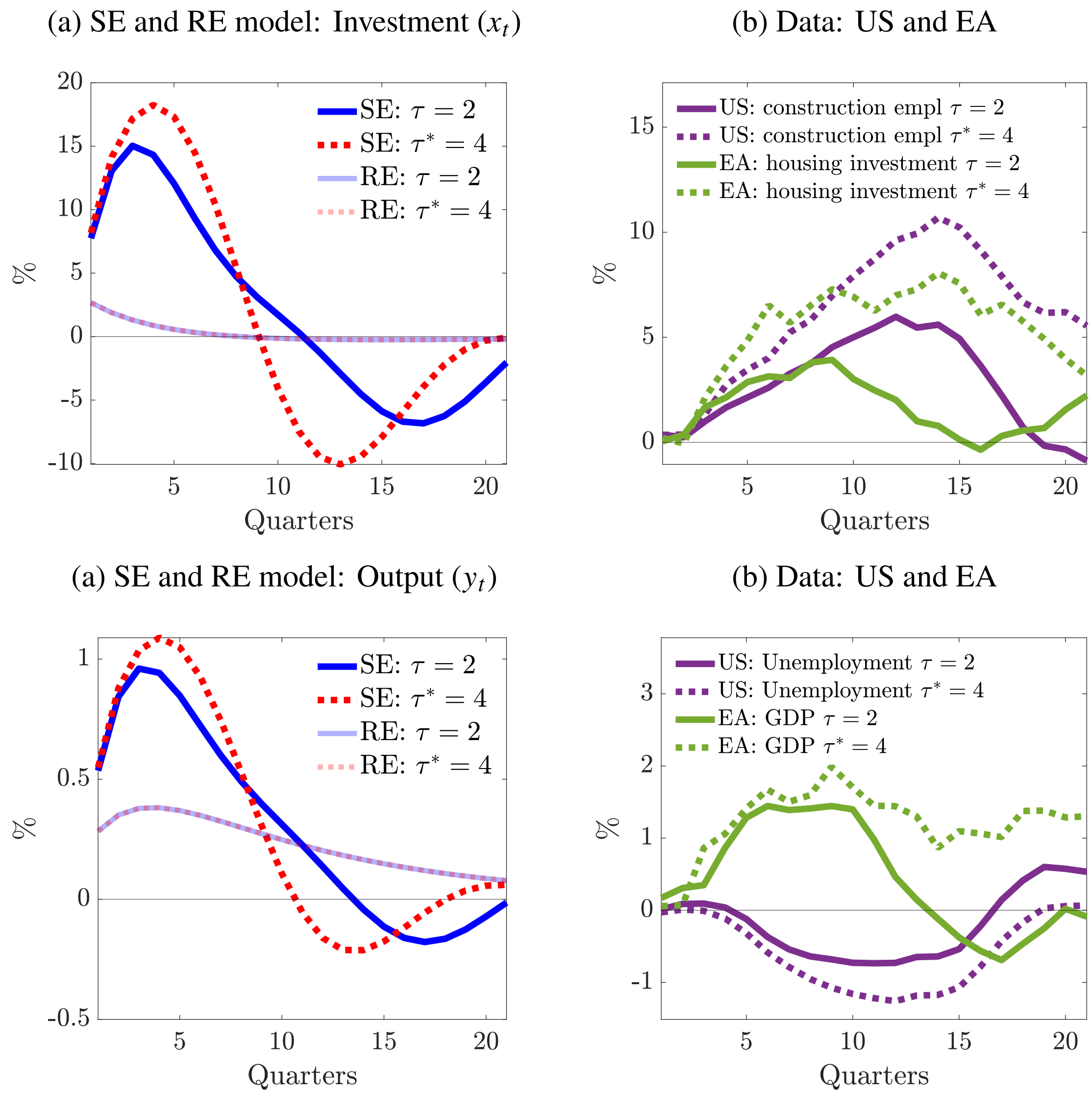
(b) Data: US and EA



Notes: Responses to expansionary MP shock (25 bp).

Moving on to economic activity, we find similar results. Figure (14) shows that housing investment and output are more responsive in more supply-constrained regions. As house prices are more responsive in more supply-constrained regions, households extrapolate a relatively higher house prices into the future. In order to make capital gains, they will increase housing investment by a higher degree compared to their more inelastic neighbours. This eventually drives output as well. This pattern is qualitatively in line with the empirical findings. On the quantitative side, however, we find that the cross-regional differences in investment and output in the model can not match their empirical counterparts. In the construction sector, employment in construction in the US and residential investment in the Euro Area, experience larger regional differences than housing investment in the model. The same holds for the aggregate economy, as unemployment differences in the US, and GDP differences in the Euro Area are more pronounced than their counterparts in the model. For the rational expectations model, we do not find any regional differences. In summary, the subjective belief model matches qualitatively all regional dynamics observed in the data. The rational expectations model matches none of them.

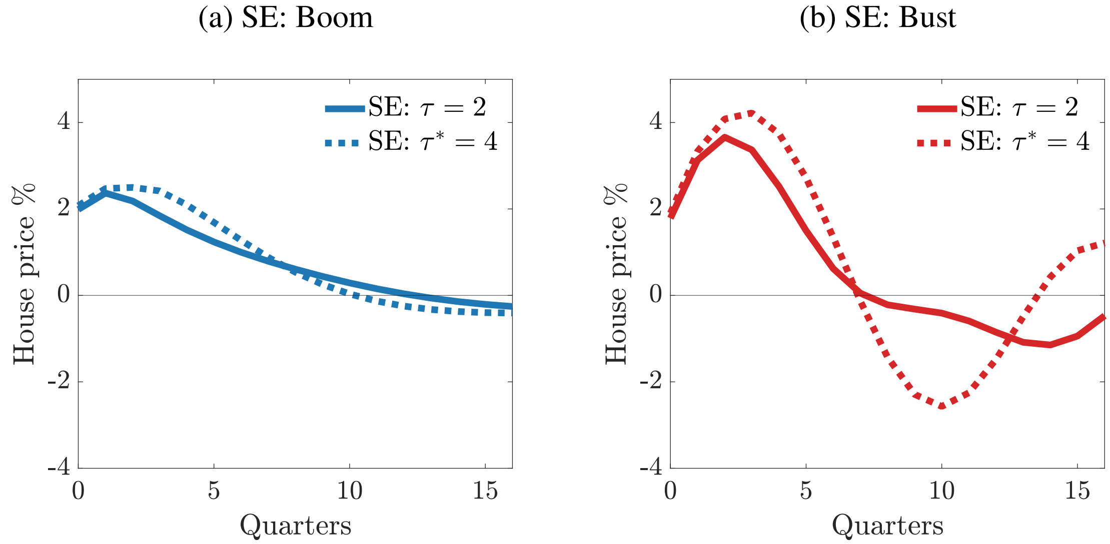
Figure 14: Economic activity response to monetary policy shock, model vs. data



Notes: Responses to expansionary MP shock (25 bp).

Cross-regional heterogeneity in booms and busts. To study cross-regional heterogeneity in booms and busts, we return to our model specification from Section (IV.B). Hence, we calibrate the linear model to match boom and bust house price belief updating. Figure (15) plots the results. The Figure clearly shows that house prices across regions are much more heterogeneous in busts compared to booms. The peak responses in the boom scenario, panel (a), are almost equivalent across regions. On the other hand, in the bust case, shown in panel (b), we do not only find a significant difference at the peak but also the dynamics more generally show large variations across regions.

Figure 15: House price response to monetary policy shock, model vs. data



Notes: Responses to expansionary MP shock (25 bp). Paramterization: $\varrho^h = 0.97$, $g^h = 0.005$ (boom), $\varrho^l = 0.97$, $g^l = 0.028$ (bust).

Unfortunately, running a comparable exercise in the data is not possible since, to the best of our knowledge, the required data is lacking. In section (II.C) we therefore relied on cross-regional standard deviations without conditioning on a specific shock. In principle we can do a similar exercise in the model by simulating data based on the realisations of shocks. Crucially, we need to allow the model to transition between boom and bust regimes. To do so we rely on a threshold model, where we allow for different house price updating processes when house prices are falling compared to when they are rising. More specifically, we substitute Equation (9) with Equation (11), and Equation (10) with Equation (12). To solve the model we rely on the "OccBin"-toolbox introduced by Guerrieri and Iacoviello (2015) for models with regime switches. As mentioned above, setting a too high Kalman gain renders the equilibrium dynamics unstable. This, in combination with the OccBin routines, prevents us from using the same parameters for the threshold model as we used above. To ensure convergence we reduce the boom/bust asymmetries in the Kalman gains calibrating them to the parameters we estimated in the short sample, see Appendix (A.2). We set $g^h = 0.011$ and $g^l = 0.021$. For the persistence parameter in the belief updating model we simply choose $\varrho^h = \varrho^l = 0.91$ in accordance with the linear model. In order to achieve comparability of the linear model with the threshold model, we choose a middle ground between the boom and bust Kalman gain for the linear model and set $g = 0.0155$. To simulate data we draw random shocks from a mean zero normal distribution and feed them into the model. We choose one supply shock, a productivity shock, and a demand shock, a consumption taste shock. Standard deviations of the shocks are chosen to ensure model convergence, and set to $\sigma^j = \frac{0.01}{8}$

with $j = c, a$ denoting the standard deviations for the preference and productivity shock. We do this exercise for the rational expectations model, the linear subjective beliefs model, and the threshold model using the same sequence of shocks. Importantly, as shock sizes and parameters in the belief updating process are not chosen in accordance with the data, this exercise can only provide qualitative insights. We simulate the models for 117 quarters, which is consistent with the sample for the US data.

Table 5: Cross-regional house price variation in booms and busts across models

$Model$	RE		SE^{lin}		SE^{th}	
	$mean$	$median$	$mean$	$median$	$mean$	$median$
$Full\ Sample$	0.0129	0.0107	0.1288	0.1144	0.0810	0.0625
$Boom:\ \Delta q_t^{agg} > 0$	0.0114	0.0100	0.1331	0.1158	0.0695	0.0589
$Bust:\ \Delta q_t^{agg} < 0$	0.0142	0.0121	0.1232	0.1095	0.0952	0.0715
$\frac{Bust}{Boom}$	1.24	1.21	0.93	0.95	1.37	1.21

Notes: The Table shows the mean and median estimates of cross-regional standard deviations of house prices. Paramterization linear model: $\varrho = 0.91$, $g = 0.0155$; Paramterization for threshold model: $\varrho^h = 0.91$, $g^h = 0.011$ (boom), $\varrho^l = 0.91$, $g^l = 0.020$ (bust).

Our analysis in section (II.C) suggests that we should expect to find larger cross-regional standard deviations in times of busts compared to booms. Table (5) shows the cross-regional standard deviations for the different model types. First, one can notice that the rational expectations model can not generate any sizeable cross-regional variation. Given our previous results, this is no surprise. In the subjective beliefs models the cross-regional variation increases by a factor of around 6 – 10 for the mean estimates. In the linear subjective beliefs model we find no large difference between variation in booms compared to busts. If anything we find larger regional variation in booms compared to busts. This is purely by chance due to the randomness in the shock sequences. The threshold model does create an increase in cross-regional variation in busts relative to booms. We find a 37% increase in the mean standard deviations and a 21% increase in the median. The counterparts in the data are 23% for the mean and 26% for the median. We can therefore conclude, that not only are subjective beliefs essential to generate sizeable cross-regional differences, but also asymmetries in the house price belief updating process are key in understanding cross-regional heterogeneities in booms and busts.

V. Policy analysis

Turning to policy implications, we study how targeting house prices in the Taylor rule changes aggregate variation and cross-regional variation of inflation, output, and house price. The linearized Taylor rule we consider is given by:

$$i_t = \phi^{\pi} \pi_t^{agg} + \phi^{q} q_t^{agg} \tag{17}$$

For our baseline case we consider a rule with $\phi_{\pi} = 1.5$ and $\phi_q = 0$, as we did for all exercises above. Under house price targeting, we increase the weight on house prices to $\phi_q = 0.25$. We will study the response of the rational expectations, the linear subjective beliefs, and the threshold subjective beliefs model to an expansionary and a contractionary 100 basis points productivity shock. The model parameterizations are the same as the ones used in the simulation exercise. We measure variations and cross-regional variations by adding up the standard deviations of the expansionary and contractionary shock. Table (6) shows the aggregate standard deviations of inflation, output, and house prices to the productivity shocks. We find that increasing the weight on house prices leads to a trade-off between inflation on the one hand, and output and house prices on the other. This pattern holds across all models, although it is less pronounced in the rational expectations model. The logic is as follows: A higher weight on house prices increases volatility in inflation but reduces volatility in output and house prices. A higher weight on house prices means that the policy rule becomes relatively less responsive to inflation. Consequently inflation increases. The same argument holds for house prices but in the opposite direction. A higher weight on house prices reduces volatility in house prices. Output volatility is decreasing in the house price weight because house prices drive housing investment and therefore aggregate activity.

Table 6: Aggregate standard deviations across policy rules

$Model$	ϕ^{π}	ϕ^{q}	π^{agg}	y^{agg}	q^{agg}
RE	1.5	0.0	0.0199	0.1603	0.1363
	1.5	0.25	0.0273	0.1207	0.0691
SE^{lin}	1.5	0.0	0.0397	0.3085	0.9653
	1.5	0.25	0.0707	0.0803	0.1720
SE^{th}	1.5	0.0	0.0398	0.3057	0.9531
	1.5	0.25	0.0714	0.0793	0.1720

Notes: The table shows standard deviations to an expansionary and contractionary 100 basis points productivity shock. Paramterization linear model: $\varrho = 0.91$, $g = 0.0155$; Paramterization for threshold model: $\varrho^{h} = 0.91$, $g^{h} = 0.011$ (boom), $\varrho^{l} = 0.91$, $g^{l} = 0.020$ (bust).

Next, we will focus on how a higher weight on house prices affects cross-regional variations accounting for boom-bust periods. We compute cross-regional standard deviations in the same manner as we have done above. Since we are looking at two isolated shocks, we define a bust period, as a period where house prices are below its' steady state value. Table (7) shows the results. We find that a higher weight on house prices reduces cross-regional variation for all variables across all models. Intuitively, the only source of heterogeneity is on the housing supply side affecting the economy through house prices. Targeting house prices mitigates the channel through which the heterogeneity affects the economy. Focusing on the rational expectations model first, we document very small cross-region variation and targeting house prices has only a limited effect. Also, due to the linearity of the model, variations across booms and busts are identical. Second, the linear subjective beliefs model experiences much larger regional variations. Targeting house prices therefore is more potent. Similar to the rational expectations model, linearity does not lead to differences in variation across booms and busts. Finally, focusing on the threshold subjective beliefs model, we find that the model can generate sizeable cross-region variation. Targeting house prices in this model is again relatively powerful. We also find differences across booms and busts. If house prices are not targeted, cross-regional variation is much larger in times of busts. Targeting house prices mitigates, and in our example eliminates, boom-bust differences. The intuition is similar to above: extrapolation amplifies movements in house prices which spill over to the rest of the economy. If house prices are not as volatile, extrapolation is also mitigated, and so is the differential response across booms and busts. This eventually leads to smaller cross-regional variation. To conclude, we find that targeting house prices reduces aggregate volatility in output and house prices as well as cross-regional variance more generally. This comes at the cost of higher aggregate inflation volatility.

Table 7: Cross-region standard deviations across policy rules and booms and bust

			Boom			*Bust*		
Model	ϕ^{π}	ϕ^{q}	$\pi^{H,F}$	$y^{H,F}$	$q^{H,F}$	$\pi^{H,F}$	$y^{H,F}$	$q^{H,F}$
RE	1.5	0.0	$7.51e^{-15}$	$3.29e^{-13}$	0.0108	$7.51e^{-15}$	$3.29e^{-13}$	0.0108
	1.5	0.25	$3.77e^{-15}$	$1.19e^{-13}$	0.0147	$3.77e^{-15}$	$1.19e^{-13}$	0.0147
SE^{lin}	1.5	0.0	0.0029	0.0672	0.3589	0.0029	0.0672	0.3589
	1.5	0.25	$7.81e^{-5}$	0.0100	0.0512	$7.81e^{-5}$	0.0100	0.0512
SE^{th}	1.5	0.0	0.0015	0.0332	0.1813	0.0040	0.0919	0.4899
	1.5	0.25	$2.85e^{-4}$	0.0074	0.0378	$2.85e^{-4}$	0.0074	0.0378

Notes: The table shows cross-regional standard deviations to an expansionary and contractionary 100 basis points productivity shock. Paramterization linear model: $\varrho = 0.91$, $g = 0.0155$; Paramterization for threshold model: $\varrho^h = 0.91$, $g^h = 0.011$ (boom), $\varrho^l = 0.91$, $g^l = 0.020$ (bust).

VI. Conclusion

In this paper, we examine regional heterogeneity in housing cycles and their implications for monetary policy. We document two novel facts. First, house price belief updating is more pronounced during busts. Second, heterogeneity in housing supply elasticities results in differential house price responsiveness across regions. These two facts, when considered together, can explain the observed higher cross-regional variation in house prices during bust episodes. We then develop a two-region New Keynesian model that aligns with the empirically documented facts. To solve the model, we introduce a novel solution method. Our findings suggest that placing a greater emphasis on house prices by the monetary authority can reduce volatility in output and house prices, as well as cross-regional variation in inflation, output, and house prices. However, this comes at the cost of increased aggregate inflation variability.

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Appendix

A. Empirical part

A.1 Data sources

US data. For the monthly house price data on the federal level we use the SP CoreLogic Case-Shiller U.S. National Home Price Index seasonally adjusted from the FRED database (CSUSHPISA). The FFR (FEDFUNDS), industrial production (INDPRO), and CPI (CPIAUCSL) on a monthly frequency are also taken from the FRED. The house price expectations data are the mean of the expected change in home value during the next year taken from the Survey of Consumers by the Michigan University. For the quarterly state level data, we use all transactions house price indices ("State code"STHPI) from the FRED and seasonally adjust them. The remaining state level data on building permits ("State code"BPPRIVSA), employment in construction ("Stade code"CONS), unemployment ("Stade code"UR), and employment in retail are all taken from the FRED and already seasonally adjusted. We omit the code for employment in retail, as these vary across states. GDP (GDP) and the GDP deflator (GDPDEF) are also obtained from the FRED. As already mentioned in the main text, the house price sensitivity indicator is taken from Guren et al. (2021), and the monetary policy shock from Bauer and Swanson (2023). When aggregating the shocks from a monthly to a quarterly frequency we weigh the shocks according to the time they occurred in the quarter. Giving a higher weight to shocks that occured at the beginning of the quarter. Hence, the weights are $1, \frac{2}{3}, \frac{1}{3}$. This aggregation method is in line with Gertler and Karadi (2015) and Almgren et al. (2022).

Euro Area data. For the Euro Area we obtain the real residential property prices from the BIS. For housing investment we use real fixed capital formation in dwellings (namq_10_an6) from Eurostat. Building permits (sts_cobp_q), real GDP (namq_10_gdp), unemployment (une_rt_q_h), HICP (prc_hicp_midx), and the EONIA (irt_st_m) are also obtained from Eurostat. All series, except the EONIA, are seasonally adjusted. As already mentioned in the main text, the time to obtain a building permit is taken from the World Bank database. And the monetary policy shcks are taken from Altavilla et al. (2019). For these, we use the same aggregation method as for the US data.

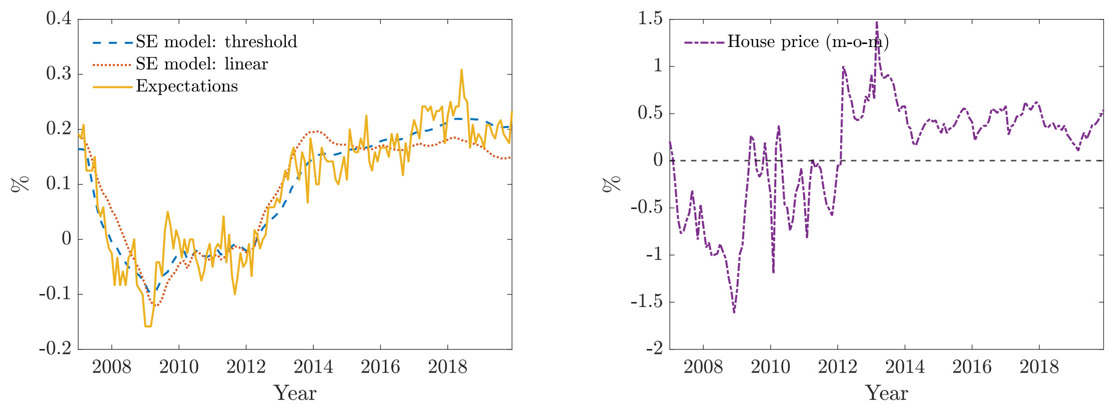
A.2 Estimation of belief model: short sample

Figure (A.1) plots the fitted values of the linear belief and the threshold model, equations (3) and (4) respectively, if one cuts the sample before the COVID episode. By doing this we implicitly ignore the episodes containing excessive volatility in house prices and house price expectations induced by COVID and the FED increasing the policy rate. Qualitatively, our results are in line with our baseline specification. We find however, the the differences in ρ^l and ρ^h, but more even more in g^l and g^h are reduced (see table (8)). As explained in the main text this results is intuitive as we do not need to match the high volatility episodes in the end of the sample. We also find that the estimated parameters seem quantitatively more realistic.

Figure A.1: House price belief model, US

(a) Expectations

(b) House price growth



Notes: SE model, threshold (blue, dashed): fitted values of equation (4); SE model, linear (red,dotted): fitted values of equation (3); Expectations (yellow, solid): mean expectations data from the Michigan Survey; House price (m-o-m) (purple, dashed-dotted): Month on month percentage change in house price.

Table 8: House price belief model: parameters, short sample

$Specification$	ρ^{lin}	ρ^h	ρ^l	g^{lin}	g^h	g^l	$Threshold$	MSE^i/MSE^{lin}
$linear$	0.95			0.018				1.0
$threshold$		0.99	0.85		0.011	0.021	0.001	0.55

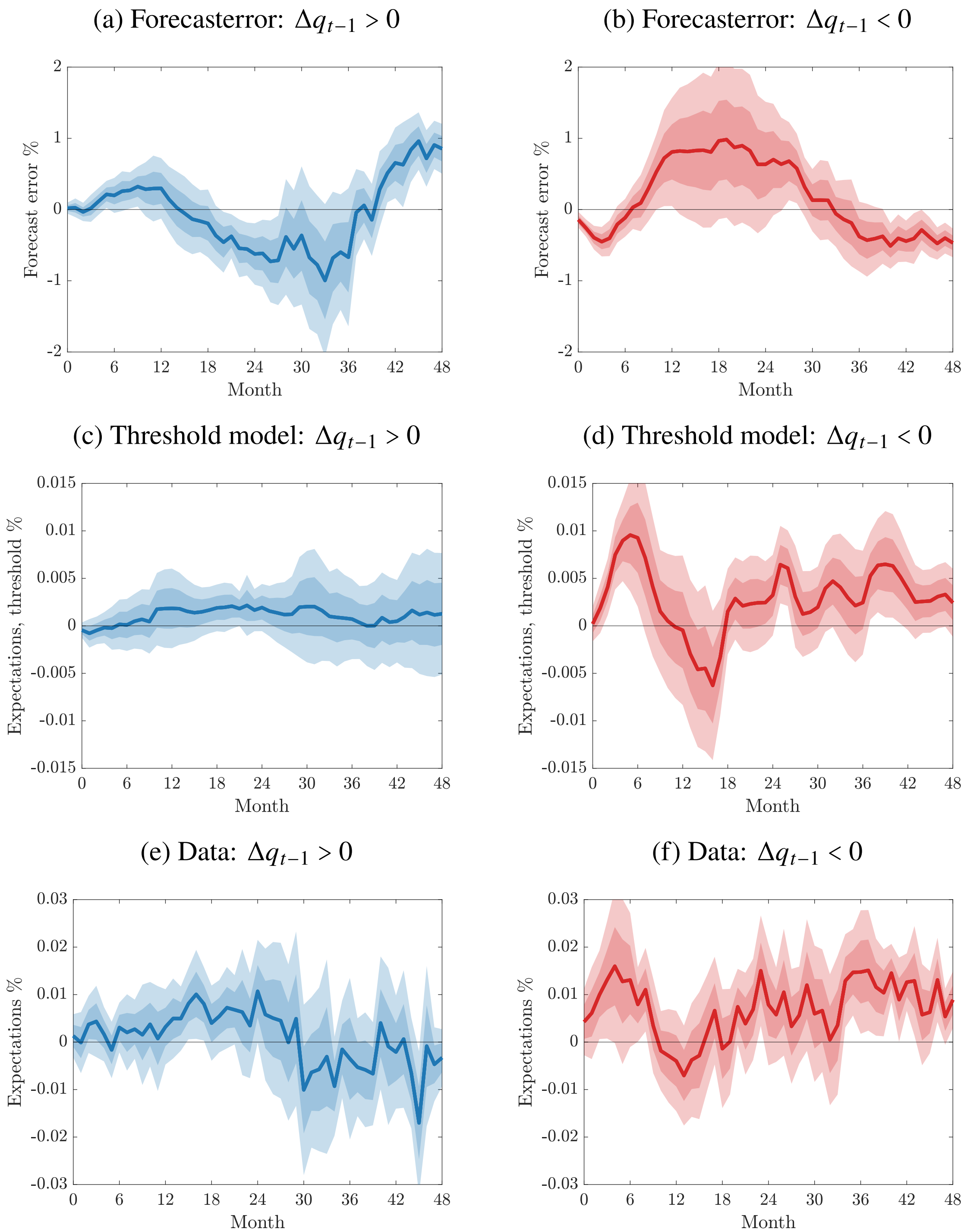
Notes: linear: estimated parameters from equation (3); threshold: estimated parameters from (4).

A.3 Response of house price expectations to a monetary policy shock in booms and busts

In the following we estimates equation (1) for house price expectations and forecast errors.

In the first specification we the forecast errors as a left-hand side variable. in the second specification, we use the fitted values of the threshold model and backcast the expectations until 1990. This allows us to use the whole sample of monetary policy shocks and largely extents our sample size (1990-2019). As a third specification, we use the actual house price expectation from the data, which spans the period from 2007 to 2019. As controls we only include the 12 lags of the left-hand side variable and the shocks. The frequency is monthly. Figure (A.2) plots the results. First, we notice, that forecast errors react differently in booms relative to busts. Second, for the threshold model we see that in busts expectations reply much faster compared to booms. This is exactly what theory would suggest. For the actual expectations data we see the same pattern, although the data is much more noisy.

Figure A.2: House price expectations in response to monetary policy shock, boom-bust

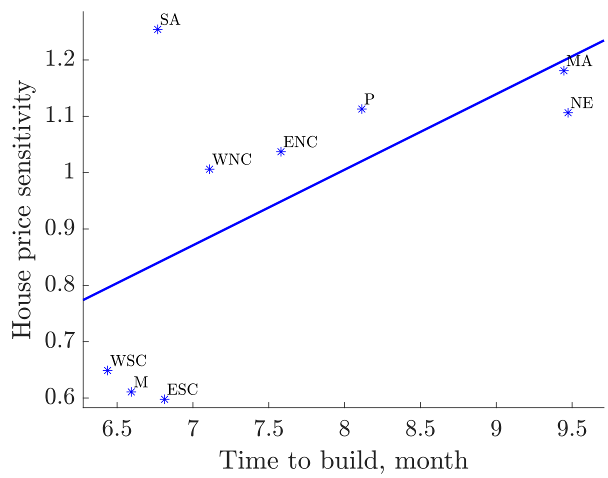


Notes: Responses to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% (Newy-West).

A.4 Time-to-build in the US

The time it takes to obtain a building permit, or generally the time it takes to build a house, is not available for the US on a state level. However, he Census Bureau documents construction times for homes on a census division level. Below we aggregated up the housing sensitivity indicator from Guren et al. (2021) to a census division level and correlate them with the construction time of housing measured from date of authorization to completion. One can see in figure (A.3) that they are clearly positively correlated.

Figure A.3: Correlation of house price sensitivity indicator with time-to-build



Correlation of house price sensitivity indicator from Guren et al. (2021) with time-to-build a house across census divisions. Time-to-build is measured from the date of the authorization until the completion of the building activities.

B. Micro-founding the debt-elastic interest rate

In the model, households in country H receive on their bond holdings the effective nominal interest rate $1+i_{t-1}-\psi b_t$, with b_t being the real value of the aggregate bond holding in country H; households in country F receive the effective nominal rate $1+i_{t-1}-\psi b_t^*$. Moreover, the intermediation of bond positions entails a real cost $\gamma(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2+(1-\gamma)(1+\pi_t)^{-1}\frac{\psi}{2}(b_t^*)^2$ of which $(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ is paid by each consumer in H and $(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ is paid by each consumer in F. In this Appendix we detail how these debt-elastic interest rates and the associated intermediation cost can be parsimoniously micro-founded. We achieve this by introducing two competitive bond clearing houses, one in each country, that represent the only access of households to financial markets and who incur a real cost that is quadratic in the size of their balance sheet. The specific market arrangement is as follows: households hold a consol and may hold liquid bonds.

Consol. Each household in H is endowed with $\bar{b} \in \mathbb{R}$ units of a non-marketable consol[25] that pays as a coupon $(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})P_t^{-1}$ units of H's consumption basket each period, per unit of consol. This implies that the nominal coupon rate, applied to the nominal coupon value $P_{t-1}\bar{b}$, is $(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})/P_{t-1}$; the real coupon rate applied to the real value $\bar{b}$ in turn is $(\beta^{-1}-1)\left(\gamma+(1-\gamma)\frac{P^*_{t-1}}{P_{t-1}}\right)(1+\pi_t)^{-1}$. The situation in country F is symmetric: each household is endowed with $\bar{b}^*$ units of a consol that pays $(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})(P^*_t)^{-1}$ units of F's consumption basket each period, per unit of consol. $\bar{b}$, $\bar{b}^*$ are model parameters selected such that (i) $\gamma\bar{b}+(1-\gamma)\bar{b}^*=0$ and (ii) all markets clear in the non-stochastic steady state with zero net inflation and terms-of-trade parity *without* households holding any liquid bonds. The latter fact ensures that there is no cost of financial intermediation in the steady state, shutting down this particular friction. The specific choice of the coupon payment scheme ensures two facts: (1) condition (i) implies that the nominal payments between H and F associated with the two consols exactly cancel out – whatever H/F receives as coupon payments on its consol endowment is paid for by F/H as a coupon service on its (endowed) short position of consol; and (2) the real coupon rates paid by/ to the consol endowment only depend on the real exchange rate and the inflation rates, not on the price levels. Households cannot trade their consol holdings.

Bonds. Household do have the possibility, though, to vary their position in the liquid bond. This liquid bond is a nominal, one-period, zero-coupon bond and the positions of the representative H-, respectively the representative F-household are denominated b_t, b^*_t. If a household wants to hold a net balance of liquid bonds different from zero, she has to go to one of the clearing houses in her country: In the H-country, there is a continuum of mass γ (respectively mass $1-\gamma$ in F) of competitive clearing houses buying and selling bonds from and to the government and from and to the respective country's citizens. Households themselves cannot directly buy/sell government bonds without having an account at the clearing house. The clearing house can costlessly buy/sell bonds but incurs an operating cost that is quadratic in the size of its balance sheet, making this a model of costly financial intermediation. Thus, the interest rate that each citizen gets on her bond holdings is determined by the nominal rate paid on government bonds and the aggregate holding of liquid bonds. Each clearing house is owned equally by all citizens of the respective country so that it pays its profits to those citizens.[26] Consider an arbitrary clearing house in H

[25] A consol is a type of bond that has infinite maturity and just keeps paying a constant or varying coupon perpetually.

[26] In equilibrium, each clearing house makes a non-negative profit, and along the transition path back to the steady state after some shock, each clearing house makes a strictly positive profit. This fact is in principle incompatible

(with symmetric arrangements in F). Denoting as $B_{c,t+1}$ the nominal value of the clearing house's net liabilities against H's citizens and as $B_{g,t+1}$ the nominal value of the clearing house's position in the government bond, the profit maximization program is:

$$\max_{B_{c,t+1},B_{g,t+1}\in\mathbb{R}} -(1+i_t^b)B_{c,t+1} + (1+i_t)B_{g,t+1} - \frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2, \text{ s.t. } B_{c,t+1} = B_{g,t+1}$$

where i_t^b is the nominal rate clearing the market for household bond positions and i_t is the nominal rate on government bonds that is set by the monetary authority. $\frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2 = \frac{\psi}{2}P_t(b_{c,t+1})^2$ is the nominal cost of intermediating – crucially, the real cost of intermediation does not directly depend on the price level. The first order conditions for this program are

$$\begin{aligned} 1+i_t^b+\psi P_t^{-1}B_{c,t+1} &= \mu_t, \\ 1+i_t &= \mu_t, \\ B_{c,t+1} &= B_{g,t+1}, \end{aligned}$$

where μ_t is the Lagrange multiplier on the balance-sheet constraint $B_{c,t+1} = B_{g,t+1}$. Market clearing in the household bond positions in H requires

$$\gamma B_{c,t+1} = \gamma P_t b_{t+1},$$

and market clearing in the government bond positions requires

$$\gamma B_{g,t+1} + (1-\gamma)B^*_{g,t+1} = 0,$$

so that by using the balance-sheet constraints $B_{c,t+1} = B_{g,t+1}, B^*_{c,t+1} = B^*_{g,t+1}$ and the clearing conditions for household bond positions in H and F we recover the market clearing condition for government bonds in the main model:

$$\gamma P_t b_{t+1} + (1-\gamma)P^*_t b^*_{t+1} = 0.$$

with the notion of competitiveness (there is an incentive to open up more clearing houses or, equivalently, it is strictly profitable to split each clearing house). Therefore, it is better to interpret the program of the clearing house as reflecting capacity constraints: the here-presented program can be thought of as the inner problem of a profit maximization program with an additional factor (say, managerial effort) that which (i) makes the intermediation service production function exhibit constant returns to scale (instead of decreasing RTS), (ii) is provided by households, and (iii) is in perfectly inelastic supply. Under this way of modeling the clearing house, it behaves exactly as modeled here, it always makes zero profits, and households get as remuneration for providing the additional factor the amount that is the profit in the current way of modeling.

In sum, the aggregate conditions implied by this market arrangement are:

$$
\begin{aligned}
1 + i_t^b + \psi b_{t+1} &= 1 + i_t, \\
1 + i_t^{b,*} + \psi b_{t+1}^* &= 1 + i_t, \\
\gamma P_t b_{t+1} + (1-\gamma) P_t^* b_{t+1}^* &= 0.
\end{aligned}
$$

The nominal profits of the typical clearing house in H in equilibrium are:

$$
\begin{aligned}
\text{Profit}_{t+1} &= (i_t - i_t^b) B_{c,t+1} - \frac{\psi}{2} P_t^{-1} (B_{c,t+1})^2 \quad \text{with optimal } B_{c,t+1} = \frac{i_t - i_t^b}{\psi} P_t \\
&= (i_t - i_t^b)^2 \psi^{-1} P_t - (i_t - i_t^b)^2 \psi^{-1} P_t \cdot \frac{1}{2} \\
&= P_t \frac{\psi}{2} (b_{t+1})^2 \quad \text{using market clearing in the household bond positions.}
\end{aligned}
$$

Of the $1 + i_t\%$ nominal interest collected on (paid for) its position of government bonds, each clearing house withholds $\psi b_{t+1}\%$ of the interest from its customers (respectively, charges $-\psi b_{t+1}\%$ of additional interest if $b_{t+1} < 0$). Half of these $\psi b_{t+1}\%$ are used for covering the operating cost (by buying this amount of H's final basket and selling it in exchange for numéraire), and the other half is paid as profit to the owners of the clearing house (which, in equilibrium, are its customers).

C. Proofs and Derivations

C.1 Derivation of Equation (8)

Equation (8) is the result of the following calculations:

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \frac{q_{t+s}}{q_t} = q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\ln q_{t+s} - \ln q_t \right) = q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\sum_{n=1}^{s} \Delta \ln q_{t+n} \right) \\
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\sum_{n=1}^{s} \ln m_{t+n} \right) \cdot \underbrace{\mathbb{E}_t^{\mathcal{P}} \left[\prod_{n=1}^{s} e_{t+n} \right]}_{= \prod_n \mathbb{E}_t^{\mathcal{P}} e_{t+n} = 1} \\
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\sum_{n=1}^{s} \left[\sum_{j=0}^{n-1} \varrho^j \ln v_{t+n-j} + \varrho^n \ln m_t \right] \right)
\end{aligned}
$$

$$
\begin{aligned}
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\ln m_t \cdot \sum_{n=1}^{s} \varrho^n \right) \cdot \mathbb{E}_t^{\mathcal{P}} \exp\Bigg(\underbrace{\sum_{n=1}^{s} \sum_{j=0}^{n-1} \varrho^j \ln v_{t+n-j}}_{\sim\mathcal{N}} \Bigg) \\
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\varrho \frac{1-\varrho^s}{1-\varrho} \ln m_t \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \\
\iff \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2 \big(\varrho \tfrac{1-\varrho^s}{1-\varrho} \big)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2
\end{aligned}
$$

C.2 Walras' law and Balance-of-Payments

C.2.1 *Walras' law*

To make sure the economics of the model with home bias checks out, we prove that Walras' law holds in our model economy.

We start by providing a list of all market clearing conditions, household budget constraints, and relevant variable definitions (assuming $T_t = T_t^* = 0$), with equations involving more than one good being in nominal terms (i.e. units of union-wide currency). In doing so, we make use of the micro-foundation of the debt-elastic interest rate rule that is presented in Appendix B. The set of market clearing-, profit-, and budget-conditions is:

$$
\begin{aligned}
(GMC) \quad & \left(1-\frac{\kappa}{2}\pi_{H,t}^2\right) y_{H,t}\gamma = \gamma \mathrm{y}_{H,t} + (1-\gamma)\mathrm{y}_{H,t}^*, \\
(GMC^*) \quad & \left(1-\frac{\kappa}{2}\pi_{F,t}^2\right) y_{F,t}^*(1-\gamma) = \gamma \mathrm{y}_{F,t} + (1-\gamma)\mathrm{y}_{F,t}^*, \\
(HMC) \quad & H(x_{t-\tau},\xi_{t-\tau}) = h_t - (1-\delta)h_{t-1}, \\
(HMC^*) \quad & H(x_{t-\tau}^*,\xi_{t-\tau}) = h_t^* - (1-\delta)h_{t-1}^*, \\
(B) \quad & \gamma P_t b_{t+1} + (1-\gamma)P_t^* b_{t+1} = 0, \\
(BC) \quad & (c_{H,t}+x_{H,t})P_{H,t} + (c_{F,t}+x_{F,t})P_{F,t} + P_t q_t(h_t-(1-\delta)h_{t-1}) + P_t b_{t+1} = W_t n_t \\
& + (1+i_{t-1}-\psi b_t)P_{t-1}b_t + H(x_{t-\tau},\xi_{t-\tau})\cdot P_t q_t + P_t\Sigma_t + (\beta^{-1}-1)\tfrac{\gamma P_{t-1}+(1-\gamma)P_{t-1}^*}{P_{t-1}}P_{t-1}\bar{b}, \\
(BC^*) \quad & (c_{H,t}^*+x_{H,t}^*)P_{H,t} + (c_{F,t}^*+x_{F,t}^*)P_{F,t} + P_t^* q_t^*(h_t^*-(1-\delta)h_{t-1}^*) + P_t^* b_{t+1}^* = W_t^* n_t^* \\
& + (1+i_{t-1}-\psi b_t^*)P_{t-1}^* b_t^* + H(x_{t-\tau}^*,\xi_{t-\tau})\cdot P_t^* q_t^* + P_t^*\Sigma_t^* + (\beta^{-1}-1)\tfrac{\gamma P_{t-1}+(1-\gamma)P_{t-1}^*}{P_{t-1}^*}P_{t-1}^*\bar{b}^*, \\
(\Sigma) \quad & P_t\Sigma_t = P_{H,t}\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t - W_t n_t + P_{t-1}\frac{\psi}{2}[b_t]^2, \\
(\Sigma^*) \quad & P_t^*\Sigma_t^* = P_{F,t}\left(1-\frac{\kappa}{2}\pi_{F,t}^2\right)y_t^* - W_t^* n_t^* + P_{t-1}^*\frac{\psi}{2}[b_t^*]^2.
\end{aligned} \tag{C.1}
$$

These are 9 conditions – Walras' law now asserts that any one of these nine conditions should be obtainable through the summation of the remaining eight conditions. We show that the collective of all equations, except for (B), implies equation (B). First, plug (HMC) and (Σ) into (BC) to get

$$
\begin{aligned}
{\scriptstyle (HMC\&\Sigma\&BC)} \quad & (c_{H,t}+x_{H,t})P_{H,t} + (c_{F,t}+x_{F,t})P_{F,t} + P_t b_{t+1} = P_{H,t}\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t \\
& + (1+i_{t-1}-\psi b_t)P_{t-1}b_t + P_{t-1}\frac{\psi}{2}[b_t]^2 + (\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P_{t-1}^*)\bar{b} \\
& = P_{H,t}\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t + (1+i_{t-1})P_{t-1}b_t - P_{t-1}\frac{\psi}{2}[b_t]^2 \\
& + (\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P_{t-1}^*)\bar{b}
\end{aligned}
$$

$$\iff_{(HMC\&\Sigma\&BC)} \quad (c_{H,t}+x_{H,t})P_{H,t}+(c_{F,t}+x_{F,t})P_{F,t}+P_t\underbrace{(1+\pi_t)^{-1}\frac{\psi}{2}[b_t]^2}_{=\Psi_t}+P_tb_{t+1}=$$

$$(c_{H,t}+x_{H,t}+\Psi_{H,t})P_{H,t}+(c_{F,t}+x_{F,t}+\Psi_{F,t})P_{F,t}+P_tb_{t+1}=$$

$$P_{H,t}\left(1-\frac{\kappa}{2}\pi^2_{H,t}\right)y_t+(1+i_{t-1})P_{t-1}b_t+(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b}$$

$$\iff_{(HMC\&\Sigma\&BC)} \quad \gamma(\mathrm{y}_{H,t}P_{H,t}+\mathrm{y}_{F,t}P_{F,t}+P_tb_{t+1})=P_{H,t}\gamma\left(1-\frac{\kappa}{2}\pi^2_{H,t}\right)y_t$$

$$+(1+i_{t-1})P_{t-1}\gamma b_t+(\beta^{-1}-1)\gamma(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b}$$

$$\implies_{(HMC\&\Sigma\&BC\&GMC)} \quad \gamma(\mathrm{y}_{H,t}P_{H,t}+\mathrm{y}_{F,t}P_{F,t}+P_tb_{t+1})=P_{H,t}\left(\gamma\mathrm{y}_{H,t}+(1-\gamma)\mathrm{y}^*_{H,t}\right)$$

$$+(1+i_{t-1})P_{t-1}\gamma b_t+(\beta^{-1}-1)\gamma(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b},$$

and symmetric derivations deliver

$$(HMC^*\&\Sigma^*\&BC^*\&GMC^*) \quad (1-\gamma)(\mathrm{y}^*_{H,t}P_{H,t}+\mathrm{y}^*_{F,t}P_{F,t}+P^*_tb^*_{t+1})=P_{F,t}\left(\gamma\mathrm{y}_{F,t}+(1-\gamma)\mathrm{y}^*_{F,t}\right)$$

$$+(1+i_{t-1})P^*_{t-1}(1-\gamma)b^*_t+(\beta^{-1}-1)(1-\gamma)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b}^*.$$

Now adding the two equations gives:

$$\begin{aligned}&{}^{(HMC\&\Sigma\&BC\&GMC)+(HMC^*\&\Sigma^*\&BC^*\&GMC^*)}\\ &[\gamma P_tb_{t+1}+(1-\gamma)P^*_tb^*_{t+1}]-(1+i_{t-1})[\gamma P_{t-1}b_t+(1-\gamma)P^*_{t-1}b^*_t]\\ &-(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\underbrace{[\gamma\bar{b}+(1-\gamma)\bar{b}^*]}_{=0\text{ by choice of }\bar{b},\bar{b}^*}=0.\end{aligned}$$

Now if the initial bond levels (which are model parameters) are chosen in agreement with the bond market clearing condition, i.e. $\gamma P_{-1}b_0+(1-\gamma)P^*_{-1}b_0=0$, a simple induction argument over t establishes (B), $\forall t$.

C.2.2 *Balance-of-Payments*

In Appendix C.2.1 we establish that

$$\begin{aligned}{}_{(HMC\&\Sigma\&BC\&GMC)} \quad &\gamma\left(\mathrm{y}_{H,t}P_{H,t}+\mathrm{y}_{F,t}P_{F,t}+P_tb_{t+1}-(1+i_{t-1})P_{t-1}b_t-(\beta^{-1}-1)\tfrac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P_{t-1}}P_{t-1}\bar{b}\right)\\ &=P_{H,t}\left(\gamma\mathrm{y}_{H,t}+(1-\gamma)\mathrm{y}^*_{H,t}\right)\end{aligned}$$

$$
(HMC^*\&\Sigma^*\&BC^*\&GMC^*) \quad (1-\gamma)\left(y^*_{H,t}P_{H,t} + y^*_{F,t}P_{F,t} + P^*_t b^*_{t+1} - (1+i_{t-1})P^*_{t-1}b^*_t - (\beta^{-1}-1)\frac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P^*_{t-1}}P^*_{t-1}\bar{b}^*\right)
$$
$$
= P_{F,t}\left(\gamma y_{F,t} + (1-\gamma)y^*_{F,t}\right).
$$

This is equivalent to

$$
(HMC\&\Sigma\&BC\&GMC) \quad \gamma y_{F,t}P_{F,t} - P_{H,t}(1-\gamma)y^*_{H,t}
$$
$$
+\gamma\left(P_t b_{t+1} - (1+i_{t-1})P_{t-1}b_t - (\beta^{-1}-1)\frac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P_{t-1}}P_{t-1}\bar{b}\right) = 0
$$
$$
(HMC^*\&\Sigma^*\&BC^*\&GMC^*) \quad (1-\gamma)x^*_{H,t}P_{H,t} - P_{F,t}\gamma y_{F,t}
$$
$$
+(1-\gamma)\left(P^*_t b^*_{t+1} - (1+i_{t-1})P^*_{t-1}b^*_t - (\beta^{-1}-1)\frac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P^*_{t-1}}P^*_{t-1}\bar{b}^*\right) = 0.
$$

Now both equations dictate that the value of net imports in the respective country (imports less of exports) be covered by an equal-sized increase in the level of debt. By Walras' law (see Appendix C.2.1), the whole list of market clearing conditions, budget constraints and relevant variable definitions, (C.1), is equivalent to (C.1) without $(BC), (BC^*)$ and augmented with the balance of payments equation

$$
(BOP) \quad \gamma(1-\lambda)\left(\frac{P_{F,t}}{P_t}\right)^{1-\frac{1}{\varsigma}}y_t - (1-\gamma)(1-\lambda^*)\left(\frac{P_{H,t}}{P^*_t}\right)^{1-\frac{1}{\varsigma}}\frac{P^*_t}{P_t}y^*_t
$$
$$
+\gamma\left(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1}b_t - (\beta^{-1}-1)\left(\gamma + (1-\gamma)\frac{P^*_{t-1}}{P_{t-1}}\right)(1+\pi_t)^{-1}\bar{b}\right) = 0.
$$

where we have used the demand schedules to substitute out the H and F good variables that do not feature in the MSV-representation of the model, and we have divided by P_t to get the representation in units of country H's final consumption basket.

C.3 Nonlinear Equilibrium Conditions

As a starting point to solving the model, we collect all equilibrium conditions in a parsimonious fashion by performing light substitutions.

C.3.1 *Expressing price levels with only inflation rates and terms of trade*

Define the terms of trade as

$$
s_t := \frac{P_{H,t}}{P_{F,t}}.
$$

This entails

$$
s_t = \frac{\Pi_{H,t}}{\Pi_{F,t}}s_{t-1},
$$

and allows us to write

$$\Pi_t = \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{1-\lambda}{\lambda} (s_t^{1-1/\varsigma})^{-1} \right)^{-1} + \left(\frac{\lambda}{1-\lambda} s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1} \right)^{-1} \right]^{\frac{1}{1-1/\varsigma}},$$

$$\Pi_t^* = \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{\lambda^*}{1-\lambda^*} (s_t^{1-1/\varsigma})^{-1} \right)^{-1} + \left(\frac{1-\lambda^*}{\lambda^*} s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1} \right)^{-1} \right]^{\frac{1}{1-1/\varsigma}},$$

$$\left(\frac{P_{H,t}}{P_t}\right)^{-\frac{1}{\varsigma}} = \left[\lambda + (1-\lambda) s_t^{\frac{1}{\varsigma}-1}\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: {}_H(s_t)^{-\frac{1}{\varsigma}}, \quad {}'_H(s) = {}_H(s)^{2-1/\varsigma}(1-\lambda)s^{1/\varsigma-1} > 0, \quad {}'_H(1) = 1-\lambda,$$

$$\left(\frac{P_{F,t}}{P_t}\right)^{-\frac{1}{\varsigma}} = \left[\lambda s_t^{1-\frac{1}{\varsigma}} + (1-\lambda)\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: {}_F(s_t)^{-\frac{1}{\varsigma}}, \quad {}'_F(s) = -{}_F(s)^{2-1/\varsigma}\lambda s^{1/\varsigma} < 0, \quad {}'_F(1) = -\lambda,$$

$$\left(\frac{P_{H,t}}{P_t^*}\right)^{-\frac{1}{\varsigma}} = \left[(1-\lambda^*) + \lambda^* s_t^{\frac{1}{\varsigma}-1}\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: {}^*_H(s_t)^{-\frac{1}{\varsigma}}, \quad {}^{*\prime}_H(s) = ({}_H(s)^*)^{2-1/\varsigma}\lambda^* s^{1/\varsigma-1} > 0, \quad {}^{*\prime}_H(1) = \lambda^*,$$

$$\left(\frac{P_{F,t}}{P_t^*}\right)^{-\frac{1}{\varsigma}} = \left[(1-\lambda^*) s_t^{1-\frac{1}{\varsigma}} + \lambda^*\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: {}^*_F(s_t)^{-\frac{1}{\varsigma}}, \quad {}^{*\prime}_F(s) = -({}^*_F(s))^{2-1/\varsigma}(1-\lambda^*)s^{1/\varsigma} > 0, \quad {}^{*\prime}_F(1) = -(1-\lambda^*),$$

$$\frac{P_t}{P_t^*} = \left[\frac{\lambda s_t^{1-1/\varsigma} + (1-\lambda)}{(1-\lambda^*)s_t^{1-1/\varsigma} + \lambda^*}\right]^{\frac{1}{1-\frac{1}{\varsigma}}} =: (s_t), \quad {}'(s) = (s)^{1/\varsigma}\frac{s^{-1/\varsigma}}{((1-\lambda^*)s^{1-1/\varsigma}+\lambda^*)^2}\cdot[\lambda\lambda^* - (1-\lambda)(1-\lambda^*)] > 0,$$
$${}'(1) = \lambda\lambda^* - (1-\lambda)(1-\lambda^*).$$

We have characterized every expression that involves any consumption price level in terms of the inflation rates and the terms of trade. This means that we need not track the price levels which are determined in equilibrium only up to a translation.

C.3.2 *Condensing the set of market clearing conditions*

Using the expressions above, and the demand schedules for varieties, we can rewrite the goods market clearing condition into

$$\left(1 - \frac{\kappa}{2}(\Pi_{H,t}-1)^2\right)\xi_{a,t}n_t\gamma = \gamma\lambda_H(s_t)^{-\frac{1}{\varsigma}}y_t + (1-\gamma)(1-\lambda^*){}^*_H(s_t)^{-\frac{1}{\varsigma}}y_t^*,$$
$$\left(1 - \frac{\kappa}{2}(\Pi_{F,t}-1)^2\right)\xi_{a,t}n_t^*(1-\gamma) = \gamma(1-\lambda)_F(s_t)^{-\frac{1}{\varsigma}}x_t + (1-\gamma)\lambda^{**}{}_F(s_t)^{-\frac{1}{\varsigma}}x_t^*.$$

We can use the expressions above also in the BOP equation (derived in C.2.2), to receive

$$(BOP) \quad \gamma(1-\lambda)_F(s_t)^{1-\frac{1}{\varsigma}}y_t - (1-\gamma)(1-\lambda^*){}^*_H(s_t)^{1-\frac{1}{\varsigma}}(s_t)^{-1}y_t^*$$
$$+\gamma\left(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1}b_t - (\beta^{-1}-1)\left(\gamma + (1-\gamma)(s_{t-1})^{-1}\right)(1+\pi_t)^{-1}\bar{b}\right) = 0.$$

We are now ready to state the set of nonlinear equilibrium conditions.

C.3.3 *Nonlinear equilibrium conditions*

Using the short-hand notation $\mathrm{y}_t := c_t + x_t + (1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ (analogously for *):

Household

$$\begin{aligned}
&(h) \quad q_t = \frac{\xi_{h,t} h_t^{-\nu}}{\xi_{c,t} c_t^{-\sigma}} + \beta(1-\delta)\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} q_{t+1}\right\},\\
&(n) \quad \frac{\chi n_t^{\varphi}}{\xi_{c,t} c_t^{-\sigma}} = w_t,\\
&(b) \quad 1 = \beta\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}(1+r_{t+1})\right\},\\
&(x) \quad 1 = q_t \cdot \xi_{x,t} x_t^{\eta-1},
\end{aligned} \tag{C.2}$$

Household*

$$\begin{aligned}
&(h^*) \quad q_t^* = \xi_{h,ss}^* \tfrac{\xi_{h,t}}{\xi_{h,ss}} \frac{(h_t^*)^{-\nu}}{\xi_{c,t}(c_t^*)^{-\sigma}} + \beta(1-\delta)\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma} q_{t+1}^*\right\},\\
&(n^*) \quad \frac{\chi (n_t^*)^{\varphi}}{\xi_{c,t}(c_t^*)^{-\sigma}} = w_t^*,\\
&(b^*) \quad 1 = \beta\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma}(1+r_{t+1}^*)\right\},\\
&(x^*) \quad 1 = q_t^* \cdot \xi_{x,ss}^* \tfrac{\xi_{x,t}}{\xi_{x,ss}}(x_t^*)^{\eta-1},
\end{aligned}$$

Interest rates

$$\begin{aligned}
&(r) \quad 1 + r_t = \frac{1 + i_{t-1} - \psi b_t}{1+\pi_t},\\
&(r^*) \quad 1 + r_t^* = \frac{1 + i_{t-1} - \psi b_t^*}{1+\pi_t^*},
\end{aligned}$$

Firm

$$\begin{aligned}
&(PC) \quad \kappa(\Pi_{H,t}-1)\Pi_{H,t} - (1-\epsilon) - \epsilon(1-\tau^{\ell})\frac{w_t}{\xi_{a,t}}{}_H(s_t)^{-1} =\\
&\qquad \mathbb{E}_t^{\mathcal{P}}\left\{\beta\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{\xi_{a,t+1} n_{t+1}}{\xi_{a,t} n_t \Pi_{t+1}}\kappa(\Pi_{H,t+1}-1)\Pi_{H,t+1}^2\right\},
\end{aligned}$$

Firm*

$$\begin{aligned}
&(PC^*) \quad \kappa(\Pi_{F,t}-1)\Pi_{F,t} - (1-\epsilon) - \epsilon(1-\tau^{\ell})\frac{w_t^*}{\xi_{a,t}}{}^*_F(s_t)^{-1} =\\
&\qquad \mathbb{E}_t^{\mathcal{P}}\left\{\beta\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma}\frac{\xi_{a,t+1} n_{t+1}^*}{\xi_{a,t} n_t^* \Pi_{t+1}^*}\kappa(\Pi_{F,t+1}-1)\Pi_{F,t+1}^2\right\},
\end{aligned}$$

Bond market clearing

$(B) \quad \gamma b_{t+1} + (1-\gamma)(s_t)b^*_{t+1} = 0,$

Goods market clearing

$$(GMC) \quad \left(1-\frac{\kappa}{2}(\Pi_{H,t}-1)^2\right)\xi_{a,t}n_t\gamma = \gamma\lambda_H(s_t)^{-\frac{1}{\varsigma}}y_t + (1-\gamma)(1-\lambda^*)^*_H(s_t)^{-\frac{1}{\varsigma}}y^*_t,$$

$$(GMC^*) \quad \left(1-\frac{\kappa}{2}(\Pi_{F,t}-1)^2\right)\xi_{a,t}n^*_t(1-\gamma) = \gamma(1-\lambda)_F(s_t)^{-\frac{1}{\varsigma}}y_t + (1-\gamma)\lambda^{**}_F(s_t)^{-\frac{1}{\varsigma}}y^*_t,$$

$$(BOP) \quad \gamma(1-\lambda)_F(s_t)^{1-\frac{1}{\varsigma}}y_t - (1-\gamma)(1-\lambda^*)^*_H(s_t)^{1-\frac{1}{\varsigma}}(s_t)^{-1}y^*_t$$
$$+\gamma\left(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1}b_t - (\beta^{-1}-1)\left(\gamma + (1-\gamma)(s_{t-1})^{-1}\right)(1+\pi_t)^{-1}\bar{b}\right) = 0,$$

Housing market clearing

$(HMC) \quad \eta^{-1}\xi_{x,t}x^\eta_t = h_t - (1-\delta)h_{t-1},$

$(HMC^*) \quad (\eta^*)^{-1}\xi^*_{x,ss}\frac{\xi_{x,t}}{\xi_{x,ss}}(x^*_t)^{\eta^*} = h^*_t - (1-\delta)h^*_{t-1}, \quad \eta^* > \eta$

Price indices

$(s) \quad s_t = \dfrac{\Pi_{H,t}}{\Pi_{F,t}}s_{t-1},$

$$(\Pi) \quad \Pi_t = \left[\left((\Pi^{1-1/\varsigma}_{H,t})^{-1} + \frac{1-\lambda}{\lambda}(s^{1-1/\varsigma}_t)^{-1}\right)^{-1} + \left(\frac{\lambda}{1-\lambda}s^{1-1/\varsigma}_t + (\Pi^{1-1/\varsigma}_{F,t})^{-1}\right)^{-1}\right]^{\frac{1}{1-1/\varsigma}},$$

$$(\Pi^*) \quad \Pi^*_t = \left[\left((\Pi^{1-1/\varsigma}_{H,t})^{-1} + \frac{\lambda^*}{1-\lambda^*}(s^{1-1/\varsigma}_t)^{-1}\right)^{-1} + \left(\frac{1-\lambda^*}{\lambda^*}s^{1-1/\varsigma}_t + (\Pi^{1-1/\varsigma}_{F,t})^{-1}\right)^{-1}\right]^{\frac{1}{1-1/\varsigma}}.$$

with the shocks $(\xi_t)_{t\geq 0}$, the allocation variables $(c_t, c^*_t, b_t, b^*_t, h_t, h^*_t, x_t, x^*_t, n_t, n^*_t)_{t\geq 0}$ and the price variables $(w_t, w^*_t, q_t, q^*_t, i_t, \Pi_t, \Pi^*_t, \Pi_{H,t}, \Pi_{F,t}, s_t)_{t\geq 0}$.

C.4 Proof of Lemma 1

In this section, we prove that there exists a unique non-stochastic steady-state with zero net inflation and parity in the terms of trade $P_{H,t}/P_{F,t}$ (which implies parity in the real exchange rate P_t/P^*_t, see the definition of). We prove this first for arbitrary parameters $\eta, \eta^*, \xi_{h,ss}, \xi_{x,ss}, \xi^*_{h,ss}, \xi^*_{x,ss}$, and then show that it is possible to select parameters such that the steady-state allocation is symmetric. The non-stochastic steady-state with zero net inflation and real exchange rate parity ("SS" for short) obtains by setting $\mathrm{Var}[\|\xi_t\|] = 0$, where the shock vector contains both actual shocks and shocks that are only perceived (and never observed) by the household within her perceived house price model: $\xi_t = (\xi_{a,t}, \xi_{c,t}, \xi_{h,t}, \xi_{x,t}, \xi_{i,t}, e_t, v_t)^\top$. Thus, the non-stochastic steady state represents the time-invariant equilibrium that obtains if agents do not expect any shock to ever materialize and indeed no shock ever does materialize, and we have $\xi_t = (1, 1, \xi_{h,ss}, \xi_{x,ss}, 1, 1, 1)^\top$ almost surely where $\xi_{h,ss}, \xi_{x,ss}$ are model parameters. We are interested in, and prove existence and uniqueness of, a non-stochastic steady-state in which the net rates of inflation are zero and in which the terms of trade are at parity. The latter assumption implies ${}_H(1) = {}^*_H(1) = {}_F(1) = {}^*_F(1) = (1) = 1$.

Beliefs are irrelevant in the non-stochastic steady state. The first relevant insight is that in the non-stochastic steady state, defined as above, the presence and precise parameterization of subjective beliefs over house prices is irrelevant. To see this, recall that the subjective house price expectation dynamics are fully characterized by

$$\begin{aligned}
\forall s > 0,\ \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2 \left(\varrho \tfrac{1-\varrho^s}{1-\varrho}\right)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \\
\ln \overline{m}_t &= (1-g)\left(\ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2}\right) + g\left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2}\right), \\
&+ \text{equations determining equilibrium-evolution of price level, } q_t.
\end{aligned} \tag{C.3}$$

Now recall that the non-stochastic steady-state represents the unique equilibrium of the economy when the variance of actual and perceived external shocks tends to zero and the initial conditions are selected such that they give rise to a constant path of equilibrium variables. Formally, the SS arises by replacing $\mathrm{Var}[\|\xi_t\|]$ with $\wp^2 \cdot \mathrm{Var}[\|\xi_t\|]$ in the model, taking the limit $\wp \to 0$, and solving for the fixed point of the equilibrium equations. Applying this logic, $\sigma_e, \sigma_v, \sigma \to 0$, to the equations (C.3) delivers

$$\begin{aligned}
q_{ss} &= q_{ss} \cdot \exp\left(\ln \overline{m}_{ss} \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + 0 \right) \cdot \exp(0) \\
\ln \overline{m}_{ss} &= (1-g) \ln \overline{m}_{ss} + g \cdot 0
\end{aligned}$$

from which we see that there exists a unique fixed point which is $\overline{m}_{ss} = 1$, regardless of the level of q_{ss}. This means that the presence of subjective beliefs has no consequences for the non-stochastic steady-state: provided that agents have the prior that house prices do not change, $\overline{m}_{ss} = 1$, the absence of perceived and actual shocks means that house prices indeed never change which, in turn, means that the prior of constant house prices is never changed.

In summary, iff a non-stochastic steady state with zero net inflation and terms of trade parity exists and is unique, then it solves the following system of equations.

$$
\begin{aligned}
&\textbf{Household} \\
&(h) \quad q_{ss} = \xi_{h,ss} h_{ss}^{-\nu} c_{ss}^{\sigma} + \beta(1-\delta) q_{ss}, \\
&(n) \quad \chi n_{ss}^{\varphi} c_{ss}^{\sigma} = w_{ss}, \\
&(b) \quad 1 = \beta(1 + r_{ss}), \\
&(x) \quad 1 = q_{ss} \cdot \xi_{x,ss} x_{ss}^{\eta-1}, \\
&\textbf{Household*} \\
&(h^*) \quad q_{ss} = \xi_{h,ss}^{*} (h_{ss}^{*})^{-\nu} (c_{ss}^{*})^{\sigma} + \beta(1-\delta) q_{ss}^{*}, \\
&(n^*) \quad \chi (n_{ss}^{*})^{\varphi} (c_{ss}^{*})^{\sigma} = w_{ss}^{*}, \\
&(b^*) \quad 1 = \beta(1 + r_{ss}^{*}), \\
&(x^*) \quad 1 = q_{ss}^{*} \cdot \xi_{x,ss}^{*} (x_{ss}^{*})^{\eta-1}, \\
&\textbf{Interest rates} \\
&(r) \quad 1 + r_{ss} = 1 + i_{ss} - \psi b_{ss}, \\
&(r^*) \quad 1 + r_{ss}^{*} = 1 + i_{ss} - \psi b_{ss}^{*}, \\
&\textbf{Firm} \\
&(PC) \quad w_{ss} = 1 \quad \text{using that } \tau^{\ell} = 1/\epsilon, \\
&\textbf{Firm*} \\
&(PC^*) \quad w_{ss}^{*} = 1 \quad \text{using that } \tau^{\ell} = 1/\epsilon, \\
&\textbf{Bond market clearing} \\
&(B) \quad \gamma b_{ss} + (1-\gamma) b_{ss}^{*} = 0, \\
&\textbf{Goods market clearing} \\
&(GMC) \quad n_{ss}\gamma = \gamma\lambda \mathrm{y}_{ss} + (1-\gamma)(1-\lambda^{*})\mathrm{y}_{ss}^{*}, \\
&(GMC^*) \quad n_{ss}^{*}(1-\gamma) = \gamma(1-\lambda)\mathrm{y}_{ss} + (1-\gamma)\lambda^{*}\mathrm{y}_{ss}^{*}, \\
&(BOP) \quad \gamma(1-\lambda)\mathrm{y}_{ss} - (1-\gamma)(1-\lambda^{*})\mathrm{y}_{ss}^{*} - \gamma(i_{ss} b_{ss} + (\beta^{-1} - 1)\bar{b}) = 0, \\
&\textbf{Housing market clearing} \\
&(HMC) \quad \eta^{-1} \xi_{x,ss} (x_{ss})^{\eta} = \delta h_{ss}, \\
&(HMC^*) \quad (\eta^{*})^{-1} \xi_{x,ss}^{*} (x_{ss}^{*})^{\eta^{*}} = \delta h_{ss}^{*}.
\end{aligned}
\tag{C.4}
$$

where $\bar{b}, \bar{b}^*$ are model parameters (see Appendix B for an interpretation) chosen so as to (i) ensure equilibrium existence, and (ii) ensure that (§1) $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$. We solve for the SS in 4 steps.

1. First, we solve a number of equations explicitly, thus substituting out a number of variables:

(a) (b) and (b^*) together with (r) and (r^*) and (B) imply $b_{ss} = b^*_{ss} = 0$ and $i_{ss} = \beta^{-1} - 1$ whence it follows $\mathrm{y}_{ss} = c_{ss} + x_{ss}$ and analogously for *. (§1) then implies $\bar{b} = -\bar{b}^* \cdot (1-\gamma)/\gamma$ where $\bar{b}^*$ is not pinned down yet. We will solve for it in the very last step.

(b) (PC) and (PC^*) imply $w_{ss} = w^*_{ss} = 1$;

(c) together with (n) and (n^*) this implies $n_{ss} = (\chi^{-1})^{1/\varphi} \cdot c_{ss}^{-\sigma/\varphi} =: \phi(c_{ss})$ with $\phi' < 0$ and analogously for n^*_{ss} with the *same* function ϕ;

(d) (HMC) and (HMC^*) imply $h_{ss} = (\delta)^{-1}\xi_{x,ss}\eta^{-1}x^{\eta}_{ss}$ and analogously for h^*_{ss};

(e) (h) and (h^*) imply (with $\bar{\beta} := \beta(1-\delta)$)

$$q_{ss} = \left(1-\bar{\beta}\right)^{-1} \xi_{h,ss} \left[\delta^{-1}\xi_{x,ss}\eta^{-1}x^{\eta}_{ss}\right]^{-\nu} c^{\sigma}_{ss}$$
$$q^*_{ss} = \left(1-\bar{\beta}\right)^{-1} \xi^*_{h,ss} \left[\delta^{-1}\xi^*_{x,ss}(\eta^*)^{-1}\left(x^*_{ss}\right)^{\eta^*}\right]^{-\nu} (c^*_{ss})^{\sigma}$$

(f) (BOP) now reads $\gamma(1-\lambda)\mathrm{y}_{ss} = (1-\gamma)(1-\lambda^*)\mathrm{y}^*_{ss} - (1-\gamma)(\beta^{-1}-1)\bar{b}^*$, and the symmetric (BOP^*) which is redundant by Walras' law reads $(1-\gamma)(1-\lambda^*)x^*_{ss} = \gamma(1-\lambda)x_{ss} + (1-\gamma)(\beta^{-1}-1)\bar{b}^*$; using this in (GMC), (GMC^*) produces

$$(GMC) \quad \phi(c_{ss}) = \mathrm{y}_{ss} + (1-\gamma)/\gamma \cdot (\beta^{-1}-1)\bar{b}^*,$$
$$(GMC^*) \quad \phi(c^*_{ss}) = \mathrm{y}^*_{ss} - (\beta^{-1}-1)\bar{b}^*,$$

2. The remaining equations are (x), (x^*) and (GMC), (GMC^*), (BOP) with unknowns $x_{ss}, x^*_{ss}, c_{ss}, c^*_{ss}, \bar{b}^*$. In this step, we show there are strictly increasing functions that yield x_{ss}, x^*_{ss} given c_{ss}, c^*_{ss} respectively. Start by plugging q_{ss} into (x):

$$\left(1-\bar{\beta}\right)^{-1} \xi_{h,ss} \left[\delta^{-1}\xi_{x,ss}\eta^{-1}x^{\eta}_{ss}\right]^{-\nu} c^{\sigma}_{ss}\xi_{x,ss}x^{\eta-1}_{ss} = 1$$

(the equation for * is symmetric.) Now since $\eta \in (0,1)$ and $\nu > 0$, the expression on the left-hand-side is a strictly decreasing function of x_{ss} for any c_{ss}. Moreover, for $x_{ss} \to 0$, the $LHS \to +\infty$ and for $x_{ss} \to \infty$, the $LHS \to 0$, whence Bolzano's intermediate value theorem (and continuity) ensures that for each c_{ss} there exists a unique x_{ss}. Call this implicitly defined mapping $x_{ss} = \psi(c_{ss})$. As the implicit function theorem shows, $\eta, \nu, \sigma > 0$ imply $\psi' > 0$. Analogous arguments hold for *.

3. We now insert our previous findings into the only remaining equations:

$$(GMC) \quad \phi(c_{ss}) - c_{ss} - \psi(c_{ss}) - (1-\gamma)/\gamma \cdot (\beta^{-1}-1)\bar{b}^* =: \zeta(c_{ss}, \bar{b}^*) = 0,$$

$$(GMC^*) \quad \phi(c^*_{ss}) - c^*_{ss} - \psi^*(c^*_{ss}) + (\beta^{-1} - 1)\bar{b}^* =: \zeta^*(c^*_{ss}, \bar{b}^*) = 0,$$

Observe now $c_{ss} \mapsto \zeta$ is continuous and strictly decreasing with $\lim_{c\to 0} \zeta = +\infty$ (by $\lim_{c\to 0} \phi = +\infty$ and $\lim_{c\to 0} \psi < +\infty$) and $\lim_{c\to +\infty} \zeta = -\infty$ (by $\lim_{c\to +\infty} c, \psi = +\infty$ and $\lim_{c\to\infty} \phi = 0$). Therefore, Bolzano's intermediate value theorem ensures there exists a unique c_{ss} for each $\bar{b}^*$. The exactly analogous argument ensures existence and uniqueness of c^*_{ss}. Call these mappings $\varpi : \bar{b}^* \mapsto c_{ss}, \varpi^* : \bar{b}^* \mapsto c^*_{ss}$. The implicit function theorem now yields:

$$\partial\varpi/\partial\bar{b}^* < 0 \text{ and } \partial\varpi^*/\partial\bar{b}^* > 0.$$

4. Finally, only one equation remains, (BOP), with only one variable, $\bar{b}^*$:

$$\begin{aligned} &\gamma(1-\lambda)[\varpi(\bar{b}^*) + \psi(\varpi(\bar{b}^*))] - (1-\gamma)(1-\lambda^*)[\varpi^*(\bar{b}^*) + \psi^*(\varpi^*(\bar{b}^*))] \\ &\quad + (1-\gamma)(\beta^{-1} - 1)\bar{b}^* =: \mathcal{H}(\bar{b}^*) = 0 \end{aligned}$$

with $\bar{b}^* \mapsto \mathcal{H}$ continuous. It also holds that $\lim_{\bar{b}^*\to -\infty} \mathcal{H}(\bar{b}^*) = -\infty$.[27] On the other hand, as $\bar{b}^* \to +\infty$, $\mathcal{H} \to +\infty$.[28] Thus, Bolzano's intermediate value theorem ensures **existence** of a $\bar{b}^* \in \mathbb{R}$ that satisfies the BOP-equation and thus existence of a non-stochastic steady state.

Uniqueness of the steady-state can be shown by establishing strict positive monotonicity of $\mathcal{H}$: (we suppress arguments for brevity)

$$\begin{aligned} \frac{\partial\mathcal{H}}{\partial\bar{b}^*} &= \gamma(1-\lambda)(1+\psi')\varpi' - (1-\gamma)(1-\lambda^*)(1+\psi^{*\prime})\varpi^{*\prime} \\ &\quad + (1-\gamma)(\beta^{-1}-1) \\ \text{step 2, cf. notes below:} \quad &> -(1-\gamma)(1-\lambda)(\beta^{-1}-1) - (1-\gamma)(1-\lambda^*)(\beta^{-1}-1) \\ &\quad + (1-\gamma)(\beta^{-1}-1) \\ &= (1-\gamma)(\beta^{-1}-1)[1-1+\lambda-1+\lambda^*] \\ &\overset{\text{sign}}{=} \lambda + \lambda^* - 1 \end{aligned}$$

---

[27]Proof: (1) $\gamma(1-\lambda)[\varpi^*(\bar{b}^*)+\psi^*(\varpi^*(\bar{b}^*))] \geq 0$ by non-negativity of consumption & housing investment; (2) $\gamma(1-\lambda)[\varpi+\psi\circ\varpi]+(1-\gamma)(\beta^{-1}-1)\bar{b}^* = \gamma(1-\lambda)[\phi-\frac{1-\gamma}{\gamma}(\beta^{-1}-1)\bar{b}^*]+(1-\gamma)(\beta^{-1}-1)\bar{b}^* = \gamma(1-\lambda)\phi+\lambda(1-\gamma)(\beta^{-1}-1)\bar{b}^*$, where the second equality is a consequence of (GMC); (3) $\lim_{\bar{b}^*\to-\infty} \varpi = +\infty$ (assuming the contrary will produce a contradiction with (GMC)); (4) $\lim_{c\to 0} \phi = 0$; (5) steps 1–4 now imply $\mathcal{H}(\bar{b}^*) \leq \gamma(1-\lambda)\phi+\lambda(1-\gamma)(\beta^{-1}-1)\bar{b}^* \to -\infty$ as $\bar{b}^* \to -\infty$. □

[28]Proof: (1) $\gamma(1-\lambda)[\varpi(\bar{b}^*) + \psi(\varpi(\bar{b}^*))] \geq 0$ by non-negativity of consumption & housing investment; (2) $-(1-\lambda^*)[\varpi^*(\bar{b}^*)+\psi^*(\varpi^*(\bar{b}^*))]+(\beta^{-1}-1)\bar{b}^* = -(1-\lambda^*)[\phi(c^*_{ss})+(\beta^{-1}-1)\bar{b}^*]+(\beta^{-1}-1)\bar{b}^* = -(1-\lambda^*)\phi(c^*_{ss})+\lambda^*(\beta^{-1}-1)\bar{b}^*$ where the substitution is made using the definition of ϖ^*; (3) $\lim_{\bar{b}^*\to+\infty} \varpi^* = +\infty$ (assuming the contrary will produce a contradiction with (GMC^*)); (4) Fact 3 and $\lim_{c\to+\infty} \phi(c) = 0$ implies $\mathcal{H}(\bar{b}^*) \geq -(1-\lambda^*)\phi(c^*_{ss}) + \lambda^*(\beta^{-1}-1)\bar{b}^* \to +\infty$ as $\bar{b}^* \to +\infty$. □

$$\begin{aligned}
\text{step 3, cf. notes below:} \quad &= \lambda + 1 - \frac{\gamma}{1-\gamma}(1-\lambda) - 1 \\
&\overset{\text{sign}}{=} (1-\gamma)\lambda - \gamma(1-\lambda) \\
&= \lambda - \gamma \\
&\geq 0 \text{ by assumption.}
\end{aligned}$$

Step 3 follows by symmetric home bias, i.e. $\gamma(1-\lambda) = (1-\gamma)(1-\lambda^*) \iff \lambda^* = 1 - \frac{\gamma}{1-\gamma}(1-\lambda)$; Step 2 requires slightly more work: First, use the implicit function theorem on $(GMC)\&(GMC^*)$, respectively, to obtain

$$\varpi' = -\frac{\partial\zeta/\partial\bar{b}^*}{\partial\zeta/\partial c_{ss}} = \frac{\frac{1-\gamma}{\gamma}(\beta^{-1}-1)}{\phi' - (1+\psi')} < 0,$$
$$\varpi^{*\prime} = -\frac{\partial\zeta^*/\partial\bar{b}^*}{\partial\zeta^*/\partial c^*_{ss}} = \frac{-(\beta^{-1}-1)}{\phi' - (1+\psi^{*\prime})} > 0;$$

Second, recognize that since $\phi' < 0$ it is

$$\frac{1+\psi'}{1+\psi'-\phi'} < 1 \iff \frac{1+\psi'}{-(1+\psi')+\phi'} > -1$$

and symmetrically for *. This shows that $\frac{\partial\mathcal{H}}{\partial\bar{b}^*} > 0$, and the SS is unique.

Finally, notice that since existence and uniqueness follow for arbitrary parameters $\eta \in (0,1), \eta^* \in (0,1), \xi_{h,ss}, \xi_{x,ss}, \xi^*_{h,ss}, \xi^*_{x,ss} > 0$, it is possible to set $\eta^* > \eta$ and then choose $\xi_{x,ss}, \xi^*_{x,ss}$ so as to ensure that both (HMC) and $(HMC)^*$ hold if $h_{ss} = h^*_{ss}$ and $x_{ss} = x^*_{ss}$. Given this choice of $\xi_{x,ss}, \xi^*_{x,ss}$, and the symmetry in housing stock and housing investment, it is then possible to select $\xi_{h,ss}, \xi^*_{h,ss}$ such that the equations $(x), (x^*)$ hold. It then follows that $c_{ss} = c^*_{ss}, n_{ss} = n^*_{ss}, \bar{b} = \bar{b}^* = 0$, and the allocation is symmetric. This completes the proof.

C.5 Derivation of the household's subjectively optimal plans

In this Appendix we provide a formal derivation of the linearized subjectively optimal household decision rules presented in equations (15) and (16). Throughout the derivation we concentrate on the representative household in H with the understanding that the situation in F is symmetric. For simplicity we omit the habit consumption and domestically produced housing investment goods. This does not affect the layout of the proof below.

Consider the household program presented in Section III.A, which is restated here for convenience.[29] We first clarify the shape of the underlying probability space to set the appropriate frame for the following derivations: denote $\Omega \ni \omega_t := (\xi_t, r_t, w_t, \Sigma_t, \pi_t, (P_{H,t}/P_{F,t}), q_t)^\top$ the vector of external decision-relevant variables that the household takes as given, denote $\Omega^t \ni \omega^t := (\omega_{t-s})_{s\geq 0}$ the one-sided infinite history of past external variables and denote $\Omega^\infty \ni \omega := (\omega_t)_{t\in\mathbb{Z}}$ the typical element from the set of possible realizations of full sequences of external variables;[30] denote $\mathcal{B}^\infty$ the Borel-sigma-algebra over Ω^∞. Each household is now endowed with a probability measure $\mathcal{P}$ over $(\Omega^\infty, \mathcal{B}^\infty)$ which encodes her subjective beliefs over the realizations of external variables ω. Rational Expectations, denoted $\mathcal{P} = \mathbb{P}$, are a special case of this setup where $\mathbb{P}$ is the (unique) measure generated by the distribution of $\xi := (\xi_t)_{t\in\mathbb{Z}}$ and the equilibrium conditions that allow to compute $\omega\backslash\xi$ as a deterministic function of ξ.[31] As explained in Section III.A, we assume $\mathcal{P}$ to be of a particular form, that is we assume that $\mathcal{P} = \mathcal{P}_q \otimes \mathbb{P}_{-q}$, where $\otimes$ is the product measure (statistical independence), $\mathcal{P}_q$ is the distribution over $(q_t)_{t\in\mathbb{Z}}$ generated by the unobserved components model (6), and $\mathbb{P}_{-q}$ is the rational expectations measure without house prices–i.e. the measure over $\omega\backslash q = (\xi_t, r_t, w_t, \Sigma_t, \pi_t, (P_{H,t}/P_{F,t}))_{t\in\mathbb{Z}}$ that which is consistent with the equilibrium-implied joint probability distribution of $\omega\backslash q$. Consequently, for any two measurable functions $f : \mathbb{R} \to \mathbb{R}, g : \mathbb{R}^{\dim\Omega - 1} \to \mathbb{R}$, we have

$$\mathbb{E}^{\mathcal{P}}[f(q_t) \cdot g(\omega_t\backslash q_t)] = \int f\mathcal{P}_q(\mathrm{d}q) \cdot \int g\mathbb{P}_{-q}(\mathrm{d}\omega\backslash q).$$

Finally, $\mathbb{E}_t^{\mathcal{P}}$ is the expectation implied by $\mathcal{P}$ conditional on the sigma-algebra generated by ω^t.

Now, at each calendar date $t \in \mathbb{Z}$, the household takes as given ω^t and chooses today's consumption, labor, housing, housing investment, and bond levels, $(c_t, n_t, h_t, x_t, b_{t+1})$, as well as contingent plans for the future, $\{(c_{t+s}, n_{t+s}, h_{t+s}, x_{t+s}, b_{t+s+1}) : \Omega^{t+s} \to \mathbb{R}_+^4 \times \mathbb{R}\}_{s\in\mathbb{N}_+}$, to maximize

$$\begin{aligned}
&\mathbb{E}_t^{\mathcal{P}} \sum_{s=0}^{\infty} \beta^s \left(\frac{\xi_{c,t+s} c_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\xi_{h,t+s} h_{t+s}^{1-\nu}}{1-\nu} - \chi \frac{n_{t+s}^{1+\varphi}}{1+\varphi} \right) \text{ subject to} \\
&c_{t+s} + q_{t+s}(h_{t+s} - (1-\delta)h_{t+s-1}) + b_{t+s+1} + x_{t+s} = w_{t+s} n_{t+s} \\
&\qquad + (1 + r_{t+s}) b_{t+s} + q_{t+s} \cdot \tfrac{\xi_{x,t+s}}{\eta} x_{t+s-\tau}^{\eta} + \Sigma_{t+s}, \quad \forall s \geq 0, \ \mathcal{P}\text{-almost surely,}
\end{aligned} \tag{C.5}$$

as well as subject to a standard no-Ponzi-game condition on b. The first-order conditions for

[29] We ignore the terms $T_{t+s}, \mathrm{b}_{t+s}$ since they will be zero in the equilibrium we analyze. All derivations go through if these terms are non-zero.

[30] We restrict Ω to be the set of absolutely summable sequences.

[31] For any three vectors x, y, z where $x = (y, z)$, we define $x\backslash y := z$.

program (C.5) (suppressing the transversality conditions) are:

$$\begin{aligned}
\forall s \geq 0, \quad (h) \quad & \xi_{c,t+s} c_{t+s}^{-\sigma} q_{t+s} = \xi_{h,t+s} h_{t+s}^{-\nu} + \beta(1-\delta)\mathbb{E}_{t+s}^{\mathcal{P}}\{\xi_{c,t+s+1} c_{t+s+1}^{-\sigma} q_{t+s+1}\}, \\
(n) \quad & \chi n_{t+s}^{\varphi} \xi_{c,t+s}^{-1} c_{t+s}^{\sigma} = w_{t+s}, \\
(b) \quad & \xi_{c,t+s} c_{t+s}^{-\sigma} = \beta \mathbb{E}_{t+s}^{\mathcal{P}}\left\{(1+r_{t+s+1})\xi_{c,t+s+1} c_{t+s+1}^{-\sigma}\right\}, \\
(x) \quad & \beta^{\tau} \mathbb{E}_{t+s}^{\mathcal{P}}\left\{q_{t+s+\tau} \xi_{c,t+s+\tau} c_{t+s+\tau}^{-\sigma} \cdot \xi_{x,t+s+\tau} x_{t+s}^{\eta-1}\right\} = \xi_{c,t+s} c_{t+s}^{-\sigma}, \\
(BC) \quad & c_{t+s} + q_{t+s}(h_{t+s} - (1-\delta)h_{t+s-1}) + b_{t+s+1} + x_{t+s} = w_{t+s} n_{t+s} \\
& \quad + (1+r_{t+s}) b_{t+s} + q_{t+s} \cdot \tfrac{\xi_{x,t+s}}{\eta} x_{t+s-\tau}^{\eta} + \Sigma_{t+s}.
\end{aligned} \tag{C.6}$$

In close analogy to the standard procedure in a model with fully rational expectations, $\mathcal{P} = \mathbb{P}$, we now derive a linear approximation to (C.6) that – together with the other linearized equilibrium conditions – allows to solve the model to first order in the amplitude of shocks. The specific challenge here, with $\mathcal{P} = \mathcal{P}_q \otimes \mathbb{P}_{-q}$, will be to compute all expectations explicitly that depend on house prices, q. This includes the house prices themselves, $(q_{t+s})_{s>0}$, as well as expectations over future contingent choices, $(c_{t+s}, n_{t+s}, h_{t+s}, x_{t+s}, b_{t+s+1})_{s>0}$.

To first order around the steady state[32] from Lemma 1, in which we have $1 + r_{ss} = \beta^{-1}, \chi n_{ss}^{\varphi} c_{ss}^{\sigma} = w_{ss}(=1), q_{ss} = \xi_{h,ss} h_{ss}^{-\nu} c_{ss}^{\sigma}/(1-\bar{\beta}), \beta^{\tau} q_{ss} \xi_{x,ss} x_{ss}^{\eta-1} = 1$, it holds that, $\forall s \geq 0$:

$$\begin{aligned}
(h) \quad & \widehat{h}_{t+s} = \tfrac{1}{\nu}\xi_{h,t+s} + \frac{\sigma}{\nu}\frac{\widehat{c}_{t+s} - \frac{1}{\sigma}\widehat{\xi}_{c,t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\left\{\widehat{c}_{t+s+1} - \frac{1}{\sigma}\widehat{\xi}_{c,t+s+1}\right\}}{1-\bar{\beta}} - \frac{1}{\nu}\frac{\widehat{q}_{t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+1}\}}{1-\bar{\beta}}, \\
(n) \quad & \varphi \widehat{n}_{t+s} + \sigma \widehat{c}_{t+s} - \widehat{\xi}_{c,t+s} = \widehat{w}_{t+s}, \\
(b) \quad & \widehat{c}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} = -\tfrac{1}{\sigma}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+1}\} + \mathbb{E}_{t+s}^{\mathcal{P}}\left\{\widehat{c}_{t+s+1} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s+1}\right\}, \\
(x) \quad & \mathbb{E}_{t+s}^{\mathcal{P}}\widehat{q}_{t+s+\tau} - \mathbb{E}_{t+s}^{\mathcal{P}}\left\{\sigma\widehat{c}_{t+s+\tau} - \widehat{\xi}_{c,t+s+\tau}\right\} \\
& + \left(\sigma\widehat{c}_{t+s} - \widehat{\xi}_{c,t+s}\right) + \mathbb{E}_{t+s}\widehat{\xi}_{x,t+s+\tau} = (1-\eta)\widehat{x}_{t+s}, \\
(BC) \quad & \widehat{b}_{t+s+1} = \tfrac{1}{\beta}\widehat{b}_{t+s} + \tfrac{w_{ss} n_{ss}}{y_{ss}}(\widehat{w}_{t+s} + \widehat{n}_{t+s}) + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_{t+s} \\
& \quad - \tfrac{q_{ss} h_{ss}}{y_{ss}}\left[\widehat{h}_{t+s} - (1-\delta)\widehat{h}_{t+s-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t+s-\tau} - \beta^{\tau}\widehat{x}_{t+s} + \widehat{\xi}_{x,t+s}\right).
\end{aligned} \tag{C.7}$$

To characterize the household's expectations over own choice variables, start with the optimality condition for liquid bonds. Iteration over future instances of the condition reveals:

$$(b) \quad \widehat{c}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} = -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \lim_{n\to\infty}\mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_{t+s+n}.$$

To keep notation concise, we define for any random process $(k_t)_t$: $\mathbb{E}_{t+s}^{\mathcal{P}} k_\infty := \lim_{n\to\infty}\mathbb{E}_{t+s}^{\mathcal{P}} k_{t+s+n}$.

[32] Technically, we are scaling $\mathrm{Var}^{\mathcal{P}}[\|\omega\|] \to 0$. This is consistent, however, with the definition of $\mathcal{P}$ and $\mathrm{Var}[\|\xi\|] \to 0$.

We implicitly assumed here that a law of iterated expectations holds for $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}$ – at the end of the section we verify that the $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$ we recover does indeed satisfy a law of iterated expectations. Careful inspection of equations (C.7) reveals that $\mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_\infty$ is the only subjective expectation of a choice variable left unknown. This is because the law of iterated expectations for $\mathcal{P}$ allows substituting (b) into

$$
\begin{aligned}
&(h) \quad \widehat{h}_{t+s} = \tfrac{1}{\nu}\xi_{h,t+s} - \tfrac{\sigma}{\nu}\frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} - \bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n+1}\}}{1-\bar{\beta}} - \tfrac{1}{\nu}\frac{\widehat{q}_{t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+1}\}}{1-\bar{\beta}} + \tfrac{\sigma}{\nu}\mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_\infty, \\
&(n) \quad \varphi\widehat{n}_{t+s} + \sigma\widehat{c}_{t+s} - \widehat{\xi}_{c,t+s} = \widehat{w}_{t+s}, \\
&(b) \quad \widehat{c}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} = -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_\infty, \\
&(x) \quad \mathbb{E}_{t+s}^{\mathcal{P}}\widehat{q}_{t+s+\tau} - \sum_{n=1}^{\tau}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \mathbb{E}_{t+s}\widehat{\xi}_{x,t+s+\tau} = (1-\eta)\widehat{x}_{t+s}, \\
&(BC) \quad \widehat{b}_{t+s+1} = \tfrac{1}{\beta}\widehat{b}_{t+s} + \tfrac{w_{ss}n_{ss}}{y_{ss}}(\widehat{w}_{t+s} + \widehat{n}_{t+s}) + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_{t+s} \\
&\qquad - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}_{t+s} - (1-\delta)\widehat{h}_{t+s-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t+s-\tau} - \beta^{\tau}\widehat{x}_{t+s} + \widehat{\xi}_{x,t+s}\right).
\end{aligned}
\tag{C.8}
$$

Now to find $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$ (and characterize the choices at calendar date t), we first use the budget constraint for some $s > \tau$. Start by plugging in the optimality conditions:

$$
\begin{aligned}
(BC) \quad \widehat{b}_{t+s+1} &= \tfrac{1}{\beta}\widehat{b}_{t+s} + \tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+s} + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\left(\widehat{c}^{*}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s}\right) - \tfrac{c_{ss}}{y_{ss}\sigma}\widehat{\xi}_{c,t+s} \\
&- \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}^{*}_{t+s} - (1-\delta)\widehat{h}^{*}_{t+s-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}^{*}_{t+s-\tau} - \beta^{\tau}\widehat{x}^{*}_{t+s} + \widehat{\xi}_{x,t+s}\right) \\
&- \tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}{y_{ss}}\mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_\infty \\
&+ \tfrac{q_{ss}h_{ss}}{y_{ss}}\tfrac{1}{\nu}\tfrac{1}{1-\bar{\beta}}\left(\widehat{q}_{t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+1}\} - (1-\delta)\left(\widehat{q}_{t+s-1} - \bar{\beta}\mathbb{E}_{t+s-1}^{\mathcal{P}}\{\widehat{q}_{t+s}\}\right)\right) \\
&+ \tfrac{x_{ss}}{y_{ss}}\tfrac{1}{\beta^{\tau}(1-\eta)}\left(\mathbb{E}_{t+s-\tau}^{\mathcal{P}}\{\widehat{q}_{t+s}\} - \beta^{\tau}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+\tau}\}\right)
\end{aligned}
$$

where we have defined the auxiliary variables

$$
\begin{aligned}
\widehat{h}^{*}_{t+s} &:= \tfrac{1}{\nu}\xi_{h,t+s} - \tfrac{\sigma}{\nu}\frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} - \bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n+1}\}}{1-\bar{\beta}}, \\
\widehat{c}^{*}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} &:= -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\}, \\
(1-\eta)\widehat{x}^{*}_{t+s} &:= -\sum_{n=1}^{\tau}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \mathbb{E}_{t+s}\widehat{\xi}_{x,t+s+\tau}.
\end{aligned}
$$

Notice that the auxiliary variables are composed only of terms over which the household has rational expectations; thus of terms that fade to zero as $s \to \infty$.

Next, we apply the operator $\mathbb{E}_t^{\mathcal{P}}$ to both sides of (BC) and consider the limit as $s \to \infty$. This delivers:

$$
\begin{aligned}
(BC) \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty = \tfrac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - \tfrac{c_{ss} + n_{ss} w_{ss} \sigma/\varphi + \delta q_{ss} h_{ss} \sigma/\nu}{y_{ss}} \mathbb{E}_{t+s}^{\mathcal{P}} \widehat{c}_\infty \\
& \quad + \tfrac{\delta q_{ss} h_{ss}/\nu \, + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \\
\iff \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty = \tfrac{y_{ss}}{c_{ss} + n_{ss} w_{ss} \sigma/\varphi + \delta q_{ss} h_{ss} \sigma/\nu} \left[\tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty + \tfrac{\delta q_{ss} h_{ss}/\nu \, + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \right].
\end{aligned}
$$

Now since $\mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty$ can be computed from the subjectively perceived house price model, the only unknown left is $\mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty$. To find it, we plug $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ into (BC). At some $s > 0$ we have

$$
\begin{aligned}
(BC) \quad \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} = {} & \tfrac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s} + \tfrac{w_{ss} n_{ss}}{y_{ss}} (1 + 1/\varphi) \mathbb{E}_t \widehat{w}_{t+s} + \mathbb{E}_t \widehat{\Sigma}_{t+s} - \tfrac{c_{ss} + n_{ss} w_{ss} \sigma/\varphi}{y_{ss}} \mathbb{E}_t \left(\widehat{c}_{t+s}^{*} - \tfrac{1}{\sigma} \widehat{\xi}_{c,t+s} \right) - \tfrac{c_{ss}}{y_{ss}\sigma} \mathbb{E}_t \widehat{\xi}_{c,t+s} \\
& - \tfrac{q_{ss} h_{ss}}{y_{ss}} \mathbb{E}_t \big[\widehat{h}_{t+s}^{*} - (1-\delta) \widehat{h}_{t+s-1}^{*} \big] + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+s-\tau} - \beta^{\tau} \widehat{x}_{t+s} + \widehat{\xi}_{x,t+s} \right) \\
& - \tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - \tfrac{\delta q_{ss} h_{ss}/\nu \, + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \\
& + \tfrac{q_{ss} h_{ss}}{y_{ss}} \tfrac{1}{\nu} \tfrac{1}{1-\bar{\beta}} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{ \widehat{q}_{t+s+1} \} - (1-\delta) \left(\widehat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_{t+s-1}^{\mathcal{P}} \{ \widehat{q}_{t+s} \} \right) \right).
\end{aligned}
$$

After defining the auxiliary variables

$$
\begin{aligned}
z_{t+s}^{*} &:= \tfrac{w_{ss} n_{ss}}{y_{ss}} (1 + 1/\varphi) \widehat{w}_{t+s} + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss} + n_{ss} w_{ss} \sigma/\varphi}{y_{ss}} \left(\widehat{c}_{t+s}^{*} - \tfrac{1}{\sigma} \widehat{\xi}_{c,t+s} \right) - \tfrac{c_{ss}}{y_{ss}\sigma} \widehat{\xi}_{c,t+s} \\
& \quad - \tfrac{q_{ss} h_{ss}}{y_{ss}} \big[\widehat{h}_{t+s}^{*} - (1-\delta) \widehat{h}_{t+s-1}^{*} \big] + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \widehat{\xi}_{x,t+s}, \\
Q_{t+s} &:= \tfrac{q_{ss} h_{ss}}{y_{ss}} \tfrac{1}{\nu} \tfrac{1}{1-\bar{\beta}} \left(\widehat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{ \widehat{q}_{t+s+1} \} - (1-\delta) \left(\widehat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_{t+s-1}^{\mathcal{P}} \{ \widehat{q}_{t+s} \} \right) \right),
\end{aligned}
\tag{C.9}
$$

we can rewrite (BC) into

$$
\begin{aligned}
(BC) \quad \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} = {} & \tfrac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s} + \mathbb{E}_t z_{t+s}^{*} + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+s-\tau} - \beta^{\tau} \widehat{x}_{t+s} \right) + \mathbb{E}_t^{\mathcal{P}} Q_{t+s} \\
& - \tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - \tfrac{\delta q_{ss} h_{ss}/\nu \, + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty.
\end{aligned}
$$

Performing backward substitution until $s = 1$ delivers

$$
\begin{aligned}
(BC) \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} = \beta^{-s} \widehat{b}_{t+1} - \beta^{-s} (1 - \beta^{s}) \tfrac{\beta}{1-\beta} \left[\tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty + \tfrac{\delta q_{ss} h_{ss}/\nu \, + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \right] \\
& \quad + \beta^{-s} \sum_{n=1}^{s} \beta^{n} (\mathbb{E}_t z_{t+n}^{*} + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \beta^{-s} \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{s} \beta^{n} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+n-\tau} - \beta^{\tau} \widehat{x}_{t+n} \right) \\
\iff \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} - \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty = \beta^{-s} \cdot \Bigg[\widehat{b}_{t+1} + \sum_{n=1}^{s} \beta^{n} (\mathbb{E}_t z_{t+n}^{*} + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{s} \beta^{n} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+n-\tau} - \beta^{\tau} \widehat{x}_{t+n} \right) \\
& \qquad - \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - (1 - \beta^{s}) \tfrac{\beta}{1-\beta} \tfrac{\delta q_{ss} h_{ss}/\nu \, + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \Bigg].
\end{aligned}
$$

Now since we know that $\lim_{s\to\infty} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s}$ exists, it must be that the left side of this equation tends to zero as $s \to \infty$. But since $\beta^{-s} \to +\infty$ as $s \to \infty$, this implies that

$$\mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty = \widehat{b}_{t+1} + \sum_{n=1}^{\infty} \beta^n (\mathbb{E}_t z_{t+n}^* + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \frac{x_{ss}}{y_{ss}} \underbrace{\beta^{-\tau} \sum_{n=1}^{\infty} \beta^n \mathbb{E}_t^{\mathcal{P}} (\widehat{x}_{t+n-\tau} - \beta^\tau \widehat{x}_{t+n})}_{= \beta^{-\tau} \sum_{n=1}^{\tau} \beta^n \widehat{x}_{t+n-\tau} \text{ by rearranging a convergent sum}} - \frac{\beta}{1-\beta} \frac{\delta q_{ss} h_{ss}/\nu + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty,$$

so that we can characterize $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ entirely in terms of variables that are either pre-determined, current choice variables (i.e. known under $\mathbb{E}_t^{\mathcal{P}}$), or variables of which we can compute expectations in closed form (be they rational or not):

$$\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty = \frac{y_{ss}}{c_{ss} + n_{ss} w_{ss} \sigma/\varphi + \delta q_{ss} h_{ss} \sigma/\nu} \frac{1-\beta}{\beta} \left[\widehat{b}_{t+1} + \sum_{n=1}^{\infty} \beta^n (\mathbb{E}_t z_{t+n}^* + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{\tau} \beta^n \widehat{x}_{t+n-\tau} \right].$$

In the last step, we explicitly characterize the subjective expectation $\mathbb{E}_t^{\mathcal{P}} \sum_{n=1}^{\infty} \beta^n Q_{t+n}$ in terms of the processes governing subjective house price expectations, namely $\widehat{q}_t, \widehat{\overline{m}}_t$.

Recall that for any $s > 0$ we have $\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} = \widehat{q}_t + (1 - \varrho^s) \frac{\varrho}{1-\varrho} \widehat{\overline{m}}_t$, so that we have

$$\begin{aligned}
\mathbb{E}_t^{\mathcal{P}} Q_{t+s} &= \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1-\bar{\beta}} \left(\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \{\widehat{q}_{t+s+1}\} - (1-\delta) \left(\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \{\widehat{q}_{t+s}\} \right) \right) \\
&= \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1-\bar{\beta}} \Big(\delta(1-\bar{\beta}) \widehat{q}_t + (1-\varrho^s) \frac{\varrho}{1-\varrho} \widehat{\overline{m}}_t - \bar{\beta}(1-\varrho^{s+1}) \frac{\varrho}{1-\varrho} \widehat{\overline{m}}_t \\
&\quad - (1-\delta)(1-\varrho^{s-1}) \frac{\varrho}{1-\varrho} \widehat{\overline{m}}_t + (1-\delta)\bar{\beta}(1-\varrho^s) \frac{\varrho}{1-\varrho} \widehat{\overline{m}}_t \Big) \\
&= \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1-\bar{\beta}} \left(\delta(1-\bar{\beta}) \widehat{q}_t + \widehat{\overline{m}}_t \frac{\varrho}{1-\varrho} \left[\delta(1-\bar{\beta}) + (1-\bar{\beta}\varrho)(1-\varrho-\delta) \varrho^{s-1} \right] \right)
\end{aligned}$$

which in turn implies

$$\sum_{s \geq 1} \beta^s \mathbb{E}_t^{\mathcal{P}} Q_{t+s} = \beta \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \left(\frac{\delta}{1-\beta} \widehat{q}_t + \widehat{\overline{m}}_t \frac{\varrho}{1-\varrho} \left[\frac{\delta}{1-\beta} + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}} \frac{1-\varrho-\delta}{1-\beta\varrho} \right] \right). \tag{C.10}$$

Notice that, as claimed in the main text, it is that

$$\frac{\delta}{1-\beta} + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}} \frac{1-\varrho-\delta}{1-\beta\varrho} > 0, \;\; \forall \beta, \delta, \varrho \in (0,1),$$ [33]

so that the coefficient of the posterior expected house price growth rate onto the series is positive.

We now have characterized the household's decisions at an arbitrary calendar date t up to first

[33] *Proof.* This follows from the claim being equivalent to the statement $f(\varrho) := \frac{\delta}{1-\beta}(1-\beta\varrho) + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}(1-\varrho-\delta) > 0, \;\forall \beta, \delta, \varrho \in (0,1)$, from f being an upward-open parabola with $f(1) = 0$, and $f'(1) < 0$. □

order around the deterministic steady state:

$$
\begin{aligned}
&(h) \quad \widehat{h}_t = \tfrac{1}{\nu}\xi_{h,t} - \tfrac{\sigma}{\nu}\frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n}\}-\bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n+1}\}}{1-\bar{\beta}} - \tfrac{1}{\nu}\widehat{q}_t + \tfrac{1}{\nu}\tfrac{\bar{\beta}}{1-\bar{\beta}}\varrho\widehat{\bar{m}}_t + \tfrac{\sigma}{\nu}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty, \\
&(n) \quad \varphi\widehat{n}_t + \sigma\widehat{c}_t - \widehat{\xi}_{c,t} = \widehat{w}_t, \\
&(b) \quad \widehat{c}_t - \tfrac{1}{\sigma}\widehat{\xi}_{c,t} = -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n}\} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty, \\
&(x) \quad \widehat{q}_t + (1-\varrho^\tau)\frac{\varrho}{1-\varrho}\widehat{\bar{m}}_t - \sum_{n=1}^{\tau}\mathbb{E}_t\{\widehat{r}_{t+n}\} + \mathbb{E}_t\widehat{\xi}_{x,t+\tau} = (1-\eta)\widehat{x}_t, \\
&(BC) \quad \widehat{b}_{t+1} = \tfrac{1}{\beta}\widehat{b}_t + \tfrac{w_{ss}n_{ss}}{y_{ss}}(\widehat{w}_t + \widehat{n}_t) + \widehat{\Sigma}_t - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_t \\
&\qquad\qquad - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t-\tau} - \beta^\tau\widehat{x}_t + \widehat{\xi}_{x,t}\right),
\end{aligned}
\tag{C.11}
$$

where, after a few rearrangements,

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = {} & \frac{\delta q_{ss}h_{ss}/\nu}{c_{ss} + \sigma/\varphi\cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu}\cdot\left[\widehat{q}_t + \widehat{\bar{m}}_t\cdot\frac{\varrho}{1-\varrho}\underbrace{\left(1 + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho}\frac{1-\beta}{\delta}\right)}_{>0\ \forall\beta,\delta,\varrho\in(0,1)}\right] \\
& + \frac{y_{ss}}{c_{ss} + \sigma/\varphi\cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu}\frac{1-\beta}{\beta}\cdot\left[\sum_{n=1}^{\infty}\beta^n\mathbb{E}_t\{z^*_{t+n}\} + \tfrac{x_{ss}}{y_{ss}}\sum_{n=0}^{\tau-1}\beta^{-n}\widehat{x}_{t-n} + \widehat{b}_{t+1}\right].
\end{aligned}
$$

Lastly, we verify that the law of iterated expectation holds for the explicit formula given for $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$ above. Define

$$
A_t := \widehat{b}_{t+1} + \sum_{n=1}^{\infty}\beta^n(\mathbb{E}_t z^*_{t+n} + \mathbb{E}_t^{\mathcal{P}}Q_{t+n}) + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\sum_{n=1}^{\tau}\beta^n\widehat{x}_{t+n-\tau}
$$

and note that the law of iterated expectations holds if and only if $\mathbb{E}_t^{\mathcal{P}}A_{t+1} = A_t$. Using equation (BC) to substitute $\widehat{b}_{t+2}$ in $\mathbb{E}_t^{\mathcal{P}}A_{t+1}$ we arrive at:

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}}A_{t+1} = A_t/\beta & - \tfrac{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\tfrac{1-\beta}{\beta}\mathbb{E}_t^{\mathcal{P}}A_{t+1} - \tfrac{q_{ss}h_{ss}\sigma/\nu}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\tfrac{1-\beta}{\beta}\mathbb{E}_t^{\mathcal{P}}A_{t+1} \\
& + \tfrac{(1-\delta)q_{ss}h_{ss}\sigma/\nu}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\tfrac{1-\beta}{\beta}A_t,
\end{aligned}
$$

which can be rearranged into $\mathbb{E}_t^{\mathcal{P}}A_{t+1} = A_t$.

C.6 Solving the model with Dynare

Although equations (C.11) serve as an explicit solution of the household decision problem at time t,[34] they are not in a form that lends itself to easy numerical implementation, i.e. to solving the model. Bringing the equations into a recursive form that – when combined with the other equations describing equilibrium – can be solved by standard methods is the goal of this appendix.

In a first step, we take care of the appearing infinite sums which are not easily recursifiable. By that, we mean the forward summation over expected real interest rates, $\widehat{c}_t^* = -1/\sigma \cdot \sum_{n\geq 1} \mathbb{E}_t \widehat{r}_{t+n}$. In principle, this variable could be recursified as

$$\widehat{c}_t^* = -1/\sigma \cdot \mathbb{E}_t \widehat{r}_{t+1} + \mathbb{E}_t \widehat{c}_{t+1}^*.$$

This representation would be incomplete, though, without the boundary condition $\lim_{s\to\infty} \mathbb{E}_t \widehat{c}_{t+s}^* = 0$. To the best of our knowledge, imposing such a boundary condition is not possible in Dynare's native `stoch_simul`-command.[35] Therefore, we employ several rearrangements of the equations (C.11) in order to eliminate $\widehat{c}_t^*$. First, consider the geometric summation over $\widehat{c}_t^*$, which is contained in $\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\}$ (z^* is defined in equation (C.9)):

$$\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{\widehat{c}_{t+n}^*\} = -\frac{\beta}{\sigma} \sum_{n\geq 0} \beta^n \sum_{s\geq 0} \mathbb{E}_t \widehat{r}_{t+2+n+s} = -\frac{\beta}{\sigma} \sum_{n\geq 0} \mathbb{E}_t \widehat{r}_{t+2+n} \cdot \sum_{s=0}^{n} \beta^s = \frac{\beta}{1-\beta} \Bigg[\underbrace{-\frac{1}{\sigma} \sum_{s\geq 0} \mathbb{E}_t \widehat{r}_{t+2+s}}_{=\mathbb{E}_t \widehat{c}_{t+1}^*} + \frac{1}{\sigma} \sum_{s\geq 1} \beta^s \mathbb{E}_t \widehat{r}_{t+1+s} \Bigg].$$

Next, consider the housing terms included in z^*:

$$\begin{aligned} \mathbb{E}_t \big[\widehat{h}_{t+s}^* - (1-\delta)\widehat{h}_{t+s-1}^*\big] &= \frac{\sigma}{\nu} \mathbb{E}_t \frac{\widehat{c}_{t+s}^* - \bar{\beta}\widehat{c}_{t+1+s}^*}{1-\bar{\beta}} - (1-\delta)\frac{\sigma}{\nu} \mathbb{E}_t \frac{\widehat{c}_{t-1+s}^* - \bar{\beta}\widehat{c}_{t+s}^*}{1-\bar{\beta}} \\ &= -\frac{1}{\nu} \mathbb{E}_t \frac{\widehat{r}_{t+s+1} + (1-\bar{\beta}) \sum_{n\geq 1} \widehat{r}_{t+1+s+n}}{1-\bar{\beta}} + (1-\delta)\frac{1}{\nu} \mathbb{E}_t \frac{\widehat{r}_{t+s} + (1-\bar{\beta}) \sum_{n\geq 1} \widehat{r}_{t+s+n}}{1-\bar{\beta}} \\ &= -\frac{1}{\nu} \frac{\beta(1-\delta)^2 + \delta}{1-\bar{\beta}} \mathbb{E}_t \widehat{r}_{r+s+1} + \frac{1}{\nu} \frac{1-\delta}{1-\bar{\beta}} \mathbb{E}_t \widehat{r}_{r+s} - \frac{\delta}{\nu} \sum_{n\geq 1} \mathbb{E}_t \widehat{r}_{t+s+1+n}. \end{aligned}$$

Symmetrically to before, the geometric summation over these terms that is contained in $\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\}$

[34]In the sense that they provide an equation system describing the household decsisions purely in terms of contemporaneous or pre-determined variables and expectations taken over the objective law.

[35]Although it is possible with the command `perfect_foresight_solver`, a routine we chose not to use due to its inability to automatically check the Blanchard-Kahn condition.

thus reads

$$\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t\big[\widehat{h}^*_{t+s}-(1-\delta)\widehat{h}^*_{t+s-1}\big]=\sum_{s=1}^{\infty}\beta^s\Big[-\tfrac{1}{\nu}\tfrac{\beta(1-\delta)^2+\delta}{1-\beta}\mathbb{E}_t\widehat{r}_{r+s+1}+\tfrac{1}{\nu}\tfrac{1-\delta}{1-\beta}\mathbb{E}_t\widehat{r}_{r+s}\Big]+\tfrac{\delta\sigma}{\nu}\tfrac{\beta}{1-\beta}\mathbb{E}_t\widehat{c}^*_{t+2}+\tfrac{\delta}{\nu}\tfrac{\beta}{1-\beta}\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t\widehat{r}_{t+s+2}.$$

We can now restate the entire geometric sum:

$$\begin{aligned}
\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t z^*_{t+s} &= \sum_{s=1}^{\infty}\beta^s\mathbb{E}_t\Big[\tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+s}+\widehat{\Sigma}_{t+s}-\tfrac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+s}+\tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+s}\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{c}^*_{t+s}-\tfrac{q_{ss}h_{ss}}{y_{ss}}\big[\widehat{h}^*_{t+s}-(1-\delta)\widehat{h}^*_{t+s-1}\big]\Big]\\
&= \sum_{s=1}^{\infty}\beta^s\mathbb{E}_t\Big[\tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+s}+\widehat{\Sigma}_{t+s}-\tfrac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+s}+\tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+s}\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\tfrac{\beta}{1-\beta}\tfrac{1}{\sigma}\widehat{r}_{t+1+s}-\tfrac{q_{ss}h_{ss}}{y_{ss}}\Big[-\tfrac{1}{\nu}\tfrac{\beta(1-\delta)^2+\delta}{1-\beta}\widehat{r}_{t+s+1}+\tfrac{1}{\nu}\tfrac{1-\delta}{1-\beta}\widehat{r}_{r+s}+\tfrac{\delta}{\nu}\tfrac{\beta}{1-\beta}\widehat{r}_{t+s+2}\Big]\Big]\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\tfrac{\beta}{1-\beta}\mathbb{E}_t\widehat{c}^*_{t+1}-\tfrac{\delta q_{ss}h_{ss}}{y_{ss}}\tfrac{\sigma}{\nu}\tfrac{\beta}{1-\beta}\underbrace{\mathbb{E}_t\widehat{c}^*_{t+2}}_{=\mathbb{E}_t\widehat{c}^*_{t+1}+\frac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+2}}.
\end{aligned}$$

The remaining geometric sums may easily be recursified:

$$\begin{aligned}
\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t z^*_{t+s} &= \beta\mathcal{Z}^*_t-\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}{y_{ss}}\tfrac{\beta}{1-\beta}\mathbb{E}_t\widehat{c}^*_{t+1}-\tfrac{\delta q_{ss}h_{ss}}{y_{ss}}\tfrac{\sigma}{\nu}\tfrac{\beta}{1-\beta}\tfrac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+2}\quad\text{where}\\
\mathcal{Z}^*_t &= \beta\mathcal{Z}^*_{t+1}+\mathbb{E}_t\Big[\tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+1}+\widehat{\Sigma}_{t+1}-\tfrac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+1}+\tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+1}\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\tfrac{\beta}{1-\beta}\tfrac{1}{\sigma}\widehat{r}_{t+2}-\tfrac{q_{ss}h_{ss}}{y_{ss}}\Big[-\tfrac{1}{\nu}\tfrac{\beta(1-\delta)^2+\delta}{1-\beta}\widehat{r}_{t+2}+\tfrac{1}{\nu}\tfrac{1-\delta}{1-\beta}\widehat{r}_{r+1}+\tfrac{\delta}{\nu}\tfrac{\beta}{1-\beta}\widehat{r}_{t+3}\Big]\Big].
\end{aligned}$$

Now, defining $\mathcal{C}_t := \mathbb{E}^{\mathcal{P}}_t\widehat{c}_\infty+\mathbb{E}_t\widehat{c}^*_{t+1}+\frac{\delta q_{ss}h_{ss}}{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}\frac{1}{\nu}\mathbb{E}_t\widehat{r}_{t+2}$ allows us to arrive at the final

Dynare-ready formulation of the household's decision rules:

$$
\begin{aligned}
&(h) \quad \widehat{h}_t = \tfrac{1}{\nu}\xi_{h,t} - \tfrac{1}{1-\bar{\beta}}\tfrac{1}{\nu}\mathbb{E}_t\widehat{r}_{t+1} - \Theta\tfrac{1}{\nu}\mathbb{E}_t\widehat{r}_{t+2} - \tfrac{1}{\nu}\widehat{q}_t + \tfrac{1}{\nu}\tfrac{\bar{\beta}}{1-\bar{\beta}}\varrho\widehat{\bar{m}}_t + \tfrac{\sigma}{\nu}\mathcal{C}_t, \\
&(b) \quad \widehat{c}_t - \tfrac{1}{\sigma}\widehat{\xi}_{c,t} = -\tfrac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+1} - \Theta\tfrac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+2} + \mathcal{C}_t, \\
&\qquad (\mathcal{C}) \quad \mathcal{C}_t = \Theta\tfrac{1}{\sigma}\cdot\left[\widehat{q}_t + \Lambda\cdot\widehat{\bar{m}}_t\right] + \Gamma y_{ss}\tfrac{1-\beta}{\beta}\cdot\left[\beta\mathcal{Z}_t^* + \tfrac{x_{ss}}{y_{ss}}\sum_{n=0}^{\tau-1}\beta^{-n}\widehat{x}_{t-n} + \widehat{b}_{t+1}\right], \\
&\qquad (\mathcal{Z}) \quad \mathcal{Z}_t^* = \beta\mathcal{Z}_{t+1}^* + \mathbb{E}_t\left\{\begin{array}{c} \frac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+1} + \widehat{\Sigma}_{t+1} - \frac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+1} + \frac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+1} \\ -\frac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\frac{\beta}{1-\beta}\frac{1}{\sigma}\widehat{r}_{t+2} - \frac{q_{ss}h_{ss}}{y_{ss}}\left[-\frac{1}{\nu}\frac{\beta(1-\delta)^2+\delta}{1-\bar{\beta}}\widehat{r}_{t+2} + \frac{1}{\nu}\frac{1-\delta}{1-\bar{\beta}}\widehat{r}_{r+1} + \frac{\delta}{\nu}\frac{\beta}{1-\beta}\widehat{r}_{t+3}\right] \end{array}\right\}, \qquad \text{(C.12)} \\
&(n) \quad \varphi\widehat{n}_t + \sigma\widehat{c}_t - \widehat{\xi}_{c,t} = \widehat{w}_t, \\
&(x) \quad \widehat{q}_t + (1-\varrho^\tau)\frac{\varrho}{1-\varrho}\widehat{\bar{m}}_t - \sum_{n=1}^{\tau}\mathbb{E}_t\{\widehat{r}_{t+n}\} + \mathbb{E}_t\widehat{\xi}_{x,t+\tau} = (1-\eta)\widehat{x}_t, \\
&(BC) \quad \widehat{b}_{t+1} = \tfrac{1}{\beta}\widehat{b}_t + \tfrac{w_{ss}n_{ss}}{y_{ss}}(\widehat{w}_t + \widehat{n}_t) + \widehat{\Sigma}_t - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_t \\
&\qquad\qquad - \tfrac{q_{ss}h_{ss}}{y_{ss}}[\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t-\tau} - \beta^\tau\widehat{x}_t + \widehat{\xi}_{x,t}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
&\Theta := \delta q_{ss}h_{ss}\sigma/\nu\cdot\Gamma \in (0,1), \quad \Lambda := \frac{\varrho}{1-\varrho}\left(1 + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho}\frac{1-\beta}{\delta}\right) > 0, \\
&\Gamma := (c_{ss} + \sigma/\varphi\cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu)^{-1}.
\end{aligned}
$$

(Notice that $\Lambda > 0$ is proven in footnote 33 in Appendix C.5.)

C.7 Derivations for Section III.C

In this Appendix we derive analytical results on the behavior of house prices in a one-region, zero liquidity endowment economy with instantaneous housing production. It is convenient to define the auxiliary variable $R_t := \sum_{n\geq 1}\mathbb{E}_t\widehat{r}_{t+n}$. L denotes the lag operator, i.e. $\mathsf{L}x_t := x_{t-1}$.

The equations describing the economy are:

$$
\begin{aligned}
&(h) \quad \widehat{q}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} = \sigma(\widehat{c}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}) - \nu(1-\bar{\beta})\widehat{h}_t, \\
&(b) \quad \widehat{c}_t = -\sigma^{-1}\mathbb{E}_t\widehat{r}_{t+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}, \\
&(x) \quad \widehat{x}_t = \widehat{q}_t/(1-\eta), \\
&(BC) \quad \widehat{b}_{t+1} = \beta^{-1}\widehat{b}_t + \widehat{y}_t - \tfrac{c}{y}\widehat{c}_t - \tfrac{qh}{y}(\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}), \\
&(B) \quad \widehat{b}_{t+1} = 0, \\
&(HMC) \quad \widehat{h}_t - (1-\delta)\widehat{h}_{t-1} = \delta\eta\widehat{x}_t,
\end{aligned} \qquad \text{(C.13)}
$$

with the endowment process $\widehat{y}_t$ being the only exogenous disturbance to the economy. Naturally,

$\mathbb{E}_t^{\mathcal{P}}$ is replaced by the objective expectation operator, $\mathbb{E}$, whenever we consider the model version with rational expectations. Whenever we consider the model version under subjective expectations, the following additional equations hold:

$$
\begin{aligned}
&(\mathbb{E}^{\mathcal{P}}) \quad \mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} = \widehat{q}_t + \tfrac{1-\varrho^s}{1-\varrho} \varrho \widehat{\overline{m}}_t, \ \forall s \geq 0, \\
&(m) \quad \widehat{\overline{m}}_t = (\varrho - g)\widehat{\overline{m}}_{t-1} + g\Delta\widehat{q}_{t-1}, \\
&(\widehat{c}_\infty) \quad \mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty = \tfrac{y}{c+\delta q h \sigma/\nu} \tfrac{1-\beta}{\beta} \left[\widehat{b}_{t+1} + \sum_{n\geq 1} \beta^n \mathbb{E}_t z^*_{t+n} + \sum_{n \geq 1} \beta^n \mathbb{E}_t^{\mathcal{P}} Q_{t+n} \right] \text{ with} \\
&\qquad\qquad z_t^* := \widehat{y}_t + \tfrac{c}{y}\sigma^{-1} R_t + \tfrac{qh}{\nu y}\left[\tfrac{R_t - \bar{\beta}\mathbb{E}_t R_{t+1}}{1-\bar{\beta}} - (1-\delta)\tfrac{R_{t-1} - \bar{\beta}\mathbb{E}_{t-1} R_t}{1-\bar{\beta}} \right] \\
&\qquad\qquad \mathbb{E}_t^{\mathcal{P}} Q_{t+n} := \tfrac{qh}{\nu y}\tfrac{1}{1-\bar{\beta}} \mathbb{E}_t^{\mathcal{P}} \left\{ (1-(1-\delta)\mathsf{L})(\widehat{q}_{t+n} - \bar{\beta}\widehat{q}_{t+n+1}) \right\} \\
&\qquad\qquad\qquad = \tfrac{qh}{\nu y}\left[\delta \widehat{q}_t + \tfrac{\varrho}{1-\varrho}\left(\delta + \tfrac{1-\bar{\beta}\varrho}{1-\bar{\beta}} (1-\varrho-\delta)\varrho^{n-1}\widehat{\overline{m}}_t \right) \right].
\end{aligned}
\tag{C.14}
$$

It is furthermore helpful to keep in mind that in the non-stochastic steady state the following relationship holds: $\delta \eta q h = x$.

Regardless of the way in which expectations are formed, we examine the house price response to an endowment shock of a particular form: $\widehat{y}_t$ is such that the real rate drops by $\varepsilon > 0$ on impact of the shock and returns to 0 afterwards; furthermore the shock is of an MIT-form – all agents understand that no shock will ever again materialize; in sum:

1. $\widehat{r}_{t+1} = -\mathbb{1}_{t=0}\varepsilon$
2. $\mathbb{E}_t \varepsilon = \mathbb{1}_{t\geq 0}\varepsilon$
3. $R_t = -\mathbb{1}_{t=0}\varepsilon$
4. $\mathbb{E}_t R_{t+n} = 0, \ \forall n > 0$

This is arguably a parsimonious way of modeling the effects of expansive monetary policy. Once we have solved for the path of house prices, housing investment, and consumption, we may back out the underlying sequence of endowments via the goods market clearing condition, $\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x}_t$.

C.7.1 *Rational Expectations*

We start by noticing that the lag operator has the inverse $\mathsf{L}^{-1} x_t = \mathbb{E}_t x_{t+1}$ on the space of real-valued, integer-indexed sequences. This permits us to write (C.13)-(h) as:

$$(1 - \bar{\beta}\mathsf{L}^{-1})\widehat{q}_t = \mathbb{1}_{t=0}\varepsilon - (1-\bar{\beta})\nu \widehat{h}_t.$$

Next, we define the parameter $\chi := \frac{(1-\bar{\beta})\nu\eta}{1-\eta} > 0$, apply the operator $(1-(1-\delta)\mathsf{L})$ and use it to substitute out the housing term for housing investment, using (HMC). Housing investment, $\widehat{x}_t$ in turn is replaced using equation (x), finally yielding:

$$(1-(1-\delta)\mathsf{L})(1-\bar{\beta}\mathsf{L}^{-1})\widehat{q}_t = \mathbb{1}_{t=0}\varepsilon - (1-\delta)\mathbb{1}_{t=1}\varepsilon - \chi\widehat{q}_t,$$

which we may rewrite as

$$\wp(\mathsf{L})\widehat{q}_t = -\mathbb{1}_{t=0}\varepsilon + (1-\delta)\mathbb{1}_{t=1}\varepsilon,$$

with $\wp : \mathbb{C} \to \mathbb{C} : z \mapsto \chi + (1-(1-\delta)z)(1-\bar{\beta}z^{-1})$. The polynomial $z \mapsto -\wp(z)z/(1-\delta)$ has the roots

$$\varpi := \tfrac{1}{2}[\Xi - \sqrt{\Xi^2 - 4\beta}], \text{ and } \beta/\varpi,$$

with $\Xi := (\chi + 1 + \bar{\beta}(1-\delta))/(1-\delta) > 1$. It can be shown that $\beta/\varpi > 1 > \varpi > 0$. This permits factorization of $\wp$ into a forward-stable operator and a backward-stable operator:

$$\wp(\mathsf{L}) = \bar{\beta}/\varpi(1-\varpi\mathsf{L}^{-1})(1-\varpi/\beta\mathsf{L}),$$

and produces the solution

$$\widehat{q}_t = \varepsilon\varpi/\bar{\beta}\cdot\left((\varpi/\beta)^t(1-\varpi(1-\delta)) - \mathbb{1}_{t\geq 1}(\varpi/\beta)^{t-1}(1-\delta)\right).$$

C.7.2 *Subjective Expectations*

We now show that selecting the endowment sequence such that the goods market clears, $\widehat{y}_t := \frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x}_t$ for given $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$, renders equation (C.14)-($\widehat{c}_\infty$) co-linear to equation (h).

To show this, start by defining $\Theta := \frac{\delta qh\sigma/\nu}{c+\delta qh\sigma/\nu}$. This allows to write (C.14)-($\widehat{c}_\infty$) as

$$\tfrac{\beta}{1-\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = (1-\Theta)\tfrac{y}{c}\left(\widehat{b}_{t+1} + \sum_{n\geq 1}\beta^n\mathbb{E}_t z^*_{t+n} + \sum_{n\geq 1}\beta^n\mathbb{E}_t^{\mathcal{P}}Q_{t+n}\right).$$

Next, we use (BC) to substitute $\widehat{b}_{t+1} = \widehat{y}_t - \frac{c}{y}\widehat{c}_t - \frac{qh}{y}(\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}) = z^*_t - \frac{c}{y}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$ $-\frac{qh}{\nu y}\left[\frac{R_t-\bar{\beta}\mathbb{E}_tR_{t+1}}{1-\bar{\beta}} - (1-\delta)\frac{R_{t-1}-\bar{\beta}\mathbb{E}_{t-1}R_t}{1-\bar{\beta}}\right] - \frac{qh}{y}(\widehat{h}_t - (1-\delta)\widehat{h}_{t-1})$. Now we apply the operator $(1-\beta\mathsf{F}^{\mathcal{P}})$ to both sides, where $\mathsf{F}^{\mathcal{P}}x_t := \mathbb{E}_t^{\mathcal{P}}x_{t+1}$.[36] This produces

$$\beta\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = (1-\Theta)\tfrac{y}{c}\Big[z^*_t + \beta\mathbb{E}_t^{\mathcal{P}}Q_{t+1} - (1-\beta)\tfrac{c}{y}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty - \tfrac{qh}{y}(\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}) + \beta\tfrac{qh}{y}(\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1} - (1-\delta)\widehat{h}_t)$$
$$- \tfrac{qh}{y}\tfrac{1}{\nu(1-\bar{\beta})}\varepsilon[-\mathbb{1}_{t=0} + (1-\delta)\mathbb{1}_{t=1} - \bar{\beta}\mathbb{1}_{t=0}].$$

[36] Notice that $\mathsf{F}^{\mathcal{P}}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = \mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$, see the discussion at the end of Appendix C.5.

Now we substitute z_t^* using its definition, we substitute $\mathbb{E}_t^{\mathcal{P}} Q_{t+1} = \frac{qh}{\nu y}\frac{1}{1-\bar{\beta}}\mathbb{E}_t^{\mathcal{P}}\left\{(1-(1-\delta)\mathsf{L})(\widehat{q}_{t+1}-\bar{\beta}\widehat{q}_{t+2})\right\}$, we substitute $\frac{qh}{y}(\widehat{h}_t-(1-\delta)\widehat{h}_{t-1}) = \frac{x}{y}\widehat{x}_t$, and we substitute $\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1} = \frac{\sigma}{\nu}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty - \frac{1}{\nu}\frac{\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}-\bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+2}}{1-\bar{\beta}}$. After cancelling terms, this delivers

$$\sigma\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = \frac{-\mathbb{1}_{t=0}\varepsilon}{1-\bar{\beta}} + \nu\left[\widehat{h}_t + \frac{1}{\nu}\frac{\widehat{q}_t-\bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}}{1-\bar{\beta}}\right],$$

which is equivalent to equation (h). This shows that as long as equation (h) holds and we choose $\widehat{y}_t$ such that the real rate behaves as described and the goods market clearing condition holds, equation $(\widehat{c}_\infty)$ holds.