United in Booms, Divided in Busts: Regional House Price Cycles and Monetary Policy

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Abstract

This paper shows that regional disparities in house price growth are more pronounced during house price busts than during booms. To explain this observation we construct a two-region currency union model incorporating a housing sector and extrapolative belief updating regarding house prices. We show that intensified extrapolation in busts and regional housing market heterogeneities jointly explain elevated regional house price growth dispersion in busts. Validating our theory, we provide empirical evidence that house price belief updating is indeed more pronounced in busts and document that regional heterogeneities on the housing-supply side affect regional house prices. Quantitatively our model can match the empirically observed elevated regional house price growth dispersion in busts. Moreover, we demonstrate that a monetary authority targeting house prices may reduce the volatility of output and house prices, as well as cross-regional disparities. This policy is welfare-improving relative to standard inflation-targeting benchmark.

JEL Codes: E31, E32, E52, F45
Keywords: Monetary Policy, Currency Area, Structural Heterogeneity, Subjective House Price Expectations, Housing Booms, Asset Price Learning

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I. Introduction

A substantial body of literature connects boom-bust cycles in house prices to over-optimism and over-pessimism regarding the future evolution of house prices.[1] The bust phases of these cycles are often associated with significant economic costs, making them a primary concern for policy-makers, particularly central bankers.[2] However, housing markets are inherently regional, and so too are housing cycles and the economic costs associated with them. While a substantial literature examines housing cycles and their implications for monetary policy at the federal level, there is comparatively little focus on regional heterogeneities in house price dynamics. This paper seeks to address this gap.

This paper contributes to the literature in several key ways. Empirically, we demonstrate that regional disparities in house price growth are more pronounced during housing busts than during booms. To account for this observation, we develop a two-region currency union model that incorporates regional heterogeneity in housing markets and extrapolative belief updating regarding house prices. In this model, intensified extrapolation in busts and regional housing market heterogeneities jointly explain cross-regional house price growth divergences in busts.

To validate our theoretical modeling choices we turn to the data. First, we show that households' expectations of future house price growth are influenced by realized growth rates, with a stronger extrapolation from past to expected rates during busts than in booms. Second, we provide evidence that cross-regional variation in house prices is associated with structural differences in housing supply. Using these insights, we calibrate our model to the US economy, demonstrating that it quantitatively matches the response of aggregate house prices and its' regional dispersion in growth rates to monetary policy shocks in both booms and busts. Finally, we illustrate that by leaning against aggregate house prices, the monetary authority can mitigate cross-regional disparities in house prices and economic activity, resulting in welfare improvements relative to the baseline case.

We begin by presenting our central empirical finding, namely that cross-regional dispersion in house price growth rates are more pronounced during busts, both unconditionally and conditionally on specific shocks. First, we document that the cross-regional standard deviation of house price growth among major US cities is 50%-60% larger during bust periods compared to booms.[3] Sec-

[1] See e.g. :Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020); Kaplan et al. (2020)

[2] During the Great Financial Crises 20% of household wealth was lost (Dettling et al., 2018), and 8 million jobs were lost (Mian and Sufi, 2015). The FED cut the rates from 5.2 to 0.12 and introduced further policy measures such as Forward Guidance and Quantitative easing.

[3] Throughout the paper, a bust period is defined as one in which house prices declined in the previous period. For

ond, conditional on monetary policy shocks, we observe a significant and sizable increase in the cross-city standard deviations of house price growth during busts, while the response is muted and far less significant during boom periods. Our unconditional findings are robust when considering US states or Euro Area countries in the cross-section. Similarly, the conditional results hold regarding other types of shocks.

To rationalize the structural forces giving rise to our empirical findings, we construct a two-region currency union New Keynesian model. We extend the framework of Benigno (2004) by (i) incorporating a housing sector, and (ii) allowing for subjective expectations about house prices. Housing markets are regional, and regions are symmetric except for the housing supply side. In this model, households form subjective expectations only about house prices. This assumption is motivated by the central role of house prices in our framework. However, households' endogenous choices, which depend on future house prices, are also shaped by subjective expectations. This raises the issue of characterizing subjective beliefs regarding these variables.

To solve this type of model, we propose a novel solution method for solving general equilibrium models with time-consistent but non-rational expectations, such as extrapolative asset price beliefs. Our approach offers the advantage of restricting subjective expectations exclusively to house prices while maintaining the rational expectations hypothesis for all other variables exogenous to agents. This method further enables us to explicitly characterize the expectations of endogenous variables, i.e. household choices. The ability to confine subjective expectations to specific variables allows us to model expectation formation in a data-consistent way where data is available while adhering to the established benchmark for variables where data is unavailable. Additionally, the first-order approximation renders our model compatible with standard solution methods used in the DSGE literature, making it easy to handle, scale, and serve as a basis for Ramsey-type analyses of optimal policy.[4]

Equipped with our model, we first demonstrate analytically that regional differences in housing markets combined with house price belief extrapolation, can account for the asymmetry in house price growth dispersion observed across the housing cycle.[5] Subjective expectations about house prices introduce a backward-looking component through extrapolation. This mechanism

this analysis, we use quarterly data.

[4]Our solution method contrasts with that introduced by Winkler (2020), which, although based on first-order approximation, does not confine subjective expectations to specified asset prices and results in belief spillovers to all other prices. Conversely, Adam et al. (2022) solve a similar model using non-linear solution techniques.

[5]In our case, we focus on monetary policy shocks, but the general proposition extends to all shocks that affect housing demand.

dynamically amplifies the responsiveness of house prices: when agents observe an increase in house prices, they anticipate further price increases in the future, leading to heightened demand for housing and, subsequently, driving prices even higher. Simultaneously, regional heterogeneity in housing markets creates regional differences in the responsiveness of house prices. The interaction of these two forces generates the following dynamics: regional heterogeneities in housing markets create differences in regional house price growth rates, which are further amplified by extrapolation. This result allows us to align our theoretical model with the empirically observed patterns. Specifically, our model is capable of generating pronounced regional dispersion in house price growth during busts, provided that extrapolation is stronger in those periods. The intuition is straightforward: extrapolation amplifies regional disparities, so when extrapolation intensifies during busts, it results in greater regional dispersion in house price growth, consistent with the empirical evidence.

The following example, highlighting stronger extrapolation during busts and differences in housing supply, illustrates this mechanism. Suppose an expansionary monetary policy shock increases housing demand. House prices rise moderately in a region with an elastic housing supply and more strongly in a region with an inelastic supply. Households extrapolate from these changes, expecting higher future price growth, particularly in the inelastic region where the initial price rise was greater. This optimism increases housing demand and further raises prices, especially in the inelastic region. Thus, extrapolation amplifies the initial regional price differences. This effect is more pronounced during busts, as expectations react more strongly to observed price movements, intensifying cross-regional dispersion in house price growth in these periods.

In a version of the model with fully rational expectations, the ability of cross-regional differences in housing markets to produce differences in house prices in response to a shock is much weaker. Additionally, this version lacks extrapolative belief updating and thus cannot account for amplified cross regional heterogeneity in busts in the absence of any additional frictions. Moreover, this model version is generally less responsive to shocks.

To test our theoretical predictions we move to the data. First, we estimate the extent to which US households' house price expectations depend on prior expectations and past house price growth, and find that the pass-through from past house price growth to expectations is stronger during busts than during booms—households update their beliefs more strongly during busts. Second, we investigate the response of regional house prices to a common monetary policy shock using a panel local projections approach. Furthermore, we differentiate these responses based on regional housing supply conditions by incorporating interaction terms. We find that in more supply-constrained regions house prices exhibit a stronger response to a monetary policy shock. These findings are consistent with our theoretical predictions and provide a basis for calibrating our model with respect to house price belief formation and heterogeneities in the housing supply.

We use the fully calibrated model to analyze the response of house prices relative to an empirical counterpart. Specifically, we target the aggregate house price response to a monetary policy shock, independent of boom-bust asymmetries in belief updating. The empirical counterpart is derived from a local projections exercise. The model accurately captures the magnitudes of aggregate house price changes in both booms and busts when accounting for asymmetries in the belief updating process. It also successfully replicates the regional house price growth dispersion observed during booms and busts in response to a monetary policy shock. In contrast, the rational expectations version of the model fails to match these dynamics, underestimating aggregate house price responses by a factor of three and generating no significant regional dispersion.

Finally, we investigate the extent to which leaning against house prices influences economic outcomes. Specifically, we analyze the effects of incorporating aggregate house prices into the monetary authority's Taylor rule on the volatility and regional dispersion of inflation, output, and house prices. Our findings suggest that placing greater emphasis on house prices within the Taylor rule reduces the volatility of both output and house prices, albeit at the cost of increased inflation volatility. Notably, a stronger response to house prices reduces the regional dispersion for all variables. In our framework, leaning against house prices therefore has the additional effect of synchronizing economic activity across regions. The underlying intuition is that regional heterogeneities originate in housing markets and are subsequently propagated throughout the economy via house prices. By targeting house prices, the central bank reduces the extent to which house price beliefs can diverge, thereby mitigating the channel through which regional heterogeneities in housing markets can spill over to the real economy. We show that house price targeting is beneficial under subjective expectations: in booms, welfare is improved by 6% relative to the baseline. In busts we observe an welfare gain of 25%.

Related literature. Our paper relates to a broad empirical literature emphasizing that house price beliefs are not formed according to rational expectations.[6] This literature highlights momentum and revisions in the belief formation of house prices. On the theoretical side, we connect to the behavioral macro-finance literature, which focuses on deviations from rational expectations in terms of asset price expectations.[7] More specifically, our paper relates to the literature on

[6] See, for instance, Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020).

[7] Among others, see: Bordalo et al. (2018); Barberis (2018); Caballero and Simsek (2019, 2020); Krishnamurthy and Li (2020); L'Huillier et al. (2023); Maxted (2024); Bianchi et al. (2024).

capital gains extrapolation.[8] Regarding house prices Glaeser and Nathanson (2017) and Schmitt and Westerhoff (2019) model house price expectations through certain forms of extrapolation in a partial equilibrium environment. In contrast, we focus on a general equilibrium New Keynesian environment, which is most closely related to Adam et al. (2012), Caines and Winkler (2021), and Adam et al. (2022). Other studies, such as Burnside et al. (2016), Guren (2018), and Kaplan et al. (2020), explain house price behavior through optimism and pessimism, concave demand curves faced by sellers in the housing market, or exogenous shifts in house price beliefs. Our contribution to this body of literature lies in emphasizing the asymmetric evolution of asset price beliefs during boom and bust episodes. Additionally, we introduce a new solution method for models incorporating capital gains extrapolation.

We also contribute to the literature examining regional housing supply variations in the United States. Studies by Mian et al. (2013), Mian and Sufi (2014), and Guren et al. (2021) leverage housing supply elasticities to explore the housing wealth effect. Our work is closely related to the studies by Aastveit and Anundsen (2022) and Aastveit et al. (2023), which demonstrate that house prices in US metropolitan areas with more inelastic housing supply exhibit greater responsiveness to monetary policy shocks. We focus on the state level instead. Furthermore, we show that similar patterns can be observed in the Euro Area. While the aforementioned studies are primarily empirical, our contribution lies in providing theoretical insights to interpret our empirical findings. We also connect to Glaeser et al. (2008), who show both theoretically and empirically that more supply-inelastic regions have a higher probability of experiencing a house price bubble. In contrast to their approach, we highlight asymmetries in house price belief formation, construct a general equilibrium New Keynesian model, and focus on monetary policy.

We focus on a setup involving multiple regions governed by a single monetary authority, thereby connecting to the literature on cross-regional heterogeneities in currency unions. Studies by Benigno (2004), Galí and Monacelli (2008), and Kekre (2022) examine optimal policy in a currency union setting. Additionally, Calza et al. (2013), Slacalek et al. (2020), Bletzinger and von Thadden (2021), Pica (2021), Almgren et al. (2022), and Corsetti et al. (2022) explore various sources of heterogeneity within currency unions and their effects on economic activity. In this strand of literature, cross-regional heterogeneities include variations in price and wage setting, the share of hand-to-mouth consumers, and mortgage market dynamics. For the US, Beraja et al. (2019), Chen (2019), and Gitti (2024) focus on regional differences in wage and price setting, exposure to fiscal policy shocks, housing markets, and migration. To our knowledge, we are the first to study supply-side housing constraints and connect these to asymmetries in the formation of

[8] Adam et al. (2017) and Winkler (2020) examine asset price learning in the context of stock markets. Fontanier (2022) and Farhi and Werning (2020) focus on asset price extrapolation more generally.

house price beliefs.

Finally, we connect to the literature on leaning against asset prices. Stein (2012) argues that leaning against the wind can address externalities arising from over-borrowing. Svensson (2017) argues that the costs of these policies outweigh their benefits, while Weidmann (2018) points out that macroprudential policies are better suited to address these concerns. Gourio et al. (2018) quantitatively outlines a trade-off between a lower crisis probability and increased cyclical inflation and output dynamics. However, once rational expectations are abandoned, leaning against the wind generally becomes beneficial (Adam and Woodford, 2021; Caines and Winkler, 2021). This holds true even if macroprudential tools are fully (Fontanier, 2022) or at least partially available (Caballero and Simsek, 2019). Our paper contributes to this literature by emphasizing that targeting asset prices can also mitigate regional dispersion and thus help harmonize an economically integrated area. To our knowledge, we are the first to make this point.

Outline. The rest of this paper is organized as follows. In Section (II), we document our main empirical observation: regional house price growth is more dispersed in busts than in booms. Section (III) outlines our theoretical model and describes our solution method. In Section (IV) we analytically show how extrapolation on house price growth and housing market heterogeneity can explain our main empirical observation. Section (V) provides evidence of stronger extrapolation in busts, housing markets supply side heterogeneity, and describes the model calibration. Section (VI) presents our quantitative results. Finally, Section (VII) contains the policy exercise. Section (VIII) concludes.

II. Regional house prices in booms and busts

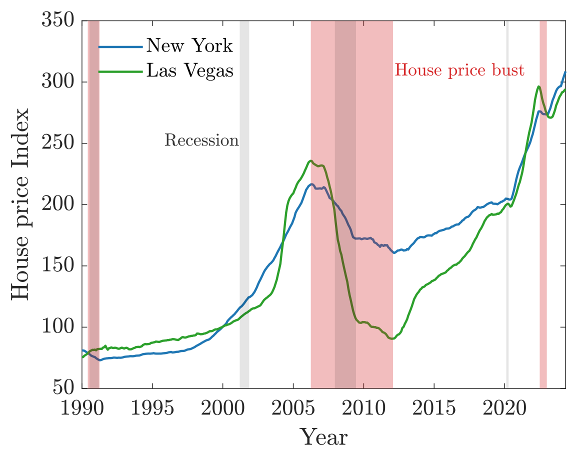
House prices experience large regional differences. This is in particular the case in times of house price busts. Figure (1) illustrates the house price index for New York and Las Vegas, with the shaded red areas indicating periods of house price busts. For the purposes of analysis, housing booms are defined as periods in which federal house price growth is positive ($\Delta q_t^{agg} > 0$), while busts are characterized by negative growth ($\Delta q_t^{agg} < 0$).[9]

The figure reveals that house prices tend to move in tandem during boom periods; however, during busts, significant divergences emerge. In particular, house prices in Las Vegas experienced a sharp decline during the Great Recession. By contrast, while house prices in New York also declined, the reduction was far less severe. Importantly, the observed divergence in house price

[9] House price growth rates on the aggregate/federal level (Δq_t^{agg}) are based on the Case-Shiller Index.

growth during busts cannot be attributed solely to economic recessions. For instance, there was minimal divergence during the Dotcom bubble and the COVID-19 crisis, both of which were periods marked by overall positive federal house price growth.

Figure 1: Regional house prices in booms and busts



Notes: Case-Shiller house price index for New York and Las Vegas.

Cross-regional house price variation, unconditional. To further investigate and generalize this pattern, we extend the analysis to regional house price growth data on different levels. We utilize regional house price growth data, specifically at the major city or state level for the United States. For each point in time, we compute the cross-regional standard deviation of house price growth rates. We then split the sample into boom and bust episodes, where these episodes are defined based on the aggregate house price growth of the economic entity. Our preferred specification is based on US major cities in the cross-section.[10] The data is given in monthly frequency and therefore allows for sufficient observations in booms and busts. On the US state level, we use quarterly data. Our sample includes all states except Alaska and Hawaii but includes the District of Columbia. Both US data sets span from 1990 until 2024. Table (1) reports the mean and median cross-regional standard deviations.

[10]We use the Case-Shiller house price index on the city level. The cities include 20 major US cities: Atlanta, Boston, Charlotte, Chicago, Cleveland, Dallas, Denver, Detroit, Las Vegas, Los Angeles, Miami, Minneapolis, New York, Phoenix, Portland, San Diego, San Francisco, Seattle, Tampa, Washington DC.

Table 1: Cross-regional house price growth standard deviation (σ_c) in booms and busts

	$US, cities$		$US, states$	
	$mean$	$median$	$mean$	$median$
$\frac{Bust}{Boom}$	**1.60**	**1.64**	**1.23**	**1.26**
$\sigma_c : \ Boom$	0.47	0.41	0.87	0.80
$\sigma_c : \ Bust$	0.74	0.67	1.08	1.01
$p - Val. \ Bust > Boom$	0.000	0.000	0.007	0.007
$Number \ regions$	20		51	
$Obs. \ boom$	331		109	
$Obs. \ bust$	82		28	
$Sample$	$1990M1 - 2024M5$		$1990Q1 - 2024Q1$	

Notes: The Table reports the mean and median, across time, of the estimates of cross-sectional standard deviations of house price growth rates within the given economic entity. The test for *Bust* > *Boom* is based on a one-sided t-test.

Our findings indicate that the cross-regional standard deviations during busts are larger for both the mean and median estimates than those observed during booms. Specifically, cross-regional standard deviations during busts are 60% larger than booms if one considers the monthly US city data. The ratio decreases to 23% – 30% for the US states. We find that a t-test rejects the H_0 hypothesis of regional standard deviations being larger in booms for both regional specifications. Table (C.1) in the Appendix shows that the same holds for the Euro Area on country level.

Cross-regional house price variation, conditional. While the previous analysis focused on unconditional variations in house price growth during booms and busts, we now move to a conditional approach. Using local projections, we estimate the response of the cross-regional standard deviation of house price growth to a monetary policy shock (MP). We utilize city-level data due to its sufficiently large time series dimension. Equation (1) depicts the estimation equation. The left-hand-side variable (y_{t+h}) denotes the three month moving average of of the cross-regional standard deviation of house price growth at horizon h after the impact of the shock. α^h denotes constant. The monetary policy shock, ϵ_t^{MP}, is the high frequency identified and orthogonalized shock from Bauer and Swanson (2023). The monetary policy shock further conditions on times of booms ($\Delta q_{t-1}^{agg} > 0$) and busts ($\Delta q_{t-1}^{agg} < 0$). We adopt this approach because monetary policy shocks are relatively small and do not influence the economy's transition between boom and bust phases.[11] We

[11] Regressing house price growth on the aggregate level, for the current period and various leads, on the monetary policy shock reveals that the R^2 is never above 0.0007 across various leads. This emphasizes that these shocks do not

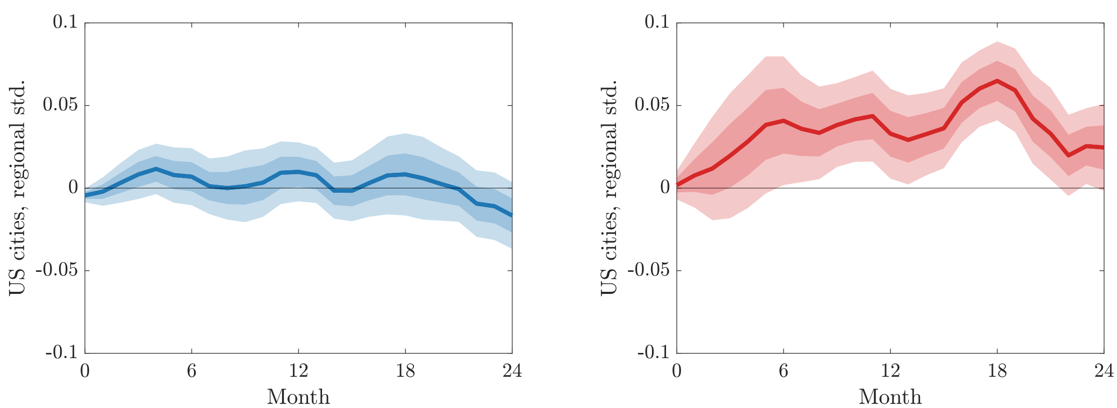
use the absolute values of the shocks, as the cross-regional dispersion should be more responsive in busts independent of the sign of the shock. The controls, x_t, contain 12 lags of the left-hand side variable, the log of the house price index on a federal level, log of industrial production, the log of CPI, the FFR, and the shocks. The sample runs from 1990 to 2019.

$$y_{t+h} = \alpha^h + \mathbb{1}(\Delta q_{t-1}^{agg} > 0) \times \beta_1 |\epsilon_t^{MP}| + \mathbb{1}(\Delta q_{t-1}^{agg} < 0) \times \beta_2 |\epsilon_t^{MP}| + x_t + u_{t+h} \tag{1}$$

Figure (2) presents the results. We see no significant change of cross-regional standard deviations in booms, see panel (a). In times of busts, panel (b), we find that a monetary policy shock leads to a significant increase in cross-regional house price growth variation over the first two years. Hence, conditional on monetary policy shocks, regional house price growth is more dispersed in busts relative to booms. This result aligns with our unconditional exercise presented above.

Figure 2: City-level std. house price growth response to MP shock, boom-bust

(a) $\Delta q_{t-1} > 0$ (b) $\Delta q_{t-1} < 0$



Notes: Responses to MP shock (1 std, absolute value) ; Confidence Intervals: 68% and 95% (Newey-West). The lhs variable is the 3 month moving average of the cross-city std. of house price growth.

Robustness. Figure (1) raises the question, of whether house price booms and busts are simply expansions and recessions if we consider all major US cities in our sample. Figure (C.1) shows that this is not the case. It plots the cross-city three-month moving average of house price growth. In particular, it shows that the standard deviations was already elevated before and remained elevated after the Great Recession. We further observe an increase in the standard deviation *after* COVID around 2022-2023. This period is associated with a house price bust, but no Recession. Further, we observe that the dotcom and the COVID recession were not associated with house price busts

drive the cycle.

and did not experience elevated cross-regional heterogeneity in house price growth. Therefore, house price busts and Recessions seem not to be perfectly correlated.

We also study the response of regional house price growth dispersion to a main business cycle shock constructed by Angeletos et al. (2020). The shock is given in quarterly frequency and we therefore move to state-level data to extend our cross-section. The main idea is that the main business cycle shock should be strong enough to affect the house price cycle.[12] Consequently, we study the response of the cross-state standard deviation of house price growth to an expansionary main business cycle shock. We observe that this shock indeed reduces house price growth heterogeneities, see Figure (C.2). As a sanity check, we also report the response of house price growth to the shock and observe that it is increasing. Our results are therefore in line with the findings reported above.

III. Model

With these empirical impressions in mind, we now describe our two-region currency union model, inspired by Benigno (2004), which we shall use to connect our empirical evidence and to conduct policy experiments. A notable difference between our model and Benigno's is the incorporation of incomplete bond markets across regions, instead of having perfect consumption insurance. In addition, we incorporate a housing sector into the model, introducing two critical elements.

First, we account for subjective expectations in the formation of house prices. Previous research has demonstrated that house price expectations deviate from rational expectations and play a crucial role in explaining boom-bust dynamics in the housing market.[13] In our model, subjective expectations are applied exclusively to house prices, as the primary focus of this study is on housing market dynamics. Furthermore, the availability of forecast data is limited to a select number of variables, making it feasible to incorporate subjective expectations only for house prices, where empirical data can effectively inform the model.

Second, we allow for regional differences in housing markets. We differentiate between a "home" region and a "foreign" region, with all foreign region variables denoted by an asterisk. The only difference between regions is on the housing supply side. The distinction between regions is the time required to construct houses, referred to as "time-to-build." This parameter enables us to model supply-side heterogeneities in a tractable manner.[14] Below we will also briefly discuss how

[12] Regressing the house price growth on the shock gives an R^2 of 0.1, meaning that this shock can explain a non-trivial part of the variation in house price growth.

[13] See, for example, Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020).

[14] In the empirical analysis below we will be more precise on why time-to-build seems a reasonable approximation for supply-side differences from an empirical point of view.

housing demand side heterogeneities may be modeled.

III.A The Economy

Households. A representative domestic household derives utility from consuming domestic and foreign varieties, leisure, and housing. The preferences are as follows:

$$\mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left(\frac{\xi_{c,t} c_t^{1-\sigma}}{1-\sigma} + \frac{\xi_{h,t} h_t^{1-\nu}}{1-\nu} - \frac{\chi n_t^{1+\varphi}}{1+\varphi} \right)$$

$$c_t = \left[\lambda^{\varsigma} c_{H,t}^{1-\varsigma} + (1-\lambda)^{\varsigma} c_{F,t}^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}}$$

$$c_{H,t} = \gamma \left[\int c_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \quad c_{F,t} = (1-\gamma) \left[\int c_{F,t}(j^*)^{\frac{\epsilon-1}{\epsilon}} dj^* \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\mathbb{E}_0^{\mathcal{P}}$ denotes the subjective expectations operator discussed below. ξ_j, $j \in \{c, h, a, x, i\}$, denote model-exogenous shock terms, h_t and n_t denote housing and hours worked respectively. $\gamma \in (0,1)$ denotes the measure of households in the home economy. Following Benigno (2004), γ is simultaneously the economic size of the home region, i.e. the mass of variety-producing firms. c_t denotes consumption of the domestic basket that is assembled from the home-good and the foreign-good which in turn are CES-aggregates of two groups of varieties. A preference bias for goods produced in the respective region of residence ("home bias") arises if $\lambda, 1-\lambda^* \neq \gamma$.[15] The slope of the housing demand curve is governed by the parameter ν. Regional differences on the housing demand side may be modeled by allowing for regional differences in ν.

The household budget constraint is then given by:

$$c_t + q_t(h_t - (1-\delta)h_{t-1}) + b_{t+1} + \frac{P_{H,t}}{P_t} x_t = w_t n_t + (1+r_t) b_t + q_t \cdot H(x_{t-\tau}, \xi_{x,t}) - T_t + \Sigma_t + \mathfrak{b}_t.$$

The budget constraint is expressed in units of the region-H final consumption basket, c. Σ_t are profits from all domestic firms, which are owned evenly by all domestic households, w_t is the real wage, T_t are government lump-sum taxes, and b_t is a one-period nominal zero-coupon bond that is traded union-wide. q_t denotes the real house price and x_t is the number of domestic consumption units dedicated to the production of new housing units.

To invest into housing production, households need to purchase domestically produced goods and transform them into housing investment units. Housing production is defined as $H(x_{t-\tau}, \xi_{x,t}) = \xi_{x,t} \frac{x_{t-\tau}^{\eta}}{\eta}, \eta \in (0,1)$. τ denotes the number of quarters it takes to construct new housing units.

[15] Throughout the paper we maintain the assumptions that (i) the degree of home bias is symmetric: $\gamma(1-\lambda) = (1-\gamma)(1-\lambda^*)$, and (ii) the bias is such that households favor domestically produced products, $\lambda \geq \gamma$.

Housing investment done in period t will therefore lead to a return in period $t + \tau$. Regional heterogeneities on the housing supply side are modeled by allowing for cross-regional differences in time-to-build (τ).

In the steady-state we will calibrate the steady-state values for ξ_h,ξ_h^*,ξ_x,ξ_x^* s.t. $b_{ss} = b_{ss}^* = 0$. Hence, in the steady-state, there are no net debtor and net creditor regions. This ensures that the only form of regional heterogeneity is situated on the housing side.

It is convenient to express the bond holdings in units of region H's final basket. The real interest rate r_t is taken as given by households and is determined in equilibrium by the following Fisher-type equation: The value of bond holdings in units of numéraire is $B_t = P_t \cdot b_t$ and the nominal bond pays $i_{t-1} - \psi b_t$ units of currency as interest.[16] The real interest rate is thus given by

$$1 + r_t = \frac{1 + i_{t-1} - \psi b_t}{1 + \pi_t}$$

where $\pi_t := P_t / P_{t-1} - 1$. Finally, $\mathfrak{b}_t := (\beta^{-1} - 1)\big(\gamma + (1-\gamma)\frac{P_{t-1}^*}{P_{t-1}}\big)(1+\pi_t)^{-1}\bar{b}$, taken as exogenous by the household, captures payment streams between H and F that guarantee that households are content with holding no bonds in the non-stochastic steady-state with zero inflation and real exchange rate parity.[17]

Subjective expectations house price setup. As is standard in the literature on capital gain extrapolation (e.g. Adam and Marcet, 2011; Adam et al., 2017), households are endowed with a set of beliefs in the form of a probability measure over the full sequence of variables that they take as given, henceforth external variables: $(\xi_t, r_t, w_t, \Sigma_t, T_t, \mathfrak{b}_t, \pi_t, (P_{H,t}/P_{F,t}), q_t)_{t \geq 0}$. This measure we denote as $\mathcal{P}$. Rational expectations are a special case of this setup in the form that households' beliefs agree with the objective, or equivalently "true" or "equilibrium-implied", distribution of external variables, $\mathcal{P} = \mathbb{P}$. Although households may hold expectations that are generally inconsistent with the equilibrium-implied (conditional) distribution of external variables, it is worth emphasizing that first they have a time-consistent set of beliefs, and second they behave optimally given their beliefs.

[16]The nominal interest rate is elastic in the aggregate holdings of bonds by domestic households. We follow Schmitt-Grohé and Uribe (2003) to ensure stationarity of the first-order dynamics. In Appendix A we provide a simple micro-foundation for debt-elastic interest rates.

[17]Given that bond holding entails a real cost in equilibrium, see footnote 16, introducing the payments $\mathfrak{b}_t$ is a way to ensure that there are no bond holding costs in the non-stochastic steady-state with zero inflation and real exchange rate parity (i.e. $1 + \pi_t = 1 + \pi_t^* = 1 + \pi_{H,t} = 1 + \pi_{F,t} = \frac{P_{H,t}}{P_{F,t}} = 1$). This ensures that this steady-state is efficient, given that fiscal policy undoes the monopolistic competition distortion. $\mathfrak{b}_t$ may be interpreted as the real interest rate paid by a non-markeTable nominal consol, that perpetually pays the nominal rate $(\beta^{-1} - 1)(\gamma + (1-\gamma)P_{t-1}^*/P_{t-1})$ and of which the household is endowed with $\bar{b}$ units. The endowments of these consols ensure that nominal payments balance, i.e. $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$, see Appendix A. Since we will linearize the model around a steady-state with zero bond holding, $\mathfrak{b}_t$ will be zero in equilibrium.

That is, households are *internally rational* in the sense of Adam and Marcet (2011). Moreover, the fact that all households are identical in beliefs and preferences is not common knowledge among agents so households cannot discover the misspecification of their beliefs, $\mathcal{P} \neq \mathbb{P}$, by eductively reasoning through the structure of the economy. Given the observed path of external variables up to period t, households then use this information and $\mathcal{P}$ to form a conditional expectation over the continuation sequence of external variables, which we denote as $\mathbb{E}_t^{\mathcal{P}}$. We denote the conditional rational expectations operator as usual by $\mathbb{E}_t$.

We assume that agents have rational expectations with respect to all external variables, except for house prices, q_{t+s}.[18] Households entertain the idea that house prices follow a simple state-space model:

$$\begin{aligned} \ln \frac{q_{t+1}}{q_t} &= \ln m_{t+1} + \ln e_{t+1} \\ \ln m_{t+1} &= \varrho \ln m_t + \ln v_{t+1}, \quad \varrho \in (0,1) \\ \begin{pmatrix} \ln e_t & \ln v_t \end{pmatrix}' &\sim \mathcal{N}\left(\begin{pmatrix} -\frac{\sigma_e^2}{2} & -\frac{\sigma_v^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix} \right) \end{aligned} \tag{2}$$

Hence, agents perceive house price growth rates as the sum of a transitory and a persistent component. Crucially, $\ln e_t$ and $\ln v_t$ are not observable to the agents, rendering $\ln m_t$ unobservable. Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system.[19]

Lemma 1 (House price belief updating)**.** *Applying the Kalman filter to the state-space model and log-linearizing around the non-stochastic steady-state gives:*

$$\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} = \widehat{q}_t + \frac{1-\varrho^s}{1-\varrho} \varrho \widehat{\overline{m}}_t \tag{3}$$

$$\widehat{\overline{m}}_t = (\varrho - g)\widehat{\overline{m}}_{t-1} + g\Delta\widehat{q}_{t-1} \tag{4}$$

where $\ln \overline{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ *is the posterior mean,* $g = \frac{\sigma^2+\sigma_v^2}{\sigma^2+\sigma_v^2+\sigma_e^2}$ *is the steady-state Kalman filter gain,* $\sigma^2 = \frac{1}{2}\left[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}\right]$ *is the steady-state Kalman filter uncertainty, and* $\ln \widehat{e}_t$ *is perceived to be a white noise process.*

Proof. See Appendix B.1 for the application of the Kalman filter. Log-linearization around the steady-state gives the result. ■

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[18]Formally, $\mathcal{P} := \mathbb{P}_{-q} \otimes \mathcal{P}_q$, where $\mathbb{P}_{-q}$ is the objective measure over sequences of external variables without house prices, $\mathcal{P}_q$ is the measure over sequences of house prices implied by the described perceived model of house prices, and $\otimes$ is the product measure. Since we are interested in a first-order solution to the model, it does not matter what households perceive to be the dependence structure between house prices and the other external variables.

[19]We assume agents' prior variance equals the steady-state Kalman variance.

Variables denoted with a "$\frown$" express the respective variables in percent deviations from its' steady-state value. Equation (3) shows that future house price beliefs depend on the current house price and today's beliefs. As $\varrho \in (0,1)$, the weight on the beliefs increases in the forecast horizon. Current house prices translate one-to-one into house price expectations. Hence, today's house price is extrapolated into the future. Turning to the belief updating Equation (3), we see that beliefs have an autoregressive component and are updated according to past observed house price changes. House price updating is increasing in the Kalman gain, g, and decreasing in the persistence of the beliefs ϱ.

Asymmetry in house price beliefs. For our analysis, we may also consider time variation in the house price updating process. Specifically, we allow for heterogeneities in the persistence of beliefs (ϱ) and the Kalman gain (g) below and above a certain threshold $\bar{\omega}$. Variations in the persistence of the belief process can be micro-founded by assuming that agents hold different perceptions of belief persistence (ϱ^h, ϱ^l). Differential Kalman gains can be modeled by allowing the relative variances of the transitory components ($(\sigma_e^h)^2, (\sigma_e^l)^2$) and the persistent components ($(\sigma_v^h)^2, (\sigma_v^l)^2$) to differ above and below the threshold. Augmenting the house price belief model (2) by incorporating differential belief updating above and below the threshold $\bar{\omega}$, and proceeding analogously to the derivation in Lemma (1), yields a threshold version of the belief updating model:

$$\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+1} = \mathbb{1}\left(\Delta\widehat{q}_{t-1} > \bar{\omega}\right)\left[q_t + \varrho^h \widehat{\widehat{m}}_t\right] + \mathbb{1}\left(\Delta\widehat{q}_{t-1} < \bar{\omega}\right)\left[q_t + \varrho^l \widehat{\widehat{m}}_t\right] \tag{5}$$

$$\widehat{\widehat{m}}_t = \mathbb{1}\left(\Delta\widehat{q}_{t-1} > \bar{\omega}\right)\left[(\varrho^h - g^h)\widehat{\widehat{m}}_{t-1} + g^h \Delta\widehat{q}_{t-1}\right] + \mathbb{1}\left(\Delta\widehat{q}_{t-1} < \bar{\omega}\right)\left[(\varrho^l - g^l)\widehat{\widehat{m}}_{t-1} + g^l \Delta\widehat{q}_{t-1}\right] \tag{6}$$

This threshold framework captures distinct dynamics in belief adjustments, conditional on whether the extrapolation parameter lies above or below the critical value $\bar{\omega}$. Consequently, agents update their beliefs asymmetrically across housing market regimes, reflecting heterogeneity in their responses to house price fluctuations. This extension provides a more nuanced representation of belief formation, particularly under varying market conditions.

Firms and price setting. We assume a continuum of monopolistically competitive firms that produce intermediate good varieties and have the same beliefs as households. Firm beliefs, however, concern only variables over which households have rational expectations. Therefore, firms are rational. Firm j buys labor $n_t(j)$ from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The variety is bought by households from both regions. The firm sets its retail price $P_{H,t}(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs. Formally the firm solves:

$$\max_{P_{H,t}(j)} \mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{P_t} \left[P_{H,t}(j) y_{H,t}(j) - (1-\tau^{\ell}) W_t n_t(j) - P_{H,t} \frac{\kappa}{2} \left(\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right)^2 y_{H,t} \right]$$

$$\text{with} \quad y_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} y_{H,t}$$

with $y_{H,t}(j) = \xi_{a,t} n_t(j)$. $\Lambda_t = u'_{c,t}/u'_{c,0}$ denotes the stochastic discount factor and τ^{ℓ} is a wage subsidy paid by the government. It is selected such that the monopolistic competition distortion is offset in the non-stochastic steady-state. The subsidy is financed through a lump-sum tax on the firm. In symmetric equilibrium, all firms choose the same price, $P_{H,t}(j) = P_{H,t} \forall j$. The solution of the firm problem results in a standard currency union version of the Phillips-Curve.

Monetary authority. The monetary authority sets the nominal interest rates according to a standard Taylor rule targeting currency union consumer price inflation:

$$i_t = \frac{1}{\beta} \left(\Pi_t^{cu}\right)^{\phi_\pi} \xi_{i,t} \tag{7}$$

Currency union inflation is the average of region-level consumer price inflation, weighted by the country size: $\Pi_t^{cu} = (\Pi_t)^{\gamma} \left(\Pi_t^*\right)^{1-\gamma}$.

Market clearing. To achieve goods market clearing, each goods market for a variety j must clear. Additionally, labor and housing markets must clear within each region. Finally, the balance-of-payments equations must hold. Aggregation and the specific market clearing conditions are relegated to Appendix (B.3).

Equilibrium. We adopt the equilibrium concept of Internally Rational Expectations Equilibrium, as defined in Adam and Marcet (2011):

Definition 1 (Internally Rational Expectations Equilibrium)**.** *An IREE consists of three bounded stochastic processes: shocks* $(\xi_t)_{t\geq 0}$*, allocations* $([c_{H,t}(j)]_{j\in[0,1]}, [c^*_{F,t}(j^*)]_{j^*\in[0,1]},$ $b_t, b_t^*, h_t, h_t^*, x_t, x_t^*, n_t, n_t^*)_{t\geq 0}$ *and prices* $(w_t, w_t^*, q_t, q_t^*, i_t, [P_{H,t}(j)]_{j\in[0,1]}, [P_{F,t}(j^*)]_{j^*\in[0,1]})_{t\geq 0}$*, such that in all t*

1. *households choose* $[c_{H,t}(j)]_{j\in[0,1]}, [c^*_{F,t}(j^*)]_{j^*\in[0,1]}, b_t, b_t^*, h_t, h_t^*, x_t, x_t^*, n_t, n_t^*$ *optimally, given their beliefs* $\mathcal{P}$*,*

2. *firms choose* $([P_{H,t}(j)]_{j\in[0,1]}, [P_{F,t}(j^*)]_{j^*\in[0,1]})_{t\geq 0}$ *optimally, given their beliefs* $\mathcal{P}$*,*

3. *the monetary authority acts according to the Taylor rule* (7)*,*

4. *markets for consumption good varieties, hours, and housing clear given the prices, and the balance-of-payments Equation holds.*

Appendix B.4 contains (i) a proof that Walras' law holds for our economy and (ii) the derivation of the Balance-of-Payments condition. Appendix B.5 presents the system of equations that characterizes the IREE.

III.B Solution method

We solve our model to first order around a non-stochastic and efficient steady-state. This preserves analytic tractability at many points in the model and allows us to derive results on household behavior under capital gain extrapolation in closed form. The model presented in Section III.A admits a unique non-stochastic steady-state with zero net inflation and parity of the terms of trade:

Lemma 2 (Non-stochastic steady-state)**.** *Consider the model economy presented in Section III.A. As the variance of shocks (actual or perceived) fades,* $\text{Var}[\|\xi_t\|] \to 0$, $\xi_t = (\xi_{a,t}, \xi_{c,t}, \xi_{h,t}, \xi_{x,t}, \xi_{i,t}, e_t, v_t)^\top$, *there exists one and only one steady-state in which net inflation is zero,* $\pi_{H,ss} = \pi_{F,ss} = \pi_{ss} = \pi^*_{ss} = 0$, *and in which the terms of trade are at parity,* $s_{ss} = 1$.

Proof. See Appendix B.6 ■

Linearizing models with capital gain extrapolation is not straightforward. In fact, to the best of our knowledge, we are the first to provide a first-order approximation to a model with capital gain extrapolation under the assumption that agents hold rational expectations outside of the asset pricing block.[20] The entire exposition focuses on the typical linearized household problem in region H with all derivations being analogous for the typical household in region F.[21] For expositional clarity, we omit the specification that housing investment occurs in domestically produced goods. This simplification does not affect the fundamental logic of our solution method.

[20] Winkler (2020) proposes the "conditionally model-consistent expectations" (CMCE) concept as a starting point for linearizing models with capital gain extrapolation. Under CMCE, however, beliefs over all external variables are distorted relative to rational expectations. In our approach, linearized decision rules may be obtained under the assumption that belief distortions apply only to asset prices, allowing to confine the deviations from rational expectations to exactly those variables where survey data allows to discipline the expectations-modeling choices.

[21] For any variable $\text{var}_t \notin \{b_t, b^*_t, \Sigma_t, \Sigma^*_t\}$ define $\widehat{\text{var}_t} := \frac{\text{var}_t - \text{var}_{ss}}{\text{var}_{ss}} \simeq \ln \text{var}_t - \ln \text{var}_{ss}$ to first order. For b_t, Σ_t (analogously for b^*_t, Σ^*_t) define $\widehat{\text{var}_t} := \frac{\text{var}_t - \text{var}_{ss}}{y_{ss}}$, which allows for the case that $\text{var}_{ss} = 0$. (That is, we scale deviations in bond holdings and profits by GDP.) Note furthermore that $\widehat{1 + r_{t+1}} \simeq \ln(1 + r_{t+1}) - \ln(1 + r_{ss}) \simeq r_{t+1} + \ln \beta$. We abuse notation slightly and write $\widehat{r}$ instead of $\widehat{1 + r}$.

External and internal variables. Standard first-order solution techniques for models with rational expectations rely on a recursive representation of the equilibrium conditions. For instance, the inter-temporal consumption decision is captured by the forward recursion, the Euler equation:

$$\widehat{c}_t = \mathbb{E}_t \widehat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t \widehat{r}_{t+1}.$$

Under subjective expectations an equivalent formulation exists:

$$\widehat{c}_t = \mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t^{\mathcal{P}} \widehat{r}_{t+1}.$$

At this point, it is important to note that first, we can characterize external variables for which households have distorted expectations. In our case this is only the house price, $\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s}$, for which we have derived a subjective expectation model. Second, for all other external variables, for which households have rational expectations, we can formulate all equilibrium conditions recursively in the usual manner. In this case, the equilibrium-implied distribution measure applies and we can drop $\mathcal{P}$ from the expectations operator. The difficulty arises concerning expectations of household choices or internal variables, in our case $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+s}$. These variables depend on external variables over which households hold subjective and rational expectations. In this case, we need to determine the subjective expectations distribution of $\widehat{c}_{t+1}$.[22] Our solution method allows us to determine these variables in closed form. We provide a detailed derivation in Appendix B.7, and concentrate here on conveying the intuition of our solution approach.[23]

Characterizing subjective expectations over internal variables. In solving for the subjectively optimal consumption plan we exploit two key insights into how households behave to first-order that are valid irrespective of which set of beliefs they hold.[24] First, there is only one inter-temporal trade-off, namely in consumption, c. Given a path for consumption, the first-order conditions for housing, hours worked, and housing investment uniquely pin down a mapping from external variables to decisions for these internal variables. This insight allows us to concentrate on finding the optimal path for consumption.

The second insight is that to first-order, the permanent income hypothesis holds and consumption

[22]The reason is that households have distorted expectations over at least one price sequence and therefore will make distorted choices; in particular they plan to make choices in the future that are inconsistent with what these choices will be in equilibrium. If we ignored this, i.e. exchanged $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+s}$ for $\mathbb{E}_t \widehat{c}_{t+s}$ in the forward iteration above, the computed equilibrium would be different from the IREE in Definition 1.

[23]For tractability, we set habit formation, $\bar{h}$, to zero and we assume that housing investment is made out of the aggregate consumption goods bundle c_t instead of only locally produced goods. Relaxing these assumptions does not affect our methodological approach.

[24]In particular, our solution method can be used to solve RE models.

depends only on the path of real interest rates, an external variable, and the subjectively expected lifetime income. By iterating forward the Euler Equation we receive

$$\widehat{c}_t = -\frac{1}{\sigma}\sum_{s\geq 0}\mathbb{E}_t\widehat{r}_{t+s+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty \tag{8}$$

where we defined $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = \lim_{s\to\infty}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}$. This shows that the only subjective expectation variable to remain is $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$.[25]

At this point the question arises whether this term is merely a technical artifact originating from our methodological approach, or if it possesses intrinsic economic meaning. For expositional purposes it is informative to consider a transitory shock to the model. Suppose that the shock raises house prices, which leads households to believe that house prices will rise further. Consequently, they believe that they will be wealthier in the future, leading to an increase in their current and planned future consumption. This expectation is reflected in the limit by an increase in $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$, thereby capturing a subjective expectations wealth effect. In essence, due to extrapolation, households believe they will be richer than they actually will be. Under rational expectations, this term also exists. However, households recognize that the shock is purely transitory and will not influence their long-term wealth. Therefore, the wealth effect under rational expectations is always zero.

Characterizing the subjective expectations wealth effect. We will now focus on deriving an analytical expression for the subjective expectations wealth effect, $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$. Using the insights from above, the household's first-order conditions are given by the equations in (9). (For expositional brevity we have dropped the exogenous shocks and set time-to-build to zero.)

$$\begin{array}{rl}
\text{Rational Expectations} & \text{Subjective Expectations} \\
\varphi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t & \varphi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t \\
\widehat{q}_t = (1-\eta)\widehat{x}_t & \widehat{q}_t = (1-\eta)\widehat{x}_t \\
\widehat{c}_t = -\frac{1}{\sigma}\sum_{s\geq 0}\mathbb{E}_t\widehat{r}_{t+s+1} + \mathbb{E}_t\widehat{c}_\infty & \widehat{c}_t = -\frac{1}{\sigma}\sum_{s\geq 0}\mathbb{E}_t\widehat{r}_{t+s+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty \\
\widehat{h}_t = \frac{\sigma}{\nu(1-\bar{\beta})}\left(\widehat{c}_t - \bar{\beta}\mathbb{E}_t\widehat{c}_{t+1}\right) - \frac{1}{\nu}\frac{\widehat{q}_t - \bar{\beta}\mathbb{E}_t\widehat{q}_{t+1}}{1-\bar{\beta}} & \widehat{h}_t = \frac{\sigma}{\nu(1-\bar{\beta})}\left(\widehat{c}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}\right) - \frac{1}{\nu}\frac{\widehat{q}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}}{1-\bar{\beta}}
\end{array} \tag{9}$$

where $\overline{\beta} := \beta(1-\delta)$ and $\mathbb{E}_t\widehat{c}_\infty = 0$ as explained above. From this representation it becomes clear, that the difference between subjective expectations and rational expectations lies in the measure $\mathcal{P}$

[25]We have directly used the fact that households have rational expectations over external variables other than house prices, $\mathbb{E}_t^{\mathcal{P}}\text{var}_{t+1} = \mathbb{E}_t\text{var}_{t+1}$ for any variable $\text{var}_{t+1} \neq q_{t+1}$.

expectations with respect to next periods consumption and house prices, and the existence of the subjective expectations wealth effect, $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$. Subjective expectations with respect to house prices are easy to characterize, as these are pinned down by the subjective expectations model. Further note, that subjective expectations on next periods consumption can be characterized by leading the Euler Equation (8) one period, once we have a representation for the subjective expectations wealth effect. Therefore, in order to solve the subjective expectations model, it suffices to find a representation for the subjective expectations wealth effect. We find this characterization by combining the first-order conditions with the linearized household budget constraint, *(BC)* in Appendix-equation (B.8). After iterating over this equation, we can find a closed form expression for the subjective expectations wealth effect.

Proposition 1 (subjective expectations wealth effect)**.** *To first-order around the non-stochastic steady-state the subjective expectations wealth effect,* $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$*, is given by:*

$$\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = \mathrm{Q}/\sigma \cdot \underbrace{\left[\widehat{q}_t + \mathrm{M}\widehat{\widehat{m}}_t\right]}_{=\Lambda_{t,1}} + \mathrm{C}\frac{y_{ss}}{c_{ss}}\frac{1-\beta}{\beta}\left[\underbrace{\sum_{n=1}^{\infty}\beta^n\mathbb{E}_t\{z^*_{t+n}\}}_{=\Lambda_{t,2}} + \widehat{b}_{t+1}\right], \tag{10}$$

where $\mathrm{Q},\mathrm{C} \in (0,1)$*,* $\mathrm{M} > 0$ *are defined below equation* (B.14)*, and* z^*_{t+n} *is a function of external variables the household has rational expectations about and is specifically stated in Appendix B.7.*

Proof. See Appendix B.7 ■

The notation "ss" in the subscript of a variable, denotes its' steady-state values. Equation (10) shows that the subjective expectations wealth effect consists of three parts. First, variables for which the household has subjective expectations, $\Lambda_{t,1}$. In our case this is only the house price, $\widehat{q}_t$, and posterior beliefs about house prices, $\widehat{\widehat{m}}_t$. Second, it depends on the expectations of all variables the household has rational expectations about, $\Lambda_{t,2}$, which are collected in z^*_{t+n}. Third, it depends on today's bond choices $\widehat{b}_{t+1}$. Intuitively, households evaluate the wealth effect under subjective expectations by applying the permanent income hypothesis. In doing so they account for their future decisions by using their first-order conditions resulting in a function that depends only on prices and today's saving choices. This formulation elucidates the previously discussed intuition regarding the role of subjective expectations in wealth effects. Specifically, an increase in house prices ($\widehat{q}_t$) or in beliefs about future house prices $(\widehat{m}_t)$ leads to a rise in $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$, given that $Q, M > 0$. Agents, anticipating higher wealth as a result of these changes, adjust their expectations accordingly and expect an increase in consumption in the long run.

Discussion. Our method has two important advantages over previous approaches to solving asset price learning models. First, we solve the model using a first-order approximation which makes it fast to solve, easily scalable, and amenable to the analysis of Ramsey-optimal policies. The literature has previously relied on non-linear solution techniques (Adam et al., 2017), or hybrid techniques (Adam et al., 2022) to solve these models. Hence, solution procedures are much more involved and limits the complexity of models that numerically can be solved. Second, our solution method confines subjective expectations to house prices. A previously developed method by Winkler (2020) and Caines and Winkler (2021), which also relies on perturbation, assumes household expectations to conform with the concept of conditionally model-consistent expectations. Under this concept, subjective expectations about one variable lead to spillovers to expectations about other variables. Thus, households will form subjective expectations across all model variables. In our approach, households only hold subjective expectations with respect to one variable, while they remain rational with respect to all other variables. This method of explicitly characterizing choices in terms of lifetime income is general in the sense that it allows solving for household decisions under any time-consistent set of beliefs.

IV. Analytic Insights

We proceed by deriving the primary channel through which house price extrapolation and heterogeneities in the housing market can produce the patterns observed in Section (II). Our analysis demonstrates that the results are robust to regional heterogeneities, whether these are located on the supply or demand side of the housing market. Furthermore, we establish that our findings do not hold under a rational expectations framework.

To do so we examine a simplified version of the model introduced above: we consider a one-region, zero liquidity endowment economy with fully sticky prices. We study the path of house prices within an exercise that, while keeping everything tractable, allows us to understand the effects of expansionary monetary policy: the consumption endowment process is selected such that the real rate drops by ε on impact of the shock and returns to 0 afterwards. This is arguably a parsimonious way to model expansionary monetary policy.

We assume that the housing supply is a function of endowment, and moves in response to the shock by $-\iota\varepsilon$. Hence, in response to an expansionary monetary policy shock housing supply increases. In the following regional differences in housing supply will be simply modelled as regional variations in ι. In Appendix (B.9.3) we also consider the case of demand-side heterogeneities and show that similar results can be attained as under supply-side heterogeneities.

Throughout this section we will consider parameter choices commonly used in the literature. Specifically, we consider parameter choices in line with Section (V), which are stated in Table (3) and (C.2). Detailed derivations are relegated to Appendix B.9. Under the given assumptions the house price and housing market clearing can be expressed as:

$$\widehat{q}_t = \mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} - \widehat{r}_{t+1} - \nu(1-\bar{\beta})\widehat{h}_t + \nu(1-\bar{\beta})^2\sum_{s=0}^{\infty}\bar{\beta}^s\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s+1} \tag{11}$$

$$\widehat{h}_t - (1-\delta)\widehat{h}_{t-1} = -\iota\varepsilon_t \tag{12}$$

IV.A Supply-side heterogeneity

As we focus on housing supply side heterogeneities we simplify the housing demand side and assume that $\nu = 1$. The house price is pinned down jointly by the equations (11) and (12). The differences between rational and subjective expectations arise due to the expectation formation on future house prices, $\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}$, and future housing choices, $\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s+1}$. Under rational expectations, we can simply use the housing market clearing condition (12) to pin down house prices. Doing so yields the following proposition.

Proposition 2 (House prices under Rational Expectations)**.** *In the outlined one-region, zero liquidity endowment economy with exogenous housing supply, and considering a shock monetary policy shock such that $\varepsilon_1 < 0$ and $\varepsilon_t = 0$ for $t > 1$. House prices are given by:*

$$\widehat{q}_t = \mathbb{E}\widehat{q}_{t+1} - \widehat{r}_{t+1} - (1-\bar{\beta})\widehat{h}_t + (1-\bar{\beta})(1-\delta)^t(1 - \frac{1-\bar{\beta}}{1-\bar{\beta}\delta})(-\iota\varepsilon_t))$$

And for the first two periods we have:

$$\widehat{q}_1 = -\varepsilon_1 + \omega_h\iota\varepsilon_1$$
$$\widehat{q}_2 = (1-\delta)\omega_h\iota\varepsilon_1$$

with $\omega_h \in (0,1)$.

Proof. See Appendix B.9.1 ∎

Proposition (2) makes clear that an expansionary monetary policy shock raises house prices. The shock fades after the first period. If housing supply increases simultaneously ($\iota > 0$), the house price increase is less pronounced in the first period. Due to the increase in housing supply, house prices will be below the steady-state level after the first period, converging back to the steady-state going forward.

Moving on to the subjective expectation case, house price expectations, $\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}$, are pinned down by the subjective expectations model, equations (3) and (4). Future expected housing choices, $\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s+1}$, can be characterized using the solution method described in Section (III.B). We arrive at the following proposition:

Proposition 3 (House prices under Subjective Expectations)**.** *In the outlined one-region, zero liquidity endowment economy with exogenous housing supply, and considering a shock monetary policy shock such that $\varepsilon_1 < 0$ and $\varepsilon_t = 0$ for $t > 1$. House prices are given by:*

$$\widehat{q}_t = -\widehat{h}_t - \omega_r\widehat{r}_{t+1} + \omega_m\widehat{\overline{m}}_t$$

And for the first two periods we have:

$$\widehat{q}_1 = -\omega_r\varepsilon_1 + \iota\varepsilon_1$$
$$\widehat{q}_2 = (1-\delta)\iota\varepsilon_1 + \omega_m g\widehat{q}_1$$

with $\omega_r > 1$, $\omega_m > 1$, and $\frac{\partial\omega_m}{\partial\rho} < 0$.

Proof. See Appendix B.9.2 ∎

Proposition (3) elucidates several key dynamics. First, an expansionary monetary policy shock leads to an increase in house prices, and given that $\omega_r > 1$, this increase is more pronounced than in the rational expectations scenario. The fact that the house price rises by more on impact under subjective expectations is a direct consequence of the fact that house prices not only convey the degree of scarcity of a good but also reveal information about the future path of the price of an asset. Under rational expectations, agents understand that house prices will mean-revert—while under subjective expectations they apply their perceived model to conclude that house prices will stay elevated. Second, a positive response in housing supply mitigates the rise in house prices induced by the monetary policy shock, and this mitigating effect is stronger compared to the rational expectations scenario, as indicated by $\omega_h < 1$. Third, belief extrapolation introduces a backward-looking component that drives up house prices. An increase in the Kalman gain (g) amplifies the effectiveness of this channel. Similarly, a decrease in the persistence of belief formation (ϱ) also enhances the channel's potency.

Let us now turn towards understanding how these dynamics come together when two regions are involved, A and B. Consider an expansionary monetary policy shock that is symmetric across regions and an increase in housing supply that is more pronounced in region A, $\iota^A > \iota^B$. Using the results from above, we can now conclude:

Proposition 4 (Differential house price growth responses, supply-side)**.** *The differential house price growth response in region A and B under rational expectations for the first two periods are* *given by:*

$$\Delta\widehat{q}_1^B - \Delta\widehat{q}_1^A = \omega_h(\iota^B - \iota^A)\varepsilon_1$$
$$\Delta\widehat{q}_2^B - \Delta\widehat{q}_2^A = -\delta\omega_h(\iota^B - \iota^A)\varepsilon_1$$

The differential house price growth response in region A and B under subjective expectations for the first two periods are given by:

$$\Delta\widehat{q}_1^B - \Delta\widehat{q}_1^A = (\iota^B - \iota^A)\varepsilon_1$$
$$\Delta\widehat{q}_2^B - \Delta\widehat{q}_2^A = (\omega_m g - \delta)(\Delta\widehat{q}_1^B - \Delta\widehat{q}_1^A)$$

where $\omega_m g >> \delta$ generally holds. And house price extrapolation amplifies regional differences:

$$\frac{\partial(\Delta\widehat{q}_1^B - \Delta\widehat{q}_1^A)}{\partial g} = \omega_m(\Delta\widehat{q}_1^B - \Delta\widehat{q}_1^A) > 0$$

Proof. Differential house price growth, $\Delta\widehat{q}_j^B - \Delta\widehat{q}_j^A$ with $j = 1, 2$, are derived by subtracting regional prices from Proposition (2), and Proposition (3) respectively. ■

Proposition (4) summarizes the paper's main argument. In the subjective expectations framework, supply-side differences lead to differential house price growth responses, reflected by $\iota^B - \iota^A$. As we consider an expansionary policy shock, $\varepsilon < 0$, house price growth in period one will be larger in region B. This holds for the rational expectations and the subjective expectations model. However, in the subjective expectations model the regional differences will be more pronounced since $\omega_h \in (0, 1)$. In the subsequent episodes, we observe a qualitative difference between the rational expectations and the subjective expectations model. Under rational expectations the regional differences flip: $\Delta\widehat{q}_2^B - \Delta\widehat{q}_2^A < 0$ as $-\delta\omega_h(\iota^B - \iota^A)\varepsilon_1 < 0$. In the subjective expectations model, this is not the case. Extrapolation introduces a backward-looking term, which overturns the dynamic observed under rational expectations: $\Delta\widehat{q}_2^B - \Delta\widehat{q}_2^A > 0$ since $\omega_m g > \delta$. Extrapolation thereby provides a dynamic amplification mechanism of regional house price growth rates. The higher g, or the lower ϱ, the higher the coefficient $\omega_m g$, and therefore the stronger the amplification channel.[26]

This mechanism explains why the dispersion in regional house price growth is greater during busts than in booms. As we will demonstrate below, the formation of house price beliefs differs

[26]Numerical explorations show that $\omega_m g$ can be as large as 73, which constitutes a significant amplification mechanism. Realistic values for the Kalman gain g range between 0.01 – 0.025. In our numerical example we assumed a Kalman gain of 0.02.

between boom and bust periods. In particular, the Kalman gain (g) is higher during busts than in booms. As a result, the amplification effect of house price extrapolation is stronger in bust periods, leading to increased regional dispersion in house price growth. Importantly, this mechanism is absent in the rational expectations model, rendering it incapable of accounting for the asymmetry in regional dispersion in house price growth rates between booms and busts. Therefore, the rational expectations framework fails to capture the differential effects observed in the dispersion of house price growth across regions during these distinct phases of the housing market cycle observed in the data.

V. Model calibration

In this section, we will describe the model calibration. We will start by focusing on the calibration of the subjective expectations belief formation process. To do so, we will turn to the data to directly estimate the belief persistence (ϱ) and the Kalman gain (g): Notably, we find that in busts the Kalman gain is larger than in booms.

We then move to regional heterogeneities and present evidence that housing supply-side differences lead to regional responses of house prices to a monetary policy shock. We use this analysis to calibrate cross-regional supply-side differences in time-to-build.

We calibrate the slope of the Phillips-Curve such that the model matches the peak response of a 25 bp monetary policy shock unconditional on a house price boom or bust. The remaining parameters a chosen in line with the literature.

V.A House price beliefs and boom-bust dynamics

In Section (III), we construct a model that breaks the rational expectations hypothesis with respect to house prices and introduces a Bayesian belief updating model to describe the house price belief formation. This modeling approach is grounded in a substantial body of empirical literature that examines the dynamics of belief formation in the context of housing markets.[27] In appendix (C.3), we conduct our own analysis and demonstrate that our modelling approach remains valid when conditioning on monetary policy shocks. Specifically, we examine the response of forecast errors to these shocks. Under the assumption of rational expectations, this response should theoretically be zero; however, our analysis reveals a substantial and statistically significant response. We observe a sluggish adjustment of beliefs following the shock, with evidence of initial over-pessimism in the

[27]See: Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020); Adam et al. (2022)

first month, which subsequently transitions to over-optimism in the longer term. These findings are consistent with existing literature and suggest that a Bayesian belief updating model is well-suited to capturing the observed dynamics in house price expectations.

House price belief formation. Our findings in Section (IV) suggested that stronger belief updating in busts are essential to explain diverging regional house price growth rates in busts. Therefore, we estimate the belief updating process described in Section (III) and allow for potential differences in the belief updating process. Specifically, we test two specifications: a linear one and threshold specification allowing for regime switching. The linear process reads:

$$\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1} = c + \varrho(\varrho - g) \times \mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t + \varrho g \times \Delta q_{t-1}. \tag{13}$$

Equation (13) can be derived by combining equations (3) and (4). $c = \varrho\bar{m}(1 - \varrho)$ is a constant with $\bar{m}$ being long-run house price growth expectations. The belief updating model is in monthly frequency, and the sample is 2007 to 2020. We obtain the month-on-month percentage change in house price expectations, $\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1}$, by dividing the year-on-year percentage changes from the Michigan Survey by 12. House price growth data is taken from Case-Shiller National House Price Index. To estimate this model, we replicate the belief updating process employed by agents in our model, which follows a recursive updating procedure. Specifically, to estimate this process we feed in realized past house price growth data, Δq_{t-1}. For a given tuple $(\varrho, g, \bar{m})$ and by recursively updating $\mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t$, we obtain a sequence of house price growth expectations. We then use a solver choosing $(\varrho, g, \bar{m})$ such that the MSE between the data and the fitted values is minimized.[28] For the estimation procedure we also impose that $\varrho, g \in (0, 1)$.

Equation (14) allows for heterogeneity in ϱ and g below and above a certain threshold. It is based on equations (5) and (6). The estimation procedure is equivalent to the linear model, only now we choose $(\varrho^h, \varrho^l, g^h, g^l, \bar{m}^h, \bar{m}^l, \omega)$ to minimize the MSE between fitted values and the data. Notice that the threshold is also estimated and that the regime depends on past observed house price growth, Δq_{t-1}.

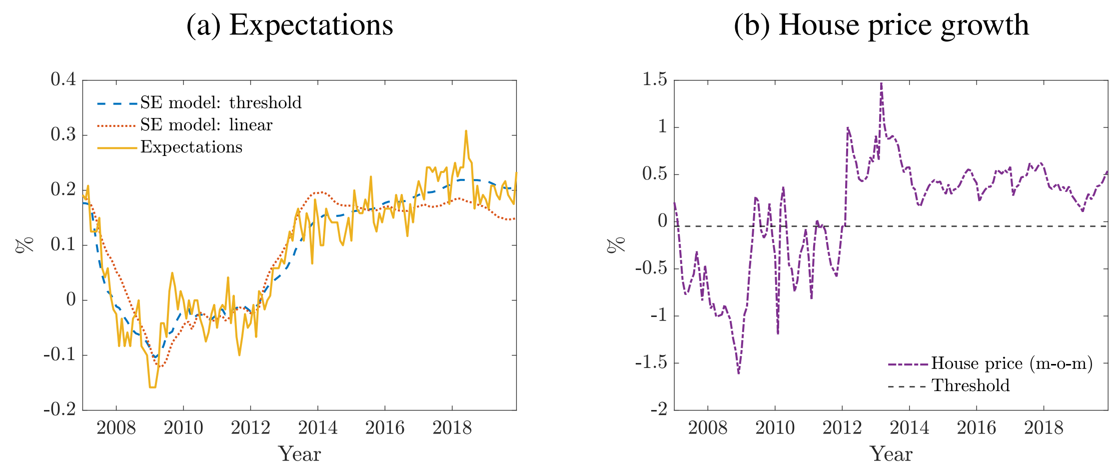
$$\begin{aligned}\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1} = \mathbb{1}(\Delta q_{t-1} > \omega) \left(c^h + \varrho^h(\varrho^h - g^h) \times \mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t + \varrho^h g^h \times \Delta q_{t-1}\right) + \\ \mathbb{1}(\Delta q_{t-1} < \omega) \left(c^l + \varrho^l(\varrho^l - g^l) \times \mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t + \varrho^l g^l \times \Delta q_{t-1}\right)\end{aligned} \tag{14}$$

The estimation results are presented in Figure (3). The solid yellow line in panel (a) plots the

[28]This procedure demands a starting value for $\mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t$ which can affect the outcomes. To minimize the effect of the starting values on the estimated parameters, we start in 1987 to feed in monthly house price growth data, thereby obtaining initial values for 2007M1. Changing the starting value in 1987 has insignificant effects on our results.

expectations data from the Michigan survey, the blue dashed line represents the threshold model estimates, and the dotted red line shows the linear model. The dashed-dotted purple line in panel (b) depicts the month-on-month house price growth, while the dashed black line indicates the threshold for the threshold model. Turning to the linear model first, we find that it does a reasonable job of explaining the expectation formation process. However, it lags behind during the bust episode and underestimates expectations for house price growth in the recovery following the Recession. In comparison, the threshold model improves on these dimensions. It captures the bust without an obvious lag and performs better in recovery episodes. The estimated threshold for this model is close to zero, suggesting that the updating behavior differs during periods when house prices rise (booms) compared to when they fall (busts).

Figure 3: House price belief model, US



Notes: SE model, threshold (blue, dashed): fitted values of Equation (14); SE model, linear (red,dotted): fitted values of Equation (13); Expectations (yellow, solid): mean expectations data from the Michigan Survey; House price (m-o-m) (purple, dashed-dotted): Month on month percentage change in house price. The black dashed line in panel (b) depicts the threshold in the threshold model.

The estimated parameters from the models are shown in Table (2). We find a ϱ of 0.97 and a Kalman gain of 0.0175 for the linear model. For the threshold model, we find a ϱ^h of 0.99, and a Kalman gain, g^h, 0.0117 in booms. In busts, ϱ^l decreases to 0.91, while g^l increases to 0.0233. All parameters are roughly in line with the literature. The differences in the parameters across regimes transparently show why the threshold model can fit the data better than the linear model: In busts when house prices and beliefs fall drastically in a short period, a higher Kalman gain, and a lower persistence parameter enable faster pass-through from observed house prices to beliefs. In booms, when house prices and beliefs recover steadily over a longer period, expectations are better matched by a slow-moving process with high persistence. This is achieved by a higher degree of persistence ϱ^h and a lower degree of updating g^h. The inclusion of the threshold component decreases the

MSE by roughly 47%.

We also perform a robustness exercise to study whether the parameter differences across regimes affect the model performance. To do so, we impose the estimated parameters from above and below the threshold, subsequently on the linear model. We find a significant increase in the MSE, implying that the differences in the estimated parameters of the threshold model matter for the model performance.

Table 2: House price belief model: parameters

$Specification$	ϱ^{lin}	ϱ^{h}	ϱ^{l}	g^{lin}	g^{h}	g^{l}	$Threshold$	MSE^{i}/MSE^{lin}
$Baseline:$								
$linear$	0.97			0.0175				1.00
$threshold$		0.99	0.91		0.0117	0.0233	−0.048	0.53
$Robustness:$								
$high\ regime\ beliefs$	0.99			0.0117				4.66
$low\ regime\ beliefs$	0.91			0.0233				8.57

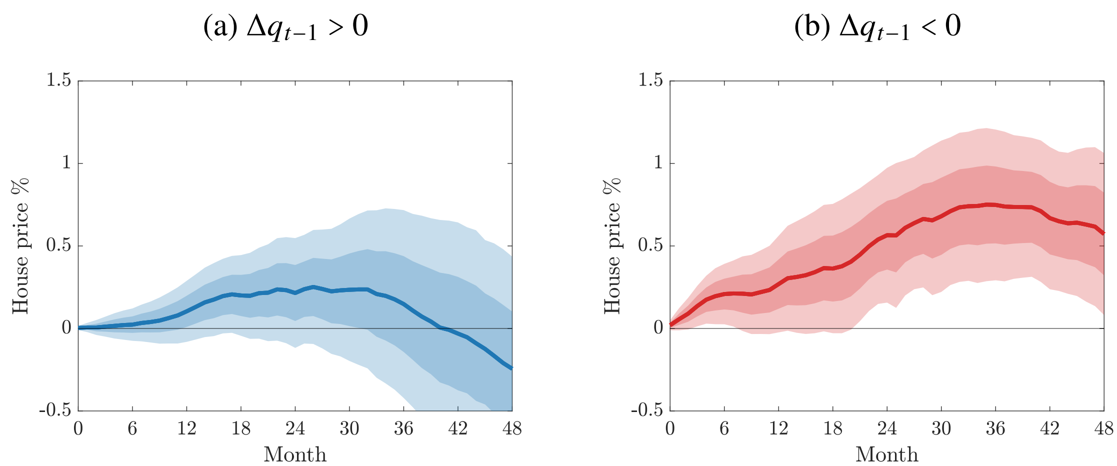
Notes: linear: estimated parameters from Equation (13); threshold: estimated parameters from (14); Robustness exercises are explained in the main text.

House prices in booms and busts. Our findings indicate that house price expectations updating is more pronounced in busts. This, in turn, can affect house prices themselves: a larger reaction in expectations translates into economic decision-making and will eventually be reflected in prices. To test whether this is the case, we run local projections on house prices responding to a monetary policy shock, conditioning on booms and busts. The estimation equation is given by:

$$q_{t+h} = \alpha^h + \mathbb{1}(\Delta q_{t-1} > 0) \times \beta_1 \epsilon_t^{MP} + \mathbb{1}(\Delta q_{t-1} < 0) \times \beta_2 \epsilon_t^{MP} + x_t + u_{t+h} \tag{15}$$

The monetary policy shock, ϵ_t^{MP}, is the high frequency identified and orthogonalized shock from Bauer and Swanson (2023). The left-hand side variable, q_{t+h}, is the log house price. The sample runs from 1990-2019 and is in monthly frequency. The controls, x_t, contain 12 lags of the left-hand side variable, log of industrial production, the log of CPI, the FFR, and the shocks. We will focus on expansionary monetary policy shocks throughout the whole empirical analysis. Figure (4) plots the results. The blue line shows the response if house prices were increasing in the past, and the red line if they were decreasing. We find that house prices are notably more responsive to monetary policy shocks in times when they have been decreasing. The peak response of the point estimates almost doubles. Also, for the most part of the dynamic response, the boom and bust confidence intervals measured at one standard deviation, do not overlap, indicating a statistically significant difference at a 32% confidence level. In connection with our previous findings about belief updating in booms and busts, this result indicates that stronger belief updating in busts indeed affects realized prices. In Appendix (C.4) we show that this result also extends to forecast errors responding to monetary policy shocks.

Figure 4: House price response to MP shock, boom-bust



Notes: Responses to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% (Newey-West).

V.B Cross-regional heterogeneity and housing supply

Our findings in Section (IV) indicated that structural regional disparities within the housing market are crucial in explaining why house price growth exhibits greater dispersion during economic downturns than in periods of expansion. In the following, we focus on a specific dimension of regional heterogeneity—variation in the housing supply—which has been shown to influence regional house price dynamics.[29] Recent research by Aastveit and Anundsen (2022) has linked regional variations in housing supply elasticities across US metropolitan areas to differential responses in house prices to monetary policy shocks. Building upon their work, we demonstrate that these findings apply at the US state level. Additionally, we provide evidence that analogous patterns are observable in the Euro Area.

Econometric setup. To study the reaction of these regional variables to a common monetary policy shock, we estimate panel local projections to an externally high-frequency identified monetary policy shock. Equation (16) represents the empirical specification:

$$q_{n,t+h} = \alpha_n^h + \beta^h \epsilon_t^{MP} + \gamma^h \epsilon_t^{MP} \times z_n + x_{n,t} + u_{n,t+h}, \qquad h = 0, 1, \ldots, H. \tag{16}$$

[29] See, for example, Glaeser et al. (2008); Saiz (2010); Mian et al. (2013); Mian and Sufi (2014); Guren et al. (2021).

For the left-hand side variable (q_n) is the log of house prices. ϵ_t^{MP} denotes the monetary policy shock. We further interact this shock with a region-specific variable capturing supply-side heterogeneities, which we denote as z_n. $x_{n,t}$ is a vector of aggregate and regional controls. A time-fixed effect and a region-fixed effect are also included. We use this empirical specification for the US, as well as for the Euro Area. For consistency, we study a one-standard deviation monetary policy shock and standardize the interaction coefficients throughout all exercises.

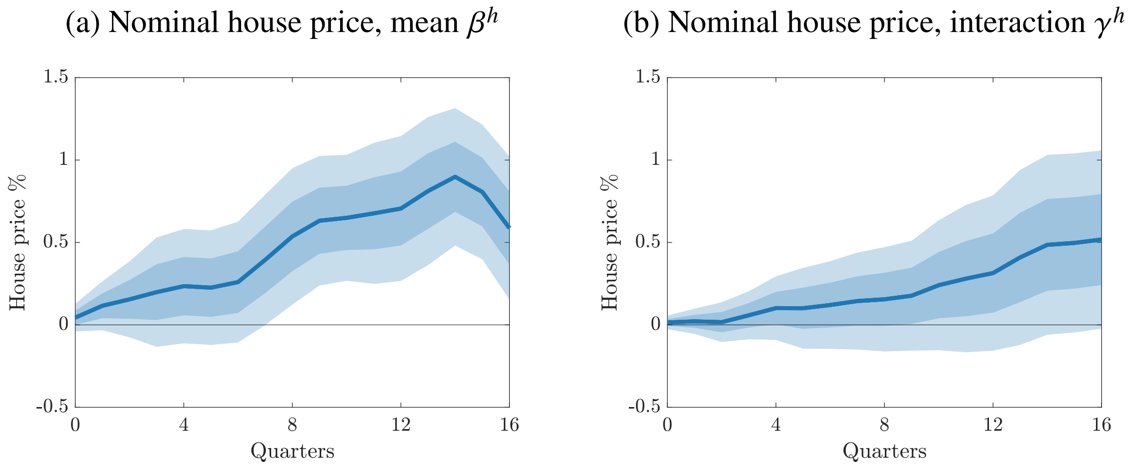
Results. For the US we focus on state-level data in the cross-section.[30] The monetary policy shock is the same as before and taken from Bauer and Swanson (2023). The interaction term is the house price sensitivity indicator from Guren et al. (2021). It measures the responsiveness of metropolitan area house prices to an increase in house prices at the Census region level controlling for a broad range of local economic conditions. It aims to capture housing supply side heterogeneities.[31] Empirically, housing supply elasticities reflect regional geographical or administrative constraints that influence the construction sector (Saiz, 2010). To facilitate the interpretation of this measure and to enable a tractable integration of our empirical estimates into the model, we interpret housing supply elasticities as a broad indicator of the time required to complete housing construction. We show that the housing sensitivity indicator is correlated with time-to-build measured at the Census division level in Appendix (C.5). To obtain state-level housing sensitivity measures, we aggregate the metropolitan-level data by weighing them according to population size. The vector of controls includes 8 lags of the left-hand side variable, the log of US GDP, the log of the GDP deflator, the FFR, the shock, and the interacted shock term. The sample runs from 1990 to 2019 and is in quarterly frequency.

Figure (5) plots the response of nominal house prices to an expansionary monetary policy shock. We find a sizeable and persistent increase in house prices in the mean coefficient, as seen in panel (a). Further, the interaction coefficient, shown in panel (b), is positive and persistent. Both IRFs are significant at least at a 90% confidence level. Our findings indicate that house prices are increasing in response to an expansionary monetary policy shock and they increase by more in states where supply is more constrained. These findings are in line with Aastveit and Anundsen (2022). Appendix (C.6) shows that these results carry over to the Euro Area.

[30] Hawaii and Alaska are not included in the sample due to insufficient data coverage.

[31] This indicator can be understood as a proxy for supply-side elasticities in the spirit of Saiz (2010). Contrary to the supply-side elasticities estimated by Saiz (2010), the house price sensitivity indicator is uncorrelated with demand-side characteristics.

Figure 5: House price response to MP shock, US



Notes: House price response to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

Calibrating supply side heterogeneities. To calibrate the model, we use the time-to-build measure presented in Appendix (C.5). A significant proportion of census divisions exhibit an average construction duration of approximately six months, measured from the issuance of building authorization to project completion. Notably, in two densely populated regions on the East Coast, the reported time-to-build extends to nearly ten months. Considering the additional time required for obtaining building permits and completing the sale of the property, we set the time-to-build parameters in the model to $\tau = 2$ and $\tau^* = 4$ quarters.[32]

V.C Remaining model parameters

Table (3) lists the parameter values, all of which are symmetric across regions. First, focusing on the household sector, the labor disutility shifter, the inverse Frisch elasticity, and the intertemporal elasticity of substitution are set to standard values in line with the literature. The discount factor is set to achieve a 1.5% steady-state interest rate. We further assume the regions are symmetric in size. We set housing depreciation to 2% per quarter. Moving to the production sector, we set the home bias, the elasticity of substitution across regions and goods in accordance with Bletzinger and von Thadden (2021). We will choose the persistence parameter of house price beliefs and the Kalman gain as presented in Table (2). The Taylor rule weight on inflation is 1.5, which is standard.

[32] The time it takes to obtain a building permit can vary largely across cities. In Dallas, for instance, it may take only a few business days, while in New York it takes between 1-3 months (see: permitflow.com). This difference is reflected in our calibration.

In terms of price adjustment, we choose the price adjustment costs such that the peak response of house prices to a monetary policy shock, unconditional on booms or busts, matches the response in the data. The slope of the Phillips Curve is given by $\frac{\epsilon-1}{\kappa} = 0.018$. This is in line with the recent literature.[33]

Table 3: Model parameters (symmetric parameters)

Parameter		Value	Description	Source/ Target
Households	χ	1.000	labor disutility shifter	standard
	φ	1.250	inverse Frisch elasticity	standard
	σ	2.000	inverse of intertemporal EOS	standard
	ν	1.000	housing utility elasticity	Iacoviello (2005)
	δ	0.020	housing depreciation	2% quarterly depreciation
	β	0.995	discount factor	standard for quarterly frequency
	γ	0.500	relative region size	symmetric regions
Goods aggregation & production	λ	0.800	home bias	
	ς	1.000	EOS across regions	Bletzinger and von Thadden (2021)
	ϵ	6.000	EOS across varieties	
	κ	272.18	price adjustment costs	slope of 0.0125 for the Phillips curve
	η	0.800	elasticity of housing production	Adam et al. (2022)
Policy	ϕ_π	1.500	Taylor coefficient	standard

Notes: All parameters depicted above are equal across countries. One period in the model is one quarter.

Table (C.2) in the Appendix shows the allocation and prices in the non-stochastic steady state. We choose a higher degree of time-to-build in the foreign region. The steady-state value for the housing preference shifter is chosen such that we attain a symmetric steady-state in the allocation variables. This modeling choice ensures that all cross-regional differences result from the structural heterogeneity on the housing supply side. The steady-state value for the house price is the only

[33] Adam and Billi (2006) set the slope of the Phillips Curve to 0.057. However, it has been argued that the slope of the Phillips Curve has decreased, see i.e Del Negro et al. (2020) and Hazell et al. (2022). Our parameter choice is in line with their estimates.

variable that differs across countries. Symmetric steady-state values for bond levels, which are zero for both countries, also imply that there is no net-borrower or net-saver country in the steady-state. Changes in monetary policy will therefore not lead to Fisherian debt revaluation effects. We parameterize the model to match the housing sector in the US economy and target a steady-state housing investment-to-consumption ratio, $\frac{x_{ss}}{c_{ss}}$, of 6.5% as in Adam et al. (2022).

VI. Quantitative Results

In this section, we will present our findings. First, we show that the model is able to replicate aggregate house price responses to a monetary policy shock, conditional and unconditional on booms and busts. Second, the model captures the documented boom-bust-asymmetry in regional house price growth dispersion.

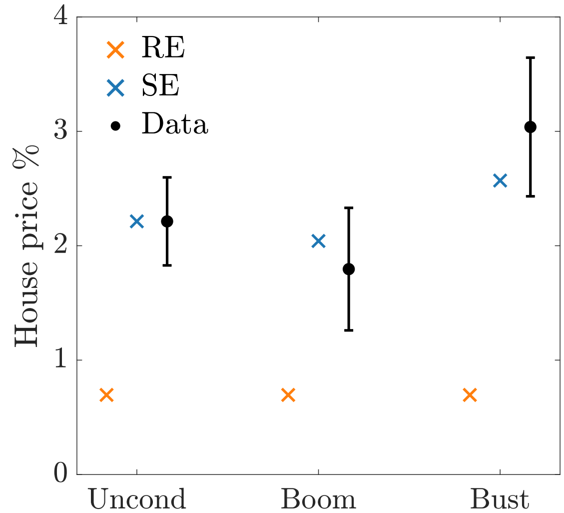
VI.A Aggregate house prices in booms and busts

We will start by studying the model performance on an aggregate level. All aggregate variables are a convex combination of the regional variables weighted by their size. Hence, the aggregate house price is given by $\widehat{q}_t^{agg} = \gamma\widehat{q}_t + (1-\gamma)\widehat{q}_t^*$. We will study the aggregate house price response in the model and the data, unconditionally and conditionally on being in a boom or a bust. As discussed above, we target the peak response unconditionally of being in a boom or bust. We then study the performance of the rational expectations model and the subjective expectations model conditional on being in a boom or bust. For the unconditional response in the model we simply consider the linear model using the parameterization described above. For the unconditional empirical response we estimate Equation (15) and drop the conditionality on the boom-bust regimes with respect to the monetary policy shocks. The conditional empirical response is given by local projections estimated from Equation (15). This setup estimates house price responses to a monetary policy shock conditional on being in a boom or a bust. An appropriate model counterpart produces the response to a monetary policy shock conditional on being in either a boom or bust regime. To construct this conditionality in the model, we use a linear model but adjust the model parameters such that the model captures either boom or bust dynamics in house price updating. We use the parameters estimated in Table (2). For the boom we have a persistence parameter of $\varrho^h = 0.99$ and a Kalman gain of $g^h = 0.0117$. For the bust period we have $\varrho^l = 0.91$ and $g^l = 0.0233$. Importantly, the respective parameters have no effect on the steady-state. Therefore, the shock will hit both models in the same steady-state but the dynamics will be different due to differential updating behavior. We consider a 25 basis points expansionary monetary policy shock across all models and their empirical counterparts.[34] As discussed previously, these shocks are empirically too small to affect the transition from one regime to the other, thereby validating our quantitative exercise. We will first focus on the magnitudes of the responses in the model and the data, after which we move on to the dynamics.

Figure (6) plots the peak responses in the model and the data, unconditional and conditional on booms and busts. First, the rational expectations model is unable to match the peak response observed in the data. It misses the peak response in the data by roughly a factor of 3. Further, the response is equivalent across boom and bust periods as there is no source of asymmetry in the rational expectations model. In contrast, the subjective expectations models conditioning on booms and busts match their empirical counterparts quite well. For the house price response in a boom, we find a peak response of 2.0 in the model and 1.8 in the data. The model peak response lies within one standard deviation of the empirical estimate. For the bust case, we find a peak response of 2.6 in the model and 3.0 in the data. Accounting for estimation uncertainty, the model lies in the range of the one standard deviation confidence intervals. The results reflect our findings from Proposition (2) and (3). Under subjective expectations, the response to a monetary policy shock is scaled up compared to the rational expectations counterpart. Additionally, extrapolation further dynamically amplifies house price responses. Both of these channels seem to be very important to match the magnitude of the house price response in the data.

[34] To determine the size of 25 bp monetary policy shock in the data, we estimate a local projection exercise regressing the FFR on the monetary policy shock. We choose the minimum of the IRF from the first 12 months as a reference point to determine the empirical shock size. We use this approach unconditional and conditional on booms/busts.

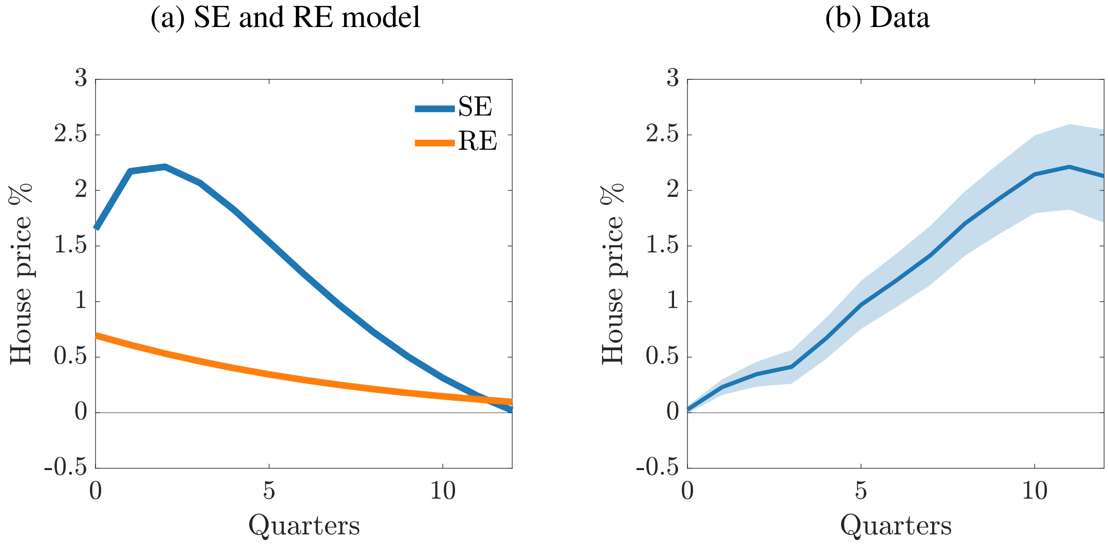
Figure 6: Peak response of house prices to a MP shock, model vs. data



Notes: Response to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newey-West); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

We will now turn to the dynamic responses in the model and the data. Figure (7) shows the response of house prices to a 25 basis point expansionary monetary policy shock in the model and in the data unconditional on booms and busts. The empirical response is depicted in panel (b). The model counterparts, in panel (a), show the response of the linear model under subjective expectations and rational expectations. In terms of dynamics we observe a very sluggish response in the data, the peak response is only reached after 10 quarters. In the rational expectations model, we observe no hump shaped pattern at all. House prices peak on impact and return to the steady-state thereafter. In the subjective expectations model we do observe some sluggishness in the house price dynamics. House prices respond on impact, increase for the first four quarters, after which the model converges back to its' steady-state. The dynamic can be explained through the extrapolative belief structure. After the initial shock, agents believe that house prices will increase further and invest into housing. They will continue to do so until their beliefs are not met, after which they will adjust their behaviour and the model returns to it's steady-state. Quantitatively, it turns out that this behaviour is too short-lived to explain the sluggishness observed in the data. The inability of DSGE models to match the persistence in the data is well known. Therefore, medium sized DSGE models such as Smets and Wouters (2007) tend to add backward looking components, for instance habit formation, to match these dynamics. While the extrapolative beliefs do improve the model performance to a certain degree, they cannot capture the full persistence of the response. This indicates that some sort of sluggish adjustment behaviour in housing demand, for instance habit formation on housing or housing search frictions, are needed to exactly match the data.

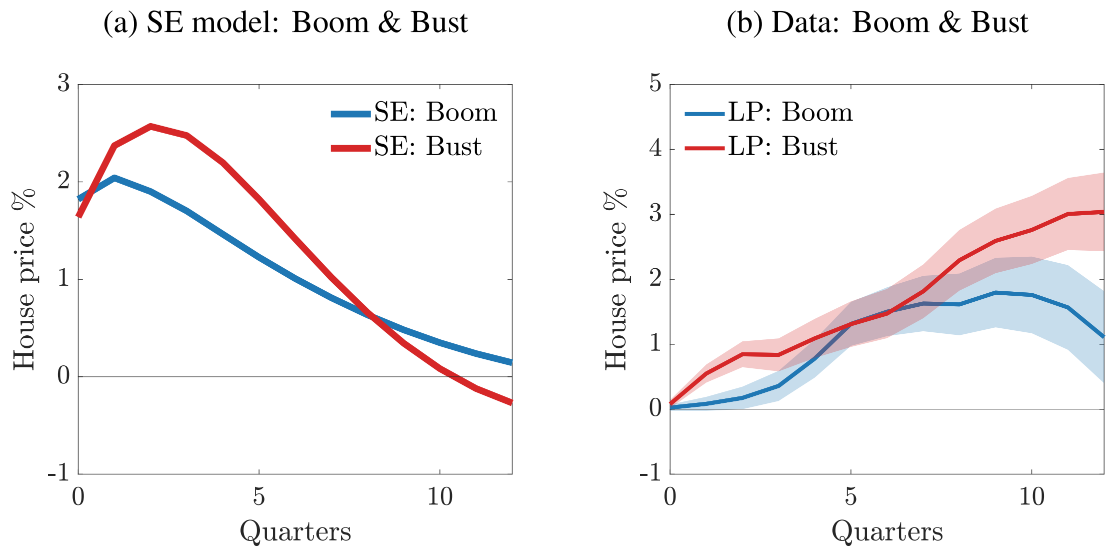
Figure 7: House price response to MP shock, model vs. data



Notes: Responses to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newey-West).

We will now turn to house price responses in booms and busts. Panel (b) in Figure (8) plots the empirically estimated impulse responses from the local projections exercise. The dynamics are equivalent to the ones depicted in Figure (4), but scaled to match a 25 basis point increase. We only plot 68% confidence bands. Panel (a) shows the subjective expectations model responses for the boom and bust parameterization discussed above. In terms of dynamics, we observe the same short-comings as in the unconditional case. The model is unable to capture the full persistence observed in the data. However, we observe a less persistent response in the boom relative to the bust. The model is able to capture this pattern qualitatively.

Figure 8: House price response to MP shock in booms and busts, model vs. data



Notes: Responses to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newey-West); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

VI.B Cross-regional heterogeneity in booms and busts

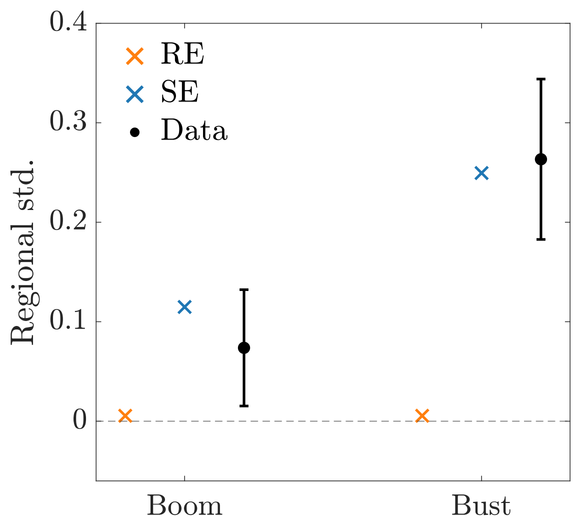
Having demonstrated that our model can replicate aggregate house price responses during both boom and bust periods, we now turn our attention to analyzing cross-regional heterogeneities. We evaluate the model's performance in response to a monetary policy shock conditional on booms and busts.

Regional house price growth dispersion. We begin by comparing the cross-regional house price growth standard deviation in the data to those in the model. Empirically, we use the local projections presented in Figure (2). As for house prices, we focus on the impulse responses in booms and busts at their respective peaks. We compare the empirically estimated peak responses with their counterparts in a rational expectations and subjective expectations model conditional on being in a booms or busts. Figure (9) presents the results.

Focusing on the rational expectations model, we find that the model is unable to generate any sizable regional differences: the peak response is almost zero. As before, the rational expectations model is also unable to generate any differences between booms and busts.Moving to the subjective expectations model, we find that the model slightly overstates the peak response in a boom relative to the data. However, accounting for estimation uncertainty, we are well within the one standard deviation confidence intervals. In the bust the model does extremely well, it only marginally understates the empirically estimated peak response.

These results are in line with Proposition (4): Housing supply side heterogeneities create differences in house price growth, which in turn is amplified by extrapolation. Larger extrapolation in busts means stronger amplification and therefore larger cross-regional differences. Under rational expectations the amplification mechanism is absent, therefore the model is only able to create minor cross-regional differences.

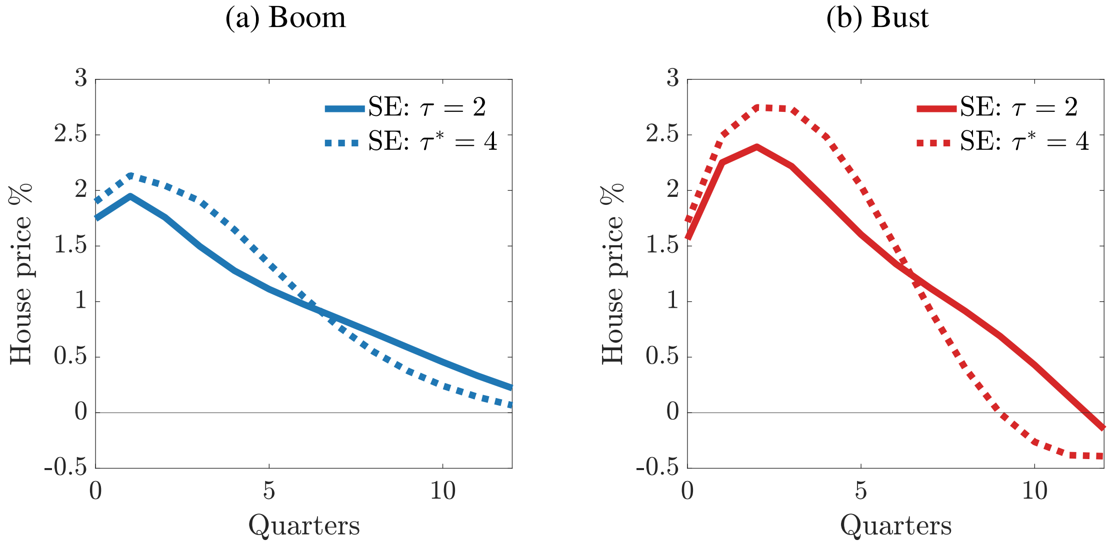
Figure 9: Peak response of reg. house price growth std. to a MP shock, model vs. data



Notes: Response to MP shock (25 bp); Confidence Intervals: 68% (Newey-West); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

House prices and economic activity. After establishing that regional house price growth responses to monetary policy shocks differ between boom and bust periods, we now shift our focus to examining house price responses in levels and, ultimately, their impact on aggregate economic activity. These dynamics are of particular interest due to their implications for welfare and, consequently, their relevance for policy considerations. In the following, we will only focus on the subjective expectations model since our earlier results showed that regional disparities in the rational expectations model are negligible. Figure (10) shows the response of house prices to a monetary policy shock conditional on booms and busts. First, we find that generally, the house price is more responsive in regions where time-to-build is larger, hence in the region where $\tau^* = 4$. Second, the differences between the regions is larger in times of a house price bust. Both of these findings are in line with the results presented in Section (IV).

Figure 10: Regional house price response to MP shock in booms and busts



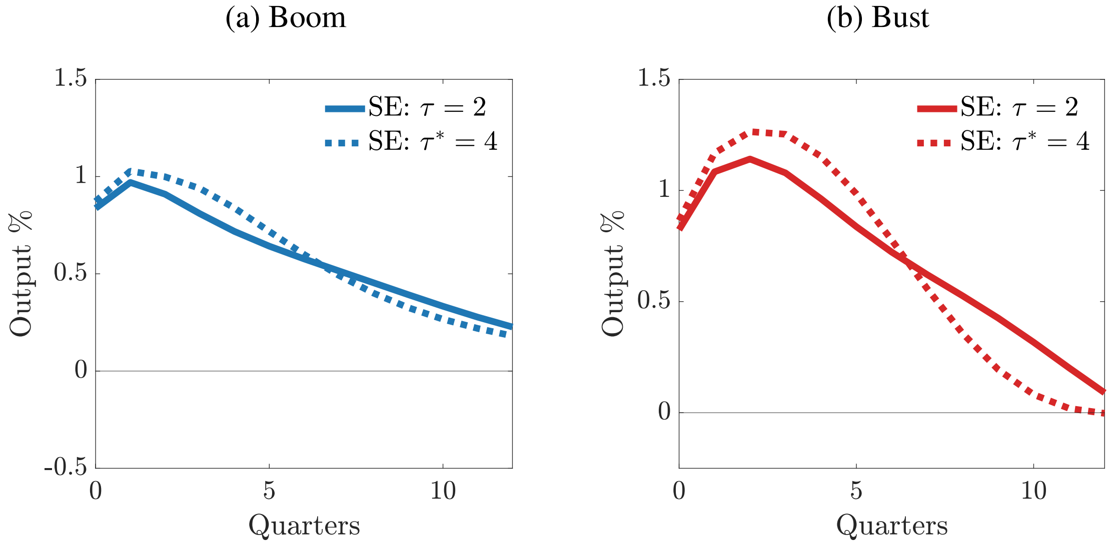
Notes: Responses to expansionary MP shock (25 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

In terms of aggregate activity, we study the response of regional output to a monetary policy shock, again conditioning on booms and busts. Figure (11) presents the results. We observe that the region in which house prices are more responsive, the region with a higher time-to-build, also experiences a larger expansion in output. The underlying intuition is as follows: as house prices rise, households expect future price increases and subsequently raise their investment in housing. This surge in housing investment leads to a corresponding increase in output. Due to stronger extrapolation during busts, the output response and regional differences in output are more pronounced in these periods.

These findings highlight that regional differences in house price growth translate into house price levels and output variations. As these regional disparities propagate through the model, they eventually affect consumption, labor markets, and inflation. Consequently, these dynamics influence welfare and become of primary importance for monetary policy. In the next section, we will discuss the policy implications of these results.

Figure (D.1) in the Appendix shows the response of regional house prices and output under rational expectations. We observe that the rational expectations model is unable to create any sizable regional differences. This result is in line with our Propositions from Section (IV).

Figure 11: Regional output response to MP shock in booms and busts



Notes: Responses to expansionary MP shock (25 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

VII. Policy analysis

Turning to policy implications, we study how targeting house prices in the Taylor rule changes aggregate variation and cross-regional variation. The linearized Taylor rule we consider is given by:

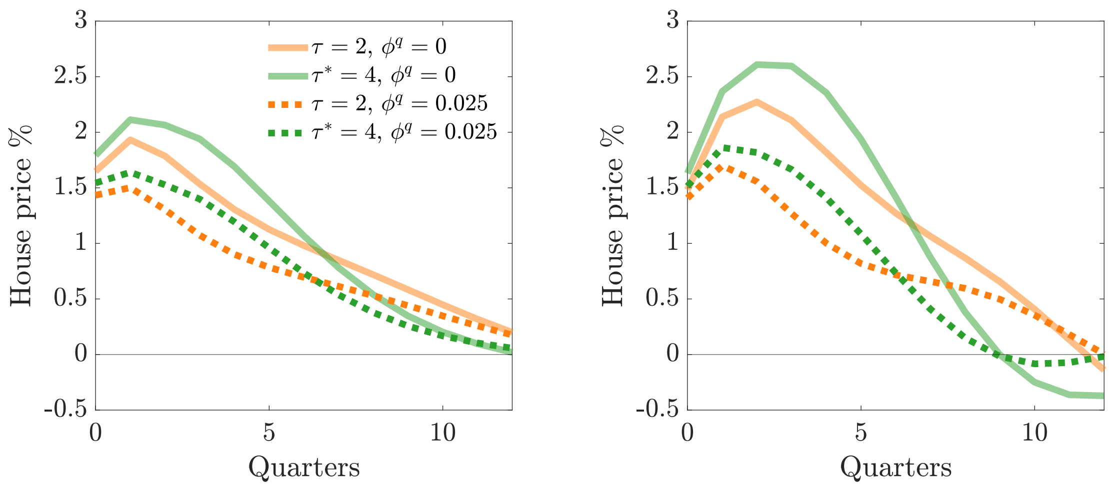
$$i_t = \phi^\pi \widehat{\pi}_t^{agg} + \phi^q \widehat{q}_t^{agg} \tag{17}$$

For our baseline case, we consider a rule with $\phi_\pi = 1.5$ and $\phi_q = 0$, as we did for all exercises above. Under house price targeting, we increase the weight on house prices to $\phi_q = 0.025$. We study the response of house prices to an expansionary productivity shock in the baseline case and under house price targeting.

Impulse responses under house price targeting. Figure (12) presents the house price responses to a productivity shock in booms and busts comparing house price targeting to our baseline. The response of house prices to a productivity shock under the baseline model closely mirrors the response to a monetary policy shock, as outlined in Section (VI). Specifically, both house prices and regional disparities in house prices tend to be more pronounced during busts than during booms. When house prices are explicitly targeted, however, the response of house prices is dampened, with a corresponding reduction in regional heterogeneity. Furthermore, a comparison of booms and busts reveals that monetary policy exhibits greater effectiveness during busts, as the relative reduction in both house prices and regional heterogeneity is more substantial in busts than in booms. The increased responsiveness of house prices can be attributed to a larger pass-through effect from house prices to expectations, which arises from a heightened degree of extrapolation. As a result, policy changes are also transmitted to house prices with greater intensity.

Figure 12: House price targeting: house price response in booms and busts

(a) Boom (b) Bust



Notes: Responses to expansionary productivity shock (100 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Figure (D.2) in the Appendix shows that these results carry over to aggregate activity. House price targeting reduces the response of output and regional heterogeneity in output. As for house prices, house price targeting is more effective in busts. Intuitively, house prices drive housing investment and thereby output. By targeting house prices the central bank implicitly reduces housing investment and consequently output.

Under rational expectations, this result breaks down (see Figure (D.3) in the Appendix): the house price and output response are reduced but to a smaller degree than under subjective expectations. Further, regional heterogeneity is unaffected, as the model is unable to create any regional differences in the first place.

To understand how house price targeting affects the economy in a broader sense, we compute standard deviations of producer price inflation, output, and house price over the first 48 quarters after the shock hits the economy. Table (4) presents the results for those variables at an aggregate level in booms and busts. It also contrasts the rational expectations model with the subjective expectations model.

Analyzing the subjective expectations model, we find that house price targeting, on the one hand, reduces aggregate volatility in both house prices and output. Moreover, this policy proves to be more effective during busts, as previously discussed. On the other hand, a trade-off emerges between stabilizing house prices and managing inflation. Increasing the emphasis on house price targeting diminishes house price volatility but leads to greater inflation volatility. The underlying intuition is that a greater focus on house prices by the monetary authority reduces the relative weight placed on inflation, thereby increasing inflation.

In the rational expectations model, we observe similar dynamics. In contrast to the subjective expectations model, the changes in volatility between the baseline and house price targeting are smaller, and there is no difference in booms and busts.

Table 4: Aggregate standard deviations across policy rules

			Boom			*Bust*		
Model	ϕ^{π}	ϕ^{q}	π^{agg}	y^{agg}	q^{agg}	π^{agg}	y^{agg}	q^{agg}
RE	1.5	0.00	0.03	0.15	0.18	0.03	0.15	0.18
	1.5	0.25	0.04	0.14	0.16	0.04	0.14	0.16
SE	1.5	0.00	0.01	0.30	0.64	0.02	0.36	0.79
	1.5	0.25	0.02	0.22	0.48	0.02	0.24	0.50

Notes: The table shows standard deviations of several variables in response to an expansionary productivity shock (100 bp); The standard deviations are computed over the first 48 quarters; Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Focusing on regional differences, we compute the cross-regional standard deviations across both policy rules and both models for producer price inflation, output, and house prices. The results are shown in table (5). Beginning with the rational expectations model, we observe no cross-regional variation in either inflation or output. However, there are minor regional variations in house prices, which are further diminished by leaning against house price fluctuations. In contrast, the subjective expectations model exhibits cross-regional differences across all variables. Here, targeting house prices significantly reduces these variations, with the policy proving more effective in times of busts. Intuitively, extrapolation amplifies movements in house prices which spill over to the rest of the economy. If house prices are not as volatile, extrapolation is also mitigated, and so is the differential regional response.

Table 5: Cross-region standard deviations across policy rules

			Boom			*Bust*		
Model	ϕ^{π}	ϕ^{q}	$\pi^{H,F}$	$y^{H,F}$	$q^{H,F}$	$\pi^{H,F}$	$y^{H,F}$	$q^{H,F}$
RE	1.5	0.00	0.000	0.000	0.0040	0.000	0.000	0.0040
	1.5	0.25	0.000	0.000	0.0037	0.000	0.000	0.0037
SE	1.5	0.00	0.001	0.016	0.055	0.002	0.045	0.135
	1.5	0.25	0.000	0.012	0.040	0.000	0.036	0.110

Notes: The table shows standard deviations of several variables in response to an expansionary productivity shock (100 bp); The standard deviations are computed over the first 48 quarters; Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Welfare consequences. We have demonstrated that house price targeting leads to a reduction in both the aggregate variance of house prices and output and a universal decrease in regional dispersion across all variables. However, this policy is associated with an increase in inflation volatility. This observation prompts the critical question of whether house price targeting is a desirable policy intervention. To assess this, we derive a welfare-based loss function and perform a comparative analysis across different policy rules. The loss function is based on a second-order approximation of the utility function of each region, with each region being weighted by its size. It is important to note that the steady-state is efficient.

Before we turn to the derivation of the loss function, we will briefly review the sources of inefficiency in this model. In the rational expectations version of the model, the only source of inefficiency is inflation which arises due to price adjustment costs. Under subjective expectations, agents' choices are distorted due to their misspecified beliefs. In particular, households may be willing to shift resources from consumption units to housing investment and housing demand to achieve capital gains in the future. Additionally, this will also distort households' labor supply. This provides a motive for the policymaker to lean against house prices. Further, in our setup their welfare losses can arise due to regional heterogeneities. We will discuss this in detail below.

A second-order approximation of the households utility gives the following consumption-equivalent loss function:

$$\frac{\mathbb{W}_{-1}}{c_{ss}} = -c_{ss}^{\sigma}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t \cdot \left[\frac{\gamma}{2}\Omega_t + \frac{1-\gamma}{2}\Omega_t^* + \frac{1}{2}\Gamma^s\widehat{s}_t^2 + O(3) + \text{t.i.p.}\right] \tag{18}$$

Ω_t collects the welfare-relevant choices and is defined as:

$$\Omega_t = \Gamma^\pi \widehat{\pi}^2_{H,t} + \Gamma^c \widehat{c}^2_t + \Gamma^h \widehat{h}^2_t + \Gamma^n \widehat{n}^2_t + \Gamma^x \widehat{x}^2_t + \Gamma^b \widehat{b}^2_t - \Gamma^{\xi_a} \widehat{\xi}_{a,t} \widehat{n}_t$$

Ω^*_t is defined equivalently. The welfare weights (Γ^j) are explicitly stated in Appendix (D.2). From equation (18) it becomes apparent that the welfare loss contains three parts: the domestic choices (Ω_t), the foreign choices (Ω^*_t), and relative prices measured by the terms of trades ($\widehat{s}_t$). The formulation further reveals that regional dispersion negatively impacts welfare. This result can be demonstrated in two distinct ways. First, consider a scenario where all variables, except one, are symmetric across regions and the regions have the same size. Suppose the policymaker could implement a policy that alters the heterogeneous variable by shifting variation from one region to another without influencing the other variables. Given the convexity of the loss function, it would be welfare-enhancing to eliminate this regional heterogeneity. Second, the terms of trade reflect relative price differentials between regions. If regions move in tandem, this term becomes zero, improving welfare. Consequently, policymakers are incentivized to promote homogeneous co-movement across regions to optimize welfare.

To illustrate this point, we can further rearrange equation (18) in terms of variation at the aggregate level and cross-regional variance.

$$\frac{\mathbb{W}_{-1}}{c_{ss}} = -c^\sigma_{ss} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \cdot \left[\frac{1}{2}\Omega^{agg}_t + \frac{1}{2}\Omega^{var}_t - 2\Omega^{cov}_t + \frac{1}{2}\Gamma^s \widehat{s}^2_t + O(3) + \text{t.i.p.} \right] \tag{19}$$

Ω^{agg}_t collects the aggregate variation on a currency area level, Ω^{var}_t the cross-regional variation, and Ω^{cov}_t covariance terms:

$$\begin{aligned}
\Omega^{agg}_t &= \Gamma^\pi (\widehat{\pi}^{agg}_t)^2 + \Gamma^c (\widehat{c}^{agg}_t)^2 + \Gamma^h (\widehat{h}^{agg}_t)^2 + \Gamma^n (\widehat{n}^{agg}_t)^2 + \Gamma^x (\widehat{x}^{agg}_t)^2 - \Gamma^{\xi_a} \widehat{\xi}_{a,t} \widehat{n}^{agg}_t \\
\Omega^{var}_t &= \Gamma^\pi \widehat{\sigma}^2_{\pi,t} + \Gamma^c \widehat{\sigma}^2_{c,t} + \Gamma^h \widehat{\sigma}^2_{h,t} + \Gamma^n \widehat{\sigma}^2_{n,t} + \Gamma^x \widehat{\sigma}^2_{x,t} + \Gamma^b \widehat{\sigma}^2_{b,t} \\
\Omega^{cov}_t &= \Gamma^\pi \lambda (1-\lambda) \widehat{\pi}^2_{H,t} \widehat{\pi}^2_{F,t}
\end{aligned}$$

The $\sigma^2_{j,t}$ terms denote the cross-regional variance of a variable j. The covariance term (Ω^{cov}_t) appears due to the presence of the home bias in consumption. Equation (19) indicates that the policymaker's objective is a function of both the aggregate economic variation and the degree of regional dispersion. By implementing a policy that promotes more synchronized economic dynamics across regions, the policymaker can effectively enhance overall welfare.

We now revisit the productivity shock and calculate the welfare loss under both our baseline model and house price targeting, comparing outcomes for the rational expectations model and the subjective expectations model. Table (6) presents the results and a decomposition into welfare loss on the aggregate level (Ω^{agg}) and the cross-regional variation (Ω^{var}). We drop the covariance and the terms of trades parts, as they are quantitatively irrelevant. We find that under rational expectations house price targeting is welfare detrimental, as house price targeting increases the loss by 12% relative to the baseline. However, under subjective expectations, house price targeting proves beneficial, yielding welfare improvements of 7% during booms and 24% during busts. This difference arises because, under subjective expectations, rising house prices lead households to anticipate further price increases. This results in a reallocation of resources from consumption to housing investment, alongside an increase in labor supply. These decisions, driven by overly optimistic projections of future house prices, generate welfare losses. House price targeting helps to stabilize price dynamics, thereby reducing the inefficiencies arising from misguided decision-making. As extrapolation is stronger during busts, the associated welfare losses are more severe, making house price targeting particularly advantageous during these periods. In contrast, under rational expectations, these channels are absent and so are the potential benefits of house price targeting.

Turning to the sources of welfare loss, we find that the welfare loss is mostly accounted for by volatility in the aggregate variables (Ω^{agg}). Under rational expectations, welfare loss through cross-regional variation (Ω^{var}) plays no role. Under subjective expectations, we find small welfare losses in the boom case. During a bust, these losses triple in the baseline case, but are still relatively small compared to the overall welfare loss. The relatively small welfare loss attributed to regional heterogeneities can be explained as follows. The primary driver of cross-regional variation in the welfare loss function arises from differences in housing investment across regions. However, the housing sector represents a relatively small share of the overall economy, resulting in a low welfare weight (Γ^x), which exerts a minimal effect on aggregate welfare. Additionally, the pass-through from the housing sector to the broader economy is weak, as it operates predominantly through general equilibrium channels. A model that amplifies these linkages, such as one incorporating heterogeneous agents, would likely generate stronger cross-regional variation in the non-housing sector. Consequently, cross-regional disparities would play a more significant role in determining aggregate welfare outcomes.

Table 6: Welfare loss across policy rules

Model		*RE*	*SE* Boom	*SE* Bust
Baseline	Ω^{agg}	−0.172	−0.147	−0.204
	Ω^{var}	0.000	−0.001	−0.003
	total	−0.172	−0.148	−0.207
$\widehat{q}_t$ *targeting*	Ω^{agg}	−0.191	−0.137	−0.154
	Ω^{var}	0.000	−0.001	−0.002
	total	−0.191	−0.138	−0.156
Ratio	Ω^{agg}	1.11	0.94	0.76
	Ω^{var}	0.86	0.57	0.57
	total	1.11	0.94	0.75

Notes: The welfare loss in response to an expansionary productivity shock (100 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust); Ratio denotes the ratio of the welfare loss between house price targeting and the baseline; The welfare loss has been scaled by 100.

VIII. Conclusion

In this paper, we examine regional heterogeneity in housing cycles and their implications for monetary policy. We document that regional dispersion in house price growth is larger in busts compared to booms. We then develop a two-region New Keynesian model and show that including stronger belief updating and regional housing supply side heterogeneities can jointly explain the increase of regional house price growth dispersion in busts. We empirically provide evidence that supports our modeling choices in terms of belief updating and housing supply side heterogeneities. Our findings suggest that placing a greater emphasis on house prices by the monetary authority can reduce volatility in output and house prices, as well as regional dispersion in inflation, output, and house prices. Under subjective beliefs, this policy proves to be welfare-improving.

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Appendix

A. Micro-founding the debt-elastic interest rate

In the model, households in country H receive on their bond holdings the effective nominal interest rate $1+i_{t-1}-\psi b_t$, with b_t being the real value of the aggregate bond holding in country H; households in country F receive the effective nominal rate $1+i_{t-1}-\psi b_t^*$. Moreover, the intermediation of bond positions entails a real cost $\gamma(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2+(1-\gamma)(1+\pi_t)^{-1}\frac{\psi}{2}(b_t^*)^2$ of which $(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ is paid by each consumer in H and $(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ is paid by each consumer in F. In this Appendix we detail how these debt-elastic interest rates and the associated intermediation cost can be parsimoniously micro-founded. We achieve this by introducing two competitive bond clearing houses, one in each country, that represent the only access of households to financial markets and who incur a real cost that is quadratic in the size of their balance sheet. The specific market arrangement is as follows: households hold a consol and may hold liquid bonds.

Consol. Each household in H is endowed with $\bar{b}\in\mathbb{R}$ units of a non-marketable consol[35] that pays as a coupon $(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P_{t-1}^*)P_t^{-1}$ units of H's consumption basket each period, per unit of consol. This implies that the nominal coupon rate, applied to the nominal coupon value $P_{t-1}\bar{b}$, is $(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P_{t-1}^*)/P_{t-1}$; the real coupon rate applied to the real value $\bar{b}$ in turn is $(\beta^{-1}-1)\left(\gamma+(1-\gamma)\frac{P_{t-1}^*}{P_{t-1}}\right)(1+\pi_t)^{-1}$. The situation in country F is symmetric: each household is endowed with $\bar{b}^*$ units of a consol that pays $(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P_{t-1}^*)(P_t^*)^{-1}$ units of F's consumption basket each period, per unit of consol. $\bar{b}$, $\bar{b}^*$ are model parameters selected such that (i) $\gamma\bar{b}+(1-\gamma)\bar{b}^*=0$ and (ii) all markets clear in the non-stochastic steady state with zero net inflation and terms-of-trade parity *without* households holding any liquid bonds. The latter fact ensures that there is no cost of financial intermediation in the steady state, shutting down this particular friction. The specific choice of the coupon payment scheme ensures two facts: (1) condition (i) implies that the nominal payments between H and F associated with the two consols exactly cancel out – whatever H/F receives as coupon payments on its consol endowment is paid for by F/H as a coupon service on its (endowed) short position of consol; and (2) the real coupon rates paid by/ to the consol endowment only depend on the real exchange rate and the inflation rates, not on the price levels. Households cannot trade their consol holdings.

[35]A consol is a type of bond that has infinite maturity and just keeps paying a constant or varying coupon perpetually.

Bonds. Household do have the possibility, though, to vary their position in the liquid bond. This liquid bond is a nominal, one-period, zero-coupon bond and the positions of the representative H-, respectively the representative F-household are denominated b_t, b_t^*. If a household wants to hold a net balance of liquid bonds different from zero, she has to go to one of the clearing houses in her country: In the H-country, there is a continuum of mass γ (respectively mass $1-\gamma$ in F) of competitive clearing houses buying and selling bonds from and to the government and from and to the respective country's citizens. Households themselves cannot directly buy/sell government bonds without having an account at the clearing house. The clearing house can costlessly buy/sell bonds but incurs an operating cost that is quadratic in the size of its balance sheet, making this a model of costly financial intermediation. Thus, the interest rate that each citizen gets on her bond holdings is determined by the nominal rate paid on government bonds and the aggregate holding of liquid bonds. Each clearing house is owned equally by all citizens of the respective country so that it pays its profits to those citizens.[36] Consider an arbitrary clearing house in H (with symmetric arrangements in F). Denoting as $B_{c,t+1}$ the nominal value of the clearing house's net liabilities against H's citizens and as $B_{g,t+1}$ the nominal value of the clearing house's position in the government bond, the profit maximization program is:

$$\max_{B_{c,t+1},B_{g,t+1}\in\mathbb{R}} -(1+i_t^b)B_{c,t+1} + (1+i_t)B_{g,t+1} - \frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2, \text{ s.t. } B_{c,t+1} = B_{g,t+1}$$

where i_t^b is the nominal rate clearing the market for household bond positions and i_t is the nominal rate on government bonds that is set by the monetary authority. $\frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2 = \frac{\psi}{2}P_t(b_{c,t+1})^2$ is the nominal cost of intermediating – crucially, the real cost of intermediation does not directly depend on the price level. The first order conditions for this program are

$$\begin{aligned} 1 + i_t^b + \psi P_t^{-1}B_{c,t+1} &= \mu_t, \\ 1 + i_t &= \mu_t, \\ B_{c,t+1} &= B_{g,t+1}, \end{aligned}$$

[36]In equilibrium, each clearing house makes a non-negative profit, and along the transition path back to the steady state after some shock, each clearing house makes a strictly positive profit. This fact is in principle incompatible with the notion of competitiveness (there is an incentive to open up more clearing houses or, equivalently, it is strictly profitable to split each clearing house). Therefore, it is better to interpret the program of the clearing house as reflecting capacity constraints: the here-presented program can be thought of as the inner problem of a profit maximization program with an additional factor (say, managerial effort) that which (i) makes the intermediation service production function exhibit constant returns to scale (instead of decreasing RTS), (ii) is provided by households, and (iii) is in perfectly inelastic supply. Under this way of modeling the clearing house, it behaves exactly as modeled here, it always makes zero profits, and households get as remuneration for providing the additional factor the amount that is the profit in the current way of modeling.

where μ_t is the Lagrange multiplier on the balance-sheet constraint $B_{c,t+1} = B_{g,t+1}$. Market clearing in the household bond positions in H requires

$$\gamma B_{c,t+1} = \gamma P_t b_{t+1},$$

and market clearing in the government bond positions requires

$$\gamma B_{g,t+1} + (1-\gamma) B^*_{g,t+1} = 0,$$

so that by using the balance-sheet constraints $B_{c,t+1} = B_{g,t+1}, B^*_{c,t+1} = B^*_{g,t+1}$ and the clearing conditions for household bond positions in H and F we recover the market clearing condition for government bonds in the main model:

$$\gamma P_t b_{t+1} + (1-\gamma) P^*_t b^*_{t+1} = 0.$$

In sum, the aggregate conditions implied by this market arrangement are:

$$\begin{aligned} 1 + i^b_t + \psi b_{t+1} &= 1 + i_t, \\ 1 + i^{b,*}_t + \psi b^*_{t+1} &= 1 + i_t, \\ \gamma P_t b_{t+1} + (1-\gamma) P^*_t b^*_{t+1} &= 0. \end{aligned}$$

The nominal profits of the typical clearing house in H in equilibrium are:

$$\begin{aligned} \text{Profit}_{t+1} &= (i_t - i^b_t) B_{c,t+1} - \frac{\psi}{2} P_t^{-1} (B_{c,t+1})^2 \quad \text{with optimal } B_{c,t+1} = \frac{i_t - i^b_t}{\psi} P_t \\ &= (i_t - i^b_t)^2 \psi^{-1} P_t - (i_t - i^b_t)^2 \psi^{-1} P_t \cdot \frac{1}{2} \\ &= P_t \frac{\psi}{2} (b_{t+1})^2 \quad \text{using market clearing in the household bond positions.} \end{aligned}$$

Of the $1 + i_t$% nominal interest collected on (paid for) its position of government bonds, each clearing house withholds ψb_{t+1}% of the interest from its customers (respectively, charges $-\psi b_{t+1}$% of additional interest if $b_{t+1} < 0$). Half of these ψb_{t+1}% are used for covering the operating cost (by buying this amount of H's final basket and selling it in exchange for numéraire), and the other half is paid as profit to the owners of the clearing house (which, in equilibrium, are its customers).

B. Proofs and Derivations

B.1 Proof of Lemma (1)

Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system:[37]

$$\ln \frac{q_{t+1}}{q_t} = \varrho \ln \overline{m}_t + \ln \widehat{e}_{t+1}$$
$$\ln \overline{m}_t = \varrho \ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2} + g \cdot \left(\ln \widehat{e}_t + \frac{\sigma_e^2 + \sigma_v^2}{2} \right)$$

where $\ln \overline{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2+\sigma_v^2}{\sigma^2+\sigma_v^2+\sigma_e^2}$ is the steady-state Kalman filter gain, $\sigma^2 = \frac{1}{2}[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ is the steady-state Kalman filter uncertainty, and $\ln \widehat{e}_t$ is perceived to be a white noise process.

To avoid simultaneity in the house price we modify the belief setup following Adam et al. (2017).[38] We obtain the same observable system but with lagged information being used in the posterior mean updating equation:

$$\ln \overline{m}_t = (\varrho - g) \left(\ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2} \right) + g \left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2} \right). \tag{B.1}$$

Under this formulation, the posterior mean is pre-determined. We may now derive the posterior mean on the $s > 0$ periods ahead of price:

$$\mathbb{E}_t^{\mathcal{P}} q_{t+s} = q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \frac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2 \left(\varrho \frac{1-\varrho^s}{1-\varrho}\right)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \tag{B.2}$$

For the derivation of equation(B.2) see Appendix (B.2). This completes the proof.

[37] We assume agents' prior variance equals the steady-state Kalman variance.

[38] q_t appears twice: in the forecast equation, and in the Kalman-updating Equation through $\ln \widehat{e}_t$. Since q_t depends on $\overline{m}_t$, but the latter also depends on the former, it is not assured that at any point an equilibrium asset price exists and whether it is unique. See Adam et al. (2017) for the details. The idea of the modification is to alter agents' perceived information setup in that they observe each period one component of the lagged transitory price growth.

B.2 Derivation of Equation (B.2)

Equation (B.2) is the result of the following calculations:

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \frac{q_{t+s}}{q_t} = q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\ln q_{t+s} - \ln q_t \right) = q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\sum_{n=1}^{s} \Delta \ln q_{t+n} \right) \\
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\sum_{n=1}^{s} \ln m_{t+n} \right) \cdot \underbrace{\mathbb{E}_t^{\mathcal{P}} \left[\prod_{n=1}^{s} e_{t+n} \right]}_{= \prod_n \mathbb{E}_t^{\mathcal{P}} e_{t+n} = 1} \\
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\sum_{n=1}^{s} \left[\sum_{j=0}^{n-1} \varrho^j \ln v_{t+n-j} + \varrho^n \ln m_t \right] \right) \\
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\ln m_t \cdot \sum_{n=1}^{s} \varrho^n \right) \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\underbrace{\sum_{n=1}^{s} \sum_{j=0}^{n-1} \varrho^j \ln v_{t+n-j}}_{\sim \mathcal{N}} \right) \\
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\varrho \frac{1-\varrho^s}{1-\varrho} \ln m_t \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \\
\iff \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + \frac{1}{2} \sigma^2 \left(\varrho \tfrac{1-\varrho^s}{1-\varrho} \right)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2
\end{aligned}
$$

B.3 Aggregation and market clearing

To achieve goods market clearing, each goods market for a variety j must clear. For notational convenience, we define $\mathrm{y}_{H,t}(j) := c_{H,t}(j) + x_{H,t}(j) + \Psi_{t,H}(j)$, as the total demand for good (H, j) coming from one typical H-consumer. $\Psi_t := (1+\pi_t)^{-1} \frac{\psi}{2} (b_t)^2$ is the real cost of intermediating the position of an H-citizen in the union-wide bond. This cost, just like consumption and housing investment, gets passed along down to the varieties: $\Psi_{t,H} := (\frac{P_{H,t}}{P_t})^{-\frac{1}{\varsigma}} \lambda \Psi_t$. Goods market clearing across all goods markets requires:

$$
y_t := \int y_{H,t}(j)dj = \gamma \int \mathrm{y}_{H,t}(j)dj + (1-\gamma) \int \mathrm{y}_{H,t}^{*}(j)dj + \int \Phi_t(j)dj
$$

where $\Phi_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \Phi_t$ and $\Phi_t = \frac{\kappa}{2} \left(\Pi_{H,t} - 1\right)^2 y_{H,t}$ account for the price adjustment costs from the firm side. Since housing investment requires domestically produced goods we have that $x_t = x_{H,t}$ and $x_t^* = x_{F,t}^*$. And therefore $x_{H,t}^* = 0$ and $x_{F,t} = 0$. Aggregation and successive substitution eventually yields the domestic and foreign aggregate good market clearing conditions:

$$\left(1-\frac{\kappa}{2}(\Pi_{H,t}-1)^2\right)y_t\gamma = \gamma \mathrm{y}_{H,t} + (1-\gamma)\mathrm{y}^*_{H,t}$$
$$\left(1-\frac{\kappa}{2}(\Pi_{F,t}-1)^2\right)y^*_t(1-\gamma) = \gamma \mathrm{y}_{F,t} + (1-\gamma)\mathrm{y}^*_{F,t}$$

Further, the bond market clearing condition is given by:

$$\gamma P_t b_{t+1} + (1-\gamma)P^*_t b^*_{t+1} = 0.$$

Market clearing in the housing sectors is given by:

$$H(x_{t-\tau}, \xi_t) = (h_t - (1-\delta)h_{t-1}),$$
$$H(x^*_{t-\tau^*}, \xi^*_{x,t}) = (h^*_t - (1-\delta)h^*_{t-1}).$$

Finally, the balance-of-payments Equation ensures that the household budget constraints hold:

$$\gamma \mathrm{y}_{F,t}P_{F,t} - P_{H,t}(1-\gamma)\mathrm{y}^*_{H,t} + \gamma\left(P_t b_{t+1} - (1+i_{t-1})P_{t-1}b_t - \mathfrak{b}_t\right) = 0.$$

B.4 Walras' law and Balance-of-Payments

B.4.1 *Walras' law*

To make sure the economics of the model with home bias checks out, we prove that Walras' law holds in our model economy.

We start by providing a list of all market clearing conditions, household budget constraints, and relevant variable definitions (assuming $T_t = T^*_t = 0$), with equations involving more than one good being in nominal terms (i.e. units of union-wide currency). In doing so, we make use of the micro-foundation of the debt-elastic interest rate rule that is presented in Appendix A. The set of market clearing-, profit-, and budget-conditions is:

$$
\begin{aligned}
(GMC) &\quad \left(1-\frac{\kappa}{2}\pi_{H,t}^2\right) y_{H,t}\gamma = \gamma \mathrm{y}_{H,t} + (1-\gamma)\mathrm{y}_{H,t}^*,\\
(GMC^*) &\quad \left(1-\frac{\kappa}{2}\pi_{F,t}^2\right) y_{F,t}^*(1-\gamma) = \gamma \mathrm{y}_{F,t} + (1-\gamma)\mathrm{y}_{F,t}^*,\\
(HMC) &\quad H(x_{t-\tau},\xi_{t-\tau}) = h_t - (1-\delta)h_{t-1},\\
(HMC^*) &\quad H(x_{t-\tau}^*,\xi_{t-\tau}) = h_t^* - (1-\delta)h_{t-1}^*,\\
(B) &\quad \gamma P_t b_{t+1} + (1-\gamma)P_t^* b_{t+1} = 0,\\
(BC) &\quad (c_{H,t}+x_{H,t})P_{H,t} + (c_{F,t}+x_{F,t})P_{F,t} + P_t q_t(h_t-(1-\delta)h_{t-1}) + P_t b_{t+1} = W_t n_t\\
&\quad + (1+i_{t-1}-\psi b_t)P_{t-1}b_t + H(x_{t-\tau},\xi_{t-\tau})\cdot P_t q_t + P_t\Sigma_t + (\beta^{-1}-1)\tfrac{\gamma P_{t-1}+(1-\gamma)P_{t-1}^*}{P_{t-1}}P_{t-1}\bar{b},\\
(BC^*) &\quad (c_{H,t}^*+x_{H,t}^*)P_{H,t} + (c_{F,t}^*+x_{F,t}^*)P_{F,t} + P_t^* q_t^*(h_t^*-(1-\delta)h_{t-1}^*) + P_t^* b_{t+1}^* = W_t^* n_t^*\\
&\quad + (1+i_{t-1}-\psi b_t^*)P_{t-1}^* b_t^* + H(x_{t-\tau}^*,\xi_{t-\tau})\cdot P_t^* q_t^* + P_t^*\Sigma_t^* + (\beta^{-1}-1)\tfrac{\gamma P_{t-1}+(1-\gamma)P_{t-1}^*}{P_{t-1}^*}P_{t-1}^*\bar{b}^*,\\
(\Sigma) &\quad P_t\Sigma_t = P_{H,t}\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t - W_t n_t + P_{t-1}\frac{\psi}{2}[b_t]^2,\\
(\Sigma^*) &\quad P_t^*\Sigma_t^* = P_{F,t}\left(1-\frac{\kappa}{2}\pi_{F,t}^2\right)y_t^* - W_t^* n_t^* + P_{t-1}^*\frac{\psi}{2}[b_t^*]^2.
\end{aligned}
\tag{B.3}
$$

These are 9 conditions – Walras' law now asserts that any one of these nine conditions should be obtainable through the summation of the remaining eight conditions. We show that the collective of all equations, except for (B), implies equation (B). First, plug (HMC) and (Σ) into (BC) to get

$$
\begin{aligned}
{\scriptstyle (HMC\&\Sigma\&BC)} &\quad (c_{H,t}+x_{H,t})P_{H,t} + (c_{F,t}+x_{F,t})P_{F,t} + P_t b_{t+1} = P_{H,t}\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t\\
&\quad + (1+i_{t-1}-\psi b_t)P_{t-1}b_t + P_{t-1}\frac{\psi}{2}[b_t]^2 + (\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P_{t-1}^*)\bar{b}\\
&= P_{H,t}\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t + (1+i_{t-1})P_{t-1}b_t - P_{t-1}\frac{\psi}{2}[b_t]^2\\
&+ (\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P_{t-1}^*)\bar{b}
\end{aligned}
$$

$$\overset{(HMC\&\Sigma\&BC)}{\iff} \quad (c_{H,t}+x_{H,t})P_{H,t}+(c_{F,t}+x_{F,t})P_{F,t}+P_t\underbrace{(1+\pi_t)^{-1}\frac{\psi}{2}[b_t]^2}_{=\Psi_t}+P_tb_{t+1}=$$

$$(c_{H,t}+x_{H,t}+\Psi_{H,t})P_{H,t}+(c_{F,t}+x_{F,t}+\Psi_{F,t})P_{F,t}+P_tb_{t+1}=$$

$$P_{H,t}\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t+(1+i_{t-1})P_{t-1}b_t+(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b}$$

$$\overset{(HMC\&\Sigma\&BC)}{\iff} \quad \gamma(\mathrm{y}_{H,t}P_{H,t}+\mathrm{y}_{F,t}P_{F,t}+P_tb_{t+1})=P_{H,t}\gamma\left(1-\frac{\kappa}{2}\pi_{H,t}^2\right)y_t$$

$$+(1+i_{t-1})P_{t-1}\gamma b_t+(\beta^{-1}-1)\gamma(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b}$$

$$\overset{(HMC\&\Sigma\&BC\&GMC)}{\Longrightarrow} \quad \gamma(\mathrm{y}_{H,t}P_{H,t}+\mathrm{y}_{F,t}P_{F,t}+P_tb_{t+1})=P_{H,t}\left(\gamma\mathrm{y}_{H,t}+(1-\gamma)\mathrm{y}^*_{H,t}\right)$$

$$+(1+i_{t-1})P_{t-1}\gamma b_t+(\beta^{-1}-1)\gamma(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b},$$

and symmetric derivations deliver

$$(HMC^*\&\Sigma^*\&BC^*\&GMC^*) \quad (1-\gamma)(\mathrm{y}^*_{H,t}P_{H,t}+\mathrm{y}^*_{F,t}P_{F,t}+P^*_tb^*_{t+1})=P_{F,t}\left(\gamma\mathrm{y}_{F,t}+(1-\gamma)\mathrm{y}^*_{F,t}\right)$$

$$+(1+i_{t-1})P^*_{t-1}(1-\gamma)b^*_t+(\beta^{-1}-1)(1-\gamma)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\bar{b}^*.$$

Now adding the two equations gives:

$$\overset{(HMC\&\Sigma\&BC\&GMC)+(HMC^*\&\Sigma^*\&BC^*\&GMC^*)}{\Longrightarrow}$$

$$[\gamma P_tb_{t+1}+(1-\gamma)P^*_tb^*_{t+1}]-(1+i_{t-1})[\gamma P_{t-1}b_t+(1-\gamma)P^*_{t-1}b^*_t]$$
$$-(\beta^{-1}-1)(\gamma P_{t-1}+(1-\gamma)P^*_{t-1})\underbrace{[\gamma\bar{b}+(1-\gamma)\bar{b}^*]}_{=0\text{ by choice of }\bar{b},\bar{b}^*}=0.$$

Now if the initial bond levels (which are model parameters) are chosen in agreement with the bond market clearing condition, i.e. $\gamma P_{-1}b_0+(1-\gamma)P^*_{-1}b_0=0$, a simple induction argument over t establishes (B), $\forall t$.

B.4.2 *Balance-of-Payments*

In Appendix B.4.1 we establish that

$$\overset{(HMC\&\Sigma\&BC\&GMC)}{\Longrightarrow} \quad \gamma\left(\mathrm{y}_{H,t}P_{H,t}+\mathrm{y}_{F,t}P_{F,t}+P_tb_{t+1}-(1+i_{t-1})P_{t-1}b_t-(\beta^{-1}-1)\tfrac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P_{t-1}}P_{t-1}\bar{b}\right)$$
$$=P_{H,t}\left(\gamma\mathrm{y}_{H,t}+(1-\gamma)\mathrm{y}^*_{H,t}\right)$$

$$\text{(HMC}^*\&\Sigma^*\&BC^*\&GMC^*) \quad (1-\gamma)\left(\mathrm{y}^*_{H,t}P_{H,t} + \mathrm{y}^*_{F,t}P_{F,t} + P^*_t b^*_{t+1} - (1+i_{t-1})P^*_{t-1}b^*_t - (\beta^{-1}-1)\frac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P^*_{t-1}}P^*_{t-1}\bar{b}^*\right)$$
$$= P_{F,t}\left(\gamma \mathrm{y}_{F,t} + (1-\gamma)\mathrm{y}^*_{F,t}\right).$$

This is equivalent to

$$\text{(HMC\&}\Sigma\text{\&BC\&GMC)} \quad \gamma \mathrm{y}_{F,t}P_{F,t} - P_{H,t}(1-\gamma)\mathrm{y}^*_{H,t}$$
$$+\gamma\left(P_t b_{t+1} - (1+i_{t-1})P_{t-1}b_t - (\beta^{-1}-1)\frac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P_{t-1}}P_{t-1}\bar{b}\right) = 0$$
$$\text{(HMC}^*\&\Sigma^*\&BC^*\&GMC^*) \quad (1-\gamma)x^*_{H,t}P_{H,t} - P_{F,t}\gamma \mathrm{y}_{F,t}$$
$$+(1-\gamma)\left(P^*_t b^*_{t+1} - (1+i_{t-1})P^*_{t-1}b^*_t - (\beta^{-1}-1)\frac{\gamma P_{t-1}+(1-\gamma)P^*_{t-1}}{P^*_{t-1}}P^*_{t-1}\bar{b}^*\right) = 0.$$

Now both equations dictate that the value of net imports in the respective country (imports less of exports) be covered by an equal-sized increase in the level of debt. By Walras' law (see Appendix B.4.1), the whole list of market clearing conditions, budget constraints and relevant variable definitions, (B.3), is equivalent to (B.3) without $(BC), (BC^*)$ and augmented with the balance of payments equation

$$(BOP) \quad \gamma(1-\lambda)\left(\frac{P_{F,t}}{P_t}\right)^{1-\frac{1}{\varsigma}}\mathrm{y}_t - (1-\gamma)(1-\lambda^*)\left(\frac{P_{H,t}}{P^*_t}\right)^{1-\frac{1}{\varsigma}}\frac{P^*_t}{P_t}\mathrm{y}^*_t$$
$$+\gamma\left(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1}b_t - (\beta^{-1}-1)\left(\gamma + (1-\gamma)\frac{P^*_{t-1}}{P_{t-1}}\right)(1+\pi_t)^{-1}\bar{b}\right) = 0.$$

where we have used the demand schedules to substitute out the H and F good variables that do not feature in the MSV-representation of the model, and we have divided by P_t to get the representation in units of country H's final consumption basket.

B.5 Nonlinear Equilibrium Conditions

As a starting point to solving the model, we collect all equilibrium conditions in a parsimonious fashion by performing light substitutions.

B.5.1 *Expressing price levels with only inflation rates and terms of trade*

Define the terms of trade as

$$s_t := \frac{P_{H,t}}{P_{F,t}}.$$

This entails

$$s_t = \frac{\Pi_{H,t}}{\Pi_{F,t}}s_{t-1},$$

and allows us to write

$$
\begin{aligned}
\Pi_t &= \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{1-\lambda}{\lambda}(s_t^{1-1/\varsigma})^{-1}\right)^{-1} + \left(\frac{\lambda}{1-\lambda}s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1}\right)^{-1}\right]^{\frac{1}{1-1/\varsigma}}, \\
\Pi_t^* &= \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{\lambda^*}{1-\lambda^*}(s_t^{1-1/\varsigma})^{-1}\right)^{-1} + \left(\frac{1-\lambda^*}{\lambda^*}s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1}\right)^{-1}\right]^{\frac{1}{1-1/\varsigma}},
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{P_{H,t}}{P_t}\right)^{-\frac{1}{\varsigma}} &= \left[\lambda + (1-\lambda)s_t^{\frac{1}{\varsigma}-1}\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: \mathfrak{p}_H(s_t)^{-\frac{1}{\varsigma}}, \quad \mathfrak{p}_H'(s) = \mathfrak{p}_H(s)^{2-1/\varsigma}(1-\lambda)s^{1/\varsigma-1} > 0, \quad \mathfrak{p}_H'(1) = 1-\lambda, \\
\left(\frac{P_{F,t}}{P_t}\right)^{-\frac{1}{\varsigma}} &= \left[\lambda s_t^{1-\frac{1}{\varsigma}} + (1-\lambda)\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: \mathfrak{p}_F(s_t)^{-\frac{1}{\varsigma}}, \quad \mathfrak{p}_F'(s) = -\mathfrak{p}_F(s)^{2-1/\varsigma}\lambda s^{1/\varsigma} < 0, \quad \mathfrak{p}_F'(1) = -\lambda, \\
\left(\frac{P_{H,t}}{P_t^*}\right)^{-\frac{1}{\varsigma}} &= \left[(1-\lambda^*) + \lambda^* s_t^{\frac{1}{\varsigma}-1}\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: \mathfrak{p}_H^*(s_t)^{-\frac{1}{\varsigma}}, \quad \mathfrak{p}_H^{*\prime}(s) = (\mathfrak{p}_H(s)^*)^{2-1/\varsigma}\lambda^* s^{1/\varsigma-1} > 0, \quad \mathfrak{p}_H^{*\prime}(1) = \lambda^*, \\
\left(\frac{P_{F,t}}{P_t^*}\right)^{-\frac{1}{\varsigma}} &= \left[(1-\lambda^*)s_t^{1-\frac{1}{\varsigma}} + \lambda^*\right]^{\frac{1}{1/\varsigma-1}\cdot\left(-\frac{1}{\varsigma}\right)} =: \mathfrak{p}_F^*(s_t)^{-\frac{1}{\varsigma}}, \quad \mathfrak{p}_F^{*\prime}(s) = -(\mathfrak{p}_F^*(s))^{2-1/\varsigma}(1-\lambda^*)s^{1/\varsigma} > 0, \quad \mathfrak{p}_F^{*\prime}(1) = -(1-\lambda^*), \\
\frac{P_t}{P_t^*} &= \left[\frac{\lambda s_t^{1-1/\varsigma} + (1-\lambda)}{(1-\lambda^*)s_t^{1-1/\varsigma} + \lambda^*}\right]^{\frac{1}{1-\frac{1}{\varsigma}}} =: \mathfrak{p}(s_t), \quad \mathfrak{p}'(s) = \mathfrak{p}(s)^{1/\varsigma}\frac{s^{-1/\varsigma}}{((1-\lambda^*)s^{1-1/\varsigma}+\lambda^*)^2}\cdot[\lambda\lambda^* - (1-\lambda)(1-\lambda^*)] > 0, \\
&\quad \mathfrak{p}'(1) = \lambda\lambda^* - (1-\lambda)(1-\lambda^*).
\end{aligned}
$$

We have characterized every expression that involves any consumption price level in terms of the inflation rates and the terms of trade. This means that we need not track the price levels which are determined in equilibrium only up to a translation.

B.5.2 *Condensing the set of market clearing conditions*

Using the expressions above, and the demand schedules for varieties, we can rewrite the goods market clearing condition into

$$
\begin{aligned}
\left(1 - \frac{\kappa}{2}(\Pi_{H,t}-1)^2\right)\xi_{a,t}n_t\gamma &= \gamma\lambda\mathfrak{p}_H(s_t)^{-\frac{1}{\varsigma}}\mathrm{y}_t + (1-\gamma)(1-\lambda^*)\mathfrak{p}_H^*(s_t)^{-\frac{1}{\varsigma}}\mathrm{y}_t^*, \\
\left(1 - \frac{\kappa}{2}(\Pi_{F,t}-1)^2\right)\xi_{a,t}n_t^*(1-\gamma) &= \gamma(1-\lambda)\mathfrak{p}_F(s_t)^{-\frac{1}{\varsigma}}x_t + (1-\gamma)\lambda^*\mathfrak{p}_F^*(s_t)^{-\frac{1}{\varsigma}}x_t^*.
\end{aligned}
$$

We can use the expressions above also in the BOP equation (derived in B.4.2), to receive

$$
\begin{aligned}
(BOP) \quad & \gamma(1-\lambda)\mathfrak{p}_F(s_t)^{1-\frac{1}{\varsigma}}\mathrm{y}_t - (1-\gamma)(1-\lambda^*)\mathfrak{p}_H^*(s_t)^{1-\frac{1}{\varsigma}}\mathfrak{p}(s_t)^{-1}\mathrm{y}_t^* \\
& + \gamma\left(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1}b_t - (\beta^{-1}-1)\big(\gamma + (1-\gamma)\mathfrak{p}(s_{t-1})^{-1}\big)(1+\pi_t)^{-1}\bar{b}\right) = 0.
\end{aligned}
$$

We are now ready to state the set of nonlinear equilibrium conditions.

B.5.3 *Nonlinear equilibrium conditions*

Using the short-hand notation $\mathrm{y}_t := c_t + x_t + (1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ (analogously for *):

Household

$$
\begin{aligned}
&(h) \quad q_t = \frac{\xi_{h,t} h_t^{-\nu}}{\xi_{c,t} c_t^{-\sigma}} + \beta(1-\delta)\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} q_{t+1}\right\},\\
&(n) \quad \frac{\chi n_t^{\varphi}}{\xi_{c,t} c_t^{-\sigma}} = w_t,\\
&(b) \quad 1 = \beta\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}(1+r_{t+1})\right\},\\
&(x) \quad 1 = q_t \cdot \xi_{x,t} x_t^{\eta-1},
\end{aligned}
\tag{B.4}
$$

Household*

$$
\begin{aligned}
&(h^*) \quad q_t^* = \xi_{h,ss}^* \tfrac{\xi_{h,t}}{\xi_{h,ss}} \frac{(h_t^*)^{-\nu}}{\xi_{c,t}(c_t^*)^{-\sigma}} + \beta(1-\delta)\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma} q_{t+1}^*\right\},\\
&(n^*) \quad \frac{\chi (n_t^*)^{\varphi}}{\xi_{c,t} (c_t^*)^{-\sigma}} = w_t^*,\\
&(b^*) \quad 1 = \beta\mathbb{E}_t^{\mathcal{P}}\left\{\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma}(1+r_{t+1}^*)\right\},\\
&(x^*) \quad 1 = q_t^* \cdot \xi_{x,ss}^* \tfrac{\xi_{x,t}}{\xi_{x,ss}} (x_t^*)^{\eta-1},
\end{aligned}
$$

Interest rates

$$
\begin{aligned}
&(r) \quad 1 + r_t = \frac{1 + i_{t-1} - \psi b_t}{1+\pi_t},\\
&(r^*) \quad 1 + r_t^* = \frac{1 + i_{t-1} - \psi b_t^*}{1+\pi_t^*},
\end{aligned}
$$

Firm

$$
\begin{aligned}
(PC) \quad & \kappa(\Pi_{H,t}-1)\Pi_{H,t} - (1-\epsilon) - \epsilon(1-\tau^{\ell})\frac{w_t}{\xi_{a,t}}\mathfrak{p}_H(s_t)^{-1} = \\
& \mathbb{E}_t^{\mathcal{P}}\left\{\beta\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{\xi_{a,t+1} n_{t+1}}{\xi_{a,t} n_t \Pi_{t+1}}\kappa(\Pi_{H,t+1}-1)\Pi_{H,t+1}^2\right\},
\end{aligned}
$$

Firm*

$$
\begin{aligned}
(PC^*) \quad & \kappa(\Pi_{F,t}-1)\Pi_{F,t} - (1-\epsilon) - \epsilon(1-\tau^{\ell})\frac{w_t^*}{\xi_{a,t}}\mathfrak{p}_F^*(s_t)^{-1} = \\
& \mathbb{E}_t^{\mathcal{P}}\left\{\beta\frac{\xi_{c,t+1}}{\xi_{c,t}}\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma}\frac{\xi_{a,t+1} n_{t+1}^*}{\xi_{a,t} n_t^* \Pi_{t+1}^*}\kappa(\Pi_{F,t+1}-1)\Pi_{F,t+1}^2\right\},
\end{aligned}
$$

Bond market clearing

$(B)\quad \gamma b_{t+1} + (1-\gamma)\mathfrak{p}(s_t) b^*_{t+1} = 0,$

Goods market clearing

$$(GMC)\quad \left(1 - \frac{\kappa}{2}(\Pi_{H,t}-1)^2\right)\xi_{a,t} n_t \gamma = \gamma\lambda \mathfrak{p}_H(s_t)^{-\frac{1}{\varsigma}} \mathrm{y}_t + (1-\gamma)(1-\lambda^*)\mathfrak{p}^*_H(s_t)^{-\frac{1}{\varsigma}} \mathrm{y}^*_t,$$

$$(GMC^*)\quad \left(1 - \frac{\kappa}{2}(\Pi_{F,t}-1)^2\right)\xi_{a,t} n^*_t (1-\gamma) = \gamma(1-\lambda) \mathfrak{p}_F(s_t)^{-\frac{1}{\varsigma}} \mathrm{y}_t + (1-\gamma)\lambda^*\mathfrak{p}^*_F(s_t)^{-\frac{1}{\varsigma}} \mathrm{y}^*_t,$$

$$(BOP)\quad \gamma(1-\lambda)\mathfrak{p}_F(s_t)^{1-\frac{1}{\varsigma}} \mathrm{y}_t - (1-\gamma)(1-\lambda^*)\mathfrak{p}^*_H(s_t)^{1-\frac{1}{\varsigma}} \mathfrak{p}(s_t)^{-1} \mathrm{y}^*_t$$
$$+ \gamma\left(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1} b_t - (\beta^{-1}-1)\big(\gamma + (1-\gamma)\mathfrak{p}(s_{t-1})^{-1}\big)(1+\pi_t)^{-1}\bar{b}\right) = 0,$$

Housing market clearing

$(HMC)\quad \eta^{-1}\xi_{x,t} x_t^{\eta} = h_t - (1-\delta)h_{t-1},$

$(HMC^*)\quad (\eta^*)^{-1}\xi^*_{x,ss}\frac{\xi_{x,t}}{\xi_{x,ss}}(x^*_t)^{\eta^*} = h^*_t - (1-\delta)h^*_{t-1}, \quad \eta^* > \eta$

Price indices

$$(s)\quad s_t = \frac{\Pi_{H,t}}{\Pi_{F,t}} s_{t-1},$$

$$(\Pi)\quad \Pi_t = \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{1-\lambda}{\lambda}(s_t^{1-1/\varsigma})^{-1}\right)^{-1} + \left(\frac{\lambda}{1-\lambda}s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1}\right)^{-1}\right]^{\frac{1}{1-1/\varsigma}},$$

$$(\Pi^*)\quad \Pi^*_t = \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{\lambda^*}{1-\lambda^*}(s_t^{1-1/\varsigma})^{-1}\right)^{-1} + \left(\frac{1-\lambda^*}{\lambda^*}s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1}\right)^{-1}\right]^{\frac{1}{1-1/\varsigma}}.$$

with the shocks $(\xi_t)_{t\geq 0}$, the allocation variables $(c_t, c^*_t, b_t, b^*_t, h_t, h^*_t, x_t, x^*_t, n_t, n^*_t)_{t\geq 0}$ and the price variables $(w_t, w^*_t, q_t, q^*_t, i_t, \Pi_t, \Pi^*_t, \Pi_{H,t}, \Pi_{F,t}, s_t)_{t\geq 0}$.

B.6 Proof of Lemma 2

In this section, we prove that there exists a unique non-stochastic steady-state with zero net inflation and parity in the terms of trade $P_{H,t}/P_{F,t}$ (which implies parity in the real exchange rate P_t/P^*_t, see the definition of $\mathfrak{p}$). We prove this first for arbitrary parameters $\eta, \eta^*, \xi_{h,ss}, \xi_{x,ss}, \xi^*_{h,ss}, \xi^*_{x,ss}$, and then show that it is possible to select parameters such that the steady-state allocation is symmetric. The non-stochastic steady-state with zero net inflation and real exchange rate parity ("SS" for short) obtains by setting $\mathrm{Var}[\|\xi_t\|] = 0$, where the shock vector contains both actual shocks and shocks that are only perceived (and never observed) by the household within her perceived house price model: $\xi_t = (\xi_{a,t}, \xi_{c,t}, \xi_{h,t}, \xi_{x,t}, \xi_{i,t}, e_t, v_t)^\top$. Thus, the non-stochastic steady state represents the time-invariant equilibrium that obtains if agents do not expect any shock to ever materialize and indeed no shock ever does materialize, and we have $\xi_t = (1, 1, \xi_{h,ss}, \xi_{x,ss}, 1, 1, 1)^\top$ almost surely where $\xi_{h,ss}, \xi_{x,ss}$ are model parameters. We are interested in, and prove existence and uniqueness of, a non-stochastic steady-state in which the net rates of inflation are zero and in which the terms of trade are at parity. The latter assumption implies $\mathfrak{p}_H(1) = \mathfrak{p}_H^*(1) = \mathfrak{p}_F(1) = \mathfrak{p}_F^*(1) = \mathfrak{p}(1) = 1$.

Beliefs are irrelevant in the non-stochastic steady state. The first relevant insight is that in the non-stochastic steady state, defined as above, the presence and precise parameterization of subjective beliefs over house prices is irrelevant. To see this, recall that the subjective house price expectation dynamics are fully characterized by

$$\begin{aligned}\forall s > 0, \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2\left(\varrho \tfrac{1-\varrho^s}{1-\varrho}\right)^2\right) \cdot \exp(V), \quad V \propto \sigma_v^2 \\ \ln \overline{m}_t &= (1-g)\left(\ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2}\right) + g\left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2}\right), \\ &+ \text{equations determining equilibrium-evolution of price level, } q_t.\end{aligned} \tag{B.5}$$

Now recall that the non-stochastic steady-state represents the unique equilibrium of the economy when the variance of actual and perceived external shocks tends to zero and the initial conditions are selected such that they give rise to a constant path of equilibrium variables. Formally, the SS arises by replacing $\mathrm{Var}[\|\xi_t\|]$ with $\wp^2 \cdot \mathrm{Var}[\|\xi_t\|]$ in the model, taking the limit $\wp \to 0$, and solving for the fixed point of the equilibrium equations. Applying this logic, $\sigma_e, \sigma_v, \sigma \to 0$, to the equations (B.5) delivers

$$\begin{aligned} q_{ss} &= q_{ss} \cdot \exp\left(\ln \overline{m}_{ss} \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + 0\right) \cdot \exp(0) \\ \ln \overline{m}_{ss} &= (1-g)\ln \overline{m}_{ss} + g \cdot 0 \end{aligned}$$

from which we see that there exists a unique fixed point which is $\overline{m}_{ss} = 1$, regardless of the level of q_{ss}. This means that the presence of subjective beliefs has no consequences for the non-stochastic steady-state: provided that agents have the prior that house prices do not change, $\overline{m}_{ss} = 1$, the absence of perceived and actual shocks means that house prices indeed never change which, in turn, means that the prior of constant house prices is never changed.

In summary, iff a non-stochastic steady state with zero net inflation and terms of trade parity exists and is unique, then it solves the following system of equations.

$$
\begin{aligned}
&\textbf{Household}\\
&(h)\quad q_{ss} = \xi_{h,ss} h_{ss}^{-\nu} c_{ss}^{\sigma} + \beta(1-\delta) q_{ss},\\
&(n)\quad \chi n_{ss}^{\varphi} c_{ss}^{\sigma} = w_{ss},\\
&(b)\quad 1 = \beta(1 + r_{ss}),\\
&(x)\quad 1 = q_{ss} \cdot \xi_{x,ss} x_{ss}^{\eta-1},\\
&\textbf{Household*}\\
&(h^*)\quad q_{ss} = \xi^*_{h,ss} (h^*_{ss})^{-\nu} (c^*_{ss})^{\sigma} + \beta(1-\delta) q^*_{ss},\\
&(n^*)\quad \chi (n^*_{ss})^{\varphi} (c^*_{ss})^{\sigma} = w^*_{ss},\\
&(b^*)\quad 1 = \beta(1 + r^*_{ss}),\\
&(x^*)\quad 1 = q^*_{ss} \cdot \xi^*_{x,ss} (x^*_{ss})^{\eta-1},\\
&\textbf{Interest rates}\\
&(r)\quad 1 + r_{ss} = 1 + i_{ss} - \psi b_{ss},\\
&(r^*)\quad 1 + r^*_{ss} = 1 + i_{ss} - \psi b^*_{ss},\\
&\textbf{Firm}\\
&(PC)\quad w_{ss} = 1 \quad \text{using that } \tau^{\ell} = 1/\epsilon,\\
&\textbf{Firm*}\\
&(PC^*)\quad w^*_{ss} = 1 \quad \text{using that } \tau^{\ell} = 1/\epsilon,\\
&\textbf{Bond market clearing}\\
&(B)\quad \gamma b_{ss} + (1-\gamma) b^*_{ss} = 0,\\
&\textbf{Goods market clearing}\\
&(GMC)\quad n_{ss}\gamma = \gamma\lambda \mathrm{y}_{ss} + (1-\gamma)(1-\lambda^*) \mathrm{y}^*_{ss},\\
&(GMC^*)\quad n^*_{ss}(1-\gamma) = \gamma(1-\lambda) \mathrm{y}_{ss} + (1-\gamma)\lambda^* \mathrm{y}^*_{ss},\\
&(BOP)\quad \gamma(1-\lambda)\mathrm{y}_{ss} - (1-\gamma)(1-\lambda^*)\mathrm{y}^*_{ss} - \gamma(i_{ss} b_{ss} + (\beta^{-1} - 1)\bar{b}) = 0,\\
&\textbf{Housing market clearing}\\
&(HMC)\quad \eta^{-1} \xi_{x,ss} (x_{ss})^{\eta} = \delta h_{ss},\\
&(HMC^*)\quad (\eta^*)^{-1} \xi^*_{x,ss} (x^*_{ss})^{\eta^*} = \delta h^*_{ss}.
\end{aligned}
\tag{B.6}
$$

where $\bar{b}, \bar{b}^*$ are model parameters (see Appendix A for an interpretation) chosen so as to (i) ensure equilibrium existence, and (ii) ensure that (§1) $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$. We solve for the SS in 4 steps.

1. First, we solve a number of equations explicitly, thus substituting out a number of variables:

(a) (b) and (b^*) together with (r) and (r^*) and (B) imply $b_{ss} = b^*_{ss} = 0$ and $i_{ss} = \beta^{-1} - 1$ whence it follows $\mathrm{y}_{ss} = c_{ss} + x_{ss}$ and analogously for *. (§1) then implies $\bar{b} = -\bar{b}^* \cdot (1-\gamma)/\gamma$ where $\bar{b}^*$ is not pinned down yet. We will solve for it in the very last step.

(b) (PC) and (PC^*) imply $w_{ss} = w^*_{ss} = 1$;

(c) together with (n) and (n^*) this implies $n_{ss} = (\chi^{-1})^{1/\varphi} \cdot c_{ss}^{-\sigma/\varphi} =: \phi(c_{ss})$ with $\phi' < 0$ and analogously for n^*_{ss} with the *same* function ϕ;

(d) (HMC) and (HMC^*) imply $h_{ss} = (\delta)^{-1}\xi_{x,ss}\eta^{-1}x^{\eta}_{ss}$ and analogously for h^*_{ss};

(e) (h) and (h^*) imply (with $\bar{\beta} := \beta(1-\delta)$)

$$q_{ss} = \left(1-\bar{\beta}\right)^{-1} \xi_{h,ss} \left[\delta^{-1}\xi_{x,ss}\eta^{-1}x^{\eta}_{ss}\right]^{-\nu} c^{\sigma}_{ss}$$
$$q^*_{ss} = \left(1-\bar{\beta}\right)^{-1} \xi^*_{h,ss} \left[\delta^{-1}\xi^*_{x,ss}(\eta^*)^{-1}\left(x^*_{ss}\right)^{\eta^*}\right]^{-\nu} (c^*_{ss})^{\sigma}$$

(f) (BOP) now reads $\gamma(1-\lambda)\mathrm{y}_{ss} = (1-\gamma)(1-\lambda^*)\mathrm{y}^*_{ss} - (1-\gamma)(\beta^{-1}-1)\bar{b}^*$, and the symmetric (BOP^*) which is redundant by Walras' law reads $(1-\gamma)(1-\lambda^*)x^*_{ss} = \gamma(1-\lambda)x_{ss} + (1-\gamma)(\beta^{-1}-1)\bar{b}^*$; using this in (GMC), (GMC^*) produces

$$(GMC) \quad \phi(c_{ss}) = \mathrm{y}_{ss} + (1-\gamma)/\gamma \cdot (\beta^{-1}-1)\bar{b}^*,$$
$$(GMC^*) \quad \phi(c^*_{ss}) = \mathrm{y}^*_{ss} - (\beta^{-1}-1)\bar{b}^*,$$

2. The remaining equations are (x), (x^*) and (GMC), (GMC^*), (BOP) with unknowns $x_{ss}, x^*_{ss}, c_{ss}, c^*_{ss}, \bar{b}^*$. In this step, we show there are strictly increasing functions that yield x_{ss}, x^*_{ss} given c_{ss}, c^*_{ss} respectively. Start by plugging q_{ss} into (x):

$$\left(1-\bar{\beta}\right)^{-1} \xi_{h,ss} \left[\delta^{-1}\xi_{x,ss}\eta^{-1}x^{\eta}_{ss}\right]^{-\nu} c^{\sigma}_{ss}\xi_{x,ss}x^{\eta-1}_{ss} = 1$$

(the equation for * is symmetric.) Now since $\eta \in (0,1)$ and $\nu > 0$, the expression on the left-hand-side is a strictly decreasing function of x_{ss} for any c_{ss}. Moreover, for $x_{ss} \to 0$, the $LHS \to +\infty$ and for $x_{ss} \to \infty$, the $LHS \to 0$, whence Bolzano's intermediate value theorem (and continuity) ensures that for each c_{ss} there exists a unique x_{ss}. Call this implicitly defined mapping $x_{ss} = \psi(c_{ss})$. As the implicit function theorem shows, $\eta, \nu, \sigma > 0$ imply $\psi' > 0$. Analogous arguments hold for *.

3. We now insert our previous findings into the only remaining equations:

$$(GMC) \quad \phi(c_{ss}) - c_{ss} - \psi(c_{ss}) - (1-\gamma)/\gamma \cdot (\beta^{-1}-1)\bar{b}^* =: \zeta(c_{ss}, \bar{b}^*) = 0,$$

$$(GMC^*)\quad \phi(c^*_{ss}) - c^*_{ss} - \psi^*(c^*_{ss}) + (\beta^{-1}-1)\bar{b}^* =: \zeta^*(c^*_{ss}, \bar{b}^*) = 0,$$

Observe now $c_{ss} \mapsto \zeta$ is continuous and strictly decreasing with $\lim_{c\to 0}\zeta = +\infty$ (by $\lim_{c\to 0}\phi = +\infty$ and $\lim_{c\to 0}\psi < +\infty$) and $\lim_{c\to+\infty}\zeta = -\infty$ (by $\lim_{c\to+\infty} c, \psi = +\infty$ and $\lim_{c\to\infty}\phi = 0$). Therefore, Bolzano's intermediate value theorem ensures there exists a unique c_{ss} for each $\bar{b}^*$. The exactly analogous argument ensures existence and uniqueness of c^*_{ss}. Call these mappings $\varpi : \bar{b}^* \mapsto c_{ss}, \varpi^* : \bar{b}^* \mapsto c^*_{ss}$. The implicit function theorem now yields:

$$\partial\varpi/\partial\bar{b}^* < 0 \text{ and } \partial\varpi^*/\partial\bar{b}^* > 0.$$

4. Finally, only one equation remains, (BOP), with only one variable, $\bar{b}^*$:

$$\begin{aligned}\gamma(1-\lambda)[\varpi(\bar{b}^*) + \psi(\varpi(\bar{b}^*))] - (1-\gamma)(1-\lambda^*)[\varpi^*(\bar{b}^*) + \psi^*(\varpi^*(\bar{b}^*))] \\ + (1-\gamma)(\beta^{-1}-1)\bar{b}^* =: \mathcal{H}(\bar{b}^*) = 0\end{aligned}$$

with $\bar{b}^* \mapsto \mathcal{H}$ continuous. It also holds that $\lim_{\bar{b}^*\to-\infty}\mathcal{H}(\bar{b}^*) = -\infty$.[39] On the other hand, as $\bar{b}^* \to +\infty$, $\mathcal{H} \to +\infty$.[40] Thus, Bolzano's intermediate value theorem ensures **existence** of a $\bar{b}^* \in \mathbb{R}$ that satisfies the BOP-equation and thus existence of a non-stochastic steady state.

Uniqueness of the steady-state can be shown by establishing strict positive monotonicity of $\mathcal{H}$: (we suppress arguments for brevity)

$$\begin{aligned} \frac{\partial\mathcal{H}}{\partial\bar{b}^*} &= \gamma(1-\lambda)(1+\psi')\varpi' - (1-\gamma)(1-\lambda^*)(1+\psi^{*\prime})\varpi^{*\prime} \\ &\quad + (1-\gamma)(\beta^{-1}-1) \\ \text{step 2, cf. notes below:}\quad &> -(1-\gamma)(1-\lambda)(\beta^{-1}-1) - (1-\gamma)(1-\lambda^*)(\beta^{-1}-1) \\ &\quad + (1-\gamma)(\beta^{-1}-1) \\ &= (1-\gamma)(\beta^{-1}-1)[1-1+\lambda-1+\lambda^*] \\ &\overset{\text{sign}}{=} \lambda+\lambda^*-1 \end{aligned}$$

[39] Proof: (1) $\gamma(1-\lambda)[\varpi^*(\bar{b}^*)+\psi^*(\varpi^*(\bar{b}^*))] \geq 0$ by non-negativity of consumption & housing investment; (2) $\gamma(1-\lambda)[\varpi+\psi\circ\varpi]+(1-\gamma)(\beta^{-1}-1)\bar{b}^* = \gamma(1-\lambda)[\phi-\frac{1-\gamma}{\gamma}(\beta^{-1}-1)\bar{b}^*]+(1-\gamma)(\beta^{-1}-1)\bar{b}^* = \gamma(1-\lambda)\phi+\lambda(1-\gamma)(\beta^{-1}-1)\bar{b}^*$, where the second equality is a consequence of (GMC); (3) $\lim_{\bar{b}^*\to-\infty}\varpi = +\infty$ (assuming the contrary will produce a contradiction with (GMC)); (4) $\lim_{c\to 0}\phi = 0$; (5) steps 1–4 now imply $\mathcal{H}(\bar{b}^*) \leq \gamma(1-\lambda)\phi+\lambda(1-\gamma)(\beta^{-1}-1)\bar{b}^* \to -\infty$ as $\bar{b}^* \to -\infty$. □

[40] Proof: (1) $\gamma(1-\lambda)[\varpi(\bar{b}^*) + \psi(\varpi(\bar{b}^*))] \geq 0$ by non-negativity of consumption & housing investment; (2) $-(1-\lambda^*)[\varpi^*(\bar{b}^*)+\psi^*(\varpi^*(\bar{b}^*))]+(\beta^{-1}-1)\bar{b}^* = -(1-\lambda^*)[\phi(c^*_{ss})+(\beta^{-1}-1)\bar{b}^*]+(\beta^{-1}-1)\bar{b}^* = -(1-\lambda^*)\phi(c^*_{ss})+\lambda^*(\beta^{-1}-1)\bar{b}^*$ where the substitution is made using the definition of ϖ^*; (3) $\lim_{\bar{b}^*\to+\infty}\varpi^* = +\infty$ (assuming the contrary will produce a contradiction with (GMC^*)); (4) Fact 3 and $\lim_{c\to+\infty}\phi(c) = 0$ implies $\mathcal{H}(\bar{b}^*) \geq -(1-\lambda^*)\phi(c^*_{ss}) + \lambda^*(\beta^{-1}-1)\bar{b}^* \to +\infty$ as $\bar{b}^* \to +\infty$. □

$$
\begin{aligned}
\text{step 3, cf. notes below:} \quad &= \lambda + 1 - \frac{\gamma}{1-\gamma}(1-\lambda) - 1 \\
&\overset{\text{sign}}{=} (1-\gamma)\lambda - \gamma(1-\lambda) \\
&= \lambda - \gamma \\
&\geq 0 \text{ by assumption.}
\end{aligned}
$$

Step 3 follows by symmetric home bias, i.e. $\gamma(1-\lambda) = (1-\gamma)(1-\lambda^*) \iff \lambda^* = 1 - \frac{\gamma}{1-\gamma}(1-\lambda)$; Step 2 requires slightly more work: First, use the implicit function theorem on $(GMC)\&(GMC^*)$, respectively, to obtain

$$
\varpi' = -\frac{\partial\zeta/\partial\bar{b}^*}{\partial\zeta/\partial c_{ss}} = \frac{\frac{1-\gamma}{\gamma}(\beta^{-1}-1)}{\phi' - (1+\psi')} < 0,
$$
$$
\varpi^{*\prime} = -\frac{\partial\zeta^*/\partial\bar{b}^*}{\partial\zeta^*/\partial c^*_{ss}} = \frac{-(\beta^{-1}-1)}{\phi' - (1+\psi^{*\prime})} > 0;
$$

Second, recognize that since $\phi' < 0$ it is

$$
\frac{1+\psi'}{1+\psi'-\phi'} < 1 \iff \frac{1+\psi'}{-(1+\psi')+\phi'} > -1
$$

and symmetrically for *. This shows that $\frac{\partial\mathcal{H}}{\partial\bar{b}^*} > 0$, and the SS is unique.

Finally, notice that since existence and uniqueness follow for arbitrary parameters $\eta \in (0,1), \eta^* \in (0,1), \xi_{h,ss}, \xi_{x,ss}, \xi^*_{h,ss}, \xi^*_{x,ss} > 0$, it is possible to set $\eta^* > \eta$ and then choose $\xi_{x,ss}, \xi^*_{x,ss}$ so as to ensure that both (HMC) and $(HMC)^*$ hold if $h_{ss} = h^*_{ss}$ and $x_{ss} = x^*_{ss}$. Given this choice of $\xi_{x,ss}, \xi^*_{x,ss}$, and the symmetry in housing stock and housing investment, it is then possible to select $\xi_{h,ss}, \xi^*_{h,ss}$ such that the equations $(x), (x^*)$ hold. It then follows that $c_{ss} = c^*_{ss}, n_{ss} = n^*_{ss}, \bar{b} = \bar{b}^* = 0$, and the allocation is symmetric. This completes the proof.

B.7 Derivation of the household's subjectively optimal plans

In this Appendix, we provide a formal derivation of the linearized subjectively optimal household decision rules presented in equations (9) and (10). Throughout the derivation, we concentrate on the representative household in H with the understanding that the situation in F is symmetric. For simplicity, we omit domestically produced housing investment goods. This does not affect the layout of the proof below.

Consider the household program presented in Section III.A, which is restated here for convenience.[41] We first clarify the shape of the underlying probability space to set the appropriate frame for the following derivations: denote $\Omega \ni \omega_t := (\xi_t, r_t, w_t, \Sigma_t, \pi_t, (P_{H,t}/P_{F,t}), q_t)^\top$ the vector of external decision-relevant variables that the household takes as given, denote $\Omega^t \ni \omega^t := (\omega_{t-s})_{s\geq 0}$ the one-sided infinite history of past external variables and denote $\Omega^\infty \ni \omega := (\omega_t)_{t\in\mathbb{Z}}$ the typical element from the set of possible realizations of full sequences of external variables;[42] denote $\mathcal{B}^\infty$ the Borel-sigma-algebra over Ω^∞. Each household is now endowed with a probability measure $\mathcal{P}$ over $(\Omega^\infty, \mathcal{B}^\infty)$ which encodes her subjective beliefs over the realizations of external variables ω. Rational Expectations, denoted $\mathcal{P} = \mathbb{P}$, are a special case of this setup where $\mathbb{P}$ is the (unique) measure generated by the distribution of $\xi := (\xi_t)_{t\in\mathbb{Z}}$ and the equilibrium conditions that allow to compute $\omega\backslash\xi$ as a deterministic function of ξ.[43] As explained in Section III.A, we assume $\mathcal{P}$ to be of a particular form, that is we assume that $\mathcal{P} = \mathcal{P}_q \otimes \mathbb{P}_{-q}$, where $\otimes$ is the product measure (statistical independence), $\mathcal{P}_q$ is the distribution over $(q_t)_{t\in\mathbb{Z}}$ generated by the unobserved components model (2), and $\mathbb{P}_{-q}$ is the rational expectations measure without house prices–i.e. the measure over $\omega\backslash q = (\xi_t, r_t, w_t, \Sigma_t, \pi_t, (P_{H,t}/P_{F,t}))_{t\in\mathbb{Z}}$ that which is consistent with the equilibrium-implied joint probability distribution of $\omega\backslash q$. Consequently, for any two measurable functions $f : \mathbb{R} \to \mathbb{R}, g : \mathbb{R}^{\dim\Omega - 1} \to \mathbb{R}$, we have

$$\mathbb{E}^{\mathcal{P}}[f(q_t) \cdot g(\omega_t\backslash q_t)] = \int f\mathcal{P}_q(\mathrm{d}q) \cdot \int g\mathbb{P}_{-q}(\mathrm{d}\omega\backslash q).$$

Finally, $\mathbb{E}_t^{\mathcal{P}}$ is the expectation implied by $\mathcal{P}$ conditional on the sigma-algebra generated by ω^t.

Now, at each calendar date $t \in \mathbb{Z}$, the household takes as given ω^t and chooses today's consumption, labor, housing, housing investment, and bond levels, $(c_t, n_t, h_t, x_t, b_{t+1})$, as well as contingent plans for the future, $\{(c_{t+s}, n_{t+s}, h_{t+s}, x_{t+s}, b_{t+s+1}) : \Omega^{t+s} \to \mathbb{R}_+^4 \times \mathbb{R}\}_{s\in\mathbb{N}_+}$, to maximize

$$\begin{aligned}
&\mathbb{E}_t^{\mathrm{p}} \sum_{s=0}^{\infty} \beta^s \left(\frac{\xi_{c,t+s} c_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\xi_{h,t+s} h_{t+s}^{1-\nu}}{1-\nu} - \chi \frac{n_{t+s}^{1+\varphi}}{1+\varphi} \right) \text{ subject to} \\
&c_{t+s} + q_{t+s}(h_{t+s} - (1-\delta)h_{t+s-1}) + b_{t+s+1} + x_{t+s} = w_{t+s}n_{t+s} \\
&\quad + (1 + r_{t+s})b_{t+s} + q_{t+s} \cdot \tfrac{\xi_{x,t+s}}{\eta} x_{t+s-\tau}^{\eta} + \Sigma_{t+s}, \quad \forall s \geq 0, \ \mathcal{P}\text{-almost surely,}
\end{aligned} \tag{B.7}$$

as well as subject to a standard no-Ponzi-game condition on b. The first-order conditions for

[41] We ignore the terms $T_{t+s}, \mathrm{b}_{t+s}$ since they will be zero in the equilibrium we analyze. All derivations go through if these terms are non-zero.

[42] We restrict Ω to be the set of absolutely summable sequences.

[43] For any three vectors x, y, z where $x = (y, z)$, we define $x\backslash y := z$.

program (B.7) (suppressing the transversality conditions) are:

$$
\begin{aligned}
\forall s \geq 0, \quad & (h) \quad \xi_{c,t+s} c_{t+s}^{-\sigma} q_{t+s} = \xi_{h,t+s} h_{t+s}^{-\nu} + \beta(1-\delta)\mathbb{E}_{t+s}^{\mathcal{P}}\{\xi_{c,t+s+1} c_{t+s+1}^{-\sigma} q_{t+s+1}\}, \\
& (n) \quad \chi n_{t+s}^{\varphi} \xi_{c,t+s}^{-1} c_{t+s}^{\sigma} = w_{t+s}, \\
& (b) \quad \xi_{c,t+s} c_{t+s}^{-\sigma} = \beta \mathbb{E}_{t+s}^{\mathcal{P}} \left\{(1+r_{t+s+1})\xi_{c,t+s+1} c_{t+s+1}^{-\sigma}\right\}, \\
& (x) \quad \beta^{\tau} \mathbb{E}_{t+s}^{\mathcal{P}} \left\{q_{t+s+\tau} \xi_{c,t+s+\tau} c_{t+s+\tau}^{-\sigma} \cdot \xi_{x,t+s+\tau} x_{t+s}^{\eta-1}\right\} = \xi_{c,t+s} c_{t+s}^{-\sigma}, \\
& (BC) \quad c_{t+s} + q_{t+s}(h_{t+s} - (1-\delta)h_{t+s-1}) + b_{t+s+1} + x_{t+s} = w_{t+s} n_{t+s} \\
& \qquad + (1+r_{t+s})b_{t+s} + q_{t+s} \cdot \tfrac{\xi_{x,t+s}}{\eta} x_{t+s-\tau}^{\eta} + \Sigma_{t+s}.
\end{aligned} \tag{B.8}
$$

In close analogy to the standard procedure in a model with fully rational expectations, $\mathcal{P} = \mathbb{P}$, we now derive a linear approximation to (B.8) that – together with the other linearized equilibrium conditions – allows to solve the model to first order in the amplitude of shocks. The specific challenge here, with $\mathcal{P} = \mathcal{P}_q \otimes \mathbb{P}_{-q}$, will be to compute all expectations explicitly that depend on house prices, q. This includes the house prices themselves, $(q_{t+s})_{s>0}$, as well as expectations over future contingent choices, $(c_{t+s}, n_{t+s}, h_{t+s}, x_{t+s}, b_{t+s+1})_{s>0}$.

To first order around the steady state[44] from Lemma 2, in which we have $1 + r_{ss} = \beta^{-1}$, $\chi n_{ss}^{\varphi} c_{ss}^{\sigma} = w_{ss}(=1)$, $q_{ss} = \xi_{h,ss} h_{ss}^{-\nu} c_{ss}^{\sigma}/(1-\bar{\beta})$, $\beta^{\tau} q_{ss} \xi_{x,ss} x_{ss}^{\eta-1} = 1$, it holds that, $\forall s \geq 0$:

$$
\begin{aligned}
& (h) \quad \widehat{h}_{t+s} = \tfrac{1}{\nu}\xi_{h,t+s} + \tfrac{\sigma}{\nu} \frac{\widehat{c}_{t+s} - \frac{1}{\sigma}\widehat{\xi}_{c,t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\left\{\widehat{c}_{t+s+1} - \frac{1}{\sigma}\widehat{\xi}_{c,t+s+1}\right\}}{1-\bar{\beta}} - \tfrac{1}{\nu} \frac{\widehat{q}_{t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+1}\}}{1-\bar{\beta}}, \\
& (n) \quad \varphi \widehat{n}_{t+s} + \sigma \widehat{c}_{t+s} - \widehat{\xi}_{c,t+s} = \widehat{w}_{t+s}, \\
& (b) \quad \widehat{c}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} = -\tfrac{1}{\sigma}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+1}\} + \mathbb{E}_{t+s}^{\mathcal{P}}\left\{\widehat{c}_{t+s+1} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s+1}\right\}, \\
& (x) \quad \mathbb{E}_{t+s}^{\mathcal{P}}\widehat{q}_{t+s+\tau} - \mathbb{E}_{t+s}^{\mathcal{P}}\left\{\sigma\widehat{c}_{t+s+\tau} - \widehat{\xi}_{c,t+s+\tau}\right\} \\
& \qquad + \left(\sigma\widehat{c}_{t+s} - \widehat{\xi}_{c,t+s}\right) + \mathbb{E}_{t+s}\widehat{\xi}_{x,t+s+\tau} = (1-\eta)\widehat{x}_{t+s}, \\
& (BC) \quad \widehat{b}_{t+s+1} = \tfrac{1}{\beta}\widehat{b}_{t+s} + \tfrac{w_{ss}n_{ss}}{y_{ss}}(\widehat{w}_{t+s} + \widehat{n}_{t+s}) + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_{t+s} \\
& \qquad - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}_{t+s} - (1-\delta)\widehat{h}_{t+s-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t+s-\tau} - \beta^{\tau}\widehat{x}_{t+s} + \widehat{\xi}_{x,t+s}\right).
\end{aligned} \tag{B.9}
$$

To characterize the household's expectations over own choice variables, start with the optimality condition for liquid bonds. Iteration over future instances of the condition reveals:

$$
(b) \quad \widehat{c}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} = -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \lim_{n\to\infty}\mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_{t+s+n}.
$$

To keep notation concise, we define for any random process $(k_t)_t$: $\mathbb{E}_{t+s}^{\mathcal{P}}k_{\infty} := \lim_{n\to\infty}\mathbb{E}_{t+s}^{\mathcal{P}}k_{t+s+n}$.

[44] Technically, we are scaling $\mathrm{Var}^{\mathcal{P}}[\|\omega\|] \to 0$. This is consistent, however, with the definition of $\mathcal{P}$ and $\mathrm{Var}[\|\xi\|] \to 0$.

We implicitly assumed here that a law of iterated expectations holds for $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+s}$ – at the end of the section we verify that the $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ we recover does indeed satisfy a law of iterated expectations. Careful inspection of equations (B.9) reveals that $\mathbb{E}_{t+s}^{\mathcal{P}} \widehat{c}_\infty$ is the only subjective expectation of a choice variable left unknown. This is because the law of iterated expectations for $\mathcal{P}$ allows substituting (b) into

$$
\begin{aligned}
&(h) \quad \widehat{h}_{t+s} = \tfrac{1}{\nu}\xi_{h,t+s} - \tfrac{\sigma}{\nu} \frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} - \bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n+1}\}}{1-\bar{\beta}} - \tfrac{1}{\nu}\frac{\widehat{q}_{t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+1}\}}{1-\bar{\beta}} + \tfrac{\sigma}{\nu}\mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_\infty, \\
&(n) \quad \varphi \widehat{n}_{t+s} + \sigma \widehat{c}_{t+s} - \widehat{\xi}_{c,t+s} = \widehat{w}_{t+s}, \\
&(b) \quad \widehat{c}_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} = -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_\infty, \\
&(x) \quad \mathbb{E}_{t+s}^{\mathcal{P}}\widehat{q}_{t+s+\tau} - \sum_{n=1}^{\tau}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \mathbb{E}_{t+s}\widehat{\xi}_{x,t+s+\tau} = (1-\eta)\widehat{x}_{t+s}, \\
&(BC) \quad \widehat{b}_{t+s+1} = \tfrac{1}{\beta}\widehat{b}_{t+s} + \tfrac{w_{ss}n_{ss}}{y_{ss}}(\widehat{w}_{t+s} + \widehat{n}_{t+s}) + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_{t+s} \\
&\qquad - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}_{t+s} - (1-\delta)\widehat{h}_{t+s-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t+s-\tau} - \beta^{\tau}\widehat{x}_{t+s} + \widehat{\xi}_{x,t+s}\right).
\end{aligned}
\tag{B.10}
$$

Now to find $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ (and characterize the choices at calendar date t), we first use the budget constraint for some $s > \tau$. Start by plugging in the optimality conditions:

$$
\begin{aligned}
(BC) \quad \widehat{b}_{t+s+1} = {} & \tfrac{1}{\beta}\widehat{b}_{t+s} + \tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+s} + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\left(\widehat{c}^*_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s}\right) - \tfrac{c_{ss}}{y_{ss}\sigma}\widehat{\xi}_{c,t+s} \\
& - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}^*_{t+s} - (1-\delta)\widehat{h}^*_{t+s-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}^*_{t+s-\tau} - \beta^{\tau}\widehat{x}^*_{t+s} + \widehat{\xi}_{x,t+s}\right) \\
& - \tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}{y_{ss}}\mathbb{E}_{t+s}^{\mathcal{P}}\widehat{c}_\infty \\
& + \tfrac{q_{ss}h_{ss}}{y_{ss}}\tfrac{1}{\nu}\tfrac{1}{1-\bar{\beta}}\left(\widehat{q}_{t+s} - \bar{\beta}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+1}\} - (1-\delta)\left(\widehat{q}_{t+s-1} - \bar{\beta}\mathbb{E}_{t+s-1}^{\mathcal{P}}\{\widehat{q}_{t+s}\}\right)\right) \\
& + \tfrac{x_{ss}}{y_{ss}}\tfrac{1}{\beta^{\tau}(1-\eta)}\left(\mathbb{E}_{t+s-\tau}^{\mathcal{P}}\{\widehat{q}_{t+s}\} - \beta^{\tau}\mathbb{E}_{t+s}^{\mathcal{P}}\{\widehat{q}_{t+s+\tau}\}\right)
\end{aligned}
$$

where we have defined the auxiliary variables

$$
\begin{aligned}
\widehat{h}^*_{t+s} &:= \tfrac{1}{\nu}\xi_{h,t+s} - \tfrac{\sigma}{\nu}\frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} - \bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n+1}\}}{1-\bar{\beta}}, \\
\widehat{c}^*_{t+s} - \tfrac{1}{\sigma}\widehat{\xi}_{c,t+s} &:= -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\}, \\
(1-\eta)\widehat{x}^*_{t+s} &:= -\sum_{n=1}^{\tau}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} + \mathbb{E}_{t+s}\widehat{\xi}_{x,t+s+\tau}.
\end{aligned}
$$

Notice that the auxiliary variables are composed only of terms over which the household has rational expectations; thus of terms that fade to zero as $s \to \infty$.

Next, we apply the operator $\mathbb{E}_t^{\mathcal{P}}$ to both sides of (BC) and consider the limit as $s \to \infty$. This delivers:

$$
\begin{aligned}
(BC) \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty = \tfrac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - \tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}{y_{ss}} \mathbb{E}_{t+s}^{\mathcal{P}} \widehat{c}_\infty \\
& \quad + \tfrac{\delta q_{ss}h_{ss}/\nu \ + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \\
\Longleftrightarrow \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty = \tfrac{y_{ss}}{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu} \left[\tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty + \tfrac{\delta q_{ss}h_{ss}/\nu \ + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \right].
\end{aligned}
$$

Now since $\mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty$ can be computed from the subjectively perceived house price model, the only unknown left is $\mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty$. To find it, we plug $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ into (BC). At some $s > 0$ we have

$$
\begin{aligned}
(BC) \quad \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} = & \tfrac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s} + \tfrac{w_{ss}n_{ss}}{y_{ss}} (1+1/\varphi) \mathbb{E}_t \widehat{w}_{t+s} + \mathbb{E}_t \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}} \mathbb{E}_t \left(\widehat{c}^*_{t+s} - \tfrac{1}{\sigma} \widehat{\xi}_{c,t+s} \right) - \tfrac{c_{ss}}{y_{ss}\sigma} \mathbb{E}_t \widehat{\xi}_{c,t+s} \\
& - \tfrac{q_{ss}h_{ss}}{y_{ss}} \mathbb{E}_t [\widehat{h}^*_{t+s} - (1-\delta) \widehat{h}^*_{t+s-1}] + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+s-\tau} - \beta^\tau \widehat{x}_{t+s} + \widehat{\xi}_{x,t+s} \right) \\
& - \tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - \tfrac{\delta q_{ss}h_{ss}/\nu \ + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \\
& + \tfrac{q_{ss}h_{ss}}{y_{ss}} \tfrac{1}{\nu} \tfrac{1}{1-\bar{\beta}} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{ \widehat{q}_{t+s+1} \} - (1-\delta) \left(\widehat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_{t+s-1}^{\mathcal{P}} \{ \widehat{q}_{t+s} \} \right) \right).
\end{aligned}
$$

After defining the auxiliary variables

$$
\begin{aligned}
z^*_{t+s} := & \tfrac{w_{ss}n_{ss}}{y_{ss}} (1+1/\varphi) \widehat{w}_{t+s} + \widehat{\Sigma}_{t+s} - \tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}} \left(\widehat{c}^*_{t+s} - \tfrac{1}{\sigma} \widehat{\xi}_{c,t+s} \right) - \tfrac{c_{ss}}{y_{ss}\sigma} \widehat{\xi}_{c,t+s} \\
& - \tfrac{q_{ss}h_{ss}}{y_{ss}} [\widehat{h}^*_{t+s} - (1-\delta) \widehat{h}^*_{t+s-1}] + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \widehat{\xi}_{x,t+s}, \\
Q_{t+s} := & \tfrac{q_{ss}h_{ss}}{y_{ss}} \tfrac{1}{\nu} \tfrac{1}{1-\bar{\beta}} \left(\widehat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{ \widehat{q}_{t+s+1} \} - (1-\delta) \left(\widehat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_{t+s-1}^{\mathcal{P}} \{ \widehat{q}_{t+s} \} \right) \right),
\end{aligned}
\tag{B.11}
$$

we can rewrite (BC) into

$$
\begin{aligned}
(BC) \quad \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} = & \tfrac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s} + \mathbb{E}_t z^*_{t+s} + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+s-\tau} - \beta^\tau \widehat{x}_{t+s} \right) + \mathbb{E}_t^{\mathcal{P}} Q_{t+s} \\
& - \tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - \tfrac{\delta q_{ss}h_{ss}/\nu \ + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty.
\end{aligned}
$$

Performing backward substitution until $s = 1$ delivers

$$
\begin{aligned}
(BC) \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} = \beta^{-s} \widehat{b}_{t+1} - \beta^{-s} (1-\beta^s) \tfrac{\beta}{1-\beta} \left[\tfrac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty + \tfrac{\delta q_{ss}h_{ss}/\nu \ + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \right] \\
& \quad + \beta^{-s} \sum_{n=1}^{s} \beta^n (\mathbb{E}_t z^*_{t+n} + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \beta^{-s} \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{s} \beta^n \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+n-\tau} - \beta^\tau \widehat{x}_{t+n} \right) \\
\Longleftrightarrow \quad & \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} - \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty = \beta^{-s} \cdot \Bigg[\widehat{b}_{t+1} + \sum_{n=1}^{s} \beta^n (\mathbb{E}_t z^*_{t+n} + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \tfrac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{s} \beta^n \mathbb{E}_t^{\mathcal{P}} \left(\widehat{x}_{t+n-\tau} - \beta^\tau \widehat{x}_{t+n} \right) \\
& \qquad - \mathbb{E}_t^{\mathcal{P}} \widehat{b}_\infty - (1-\beta^s) \tfrac{\beta}{1-\beta} \tfrac{\delta q_{ss}h_{ss}/\nu \ + x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_\infty \Bigg].
\end{aligned}
$$

Now since we know that $\lim_{s\to\infty}\mathbb{E}_t^{\mathcal{P}}\widehat{b}_{t+s}$ exists, it must be that the left side of this equation tends to zero as $s\to\infty$. But since $\beta^{-s}\to+\infty$ as $s\to\infty$, this implies that

$$\mathbb{E}_t^{\mathcal{P}}\widehat{b}_\infty=\widehat{b}_{t+1}+\sum_{n=1}^{\infty}\beta^n(\mathbb{E}_t z^*_{t+n}+\mathbb{E}_t^{\mathcal{P}}Q_{t+n})+\frac{x_{ss}}{y_{ss}}\underbrace{\beta^{-\tau}\sum_{n=1}^{\infty}\beta^n\mathbb{E}_t^{\mathcal{P}}\left(\widehat{x}_{t+n-\tau}-\beta^{\tau}\widehat{x}_{t+n}\right)}_{=\beta^{-\tau}\sum_{n=1}^{\tau}\beta^n\widehat{x}_{t+n-\tau}\text{ by rearranging a convergent sum}}-\frac{\beta}{1-\beta}\frac{\delta q_{ss}h_{ss}/\nu+x_{ss}(\beta^{-\tau}-1)/(1-\eta)}{y_{ss}}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_\infty,$$

so that we can characterize $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$ entirely in terms of variables that are either pre-determined, current choice variables (i.e. known under $\mathbb{E}_t^{\mathcal{P}}$), or variables of which we can compute expectations in closed form (be they rational or not):

$$\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty=\frac{y_{ss}}{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}\frac{1-\beta}{\beta}\left[\widehat{b}_{t+1}+\sum_{n=1}^{\infty}\beta^n(\mathbb{E}_t z^*_{t+n}+\mathbb{E}_t^{\mathcal{P}}Q_{t+n})+\frac{x_{ss}}{y_{ss}}\beta^{-\tau}\sum_{n=1}^{\tau}\beta^n\widehat{x}_{t+n-\tau}\right].$$

In the last step, we explicitly characterize the subjective expectation $\mathbb{E}_t^{\mathcal{P}}\sum_{n=1}^{\infty}\beta^nQ_{t+n}$ in terms of the processes governing subjective house price expectations, namely $\widehat{q}_t,\widehat{\overline{m}}_t$.

Recall that for any $s>0$ we have $\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s}=\widehat{q}_t+(1-\varrho^s)\frac{\varrho}{1-\varrho}\widehat{\overline{m}}_t$, so that we have

$$\begin{aligned}\mathbb{E}_t^{\mathcal{P}}Q_{t+s}&=\frac{q_{ss}h_{ss}}{y_{ss}}\frac{1}{\nu}\frac{1}{1-\bar{\beta}}\left(\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s}-\bar{\beta}\mathbb{E}_t^{\mathcal{P}}\{\widehat{q}_{t+s+1}\}-(1-\delta)\left(\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s-1}-\bar{\beta}\mathbb{E}_t^{\mathcal{P}}\{\widehat{q}_{t+s}\}\right)\right)\\&=\frac{q_{ss}h_{ss}}{y_{ss}}\frac{1}{\nu}\frac{1}{1-\bar{\beta}}\Big(\delta(1-\bar{\beta})\widehat{q}_t+(1-\varrho^s)\frac{\varrho}{1-\varrho}\widehat{\overline{m}}_t-\bar{\beta}(1-\varrho^{s+1})\frac{\varrho}{1-\varrho}\widehat{\overline{m}}_t\\&\quad-(1-\delta)(1-\varrho^{s-1})\frac{\varrho}{1-\varrho}\widehat{\overline{m}}_t+(1-\delta)\bar{\beta}(1-\varrho^s)\frac{\varrho}{1-\varrho}\widehat{\overline{m}}_t\Big)\\&=\frac{q_{ss}h_{ss}}{y_{ss}}\frac{1}{\nu}\frac{1}{1-\bar{\beta}}\left(\delta(1-\bar{\beta})\widehat{q}_t+\widehat{\overline{m}}_t\frac{\varrho}{1-\varrho}\left[\delta(1-\bar{\beta})+(1-\bar{\beta}\varrho)(1-\varrho-\delta)\varrho^{s-1}\right]\right)\end{aligned}$$

which in turn implies

$$\sum_{s\geq1}\beta^s\mathbb{E}_t^{\mathcal{P}}Q_{t+s}=\beta\frac{q_{ss}h_{ss}}{y_{ss}}\frac{1}{\nu}\left(\frac{\delta}{1-\beta}\widehat{q}_t+\widehat{\overline{m}}_t\frac{\varrho}{1-\varrho}\left[\frac{\delta}{1-\beta}+\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho}\right]\right).\tag{B.12}$$

Notice that, as claimed in the main text, it is that

$$\frac{\delta}{1-\beta}+\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho}>0,\ \ \forall\beta,\delta,\varrho\in(0,1),$$ [45]

so that the coefficient of the posterior expected house price growth rate onto the series is positive.

We now have characterized the household's decisions at an arbitrary calendar date t up to first

[45] *Proof.* This follows from the claim being equivalent to the statement $f(\varrho):=\frac{\delta}{1-\beta}(1-\beta\varrho)+\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}(1-\varrho-\delta)>0,\ \forall\beta,\delta,\varrho\in(0,1)$, from f being an upward-open parabola with $f(1)=0$, and $f'(1)<0$. □

order around the deterministic steady state:

$$
\begin{aligned}
&(h) \quad \widehat{h}_t = \tfrac{1}{\nu}\xi_{h,t} - \tfrac{\sigma}{\nu}\frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n}\}-\bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n+1}\}}{1-\bar{\beta}} - \tfrac{1}{\nu}\widehat{q}_t + \tfrac{1}{\nu}\tfrac{\bar{\beta}}{1-\bar{\beta}}\varrho\widehat{\bar{m}}_t + \tfrac{\sigma}{\nu}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty, \\
&(n) \quad \varphi\widehat{n}_t + \sigma\widehat{c}_t - \widehat{\xi}_{c,t} = \widehat{w}_t, \\
&(b) \quad \widehat{c}_t - \tfrac{1}{\sigma}\widehat{\xi}_{c,t} = -\tfrac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n}\} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty, \\
&(x) \quad \widehat{q}_t + (1-\varrho^\tau)\frac{\varrho}{1-\varrho}\widehat{\bar{m}}_t - \sum_{n=1}^{\tau}\mathbb{E}_t\{\widehat{r}_{t+n}\} + \mathbb{E}_t\widehat{\xi}_{x,t+\tau} = (1-\eta)\widehat{x}_t, \\
&(BC) \quad \widehat{b}_{t+1} = \tfrac{1}{\beta}\widehat{b}_t + \tfrac{w_{ss}n_{ss}}{y_{ss}}(\widehat{w}_t + \widehat{n}_t) + \widehat{\Sigma}_t - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_t \\
&\qquad\qquad - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t-\tau} - \beta^\tau\widehat{x}_t + \widehat{\xi}_{x,t}\right),
\end{aligned}
\tag{B.13}
$$

where, after a few rearrangements,

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = {} & \frac{\delta q_{ss}h_{ss}/\nu}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\cdot\left[\widehat{q}_t + \widehat{\bar{m}}_t\cdot\underbrace{\frac{\varrho}{1-\varrho}\left(1+\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho}\frac{1-\beta}{\delta}\right)}_{>0\ \forall\beta,\delta,\varrho\in(0,1)}\right] \\
& + \frac{y_{ss}}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\frac{1-\beta}{\beta}\cdot\left[\sum_{n=1}^{\infty}\beta^n\mathbb{E}_t\{z^*_{t+n}\} + \tfrac{x_{ss}}{y_{ss}}\sum_{n=0}^{\tau-1}\beta^{-n}\widehat{x}_{t-n} + \widehat{b}_{t+1}\right].
\end{aligned}
$$

Lastly, we verify that the law of iterated expectation holds for the explicit formula given for $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty$ above. Define

$$
A_t := \widehat{b}_{t+1} + \sum_{n=1}^{\infty}\beta^n(\mathbb{E}_t z^*_{t+n} + \mathbb{E}_t^{\mathcal{P}}Q_{t+n}) + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\sum_{n=1}^{\tau}\beta^n\widehat{x}_{t+n-\tau}
$$

and note that the law of iterated expectations holds if and only if $\mathbb{E}_t^{\mathcal{P}}A_{t+1} = A_t$. Using equation (BC) to substitute $\widehat{b}_{t+2}$ in $\mathbb{E}_t^{\mathcal{P}}A_{t+1}$ we arrive at:

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}}A_{t+1} = A_t/\beta & - \tfrac{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\tfrac{1-\beta}{\beta}\mathbb{E}_t^{\mathcal{P}}A_{t+1} - \tfrac{q_{ss}h_{ss}\sigma/\nu}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\tfrac{1-\beta}{\beta}\mathbb{E}_t^{\mathcal{P}}A_{t+1} \\
& + \tfrac{(1-\delta)q_{ss}h_{ss}\sigma/\nu}{c_{ss}+\sigma/\varphi\cdot n_{ss}w_{ss}+\delta q_{ss}h_{ss}\sigma/\nu}\tfrac{1-\beta}{\beta}A_t,
\end{aligned}
$$

which can be rearranged into $\mathbb{E}_t^{\mathcal{P}}A_{t+1} = A_t$.

B.8 Solving the model with Dynare

Although equations (B.13) serve as an explicit solution of the household decision problem at time t,[46] they are not in a form that lends itself to easy numerical implementation, i.e. to solving the model. Bringing the equations into a recursive form that – when combined with the other equations describing equilibrium – can be solved by standard methods is the goal of this appendix.

In a first step, we take care of the appearing infinite sums which are not easily recursifiable. By that, we mean the forward summation over expected real interest rates, $\widehat{c}_t^* = -1/\sigma \cdot \sum_{n\geq 1} \mathbb{E}_t \widehat{r}_{t+n}$. In principle, this variable could be recursified as

$$\widehat{c}_t^* = -1/\sigma \cdot \mathbb{E}_t \widehat{r}_{t+1} + \mathbb{E}_t \widehat{c}_{t+1}^*.$$

This representation would be incomplete, though, without the boundary condition $\lim_{s\to\infty} \mathbb{E}_t \widehat{c}_{t+s}^* = 0$. To the best of our knowledge, imposing such a boundary condition is not possible in Dynare's native `stoch_simul`-command.[47] Therefore, we employ several rearrangements of the equations (B.13) in order to eliminate $\widehat{c}_t^*$. First, consider the geometric summation over $\widehat{c}_t^*$, which is contained in $\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\}$ (z^* is defined in equation (B.11)):

$$\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{\widehat{c}_{t+n}^*\} = -\frac{\beta}{\sigma} \sum_{n\geq 0} \beta^n \sum_{s\geq 0} \mathbb{E}_t \widehat{r}_{t+2+n+s} = -\frac{\beta}{\sigma} \sum_{n\geq 0} \mathbb{E}_t \widehat{r}_{t+2+n} \cdot \sum_{s=0}^{n} \beta^s = \frac{\beta}{1-\beta} \Bigg[\underbrace{-\frac{1}{\sigma} \sum_{s\geq 0} \mathbb{E}_t \widehat{r}_{t+2+s}}_{=\mathbb{E}_t \widehat{c}_{t+1}^*} + \frac{1}{\sigma} \sum_{s\geq 1} \beta^s \mathbb{E}_t \widehat{r}_{t+1+s} \Bigg].$$

Next, consider the housing terms included in z^*:

$$\begin{aligned} \mathbb{E}_t [\widehat{h}_{t+s}^* - (1-\delta) \widehat{h}_{t+s-1}^*] &= \frac{\sigma}{\nu} \mathbb{E}_t \frac{\widehat{c}_{t+s}^* - \bar{\beta} \widehat{c}_{t+1+s}^*}{1-\bar{\beta}} - (1-\delta) \frac{\sigma}{\nu} \mathbb{E}_t \frac{\widehat{c}_{t-1+s}^* - \bar{\beta} \widehat{c}_{t+s}^*}{1-\bar{\beta}} \\ &= -\frac{1}{\nu} \mathbb{E}_t \frac{\widehat{r}_{t+s+1} + (1-\bar{\beta}) \sum_{n\geq 1} \widehat{r}_{t+1+s+n}}{1-\bar{\beta}} + (1-\delta) \frac{1}{\nu} \mathbb{E}_t \frac{\widehat{r}_{t+s} + (1-\bar{\beta}) \sum_{n\geq 1} \widehat{r}_{t+s+n}}{1-\bar{\beta}} \\ &= -\frac{1}{\nu} \frac{\beta(1-\delta)^2 + \delta}{1-\bar{\beta}} \mathbb{E}_t \widehat{r}_{r+s+1} + \frac{1}{\nu} \frac{1-\delta}{1-\bar{\beta}} \mathbb{E}_t \widehat{r}_{r+s} - \frac{\delta}{\nu} \sum_{n\geq 1} \mathbb{E}_t \widehat{r}_{t+s+1+n}. \end{aligned}$$

Symmetrically to before, the geometric summation over these terms that is contained in $\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\}$

[46] In the sense that they provide an equation system describing the household decsisions purely in terms of contemporaneous or pre-determined variables and expectations taken over the objective law.

[47] Although it is possible with the command `perfect_foresight_solver`, a routine we chose not to use due to its inability to automatically check the Blanchard-Kahn condition.

thus reads

$$\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t[\widehat{h}^*_{t+s}-(1-\delta)\widehat{h}^*_{t+s-1}] = \sum_{s=1}^{\infty}\beta^s\left[-\tfrac{1}{\nu}\tfrac{\beta(1-\delta)^2+\delta}{1-\beta}\mathbb{E}_t\widehat{r}_{r+s+1}+\tfrac{1}{\nu}\tfrac{1-\delta}{1-\beta}\mathbb{E}_t\widehat{r}_{r+s}\right]+\tfrac{\delta\sigma}{\nu}\tfrac{\beta}{1-\beta}\mathbb{E}_t\widehat{c}^*_{t+2}+\tfrac{\delta}{\nu}\tfrac{\beta}{1-\beta}\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t\widehat{r}_{t+s+2}.$$

We can now restate the entire geometric sum:

$$\begin{aligned}\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t z^*_{t+s} &= \sum_{s=1}^{\infty}\beta^s\mathbb{E}_t\Big[\tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+s}+\widehat{\Sigma}_{t+s}-\tfrac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+s}+\tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+s}\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{c}^*_{t+s}-\tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}^*_{t+s}-(1-\delta)\widehat{h}^*_{t+s-1}\right]\Big]\\
&= \sum_{s=1}^{\infty}\beta^s\mathbb{E}_t\Big[\tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+s}+\widehat{\Sigma}_{t+s}-\tfrac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+s}+\tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+s}\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\tfrac{\beta}{1-\beta}\tfrac{1}{\sigma}\widehat{r}_{t+1+s}-\tfrac{q_{ss}h_{ss}}{y_{ss}}\left[-\tfrac{1}{\nu}\tfrac{\beta(1-\delta)^2+\delta}{1-\beta}\widehat{r}_{t+s+1}+\tfrac{1}{\nu}\tfrac{1-\delta}{1-\beta}\widehat{r}_{r+s}+\tfrac{\delta}{\nu}\tfrac{\beta}{1-\beta}\widehat{r}_{t+s+2}\right]\Big]\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\tfrac{\beta}{1-\beta}\mathbb{E}_t\widehat{c}^*_{t+1}-\tfrac{\delta q_{ss}h_{ss}}{y_{ss}}\tfrac{\sigma}{\nu}\tfrac{\beta}{1-\beta}\underbrace{\mathbb{E}_t\widehat{c}^*_{t+2}}_{=\mathbb{E}_t\widehat{c}^*_{t+1}+\frac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+2}}.\end{aligned}$$

The remaining geometric sums may easily be recursified:

$$\begin{aligned}\sum_{s=1}^{\infty}\beta^s\mathbb{E}_t z^*_{t+s} &= \beta\mathcal{Z}^*_t-\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}{y_{ss}}\tfrac{\beta}{1-\beta}\mathbb{E}_t\widehat{c}^*_{t+1}-\tfrac{\delta q_{ss}h_{ss}}{y_{ss}}\tfrac{\sigma}{\nu}\tfrac{\beta}{1-\beta}\tfrac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+2}\quad\text{where}\\
\mathcal{Z}^*_t &= \beta\mathcal{Z}^*_{t+1}+\mathbb{E}_t\Big[\tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+1}+\widehat{\Sigma}_{t+1}-\tfrac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+1}+\tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+1}\\
&\qquad -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\tfrac{\beta}{1-\beta}\tfrac{1}{\sigma}\widehat{r}_{t+2}-\tfrac{q_{ss}h_{ss}}{y_{ss}}\left[-\tfrac{1}{\nu}\tfrac{\beta(1-\delta)^2+\delta}{1-\beta}\widehat{r}_{t+2}+\tfrac{1}{\nu}\tfrac{1-\delta}{1-\beta}\widehat{r}_{r+1}+\tfrac{\delta}{\nu}\tfrac{\beta}{1-\beta}\widehat{r}_{t+3}\right]\Big].\end{aligned}$$

Now, defining $\mathcal{C}_t := \mathbb{E}^{\mathcal{P}}_t\widehat{c}_\infty+\mathbb{E}_t\widehat{c}^*_{t+1}+\frac{\delta q_{ss}h_{ss}}{c_{ss}+n_{ss}w_{ss}\sigma/\varphi+\delta q_{ss}h_{ss}\sigma/\nu}\frac{1}{\nu}\mathbb{E}_t\widehat{r}_{t+2}$ allows us to arrive at the final

Dynare-ready formulation of the household's decision rules:

$$
\begin{aligned}
&(h) \quad \widehat{h}_t = \tfrac{1}{\nu}\xi_{h,t} - \tfrac{1}{1-\bar{\beta}}\tfrac{1}{\nu}\mathbb{E}_t\widehat{r}_{t+1} - \mathrm{Q}\tfrac{1}{\nu}\mathbb{E}_t\widehat{r}_{t+2} - \tfrac{1}{\nu}\widehat{q}_t + \tfrac{1}{\nu}\tfrac{\bar{\beta}}{1-\bar{\beta}}\varrho\widehat{\bar{m}}_t + \tfrac{\sigma}{\nu}\mathcal{C}_t,\\
&(b) \quad \widehat{c}_t - \tfrac{1}{\sigma}\widehat{\xi}_{c,t} = -\tfrac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+1} - \mathrm{Q}\tfrac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+2} + \mathcal{C}_t,\\
&\qquad (\mathcal{C}) \quad \mathcal{C}_t = \mathrm{Q}\tfrac{1}{\sigma}\cdot\left[\widehat{q}_t + \mathrm{M}\cdot\widehat{\bar{m}}_t\right] + \mathrm{C}\tfrac{y_{ss}}{c_{ss}}\tfrac{1-\beta}{\beta}\cdot\left[\beta\mathcal{Z}_t^* + \tfrac{x_{ss}}{y_{ss}}\sum_{n=0}^{\tau-1}\beta^{-n}\widehat{x}_{t-n} + \widehat{b}_{t+1}\right],\\
&\qquad (\mathcal{Z}) \quad \mathcal{Z}_t^* = \beta\mathcal{Z}_{t+1}^* + \mathbb{E}_t\left\{\begin{array}{c} \tfrac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+1} + \widehat{\Sigma}_{t+1} - \tfrac{c_{ss}(1+1/\sigma)+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{\xi}_{c,t+1} + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\widehat{\xi}_{x,t+1}\\ -\tfrac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\tfrac{\beta}{1-\beta}\tfrac{1}{\sigma}\widehat{r}_{t+2} - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[-\tfrac{1}{\nu}\tfrac{\beta(1-\delta)^2+\delta}{1-\bar{\beta}}\widehat{r}_{t+2} + \tfrac{1}{\nu}\tfrac{1-\delta}{1-\bar{\beta}}\widehat{r}_{r+1} + \tfrac{\delta}{\nu}\tfrac{\beta}{1-\beta}\widehat{r}_{t+3}\right]\end{array}\right\},\\
&(n) \quad \varphi\widehat{n}_t + \sigma\widehat{c}_t - \widehat{\xi}_{c,t} = \widehat{w}_t,\\
&(x) \quad \widehat{q}_t + (1-\varrho^\tau)\frac{\varrho}{1-\varrho}\widehat{\bar{m}}_t - \sum_{n=1}^{\tau}\mathbb{E}_t\{\widehat{r}_{t+n}\} + \mathbb{E}_t\widehat{\xi}_{x,t+\tau} = (1-\eta)\widehat{x}_t,\\
&(BC) \quad \widehat{b}_{t+1} = \tfrac{1}{\beta}\widehat{b}_t + \tfrac{w_{ss}n_{ss}}{y_{ss}}(\widehat{w}_t + \widehat{n}_t) + \widehat{\Sigma}_t - \tfrac{c_{ss}}{y_{ss}}\widehat{c}_t\\
&\qquad\quad - \tfrac{q_{ss}h_{ss}}{y_{ss}}\left[\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}\right] + \tfrac{x_{ss}}{y_{ss}}\beta^{-\tau}\left(\widehat{x}_{t-\tau} - \beta^\tau\widehat{x}_t + \widehat{\xi}_{x,t}\right),
\end{aligned}
\tag{B.14}
$$

where

$\mathrm{Q} := \delta q_{ss}h_{ss}\sigma/\nu(c_{ss} + \sigma/\varphi\cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu)^{-1} \in (0,1)$, $\quad \mathrm{M} := \dfrac{\varrho}{1-\varrho}\left(1 + \dfrac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\dfrac{1-\varrho-\delta}{1-\beta\varrho}\dfrac{1-\beta}{\delta}\right) > 0,$
$\mathrm{C} := (c_{ss} + n_{ss}w_{ss}\sigma/\varphi)(c_{ss} + \sigma/\varphi\cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu)^{-1}.$

(Notice that $\mathrm{M} > 0$ is proven in footnote 45 in Appendix B.7.)

B.9 Derivations for Section IV

In this Appendix we derive analytical results on the behavior of house prices in a one-region, zero liquidity endowment economy with instantaneous housing production.

The equations describing the economy are:

$$
\begin{aligned}
&(h) \quad \widehat{q}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} = \sigma(\widehat{c}_t - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}) - \nu(1-\bar{\beta})\widehat{h}_t,\\
&(b) \quad \widehat{c}_t = -\sigma^{-1}\mathbb{E}_t\widehat{r}_{t+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1},\\
&(BC) \quad \widehat{b}_{t+1} = \beta^{-1}\widehat{b}_t + \widehat{y}_t - \tfrac{c}{y}\widehat{c}_t - \tfrac{qh}{y}(\widehat{h}_t - (1-\delta)\widehat{h}_{t-1}) + \tfrac{qh\delta}{y}\widehat{q}_t,\\
&(B) \quad \widehat{b}_{t+1} = 0,\\
&(HMC) \quad \widehat{h}_t - (1-\delta)\widehat{h}_{t-1} = -\iota\varepsilon_t
\end{aligned}
\tag{B.15}
$$

with the endowment process $\widehat{y}_t$ being the only exogenous disturbance to the economy. Iterating on

(h) and plugging the results into the Euler equation gives our key equation:

$$\widehat{q}_t = \mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} - \widehat{r}_{t+1} - \nu(1-\bar{\beta})\widehat{h}_t + \nu(1-\bar{\beta})^2\sum_{s=0}^{\infty}\bar{\beta}^s\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s+1}$$

B.9.1 *Proof of Proposition (2)*

Under rational expectations, and using the fact that the shocks only hit in period one, we can use housing market clearing to rewrite the house price equation as:

$$\widehat{q}_t = \mathbb{E}\widehat{q}_{t+1} - \widehat{r}_{t+1} - (1-\bar{\beta})\widehat{h}_t + (1-\bar{\beta})(1-\delta)^t(1-\frac{1-\bar{\beta}}{1-\bar{\beta}\delta})(-\iota\varepsilon_t)$$

Using the fact that under rational expectations expected house prices have the following form

$$\mathbb{E}_1\widehat{q}_2 = \mathbb{E}_1\widehat{q}_3 - (1-\bar{\beta})(1-\delta)\varepsilon_1^h + (1-\bar{\beta})(1-\delta)^2(1-\frac{1-\bar{\beta}}{1-\bar{\beta}\delta})(-\iota\varepsilon_t)$$
$$\mathbb{E}_1\widehat{q}_3 = \mathbb{E}_1\widehat{q}_4 - (1-\bar{\beta})(1-\delta)^2\varepsilon_1^h + (1-\bar{\beta})(1-\delta)^3(1-\frac{1-\bar{\beta}}{1-\bar{\beta}\delta})(-\iota\varepsilon_t)$$
$$\ldots$$

Iteration on this equation and using the fact that the economy returns to the steady-state in expectations gives:

$$\mathbb{E}_1\widehat{q}_2 = -\frac{1}{\delta}(1-\bar{\beta})((1-\delta)\frac{1-\bar{\beta}}{1-\bar{\beta}\delta} + \delta)(-\iota\varepsilon_t)$$

Also note that $\mathbb{E}_1\widehat{q}_3 = (1-\delta)\mathbb{E}_1\widehat{q}_2$. Finally, we can derive the house price for the first to periods as a function of shocks:

$$\widehat{q}_1 = -\varepsilon_1 - \omega_h(-\iota\varepsilon_t)$$
$$\widehat{q}_2 = -(1-\delta)\omega_h(-\iota\varepsilon_t)$$

With $\omega_h = (1-\bar{\beta})\Big[(1-\delta)(\frac{1-\bar{\beta}}{1-\bar{\beta}\delta}(1+\frac{1}{\delta})) + \delta\Big]$. This completes the proof.

B.9.2 *Proof of Proposition (3)*

Under subjective beliefs we need to specifically characterize $\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s+1}$. We can make use of the method described in section (III.B). Importantly, we can make use of the fact that the shocks on the endowment, and hence the interest rate only hit in the first period. As households have rational expectations about these processes, they posses knowledge of this fact. Applying our solution method, and substituting the subjective beliefs housing model, we get:

$$\widehat{q}_t = \widehat{q}_t + \varrho\widehat{\widehat{m}}_t - (1-\bar{\beta})\widehat{h}_t - \widehat{r}_{t+1} + (1-\bar{\beta})\Bigg[-\widehat{q}_t - \frac{y}{y+\delta qh}\frac{1-\beta}{1-\bar{\beta}}\widehat{r}_{t+1}$$
$$+\Big(\beta\frac{qh}{y}\frac{\varrho}{1-\varrho}(\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho} + \frac{1}{1-\beta\varrho}) + \frac{1-\varrho}{1-\varrho\bar{\beta}} - \frac{1}{1-\varrho}\Big)\widehat{\widehat{m}}_t\Bigg]$$

Rearranging yields:

$$\widehat{q}_t = -\widehat{h}_t - \omega_r\widehat{r}_{t+1} + \omega_m\widehat{\widehat{m}}_t$$

where $\omega_r = 1 + \frac{y(1-\beta)}{y+hq\delta}$ and $\omega_m = (1-\bar{\beta})^{-1}\Big[\varrho + (1-\bar{\beta})(\beta\frac{qh}{y}\frac{\varrho}{1-\varrho}(\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho} + \frac{1}{1-\beta\varrho}) + \frac{1-\varrho}{1-\varrho\bar{\beta}} - \frac{1}{1-\varrho})\Big]$. Further, we have that

$$\frac{\partial\omega_m}{\partial\varrho} = (1-\bar{\beta})^{-1}\Bigg[1 + (1-\bar{\beta})\beta\frac{qh}{y}\Big(\frac{\varrho}{1-\varrho}(\frac{(1-\bar{\beta})(\beta\bar{\beta}\rho(\varrho+\delta)-1)}{((1-\bar{\beta})(1-\beta\varrho))^2} + \frac{\beta}{(1-\beta\varrho)^2})$$
$$\frac{1-2\varrho}{(1-\varrho)^2}(\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho} + \frac{1}{1-\beta\varrho}) - \frac{1-\bar{\beta}}{(1-\bar{\beta}\varrho)^2} + \frac{1}{(1-\varrho)^2}\Big)\Bigg]$$

which is smaller than zero for standard parameter choices. THe first to periods of the house price are given by:

$$\widehat{q}_1 = -\omega_r\varepsilon_1 - (-\iota\varepsilon_t)$$
$$\widehat{q}_2 = -(1-\delta)(-\iota\varepsilon_t) + \omega_m g\widehat{q}_1$$

This completes the proof.

B.9.3 *Demand-side heterogeneity*

Turning to regional differences on the housing demand side, we focus on differences in the housing demand elasticity (ν). Following the same logic as in Propositions (2) and (3), we can derive closed-form characterizations of the house price with respect to a monetary policy shock.

Under rational expectations, the house price is given by

$$\widehat{q}_t = \mathbb{E}\widehat{q}_{t+1} - \widehat{r}_{t+1} - \nu(1-\bar{\beta})\widehat{h}_t + \nu(1-\bar{\beta})(1-\delta)^t(1 - \frac{1-\bar{\beta}}{1-\bar{\beta}\delta})(-\iota\varepsilon_t)$$

Using the same steps as described in the Proof of Proposition (B.9.1) we get:

$$\mathbb{E}_1\widehat{q}_2 = -\nu\frac{1}{\delta}(1-\bar{\beta})((1-\delta)\frac{1-\bar{\beta}}{1-\bar{\beta}\delta} + \delta)(-\iota\varepsilon_t)$$

The house price response in the first two periods is given by

$$\widehat{q}_1 = -\varepsilon_1 - \omega_h \nu(-\iota\varepsilon_t)$$
$$\widehat{q}_2 = -(1-\delta)\omega_h \nu(-\iota\varepsilon_t)$$

Given that $\iota > 0$, a higher elasticity of housing demand, hence a lower (ν), leads to a stronger response in house prices to a monetary policy shock.

Moving to the subjective beliefs model, we can use the same approach as in the proof of Proposition (B.9.2) and arrive at:

$$\widehat{q}_t = -\nu\widehat{h}_t - \tilde{\omega}_r \widehat{r}_{t+1} + \tilde{\omega}_m \widehat{\bar{m}}_t$$

where $\tilde{\omega} = 1 + \frac{y(1-\beta)}{y+hq\delta\nu^{-1}}$ and $\tilde{\omega}_m = (1-\bar{\beta})^{-1}\Big[\varrho + (1-\bar{\beta})(\beta\frac{qh\nu^{-1}}{y}\frac{\varrho}{1-\varrho}(\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho} + \frac{1}{1-\beta\varrho}) + \frac{1-\varrho}{1-\varrho\bar{\beta}} - \frac{1}{1-\varrho})\Big]$.
Assuming that $\iota = 0$, the house price response to the shock in the first two periods is given by:

$$\widehat{q}_1 = -\tilde{\omega}_r \varepsilon_1$$
$$\widehat{q}_2 = \tilde{\omega}_m g \widehat{q}_1$$

Under subjective beliefs, we arrive at the same result without conditioning on a positive supply response ($\iota \geq 0$). As under the supply side differences, the subjective beliefs model is more responsive to a monetary policy shock.

To examine the regional disparities in house price growth driven by variations in housing demand, we make the simplifying assumption that $\iota = 0$. As before, we consider two regions (A, B) facing the same monetary policy shock. The regions differ in their elasticity of housing demand $\nu^A > \nu^B$. The following statement can be made:

Proposition 5 (Differential house price growth responses, demand-side)**.** *The differential house price growth response in region A and B under rational expectations with $\iota = 0$ is zero.*
The differential response of house prices in regions A and B under subjective expectations, with $\iota = 0$ for the first two periods, is given by:

$$\Delta\widehat{q}_1^B - \Delta\widehat{q}_1^A = (\tilde{\omega}_r^B - \tilde{\omega}_r^A)\varepsilon_1$$
$$\Delta\widehat{q}_2^B - \Delta\widehat{q}_2^A = g(\tilde{\omega}_m^B - \tilde{\omega}_m^A)(\Delta\widehat{q}_1^B - \Delta\widehat{q}_1^A)$$

where $\tilde{\omega}_r^B \geq \tilde{\omega}_r^A > 0$ and $\tilde{\omega}_m^B \geq \tilde{\omega}_m^A > 0$.

Proof. Differential house price growth, $\Delta\widehat{q}_j^B - \Delta\widehat{q}_j^A$ with $j = 1, 2$, are obtained by differentiating regional house price responses to a monetary policy shock derived above. ∎

Proposition (5) demonstrates that under demand-side heterogeneities a similar result can be obtained compared to supply-side heterogeneities. On impact of the shock demand-side heterogeneities creates differential house price growth rates. This difference is dynamically amplified through extrapolation. Additionally, differences in housing demand elasticities affect the subjective beliefs path of housing demand ($\tilde{\omega}_m^B \geq \tilde{\omega}_m^A$). The region with a more elastic housing demand elasticity is more responsive to the shock, as subjective beliefs about future housing demand are more responsive. Under the simplifying assumptions we made a similar channel is absent in the case of supply-side heterogenities.

C. Empirical part and model calibration

C.1 Data sources

US data. For the monthly house price data on the federal level we use the SP CoreLogic Case-Shiller U.S. National Home Price Index seasonally adjusted from the FRED database (CSUSHPISA). The FFR (FEDFUNDS), industrial production (INDPRO), and CPI (CPIAUCSL) on a monthly frequency are also taken from the FRED. The house price expectations data are the mean of the expected change in home value during the next year taken from the Survey of Consumers by the Michigan University. For the quarterly state level data, we use all transactions house price indices ("State code"STHPI) from the FRED and seasonally adjust them. The remaining state level data on building permits ("State code"BPPRIVSA), employment in construction ("Stade code"CONS), unemployment ("Stade code"UR), and employment in retail are all taken from the FRED and already seasonally adjusted. We omit the code for employment in retail, as these vary across states. GDP (GDP) and the GDP deflator (GDPDEF) are also obtained from the FRED. As already mentioned in the main text, the house price sensitivity indicator is taken from Guren et al. (2021), and the monetary policy shock from Bauer and Swanson (2023). When aggregating the shocks from a monthly to a quarterly frequency we weigh the shocks according to the time they occurred in the quarter. Giving a higher weight to shocks that occured at the beginning of the quarter. Hence, the weights are $1, \frac{2}{3}, \frac{1}{3}$. This aggregation method is in line with Gertler and Karadi (2015) and Almgren et al. (2022).

Euro Area data. For the Euro Area we obtain the real residential property prices from the BIS. For housing investment we use real fixed capital formation in dwellings (namq_10_an6) from Eurostat. Building permits (sts_cobp_q), real GDP (namq_10_gdp), unemployment (une_rt_q_h), HICP (prc_hicp_midx), and the EONIA (irt_st_m) are also obtained from Eurostat. All series, except the EONIA, are seasonally adjusted. As already mentioned in the main text, the time to obtain a building permit is taken from the World Bank database. And the monetary policy shcks are taken from Altavilla et al. (2019). For these, we use the same aggregation method as for the US data.

C.2 Robustness: Cross-regional heterogeneity in booms and busts

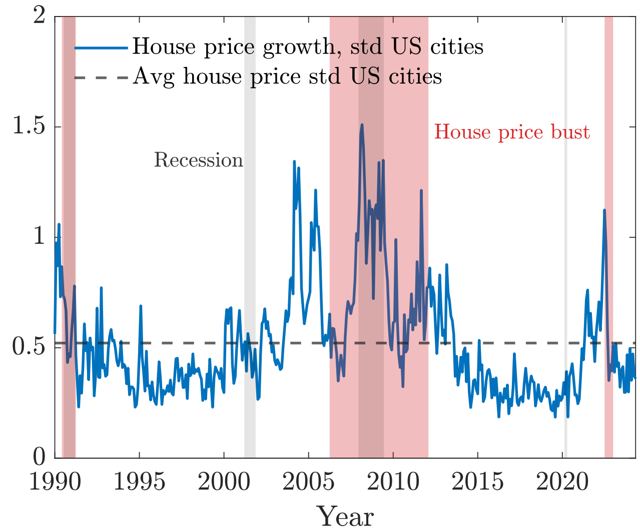
For the Euro Area, the sample covers 2000 to 2019, also on a quarterly basis. The countries included in the Euro Area sample are Austria, Germany, Spain, Finland, France, Ireland, Italy, the Netherlands, and Portugal.

Table C.1: Cross-regional house price growth standard deviation (σ_c) in booms and busts, Euro

	mean	*median*
$\frac{Bust}{Boom}$	**1.30**	**1.27**
σ_c : *Boom*	1.57	1.59
σ_c : *Bust*	2.04	2.02
$p - Val.\ Bust > Boom$	0.002	0.004
Number regions	8	
Obs. boom	55	
Obs. bust	24	
Sample	$2000Q1 - 2019Q3$	

Notes: The Table reports the mean and median, across time, of the estimates of cross-sectional standard deviations of house price growth rates for the Euro Area. The test for *Bust* > *Boom* is based on a one-sided t-test.

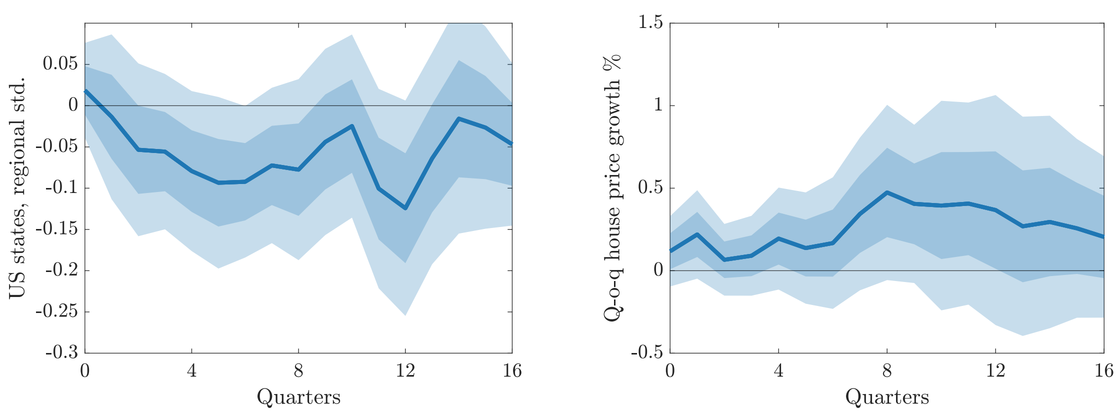
Figure C.1: Regional house price variations in booms and busts



Notes: 3 Month moving average of cross-city std.

Figure C.2: State-level std. house price growth response to Main business cycle shock

(a) *Std. of House price growth* (b) *House price growth*



Notes: Responses to a main business cycle shock (1 std) ; Confidence Intervals: 68% and 95% (Newey-West); 3 month moving average of std. across states of quarter-on-quarter house prices growth (left); Q-o-q house price growth at a federal level (right.)

C.3 Forecast error response to monetary policy shock

We are first interested in whether house price expectations are formed according to rational expectations. To answer this question, we study the response of forecast errors to a monetary policy shock. Forecast errors are interesting for two reasons. First, they show whether house price expectations follow rational expectations. Under rational expectations forecast errors should not respond to the shock. This becomes obvious in our local projections setup. We define forecast errors as as follows:

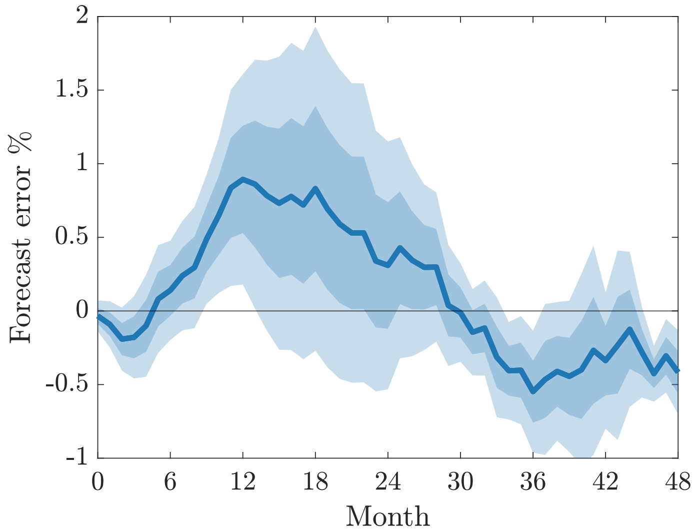
$$fe_t = \Delta q^y_{t+12} - \mathbb{E}^{\mathcal{P}}_t \Delta q^y_{t+12}$$

where Δq^y_{t+12} is the year-on-year percentage change in house prices. $\mathbb{E}^{\mathcal{P}}_t \Delta q^y_{t+12}$ is the expected year-on-year percentage change in 12 month formed today. The local projections we estimate are given by Equation (C.1). At the time the shock hits, in period t, the forecast errors seen on the left-hand side already contain the forecast formed today for 12 month ahead. In other words, the shock is contained in the information set of the agent and should be taken into account. Under rational expectations agents will perfectly use this information and forecast errors should not respond. The second insight we may gain form this exercise is on how house price expectations are formed. Given the response of the forecast errors is non-zero, the dynamics reveal how households' expectations evolve over time. This will enable us to zoom in on a specific belief updating process that explains our observations. Due to data availability we will focus on the US in this exercise. The monetar policy shock is taken from Bauer and Swanson (2023). We use the expectations data from the Michigan Survey on Consumer Sentiment. The frequency is monthly and the sample runs from 2007 to 2019. As controls, we include 6 lags of the forecast error and the monetary policy shock. We estimate the following equation:

$$fe_{t+h} = \alpha^h + \beta^h \epsilon^{MP}_t + x_t + u_{t+h} \qquad h = 0, 1, ..., H \tag{C.1}$$

Figure (C.3) shows the results. The impulse response reveals a non-zero and highly significant response in forecast errors. We can therefore reject rational expectations. On the dynamic behaviour we find that for the first 30 months the forecast error is positive, indicating over-pessimism on the side of the agents. After 30 months the forecast error turns negative, indicating over-optimism. For both cases, over pessimism and over optimism, the IRF is significant at a 95% confidence level. Further, the hump-shaped dynamics in the forecast errors indicate sluggish updating in the expectation formation process. These findings are in line with Adam et al. (2022). Angeletos et al. (2021) document similar dynamics in a SVAR setup for unemployment and inflation in response to an unemployment and inflation shock.

Figure C.3: Forecast error response to a monetary policy shock, US



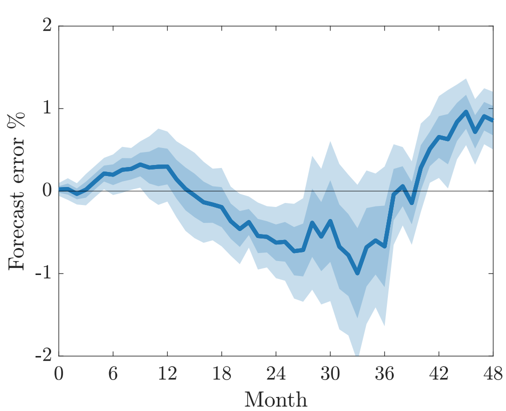
Notes: Response to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% Newey-West).

C.4 Forecast error response to a monetary policy shock in booms and busts

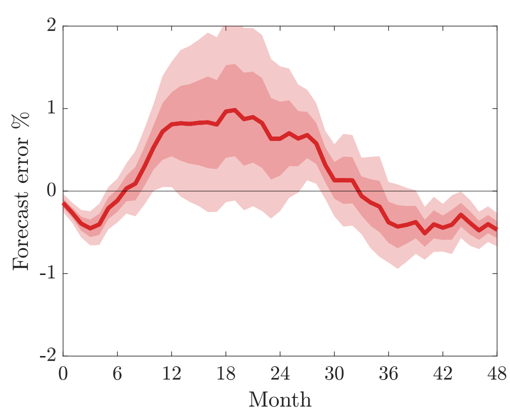
In the following we estimate equation (15) for forecast errors. Figure (C.4) plots the results. We notice, that forecast errors react notably different in booms relative to busts.

Figure C.4: Forecast errors in response to monetary policy shock, boom-bust

(a) Forecasterror: $\Delta q_{t-1} > 0$



(b) Forecasterror: $\Delta q_{t-1} < 0$

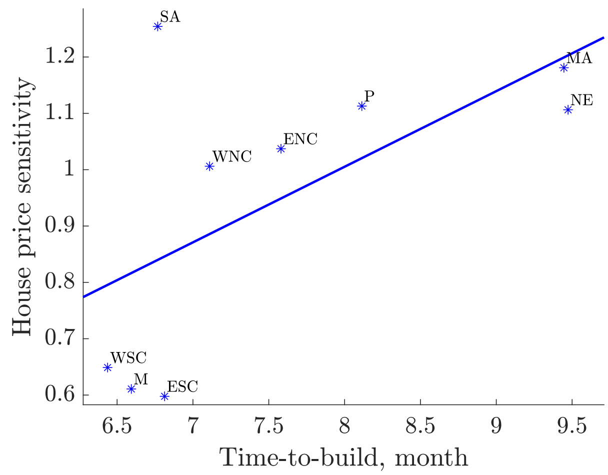


Notes: Responses to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% (Newey-West).

C.5 Time-to-build in the US

The time it takes to obtain a building permit, or generally the time it takes to build a house, is not available for the US on a state level. However, he Census Bureau documents construction times for homes on a census division level. Below we aggregated up the housing sensitivity indicator from Guren et al. (2021) to a census division level and correlate them with the construction time of housing measured from date of authorization to completion. One can see in figure (C.5) that they are clearly positively correlated.

Figure C.5: Correlation of house price sensitivity indicator with time-to-build



Correlation of house price sensitivity indicator from Guren et al. (2021) with time-to-build a house across census divisions. Time-to-build is measured from the date of the authorization until the completion of the building activities.

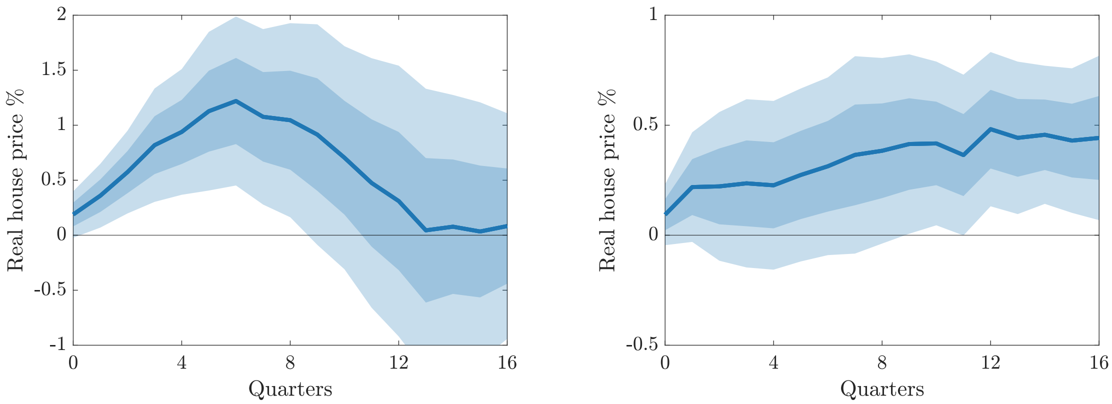
C.6 House price response and housing supply side heterogeneity in the Euro Area

For the Euro Area we rely on the same setup as for the US: We estimate Equation (16) for house prices. The cross-section is taken at a country level. The monetary policy shock is high-frequency identified from overnight interest swaps at a one-year horizon and taken from Altavilla et al. (2019). As an interaction term, we use the days it takes to obtain a building permit in a given country, provided by the World Bank database. The sample runs from 2000 to 2019 and is in quarterly frequency. The countries contained in the sample are Austria, Germany, Spain, Finland, France, Ireland, Italy, the Netherlands, and Portugal. The vector of controls consists of 6 lags of the following variables: The left-hand-side variable, log GDP, log HICP, the EONIA, the shock, and the shock interacted with the interaction term.

Figure (C.6) plots the response of real property prices to an expansionary monetary policy shock. We find that house prices increase (panel (a)) and that they increase more in countries where supply is more inelastic (panel (b)). The IRFs are significant at a 95% confidence level. These results mirror the ones we found for the US.

Figure C.6: House price response to monetary policy shock, Euro

(a) Real property price, mean β^h (b) Real property price, interaction γ^h



Notes: House price response to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

C.7 Model: steady-state values

Table C.2: Steady-state values

Domestic	Value	Foreign	Value	Description
$\xi_{x,ss}$	0.800	$\xi^*_{x,ss}$	0.800	housing productivity shifter
$\xi_{h,ss}$	0.104	$\xi^*_{h,ss}$	0.105	housing preference shifter
c_{ss}	0.976	c^*_{ss}	0.976	consumption
x_{ss}	0.063	x^*_{ss}	0.063	housing investment
h_{ss}	5.506	h^*_{ss}	5.506	housing
y_{ss}	1.040	y^*_{ss}	1.040	output
b_{ss}	0.000	b^*_{ss}	0.000	bond holdings
q_{ss}	0.735	q^*_{ss}	0.727	house price
w_{ss}	1.000	w^*_{ss}	1.000	wage

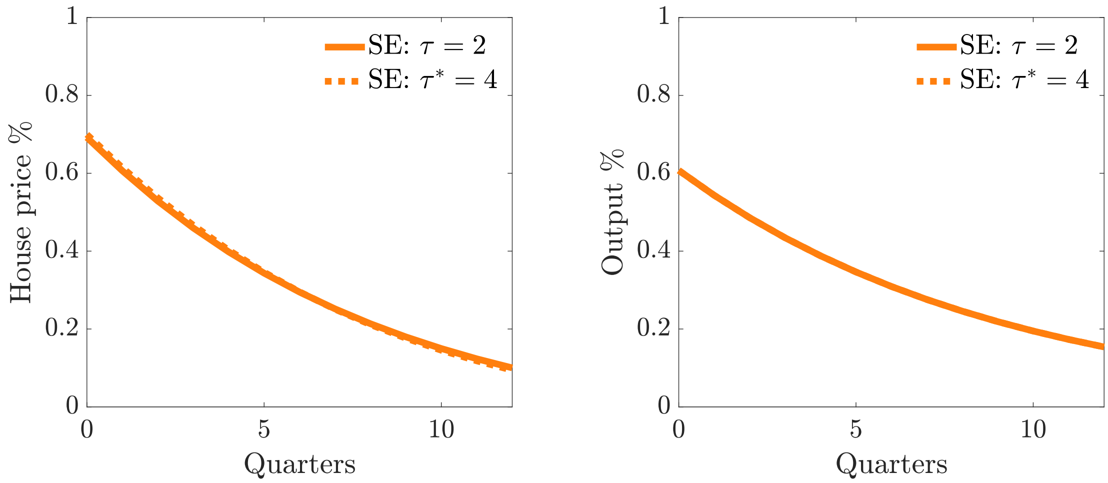
Notes: All steady-states except the house price are symmetric across countries. Time-to-build and the steady state housing preference shifter are not symmetric across countries. All elements of ξ_{ss} not explicitly mentioned assume the value 1.

D. Quantitative model

D.1 Further results

Figure D.1: Regional house price and output response under RE

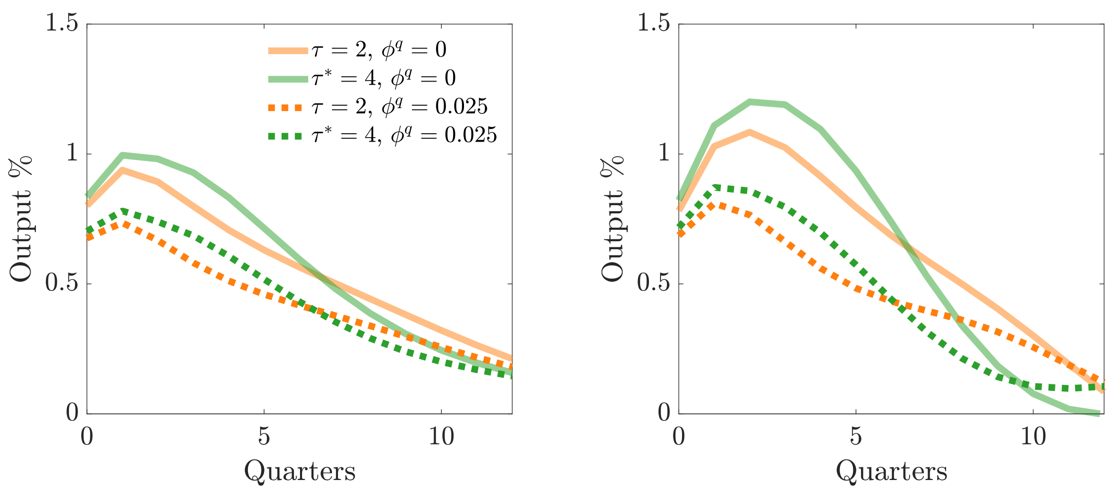
(a) House prices (b) Output



Notes: Responses to expansionary MP shock (25 bp).

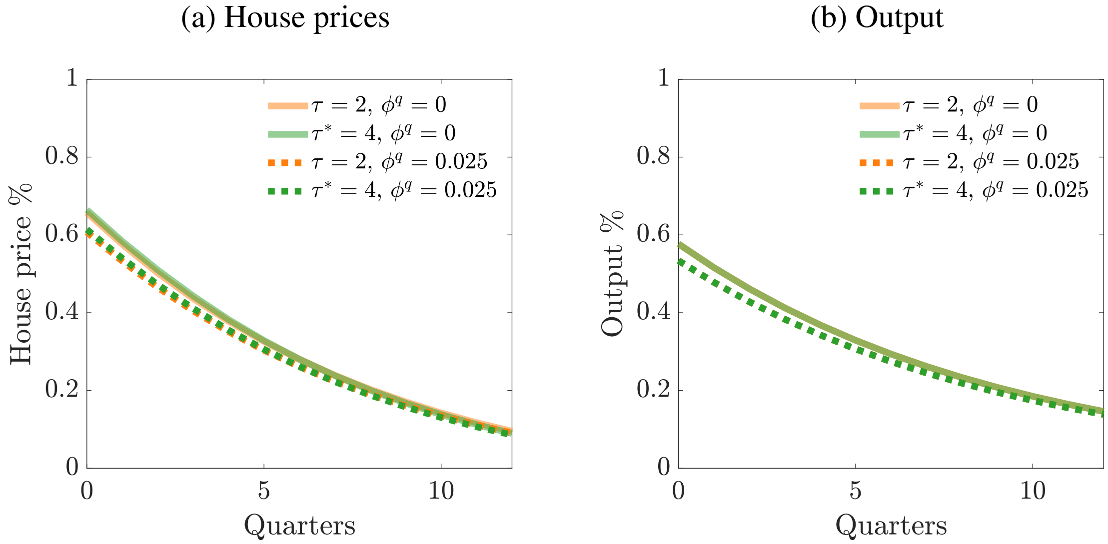
Figure D.2: House price targeting: output response in booms and busts

(a) Boom (b) Bust



Notes: Responses to expansionary productivity shock (100 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Figure D.3: House price targeting: house price and output response under RE



Notes: Responses to expansionary productivity shock (100 bp).

D.2 Loss function

We assume that the policy-maker's objective is the Utilitarian average welfare in the economy, evaluated under rational expectations. This definition leaves open the question of what information the policy-maker possesses. Conditional welfare, $\mathbb{W}_{-1}$, is the average expected welfare in the union, conditional on (i) the economy being in steady state in the current and all past periods $t \leq 0$, and on (ii) the policy-maker knowing that in $t = 0$ a set of shocks $(\xi_t)_{t\geq 0} \in (\mathbb{R}^n)^\infty$ will realize with a Gaussian probability distribution but not knowing about the exact realization of the shocks. The experiment here is that the policy-maker decides at the end of the period $t = -1$ about her instruments, maximizing the average utility in the union under the expectation that a set of surprise shocks might materialize in the next period according to a known distribution—let $\mathbb{E}_{-1}$ denote the operator drawing the expectation with respect to the rational expectations measure and conditional on (i) and (ii). Specifically,

$$\mathbb{W}_{-1} := \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t [\gamma u_t + (1-\gamma) u_t^*]$$

$$\mathbb{W} := \mathbb{E}[\gamma u_t + (1-\gamma) u_t^*]$$

where u_t, u_t^* are the utility functions of the home and foreign household, respectively.[48] We approximate both criteria to second order around the non-stochastic steady state: Assuming that the allocation in the non-stochastic steady state is symmetric ($c_{ss} = c_{ss}^*$, $n_{ss} = n_{ss}^*$, $h_{ss} = h_{ss}^*$ and so on), we have that

$$\begin{aligned}\frac{\mathbb{W}_{-1}}{c_{ss}^{1-\sigma}} = -\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t \cdot \Bigg[& \frac{\gamma}{2}\left(\begin{matrix}\kappa\frac{n_{ss}}{c_{ss}}\pi_{H,t}^2 + \sigma\widehat{c}_t^2 + \nu\frac{h_{ss}^{1-\nu}}{c_{ss}^{1-\sigma}}\widehat{h}_t^2 + \varphi\chi\frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}}\widehat{n}_t^2 + \frac{x_{ss}}{c_{ss}}(1-\eta)\widehat{x}_t^2 + \frac{n_{ss}}{c_{ss}}\psi n_{ss}\widehat{b}_t^2 \\ -2\frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}}\widehat{\xi}_{a,t}\widehat{n}_t\end{matrix}\right) \\ & + \frac{1-\gamma}{2}\left(\begin{matrix}\kappa\frac{n_{ss}}{c_{ss}}\pi_{F,t}^2 + \sigma(\widehat{c}_t^*)^2 + \nu\frac{h_{ss}^{1-\nu}}{c_{ss}^{1-\sigma}}(\widehat{h}_t^*)^2 + \varphi\chi\frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}}(\widehat{n}_t^*)^2 + \frac{x_{ss}}{c_{ss}}(1-\eta^*)(\widehat{x}_t^*)^2 + \frac{n_{ss}}{c_{ss}}\psi n_{ss}(\widehat{b}_t^*)^2 \\ -2\frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}}\widehat{\xi}_{a,t}\widehat{n}_t^*\end{matrix}\right) \\ & + \frac{1}{2}\frac{1}{\varsigma^2}\upsilon \cdot \widehat{s}_t^2 \Bigg] + O(3) + \text{t.i.p.}\end{aligned}$$

where $\upsilon = (1-\lambda)(1+\lambda+[2-\lambda]\varsigma)+\lambda(1+[2+\varsigma]\lambda)+\lambda^*(2-\lambda^*+[1+\lambda^*]\varsigma)+(1-\lambda^*)(1+(2+\varsigma)[1-\lambda^*])$, $O(3)$ are terms of third or higher order and “t.i.p.” stands for “terms independent of policy”, i.e. variables that are beyond the policy-maker’s control and thus irrelevant for the selection of policy.

[48]Note that it holds that $\mathbb{E}\mathbb{W}_{-1} = \mathbb{W}$.