1. Introduction

1.1 Economic Intuition

As detailed by this research paper, copper-gold ratio can serve as an indicator of the market's appetite for risk assets versus the perceived safety of Treasuries. In particular, it can serve as a leading indicator for the 10-year US treasury yield. The two economic variables have close correlations and tend to move in tandem. The relationship is not perfect however, and in certain episodes the two diverged with each other. But in those occasions the 10-year treasury yield tends to eventually "capitulate" and follow suit of copper-gold ratio.

For example, on June 7, 2010, copper-gold raito made a relative low of 0.0022 with the 10-year Treasury yield at around 3%. Copper-gold ratio started to bounce back afterwards while TY yield kept falling to a regional low of around 2.4%. TY yield then finally "capitulated" and started climing up until a regional top of 3.5%.

Another example would be the episode during COVID-19 as shown on the bottom right of Figure 2. Copper-gold ratio went up violently during late 2020 and early 2021 in response to the COVID commodity supply constraints in LatAm countries. Treasury yield in contrast stayed relatively flat. However, at late 2021 Treasury yield again "capitulated" and followed suit of copper-gold and made a top at around 3%.

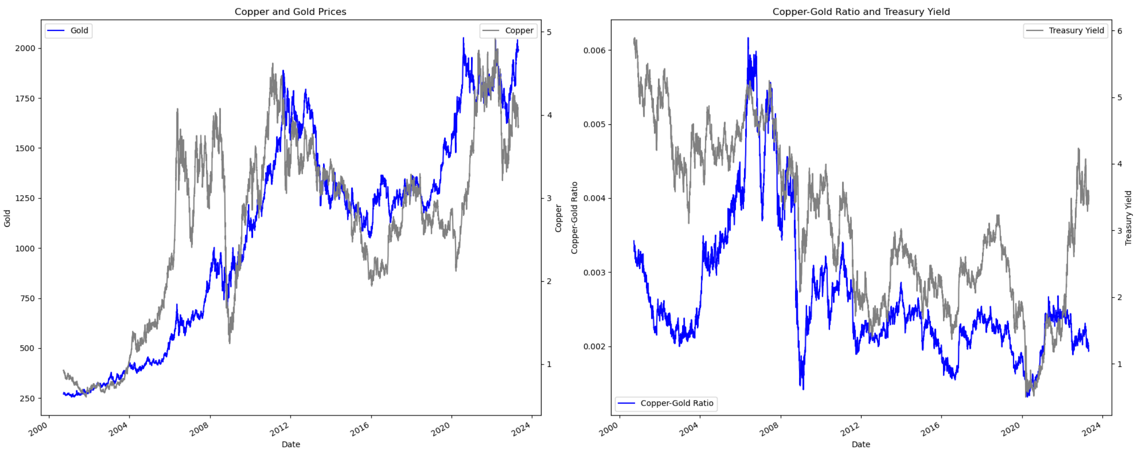

Fig. 1 Copper, Gold and Treasury Yield

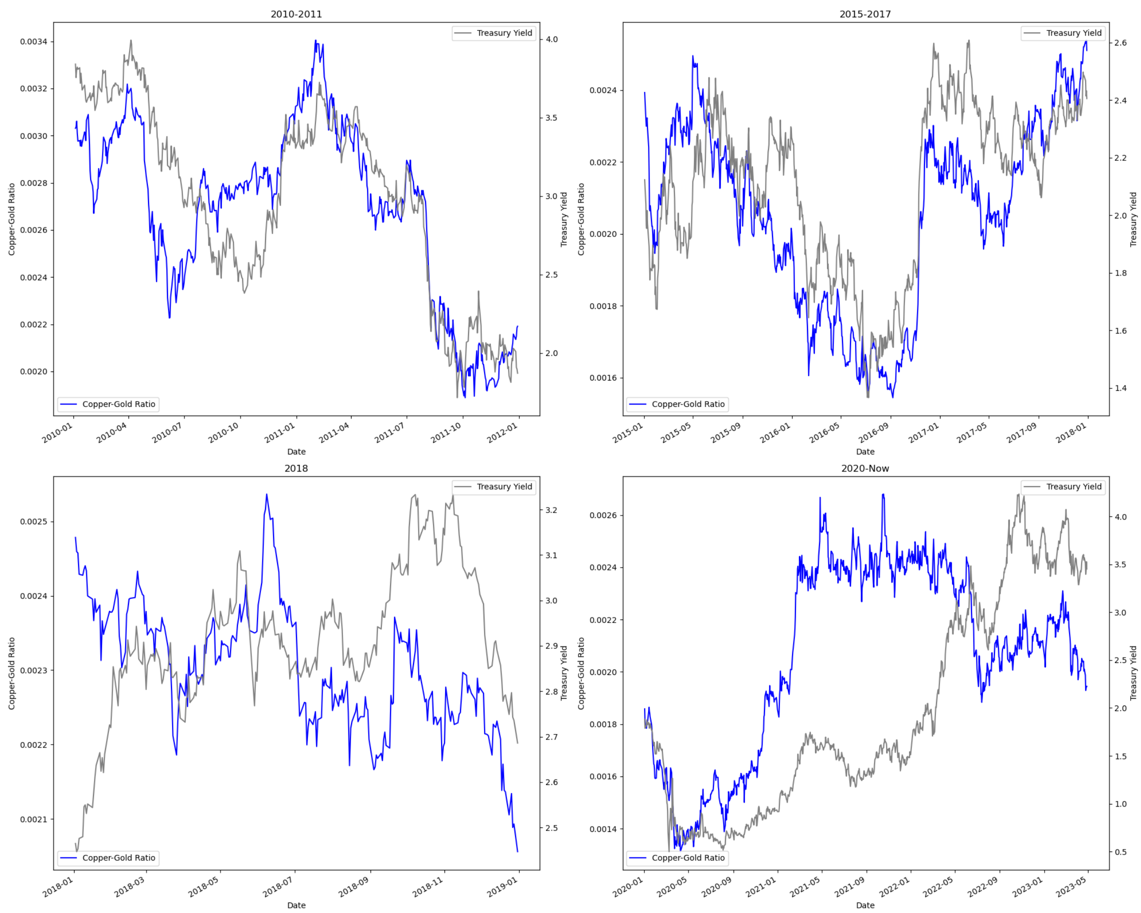

Fig. 2 Periods of Divergences

With this relationship in mind, the strategy tries to capitalize on the timing of Treasuries according to the divergence signal and also hedging with gold and copper appropriately.

1.2 Kalman Filter

We use Kalman Filters to model the relationship between copper-gold ratio and treasury yield. We assume a linear form of relationship as we can hardly trade nonlinear ones. Let y_t be the copper-gold ratio, x_t be the treasury yield. We try to model the spread between the two as a residual process ϵ_t by the following:

$$y_t = \beta_t x_t + \epsilon_t \tag{1}$$

where β_t is the hedge ratio. As can be seen from the above charts, β_t is likely to change over time. We overlay a simple state transition model on it by having:

$$\beta_t = a_t \beta_{t-1} + g_t w_t \tag{2}$$

where w_t is a noise process inherent in the state estimation. For simplicity, we assume the state estimation noise is uncorrelated with the spread:

$$\epsilon_t \sim F_1(\mu_1, \sigma_1^2) \tag{3}$$

$$w_t \sim F_2(\mu_2, \sigma_2^2) \tag{4}$$

$$\mathbb{E}(\epsilon_t w_t) = 0 \tag{5}$$

$$\mathbb{E}(\epsilon_t \epsilon_s) = \delta(t-s) \tag{6}$$

$$\mathbb{E}(w_t w_s) = \delta(t - s) \tag{7}$$

where $\delta(\cdot)$ is the Dirac delta function.

Equations (1) - (7) give us a simple one dimensional Kalman filter, where β_t is the hidden state and y_t the measurement. Given a series of observations and measurements (x_t, y_t), the goal is to find an optimal state estimation despite the inherent noises (ϵ_t, w_t):

$$\min_{\hat{\beta}_t} J = \mathbb{E}[(\hat{\beta}_t - \beta_t)^T(\hat{\beta}_t - \beta_t)] \tag{8}$$

with state estimate transition and output equations:

$$\hat{\beta}_{t|t-1} = \mathbb{E}[a_t\hat{\beta}_{t-1} + g_t w_t] = a_t\hat{\beta}_{t-1} \tag{9}$$

$$\hat{y}_t = \mathbb{E}[x_t\hat{\beta}_{t|t-1} + \epsilon_t] = x_t\hat{\beta}_{t|t-1} \tag{10}$$

estimation correction based on the new measurement y_t:

$$\hat{\beta}_t = \hat{\beta}_{t|t-1} + k_t(y_t - \hat{y}_t) \tag{11}$$

where k_t is the one dimensional Kalman gain matrix.

The solution to problem (8) - (11) can be found by differentiating the error function w.r.t the Kalman gain matrix. This is given by the following:

$$k_t = \frac{x_t p_{t|t-1}}{x_t^2 p_{t|t-1} + \sigma_1^2} \tag{12}$$

$$p_t = (1 - k_t x_t)p_{t|t-1} \tag{13}$$

$$p_{t+1|t} = a_t^2 p_t + g_t^2\sigma_2^2 \tag{14}$$

where $p_{t|t-1} = \mathbb{E}[(\hat{\beta}_{t|t-1} - \beta_t)(\hat{\beta}_{t|t-1} - \beta_t)^T]$ the error variance of the prior state estimation and $p_t = \mathbb{E}[(\hat{\beta}_t - \beta_t)(\hat{\beta}_t - \beta_t)^T]$ the posterior variance.

In a nutshell, equations (12) - (14) give the update rules to recursively compute the optimal state estimation as more data come in.

1.3 Preliminary Analysis

All data used in the following analysis come from yahoo.finance. We fit the above derived Kalman Filters to the copper-gold-treasury relationship. The following chart shows the fitted parameters as well as the predicted measurements. The fit result looks decent. Long term R^2 is approaching 90%, although there were certain deviation espisodes especially after COVID:

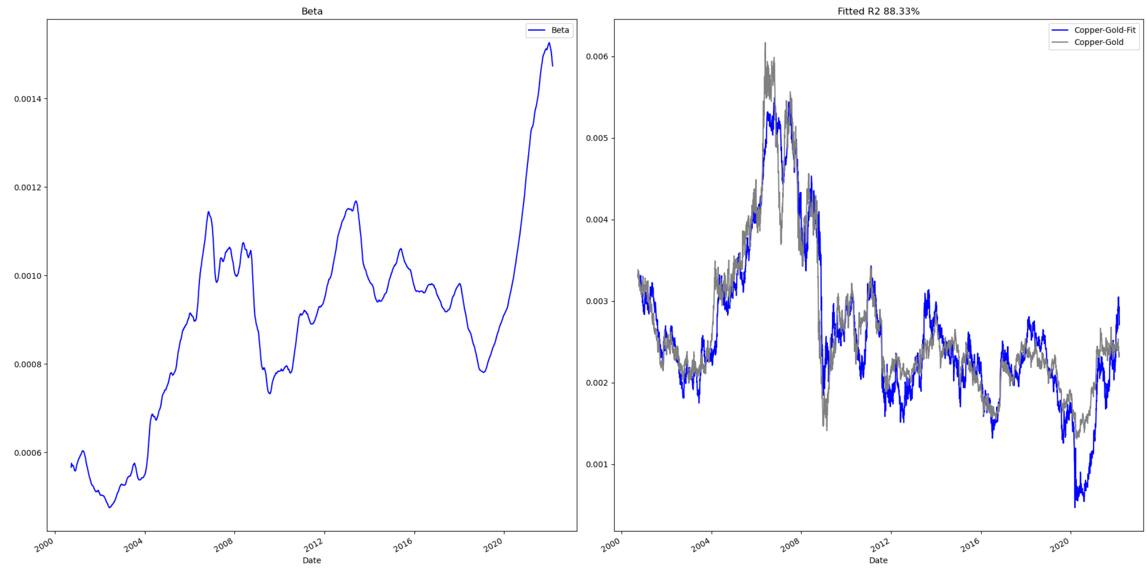

Fig. 3 Fitted Beta and Measurement

We then extract the residual time series and run ADF test on it to check whether it is stationary / mean reverting. As can be seen from below, ADF test result over the long term is highly statistically significant, with p-value well below 0.01 or even approaching zero. This is credited to the more stable relation in the early years from 2000 to around 2021. After COVID there is a more pronounced trend in the residual process, suggesting the fitted linear relationship starts to be unstable. The ADF test covering the years from 2021 to 2023 is actually not statistically significant, with p-value higher than 0.1. There's a lot more rates volatility during this period. We may see a bit of deterioration in terms of strategy performance.

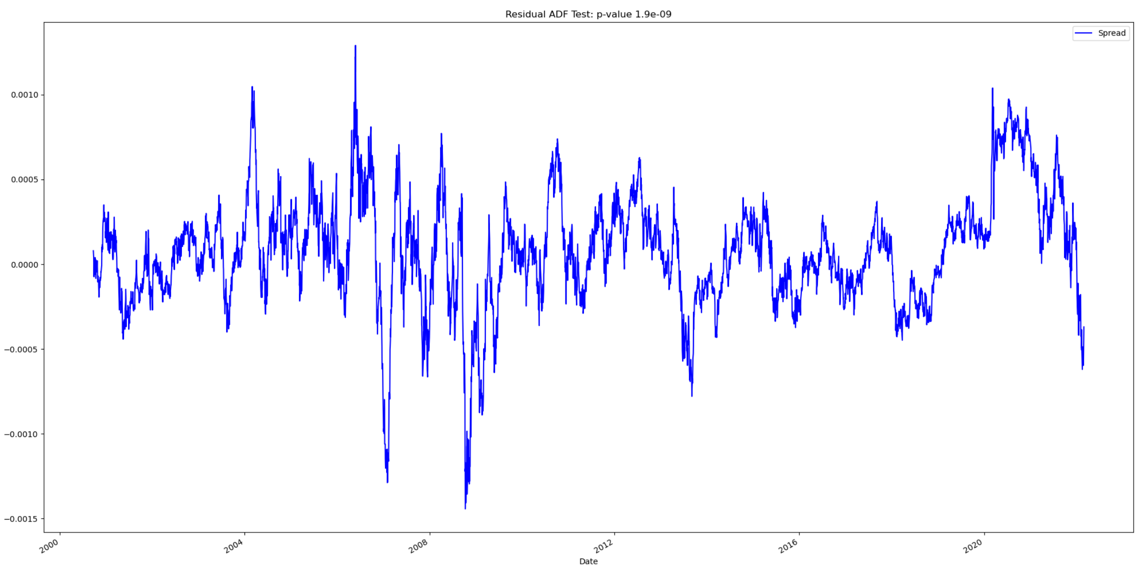

Fig. 4 Residual and ADF Test

2. Backtest

2.1 Parameters

From the above stationary residual / spread we then formulate a trading strategy by going long / short the spread when it deviates too much from the mean and closing the trade when it comes back. Specifically,

$$z = coppergold - \hat{\beta} yield$$

Ideally, to long 1 unit of z means to long 1 unit of copper-gold ratio and short $\hat{\beta}$ unit of treasury yield. But of course the ratio or yield itself is not tradable, so we will have to find some proxy instruments. We use TY futures as the proxy instrument for trading treasury yield. This means we will short 1 unit of TY futures when the signal tells us to long 1 unit of yield. We will also hedge the trade with copper / gold futures but gain we cannot trade the ratio directly so the final hedge units to use will be a set of preset constants / parameters. This is an imperfect but quick and easy approximation.

Furthermore, we define "deviates too much from the mean" by the spread deviating n_1 standard deviation from its m day moving average and "comes back" by it falling back into its m day n_2 standard deviation envelope.

This leaves a couple of hyperparameters to tune for strategy performance: m, n_1, n_2 and hedge ratios. Ideally, we should pick a parameter set that sits in the center of a "plateau", a region where nearby parameter set also gives similar high sharpe ratio. In this case, it's a bit difficult to visualize a multi-dimensional dataset. We just picked the parameter set that maximizes in-sample sharpe ratio.

We split the sample data into in-sample and out-sample portions. The in-sample dataset covers data from 2000 to 2022. Out-sample dataset covers the rest from 2022 onwards until 2023-05-01. The following table summarizes the optimal in sample parameters to use:

m	n1	n2	Hedge Ratio		
			Copper	Gold	TY
10	1.7	1.5	2	3	6

Fig. 5 Optimal In-Sample Parameters

2.2 Performance

We apply the above optimal parameters to backtest the entire sample history. The following charts summarize the strategy performance across time[1], with the blue line portion being the out-sample period. We have adjusted leverage for the strategy to target 10% annual volatility.

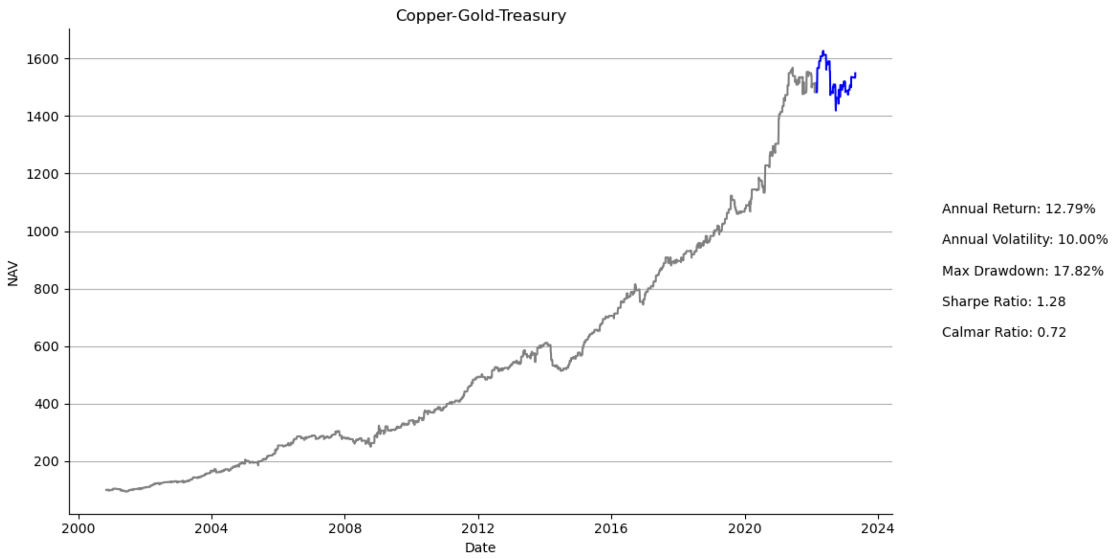

Fig. 6 Full History NAV

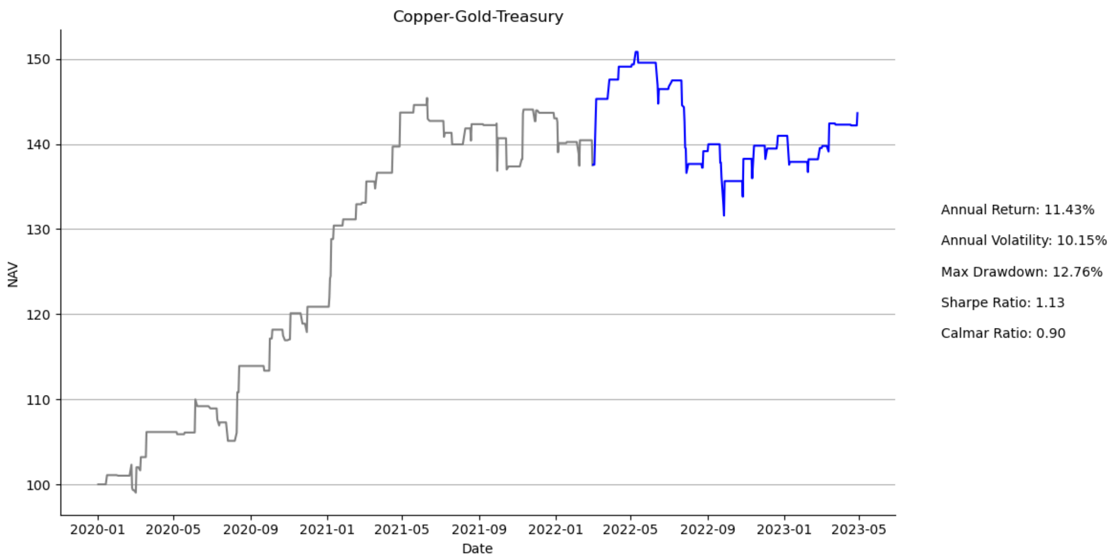

Fig. 7 NAV Since 2020

Note that as we have observed from Figure 3 / 4, the fitted linear relationship starts to become unstable after late 2020 / early 2021. This has a pronounced negatvie impact on the strategy performance as it has been staying flat ever since mid 2021. Hopefully as rates volatility continues to normalize the linear relationship will become more stable. Only then can we see the strategy starting to pick up momentum again.

3. Conclusion

In this article we investigated a potential systematic trading strategy that is based on the spread between the copper-gold ratio and the 10 year treasury yield. We used Kalman Filters to help establish the spread / linear relations between the two economic variables. Trading signals originate from the occassions when the spread deviates too much from the mean.

We looked at the backtested performance over the past 20+ years. The strategy performed well during the 21 years from 2000 to 2021, registering a 1.3+ Sharpe Ratio.

After mid-2021, the strategy started to become stagnant and remained flattish. However, as market volatility continues to normalize, we should hopefully see the strategy starting to pick up momentum again.

The strategy also has low correlations with common asset classes like SPX and TY. It can serve as a diversification source for a traditional 60/40 portfolio.

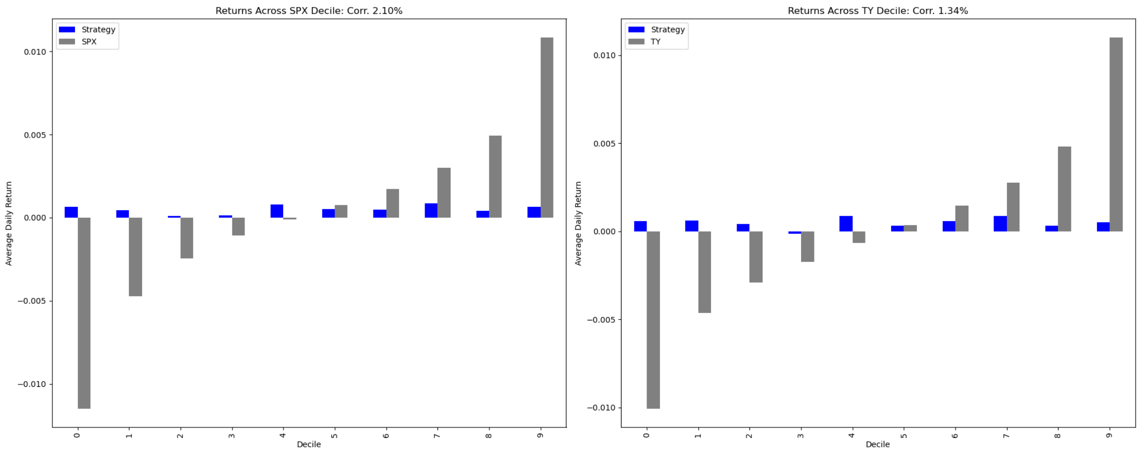

Fig. 8 Correlations with SPX and TY

1. We assume 100% margin for futures trading and performance numbers are before cost.