Worked Example 1.3 shows how to calculate density, and how to use density in the calculation of volume.

WORKED EXAMPLE 1.3

Ice cubes float in a glass of water because solid water is less dense than liquid water. (a) Calculate the density of ice given that, at 0°C, a cube that is 2.0 cm on each side has a mass of 7.36 g, and (b) determine the volume occupied by 23 g of ice at 0°C.

Strategy (a) Determine density by dividing mass by volume (Equation 1.3), and (b) use the calculated density to determine the volume occupied by the given mass.

Setup (a) We are given the mass of the ice cube, but we must calculate its volume from the dimensions given. The volume of the ice cube is $(2.0\text{ cm})^3$, or 8.0 cm^3. (b) Rearranging Equation 1.3 to solve for volume gives $V = m/d$.

Solution

(a) $d = \dfrac{7.36\text{ g}}{8.0\text{ cm}^3} = 0.92\text{ g/cm}^3$ or 0.92 g/mL (b) $V = \dfrac{23\text{ g}}{0.92\text{ g/cm}^3} = 25\text{ cm}^3$ or 25 mL

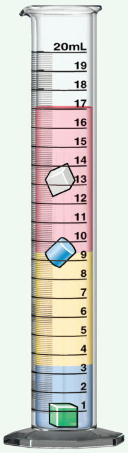
Think About It

For a sample with a density *less* than 1 g/cm^3, the number of cubic centimeters should be *greater* than the number of grams. In this case, 25 (cm^3) > 23 (g).

Practice Problem ATTEMPT Given that 20.0 mL of mercury has a mass of 272 g, calculate (a) the density of mercury and (b) the volume of 748 g of mercury.

Practice Problem BUILD Calculate (a) the density of a solid substance if a cube measuring 2.33 cm on one side has a mass of 117 g and (b) the mass of a cube of the same substance measuring 7.41 cm on one side.

Practice Problem CONCEPTUALIZE Using the picture of the graduated cylinder and its contents, arrange the following in order of increasing density: blue liquid, pink liquid, yellow liquid, grey solid, blue solid, green solid.



Environmental Aspects

Global Climate Change

Those who describe themselves as "skeptical" about climate change sometimes posit that global temperature change is normal, and that any observed increase in temperature is simply the result of natural processes—outside the control of humans. However, there is an enormous body of climate research that clearly demonstrates otherwise. One line of inquiry that has helped to established the connection between human activity and so-called "global warming" involves what is known as *vertical structure of temperature*.

Earth's atmosphere is divided into a series of altitudinal layers: the troposphere (ground-level to 8–14.5 km), the stratosphere (top of the troposphere–50 km), the mesosphere (50–80 km), the thermosphere (80–700 km), and the exosphere (700–10,000 km). The troposphere is where we live, where weather events occur, and where nearly all human activity takes place. When we burn fossil fuels, we increase the amount of CO_2 in the *troposphere*.

In 1988, atmospheric scientist V. Ramanathan, now of the Scripps Institution of Oceanography at the University of California, San Diego, proposed that global temperature change caused by the anthropogenic increase in atmospheric CO_2 could be readily distinguished from that caused by *natural* events, such as increased solar activity. Global temperature increase caused by the Sun, he reasoned, would occur in both the troposphere *and* the stratosphere. Conversely, changes caused by the enhanced greenhouse effect (the result of increased atmospheric CO_2 concentration) would cause warming of the troposphere; but *cooling* of the stratosphere—because more of the heat radiating from Earth's surface would be trapped by greenhouse gases in the troposphere, thus never reaching the stratosphere. Indeed, temperature monitoring over several decades has demonstrated an *increase* in tropospheric temperature, and a *decrease* in stratospheric temperature. This is one of the observations that climate scientists refer to as a *human fingerprint* on global climate change.

Section 1.2 Review

Scientific Measurement

1.2.1 The lowest body temperature ever recorded for a human (who survived) was 13.7°C when a skier spent over an hour submerged after falling headfirst through the ice on a frozen river. Convert this body temperature to degrees Fahrenheit.

(a) 24.7°F
(b) 45.7°F
(c) 56.7°F
(d) 82.3°F
(e) −10.2°F

1.2.2 What is the density of an object that has a volume of 69.5 cm^3 and a mass of 121 g?

(a) 0.573 g/cm^3
(b) 1.74 g/cm^3
(c) 670 g/cm^3
(d) 53.8 g/cm^3
(e) 14.6 g/cm^3

1.2.3 Which correctly represents the relative magnitudes of a single kelvin, a degree Celsius, and a degree Fahrenheit?

(a) 1 kelvin = 1°C > 1°F
(b) 1 kelvin = 1°C = 1°F
(c) 1 kelvin < 1°C < 1°F
(d) 1 kelvin = 1°C < 1°F
(e) 1 kelvin > 1°C > 1°F

1.2.4 Given that the density of gold is 19.3 g/cm^3, calculate the volume (in cm^3) of a gold nugget with a mass of 115 g.

(a) 3.23 cm^3
(b) 5.96 cm^3
(c) 115 cm^3
(d) 0.310 cm^3
(e) 13.3 cm^3

1.3 UNCERTAINTY IN MEASUREMENT

Science makes use of two types of numbers: exact and inexact. *Exact* numbers include numbers with defined values, such as 2.54 in the definition 1 inch (in) = 2.54 cm, 1000 in the definition 1 kg = 1000 g, and 12 in the definition 1 dozen = 12 objects. (The number 1 in each of these definitions is also an exact number.) Exact numbers also include those that are obtained by counting. Numbers measured by any method other than counting are *inexact.*

Measured numbers are inexact because of the measuring devices that are used, the individuals who use them, or both. For example, a ruler that is poorly calibrated will result in measurements that are in error—no matter how carefully it is used. Another ruler may be calibrated properly but have insufficient resolution for the necessary measurement. Finally, whether or not an instrument is properly calibrated or has sufficient resolution, there are unavoidable differences in how different people see and interpret measurements.

Significant Figures

An inexact number must be reported in such a way as to indicate the uncertainty in its value. This is done using significant figures. ***Significant figures*** are the *meaningful digits* in a reported number. Consider the measurement of the USB plug in Figure 1.5 using the ruler above it. The plug's width is slightly greater than 1 cm. We may record the width as 1.2 cm, but because there are no gradations between 1 and 2 cm on this ruler, we are *estimating* the second digit. Although we are certain about the 1 in 1.2, we are *not* certain about the 2. The last digit in a measured number is referred to as the *uncertain digit;* and the uncertainty associated with a measured number is generally

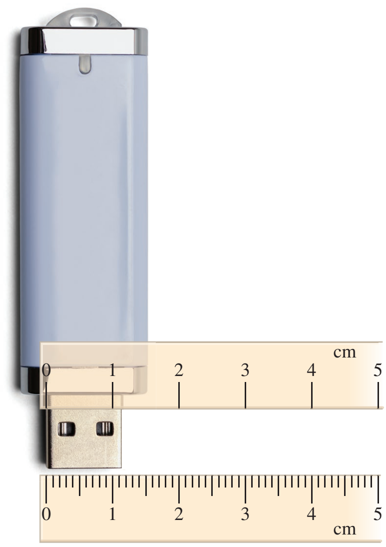


Figure 1.5 The width we report for the USB plug depends on which ruler we use to measure it.
Mega Pixel/Shutterstock

considered to be ± 1 in the place of the last digit. Thus, when we report the width of the USB plug to be 1.2 cm, we are implying that its width is 1.2 ± 0.1 cm—and that its actual width may be as low as 1.1 cm or as high as 1.3 cm. Each of the digits in a measured number, including the uncertain digit, is a significant figure. The reported width of the USB plug, 1.2 cm, contains *two* significant figures.

A ruler with millimeter gradations would enable us to be certain about the second digit in this measurement and to estimate a third digit. Now consider the measurement of the USB plug using the ruler below it. We may record the width as 1.15 cm. Again, we estimate one digit beyond those we can read. The reported width of 1.15 cm contains three significant figures. Reporting the width as 1.15 cm implies that the width is 1.15 ± 0.01 cm.

Student Annotation: It is important not to imply greater certainty in a measured number than is realistic. For example, it would be inappropriate to report the width of the USB plug in Figure 1.5 as 1.1500 cm, because this would imply an uncertainty of ± 0.0001 cm.

The number of significant figures in any number can be determined using the following guidelines:

1. Any digit that is not zero is significant (112.1 has four significant figures).
2. Zeros located between nonzero digits are significant (305 has three significant figures, and 50.08 has four significant figures).
3. Zeros to the left of the first nonzero digit are not significant (0.0023 has two significant figures, and 0.000001 has one significant figure).
4. Zeros to the right of the last nonzero digit are significant if the number contains a decimal point (1.200 has four significant figures).
5. Zeros to the right of the last nonzero digit in a number that does not contain a decimal point may or may not be significant (100 may have one, two, or three significant figures—it is impossible to tell without additional information). To avoid ambiguity in such cases, it is best to express such numbers using scientific notation. If the intended number of significant figures is one, the number is written as 1×10^2; if the intended number of significant figures is two, the number is written as 1.0×10^2; and if the intended number of significant figures is three, the number is written as 1.00×10^2.

Student Annotation: Appendix 1 reviews scientific notation.

Student Hot Spot

Student data indicate you may struggle with significant figures. Access your eBook to view additional Learning Resources on this topic.

Worked Example 1.4 lets you practice determining the number of significant figures in a number.

WORKED EXAMPLE 1.4

Determine the number of significant figures in the following measurements: (a) 443 cm, (b) 15.03 g, (c) 0.0356 kg, (d) 3.000×10^{-7} L, (e) 50 mL, (f) 0.9550 m.

Strategy All nonzero digits are significant, so the goal will be to determine which of the zeros is significant.

Setup Zeros are significant if they appear between nonzero digits or if they appear after a nonzero digit in a number that contains a decimal point. Zeros may or may not be significant if they appear to the right of the last nonzero digit in a number that does not contain a decimal point.

Solution (a) 3; (b) 4; (c) 3; (d) 4; (e) 1 or 2, an ambiguous case; (f) 4.

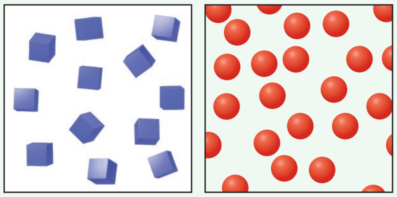
Think About It

Be sure that you have identified zeros correctly as either significant or not significant. They are significant in (b) and (d); they are not significant in (c); it is not possible to tell in (e); and the number in (f) contains one zero that is significant, and one that is not.

Practice Problem ATTEMPT Determine the number of significant figures in the following measurements: (a) 1129 m, (b) 0.0003 kg, (c) 1.094 cm, (d) 3.5×10^{12} atoms, (e) 150 mL, (f) 9.550 km.

Practice Problem BUILD Using scientific notation, express the number *one million* to (a) two significant figures, (b) four significant figures, (c) seven significant figures; and using decimal notation, express the number *one tenth* to (d) three significant figures, (e) five significant figures, and (f) one significant figure.

Practice Problem CONCEPTUALIZE Report the number of colored objects contained within each square and, in each case, indicate the number of significant figures in the number you report.



(i) (ii)

Calculations with Measured Numbers

Because we often use one or more measured numbers to calculate a desired result, a second set of guidelines specifies how to handle significant figures in calculations.

1. In addition and subtraction, the answer cannot have more digits to the right of the decimal point than any of the original numbers. For example:

$$\begin{array}{rl} 102.50 & \longleftarrow \text{two digits after the decimal point} \\ \underline{+\ 0.231} & \longleftarrow \text{three digits after the decimal point} \\ 102.731 & \longleftarrow \text{round to } 102.73 \end{array}$$

$$\begin{array}{rl} 143.29 & \longleftarrow \text{two digits after the decimal point} \\ \underline{-20.1} & \longleftarrow \text{one digit after the decimal point} \\ 123.19 & \longleftarrow \text{round to } 123.2 \end{array}$$

   The rounding procedure works as follows. Suppose we want to round 102.13 and 54.86 each to one digit to the right of the decimal point. To begin, we look at the digit(s) that will be dropped. If the leftmost digit to be dropped is less than 5, as in 102.13, we *round down* (to 102.1), meaning that we simply drop the digit(s). If the leftmost digit to be dropped is equal to or greater than 5, as in 54.86, we *round up* (to 54.9), meaning that we add 1 to the preceding digit.
2. In multiplication and division, the number of significant figures in the final product or quotient is determined by the original number that has the smallest number of significant figures. The following examples illustrate this rule:

$1.4 \times 8.011 = 11.2154$ $\longleftarrow$ round to 11
(limited by 1.4 to *two* significant figures)

$11.57/305.88 = 0.037825290964$ $\longleftarrow$ round to 0.03783
(limited by 11.57 to *four* significant figures)

3. *Exact numbers* can be considered to have an infinite number of significant figures and do not limit the number of significant figures in a calculated result. For example, a penny minted after 1982 has a mass of 2.5 g. If we have three such pennies, the total mass is

$$\mathbf{3 \times 2.5\ g = 7.5\ g}$$

   The answer should *not* be rounded to one significant figure because 3, having been determined by counting, is an *exact* number.
4. In calculations with multiple steps, rounding the result of each step can result in "rounding error." Consider the following two-step calculation:

   First step: $A \times B = C$
   Second step: $C \times D = E$

   Suppose that $A = 3.66$, $B = 8.45$, and $D = 2.11$. The value of E depends on whether we round off C prior to using it in the second step of the calculation.

Method 1	**Method 2**
$C = 3.66 \times 8.45 = 30.9$	$C = 3.66 \times 8.45 = 30.93$
$E = 30.9 \times 2.11 = 65.2$	$E = 30.93 \times 2.11 = 65.3$

   In general, it is best to retain at least one extra digit until the end of a multistep calculation, as shown by method 2, to minimize rounding error.

Worked Examples 1.5 and 1.6 show how significant figures are handled in arithmetic operations.

Student Hot Spot
Student data indicate you may struggle with determining significant figures in calculations. Log in to Connect to view additional Learning Resources on this topic.

WORKED EXAMPLE 1.5

Perform the following arithmetic operations and report the result to the proper number of significant figures: (a) 317.5 mL + 0.675 mL, (b) 47.80 L − 2.075 L, (c) 13.5 g ÷ 45.18 L, (d) 6.25 cm × 1.175 cm, (e) 5.46×10^2 g + 4.991×10^3 g.

Strategy Apply the rules for significant figures in calculations, and round each answer to the appropriate number of digits.

Setup (a) The answer will contain one digit to the right of the decimal point to match 317.5, which has the fewest digits to the right of the decimal point. (b) The answer will contain two digits to the right of the decimal point to match 47.80. (c) The answer will contain three significant figures to match 13.5, which has the fewest number of significant figures in the calculation. (d) The answer will contain three significant figures to match 6.25. (e) To add numbers expressed in scientific notation, first write both numbers to the same power of 10. That is, $4.991 \times 10^3 = 49.91 \times 10^2$, so the answer will contain two digits to the right of the decimal point (when multiplied by 10^2) to match both 5.46 and 49.91.

Solution

(a)
$$\begin{array}{r} 317.5\text{ mL} \\ +\ 0.675\text{ mL} \\ \hline 318.175\text{ mL} \end{array} \longleftarrow \text{round to 318.2 mL}$$

(b)
$$\begin{array}{r} 47.80\text{ L} \\ -\ 2.075\text{ L} \\ \hline 45.725\text{ L} \end{array} \longleftarrow \text{round to 45.73 L}$$

(c) $$\frac{13.5\text{ g}}{45.18\text{ L}} = 0.298804781\text{ g/L} \longleftarrow \text{round to 0.299 g/L}$$

(d) $$6.25\text{ cm} \times 1.175\text{ cm} = 7.34375\text{ cm}^2 \longleftarrow \text{round to 7.34 cm}^2$$

(e)
$$\begin{array}{r} 5.46 \times 10^2\text{ g} \\ +\ 49.91 \times 10^2\text{ g} \\ \hline 55.37 \times 10^2\text{ g} \end{array} = 5.537 \times 10^3\text{ g}$$

Think About It

It may look as though the rule of addition has been violated in part (e) because the final answer (5.537×10^3 g) has three places past the decimal point, not two. However, the rule was applied to get the answer 55.37×10^2 g, which has *four* significant figures. Changing the answer to correct scientific notation doesn't change the number of significant figures, but in this case it changes the number of places past the decimal point.

Practice Problem ATTEMPT Perform the following arithmetic operations, and report the result to the proper number of significant figures: (a) 105.5 L + 10.65 L, (b) 81.058 m − 0.35 m, (c) 3.801×10^{21} atoms + 1.228×10^{19} atoms, (d) 1.255 dm × 25 dm, (e) 139 g ÷ 275.55 mL.

Practice Problem BUILD Perform the following arithmetic operations, and report the result to the proper number of significant figures: (a) 1.0267 cm × 2.508 cm × 12.599 cm, (b) 15.0 kg ÷ 0.036 m^3, (c) 1.113×10^{10} kg − 1.050×10^9 kg, (d) 25.75 mL + 15.00 mL, (e) 46 cm^3 + 180.5 cm^3.

Practice Problem CONCEPTUALIZE A citrus dealer in Florida sells boxes of 100 oranges at a roadside stand. The boxes routinely are packed with one to three extra oranges to help ensure that customers are happy with their purchases. The average weight of an orange is 7.2 ounces, and the average weight of the boxes in which the oranges are packed is 3.2 pounds. Determine the total weight of five of these 100-orange boxes.

WORKED EXAMPLE 1.6

An empty container with a volume of 9.850×10^2 cm^3 is weighed and found to have a mass of 124.6 g. The container is filled with a gas and reweighed. The mass of the container and the gas is 126.5 g. Determine the density of the gas to the appropriate number of significant figures.

Strategy This problem requires two steps: subtraction to determine the mass of the gas, and division to determine its density. Apply the corresponding rule regarding significant figures to each step.

Setup In the subtraction of the container mass from the combined mass of the container and the gas, the result can have only one place past the decimal point: 126.5 g − 124.6 g = 1.9 g. Thus, in the division of the mass of the gas by the volume of the container, the result can have only two significant figures.

Solution

$$\begin{array}{rl} & 126.5\ \text{g} \\ & \underline{-124.6\ \text{g}} \\ \text{mass of gas} = & 1.9\ \text{g} \longleftarrow \text{one place past the decimal point (two significant figures)} \end{array}$$

$$\text{density} = \frac{1.9\ \text{g}}{9.850 \times 10^2\ \text{cm}^3} = 0.00193\ \text{g/cm}^3 \longleftarrow \text{round to } 0.0019\ \text{g/cm}^3$$

The density of the gas is 1.9×10^{-3} g/cm^3.

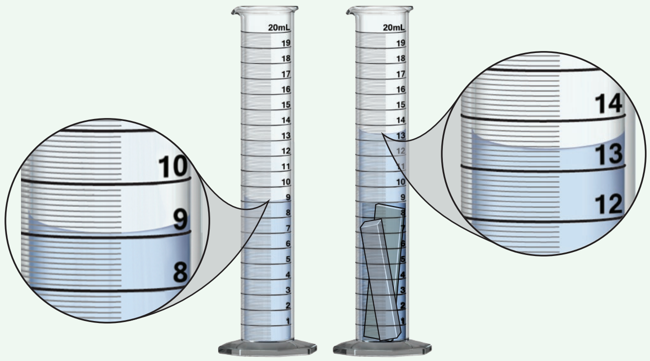
Think About It

In this case, although each of the three numbers we started with has *four* significant figures, the solution has only *two* significant figures.

Practice Problem ATTEMPT An empty container with a volume of 150.0 cm^3 is weighed and found to have a mass of 72.5 g. The container is filled with a liquid and reweighed. The mass of the container and the liquid is 194.3 g. Determine the density of the liquid to the appropriate number of significant figures.

Practice Problem BUILD Another empty container with an unknown volume is weighed and found to have a mass of 81.2 g. The container is then filled with a liquid with a density of 1.015 g/cm^3 and reweighed. The mass of the container and the liquid is 177.9 g. Determine the volume of the container to the appropriate number of significant figures.

Practice Problem CONCEPTUALIZE Several pieces of aluminum metal with a total mass of 11.63 g are dropped into a graduated cylinder of water to determine their combined volume. The graduated cylinder is shown before and after the metal has been added. Use the information shown here to determine the density of aluminum. Be sure to report your answer to the appropriate number of significant figures.



Accuracy and Precision

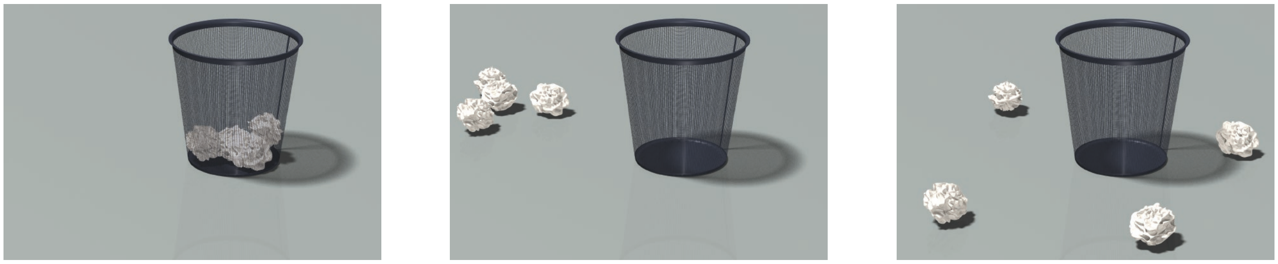
Accuracy and precision are two ways to gauge the quality of a set of measured numbers. Although the difference between the two terms may be subtle, it is important. ***Accuracy*** tells us how close a measurement is to the *true* value. ***Precision*** tells us how close a series of replicate measurements (measurements of the same thing) are to one another (Figure 1.6).

Suppose that three students are asked to determine the mass of an aspirin tablet. Each student weighs the aspirin tablet three times. The results (in grams) are tabulated here.

	Student A	Student B	Student C
	0.335	0.357	0.369
	0.331	0.375	0.373
	0.333	0.338	0.371
Average value	0.333	0.357	0.371

The true mass of the tablet is 0.370 g. Student A's results are more precise than those of student B, but neither set of results is very accurate. Student C's results are both precise (very small deviation of individual masses from the average mass) and accurate (average value very close to the true value). Figure 1.7 shows all three students' results in relation to the true mass of the tablet. Highly accurate measurements are usually precise, as well, although highly precise measurements do not necessarily guarantee *accurate* results. For example, an improperly calibrated meterstick or a faulty balance may give precise readings that are significantly different from the correct value.

Student Annotation: Even properly calibrated measuring devices can give varied results. Replicate measurements, such as those represented in the table, are used to determine the variability in the value of a measured quantity.



(a) (b) (c)

Figure 1.6 The distribution of papers shows the difference between accuracy and precision. (a) Good accuracy and good precision. (b) Poor accuracy but good precision. (c) Poor accuracy and poor precision.

	Student A	Student B	Student C
Measurement 1	0.335 g	0.357 g	0.369 g
Measurement 2	0.331 g	0.375 g	0.373 g
Measurement 3	0.333 g	0.338 g	0.371 g

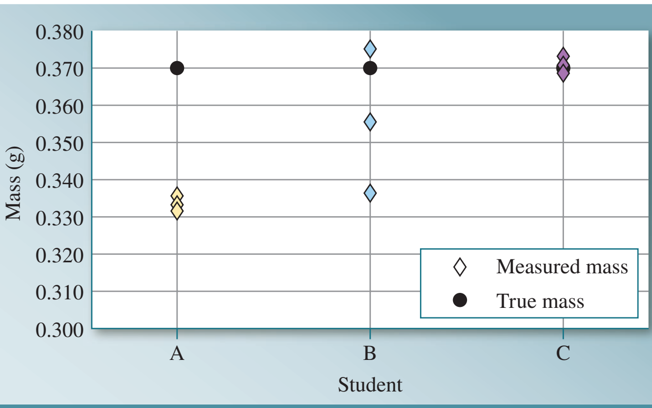


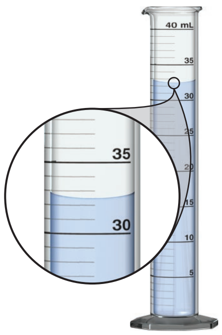
Figure 1.7 Graphing the students' data illustrates the difference between precision and accuracy. Student A's results are precise (values are close to one another) but not accurate because the average value is far from the true value. Student B's results are neither precise nor accurate. Student C's results are both precise and accurate.

Section 1.3 Review

Uncertainty in Measurement

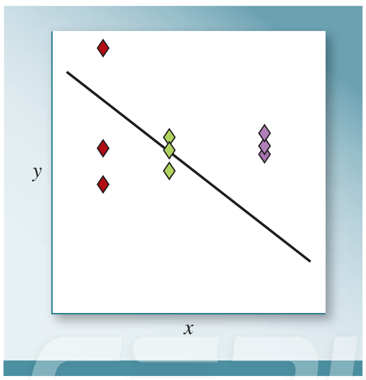
1.3.1 To the proper number of significant figures, what volume of water does the graduated cylinder contain? Note: The volume of water in a graduated cylinder should be read at the *bottom* of the meniscus (the curved surface at the top).

(a) 32.2 mL
(b) 30.25 mL
(c) 32.5 mL
(d) 32.50 mL
(e) 32.500 mL



1.3.2 The true dependence of *y* on *x* is represented by the black line. Three students measured *y* as a function of *x* and plotted their data on the graph. Which set of data (red, green, or purple) has the best accuracy? Which has the best precision?

(a) red, green
(b) green, green
(c) green, purple
(d) purple, purple
(e) purple, green



1.3.3 Specify the number of significant figures in each of the following numbers and determine the result of the following calculation to the correct number of significant figures.

$$63.102 \times 10.18 =$$

(a) 5, 4, 642.4
(b) 5, 5, 642.38
(c) 4, 3, 640
(d) 4, 4, 642.4
(e) 5, 4, 642.38

1.3.4 Specify the number of significant figures in each of the following numbers and determine the result of the following calculation to the correct number of significant figures.

$$3.115 + 0.2281 + 712.5 + 45 =$$

(a) 4, 5, 4, 2, 760.8
(b) 4, 4, 4, 2, 760.84
(c) 4, 4, 4, 2, 761
(d) 4, 5, 4, 2, 760.84
(e) 4, 5, 4, 2, 761

1.3.5 What is the result of the following calculation to the correct number of significant figures?

$$153.1 \div 5.300 =$$

(a) 28.887
(b) 28.89
(c) 28.9
(d) 29
(e) 30

1.3.6 What is the result of the following calculation to the correct number of significant figures?

$$(7.6 - 6.10) \div 0.500 =$$

(a) 3.0
(b) 3.00
(c) 2.6
(d) −5
(e) −4.6

1.4 USING UNITS AND SOLVING PROBLEMS

Solving problems correctly in chemistry requires careful manipulation of both numbers and units. Paying attention to the units will benefit you greatly as you proceed through this, or any other, chemistry course.

Conversion Factors

A ***conversion factor*** is a fraction in which the same quantity is expressed one way in the numerator and another way in the denominator. By definition, for example, 1 in = 2.54 cm. We can derive a conversion factor from this equality by writing it as the following fraction:

$$\frac{1 \text{ in}}{2.54 \text{ cm}}$$

Because the numerator and denominator express the same length, this fraction is equal to 1; as a result, we can equally well write the conversion factor as:

$$\frac{2.54 \text{ cm}}{1 \text{ in}}$$

Further, because both forms of this conversion factor are equal to 1, we can multiply a quantity by either form without changing the value of that quantity. This is useful for changing the units in which a given quantity is expressed—something you will do often throughout this text. For instance, if we need to convert a length from inches to centimeters, we multiply the length in inches by the form of the conversion factor with the unit inches in the *denominator*:

$$12.00\ \cancel{\text{in}} \times \frac{2.54\ \text{cm}}{1\ \cancel{\text{in}}} = 30.48\ \text{cm}$$

Our choice of this form of the conversion factor allows us to cancel the unit inches—and gives us the desired unit, centimeters. The result contains four significant figures because exact numbers, such as those obtained from definitions, do not limit the number of significant figures in the result of a calculation. Thus, the number of significant figures in the answer to this calculation is based on the number 12.00, not the number 2.54.

Dimensional Analysis—Tracking Units

The use of conversion factors in problem solving is called ***dimensional analysis*** or the *factor-label method*. Many problems require the use of more than one conversion factor. The conversion of 12.00 into meters, for example, takes two steps: one to convert inches to centimeters, which we have already demonstrated; and one to convert centimeters to meters. The additional conversion factor required is derived from the equality:

$$1\ \text{m} = 100\ \text{cm}$$

and is expressed as either:

$$\frac{100\ \text{cm}}{1\ \text{m}} \quad \text{or} \quad \frac{1\ \text{m}}{100\ \text{cm}}$$

We choose the conversion factor that will introduce the unit meter and cancel the unit centimeter (i.e., the one on the right). We can set up a problem of this type as the following series of unit conversions so that it is unnecessary to calculate an intermediate answer at each step:

$$12.00\ \cancel{\text{in}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{1\ \text{m}}{100\ \cancel{\text{cm}}} = 0.3048\ \text{m}$$

Careful tracking of units and their cancellation can be a valuable tool in checking your work. If we had accidentally used the *reciprocal* of one of the conversion factors, the resulting units would have been something other than meters. Unexpected or nonsensical units can reveal an error in your problem-solving strategy.

Student Annotation: If we had accidentally used the reciprocal of the conversion from centimeters to meters, the result would have been 3048 cm^2/m, which would make no sense—both because the units are nonsensical and because the numerical result is not reasonable. You know that 12 inches is a foot and that a foot is not equal to *thousands* of meters!

Student Hot Spot

Student data indicate you may struggle with conversion factors. Access your eBook to view additional Learning Resources on this topic.

Worked Example 1.7 shows how to derive conversion factors and use them to do unit conversions.

WORKED EXAMPLE 1.7

The Food and Drug Administration (FDA) recommends that dietary sodium intake be no more than 2400 mg per day. What is this mass in pounds (lb), if 1 lb = 453.6 g?

Strategy This problem requires a two-step dimensional analysis, because we must convert milligrams to grams and then grams to pounds. Assume the number 2400 has four significant figures.

Setup The necessary conversion factors are derived from the equalities 1 g = 1000 mg and 1 lb = 453.6 g:

$$\frac{1\ \text{g}}{1000\ \text{mg}} \quad \text{or} \quad \frac{1000\ \text{mg}}{1\ \text{g}} \quad \text{and} \quad \frac{1\ \text{lb}}{453.6\ \text{g}} \quad \text{or} \quad \frac{453.6\ \text{g}}{1\ \text{lb}}$$

From each pair of conversion factors, we select the one that will result in the proper unit cancellation.

Solution

$$2400\ \cancel{\text{mg}} \times \frac{1\ \cancel{\text{g}}}{1000\ \cancel{\text{mg}}} \times \frac{1\ \text{lb}}{453.6\ \cancel{\text{g}}} = 0.005291\ \text{lb}$$

Think About It

Make sure that the magnitude of the result is reasonable and that the units have canceled properly. If we had mistakenly multiplied by 1000 and 453.6 instead of dividing by them, the result (2400 mg × 1000 mg/g × 453.6 g/lb = 1.089×10^9 mg^2/lb) would be unreasonably large—and the units would not have canceled properly.

Student Annotation: Because pounds are much larger than milligrams, a given mass will be a much smaller number of pounds than of milligrams.

Practice Problem ATTEMPT The American Heart Association recommends that healthy adults limit dietary cholesterol to no more than 300 mg per day. Convert this mass of cholesterol to ounces (1 oz = 28.3459 g). Assume 300 mg has just one significant figure.

Practice Problem BUILD A gold nugget has a mass of 0.9347 oz. What is its mass in milligrams?

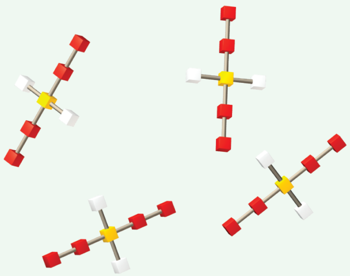
Practice Problem CONCEPTUALIZE The diagram contains several objects that are constructed using colored blocks and grey connectors. Each of the objects is essentially identical, consisting of the same number and arrangement of blocks and connectors. Give the appropriate conversion factor for each of the specified operations.

(a) We know the number of objects and wish to determine the number of red blocks.

(b) We know the number of yellow blocks and wish to determine the number of objects.

(c) We know the number of yellow blocks and wish to determine the number of white blocks.

(d) We know the number of grey connectors and wish to determine the number of yellow blocks.



Many familiar quantities require units raised to specific powers. For example, an area may be expressed in units of *length* squared (e.g., square meters, m^2; or square inches, in^2). Volumes sometimes are expressed in units of length *cubed* (e.g., cubic feet, ft^3; or cubic centimeters, cm^3). More often, though, volumes are expressed in liters (L) or milliliters (mL). It's important to remember that these are the common names given to specific units of length cubed. The liter is defined as a decimeter (dm) cubed: 1 L = 1 dm^3; and the milliliter is defined as the centimeter cubed: 1 mL = 1 cm^3. (See Figure 1.4.) When units are squared or cubed, special care must be taken when using them in dimensional analysis. For example, converting from cubic meters to cubic centimeters requires the following operation:

$$2.75\ \text{m}^3 \times \frac{100\ \text{cm}}{1\ \text{m}} \times \frac{100\ \text{cm}}{1\ \text{m}} \times \frac{100\ \text{cm}}{1\ \text{m}} = 2.75 \times 10^6\ \text{cm}^3$$

or

$$2.75\ \text{m}^3 \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 2.75 \times 10^6\ \text{cm}^3$$

Failing to raise the conversion factor to the same power as the unit itself is a common error—and one that can happen easily when the units L or mL appear—because they do not explicitly show the power 3.

Worked Example 1.8 shows how to handle problems in which conversion factors are squared or cubed in dimensional analysis.

WORKED EXAMPLE 1.8

An average adult has 5.2 L of blood. What is the volume of blood in cubic meters?

Strategy There are several ways to solve a problem such as this. One way is to convert liters to cubic centimeters and then cubic centimeters to cubic meters.

Setup $1\ \text{L} = 1000\ \text{cm}^3$ and $1\ \text{cm} = 1 \times 10^{-2}\ \text{m}$. When a unit is raised to a power, the corresponding conversion factor must also be raised to that power in order for the units to cancel appropriately.

Solution

$$5.2\ \text{L} \times \frac{1000\ \text{cm}^3}{1\ \text{L}} \times \left(\frac{1 \times 10^{-2}\ \text{m}}{1\ \text{cm}}\right)^3 = 5.2 \times 10^{-3}\ \text{m}^3$$

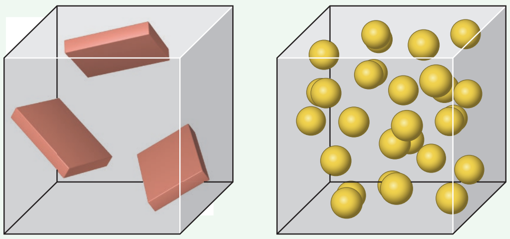
Think About It

Based on the preceding conversion factors, $1\ \text{L} = 1 \times 10^{-3}\ \text{m}^3$. Therefore, 5 L of blood would be equal to $5 \times 10^{-3}\ \text{m}^3$, which is close to the calculated answer.

Practice Problem ATTEMPT The density of silver is $10.5\ \text{g/cm}^3$. What is its density in kg/m^3?

Practice Problem BUILD The density of mercury is $13.6\ \text{g/cm}^3$. What is its density in pounds per cubic foot (lb/ft^3)? (1 lb = 453.6 g, 1 in = 2.54 cm)

Practice Problem CONCEPTUALIZE Each diagram [(i) or (ii)] shows the objects contained within a cubical space. In each case, determine to the appropriate number of significant figures the number of objects that would be contained within a cubical space in which the length of the cube's edge is exactly five times that of the cube shown in the diagram.



(i) (ii)

Section 1.4 Review

Using Units and Solving Problems

1.4.1 Convert 43.1 liters to cm^3.
(a) $43.1\ \text{cm}^3$
(b) $43{,}100\ \text{cm}^3$
(c) $0.0431\ \text{cm}^3$
(d) $4310\ \text{cm}^3$
(e) $0.00431\ \text{cm}^3$

1.4.2 What is the volume of a 5.75-g object that has a density of $0.252\ \text{g/cm}^3$?
(a) $1.45\ \text{cm}^3$
(b) $0.690\ \text{cm}^3$
(c) $22.8\ \text{cm}^3$
(d) $0.0438\ \text{cm}^3$
(e) $5.75\ \text{cm}^3$

1.4.3 The density of lithium metal is $535\ \text{kg/m}^3$. What is this density in mg/cm^3?
(a) $0.000535\ \text{mg/cm}^3$
(b) $0.535\ \text{mg/cm}^3$
(c) $0.0535\ \text{mg/cm}^3$
(d) $0.54\ \text{mg/cm}^3$
(e) $535\ \text{mg/cm}^3$

1.4.4 How many liters are there in a cubic meter?
(a) 10
(b) 100
(c) 1000
(d) 1×10^4
(e) 1×10^6