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From Taylor's Rule to Bernanke's Temporary Price Level Targeting*

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Abstract

Bernanke's strategies for integrating forward guidance into conventional instrument rules anticipate that effective lower bound (ELB) episodes may become part a regular occurrence and that monetary policy should recognize this likelihood (Bernanke (2017a,b)). Bernanke's first proposal is a form of flexible temporary price level targeting (TPLT), in which a lower-for-longer policy path is prescribed through a "shadow rate". This shadow rate accounts for cumulative shortfalls in inflation and output relative to exogenous trends, and the policy rate is kept at the ELB until the joint shortfall is made up. Bernanke's second proposal adds only the cumulative inflation shortfall since the beginning of an ELB episode directly to an otherwise standard Taylor rule. This cumulative shortfall in inflation from the 2 percent objective can be restated in terms of deviations of the price level from a price level target that increases at 2 percent annually.

We evaluate the performance of these strategies, which we call Bernanke's TPLT rules, using a small version of the FRB/US model. We then optimize these rules, computing efficient policy frontiers that trace out the best (minimum) obtainable combinations of output and inflation volatility given the effective lower bound constraint on the policy rate. The results suggest that Bernanke's rules give better macroeconomic outcomes than most of the other rules considered in the literature (including Taylor (1993) and Taylor (1999)) by stabilizing inflation and unemployment during severe recessions. Under these TPLT strategies, when the policy rate is made more responsive to shortfalls in inflation, the the likelihood of below-target inflation occurring alongside high unemployment rates decreases. However, the probability of an overheated economy, with temporarily above-target inflation and low unemployment rate, increases.

JEL Codes: E32, E52, E58.

Keywords: zero lower bound, Taylor rule, history-dependent policy, price level targeting.

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1 Introduction

Soon after John Taylor's seminal work on simple policy rules (Taylor (1993)), his proposal was successfully incorporated into macroeconomic models and, until the global financial crisis, the rule characterized central banks' behavior fairly well. Because of the effective lower bound, prescriptions of negative rates by Taylor-type rules—as occurred in the aftermath of the crisis—cannot be followed. The meaning of "following a Taylor-type rule" became adherence to the rule except in the event of prescriptions of negative rates, in which case the policy rate is set to its effective lower bound. However, this adaptation of Taylor-type rules to the ELB constraint may lead to poor economic outcomes, largely because the ELB creates a range of asymmetries—in particular, the inability to lower the short-term rate in response to adverse economic shocks—that pose challenges for central banks' communication strategy and ability to achieve macroeconomic stability.

To be sure, interest rate rules remain an important part of the policy framework, but the previous considerations may suggest that simple rules are less useful when the policy rate is constrained by the effective lower bound.¹ In particular, many academics and central bankers have emphasized the need to depart substantially from prescriptions of such rules in favor of more policy accommodation coming out of a prolonged ELB episode. Following a protracted recession, there could be a material risk of downward drift in long-run inflation expectations and a slow recovery. In addition, recent estimates of the neutral rate of interest are quite low, and suggest frequent occurrence of ELB episodes and accompanying concerns over communication, the costs of policy reversals, and risk management.² The likely recurrence of these concerns underscores the need for policymakers to better integrate forward guidance with their conventional policy tools. An important suggestion along these lines is the proposal of Bernanke (2017a,b).

Whether the policy reaction function should embed some form of forward guidance during ELB episodes is at the heart of Bernanke's suggestions. His recommendations hinge on how policymakers can augment simple policy rules to mitigate the impact of the ELB by providing clarity and guidance about the likely future path of the funds rate, including the timing of the exit from the ELB. In this way, his rule-based approach is a natural complement to John Taylor's.

Interestingly, in Bernanke's first proposal, the history-dependence characteristic of a Taylor-type rule with inertia in normal times is carried over to ELB episodes, as the criterion for exit from the ELB becomes a function of the cumulative inflation shortfall and output gap losses since the onset of the ELB episode. Hence, this strategy is a form of what can be defined as "full tilt" temporary price level targeting (TPLT), under which the policy instrument is maximally accommodative, given the ELB constraint, to shortfalls in inflation and output relative to exogenous trends until the joint shortfall is made up. This strategy will also impose communications challenges for policymakers due to the unobservability of the shadow rate—defined as the policymaker's desired interest rate

¹See, for instance, the recent notes on principles of sound monetary policy and central banks' practices posted on the Federal Reserve Board's web page: https://www.federalreserve.gov/monetarypolicy/monetary-policy-principles-and-practice.htm.

²See, for instance, Yellen (2015), Williams (2017) and Holston et al. (2017).

were policy unconstrained by the ELB.³ Although not directly observable, the shadow rate can be related to economic outcomes occurring around the ELB episode, owing to the reaction function operating during normal times. Thus, there is some scope for the central bank to at least partially communicate its policy intentions by referring to an underlying (shadow) rate that is consistent with practices in normal times.

Temporary price level targeting is also the key aspect of Bernanke's second proposal; however, in this case, the policy instrument responds incrementally and only to the inflation shortfall during ELB episodes. This cumulative shortfall in inflation from the 2 percent objective can be restated in terms of deviations of the price level from a price level target that follows a time trend which increases at 2 percent annually. Bernake's second approach can be described as "strict" temporary price level targeting, as opposed to his first proposal in which the shadow rate responds to output as well as inflation akin to "flexible" temporary price level targeting.

Bernanke's proposals draw from key elements of the recent literature. In particular, his suggestions are consistent with the touted macroeconomic benefits of "history dependence" in monetary policy making, especially when the policy rate is at the effective lower bound. There are two important considerations about the nature of this commitment.⁴ First, rules that display history-dependence in normal times, or inertia, are those in which a change in the policy rate today signals a persistent change in the stance of policy.⁵ Implicit in the dependency on the past interest rate is dependency on past deviations of inflation from its target and output from potential. Bernanke's proposals carry this commitment into ELB episodes either through a shadow rate or the strict but temporary targeting of the price level. The macroeconomic benefits of these commitments depend not only on the central bank setting policy in a manner consistent with a TPLT rule but also on the public's belief that the central bank will do so.⁶

Second, the effective lower bound constraint may cause policymakers to systematically undershoot their inflation target,⁷ particularly if policymakers are unable or unwilling to influence private sector expectations through credible commitments. In Bernanke's (strict or flexible) TPLT, the

³This idea was developed out of his comments on Kiley and Roberts (2017)'s proposal. Different from these authors, the main characteristic of Bernanke's shadow-rate proposal is the combination of an inertial Taylor rule (to characterize normal times policy away from the ELB) and a rule for signaling changes in the federal funds rate (to define the shadow rate during the period over which the interest rate is at the ELB). Both approaches clearly resonate with the adjustment proposed by Reifschneider and Williams (2000a). Later we will be explicit about the formalization of Bernanke's approach in the context of these papers. Hall (1984) is a very early, still unappreciated, contribution to this literature.

⁴As the literature has emphasized, this will induce potentially important time inconsistency (or credibility) problems to the policy making during and after an ELB episode. See, for instance, Eggertsson and Woodford (2003), Woodford (2012), and more recently Kiley and Roberts (2017) (which includes a review of the literature.)

⁵Accordingly, an adjustment in the policy rate today *anticipates* (or signals) changes in the future path of short-term interest rates, thus prompting an immediate reaction of longer-term interest rates, asset prices, and other forward looking variables.

⁶As we will show below, in Bernanke's proposals the duration of an ELB episode depends not only on the economic outcomes prevailing when the policy rate leaves the ELB, but also on past economic outcomes so the central bank does not treat "bygones as bygones."

⁷Gust et al. (2017) show that, if monetary policy is constrained by the effective lower bound, optimal policymakers minimizing a standard quadratic loss function involving deviations of inflation and output from targets under discretion may systematically undershoot their inflation target. For a similar point, see Evans (2017).

policymaker responds to deviations of prices relative to a time trend that grows 2 percent per year, preserving the commitment to a symmetric 2 percent inflation objective despite the ELB constraint.

Against this backdrop, we evaluate Bernanke's proposals using a small scale approximation of the FRB/US model. The model is of similar size to DSGE models, and its equations can be related to the macroeconomic framework built into these models: a neoclassical core underlying the medium to longer-run dynamics and short-run nominal price and wage rigidities of New-Keynesian models. For the purposes of this paper, the smaller size and linear structure of the model reduces the technical burden of the simulations required for our analysis.⁸

We show that strict adherence to Taylor-type rules would usually lead to early departures from effective-lower-bound episodes, but with a high likelihood of a relatively prompt return to the bound. Bernanke's amendments to Taylor's 1999 rule delay the anticipated first policy firming as well as lower the expected path taken by the short-term interest rate once it leaves the ELB.

Overall, Bernanke's strategies give better macroeconomic outcomes than most of the other rules in the literature. In particular, Bernanke's policy proposals (either in the form of a flexible or strict TPLT) alleviate the fall in inflation and reduce the increase in the unemployment rate during severe recessions and the early stages of the subsequent recoveries. Thus, both the variability of inflation and unemployment relative to their longer-run targets are substantially reduced, hence lowering the likelihood of very poor economic performance. The strategies ameliorate severe recessions, bringing inflation close to target and unemployment substantially below the otherwise-prevailing recession level and close to (or even below) its longer-run level. Under Bernanke's TPLT strategies, when the policy rate is made more responsive to shortfalls in inflation, the likelihood of below-target inflation occurring alongside high unemployment rates decreases. However, the probability of an overheated economy, with temporarily above-target inflation and low unemployment rates, increases.

This tradeoff suggests studying whether history-dependent TPLT policies could be credible in the aftermath of a severe recession. Accordingly, we try to assess how much history dependence there should be, and the factors this assessment depends on. To do that, we optimize these rules, computing efficient policy frontiers that trace out the *best* (minimum) obtainable combinations of output and inflation volatility, given the effective lower bound constraint on the policy rate and the assumed structure of the model economy. Each point on a frontier corresponds to an optimal tradeoff for a central bank with a particular preference for minimizing inflation volatility relative to unemployment volatility.

Over the business cycle, the policy frontier indicates that an inflation-averse policymaker will have to tolerate greater and greater increases in unemployment volatility to achieve negligible improvements in stabilizing inflation. The frontier also indicates an unemployment-averse policymaker will face a steep tradeoff in accepting higher inflation volatility if exclusively concerned with min-

⁸We compute different moments by running stochastic simulations using the so-called small FRB/US, a model that closely resemble the approach used, among others, by Reifschneider and Williams (2000a), Williams (2009), and Kiley and Roberts (2017).

⁹The analysis of this optimal variability tradeoff is closely patterned after those in Taylor (1993), Fuhrer (1994), Rotemberg and Woodford (1997), Levin et al. (1999), Williams (2003) and Levin et al. (2003).

imizing unemployment volatility. The frontier suggests that a central bank will fare better with Bernanke's shadow rate rule (flexible TPLT) or his strict TPLT approach than with the inertial or non-inertial variants of Taylor (1999) – and, in particular, fare much better if aversions to inflation and unemployment volatility are relatively balanced. While Taylor (1993) and the inertial and non-inertial variants of Taylor (1999) produce inflation and unemployment volatilities that lie inside the optimal frontier, the Reifschneider and Williams and Kiley and Roberts strategies lie just a bit outside the frontier, unless the policymaker's preferences are somewhat more unbalanced.

Nevertheless, during and in the immediate aftermath of severe recessions, even an inflation-averse policymaker who announces a large temporary response to the price level gap is successful in reducing inflation volatility to 1.95 percent without inducing excessive unemployment variability in the near-term; that is, the policy frontier is very flat in these times. For a policymaker following Bernanke's shadow rate rule with equal concern for inflation and unemployment volatility, similar levels of inflation and unemployment volatility will result compared to those produced by a policymaker following the Kiley and Roberts policy rule.

Forward guidance allows the policymaker to lower the real interest rate despite the ELB on the nominal rate. Through a lower-for-longer commitment about the future path of the policy rate, this policy raises inflation expectations. This mechanism, which is contingent on the responsiveness of inflation to the path of the policy rate, is limited when the short-run Phillips curve is flat—that is, when the response of inflation to the amount of slack in the labor market is very slow. We show that for a flatter Phillips curve, which hence diminishes the efficiency of forward guidance, the short-term benefit of aggressively reducing inflation variability during severe recessions and the early stages of the recovery under Bernanke's TPLT rules could become somewhat pernicious, as this might bring about larger than desirable unemployment volatility later on in the business cycle—assuming the perfect credibility of the strategy. The final section of the paper then evaluates how the optimal policy frontiers for Bernanke's rules change given a steeper Phillips curve. In particular, for a policymaker weighing forward guidance à la Bernanke, a steeper Phillips curve yields a better tradeoff between a desirable rise in inflation expectations at the onset of an ELB episode (which lowers real rates in the face of the ELB constraint on the nominal rate), and the undesirable overshooting in unemployment in the episode's aftermath.

The structure the paper continues as follows. The next section discusses the effects of the effective lower bound for simple rules, and formally characterizes Bernanke's temporary price level targeting rules. Section 3 presents our results and the final section offers some conclusions.

2 Policy Rules and the ELB

This section reviews the use of simple rules as monetary policy benchmarks when the short-term nominal interest rate is constrained by the ELB. Most of the analysis hinges on how these rules mitigate the impact of the ELB by providing *forward guidance* about the likely future path of the funds rate, including the timing of the exit from the ELB.

We distinguish between forward guidance associated with either the introduction of a "shadow" nominal rate that remains consistent with the policy rule in normal times, or with an "ad-hoc" adjustment that can be seen as a (temporary) change in the policy rule only during ELB episodes. In this context, we evaluate Bernanke's recent strategies for providing forward guidance during ELB episodes: one strategy based on a shadow rate and the other based on a temporary change in the rule (Bernanke (2017a,b)).

2.1 Shadow Rates at the ELB

Here we discuss how monetary policymaking centered on Taylor-type rules can incorporate a shadow or notional rate, defined as the policymaker's desired interest rate were policy unconstrained by the ELB, which also provides information about the path of future policy when the ELB constraint binds. Unlike the actual interest rate, this conceptual rate can be negative but cannot be directly observed by the public. Accordingly, at the ELB, the policymaker's strategy will consist of communicating estimates of this shadow rate to the public and maintaining interest rates at the ELB until the shadow rate rises above the ELB. If the policymaker has as her shadow rate the negative rate prescriptions coming from its preferred Taylor-type rule, this form of forward guidance does not necessarily imply a change in the underlying monetary policy strategy at the ELB relative to normal times. This approach still suffers from potential communications challenges due to the unobservability of the shadow rate during ELB spells. However, although the shadow rate is not directly observable, if the shadow rate is rule-based, it can be related to economic outcomes occurring during the ELB episode, and those outcomes are readily communicable.

The next subsection frames these issues in a general form, and the subsequent section presents the recent proposal by Kiley and Roberts (2017) which can be nested as a special case in our framework.

2.1.1 Inertial Taylor-type Rules

Following Bernanke's notation (Bernanke (2017b)), our starting point is a Taylor-type rule in which the central bank in normal times sets the nominal interest rate, i_t , according to the following expression:

$$i_t = r + \pi_t + a(\pi_t - 2) + by_t + ci_{t-1},\tag{1}$$

where π_t denotes the inflation rate, y_t denotes the output gap, r denotes the equilibrium real interest rate (assumed to be constant), and 2 represents the (annualized percent) inflation target. The appropriate response to inflation and the output gap (i.e., the parameters a > 0 and b > 0, respectively) will depend on how the central bank balances inflation and output stabilization in

¹⁰We make a small distinction between a shadow rate and notional rate. We take the former to be in principle any statistic that determines the liftoff date from the ELB, which can be different from the negative notional rate prescriptions coming from the rule of ordinary times. This is the case in Bernanke's shadow rate rule where the shadow rate is constructed differently than the base rule, including different coefficients on movements in inflation and output.

response to different economic shocks.

Critically for our discussion, expression (1) allows for the presence of inertia or interest rate smoothing. That is, the current policy rate also depends on its lagged value, with a possibly high coefficient (i.e., $0 \le c \le 1$). The presence of interest rate smoothing might suggest that policymakers prefer to avoid large changes and reversals in the policy rate, or that they set the rate to hedge themselves against uncertainty and the policy errors that a less gradual policy response might entail. Alternatively, inertial rules might arise if the central bank were to actually set policy in a non-inertial manner but respond to some persistent variable not included in expression (1).¹¹

Most important for the analysis in this paper, we show that when monetary policy displays inertia, policy decisions regarding the path of the policy rate become history-dependent. As noted above, the nature of this commitment is rooted in the underlying policy rule: implicit in the dependency on the past interest rate is dependency on past deviations of inflation from target, output from potential, and possibly changes in the equilibrium real rate or the inflation target.¹²

The presence of the effective lower bound (taken here as zero) introduces an important modification to expression (1), which now becomes:

$$i_t = \max\{0, i_t^T = r + \pi_t + a(\pi_t - 2) + by_t + ci_{t-1}^*\},$$
(2)

where i_{t-1}^* denotes, using the words of Taylor and Williams (2011), "the preferred setting of the policy rate in period t-1 that would occur in the absence of the effective lower bound." At the ELB, for a rule in which the lagged interest rate appears, the distinction between the *actual* (constrained) interest rate and the *shadow* interest rate (that is, the rate that would prevail in the absence of the ELB) is crucial for setting the intended path of future policy rates and communicating such intentions to the public during an ELB episode.

In normal times, the rule implicitly depends on past inflation and output through the lagged interest rate term. If the lagged constrained rate appears in the rule, i.e. if $i_{t-1}^* = i_{t-1}^T$, then at the ELB, the non-negativity constraint also breaks the implicit history-dependence on past inflation and output. This compounds the policy misses induced by the ELB, and results in a premature return to positive rates, compared with a policymaker who references the notional rate in policy setting, which will carry the history dependence of normal times through to ELB episodes.

Accordingly, we now restrict our attention to the case in which the inertia term keys off the notional or shadow rate. This implies a condition for leaving the ELB consistent with the principles guiding policy during normal times. Although the notional rate is unobserved during the ELB spell, the presence of inertia allows for a recasting of the duration and time of departure from the ELB in terms of macroeconomic outcomes during an ELB episode (e.g., Gust et al. (2017)).

¹¹Rudebusch (2006) presents arguments and evidence against true inertia as the primary explanation. English et al. (2003) argue that both inertia and other causes seem to be at work empirically. At an analytical level, Woodford (2003) emphasizes that inertia would be consistent with optimal policy in many models. Evidence suggests that each story plays some role, but we do not take a strong position on the source of this historical phenomenon.

¹²Formally, this follows from using expression (1) to solve for the current interest rate by using backward substitution. Below, we extensively make use of this approach.

To see this, we solve expression (2) recursively, assuming that the policy rate is constrained by the lower bound at time t_1 , and that the ELB episode lasts $\tau - t_1$ periods. Accordingly, when smoothing is based on the lagged notional rate, the ELB-departure date τ is determined by the following condition:

$$c^{\tau - t_1 + 1} i_{t_1 - 1} + \sum_{j = t_1}^{\tau} c^{\tau - j} (r + \pi_j + a(\pi_j - 2) + by_j) > 0.$$
(3)

This expression characterizes the lower-for-longer commitment of a policymaker who adheres to a rule like (1) in normal times, and communicates as its shadow rate policy during ELB episodes the negative notional rate prescriptions of that same rule. Some comments about the history dependence embedded in this commitment are in order.

First, the presence of inertia induces history dependence in the policy decisions and makes it possible, but difficult, to link the exit condition to the average inflation or the average output gap during the ELB episode. This follows from a close examination of the ELB-exit condition (3). This expression depends upon past misses of inflation from target, the depth and persistence of the recession (as captured in the sequence of negative output gaps), as well as potential changes over time in the equilibrium real rate (that we have assumed to be constant in the specification of the rule). Because of inertia, the shadow rate accumulates forgone accommodation associated with the ELB, and does not move above zero until this accumulation has been run off, such that adverse economic conditions during an ELB episode trigger additional accommodation through a longer spell at the ELB.

Second, these past misses from targets are not equally weighted: They are discounted by the rule's degree of inertia during normal times (i.e., the parameter c), which amounts to the history dependence embedded in the policy rule. A highly inertial policy rule (i.e., with parameter $c\rightarrow 1$) implies that all misses are almost equally weighted while the policy rate is at the bound—thus prolonging the period over which the policy rate will remain low. As a corollary, the longer memory that comes with high inertia may require keeping the policy rate at the bound, even as nominal aggregate demand grows rapidly—a scenario that would otherwise lead policymakers to raise interest rates.¹³

Third, and in contrast, a rule with little inertia has reduced memory concerning previous misses: Only the most recent deviations of inflation and output from their targets matter for the setting of monetary policy, and thus the duration of the policy rate at the lower bound is generally reduced. Indeed, in the absence of inertia interacting with a shadow rate, the rule will provide little guidance regarding the exit date from an ELB episode in terms of past inflation and output. Formally, this clearly follows from evaluating expression (2) as $c\rightarrow 0$.

An important caveat is that while policymakers can potentially stimulate the economy—and

¹³In this discussion,we are omitting the first term of expression (3) that corresponds to the effect of the last reduction in the policy rate before the economy enters the effective lower bound zone. That value of the policy rate acts like a state variable that reflects all previous policy decisions.

thereby mitigate the impact of the ELB constraint—by making commitments to implement a lower-for-longer path for the policy rate, these benefits depend on the public believing that the central bank will do so. Importantly, the credibility of the lower-for-longer commitment matters as soon as the ELB binds, not only in the waning quarters of an ELB episode when economic conditions begin to call for higher rates (e.g., Bodenstein et al. (2012)). That is, not only is a shadow rate unobserved, but the credibility of a shadow rate policy matters long before agents can even verify that rates are being held down in a way that is at least *consistent* with an announced shadow rate policy.

2.1.2 A Rule for the Change in the Interest Rate

Recent work by Kiley and Roberts (2017) uses a version of the policy rule (1) in which c = 1, and $i_{t-1}^* = i_{t-1}^T$. This corresponds to the case in which the policymaker uses a rule to describe the *change* in the policy rate rather than rate's level. Kiley and Roberts assume that the change in the nominal interest rate responds to deviations of inflation from a 2 percent objective and the output gap, and that these responses are equally weighted, in particular with a = b = 0.125. Accordingly, their rule is a special case of expression (1) that can be written as follows:

$$i_t^T = i_{t-1}^T + a[(\pi_t - 2) + y_t]. (4)$$

Outside the ELB, this rule shares some similarities with most commonly used first-differences rules but also has an important difference (Orphanides (2003)). In particular, although the change in the policy rate does not depend on the level of the equilibrium real short-term interest rate, it does depend on the level of the output gap, and hence on the level of potential output (as in some of the rules considered in Taylor (1999) and Taylor and Williams (2011)).¹⁴ This feature contrasts with other first-differences rules that have been advanced on grounds of robustness (see, e.g., Orphanides (2003) and Orphanides and Williams (2002)). This dependency has important implications for how to set and communicate policy at the lower bound.

Kiley and Roberts emphasize that their specification can be interpreted as a form of a "flexible price level" instrument rule. That is, the *intended level* of the policy rate responds to the entire history (i.e., since the inception of the rule) of deviations of output from potential as well as to the average deviations of inflation from the 2 percent target—in effect, to the gap between deviations of the price level and a 2 percent trend. Formally, this follows from integrating backward expression (4) so that the level of policy rate can be written as follows:

$$i_t^T = a[\sum_{j=0}^{\infty} (\pi_{t-j} - 2) + \sum_{j=0}^{\infty} y_{t-j}].$$
 (5)

¹⁴As with other policy rules that respond to output, it has been recognized that errors in estimating the level of potential can accumulate inducing potentially persistent policy mistakes (e.g., Orphanides (2003)), something that seems particularly relevant during ELB episodes.

If the inflation rate in quarter t is defined as the annualized quarterly price change, then the first term would correspond to the current quarter's "price level gap," defined as $\sum_{i=0}^{\infty} (\pi_t - 2) =$ $4(p_t - p_t^*)$, where p_t is the log price level and p_t^* is the price level target which increases at 2 percent annually, such that every quarter $p_t^* = \frac{1}{2}t^{15}$. Finally, because of the presence of the output gap, this rule has been also described as a flexible price level instrument rule, in contrast to strict price level targeting.

An important property of the Kiley-Roberts policy rule is that expression (5) also defines the shadow rate at the ELB. That is, allowing for the presence of the ELB, the rule can be written as follows:

$$i_t = \max\{0, \ i_t^T - i_{t-1}^T = a[(\pi_t - 2) + y_t]\},\tag{6}$$

and the condition for leaving the ELB is given by:

$$i_{t_1-1} + a \sum_{j=t_1}^{\tau} [(\pi_j - 2) + y_j] > 0,$$
 (7)

which is an interesting special case of the general expression (2).

This "lower for longer" condition captures the forward guidance associated with this rule, stipulating that the policy rate will be kept low (at the bound) until average inflation (from period t_1 until the exit period τ) is close to 2 percent and the cumulative output gap is sufficiently positive. ¹⁶ The condition thus depends on the cumulative output gap and hence on the non-observable level of potential output. Accordingly, errors in measuring the level of potential – especially during ELB episodes—can accumulate, inducing potentially persistent policy mistakes (e.g., Orphanides (2003)). Because it is the change in the shadow rate that is linked to inflation and the output gap, the shadow rate will not begin to climb out of negative territory until at least one of inflation and output has overshot its objective. This is a clearly distinct feature of this rule relative to the level-rule policy with inertia (expression (1)).

A common feature of two of the strategies discussed above is that they do not see the advent of the ELB as heralding a regime change for monetary policy, and they instead postulate that the shadow rates are the negative notional prescriptions of the preexisting reaction function of normal times. Such negative rates cannot be followed and are hence unobservable, but can be linked to economic outcomes through the reaction function. Importantly, the duration of the ELB and the time when the policy rate leaves the ELB depend not only on the economic outcomes occurring at

To see this, notice that $p_t = p_0 + \sum_{j=0}^{t} \pi_t$, with an initial (log) price level p_0 normalized to zero. Moreover, it is also easy to show that, if rule (4) is specified in terms of the output growth, the underlying intended policy rate is closely related to nominal income targeting. Notice that these deviations of the price level from a 2 percent target will depend on how the inflation rate is actually measured in the policy rule. In their implementation, Kiley and Roberts define π_t as the four-quarter rate of inflation (in quarter t) and hence the first term of the rule can be written in deviations of the price level from a trend that increases at 2 percent during the most recent four quarters—i.e., $\sum_{j=0}^{\infty} (\pi_t - 2) = \sum_{j=0}^{3} [p_{t-j} - \frac{1}{2}t].$ 16 Alternatively, this condition for average inflation is akin to targeting the price level relative to a deterministic

trend increasing at 2 percent annually.

that time but also on (all) past economic outcomes, and hence policymakers will be faced with a time-inconsistency problem that will test their credibility.

In this section, notional rule prescriptions, taken as shadow rates, combined with inertia provide guidance by preserving the history-dependent strategy of normal times at the ELB. In the next subsection, we discuss guidance during the ELB as a deviation or change from the monetary policy strategy pursued during normal times. After that we will revisit shadow rates, allowing the shadow rate to differ from the negative notional prescriptions of the policy rule of normal times.

2.2 Temporary Deviations from Simple Rules

In this section, we consider approaches that introduce temporary deviations from a Taylor-type rule during ELB episodes, resulting in what we refer to as an augmented simple rule. While shadow rates preserve, with as much fidelity as possible, a history-dependent strategy from normal times during an ELB episode, augmented simple rules can allow for a different treatment of ELB episodes from normal times, as may be appropriate because of the unique and asymmetric risks posed by the bound.

An important proposal along these lines was introduced by Reifschneider and Williams (2000a), which serves as a useful benchmark for rule-based forward guidance. They argued that, in the aftermath of a prolonged period when the policy rate had been constrained by the effective lower bound, rates should be held lower for longer than would be suggested by prescriptions of the conventional rule. In particular, as recovery from an effective lower bound episode proceeds, the interest rate is kept lower than the rule would ordinarily call for in order to make up for the prior period when the interest rate was necessarily higher, on account of the ELB constraint, than the negative rates the rule called for.

Formally, this approach can be described by the following specification of the monetary policy strategy:

$$i_t = max[0, i_t^T - \gamma Z_t, 0],$$
 (8)

where i_t^T is given by expression (1) assuming no inertia (c=0), γ is a parameter $0 < \gamma \le 1$ that controls the speed at which foregone accommodation is made up, and the variable $Z_t = \sum_{j=1}^{t-1} [i_j - i_j^T]$ is defined as cumulative past deviations of the actual constrained rate from the unconstrained prescription of the non-inertial Taylor rule. Thus, when i_t^T prescribes a negative notional rate and the effective lower bound is reached, deviations of the actual interest rate i_t from the rule prescription i_t^T accumulate in the term $Z_t > 0$.

This strategy only succeeds if it is internalized by the public, but because Z_t is unobserved and the interest rate set to its effective lower bound is not initially indicative of whether this strategy is in use, similar credibility and communication concerns to those of a shadow rate strategy also apply here. Policymakers keep track of the forgone decline in interest rates implied by the ELB and commit to lower rates in the future until the stock of forgone cuts in interest rates has been

exhausted. Formally, it can be shown that this adjustment results in the following condition for leaving the ELB:

$$i_{\tau}^{T} + \gamma \sum_{j=t_{1}}^{\tau-1} [r + \pi_{j} + a(\pi_{j} - 2) + by_{j}] > 0.$$
 (9)

This expression highlights how the criteria to characterize a lower-for-longer interest rate policy under the Reifschneider-Williams approach has important similarities with the criteria previously discussed in the context of shadow rate policies.

First, compared with rules for the level of the policy rate (expression (3)), this condition avoids weighting the cumulative inflation and output deviations from their targets differently over time. Thus, forward guidance can be communicated in terms of the cumulative deviations (average inflation and output gap) experienced over the ELB episode as well as the expected evolution of the equilibrium real interest rate (r)—a variable that can change over time and will add challenges to the communication of the future policy rate path during the ELB episode (e.g., Yellen (2015)).

Second, compared with the difference rule by Kiley and Roberts (2017), this condition introduces a couple of extra parameters: γ and the equilibrium real rate r. The first parameter governs the timing of the exit from the ELB and the subsequent transition back to the Taylor rule. In particular, when economic conditions warrant i_t^T being positive again, the parameter γ controls the rate at which the stock variable Z_t is unwound through additional accommodation (lower-for-longer conditions). If $\gamma = 1$, Z_t is unwound as quickly as possible.¹⁷ Also note the cumulative deviations of the actual policy rate, i_t , from the notional rate, i_t^T , sum to zero once Z_t is unwound. Because deviations from i_t^T that occur when the constraint binds are always made up one-for-one by later accommodation for any $0 < \gamma \le 1$, the parameter γ only hastens or delays the timing of additional accommodation, but does not directly allow for more or less accommodation. As a result, this form of an augmented rule does not permit much fine tuning.¹⁸

Third, regardless of the state of the economy, the promise of additional accommodation implicit in Z_t will eventually be realized. Because Z_t only decreases when additional accommodation is provided, any future promised accommodation implicit in the size of Z_t will still be enacted even if there are positive surprises to the economy, reflecting the nature of the commitment embedded in this adjustment. In addition, because Z_t only increases if $i_t^T < 0$, negative shocks that occur prior to exiting the ELB but after $i_t^T > 0$ will not trigger additional accommodation through the variable Z_t .

¹⁷The proximity of i_t^T from the lower bound also constrains how quickly Z_t can be unwound, and ELB proximity is the limiting factor on unwinding Z_t whenever $i_t^T < \gamma Z_t$.

¹⁸ If $0 < \gamma < 1$, there will be an infinite sequence of periods in which $0 < i_t < i_t^T$, with i_t rapidly converging to i_t^T (the discrepancy halves each period if $\gamma = 0.5$).

2.3 Bernanke's Temporary Price Level Targeting

In this next section we examine the two proposals introduced by Bernanke (2017b) that can be seen as hybrids of the previously discussed monetary policy strategies. The main characteristic of his first proposal is the combination of an inertial Taylor rule (to characterize normal times) and a change rule (to define a shadow rate over the period in which the interest rate is at the ELB). The second proposal also introduces history dependence during ELB episodes but through a temporary adjustment to the monetary policy strategy of normal times.

2.3.1 Shadow Rate

Bernanke's first approach consists of postulating that the central bank follows an inertial Taylor rule (similar to expression (1)) in normal times, but, once the policy rate is constrained by the ELB, a shadow rate strategy similar to Kiley and Roberts is enacted as follows:

$$i_t^T = i_{t-1}^T + d(\pi_t - 2) + ey_t. (10)$$

But, in contrast to Kiley and Roberts, the non-negative parameters d and e in principle differ from the parameters a and b defining how the policy rate responds to inflation and output in normal times (e.g., expression (1)). As noted above, for an economy in which the policymaker is constrained at time t_1 , the exit period τ can be characterized as the one satisfying this expression:

$$\sum_{j=t_1}^{\tau} [d(\pi_j - 2) + ey_j] > 0, \tag{11}$$

which generalizes expression (7) in Kiley and Roberts (2017)—although this is only an occasionally binding condition, as we will discuss. Because the parameters d and e are not necessarily the same as the parameters a and b which govern the rule in normal times, the criterion for exit from the ELB is a weighted average of the inflation shortfall and the cumulative output gap, with an explicit relative weight, only during the ELB. This differs from Kiley and Roberts (2017) who impose the condition that the policymaker uses the same parameters balancing its inflation and output tradeoff in normal times and during the ELB. As we now discuss, the choice of these parameters d and e could introduce some problems in implementing this form of the shadow rate. ¹⁹

Several comments are in order. First, if c = 0, that is if the conventional interest rate rule of normal times is non-inertial, then this strategy will prescribe abrupt rate hikes at the end of an ELB episode. Second, because in this framework we smooth on the actual, not the notional lagged rate, if the interest rate is binding in period t then the non-inertial components of the rule must be

 $^{^{19}}$ Notice that, because the d/e effects would apply to all recessions, the policymaker is forced to choose between handling "disinflation-dominated" recessions well and "unemployment-dominated" recessions poorly or vice versa. One could imagine having two independent liftoff criteria depending on the type of recession. This would allow the policymaker to offer extra forward guidance in either type of recession. Actually, Bernanke entertains the possibility of multiple exit criteria (Bernanke (2017b) p.40).

negative in total: $r+\pi_t+a(\pi_t-2)+by_t<0$. Rearranging this, we obtain $(a+1)(\pi_t-2)+by_t<-2-r$ which is always negative for positive r, suggesting the choice of the ratio $\frac{d}{e}=\frac{a+1}{b}$, which guarantees that the shadow rate is negative whenever the ordinary rule prescription i_t^T is negative, so that the shadow rate is the sole determinant of the time of exit from the ELB.

Third, barring the choice of $\frac{d}{e} = \frac{a+1}{b}$, the ELB-exit condition only occasionally binds, such that either the exit condition or the ordinary policy rule could govern the exit from the ELB which may create a communication and credibility hurdle. Agents who incorrectly believe that this condition exclusively determines exit may expect rates to rise sooner than the policymaker intends. Agents may also perceive the policymaker as having reneged if the condition does not bind. We find that in practice $\frac{d}{e} = 1$ also does not create an inconsistency between the inertial rule as the conventional policy rule in normal times and the shadow rate during the ELB. However, when the shadow rate only responds to inflation, that is d > 0 and e = 0, the condition (11) does not solely determine the end of ELB episodes, as inflation may be close to target while the inertial Taylor rule, responding to a negative output gap, still prescribes negative rates. Finally, unlike agents who assume no future shocks, agents who understand that shocks will hit the economy in the future will also understand that positive surprises could cause the extra accommodation to be withdrawn earlier.

2.3.2 Temporary Price-Level Targeting (TPLT)

Interestingly, under the previous shadow rate policy for the case of e=0 (assuming the shadow rate is binding), the average shortfall of inflation since the onset of the ELB episode also determines its duration. Hence, this strategy is a form of what can be defined as "full tilt" TPLT under which the policy instrument maximally responds, given the ELB constraint, to shortfalls in inflation until the shortfall is made up.

This element is also the key aspect of Bernanke's second proposal, but in this case the policy instrument responds *incrementally* to the inflation shortfall. In particular, Bernanke's second proposal adds the cumulative inflation shortfall since the beginning of the ELB directly to an otherwise standard Taylor rule (non-inertial). Formally:

$$i_t = r + \pi_t + a(\pi_t - 2) + by_t + f\left[\sum_{j=t_1}^t \frac{(\pi_j - 2)}{j - t_1 + 1}\right],$$
 (12)

where as before t_1 is the period in which the economy last entered the ELB regime and f is a positive parameter.²⁰ Notice that the ELB-exit condition in period τ resembles the previously

Notice that this rule can be also described as a temporary deviation from the Taylor rule as follows: $i_t = max[min(i_t^T, i_t^T + fZ_t), 0]$, where $Z_t = \sum_{j=t_1}^t \frac{\pi_j - 2}{j - t_1 + 1}$, and Z_t drops out when $\{Z_t > 0 \text{ and } i_t^T > 0\}$. This notation also shows that we only impose the inflation shortfall term when that is in fact a shortfall, but not when inflation is above target, and shows the conditions under which the term eventually drops out after an ELB episode.

discussed expression (11) under (e = 0). That is,

$$i_{\tau}^{T} + f \left[\sum_{j=t_{1}}^{\tau} \frac{(\pi_{j} - 2)}{j - t_{1} + 1} \right] > 0.$$
 (13)

However, raising the policy rate from its ELB does not necessarily mean the end of additional accommodation.²¹ There is a phase of post-liftoff adjustments in which the interest rate has left the ELB but still provides more accommodation than the non-augmented rule prescription i_t^T . The parameter f allows more or less accommodation for the same inflation shortfall. For smaller values of the parameter f, there is a longer sequence of smaller deviations of i_t from i_t^T during the phase out, and for larger values of f, there is a shorter sequence of larger deviations.

As before, in contrast to agents who assume no future shocks, agents who understand that unexpected shocks will hit the economy in the future will also understand that positive surprises could cause the extra accommodation to be scaled back or withdrawn earlier. In the event of consecutive ELB episodes in which the post-liftoff phase period from an initial episode has not elapsed before the ELB binds again, t_1 is defined such that it refers to the period in which the ELB was initially binding; this ensures that the policymaker achieves an average of 2 percent inflation across consecutive ELB episodes.

Finally, we note that achieving 2 percent inflation in ELB episodes, where unconstrained policy would precribe below average inflation, leads the policymaker to exceed 2 percent inflation on average by a small amount. To remedy this, an alternative would be to target not 2 percent inflation, but what average inflation would have been absent the ELB.

3 Results

3.1 The Model and the Simulation Approach

We evaluate the policy rules using simulations of the small FRB/US (sFRB) model. The sFRB model has been developed by Brayton (2018) and it is a simplified, linear version of FRB/US with similar properties. As with her big brother FRB/US, in sFRB firms' investment depends on aggregate demand as well as current and future user costs, and consumer spending depends on disposable income, wealth, and long-term interest rates. The sFRB model places yields on long-term Treasury securities at the center of monetary transmission, implying that both the current and the expected future values of short-term interest rates have a fundamental role in the behavior of aggregate demand. The supply side also incorporates price and nominal wage inflation dynamics that reflect both nominal and real rigidities. In line with a number of recent papers that have produced new empirical evidence on a weak relationship between resource utilization and inflation, the model incorporates a fairly flat short-run Phillips curve relationship, but inflation dynamics

Notice that as the coefficient f becomes large enough, this policy approximates the shadow rate policy when e approaches zero. Still, this comparison is not totally exact, but just an approximation, because the f term is introduced in a non-inertial rule, while e = 0 is based upon an inertial policy rule during normal times.

are still consistent with well-anchored longer-run inflation expectations near 2 percent (see, for instance, the recent discussion in Kiley (2015) and Blanchard (2016)).

More generally, the dynamics of the model are affected by the presence of adjustment costs that introduce substantial inertia in aggregate demand as well as in prices and wages. Moreover, the model incorporates a flexible treatment of agents' expectations—a crucial element of the transmission mechanism of monetary policy. In our simulations, we assume model-consistent expectations (MCE), which means that agents are forward-looking and they form expectations with knowledge of the structure of the economy, including the monetary policy rule followed by the central bank. In particular, we assume MCE in those sectors that are central to the monetary policy transmission mechanism, namely in asset pricing and wage and price determination, but not elsewhere. This assumption of MCE conveys considerable power to monetary policy via the central bank's ability to affect the expectations of wage and price setters and financial market participants directly through changes in current and future policy, but it limits any forward-looking element in consumption and investment. On the spending side, key frictions include the presence of some liquidity-constrained households, and firms' investment behavior also embeds accelerator terms that may capture the impact of sales on small and liquidity-constrained firms' ability to invest.

The model is of similar size to DSGE models, and its equations can be related to the macroeconomic framework built into these models: a neoclassical core underlying the medium to longer-run dynamics, plus the short-run nominal price and wage rigidities of New-Keynesian models. For the purposes of this paper, the smaller size and linear structure of the model reduce the technical burden of the simulations required for our analysis. We compute different moments by running stochastic simulations using sFRB; this closely resembles the approach used by Reifschneider and Williams (2000a), Williams (2009), and Kiley and Roberts (2017). In particular the simulations: (i) are arranged as 5000 (simulated) samples of 140 quarters with a burn in period of 40 quarters (i.e., 10 years) after initialization at the model's deterministic steady state; (ii) are subject to demand and supply shocks drawn from the period 1970 to 2016 (via bootstrap of the residuals from the model), but no shocks to the monetary policy rule; (iii) impose the condition that economic agents never expect an ELB event of duration longer than 10 years.²²

3.2 The Performance of Bernanke's Rules

Before turning to the implications that alternative rules have for key moments and distributions of macroeconomic variables emerging from simulating many different economic histories (generated by random draws of many economic shocks), the next section first illustrates how macroeconomic outcomes will change across alternative rules for a particular history of shocks.

²²The bootstrap method sequentially re-samples shocks from recessions and the Great Recession, and it preserves the correct unconditional distribution. That is, recessions are not over- or under-sampled as a result of bootstrapping. There are 11 demand-side and supply-side shocks in total.

3.2.1 A Great Recession Simulation

To assess the macroeconomic consequences of alternative monetary policy rules, the literature usually presents what is called an *impulse-response* analysis. To do that, it is typical to start with a baseline describing how the economy will evolve after an unexpected shock or combination of shocks in an initial period. The baseline is constructed to approximate a certain type of economic episode (e.g., a recession) and it embodies a reference monetary policy rule to describe the behavior of the policymaker. Then, a counterfactual is constructed by switching to a different rule under the assumption that the economic agents have perfect foresight, and hence the monetary policy is perfectly credible. Thus, private agents know that the expected path of the policy rate will not change over time. Different policy rules will generally imply different policy rate paths and different amounts of stimulus. The stimulus provided by different rules then depends on what agents expect the central bank to do from an initial point onward and if the policy will remain unrevised thereafter. Although this type of analysis provides useful insights, it constrains the comparison across rules by selecting initial conditions beyond which the future path of short-term rates is perfectly anticipated by economic agents. Such an analysis overlooks the fact that the economy is subject to many unanticipated contingencies that will likely force the central bank to respond and thus update the policy path as new information about the state of the economy arrives. To control for these effects, we use a slightly different simulation design.

Specifically, we assume that the economy is hit not just by an initial shock, but by a sequence of unanticipated, large, and persistent economic shocks similar to those occurring during the Great Recession and the subsequent recovery. This hypothetical stochastic scenario is constructed as follows. We start from a baseline taken from the long-run values in the 2017 December public baseline.²³ Then, every period, economic agents are surprised by a new set of shocks causing the economy to deviate from that baseline. Accordingly, the duration/severity of the effective lower bound episode is not (ex-ante) known but reevaluated—and hence changing—period-by-period and across different policy rules. In this exercise, among the shocks hitting the economy are an array of demand, supply, and financial shocks, but not monetary policy shocks – akin to a particular slice of one of our stochastic simulations that resamples shocks from 2006 to 2016. Thus, every period, different rules will prescribe alternative expected paths of the policy rate, including the length of time at the ELB, and these revised paths will take account of any new shocks as well as any history-dependence embedded in a particular rule. Although private agents fully understand the future economic implications of each shock and each rule, and the central bank enjoys complete credibility, the stochastic or unanticipated elements of our exercise will figure importantly for the efficacy of alternative policy rules to stabilize inflation and output in an uncertain environment.²⁴

Figures 1 and 3 illustrate the performance of two sets of rules, non-inertial and inertial, under the stochastic simulation that resembles a recession similar to the Great Recession. The top left and

²³Public datasets are available as part of the FRB/US model package on https://www.federalreserve.gov/econres/us-models-package.htm. The public dataset is constructed to align with the five key macroeconomic variables of the Summary of Economic Projections.

²⁴In the Appendix we compare our stochastic exercise with a more standard impulse-response analysis.

bottom two panels of Figure 1 show the policy prescriptions and economic outcomes for alternative non-inertial rules. In particular, the figure displays the macroeconomic outcomes under the Taylor 1999 rule, and two augmented non-inertial rules: Reifschneider and Williams (RW) and Bernanke's strict temporary price level targeting rule (B-TPLT). To complement the analysis of these two types of rules, the top right of Figure 1 also displays the forgone decline in interest rates implied by the ELB that accumulates to the stock variable Z_t included in the RW rule, and the average shortfall of inflation since the onset of the ELB that is the key variable of B-TPLT rule.

Under Taylor 1999 (the red lines plotted in Figure 1), at the beginning of the simulation—the end of 2006—the economy was close to the steady state, with an inflation rate of 2 percent and output close to its potential (i.e., an output gap around zero). Then, starting late in 2007 negative shocks cause the output gap to deepen sharply to almost minus 8 percent by the end of 2009, recovering gradually thereafter until reaching positive territory during 2013, and then increasing to about 2 percent through the end of 2016. Initially, inflation drops by more than two percentage points into negative territory and then slowly recovers, eventually moving toward 2 percent by the end of 2018. These two outcomes are consistent with a federal funds rate constrained by the ELB beginning in the fourth quarter of 2008. Under the Taylor 1999 rule, the federal funds rate remains at its effective lower bound until early 2014 with the output gap almost closed but inflation still hovering around 1.5 percent—below the 2 percent target. Beyond this point, Taylor (1999) calls for a substantial increase (temporarily above its longer-run level) in the federal funds rate. The stochastic nature of our simulations implies that underlying this sharp increase in the federal funds rate is not below-target inflation but rather a sequence of (positive) unexpected shocks, the effect of which is to make agents believe that the increase in short-term interest rates is consistent with narrowing the protracted, positive output gap—and hence supporting the monotonic return of inflation toward its long-run target. As we will show below, this is consistent with an initial projection that the period of low interest rates will be extensive—a projection that is scaled back as the central bank adjusts its forecast to positive shocks that boost economic recovery.

Figure 1 shows that rules that incorporate greater history dependence generally keep the federal funds rate at its lower bound for a longer period. This history dependence takes the form either of accounting for the forgone decline in interest rates stemming from the ELB as in the RW rule (the green lines) or the shortfall of inflation accumulated since the onset of the ELB as in the Bernanke TPLT rule (the blue lines). Under the RW rule, when the embedded Taylor rule prescribes a negative (notional) rate and the ELB binds, deviations of the actual interest rate from the notional prescriptions accumulate in what the RW rule denotes by the variable Z_t (shown in the upper right panel). These cumulative deviations are driven back to zero as Z_t is unwound. As shown in the previous section, for any value of $0 < \gamma \le 1$, deviations due to the binding of the ELB are always made up one-for-one by later accommodation. We illustrate this by presenting two versions of this rule for two values of the parameter γ , $\gamma = 1$ and $\gamma = 0.5$.

Because the forgone reduction in the interest rate Z_t only decreases when additional accommodation is provided, all the promised accommodation implicit in Z_t is still enacted despite the

positive surprises to the economy. As shown by the green lines of Figure 1, when the prescription of the base Taylor rule embedded in the RW rule is positive again, the parameter γ controls the rate at which the forgone decline in the short-term interest rate is unwound through additional accommodation. In the case of $\gamma = 1$, Z_t is unwound as quickly as possible, resulting in a sharp increase in short-term interest rates once Z_t is fully unwound and the ELB episode ends. Because the parameter γ does not control the amount of additional accommodation—merely its timing— $\gamma = 1$ will frontload the accommodation as much as possible which results, relative to the other value considered of $\gamma = 0.5$, in slightly higher inflation (and a minor overshooting of 2 percent target later on, bottom left panel) and a faster and stronger recovery (the bottom right panel) than under Taylor 1999.

Two versions of the Bernanke TPLT rule are also presented in Figure 1. Two different values of the parameter f allow tuning for more or less accommodation as the value of this parameter controls the central bank's reaction to the inflation shortfall occurring during the ELB episode. ²⁵ The amount of this inflation shortfall, for f = 4 and f = 8, is presented by the blue lines of the top-right panel of Figure 1. ²⁶ Relative to the standard Taylor rule, Bernanke's TPLT rule pushes the ELB exit into the future, but a bit less so than under the RW approach. The reasons are relatively straightforward. In the case of RW, the stock of forgone accommodation Z_t takes account of the negative output gap, which Bernanke's TPLT rule does not. Additionally, the future promised accommodation implicit in Z_t , once accumulated, is always eventually fully realized. Under Bernanke's TPLT rule, positive surprises to inflation can cause the inflation-shortfall term to be smaller or drop out sooner than originally anticipated. Finally, for $\gamma \geq 0.5$, policy under the RW rule has a steep liftoff from the ELB. Taking account of the entire policy path, a gradual liftoff enables Bernanke's TPLT rule to raise inflation closer to target despite less time at the ELB relative to RW.

These different policy strategies for keeping short-term rates low (at the bound) also translate to post-liftoff guidance during the uncertain policy normalization phase. In particular, the Bernanke TPLT rule would hold the short-term interest rate lower than the Taylor 1999 (and the RW) rule well into the projected recovery. This commitment is intended to ensure that the inflation shortfall during the ELB is not forgotten and it implies that output will increase well above potential and hence inflation will rise above its long-run objective. In comparison with the standard Taylor 1999 rule, this promise of further accommodation under the augmented rules (especially B-TPLT) provides near-term stimulus, but also requires substantial credibility because it restrains future tightening at a time when inflation and output will run above their longer-run objectives.

Our simulation strategy allows us to infer changes in private agents' perception of the implicit

²⁵In the simulations, if the inflation-shortfall term is initially positive upon entry into an ELB episode (because inflation is above target but the output gap is negative), and later becomes negative, we always "start the clock" on average inflation when the economy enters the ELB. Hence those initially above target inflation values will feed into the calculation of the average inflation term, but the term will only be active in the rule when its contribution is negative.

 $^{^{26}}$ Below we will discuss how f is optimally chosen, given the model, over different specifications of the policymaker's preferences.

expected path for short-run nominal interest rates across rules, including normalization after the rate leaves the effective lower bound. As noted above, a key issue in the evaluation of the effectiveness of the policy rules is whether the agents interpret the guidance as a credible signal that, as the economy recovers, the policymaker would keep the federal funds rate lower for longer than would otherwise be expected—given the historical responses to movements in resource utilization and inflation.

Figure 2 summarizes the evolution of the anticipated paths for the federal funds rate, the expected year-over-year rate of core PCE inflation, and the expected evolution of the output gap at two different dates (2008:4 in blue, and 2011:4 in green) for two different rules—Taylor 1999 (the dashed lines) and Bernanke's TPLT rule (the continuous lines) assuming a value of the parameter f = 8. After the federal funds rate hits the ELB in late 2008, given monetary policy adhering to Taylor 1999, the ELB constraint is projected to bind until the end of 2013, while under Bernanke's TPLT rule the period at the ELB is lengthened, stretching out into the middle of 2014 (the blue dashed and continuous lines in the top panel). Moreover, under Bernanke's TPLT rule the federal funds rate would begin rising at a shallower pace, supporting a less pronounced (expected) fall in output and a faster rebound of inflation (the middle and lower panels). As the simulation unfolds, a sequence of better-than-expected shocks induces substantial revisions to the original strategies, albeit with important differences. Under the Taylor (1999) rule, the policy rate leaves the ELB much earlier than under Bernanke's TPLT rule. The latter rule accounts for the inflation shortfall by not letting bygones be bygones. The lower-for-longer strategy implemented under Bernanke's rule brings about a larger than initially anticipated output gap that also leads to a larger and more persistent overshooting of inflation above the 2 percent target.

The previous analysis was based on non-inertial policy rules. Figure 3 compares Bernanke's shadow rate policy rule with other inertial rules chosen to cover other alternatives discussed in the literature. As a benchmark, we include the inertial Taylor (1999) rule specification. This adds a moderate degree of interest rate inertia to the original Taylor 1999 rule by setting the coefficient on the lagged nominal interest rate to 0.85 (as in English et al. (2015)). The figure also shows the first-difference rule that incorporates the shadow rate during the ELB—the rule recently suggested by Kiley and Roberts (2017).²⁷ For Bernanke's shadow rate rule, Bernanke suggests two alternative means of computing the shadow rate depending on the relative weight given to the shortfalls in output and inflation from their target values once the policy rate is constrained by the effective lower bound (that is, the parameters d and e in expression (11)).²⁸ As in Figure 1, the performance of these rules is studied by generating a sequence of unexpected shocks that produce a recession similar to the Great Recession. The top left and the two bottom panels show the policy prescriptions and economic outcomes for alternative inertial rules, and the top-right panel shows the underlying

²⁷The first-difference rule without a shadow rate is to Kiley and Roberts (2017) as the standard Taylor rule with the max operator is to introducing smoothing by responding to the notional interest rate.

 $^{^{28}}$ As noted earlier, in this rule only the ratio of d and e matters, as all that is relevant is the sign of the shadow rate, not its value. However, we do show a level for the shadow value during the Great Recession plots (the top-right panel of Figure 3), for which the actual d and e values are the same as in Kiley and Roberts (2017) (purely to facilitate comparison with their paper).

evolution of the shadow interest rates of the relevant rules.

Relative to the inertial Taylor 1999 rule, rules that incorporate greater history dependence (in the form of an ELB-exit criterion based on an underlying shadow rate) generally involve a longer period over which the federal funds rate is kept at its lower bound. Because private, rational agents have full understanding of these rules and the policymakers enjoy full credibility regarding their actions, agents anticipate a low path for the federal funds rate beyond the period promised by the inertial Taylor rule. As a result, inflation under these shadow-rate rules is higher and thus current and expected real rates are lower, thereby driving output up faster than under the inertial Taylor 1999 rule. If the shadow rate is mainly a function of the inflation shortfall at the ELB—as suggested by Bernanke's specification with d > 0 and e = 0, the federal funds rate will be low in the years beyond the period shown for the other policy rules (light blue lines).²⁹ That said, this rule calls for a relatively rapid tightening, as inflation is clearly above its 2 percent target and the output gap is substantially positive. This is consistent with the weak short-run relationship between inflation and resource utilization in the model, which implies that the stabilization of inflation requires substantial policy accommodation (in the form of a long period of low nominal short-term interest rates) thus keeping unemployment low. Rules in which the shadow rate is a function of both inflation and output, such as under Kiley-Roberts and Bernanke's specification with d = e, produce a notable fall in the shadow rate but, as the recovery proceeds (partially due to positive unexpected news), the shadow rate quickly rebounds, fostering an earlier departure from the ELB (green and dark blue lines in the top-right panel).³⁰ These two rules produce broadly similar paths for the federal funds rate, output, and inflation.

Overall, the analysis in this section illustrates that the projected timing and pace of policy firming would likely be quite sensitive to modest differences in views regarding the outlook, modest revisions over time in projections of real activity and inflation, and the details of the rule. In the next section, we describe how alternative rules have implications for the probability of hitting the effective lower bound and for the stochastic behavior (key moments and distributions) of macroeconomic variables emerging from simulating many different economic trajectories (histories), generated by random draws from many economic shocks.

 $^{^{29}}$ Note that incremental changes to the parameters d and e can lead to substantially different realized paths in a particular stochastic simulation. For example, suppose that significantly large adverse shocks are followed by a sequence of milder adverse shocks. If d and e are such that the shadow rate is still negative and the ELB is thus still binding when the milder shocks hit, these relatively small shocks will depress the shadow rate and the ELB will continue to bind. However, a small change to the parameters defining the shadow rate can lead to policy firming and hence "turn off" the shadow rate mechanism before the milder shocks hit. Absent the shadow rate mechanism that keeps the interest rate low amid improved economic conditions, these milder shocks will not be sufficient for the ELB to re-bind. Thus, a relatively small change in the parameters can lead to a large (discrete) change in the amount of accommodation offered in a particular stochastic scenario. This feature is also present for the f parameter in Bernanke's TPLT rule.

 $^{^{30}}$ This trajectory for the shadow rate is in keeping with one of its general features: a stronger response to a variable such as the output gap means that the shadow rate falls more sharply at the onset of the recession, but also means that the shadow rate unwinds faster as recovery proceeds, illustrative of how only the ratio of d and e matters.

3.2.2 Bernanke's Rules, Macroeconomic Volatility and the Effective Lower Bound

Faced with the prospect of ELB and recession events, it is possible that policymakers may be tempted "to keep their powder dry" as a precaution against additional adverse shocks, leaving them space to cut rates further when the occasion arises. Likewise, in the aftermath of a recession in which the ELB binds, policymakers may wish to hasten policy normalization to generate space for accommodation before the next recession occurs. Lower-for-longer strategies, including Bernanke's TPLT, correctly reject that reasoning: the precautionary choice that better achieves objectives for inflation and unemployment rates is to (deliberately) embrace rather than avoid the ELB.³¹

Bernanke's first and second proposals employ this type of deliberate choice, by augmenting simple rules with a TPLT to keep rates at or near the ELB longer than those simple rules would otherwise prescribe. In this way, the TPLT acts as a precautionary commitment or forward guidance. Because TPLT and the other proposals we examine for mitigating the effects of the ELB all invoke this lower for longer commitment, these proposals keep the nominal interest rate at the ELB even as recovery is well underway. Thus, an average of conditions prevailing when the ELB binds is a metric that would conflate any improvement in outcomes early in the ELB episode with the subsequent recovery phase (when the funds rate is held at the ELB as follow through on the lower for longer commitment). As a result, we condition our ELB results on a common set of periods across all proposals and simulations: we take the time periods when the non-inertial Taylor 1999 rule is binding as an index for (exogenously) poor economic conditions (bad times) and compare how TPLT and other proposals perform in those times.

Table 1 presents macroeconomic outcomes from stochastic simulations for different rules. These outcomes are means and volatilities (root mean square error deviations of inflation and unemployment from their objectives) of inflation and unemployment during poor economic conditions: Thus, the time window over which we analyze the performance across rules is consistent and dictated by the benchmark of the Taylor 1999 rule. An analysis of this type is useful for the reasons given above, but below (Table 2) we also show how these macroeconomic outcomes vary across rules once we condition on each rule's prescription regarding the duration of the effective lower bound episode. In other words, while Table 1 displays how the economy improves during the recession—and the very early stages of the recovery—if the policymaker follows Bernanke's rule or other proposals in the literature, Table 2 offers a broader time perspective. This table displays how the economy performs over the recession plus a (rule-dependent) subsequent recovery during which the policymaker implemented a lower-for-longer strategy relative to a policy following Taylor 1999.

Overall, the results suggest that Bernanke's rules give better macroeconomic outcomes than most of the other rules. In particular, Table 1 shows that Bernanke's policy proposals (either through the introduction of a shadow rate or strict TPLT) cushion the fall in inflation and reduce the increase in the unemployment rate during the recession and the early stages of the recovery.

³¹However, the "lower-for-longer" strategy can affect financial conditions (such as "searching for yields strategies") in ways that can potentially trigger undesirable macroeconomic outcomes with implications for financial stability. These elements, albeit important, are outside the scope of our analysis.

In addition, the variability of both inflation and unemployment relative to their longer-run targets is substantially reduced, lowering the likelihood of very poor economic performance. If we instead condition on each rule prescription regarding the duration at the effective lower bound, Table 2 also confirms rules that incorporate substantial history-dependence, such as in Kiley and Roberts (2017) and Bernanke's shadow rate rules, generate substantial improvement in macroeconomic outcomes: On average, inflation is close to target and unemployment moves substantially below the recession level and close to (or even below) its longer-run level. As noted earlier, the γ parameter in the RW rule only affects the timing of additional accommodation, and has minimal impact on losses absent an explicit policymaker preference for a smooth interest rate path.

Figures 4 and 5 show, for the non-inertial and inertial rules considered in this paper, respectively, the probabilities associated with the joint occurrence of certain combinations of inflation and unemployment in our simulations. These figures aim to assess the risks, as measured by the probabilities of specific combinations of inflation and unemployment, associated with the central bank following a given policy rule. These probabilities are constructed to characterize the distribution of inflation and unemployment during "bad times and 8 quarters in the recovery phase," as a metric for how the rules are performing in quarters with bad shocks and the subsequent two years. By spanning periods of bad shocks and recovery, the benefit of forward guidance to ameliorate poor economic conditions early in ELB episodes will be pitted against the later cost of overshooting objectives caused by lower-for-longer accommodation. Each panel corresponds to a particular range for inflation and unemployment; for instance, the top-left panel shows states of the economy in which low inflation is paired with low unemployment. The height of the bars indicates the probabilities associated with these combinations of macroeconomic outcomes for each rule.

Figure 4 presents the results for the non-inertial rules, including Taylor 1999 as a benchmark, along with the RW rule and Bernanke's TPLT rule. As can be seen in the bottom left panels, the more aggressive the commitment approach taken by the Bernanke TPLT rule (i.e. higher f), the more effective the strategy is in reducing the likelihood of poor economic performance—as measured by below target inflation coupled with high unemployment rates. The Taylor 1999 rule gives rise to more adverse outcomes with inflation below target coupled with high unemployment (bottom left panel) implying about a 20 percent chance of a substantial deterioration in macroeconomic outcomes.³² These risks are substantially reduced under Bernanke's TPLT strategy. In particular, the analysis suggests that the previous probability can be reduced by half under a relatively aggressive temporary response to forgone inflation associated with ELB episodes. Despite the low probability that unemployment rises substantially and inflation falls below target, the model simulations indicate a fairly large probability that both inflation and unemployment remain low (see the top left panel). However, for higher f, Bernanke's TPLT rule supports a relatively sizable probability of an overheated economy—that is, a situation in which inflation will run above the target with relatively low unemployment (top right panel). Figure 5 confirms that the previous results also

 $^{^{32}}$ In periods with bad shocks but excluding the subsequent 2 years, the corresponding percent chance is about 40 percent.

apply to the case of inertial rules.

These results suggest that it may be reasonable to question whether these history-dependent policies could be credibly sustained during the first two years after a recession. However, these results are consistent with a flat Phillips curve slope and an important role for exogenous supply shocks driving volatility in inflation. This is a theme that we will explore later on.

3.3 Optimal Version of Bernanke's TPLT

In this section we follow the suggestion of Taylor (1980) and Taylor (1982) to evaluate the performance of monetary policy rules by considering the inflation-output variability trade-offs. The rationale behind this approach is that keeping the inflation rate closer to a target may entail accepting (much) greater fluctuations of real GDP about potential (or unemployment about the natural rate). That being the case, monetary policy may wish to balance its effects on inflation and output variability. If inflation exceeds the target, monetary policy will raise interest rates and depress output. The opposite situation can be exacerbated if the effective lower bound limits current policy actions.

As already mentioned, Bernanke's proposals to adopt a temporary price level target—either through introducing a shadow interest rate or a temporary adjustment to a policy rule—combat ELB recessionary episodes and their aftermath by committing to follow a history-dependent monetary policy that does not let bygones be bygones. But what remains is to ascertain how much history-dependence there should be, and what this response depends on. We study how Bernanke's rules affect this variability tradeoff using exercises closely patterned after those in Fuhrer (1994), Levin et al. (1999), and Levin et al. (2003). Formally, for the two functional forms of the interest rate rule proposed by Bernanke (expressions (10) and (12)), the parameters d/e and f—setting the rest of the parameters equal to those in Taylor 1999 rule—are chosen to minimize the average loss per period given by:³³

$$L = \frac{1}{N} \sum_{j=1}^{N} [(\lambda)(u_j - u_j^*)^2 + (1 - \lambda)(\pi_j - 2)^2] = (\lambda)(RMSE_u)^2 + (1 - \lambda)(RMSE_\pi)^2$$
 (14)

where j is an index over all quarters.³⁴ The loss function implies that the central bank's preferences

 $^{^{33}}$ The nonlinearity of the ELB demands the use of simulated moments. Note a traditional variance frontier would plot the standard deviation of unemployment σ_u and headline inflation σ_π stemming from minimization of $\lambda \sigma_u^2 + (1-\lambda)\sigma_\pi^2$. We will analogously produce the the loss frontier in terms of $RMSE_u$ and $RMSE_\pi$ stemming from the minimization of mean period loss in (14). We burden the reader with this convention because in examination of policymaker tradeoffs in subsets of periods with bad shocks, conditional losses (but not conditional variances) will take into account that mean outcomes in bad times are already poor. Also note discounting is innocuously omitted from (14) because a 10 year burn in preceding 100 quarters of tabulated results means that within those quarters, outcomes in quarter t and t+1 both stem from the ergodic distribution. Beyond increasing the efficiency of our estimates, omitting discounting also makes plain that loss frontier for the full sample (not conditioned on bad times) is equivalent to the traditional variance frontier, barring the shock process to the natural rate u^* .

³⁴To determine each policy frontier, we perform sets of stochastic simulations of the sFRB model for alternative values of the parameter of the rule (i.e., either d/e or f) which minimize the losses (14) subject to a range of λ ,

can be described by a quadratic loss function in two arguments: First, the (squared) deviation of the unemployment rate from the natural rate of unemployment (assumed to be 4.6 percent plus a shock process in the sFRB model), and second, the (squared) deviation of PCE inflation from a target rate of 2 percent. The parameter $0 \le \lambda \le 1$ reflects the policymaker's preference for minimizing inflation variability relative to unemployment variability.

To compute the (efficient or optimal) policy frontier for a particular functional form of the interest rate rule, we calculate the parameters of the rule (d/e or f) that minimize the loss function for each value of λ over the range zero to one. Thus, for a given form of the interest-rate rule, the policy frontier traces out the *best* (minimum) obtainable combinations of output and inflation volatility for all possible preferences (i.e., weights on inflation and unemployment volatility), taking into account the effective lower bound on the federal funds rate and the structure of the model economy.

Figure 6 plots the inflation-unemployment trade-off delivered by Bernanke's shadow rate rule, and the optimal ratio (d/e) under different policymaker preferences governed by the parameter λ . The other parameters of the rule are kept fixed at their values under Taylor 1999 (with a smoothing coefficient ρ equal to 0.85). As expected, the frontier is convex toward the origin. Along this locus, λ varies between a value close to one—that is, the horizontal asymptote—that reflects exclusive concern for minimizing the volatility of unemployment, and a value close to zero—the upper-left vertical limit asymptote—reflecting policymakers whose preferences are exclusively concerned with minimizing the volatility of inflation (inflation-averse policymakers).³⁵ These loci then represent an efficiency frontier for Bernanke's shadow rule given the sFRB model: Points closer to the origin are welfare superior (lower losses), while points in the opposite direction are inferior. The top panel displays the frontier by computing how inflation and unemployment variability changes over the entire history of business cycles. The bottom panel evaluates how this tradeoff changes when only looking at severe recessions (i.e., using the index for "bad times" when the prescriptions of the Taylor 1999 rule are constrained by the ELB) plus two years into the recovery phase. Since the inertial Taylor rule—and Bernanke's related shadow rules based on it—differs from the Kiley-Roberts rule even in normal times, the tradeoff in this subset of periods focuses on the benefits and costs of the forward guidance these rules offer at the ELB. As earlier noted, we regard the latter as particularly relevant in light of the risks of these episodes.

Several comments follow from the top panel of Figure 6. First, over the business cycle, the efficient frontier displays substantial steepness: If policymakers put substantial weight on the costs of inflation volatility (λ near zero), they would quickly create unnecessarily high labor market volatility. But unemployment-averse policymakers (those with λ close to one) would not induce substantial inflation variability. This result is consistent with the constraint that the estimated relationship between inflation and unemployment imposes on monetary policy. That is, the short-

 $^{0 \}le \lambda \le 1$. As discussed in section 3.1, within each set of 5000 simulations, results are tabulated over a 100 quarter interval following a 10 year burn in period.

³⁵In this section, we use volatility to mean deviations from the policymaker's objectives as captured by $RMSE_u$ and $RMSE_{\pi}$.

run Phillips curve is estimated to be quite flat which, combined with the fairly large exogenous components that the model attributes to the behavior of inflation, means that even an inflation-averse policymaker struggles to keep inflation close to target. In particular, small gains in reducing inflation variability require substantial unemployment volatility, given that a substantial fraction of the inflation variability is independent of policy.

We next address the implications for the optimal choice of the parameters d/e in Bernanke's shadow rate rule along the efficient frontier. As expected, a shadow rate that embeds a response primarily to forgone inflation during recessions (i.e. high d/e) tends to reduce inflation volatility, but it becomes too costly in terms of the induced unemployment volatility. Thus, if we consider extreme inflation aversion, the optimized value of d/e is around 10 times larger than the optimized value if the policymaker places equal weight on inflation and unemployment stabilization (in this case the optimized value for d/e is around one as shown in the upper panel of Figure 6). The flat Phillips curve in sFRB limits the efficiency of forward guidance, and suggests modestly increased forward guidance would be desirable at the ELB, but the d/e ratio in Bernake's shadow rate rule does not permit such fine tuning. Consider two recessions, one predominately associated with a shortfall in inflation and the other predominately with a fall in output. Any change in d/e will foster more forward guidance in one scenario and less in the other—there is no adjustment of d/e that would allow dialing back the amount of forward guidance offered in all types of recessions.

Figure 6 also displays the points in the unemployment-inflation space associated with the inertial Taylor 1999 and the Kiley-Roberts rules (the red diamond and the green square in the figure, respectively). The point corresponding to the inertial Taylor 1999 rule is inside the frontier of Bernanke's shadow rate rule—and hence is suboptimal. The Kiley-Roberts rule, by contrast, is slightly closer to the origin. On the one hand, the Kiley-Roberts rule leads to lower unemployment volatility than does the Bernanke shadow rate policy, but a sufficiently inflation-averse policymaker with a strong response to the forgone output and inflation during severe recessions can generate similar inflation volatility.

Rather than focusing on the performance of the optimized rules over the entire business cycle, we now present the sensitivity of these results by looking at the policy frontiers only during severe recessions and two years into the recovery phase. These results are plotted in the lower panel of Figure 6. Focusing on this period yields a substantially flatter (less convex) efficiency frontier, implying that the tradeoffs between unemployment and inflation are less extreme than over the entire business cycle—that is, in announcing a credible shadow-rate policy plan at the onset of a severe recession, a policymaker would face a choice of whether to stabilize inflation or output in the near term. Bernanke's shadow rate rule is therefore associated with substantially improved macroeconomic outcomes over inertial Taylor 1999. In fact, during these episodes, an inflation-averse policymaker (i.e., mainly concerned with subdued inflation dynamics) can substantially improve on inflation variability by aggressively responding to potential deflation without much increase in unemployment variability in the near term. Finally, and not surprisingly, a policymaker with a balanced loss function—which weights inflation and unemployment equally—following Bernanke's

shadow rate produces very similar inflation and unemployment volatility to a policymaker following the Kiley and Roberts policy rule.

Figure 7 presents the policy frontier for the optimized Bernanke TPLT rule. As before, the optimization is only with respect to the coefficient f, while keeping the rest of the rule – the normal times reaction function – the same as the (non-inertial) Taylor 1999 rule. In the top panel, the curve traces the lowest combinations of inflation and output gap volatilities that can be achieved with Bernanke's TPLT rule over the full sample period – that is, when the parameter f is globally (i.e., over multiple business cycles) optimized. The bottom panel displays the policy frontiers only during the severe portion of the recessions and two years into the recovery phase, evaluated at the globally-optimized f coefficients along the points of the policy frontier.

The top panel of Figure 7 illustrates similar points to those previously discussed in the context of Bernanke's shadow rate policy, but for a TPLT adjustment to the Taylor 1999 rule. Introducing a temporary response to avoid dismissing bygones as bygones tends to reduce the variability of inflation, but only up to a point. The more aggressive the temporary response to inflation, the more it will result in a very pernicious increase in unemployment volatility. Thus, from the perspective of the entire business cycle, the policy frontier is considerably steeper than when performance is only considered around recessions and the early stages of recovery.

The bottom panel of the figure illustrates fairly persuasively the case for temporary price level targeting during recessions and their aftermath. In particular, policymakers who temporarily respond to the forgone inflation generate economic outcomes to the west and the south of the graph—i.e., welfare superior, as both inflation and unemployment are better stabilized. As shown in the graph, Bernanke's TPLT rule welfare dominates the Taylor 1999 rule. From the standpoint of a policymaker with an equal weights loss function, Bernanke's TPLT rule would foster lower inflation volatility than a policymaker following the Reifschneider and Williams approach. However, unemployment variability will remain a bit higher, but not excessively so.

From the perspective of severe recessions and the initial phase of recoveries, even a strict inflation-targeting policymaker (corresponding to an optimal value of f of around 20) would successfully stabilize inflation without inducing excessive unemployment variability in the near term (i.e., the policy frontier is very flat in those times). Nevertheless, with a transmission mechanism featuring a fairly flat short-run Phillips curve, the short-term benefit of aggressively reducing inflation variability during severe recessions and the early stages of the recovery under Bernanke's TPLT rule could become much more pernicious later on in the business cycle through larger than desirable unemployment volatility – assuming the perfect credibility of the strategy.

Figure 7 also shows the loss frontiers for Bernanke's TPLT rule when the base rule to which TPLT is applied is either Taylor 1993 (the black line) or has double the output gap response of the Taylor 1999 rule (the purple line). Across all three output gap responses considered, the range of feasible inflation losses is similar.³⁶

³⁶Note that the seeming superiority of the purple line stems from the indirect ability to optimize over inflation through both the TPLT effect as well as the output gap response. The ranking of the purple line relative to RW (in green) is irrelevant since the RW shown is based on Taylor (1999).

We now summarize the main results in this section. Over the business cycle, the policy frontier implies a very large unemployment volatility for inflation standard deviations below 1.9 percent; correspondingly, the frontier implies large inflation variability penalties for unemployment standard deviations below 1.5 percent. The inertial and non-inertial variants of Taylor 1993 as well as Taylor 1999 produce inflation and unemployment volatilities that lie inside the optimal frontiers of TPLT-augmented rules. The Reifschneider-Williams and Kiley-Roberts strategies lie just a bit outside the frontiers, unless the policymaker's preferences are somewhat more unbalanced.

Nevertheless, from the perspective of severe recessions and their immediate aftermath, even a strict inflation-targeting policymaker who implements a large temporary response to the price level gap to successfully reduce inflation volatility to 1.75 percent will not immediately induce excessive unemployment variability (i.e., the policy frontier is very flat in those times). In fact, during these recessionary episodes, an inflation-averse policymaker (i.e., mainly concerned with subdued inflation dynamics) can substantially improve on inflation variability by aggressively responding to potential deflation without much increase in unemployment variability. A policymaker with a balanced loss function—weighting inflation and unemployment equally—who follows Bernanke's shadow rate rule produces very similar inflation and unemployment volatility to that of a policymaker following the Kiley-Roberts policy rule.

3.4 Robustness Under Alternative Inflation Dynamics

Forward guidance allows the policymaker to lower the real interest rate despite the ELB on the nominal rate. Through a lower-for-longer commitment about the future path of the policy rate, this policy raises inflation expectations. This mechanism, which is contingent on the responsiveness of inflation to the path of the policy rate, is limited when the short-run Phillips curve is flat—that is, when the response of inflation to the amount of slack in the labor market is very slow. In sFRB, the flat short-run Phillips curve therefore limits the efficiency of forward guidance. The alternative inflation dynamics explored in this section increase the central bank's ability to affect current and expected inflation. These robustness results are provided with the caveat that the literature questions whether forward guidance in forward-looking models gives monetary policy too much power over current and future inflation, in particular if a central bank can engineer a temporary but protracted overshooting of its inflation objective while keeping long-run inflation expectations well anchored (i.e., the so-called forward guidance puzzle; see, for instance, McKay et al. (2016)).³⁷

To illustrate the effects of a relatively steeper short-run Phillips curve, Figure 8 displays the impulse responses of output and inflation to alternative monetary policy actions for two calibrations of the Phillips curve.³⁸ The first row displays the response to an immediate 100 basis point rise

³⁷However, we note that Bernanke's rule-based approach may do better in this regard than a state-contingent optimal policy plan. The latter is more difficult to communicate, and any discrepancies between a misperceived policy communication and the implemented policy may be misattributed to a change in the policymaker's inflation target or tolerance of inflation volatility.

³⁸As mentioned earlier, recent research has documented the substantial reduction in the slope of the Phillips curve

in the policy rate governed by the inertial Taylor 1999 rule.³⁹ The second row shows the power of forward guidance by considering the effects of an anticipated decline in the policy rate of 100 basis points 12 quarters in the future. Under the baseline specification of the model (i.e., a very flat Phillips curve), in response to the immediate shock, output falls modestly and inflation responds very little (top-left panel). The bottom-left panel also confirms that the small response of inflation to unemployment greatly limits the effectiveness of any given amount of forward guidance as compared with other DSGE models (e.g., Lindé et al. (2016)).⁴⁰ Motivated by this result, we double the slope of the Phillips curve. The second column of Figure 8 confirms the stronger—although still modest—response of inflation and output to both monetary policy actions. We see this calibration as a way of illustrating the importance of the Phillips curve for the analysis of Bernanke's rules, without giving too much macro-stabilization power to monetary policy through managing future inflation expectations (as in some DSGE models).

We now evaluate how the location and shape of the optimal policy frontiers for Bernanke's rules change when the model used to construct the frontier has a steeper Phillips curve. First note for non-augmented rules like Taylor 1999 which do not embed any lower-for-longer commitment, the ELB will be more deflationary under a steeper short-run Phillips curve, leading to a larger volatility of inflation. This becomes apparent when comparing the blue and the red diamonds in Figure 9. As a consequence, forward guidance becomes more essential to offset the stronger deflationary effects of the ELB.

Conveniently, a steeper short-run Phillips curve will be a net boon to policymakers who offer forward guidance by increasing the effectiveness of that policy-contingent on our ongoing assumptions that agents are forward looking and the central bank enjoys credibility about its policy announcements. In particular, for a policymaker weighing forward guidance, a steeper Phillips curve yields a better tradeoff between a desirable rise in inflation expectations at the onset of an ELB episode (which lowers real rates in the face of the ELB constraint on the nominal rate), and the undesirable undershooting in unemployment in the episode's aftermath. The enhanced effects of forward guidance improve outcomes under the augmented rules which offer it, more than offsetting the otherwise stronger deflationary effects of the ELB. Accordingly, Bernanke's TPLT rule lowers the volatilities of inflation and unemployment (in relation to their objectives), thus shifting the policy frontier noticeably inward, as shown in Figure 9. The new frontier suggests that balanced policies—those that put equal weight on unemployment and inflation variability—favor a larger optimal response to price-level deviations during ELB episodes when resource utilization gives rise to larger responses of prices. The effect of a steeper short-run Phillips curve leads to a significant increase in the optimal value of the parameter f given balanced preferences, more than double the 2.4 value obtained under the baseline (flat) Phillips curve.

as well as the anchoring of longer-run inflation expectations near 2 percent (e.g., Blanchard (2016) and Kiley (2015)). ³⁹Note that when the inertial Taylor (1999) rule prescribes a negative notional rate, the ELB induces an immediate positive monetary policy shock such as the one shown, as well as anticipated positive shocks if the notional rate is expected to remain negative for additional periods in the future.

⁴⁰These impulse responses are very similar to Kiley and Roberts (2017), although they use the full version of the FRB/US model.

While the f parameter in the Bernanke-TPLT rule allows scaling back forward guidance when it is less effective under a flatter Phillips curve or less desirable under policymaker preferences that do not penalize inflation volatility, as noted earlier the d/e ratio in Bernake's shadow rate rule always introduces some added forward guidance in at least some types of recessions. As a consequence, allowing for a steeper Phillips curve, as in Figure 10, notably improves the optimal policy frontier for the Bernanke shadow rate policy. This is not a surprising result: Responsive inflation means that less unemployment must be generated to get prices back on target after the shock.

4 Conclusions

In this paper, we first characterized Ben Bernanke's two possibilities for integrating forward guidance in conventional policy tools in a world in which the effective lower bound (ELB) may become part of the convention (Bernanke (2017a,b)). In Bernanke's first proposal, the criterion for exit from the ELB is a form of what can be defined as "full tilt" (flexible) but temporary price level targeting (TPLT) under which the duration at the bound is prescribed by a "shadow rate" that responds to shortfalls in inflation and output (relative to exogenous trends) occurring during the ELB. In Bernanke's second proposal, the policy rate responds incrementally and only to the inflation shortfall during ELB episodes. In both cases, this cumulative shortfall in inflation from the 2 percent objective can be restated in terms of the deviations of the (log) price level from the (log) price level target—characterized by a time trend that increases at 2 percent annually.

We then evaluated the performance of these proposals using the FRB/US model. The results suggest that Bernanke's rules give better macroeconomic outcomes than most of other rules considered in the literature (including Taylor 1993 and Taylor 1999). We also explore the characteristics and performance of Bernanke's rules designed to minimize fluctuations in inflation and output, taking into account the effective lower bound on the federal funds rate and the structure of the model economy. Efficient Bernanke TPLT rules tend to stabilize inflation and unemployment over the business cycle, and especially during severe recessions and the subsequent recoveries during which the policy rate is constrained by the effective lower bound. The more aggressive the commitment to TPLT, the more effective the strategy is in reducing the likelihood of a period in which below-target inflation is coupled with high unemployment rate. However, this strategy increases the probability of an overheated economy—that is, a situation in which inflation will run above the target and the unemployment rate will undershoot its long-run equilibrium level.

References

- Bernanke, B. (2017a). Monetary policy in a low interest rate world: Comment. In *Brookings Papers* on *Economic Activivity*. Brookings Institution.
- Bernanke, B. (2017b). Monetary Policy in a New Era. in Rethinking Monetary Policy in a New Normal.
- Blanchard, O., G. Lorenzoni, and J.-P. L'Huillier (2017). Short-run effects of lower productivity growth. A twist on the secular stagnation hypothesis. *Journal of Policy Modeling* 39-4, 639-649.
- Blanchard, O. J. (2016, January). The US Phillips Curve: Back to the 60s? Policy Briefs PB16-1, Peterson Institute for International Economics.
- Bodenstein, M., J. Hebden, and R. Nunes (2012). Imperfect credibility and the zero lower bound. Journal of Monetary Economics 59(2), 135–149.
- Brayton, F. (July, 2018). sfrb: A small linear version of frb/us.
- Coibion, O., Y. Gorodnichenko, and J. Wieland (2012). The optimal inflation rate in new keynesian models: Should central banks raise their inflation targets in light of the zero lower bound? *The Review of Economic Studies* 79(4), 1371–1406.
- Eggertsson, G. B. and M. Woodford (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity* 34(1), 139–235.
- English, W. B., J. D. López-Salido, and R. J. Tetlow (2015, May). The Federal Reserve Framework for Monetary Policy: Recent Changes and New Questions. *IMF Economic Review* 63(1), 22–70.
- English, W. B., W. R. Nelson, and B. P. Sack (2003, April). Interpreting the Significance of the Lagged Interest Rate in Estimated Monetary Policy Rules. *The B.E. Journal of Macroeconomics* 3(1), 1–18.
- Evans, C. (2017). Low inflation and the symmetry of the 2 percent target. Speech delivered on November 15, 2017, before the UBS European Conference in London, England.
- Fuhrer, J. C. (1994). Optimal monetary policy and the sacrifice ratio. Conference Series; [Proceedings, Federal Reserve Bank of Boston] 38, 43–84.
- Gust, C., E. Herbst, D. López-Salido, and M. E. Smith (2017, July). The empirical implications of the interest-rate lower bound. *American Economic Review* 107(7), 1971–2006.
- Gust, C., D. López-Salido, and S. Meyer (2017). Asymmetric monetary policy and the effective lower bound. Research in Economics 71(3), 441–451.
- Hall, R. (1984). Monetary strategy with an elastic price standard. *Proceedings Economic Policy Symposium Jackson Hole*, 137–167.
- Holston, K., T. Laubach, and J. C. Williams (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics* 108, S59 S75. 39th Annual NBER International Seminar on Macroeconomics.
- Kiley, M. T. (2015, November). Low Inflation in the United States: A Summary of Recent Research. FEDS Notes 2015-11-23, Board of Governors of the Federal Reserve System (U.S.).

- Kiley, M. T. and J. M. Roberts (2017). Monetary policy in a low interest rate world. In *Brookings Papers on Economic Activity*. Brookings Institution.
- Laubach, T. and J. C. Williams (2003). Measuring the Natural Rate of Interest. *The Review of Economics and Statistics* 85(4), 1063–1070.
- Levin, A., V. Wieland, and J. C. Williams (2003, June). The performance of forecast-based monetary policy rules under model uncertainty. *American Economic Review* 93(3), 622–645.
- Levin, A. T., V. Wieland, and J. Williams (1999, January). Robustness of Simple Monetary Policy Rules under Model Uncertainty, pp. 263–318. University of Chicago Press.
- Lindé, J., F. Smets, and R. Wouters (2016). Challenges for Central BanksZ Macro Models. In J. Taylor and H. Uhlig (Eds.), Handbook of Macroeconomics, Volume 3 of Handbook of Macroeconomics, Chapter Volume 2, pp. 829–859. Elsevier.
- McKay, A., E. Nakamura, and J. Steinsson (2016, October). The Power of Forward Guidance Revisited. *American Economic Review* 106(10), 3133–3158.
- Orphanides, A. (2003, July). Historical monetary policy analysis and the Taylor rule. *Journal of Monetary Economics* 50(5), 983–1022.
- Orphanides, A. and J. C. Williams (2002). Robust monetary policy rules with unknown natural rates. *Brookings Papers on Economic Activity* 2002(2), 63–118.
- Reifschneider, D. and J. C. Williams (2000a). Three Lessons for Monetary Policy in a Low-Inflation Era. *Journal of Money, Credit and Banking 32*, 936–66.
- Reifschneider, D. and J. C. Williams (2000b). Three Lessons for Monetary Policy in a Low-Inflation Era. *Journal of Money, Credit and Banking 32*, 936–66.
- Reifschneider, D. L. and J. M. Roberts (2006, September). Expectations formation and the effectiveness of strategies for limiting the consequences of the zero bound. *Journal of the Japanese and International Economies* 20(3), 314–337.
- Rotemberg, J. and M. Woodford (1997). An optimization-based econometric framework for the evaluation of monetary policy. In *NBER Macroeconomics Annual 1997, Volume 12*, pp. 297–361. National Bureau of Economic Research, Inc.
- Rudebusch, G. D. (2006, December). Monetary Policy Inertia: Fact or Fiction? *International Journal of Central Banking* 2(4).
- Taylor, J. B. (1980). Output and price stability: An international comparison. *Journal of Economic Dynamics and Control* 2, 109 132.
- Taylor, J. B. (1982). Policy choice and economic structure. Occassional Paper, Group of Thirty Number 9.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy 39, 195 214.
- Taylor, J. B. (1999, June). The robustness and efficiency of monetary policy rules as guidelines for interest rate setting by the European central bank. *Journal of Monetary Economics* 43(3), 655–679.

- Taylor, J. B. and J. C. Williams (2011). Simple and Robust Rules for Monetary Policy. In B. M. Friedman and M. Woodford (Eds.), Handbook of Monetary Economics, Volume 3 of Handbook of Monetary Economics, Chapter 15, pp. 829–859. Elsevier.
- Williams, J. (2009). Heeding daedalus: Optimal inflation and the zero lower bound. *Brookings Papers on Economic Activity* 40(2 (Fall)), 1–49.
- Williams, J. (2017). Preparing for the next storm: Reassessing frameworks and strategies in a low r-star world. FRBSF Economic Letter, May 8.
- Williams, J. C. (2003). Simple rules for monetary policy. *Economic Review*, 1–12.
- Woodford, M. (2003). Optimal interest-rate smoothing. The Review of Economic Studies 70(4), 861–886.
- Woodford, M. (2012). Methods of policy accommodation at the interest-rate lower bound. *Proceedings Economic Policy Symposium Jackson Hole*, 185–288.
- Yellen, J. (2015). Normalizing monetary policy: Prospects and perspectives. At the "The New Normal Monetary Policy," a research conference sponsored by the Federal Reserve Bank of San Francisco, San Francisco, California.

Table 1: Performance of Policy Rules at the Effective Lower Bound Conditional on Taylor (1999) being at the ELB

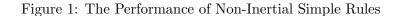
<u> </u>	(1000) 80				
$Macroeconomic\ Outcomes$					
\overline{u}	$RMSD_u$	$\overline{\pi}$	$RMSD_{\pi}$		
6.10	1.85	0.13	2.32		
5.81	1.61	0.29	2.19		
5.81	1.62	0.29	2.19		
5.87	1.70	0.31	2.16		
5.51	1.56	0.59	1.93		
5.29	1.56	0.74	1.82		
6.06	1.82	0.20	2.26		
5.42	1.48	0.62	1.98		
5.46	1.49	0.57	2.01		
5.18	1.67	0.80	1.83		
	$ \overline{u} $ 6.10 5.81 5.81 5.87 5.51 5.29 6.06 5.42 5.46	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		

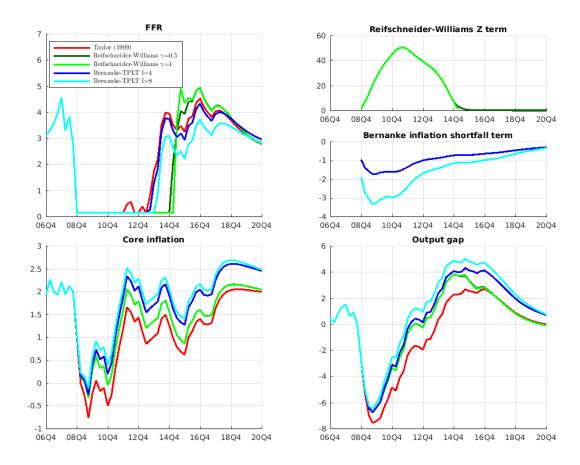
Note: For each rule, we perform a set of 5000 stochastic simulations in which the economy is subject to different realizations of shocks, with each rule subject to the same portfolio of shocks. We use quarters for which Taylor 1999 is at the ELB as an index for bad shock realizations. Within each stochastic simulation, statistics are tabulated for 100 quarters following initialization at steady state and a 40 quarter burn in. The root mean square deviation of unemployment is calculated relative to the natural rate, $u^* = 4.6 + shock$, as $\sqrt{\frac{1}{n} \sum_{j=1}^{n} (u_j - u_j^*)^2}$ where n is the count of all quarters out of 500000 quarters for which Taylor 1999 is at the ELB. Mean unemployment \overline{u} is also conditional on this index. $\overline{\pi}$ and $RMSD_{\pi}$ reflect the corresponding calculations and conditioning but for the four quarter average of headline inflation with respect to 2 percent. Shock realizations are resampled model residuals from 1969 to 2016. Shock draws are governed by a Markov-switching process such that residuals from historical recessions are replayed in sequence without over- or under-sampling rececession quarters relative to the 1969 to 2016 period.

Table 2: Performance of Policy Rules at the Effective Lower Bound Conditional on Respective Rule Prescription at Bound

Policy Rules	Macroeconomic Outcomes			Mean ELB Duration	
	\overline{u}	$RMSD_u$	$\overline{\pi}$	$RMSD_{\pi}$	
Taylor (1999)	6.10	1.85	0.13	2.32	5.79
Reifsch-Williams					
$\gamma = 0.5$	5.44	1.48	0.71	2.03	10.08
$\gamma = 1.0$	5.39	1.46	0.78	2.03	10.95
Benanke-TPLT					
f = 1	5.80	1.66	0.30	2.15	6.07
f = 4	5.20	1.49	0.66	1.84	6.92
f = 8	4.72	1.59	0.96	1.65	8.01
Inertial Taylor	6.59	2.15	-0.02	2.43	7.24
Kiley-Roberts	4.70	1.43	1.48	1.84	15.69
Bernanke-Shadow					
d = e	4.61	1.51	1.49	1.94	22.49
e = 0, d > 0	3.37	2.51	1.84	1.58	24.23

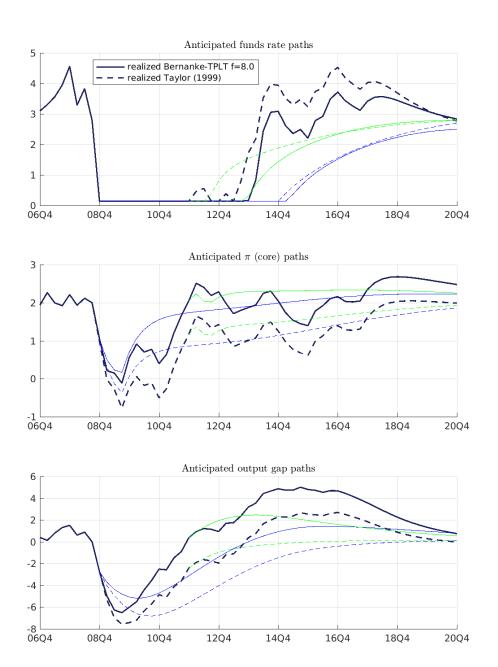
Note: For each rule, we perform a set of 5000 stochastic simulations in which the economy is subject to different realizations of shocks, with each rule subject to the same portfolio of shocks. Reported statistics for each rule are conditional on quarters for which that rule is at the ELB. Within each stochastic simulation, statistics are tabulated for 100 quarters following initialization at steady state and a 40 quarter burn in. The root mean square deviation of unemployment is calculated relative to the natural rate, $u^* = 4.6 + shock$, as $\sqrt{\frac{1}{n}\sum_{j=1}^{n}(u_j - u_j^*)^2}$ where n is the count of all quarters out of 500000 quarters for which the respective rule is at the ELB. Mean unemployment \overline{u} is also conditional on this index. $\overline{\pi}$ and $RMSD_{\pi}$ reflect the corresponding calculations and conditioning but for the four quarter average of headline inflation with respect to 2 percent. Shock realizations are resampled model residuals from 1969 to 2016. Shock draws are governed by a Markov switching process such that residuals from historical recessions are replayed in sequence without over- or under-sampling receession quarters relative to the 1969 to 2016 period.





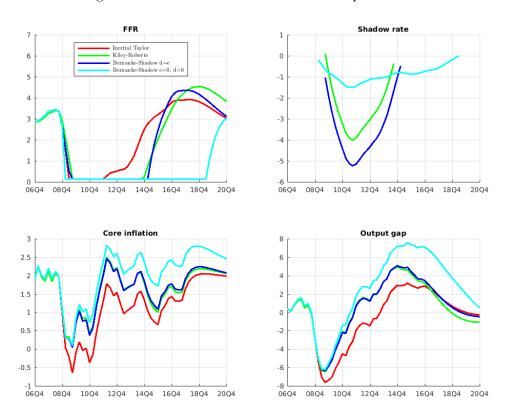
Note: Shown are realized outcomes under different policy rules given historical model residuals from 2006 through 2016 applied as a sequence of shocks to the economy initially in its steady state. There are no additional shock realizations after 2016.

Figure 2: The Performance of Non-Inertial Simple Rules



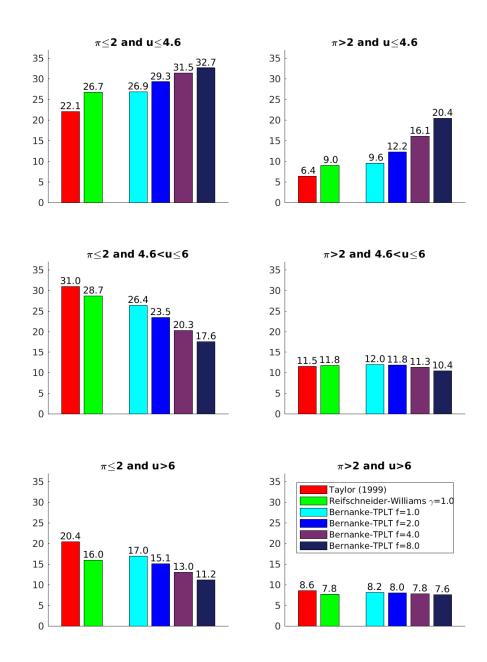
Note: Shown are anticipated outcomes as of 2008:4 [light blue line] and 2011:4 [green] under Bernanke-TPLT with f equals to 8.0 [all solid lines] and Taylor (1999) [all dashed lines], given historical model residuals from 2006 through 2016 applied as a sequence of shocks to the economy initially in its steady state. There are no additional shock realizations after 2016.

Figure 3: The Performance of Inertial Simple Rules



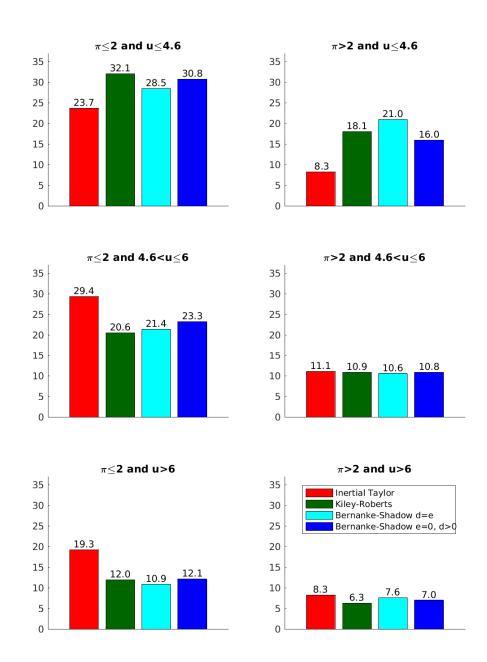
Note: Shown are realized outcomes under different policy rules given historical model residuals from 2006 through 2016 applied as a sequence of shocks to the economy initially in its steady state. There are no additional shock realizations after 2016.

Figure 4: Probabilities of Dual Mandate Outcomes under Non-Inertial Rules



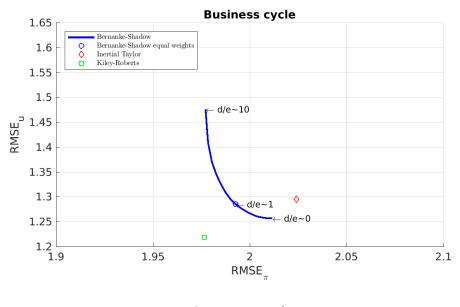
Note: The figure shows the joint distribution of outcomes for core inflation and unemployment under different policy rules. For each rule, we perform a set of 5000 stochastic simulations in which the economy is subject to different realizations of shocks, with each rule subject to the same vector of shocks. Reported outcomes for all rules are conditional on quarters when Taylor (1999) is at the ELB and 8 quarters after, taken as an index for bad shock realizations and subsequent recovery.

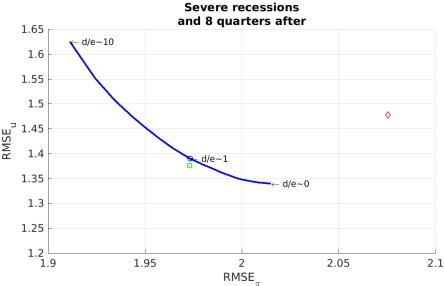
Figure 5: Probabilities of Dual Mandate Outcomes under Inertial Rules



Note: The figure shows the joint distribution of outcomes for core inflation and unemployment under different policy rules. For each rule, we perform a set of 5000 stochastic simulations in which the economy is subject to different realizations of shocks, with each rule subject to the same portfolio of shocks. Reported outcomes for all rules are conditional on quarters when Taylor (1999) is at the ELB and 8 quarters after, taken as an index for bad shock realizations and subsequent recovery.

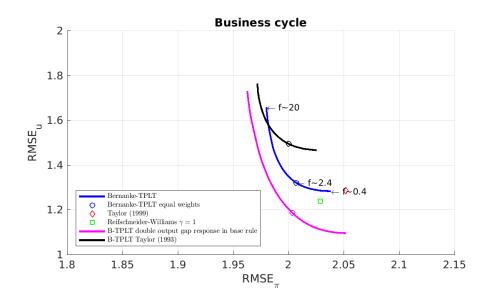
Figure 6: Policy Frontiers: Bernanke's Shadow Rate Rule

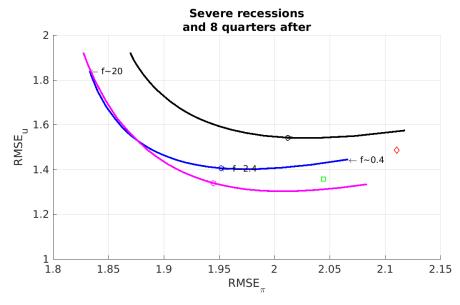




Note: The figure shows the loss frontier for Bernanke-Shadow rule where $RMSD_{\pi}$ is calculated for the four quarter average of headline inflation with respect to 2 percent and $RMSD_u$ is similarly for unemployment with respect to the natural rate, based on a set of 5000 stochastic simulations for each rule and parameterization. We optimize over the d/e ratio in Bernanke-Shadow holding fixed the rule's coefficients on inflation, output gap, and lagged interest rate. Because the inertial Taylor rule and Bernake-Shadow rule differ from Kiley-Roberts in all periods even apart from the ELB, we report results for all periods (top panel) and results conditional on when Taylor (1999) is binding and some quarters after (middle and bottom panels), which we use as indices for bad shock realizations and subsequent recovery.

Figure 7: Policy Frontiers: Bernanke's Temporary PLT Rule





Note: The figure shows the loss frontier for Bernanke-TPLT where $RMSD_{\pi}$ is calculated for the four quarter average of headline inflation with respect to 2 percent and $RMSD_u$ is similarly for unemployment with respect to the natural rate, based on a set of 5000 stochastic simulations for each rule and parameterization. We optimize over the f coefficient in Bernanke-TPLT holding fixed the rule's coefficients on inflation and output gap. Because we implement Bernanke-TPLT and Reifschneider-Williams as augmented versions of Taylor (1999), any differences among these rules always stem from the ELB.

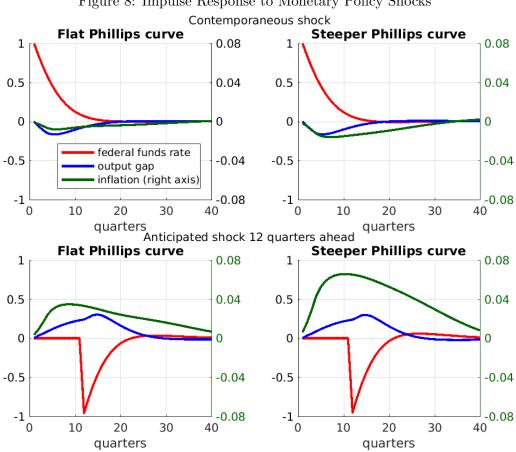
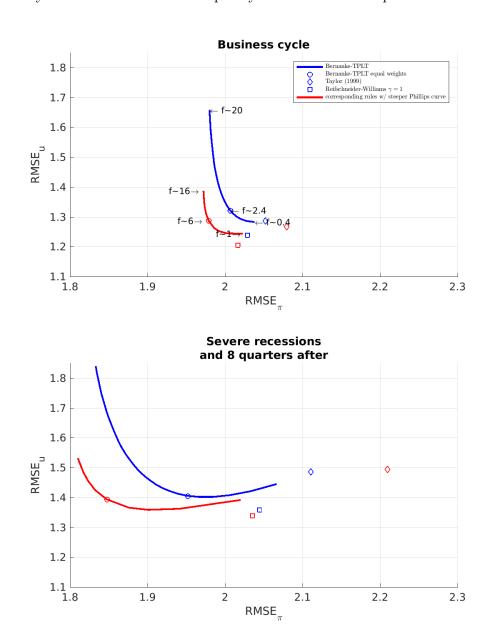


Figure 8: Impulse Response to Monetary Policy Shocks

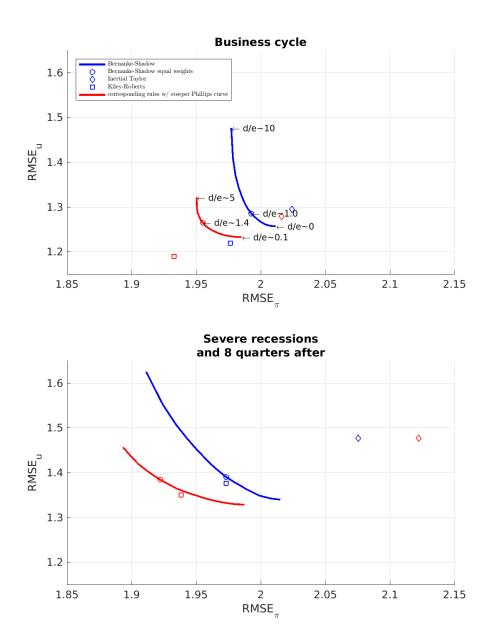
Note: Obtained from simulations using the small FRB/US model.

Figure 9: Policy Frontiers: Bernanke's Temporary PLT Rule with Steeper Short-Run Phillips Curve



Note: The figure shows the loss frontier for Bernanke-TPLT where $RMSD_{\pi}$ is calculated for the four quarter average of headline inflation with respect to 2 percent and $RMSD_u$ is similarly for unemployment with respect to the natural rate, based on a set of 5000 stochastic simulations for each rule and parameterization. We optimize over the f coefficient in Bernanke-TPLT holding fixed the rule's coefficients on inflation and the output gap. Because we implement the Bernanke-TPLT and Reifschneider-Williams rules as augmented versions of the Taylor (1999) rule, any differences among these rules always stem from the ELB.

Figure 10: Policy Frontiers: Bernanke's (Shadow Rate) Rule with Steeper Short-Run Phillips Curve



Note: The figure shows the loss frontier for the Bernanke-Shadow rule where $RMSD_{\pi}$ is calculated for the four quarter average of headline inflation with respect to 2 percent and $RMSD_u$ is similarly for unemployment with respect to the natural rate, based on a set of 5000 stochastic simulations for each rule and parameterization. We optimize over the d/e ratio in the Bernanke-Shadow rule holding fixed the rule's coefficients on inflation, the output gap, and the lagged interest rate. Because the inertial Taylor rule and the Bernanke-Shadow rule differ from the Kiley-Roberts rule in all periods even apart from the ELB, we report results for all periods (top panel) and results conditional on when the Taylor (1999) rule is binding and some quarters after (middle and bottom panels), which we use as indices for bad shock realizations and subsequent recovery.