Day 8: Convolutional Neural Networks

Summer STEM: Machine Learning

Department of Electrical and Computer Engineering
NYU Tandon School of Engineering
Brooklyn, New York

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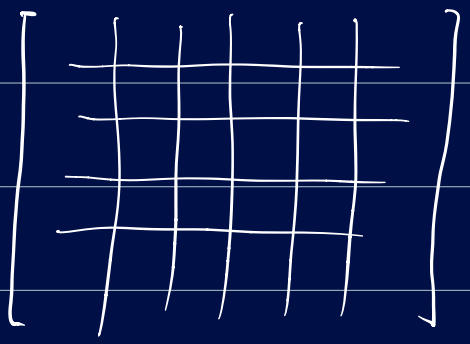
Outline

Better performance with images

- Encoding locality
- How does an MLP see an image?
- Is this how we see images?

Examples: Lena & Mandrill

How are images stored?



- pixels 0 – 255
  - just a number
  - represents intensity
- 3 matrices to store Red / Green / Blue channels

- In deep learning, we stack these 3 matrices in one object: a three dimensional array (Tensor)

features X Tensors

$X[i, j, k]$

↑ width ↑ height ↖ depth? channels R/G/B

MLPs only accept vector-valued inputs

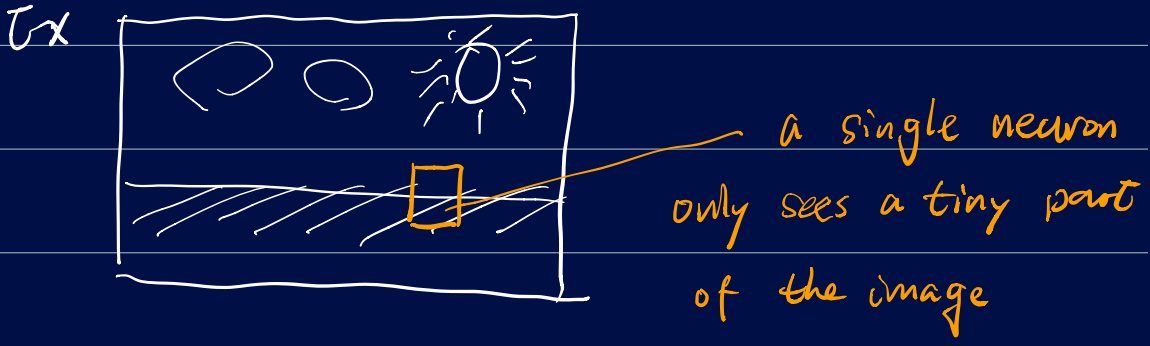
- Flatten the Tensor

Ex. MNIST numbers $(28, 28, 1) \longrightarrow (28^2, 1)$

Then, "each pixel is seen by EVERY neuron" due to fully-connected layer

· Receptive field

A retinal neuron (ganglion cell) only reacts to a certain part of the image with an area called receptive field

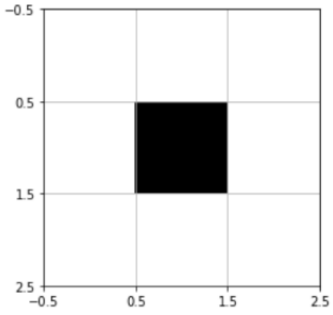


Outline

Images in Computer

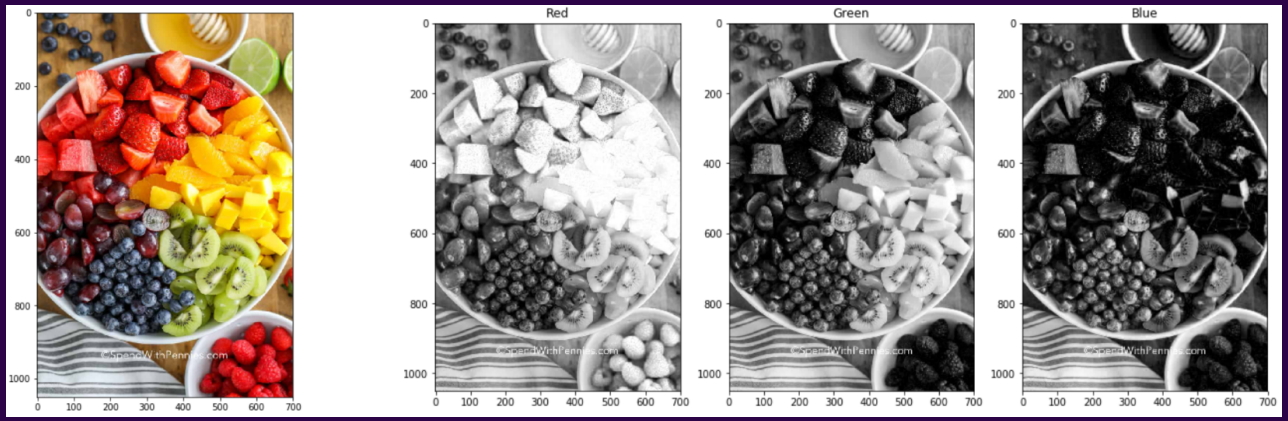
- Images are stored as arrays of quantized numbers in computers
- Gray scale image: 2D matrices with each entry specifying the intensity (brightness) of a pixel
  - Pixel values range from 0 to 255, 0 being the darkest, 255 being the brightest

```
[[255 255 255]
 [255   0 255]
 [255 255 255]]
```



Color Images

- Color image: 3D array, 2 dimensions for space, 1 dimension for color
  - Can be thought of as three 2D matrices stacked together into a cube, each 2D matrix specify the amount of each color: Red ,Green ,Blue value at each pixel



- Shape of this image: (1050,700,3)
- There are 1050x700 pixels, 3 channels: R,G,B

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Limitations of Fully Connected Network

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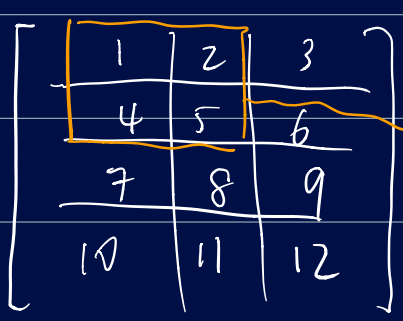
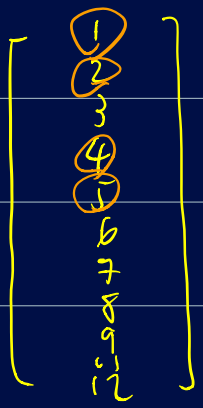
- In Fashion MNIST, we used a fully connected network, in which each neuron in the hidden layer is connected to all 28x28 = 784 pixels
- Higher definition images often contain millions of pixels → It is not practical to use fully connected network
- Fully connected network treat each individual pixel as a feature, it does not utilize the positional relationship between pixels



Represent "similar things"



Convolution

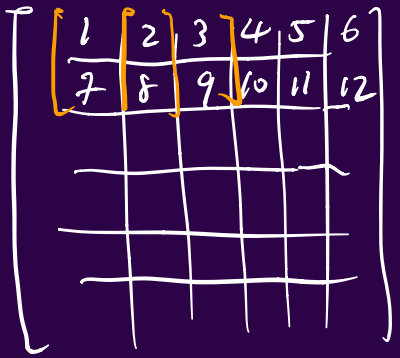
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \rightarrow 6$$

$$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

1. Stride : how much you slide the filter
2. size of the filter

- Introducing a new operation: Convolution ?
  - filter

$$\begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix}$$



$$w_1 + 2w_2 + 7w_3 + 8w_4 = o_1$$

$$2w_1 + 3w_2 + 8w_3 + 9w_4 = o_2$$

$$\begin{bmatrix} o_1 & o_2 \cdots \\ \vdots & \end{bmatrix}$$

Convolution

- Introducing a new operation: Convolution
- An operation on an image(matrix) X with a kernel W

Convolution

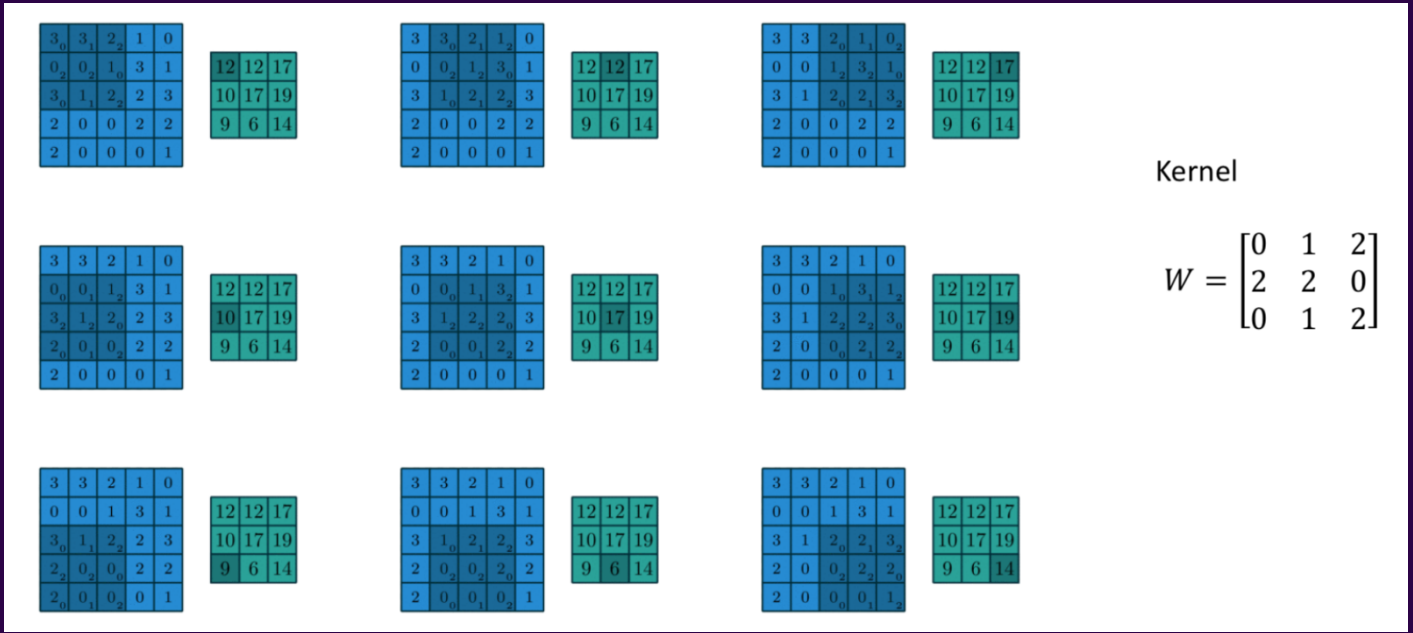
- Introducing a new operation: Convolution

- An operation on an image(matrix) X with a kernel W
- $Z = X \circledast W$

Some Animations, Source: https://towardsdatascience.com

Some Animations, Source: https://cs231n.github.io/convolutional-networks/

Example of a Convolultion



Why Convolution?

- With convolution, each output pixel depends on only the neighboring pixels in the input

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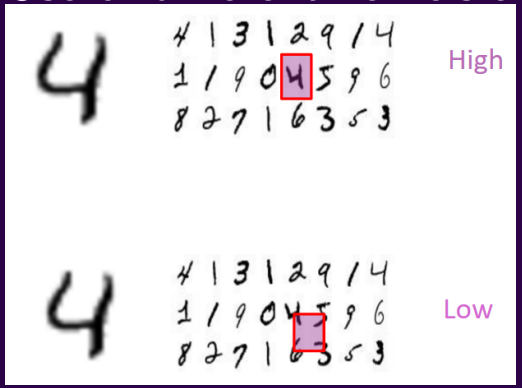
- This allows us to learn the positional relationship between pixels
- Use of different kernels allows us to detect features



Convolution for Multiple Channels

- A kernel for each channel. Could be same kernel, or different
- Perform a convolution for each of the channel, with the respective kernel
- Sum the results

Feature Maps

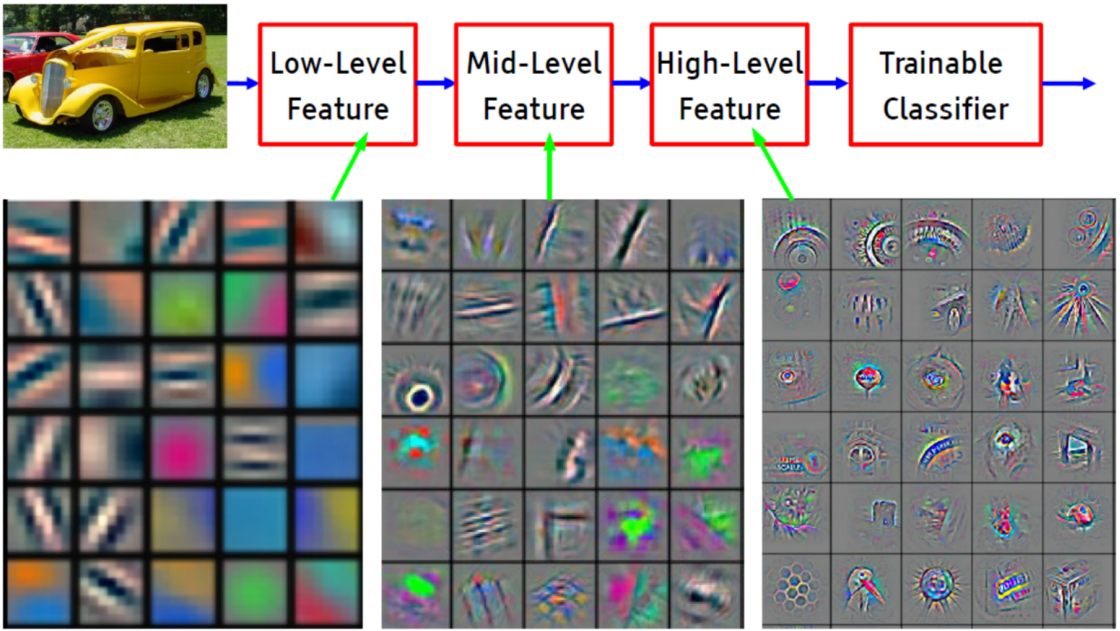


Figure: Figure from Y. LeCun Presentation, "What's wrong with deep learning?". Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013].

Demo: Fashion MNIST

Open `demo_fashion_mnist.ipynb`

Outline

Recap: how to regularize linear models?

- $+ \lambda \|w\|_p^2$
- For polynomial features.

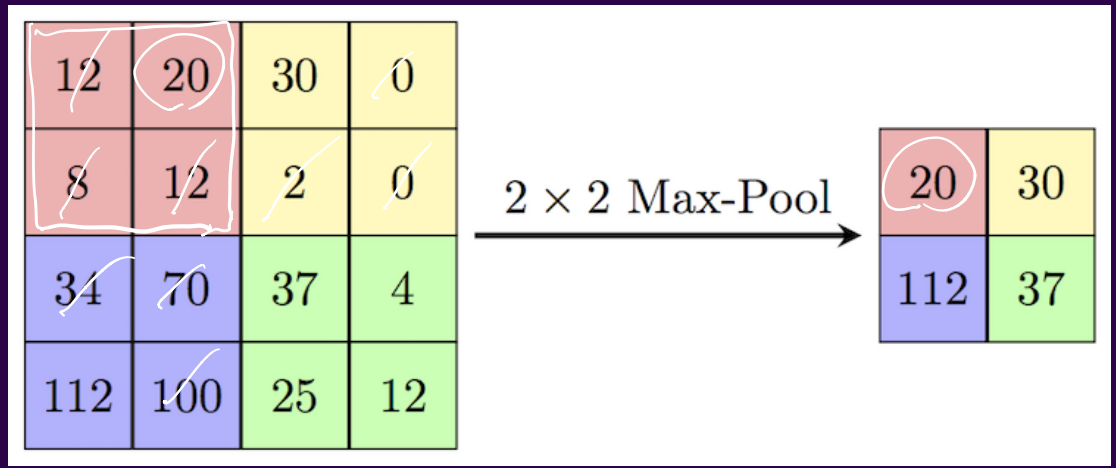
$\phi(x) = [1 \; x \; x^2 \ldots x^M]^T$

$w^T \phi(x)$

- limit the degree of the polynomial M

Max-Pooling

- Down-samples the inputs
- Provides translation invariance. Why?
- Apply after activation!



Batch Normalization

Normalized input x' has zero mean and unit std-dev

Recap, given input

$$\bar{x}' = 0 \qquad \sigma_{x'} = 1$$

NOT $\frac{x}{\|x\|}$

$$x' = \frac{x - \bar{x}}{\sigma_x}$$

σ_x std-dev $= \sqrt{\sigma_x^2}$ variance

- We normalize the inputs to the network. Why not do that for the inputs to the hidden layers?
- Batch norm: normalize the inputs to a layer for each mini-batch.
- Apply before activation!

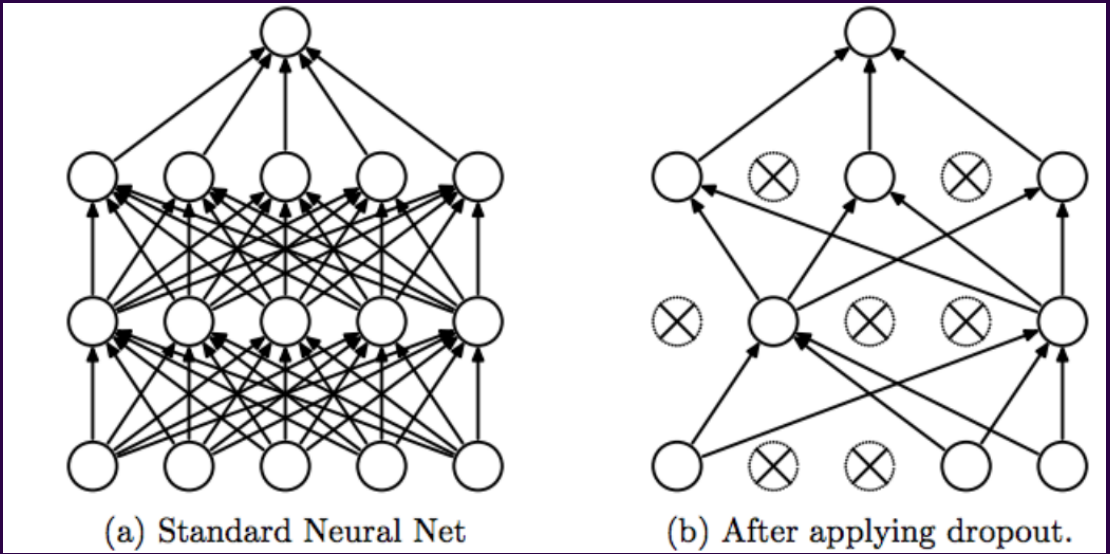
$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

For BN

$$\frac{1}{B} \sum_{i=1}^{B} x_i$$

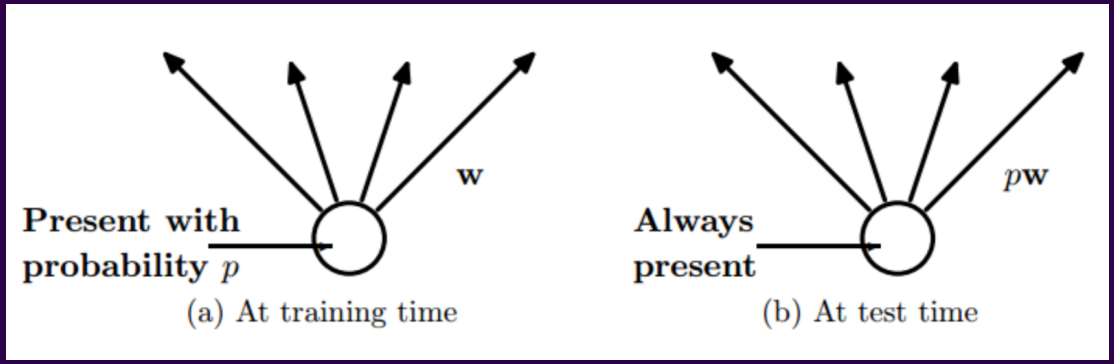
Dropout

- Patented by Google
- Randomly disable neurons and their connections between each other.



Dropout

- In train() mode, each neuron is present with probability p.
- In eval() mode, multiply the weights with p.
- Apply after activation!



Lab: Cats vs. Dogs

Open `lab_cats_vs_dogs.ipynb`