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# Time-varying pattern causality inference in global stock markets

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ABSTRACT

Causality analysis can reveal the intrinsic interactions in financial markets. Though Granger causality test and transfer entropy method have successfully determined positive and negative causal interactions, they fail to reveal a more complex causal interaction, dark causality. Moreover, the causal relationship between variables may be time-varying. Thus, in this work, we are dedicated to determining the nature of causal interaction and explore the time-varying causality in global stock markets. To achieve this goal, pattern causality (PC) theory, cross-convergent mapping (CCM) theory, the sliding window method and complex networks are applied. By them, three causal interactions with different strength are revealed in global stock markets, and the causal strength is time-varying in different periods both in simulated systems and financial markets. While the dominant causal interaction is stable except for some stock pairs in frontier and emerging markets. In total, we determine the positive dominant causality in global stock markets; that is, the overall consistent trend among stocks can be explored. Additionally, we discover some exceptions that show negative dominant causality, where the reverse trend can be revealed among them; moreover, their dominant causality is time-varying. These uncertainties should receive great attention from investors and government managers.

## 1. Introduction

The study of the relationships between multivariate time series is of great significance to reveal their dynamic laws, and several methods has been used, such as correlation, cointegration, and causality. Among these, correlation and causality are widely used in finance fields. For example, correlation analysis can explore the structure and the dynamic transmission mechanisms to measure the systemic risk in stock markets (Acemoglu, Ozdaglar, & Tahbaz-Salehi, 2015; Bein & Tuna, 2015; Chen, Han, & Qiao, 2019; Das, Demirer, Gupta, & Mangisa, 2019; Neaime, 2012; Sun, Wang, Yao, Li, & Li, 2020). Moreover, many studies have built networks to detect the characteristics based on correlation (Gebarowski, Oswiecimka, Watorek, & Drozdz, 2019; Huang, An, Gao, Hao, & Liu, 2015; Song, Tumminello, Zhou, & Mantegna, 2011; Wen, Yang, & Zhou, 2019). Cointegration analysis describes the nonlinear adjustment mechanism between stocks, where stationary equilibrium relationships can be revealed between partial stocks (Ausloos, Zhang, & Dhesi, 2020; Caporale, You, & Chen, 2019; Shi, Ahmed, & Shi, 2019), which considers the long term equilibrium relationship, while fails to uncover the hidden dynamics behind the equilibrium relationship, and it heavily relies on the time length. Furthermore, causality analysis is often used to detect the volatility between variables (Balcilar & Ozdemir, 2013; Wang, Si, Chen, Xie, & Chevallier, 2020; Wang, Wang, Ye, Pei, & Li, 2020; Zhang, Qin, & Liu, 2019; Tang, Xiong, Luo, & Zhang, 2019). Which can reveal the intrinsic laws between variables.

Several models have been proposed to detect causality. Among them, the linear Granger causality test is a traditional method and it is widely used in economics, finance, ecological environmental studies and so on (Li, Zhang, & Yuan, 2019; Vyrost, Lyocsa, & Baumohl, 2015; Zhuo, 2011). As we realize that nonlinearity is ubiquitous in practice applications, nonlinear Granger causality test was proposed, which has successfully revealed nonlinear causal relationship in diverse fields (Breitung & Candelon, 2006; He, 2020; Tao & Feng, 2016; Zhao, Wen, & Wang, 2020). However, both of them are based on prediction, an important premise is separability. Namely, if variable $X$ Granger-causes variable $Y$, the prediction of target variable $Y$ declines if $X$ is removed from the universe of all possible causative variables. However, in real nonlinear complex systems, each variable could carry all information of the system. Thus, nonlinear systems are generally nonseparable. Transfer entropy is another tool that can detect causality based on the

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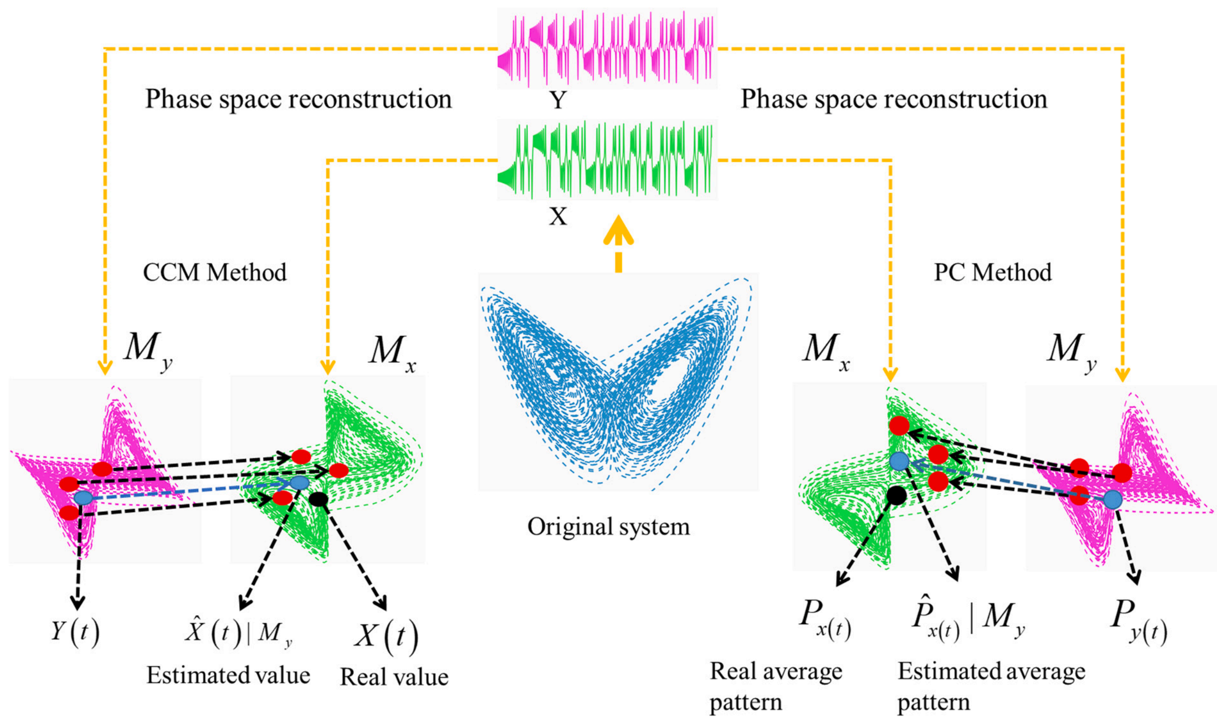



**Fig. 1.** Sketch of the CCM and PC methods to detect the causality from X to Y. A Lorenz system is applied to show the main procedures of the CCM and PC methods. The original system is constituted by the dynamics of variables X, Y, and Z. Based on these steps we can also detect the causality to variable Z. Note: X, Y and Z are time series, $M_x$and$M_y$are the attractors reconstructed by the time series of X and Y. And the blue dot is any point (for CCM) or pattern (for PC) on$M_y$, its neighbors are represented by red dots, the black dot is the point or pattern on $M_x$, whose time index is correspond to point or pattern on $M_y$. And the directed dotted line between points denotes the one-to-one map. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

information flowing between variables, and it is essentially an extension of the Granger causality test (Bekiros, Nguyen, Sandoval, & Uddin, 2017; Gao & Su, 2019; Papana, Kyrtsou, Kugiumtzis, & Diks, 2016). From perspective of dynamics, Sugihara proposed cross-convergent mapping (CCM) whose core strategy is that the time series variables are causally linked if they are from the same dynamical system, namely, they share a common attractor. Thus, each variable can identify the state of another (Sugihara, May, & Ye, 2012). The CCM method has a satisfactory ability to detect causality in complex nonlinear systems, which solves the shortage of the Granger causality test; moreover, it can distinguish a causal relationship from correlation. However, in real systems, the influence of noise is inevitable; therefore, Stavroglou et al. proposed the pattern causality algorithm (PC) on the basis of the CCM, which used the symbolic dynamics to decease the impact of noise (Stavroglou, Pantelous, Stanley, & Zuev, 2019). The PC algorithm can not only detect the causality, but it can also quantify causal strength.

Previous studies often detect the causality over the whole time period, which ignores that causality is time-varying between some time series variables. Some studies have obtained time-varying causal relationships based on the Granger causality test (Zhang & Broadstock, 2016; Balcilar and Ozdemir, 2013; Nataf & De Moor, 2019). However, a more complex causal relationships cannot be determined based on Granger causality test, such as dark causality.

Stock markets are important markets in the global economy, and they are closely connected to other markets. Yan and Xu (2013) studied the relationship between Chinese stock markets and the international commodity futures market based on correlation analysis. Ren, Ji, Cai, Li, and Jiang (2016) studied the dynamic lead-lag relationship between stock indices and their derivatives. Zeren and Koc (2016) revealed the causal relationship between stock markets and foreign exchange markets. Another important issue is to predict the trend of a stock. Hassan, Nath, and Kirley (2007) proposed a fusion model that combines the HMM, ANN and GA to forecast a stock. Chong, Han, and Park (2017) adopted the deep learning method to make predictions. In addition, the detection of the system risk in a stock market is also significant. Caetano and Yoneyama (2011) built a model based on the catalytic chemical reaction model to evaluate the system risk in stock markets. Mensi, Hammoudeh, Shahzad, and Shahbaz (2017) used the decomposition-based copula method to model the system risk in stock markets and oil markets. However, it is helpful to detect the system risk and make predictions by revealing the inherent causal relationships in stock markets.

Thus, in this paper, we are dedicated to revealing the causality from stock markets and detecting the characters of the time-varying causality among them, including the nature of the causality and the strength of the causality. To achieve the goal, the PC method, the CCM method, complex networks and sliding windows are applied here. The PC method can determine the nature and the strength of causality. The CCM method can distinguish causality from correlation. Complex networks can provide good visualizations. Finally, a sliding window has a good effect when extracting time series segments and can provide a solid foundation for investigating the structural characteristics of the whole time series (Scruggs, 2007; Gao et al., 2014). In this paper, we first study the simulated deterministic systems, and determine the time-varying causality. Then, we study the practical system and global stock markets. To reveal their inherent causal laws, we assess both the whole period and different time periods and build the causal networks. By analyzing the topological characteristics, we reveal the causal relationships in the markets.

The remainder of this paper is organized as follows. Sections 2 and 3 describe the methods and data. In Section 4, we investigate the causal relationships in the global stock markets. Section 5 summarizes our main findings.

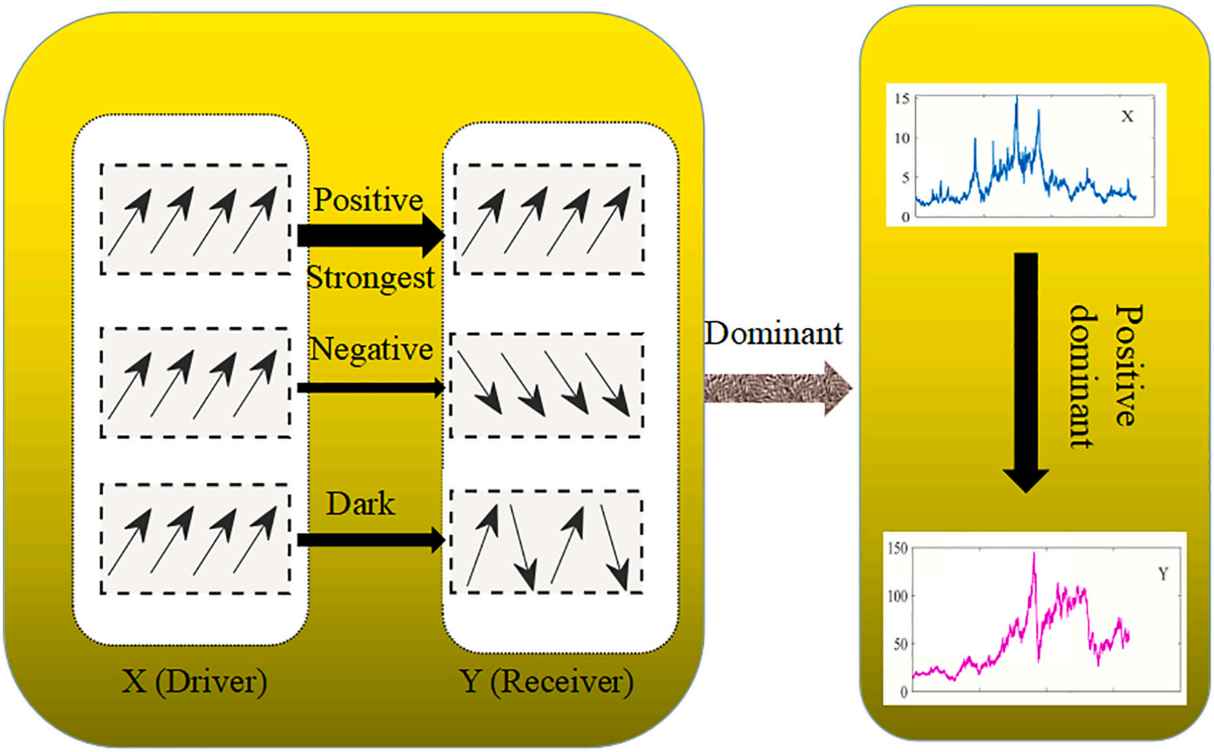



**Fig. 2.** An example of causality from time series variable X to Y. From the left part, positive causality can be determined when the increase of X causes the same trend for Y (Consistent). Negative causality can be determined when the increase of X causes the inverse trend for Y (Inverse). Dark causality can be determined when the increase of X causes the fluctuation of Y (Disorder). From the right part, dominant causality can be determined from X to Y according to causal strength.

## 2. Methods

Several methods have been proved to be able to detect the causality between multivariate time series, such as the Granger causality test (Vyrost et al., 2015) and transfer entropy (Bekiros et al., 2017). However, cross-convergent mapping (CCM) and pattern causality (PC) achieve satisfactory performance in nonlinear systems and have been applied in ecology and finance (Stavroglou et al., 2019; Wu et al., 2021; Sugihara et al., 2012). In this paper, 46 MSCI stock indices can be regarded as a high-dimensional nonlinear system. Thus, both CCM and PC are applied in this paper, and PC is more efficient at quantifying the strength of the causality. In addition, the sliding window is applied to detect the evolution of the causality over time, and the length of window can be determined by $L/2 \le l_{win} \le L$, where $L$ is the length of the data (Finkle, Wu & Bagheri, 2018). Moreover, complex networks are also used here, which are a good tool for visualization. The main steps of the CCM and PC methods are shown in Fig. 1.

### 2.1. Cross convergent mapping

CCM was proposed by George Sugihara (Sugihara et al., 2012). It corrects the shortage of the Granger causality test when detecting the causality in nonlinear systems. Consider the causality from time series $X$ to $Y$ (here, variable $X$ is called the causal driver and variable $Y$ is called the causal receiver). First, according to the theory of space reconstruction, we obtain the attractors$M_x$and$M_y$built from $X$ and $Y$, respectively. Each point on $M_x$ is $x(t) = [X(t), X(t-\tau), \ldots, X(t-(E-1)\tau)]$ for $t = 1 + (E-1)\tau$ to $t = L$($L$ is the length of $X$), where $E$ is the embedding dimension, and $\tau$ is the time lag. In this paper, the false nearest neighbors algorithm is used to determine $E$ and $\tau$ simultaneously (Krakovská, Mezeiová, & Budáčová, 2015). Then, we estimate $X$ according to $M_y$, where $\widehat{X}(t)|\mathrm{M}_y$. To achieve this goal, we begin by finding the $E$+1 nearest points to $y(t)$, which is based on the Euclidean distance, and we keep the time indices. These time indices corresponding to the nearest neighbors to $y(t)$ on $M_y$ are used to identify the points (neighbors) in $X$ to estimate $X$ from a locally weighted mean of the $E$+1 $X$ values.

$$\widehat{X}(t)|M_y = \sum_{i=1}^{E+1} w_i X(t_i) \tag{1}$$

$$\text{where } w_i = \frac{k_i}{\sum_{j=1}^{E+1} k_j}, \tag{2}$$

$$k_i = exp\{-d[x(t), x(t_i)]/d[x(t), x(t_1)]\}, \text{and } d: \text{the Euclidean distance.} \tag{3}$$

If there is causality from $X$ to $Y$, the estimated $\widehat{X}(t)|\mathrm{M}_y$ will converge to real values of $X$. We use the correlation coefficient to weight the convergence.

### 2.2. Pattern causality

Although CCM can effectively detect causality, it cannot quantify causal strength. Therefore, PC theory is also applied and it can quantify causal strength (Stavroglou et al., 2019). PC theory combines the symbolic dynamics that can decrease the impact of noise, especially in real systems. According to PC, the nature of the causality between each variable pair may be one of three types, namely, positive causality, negative causality and dark causality. Positive causality means the 'mutualism', negative causality means the 'competition', and the dark causality is a more complex relationship that is not 'mutualism' or 'competition', as shown in Fig. 2. Furthermore, the strongest causality (by comparing causal strengths) is the dominant causality that is expressed in reality.

To detect the causality from time series $Y$ to $X$, first we should reconstruct the attractors $M_y$ and $M_x$, respectively. Then, we need to calculate the average pattern of each $x(t)$ on $M_x$ according to its nearest

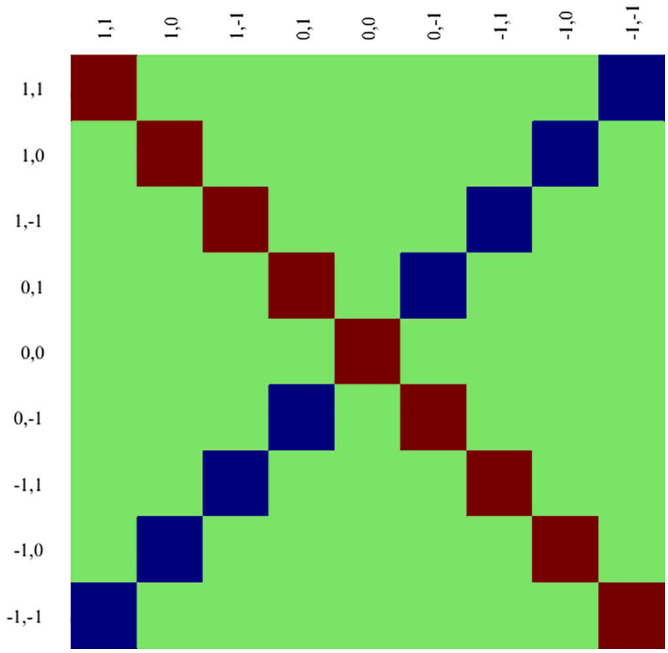



**Fig. 3.** The CS matrix when $E = 3$.

neighborhoods and keep the time indices. These indices are used to estimate the average pattern of $y(t)$. Then, we calculate the percentage of the occasions that the contemporaneous estimated average pattern of $y(t)$ equals the real average pattern of $y(t)$, namely, causal strength (CS) from $Y$ to $X$. The value of CS ranges from 0 to 1, where a smaller value indicates weaker causality and a larger value indicates stronger causality.

The average pattern of $x(t)$ can be determined as (4, 5, 6, 7, 8), and the average pattern of $y(t)$ can be determined in the same way.

$$P_{x(t)} = f\left(H_{x(t)}\right) \tag{4}$$

$$H_{x(t)} = \sum_{i=1}^{E+1} w_i^x h_i^x \tag{5}$$

$$h_i^x = \left(\frac{X(t_i - \tau) - X(t_i)}{X(t_i)}, \cdots, \frac{X(t_i - (E-1)\tau) - X(t_i - (E-2)\tau)}{X(t_i - (E-2)\tau)}\right) \tag{6}$$

$$w_i^x = \frac{e^{-d(x(t), x(t_i))}}{\sum_i e^{-d(x(t), x(t_i))}} \tag{7}$$

$$x(t_i) = \left(X(t_i), X(t_i - \tau), \cdots, X(t_i - (E-1)\tau)\right) \tag{8}$$

where $P_{x(t)}$ represents the average pattern of $x(t)$, and $f$ is a function that converts a vector to another vector whose elements are $-1$, 1, or 0, such as $f(0.3, 4.8) = (1,1), f(-0.9, 2) = (-1,1), f(1.3, 0) = (1,0)$.

In addition, the estimated average pattern of $y(t)$ is determined as (9, 10, 11, 12).

$$\hat{P}_{y(t)} = f\left(\hat{H}_{y(t)}\right) \tag{9}$$

$$\hat{H}_{y(t)} = \sum_{i=1}^{E+1} w_i^x \hat{h_i^y}, \text{where} \quad d: \text{Euclidean distance} \tag{10}$$

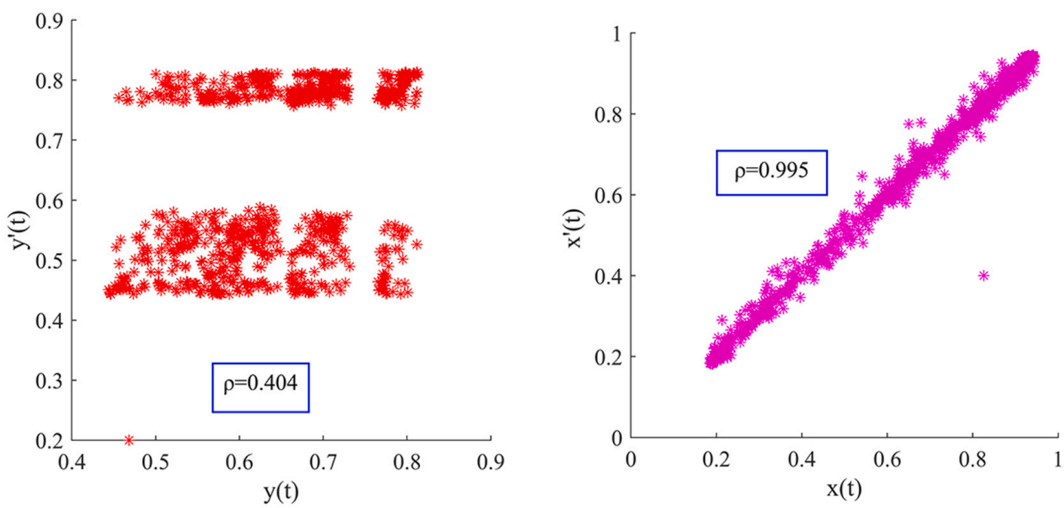
$$h_i^{\hat{y}} = \left(\frac{\hat{Y}(t_{x_2}) - \hat{Y}(t_{x_1})}{\hat{Y}(t_{x_1})}, \cdots, \frac{\hat{Y}(x_{t_E}) - \hat{Y}(x_{t_{E-1}})}{\hat{Y}(x_{t_{E-1}})}\right) \tag{11}$$

$$\hat{y}(t_i) = \left(\hat{Y}(t_{x_i}), \hat{Y}(t_{xi} - \tau) \cdots, \hat{Y}(t_{xi} - (E-1)\tau)\right) \tag{12}$$

where $t_{x_i}$ is the time index corresponding to the nearest neighbor of $x(t)$.

Repeat the procedure for every $x(t)$ in $M_x$, and fill the CS (Causal strength) matrix with the ratio of the estimated average pattern to the real average pattern. Fig. 3 shows the CS matrix for $E = 3$, and it is the same for other situations. The column vectors represent the average patterns of $x(t)$ in its nearest neighborhoods, and the row vectors represent the average pattern of $y(t)$ (contemporaneous to$x(t)$) in its nearest neighborhoods. Each element of the matrix represents the ratio of the estimated average pattern to the real average pattern.

The main diagonal elements of the matrix and the back-diagonal elements of the matrix correspond to the positive causality (brown areas) and the negative causality (blue areas), respectively, while the other elements correspond to the dark causality (green areas). Naturally,




(a) The convergence from $y'(t)$ to $y(t)$

(b) The convergence from $x'(t)$ to $x(t)$

**Fig. 4.** The convergence from the estimated values to real values. The correlation coefficient is applied to weight the convergence ($\rho$).

**Table 1**
The correlation and causality between ISR and AUT.

| Driver | Receiver | Correlation | Causality |
|---|---|---|---|
| ISR | AUT | −0.099 | 0.16 |
| AUT | ISR | | 0.62 |

Note: The direction of causality is from the 'Driver' to 'Receiver'.

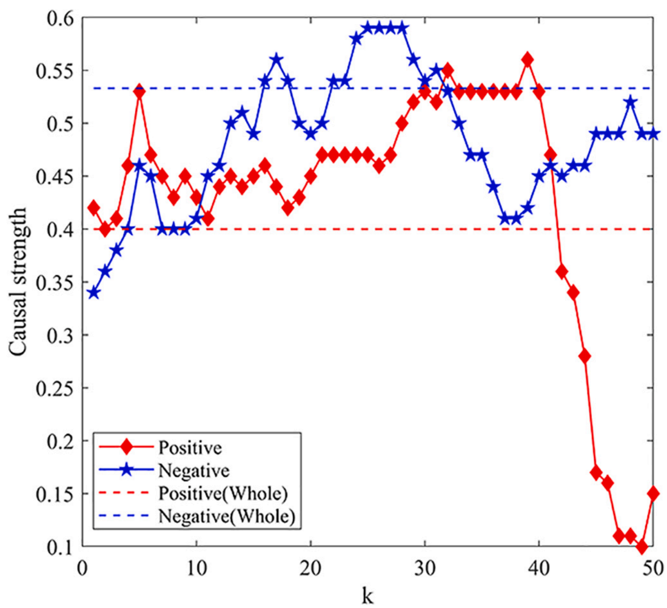



**Fig. 5.** The causality from Y to X. The horizontal axis represents the k-th sliding window. The vertical axis is causal strength. The horizontal line represents the average causality, which is obtained from the whole period.

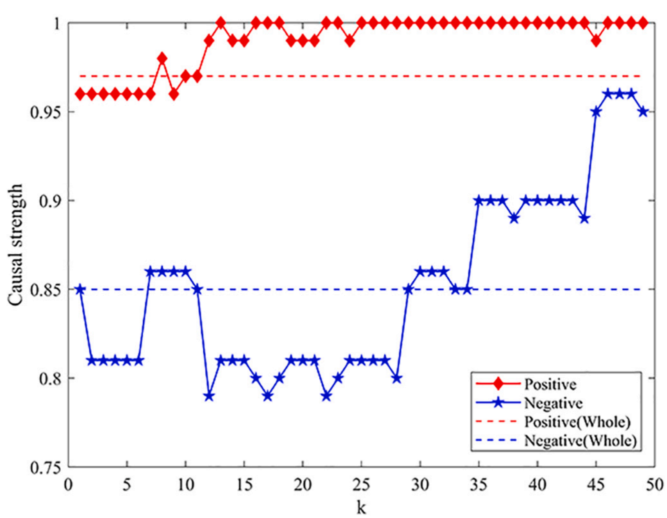



**Fig. 6.** The causality from $Y_1$ to $Y_2$. The horizontal axis represents the k-th sliding window. The vertical axis is causal strength. The horizontal line represents the average causal strength, which is obtained from the whole period.

we can calculate the three causalities from $Y$ to $X$ as follows.

Positive causal strength from $Y$ to $X$:

$$CS(positive) = \frac{1}{n}\sum_{main}(matrix(CS)) \tag{13}$$

Negative causal strength from $Y$ to $X$:

**Table 2**
The selected MSCI stock indices in global stock markets.

| Abbreviation | Region | Max. | Min. | Mean | SD |
|---|---|---|---|---|---|
| ISR | Asia | 221.69 | 166.38 | 195.03 | 10.93 |
| AUS | Oceania | 883.08 | 698.13 | 802.61 | 31.35 |
| AUT | Europe | 1723.39 | 981.85 | 1285.23 | 174.84 |
| CAN | North America | 1833.1 | 1399.50 | 1674.57 | 70.89 |
| DNK | Europe | 10,773.22 | 7733.53 | 9389.82 | 657.59 |
| FIN | Europe | 593.33 | 438.74 | 529.77 | 35.25 |
| FRA | Europe | 2085.24 | 1528.25 | 1841.51 | 125.22 |
| DEU | Europe | 2545.59 | 1773.21 | 2105.56 | 174.08 |
| HK | Asia | 13,153.68 | 9346.12 | 11,666.44 | 786.35 |
| IRL | Europe | 222.91 | 151.37 | 189.36 | 14.82 |
| ITA | Europe | 323.8 | 217.2 | 266.74 | 24.76 |
| JPN | Asia | 3725.75 | 2774.19 | 3200.71 | 187.89 |
| NZL | Oceania | 172.12 | 128.35 | 145.29 | 8.7 |
| NOR | Europe | 2869.7 | 2053.0 | 2463.54 | 192.52 |
| PRT | Europe | 76.59 | 57.88 | 67.26 | 4.5 |
| SWE | Europe | 8450.64 | 6354.2 | 7364.67 | 471.87 |
| CHE | Europe | 6050.16 | 4586.46 | 5324.19 | 308.03 |
| GBR | Europe | 1316.1 | 1003.45 | 1150.27 | 60.9 |
| USA | North America | 2971.24 | 2129.36 | 2555.15 | 202.38 |
| SGP | Asia | 4511.22 | 3164.02 | 3853.26 | 261.74 |
| RUS | Europe | 792.8 | 497.24 | 619.3 | 59.29 |
| EGY | Africa | 691.15 | 457.28 | 582.27 | 42.04 |
| IDN | Asia | 965.35 | 677.79 | 821.58 | 55.12 |
| MYS | Asia | 419.11 | 316.64 | 359.178 | 24.38 |
| THA | Asia | 526.56 | 362.829 | 452.718 | 40.64 |
| CHN | Asia | 101.3 | 58.5589 | 79.178 | 8.9 |
| MEX | Latin America | 6145.47 | 4001.99 | 5111.78 | 514.68 |
| PHL | Asia | 623.99 | 447.22 | 536.28 | 34.88 |
| BRA | Latin America | 2393.48 | 1561.00 | 1997.03 | 194.17 |
| CHI | Latin America | 2182.26 | 1164.23 | 1722.98 | 208.21 |
| TW | Asia | 417.46 | 304.9 | 364.51 | 22.56 |
| COL | Latin America | 757.91 | 532.33 | 636.80 | 44.98 |
| CZE | Europe | 343.05 | 236.43 | 289.38 | 23.26 |
| GRC | Europe | 33.57 | 18.05 | 24.64 | 3.91 |
| HUN | Europe | 868.64 | 572.84 | 716.87 | 64.94 |
| IND | Asia | 641.55 | 445.03 | 559.95 | 35.74 |
| KOR | Asia | 583.31 | 380.85 | 474.95 | 50.32 |
| PER | Latin America | 1917.27 | 1245.00 | 1635.12 | 169.11 |
| POL | Europe | 888.33 | 525.99 | 696.16 | 65.21 |
| ZAF | Africa | 656.16 | 403.59 | 498.87 | 52.02 |
| TUR | Asia | 456.53 | 182.00 | 313.67 | 81.25 |
| MAR | Africa | 351.69 | 271.66 | 303.72 | 20.34 |
| ARG | Latin America | 4529.49 | 1213.71 | 2863.23 | 894.26 |
| LKA | Asia | 209.66 | 140.07 | 173.76 | 19.95 |
| JOR | Asia | 99.65 | 71.33 | 81.82 | 5.61 |
| PAK | Asia | 160.78 | 48.66 | 98.30 | 31.53 |

Note: TW represents Tai Wan, China. HK represents Hong Kong, China. SD is the standard deviation. And in this table, the countries or regions from ISR to SGP are developed markets, from RUS to TUR are emerging markets and from MAR to PAK are frontier markets.

$$CS(negative) = \frac{1}{n}\sum_{back}(matrix(CS)) \tag{14}$$

Dark causal strength from $Y$ to $X$:

$$CS(dark) = \frac{1}{n}\sum_{others}(matrix(CS)) \tag{15}$$

There may be three types of causal interactions between each variable pair, each with a different causal strength. Thus, the causal relationship with the strongest causal strength is regarded as the dominant causality between them, which is just expressed in reality (Fig. 2).

### 2.3. A CCM approach: correlation and causality

There are several practical tools to describe the relationships between time series variables, among which correlation and causality are more widely used. However, correlation is different from causality, and zero correlation does not mean no causality. For example, in system (16), we can obtain the correlation between $X$ and $Y$ as $corr(X, Y) =$

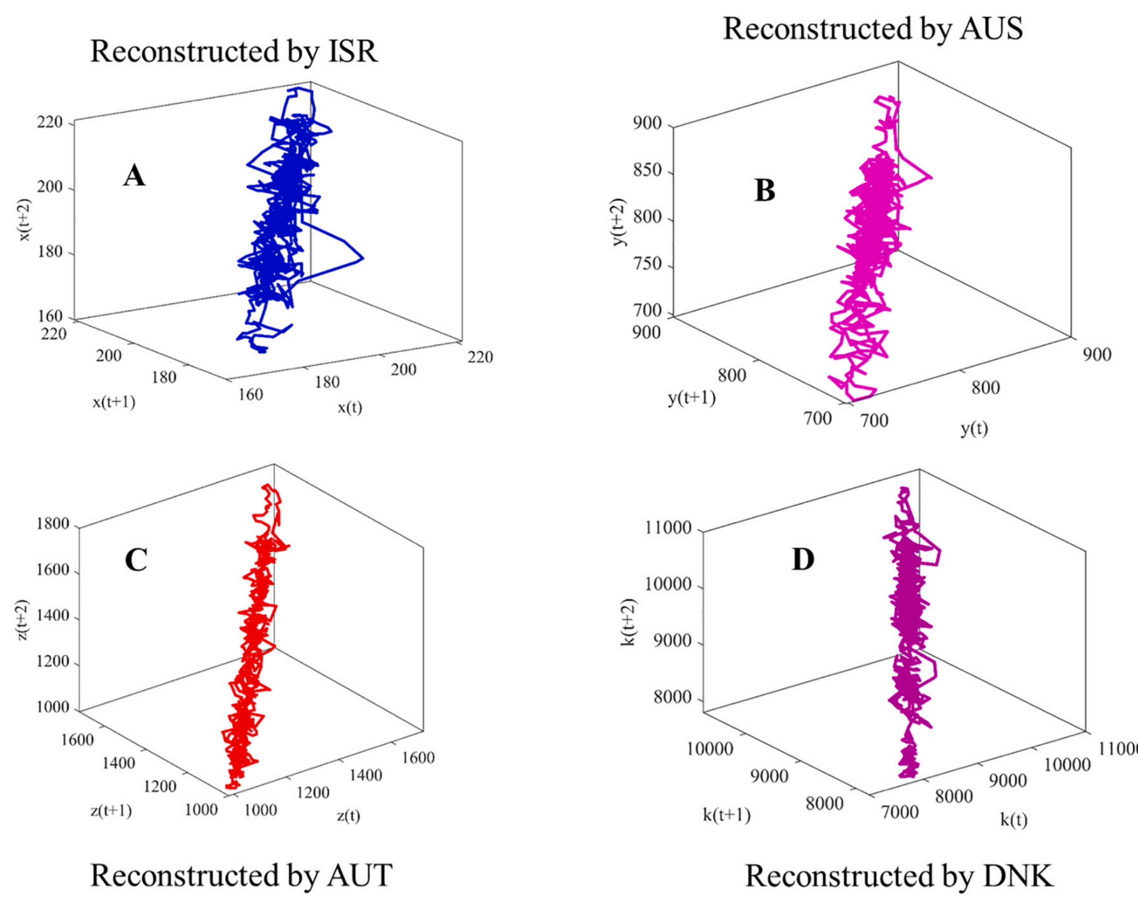



**Fig. 7.** The reconstructed attractors by the time series from stock markets.

0.025. However, through CCM, we can reveal the convergence of $x$ ' $(t)$ to $x(t)$ ($\rho = 0.995$) and the convergence of $y$ ' $(t)$ to $y(t)$ ($\rho = 0.404$), that is, there is strong causality from $X$ to $Y$ and weak causality from $Y$ to $X$, as shown in Fig. 4. And we can obtain weak correlation relationship between ISR and AUT, but there actually exists causality between them, as shown in Table 1. Therefore, in this paper, we detect the causal relationship to reveal the laws between time series variables.

$$X(t+1) = X(t)(3.8 - 3.8X(t) - 0.02Y(t)) \tag{16}$$

$$Y(t+1) = Y(t)(3.3 - 3.3Y(t) - 0.1X(t))$$

where $X(0) = 0.4$, $Y(0) = 0.2$, $L = 200$, $E = 2$ and $\tau = 1$.

### 2.4. A PC approach: time-varying causality

To certify the time-varying causality between variables, a simulated system is built as in Eq. (17). PC theory provides us with ways to detect the type and strength of causality. Here, we detect the causality from Y to X using PC theory, the width of sliding window is 151, and the lag is 1. From Fig. 5, on the one hand, the average strength of the negative causality (blue horizontal line) is strong than the positive causality (red horizontal line), which represents that the dominant causality is negative causality over the whole period. On the other hand, causal strength changes in different periods both for positive and negative causality, and the two types lead alternately, which means that the dominant causality is time-varying. In addition, their laws are not consistent with the average causality all the time. Therefore, we can draw a conclusion that the causal strength and the type of dominant causality are time-varying, even for a deterministic system.

$$\begin{aligned} X(t+1) &= X(t)(3.8 - 3.8X(t) - 0.02Y(t)) \\ Y(t+1) &= Y(t)(3.5 - 3.5Y(t) - 0.1X(t)) \end{aligned} \tag{17}$$

where $X(0) = 0.4$, $Y(0) = 0.2$, $L = 200$, $E = 2$ and $\tau = 1$.

### 2.5. A PC approach: consistent causality

In the last deterministic system, the dominant causality is time-varying, but the same situation is not always the same. In Eq. (18), we detect the causality from $Y_1$ to $Y_2$, and PC theory is also applied. The width of the sliding window is 151 and the lag is 1. The results show that causal strength is time-varying, but the dominant causality is positive causality all the time. Moreover, the dominant causality remains consistent with the average dominant causality (Fig. 6).

$$Y_1(t+1) = Y_1(t)(4 - 4Y_1(t) - 2Y_2(t) - 0.4Y_3(t))$$

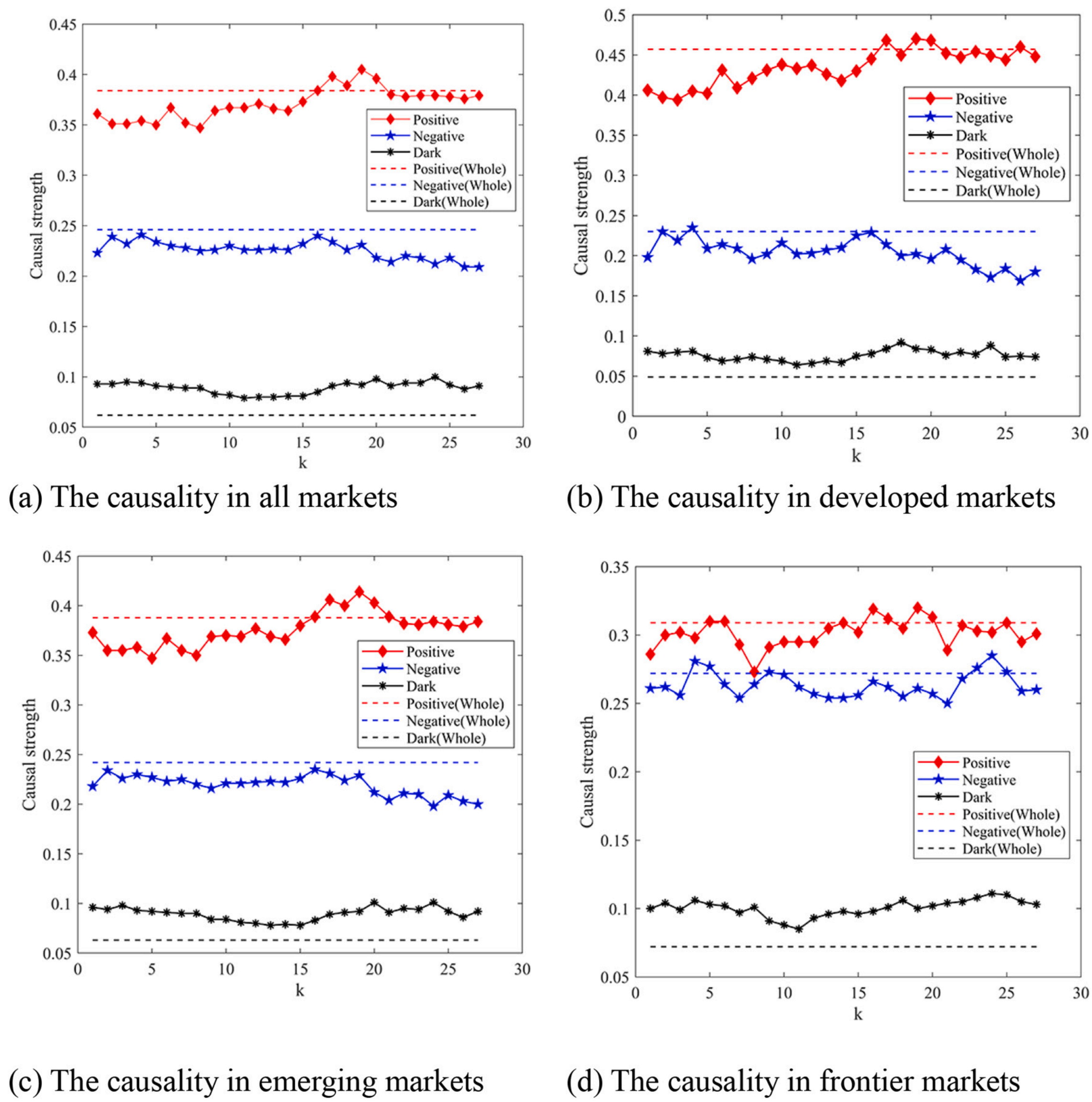
$$Y_2(t+1) = Y_2(t)(3.1 - 0.31Y_1(t) - 3.1Y_2(\mathrm{t}) - 0.93Y_3(t)) \tag{18}$$

$$Y_3(t+1) = Y_3(t)(2.12 + 0.636Y_1(t) + 0.636Y_2(t) - 2.12Y_3(t))$$

where $Y_1 = Y_2 = Y_3 = 0.2$, $L = 200$, $E = 2$ and $\tau = 1$.

## 3. Data

To study the causal relationships in global stock markets, we choose the MSCI (Morgan Stanley Capital International) stock index of each country or region, which is the most widely used benchmark index for portfolio managers in the world and has become an important benchmark to measure the performance of capital markets in various countries. According to the MSCI, the global stock markets contain three




(a) The causality in all markets

(b) The causality in developed markets

(c) The causality in emerging markets

(d) The causality in frontier markets

**Fig. 8.** The causality in the whole markets, developed markets, emerging markets, and frontier markets. The horizontal axis represents the kth sliding window. The vertical axis is causal strength. The horizontal line represents the average causality, which is obtained from the whole period.

types of markets, namely, developed markets, emerging markets and frontier markets. In this paper, we select 20 countries or regions in developed markets, 21 countries or regions in emerging markets and 5 countries in frontier markets. The daily closing prices from 2017-01-01 to 2019-11-16 are selected as the sample. The data are downloaded from the Wind database.

From the Table 2, we can find that the MSCI of HK is with highest SD in developed markets, while PRT is with lowest SD, which indicates low stability of MSCI for HK, while high stability of PRT. In emerging markets, SD of MSC for MEX is the highest, while lowest for MSCI of GRC. And in frontier markets, SD of MSCI for ARG is with the highest standard deviation, while lowest for JOR.

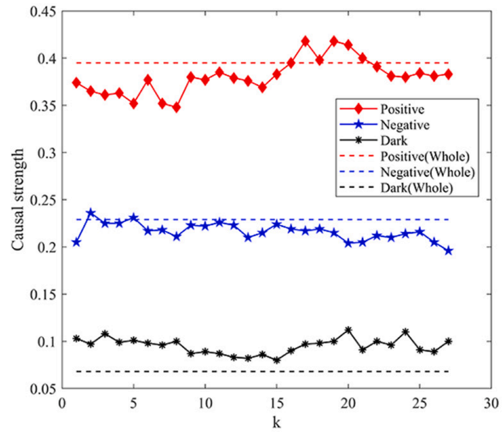
## 4. Results

From the above simulations, it can be seen that even for deterministic systems, some of the dominant causal relationships among variables change with time, and some are constant all the time. However, what happens in real systems? Now we consider the global stock markets. PC theory is also applied. The width of the sliding window is 530, the lag is 20, $E = 3$, $\tau = 1$. To make a distinction to the causality from each time period, the average causality and average dominant causality are called here, which are determined from the whole time period. And an important premise of PC method is that there exists a one to one map between attractors that are based on stock X and stock Y, respectively. Which can be certified by the topological structure of the attractors. In Fig. 7, we show four stocks, and we can find that the four attractors reconstructed by four stocks are topologically isomorphic, so there can built one to one map between them. Then it is suitable to apply the PC and CCM methods to detect the causality between them.

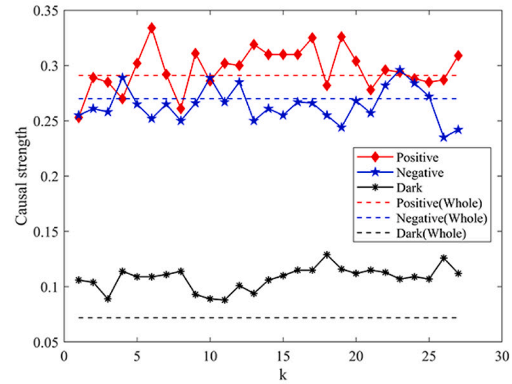
### *4.1. The causality in different markets*

The global stock markets include developed markets, emerging markets, and frontier markets. Fig. 8 exhibits the causality in whole markets and three different types of markets. The horizontal lines represent the average causal strength, which is obtained from the whole period, while the polylines are obtained in different periods.

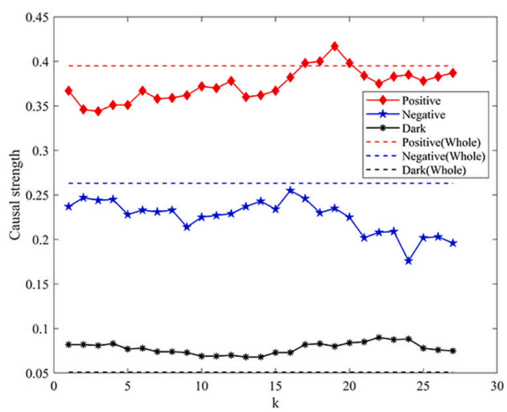
For the whole stock markets, the average positive causal strength is 0.38, which is stronger than the average negative (0.25) and dark (0.06) causality; and the positive causal strength increases as time passes. Positive dominant causality can be revealed in the global stock markets.



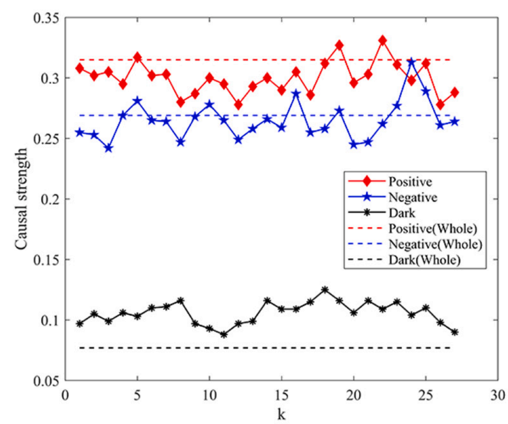

(a) The causality from developed markets to emerging markets.



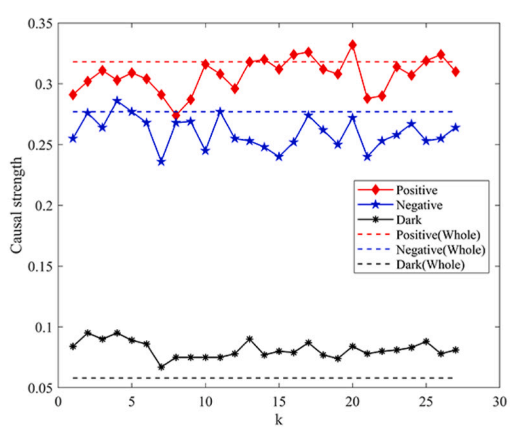

(b) The causality from developed markets to frontier markets.



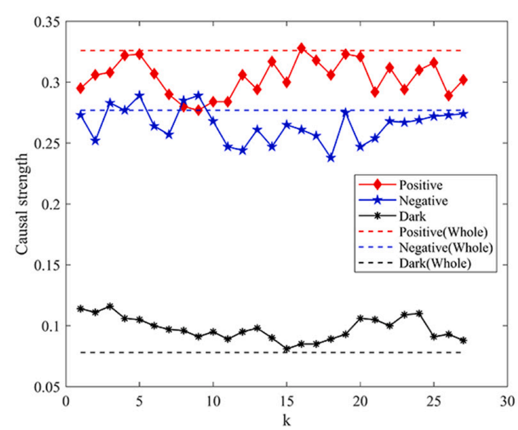

(c) The causality from emerging markets to developed markets.




(d) The causality from emerging markets to frontier markets.

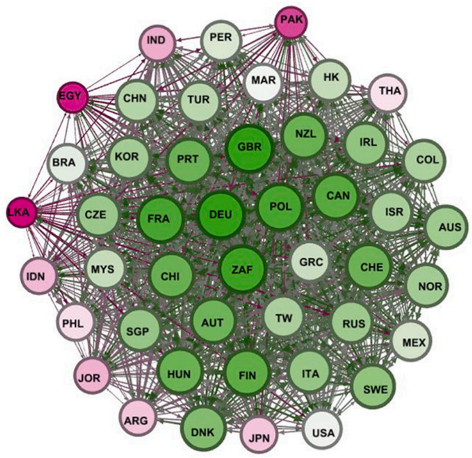



(e) The causality from frontier markets to developed markets.

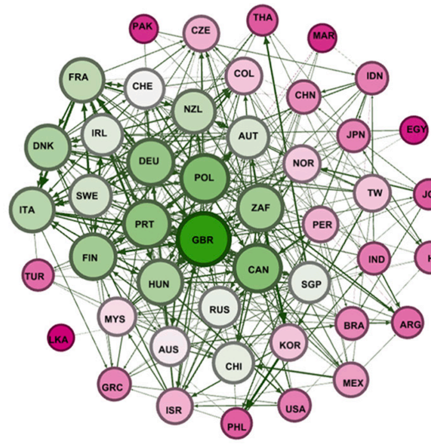



(f) The causality from frontier markets to emerging markets.

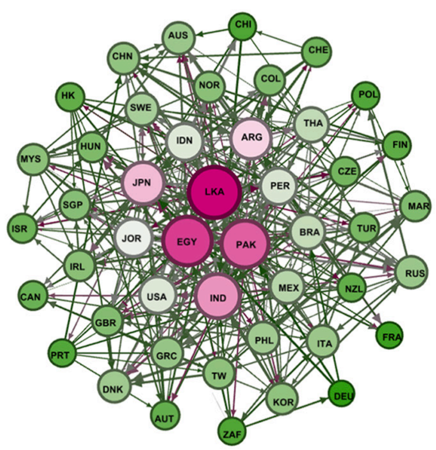
**Fig. 9.** The causality among the three types of markets.



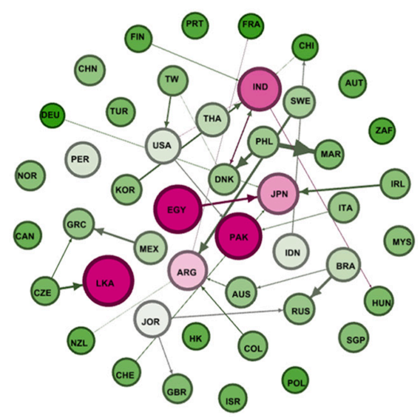

(a) The average positive dominant causal network in global stock markets



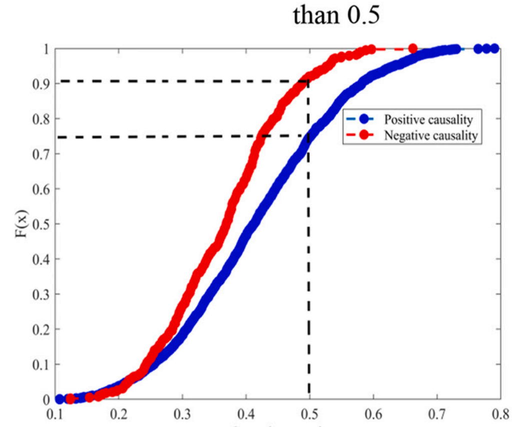

(b) The average positive dominant causal network whose causal strengths are more than 0.5




(c) The average negative dominant causal network in global stock markets.




(d) The average negative dominant causal network whose causal strengths are more than 0.5




(e) The cumulative distribution of the average positive and negative dominant causal strengths.

**Fig. 10.** The average dominant causal networks in stock markets. (a) The average positive dominant causal network. (c) The average negative dominant causal network. (e) The cumulative distribution of the dominant causal strength.

**Table 3**
Topological indices of average positive dominant network.

| Abbreviation | Submarkets | Region | Weighted degree | Weighted in-degree | Weighted out-degree |
|---|---|---|---|---|---|
| GBR | Developed | Europe | 38.83 | 19.51 | 19.32 |
| DEU | Developed | Europe | 38.78 | 20.42 | 18.37 |
| ZAF | Emerging | Africa | 38.26 | 19.03 | 19.23 |
| POL | Emerging | Europe | 37.81 | 19.39 | 18.41 |
| FRA | Developed | Europe | 37.74 | 18 | 19.73 |
| CAN | Developed | North America | 36.97 | 19.57 | 17.4 |
| FIN | Developed | Europe | 36.51 | 18.15 | 18.36 |
| CHI | Emerging | Latin America | 35.95 | 18.53 | 17.41 |
| NZL | Developed | Oceania | 35.87 | 19.18 | 16.69 |
| HUN | Emerging | Europe | 35.82 | 18.85 | 16.97 |

**Table 4**
Topological indices of average negative dominant network.

| Abbreviation | Submarkets | Region | Weighted degree | Weighted in-degree | Weighted out-degree |
|---|---|---|---|---|---|
| LKA | Frontier | Asia | 16.63 | 10.83 | 5.8 |
| EGY | Emerging | Africa | 15.25 | 10.38 | 4.87 |
| PAK | Frontier | Asia | 14.29 | 5.88 | 8.41 |
| IND | Emerging | Asia | 12.67 | 6.97 | 5.7 |
| JPN | Developed | Asia | 11.38 | 7.26 | 4.12 |
| ARG | Frontier | Latin America | 10.48 | 6.2 | 4.28 |
| JOR | Frontier | Asia | 9.01 | 2.46 | 6.55 |
| IDN | Emerging | Asia | 8.41 | 4.78 | 3.63 |
| USA | Developed | North America | 8.33 | 5.64 | 2.69 |
| PER | Emerging | Latin America | 8.19 | 3.96 | 4.23 |

In different time periods, causal strength changes but the positive causal strength is still the strongest all the time. Therefore, we can conclude that the dominant causality in different periods remains consistent with that on the whole. Next, we assess the specific types of markets.

In developed markets, the average positive causal strength is 0.46, which is stronger than other two types. Positive dominant causality is still revealed in these markets. In addition, the average positive causal strength is stronger than that in global markets. Furthermore, in these markets, the dominant causality in different time windows remains consistent with that on the whole.

In emerging markets and frontier markets, the dominant causality in different periods is equal to the average dominant causality, namely, the positive causality.

### 4.2. The causality between different markets

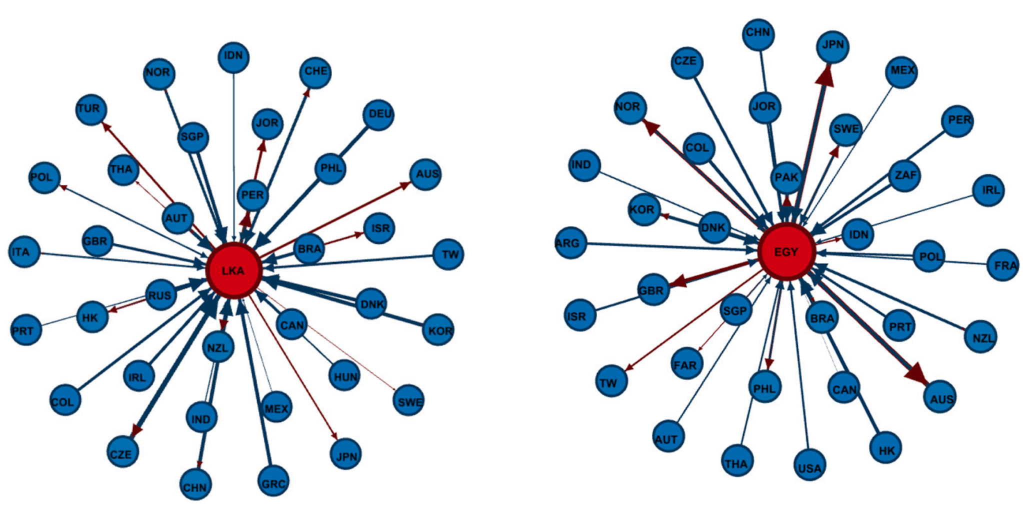
We have studied the causality in separate markets. Naturally, it is of importance to discuss the causal relationships between markets. Fig. 9 depicts the causality among developed markets, emerging markets and frontier markets.

A common character can be obtained that the average positive causal strength is the strongest between each market pairs. Thus, the positive dominant causality can be inferred among three markets. Furthermore, it is obvious that the average causal strength from developed markets to emerging markets is stronger than others, as shown in Fig. 9(a, c). Therefore, stronger causal connections can be revealed from developed markets to emerging markets.

For different time periods, the positive dominant causality can also be revealed. Although causal strength is time-varying, it remains stable with respect to the average causal strength. Unfortunately, there are several exceptions that happen in frontier markets, where negative dominant causality is found in some periods. These uncertainties should receive the attention of investors and government managers.

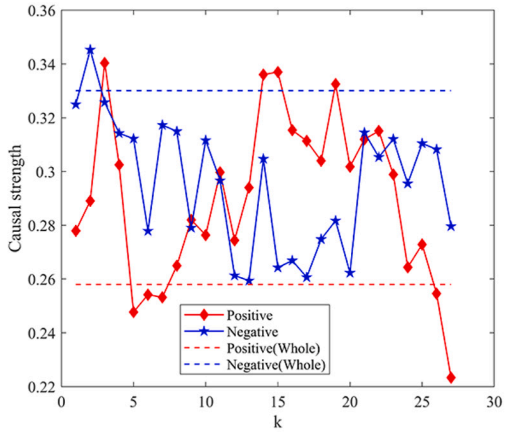
### 4.3. Uncertain causality in global stock markets

Previous studies have shown that positive dominant causality is revealed in whole markets and specific types of markets. Here, the causal characteristics look deterministic and "surgeless". However, from the micro perspective, we disclose some exceptions whose causal relationships are inconsistent with the markets. Although the average dominant causality between most stock index pairs is positive dominant causality, negative dominant causality is found between some stock

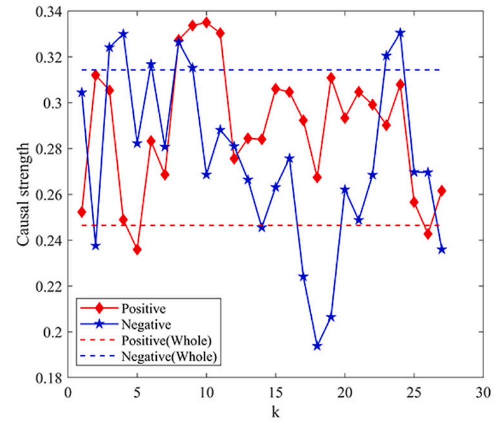



(a) The negative dominant causal network for LKA. (b)The negative dominant causal network for EGY.

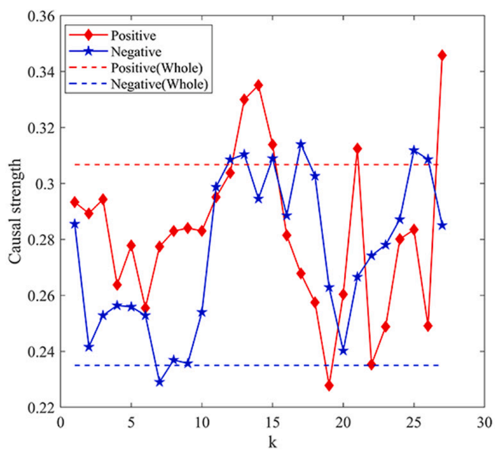
**Fig. 11.** The negative dominant causal networks of typical stock indices pairs in the markets. (a and b) The causal networks for LKA and EGY, respectively, whose causal connections are denser and stronger.



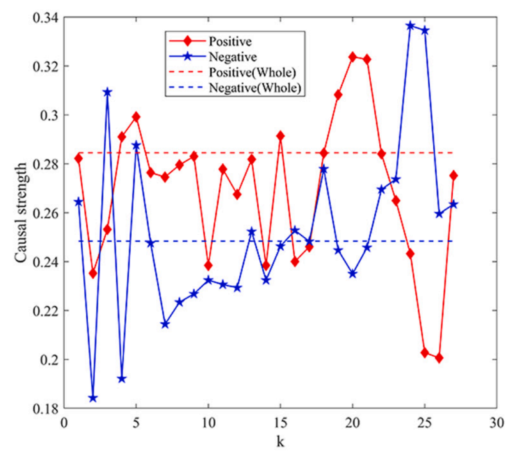

(a) The average weighted out degree of LKA in different time periods.



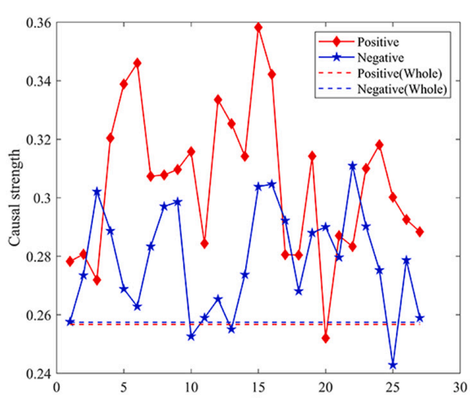

(b) The average weighted in degree of LKA in different time periods.



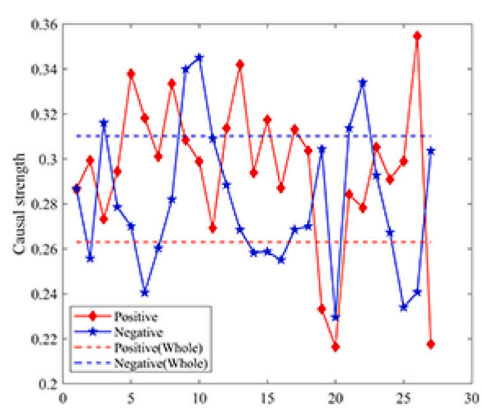

(c) The average weighted out degree of EGY in different time periods.




(d) The average weighted in degree of EGY in different time periods.




(e) The average weighted out degree of PAK in different time periods.




(f) The average weighted in degree of PAK in different time periods.

**Fig. 12.** Some exceptions in the global stock markets whose causalities are uncertain.

index pairs.

To view the average dominant causality in the global markets, complex networks are used. The nodes represent countries or regions whose sizes are ordered by their weights, and the directed edges represent causal strength. Fig. 10(a) depicts the average dominant positive causal network whose density is 81.3%. In addition, the average causal strength is 0.42, and nearly 76% of the causal strengths are distributed from 0.2 to 0.5. Nevertheless, 24% of them are more than 0.5, which indicate strong causal relationships, as shown in Fig. 10(b, e). Based on the weighted degree, we select the top ten stocks, all of them are from developed markets and emerging markets, as shown in Table 3. Although the average negative dominant network contains 46 stock indices, it is not fully connected and its density is only 18.7%. The average causal strength is 0.37. More than 90% of them are less than 0.5, and 10% are from 0.5 to 0.66, as shown in Fig. 10(d, e). And based on the weighted degree, we select the top ten stocks, most of them come from emerging markets and frontier markets, as shown in Table 4.

Thus, the average positive dominant causality is stronger in these markets. Regardless of the weakness of the average negative dominant causality here, it can bring some hidden risk if the relationships are not revealed. Among them, the negative dominant causal connections to LKA and EGY are denser and stronger than others, as shown in Fig. 11(a, b). There are 35 nodes connected to LKA, 34 nodes connected to EGY, and the average causal strengths are 0.46 and 0.43, respectively.

We have exposed some exceptions whose dominant causality violates that of the whole markets. Moreover, some more complex situations can be revealed when consider these exceptions from different time periods.

In Fig. 12(a, b), the average negative dominant causality can be obtained between the LKA and others (both as a causal driver and as a causal receiver), which is different from the whole markets. In addition, the dominant causality changes in different time periods; moreover, causal strength fluctuates heavily.

In Fig. 12(c, d), although the average dominant causality between EGY and others is consistent with the whole markets (positive dominant causality), it is unstable as time passes, and causal strength oscillates heavily.

In Fig. 12(e, f), it is more complex when PAK is a causal driver, the average dominant causality is not clear, and causal strength is weak. Moreover, the dominant causality changes both when it is a driver and a receiver.

## 5. Conclusions and discussion

In this work, we employ PC method to reveal the diverse causal interactions in global markets. And combining the complex network method and sliding window method, we detect the time-varying feature of these causal interactions. Base on these methods, numerous results are found as follows:

Causal relationship is actually different from correlation relationship, even when the correlation coefficient between time series variables is zero, strong causality can be determined, and when strong correlation is determined, no causality may be revealed between variables.

In global stock markets, besides positive and negative causality, a more complex causal interaction is revealed, namely dark causality. Which can explain that the fluctuations between two stocks are not always acting as the same or opposite direction, but acting in disordered direction sometimes. And the causal strength between most stocks is time-varying, but the positive dominant causality can be determined in stock markets, as well as in three different types of markets, for the whole period and also different time periods. In total, the feature of positive dominant causality tells us that the fluctuation in these submarkets are consistent. Which means a rise of a stock will likely cause the rise of other stocks. And a drop of a stock will likely cause the drop of other stocks. Strong and positive dominant causality between two stocks can be interpreted that their fluctuation trends are highly consistent, which can be risk for long periods investment, but it can be a high return selection for risk-takers especially in short-time or in boom periods. Where negative dominant causality between stocks can hedge the loss, the fluctuation will be likely opposite. So, the causality analysis can reveal the driving trend and provides a basic theory to guide to invest and manage. Although a stable character is obtained in stock markets, some exceptions can be revealed whose dominant causality is negative. The more complex situation is that the dominant causality is time-varying, and both positive dominant causality and negative dominant causality exist here, which may due to the degree of perfection of financial markets, where high degree of marketization and more perfect mechanism are in developed markets, while weak in frontier markets. Thus, it is risky to make decisions only based on the average dominant causal relationships.

Overall, the causal relationships in developed markets are stronger and more stable, and there are closer causal connections to emerging markets. In addition, in frontier markets, the causal relationships are weak, and many uncertainties are hidden here. Thus, regarding causality, developed markets are low uncertain, with clear and stable causal relationships. Which can be a low-risk markets for investors comparing to others. However, more careful attention should be given to when to invest in frontier markets, where causal laws are not easy to obtain because of uncertain dominant causality and weak causal strength.

However, though diverse causal interactions are revealed in global stock markets, we fail to uncover the driving force behind the time-varying causality. In other hand, we fail to distinguish the direct, indirect and spurious causality, which can be a more comprehensive discussion on the nature of causality. We will consider them in our future research.

## Author statement

The authors declare that they have not been published previously.

## Conflicts of interest

The authors declare that they have no conflicts of interest.

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