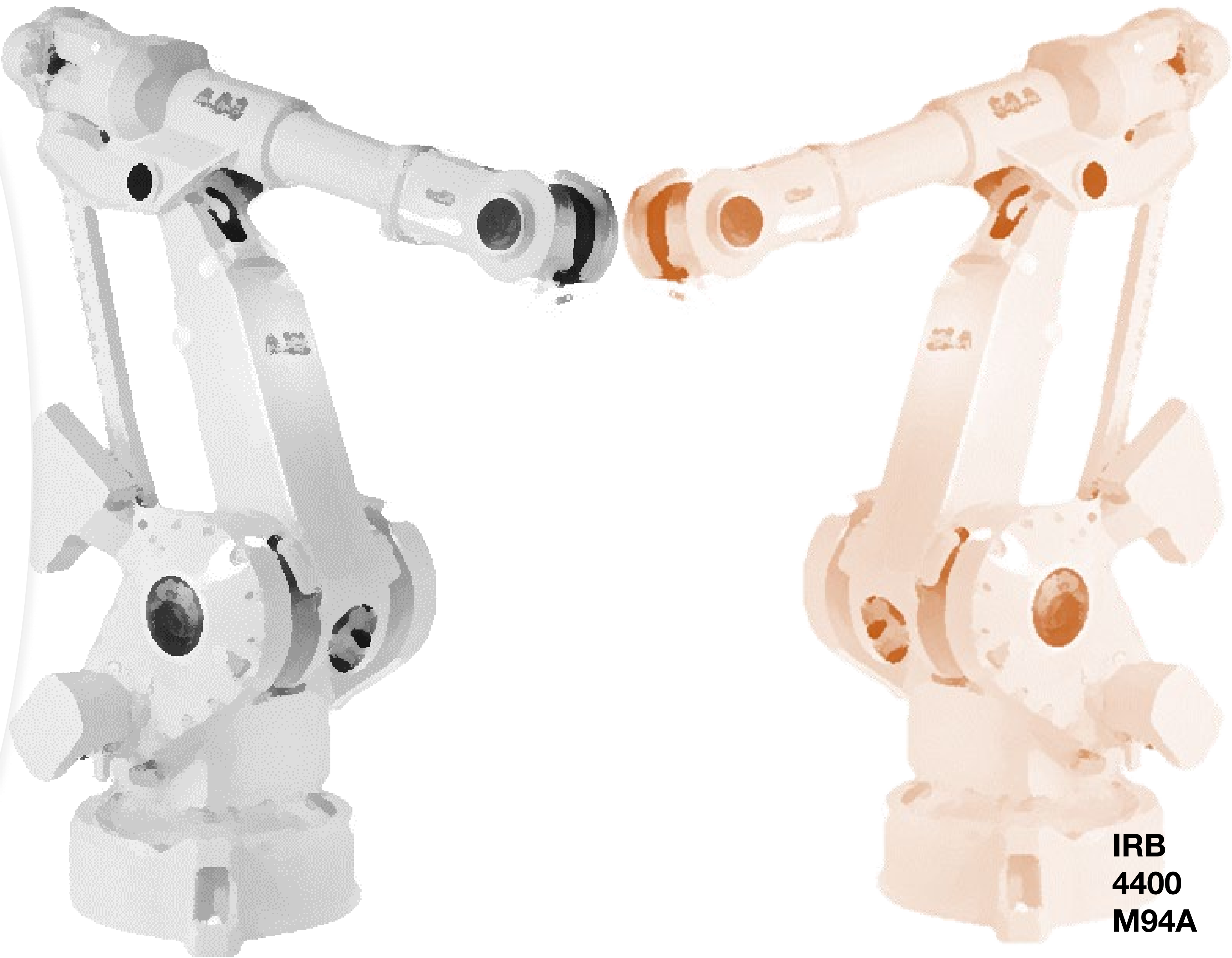
INTRODUCTION TO ROBOTICS

Saksorn Ruangtanusak
61070505063
Mechanical Engineering



IRB
4400
M94A

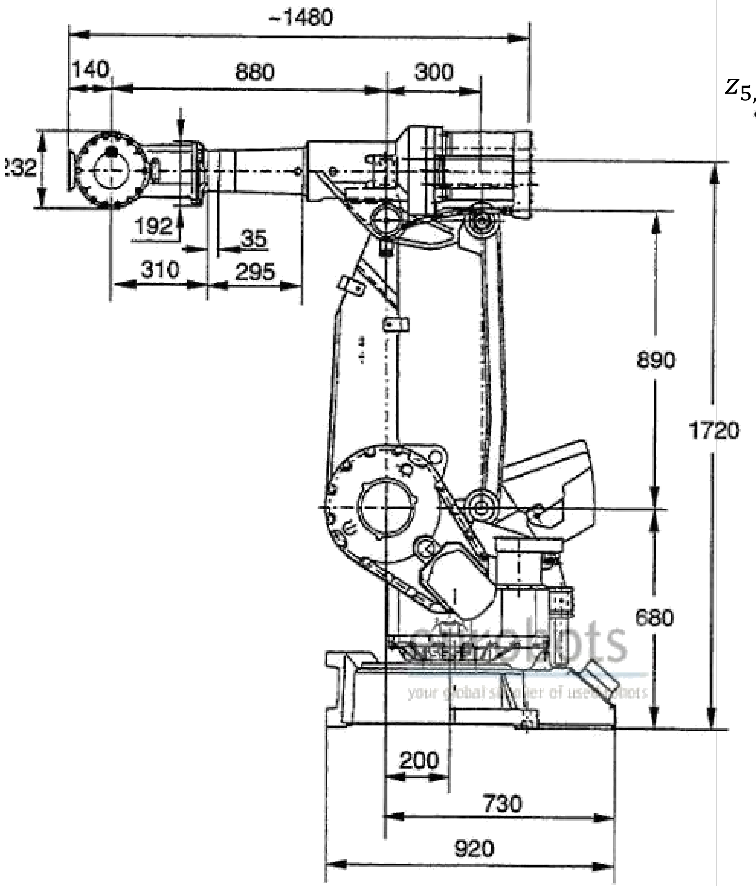
FORWARD KINEMATICS

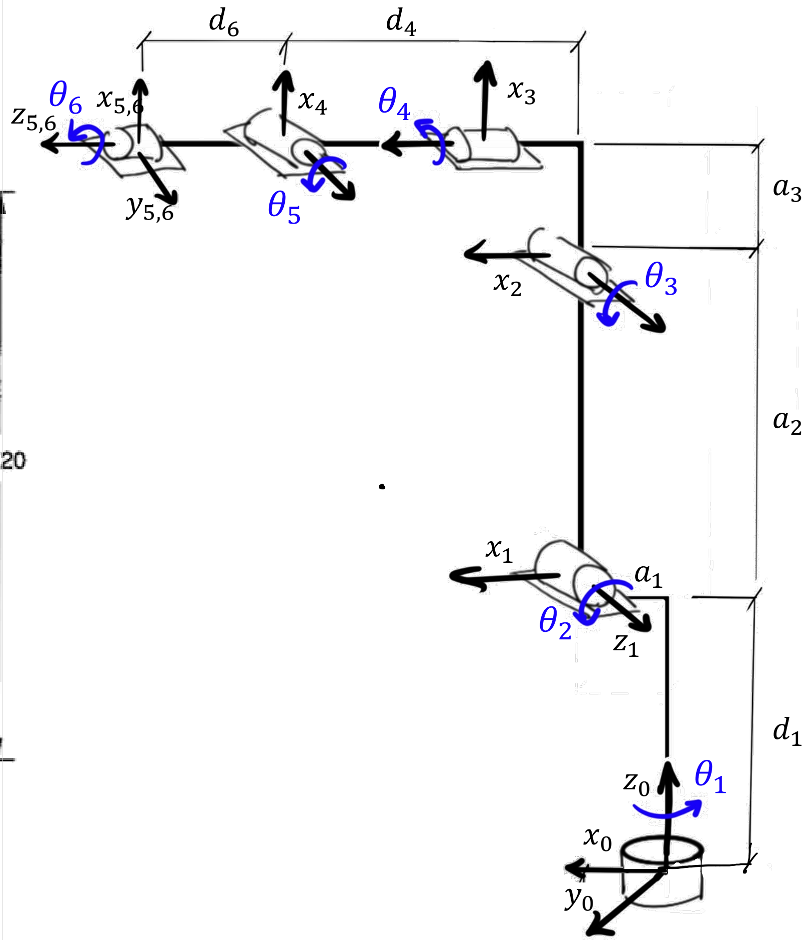
Denavit–Hartenberg Parameters

Product specification
IRB 4400/60
IRB 4400/L10

Document ID: 3HAC042478-001
Revision: P

Axis movement	Working range
Axis 1, Rotation	+165° to -165°
Axis 2, Arm	+95° to -70°
Axis 3, Arm	+65° to -60°
Axis 4, Rotation	+200° to -200°
Axis 5, Bend	+120° to -120°
Axis 6, Turn	+400° to -400°

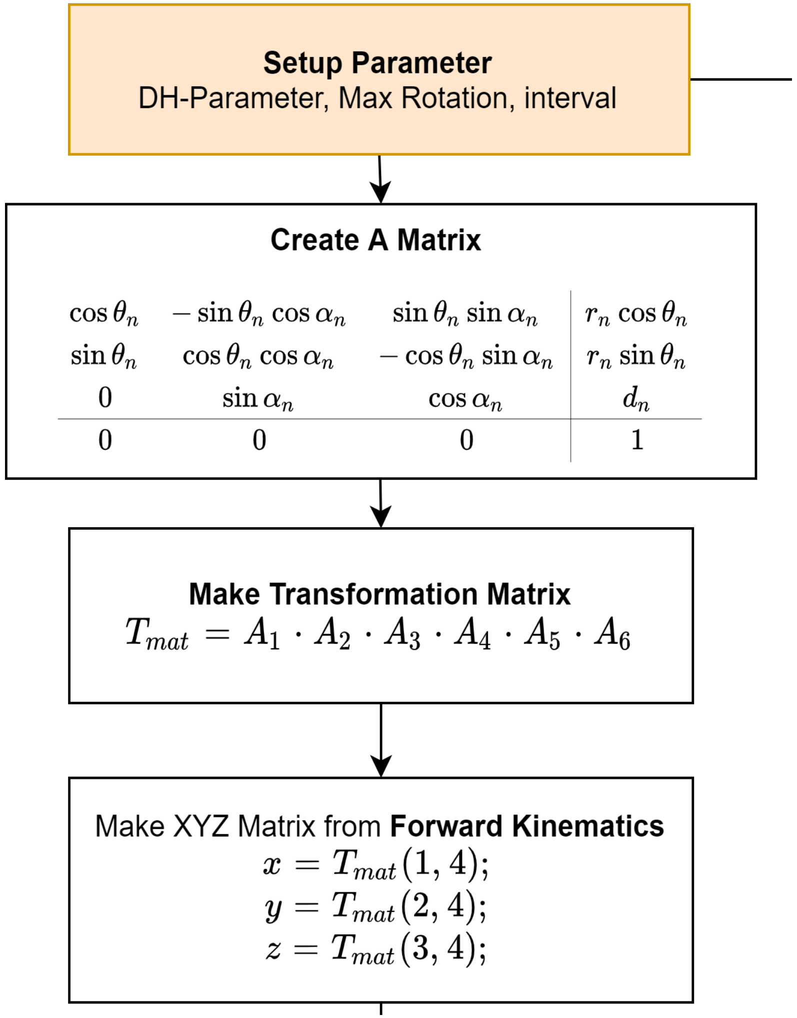




Link	a_i	α_i	d_i	θ_1
1	a_1	$-90°$	d_1	θ_1^*
2	a_2	0	0	$\theta_2^* - 90°$
3	a_3	$-90°$	0	θ_3^*
4	0	$+90°$	d_4	θ_4^*
5	0	$-90°$	0	θ_5^*
6	0	0	d_6	θ_6^*

FORWARD KINEMATICS

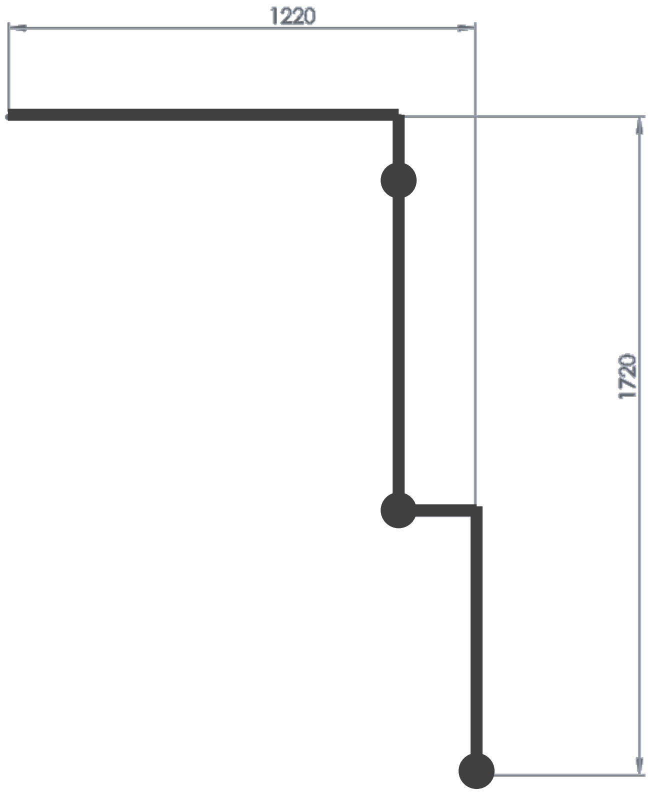
Denavit–Hartenberg Parameters



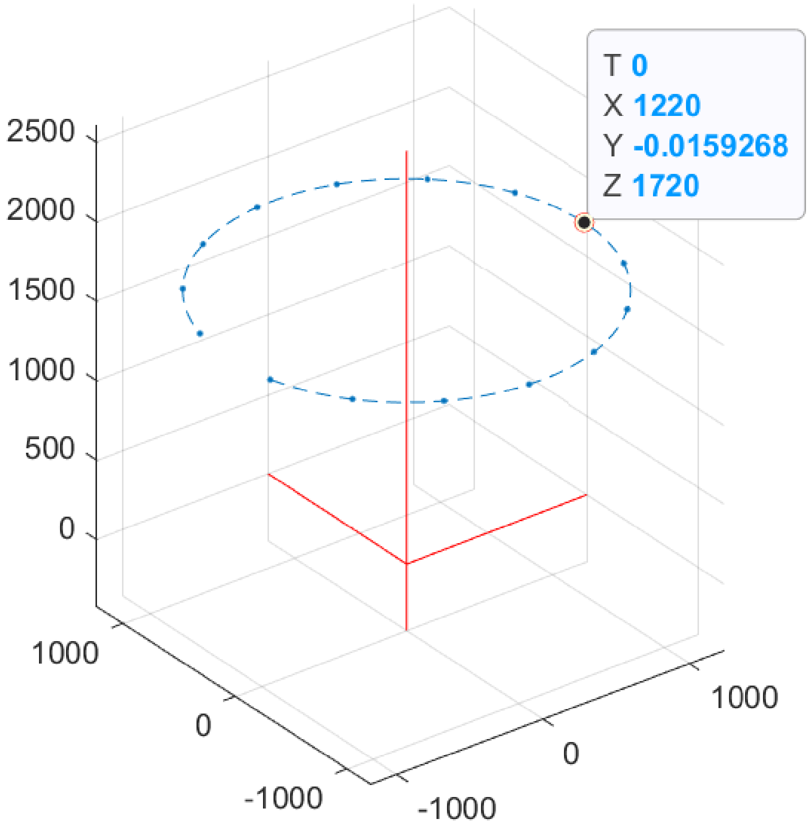
Link	a_i	α_i	d_i	θ_1
1	a_1	$-90°$	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	$-90°$	0	θ_3^*
4	0	$+90°$	d_4	θ_4^*
5	0	$-90°$	0	θ_5^*
6	0	0	d_6	θ_6^*

CHECK FWD 1

$\theta_2(deg)$	0
$\theta_3(deg)$	0
$x\ (mm)$	1220
$z\ (mm)$	1720

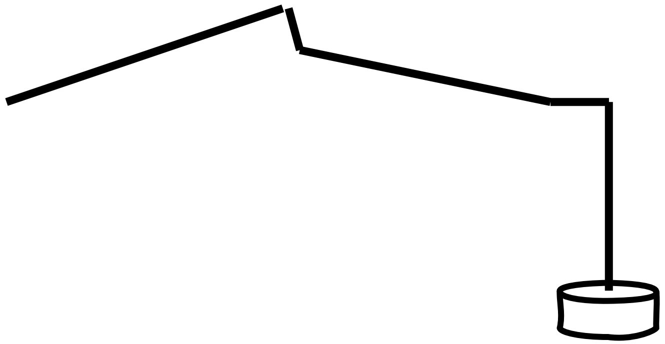


Geometry Method

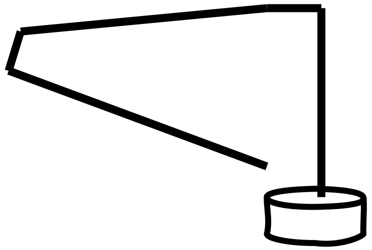


DH-Parameter Method

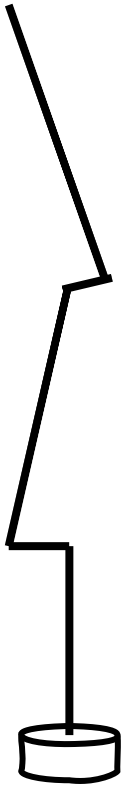
CHECK FWD 2



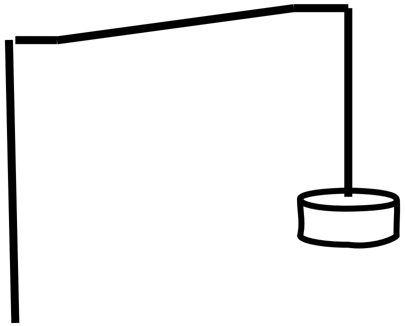
Max X
$[x, z] = [2087, 710]$



Min X
$[x, z] = [179, 113]$



Max Z
$[x, z] = [1102, 2120]$



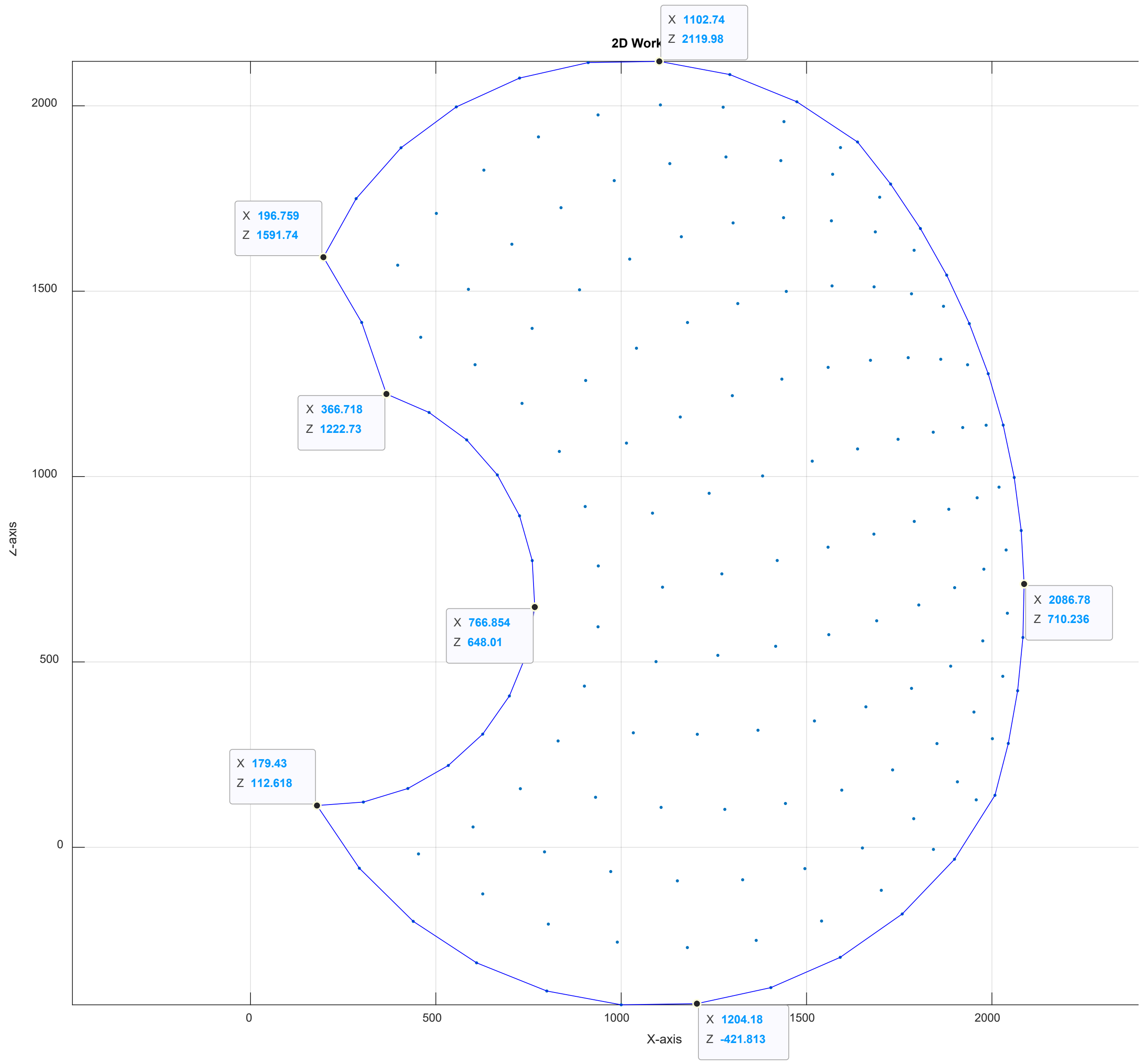
Min z
$[x, z] = [1237, -421]$

CHECK FWD 3

Check Min And Max 2D

Coor	X	Z
Max X	**2086.78**	710.24
Min X	**179.43**	112.618
Max Z	1102.74	**2119.98**
Min Z	1204.18	**-421.81**

Axis movement	Working range
Axis 1, Rotation	+165° to -165°
Axis 2, Arm	+95° to -70°
Axis 3, Arm	+65° to -60°
Axis 4, Rotation	+200° to -200°
Axis 5, Bend	+120° to -120°
Axis 6, Turn	+400° to -400°

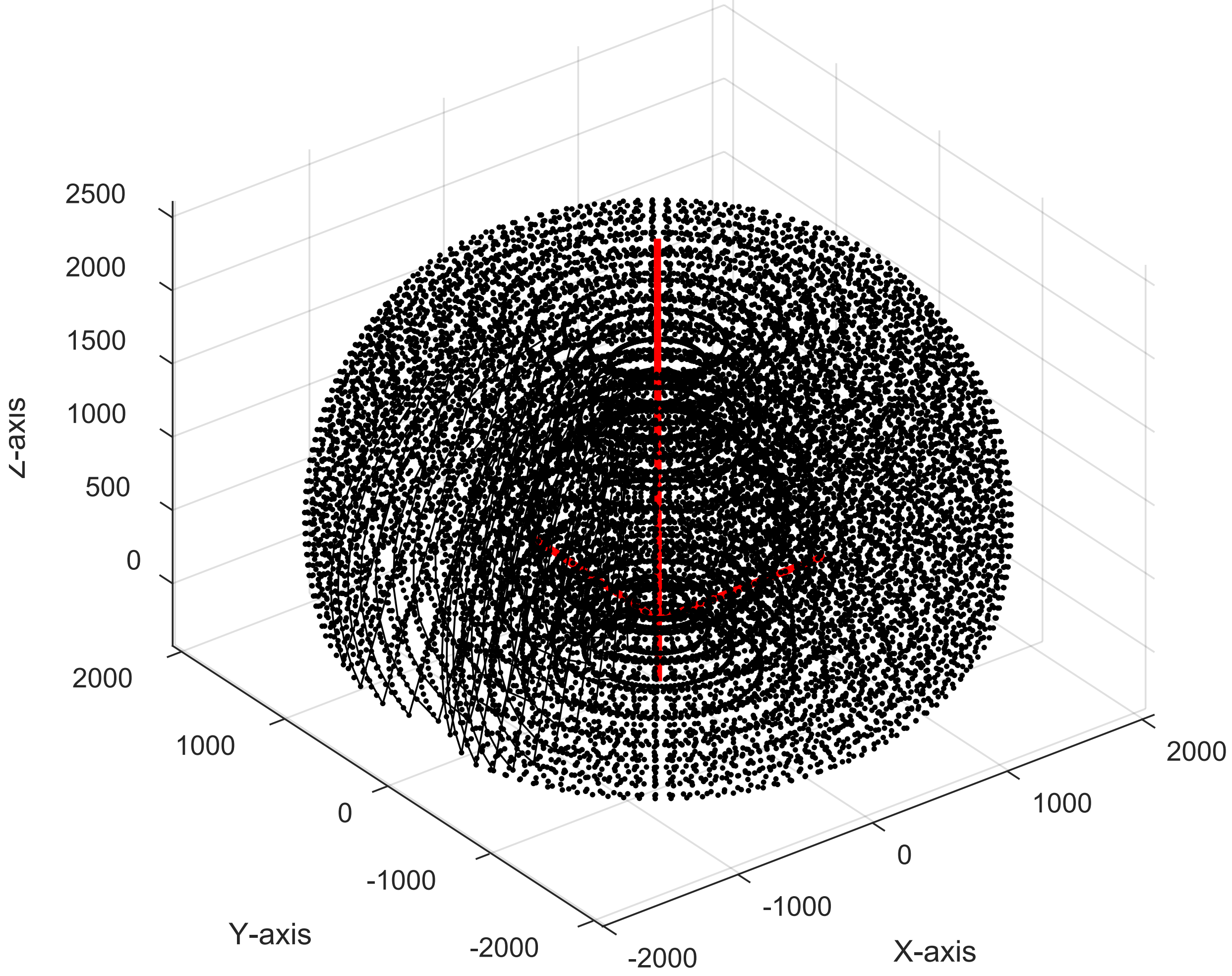


CHECK FWD 4

Check Min And Max 3D

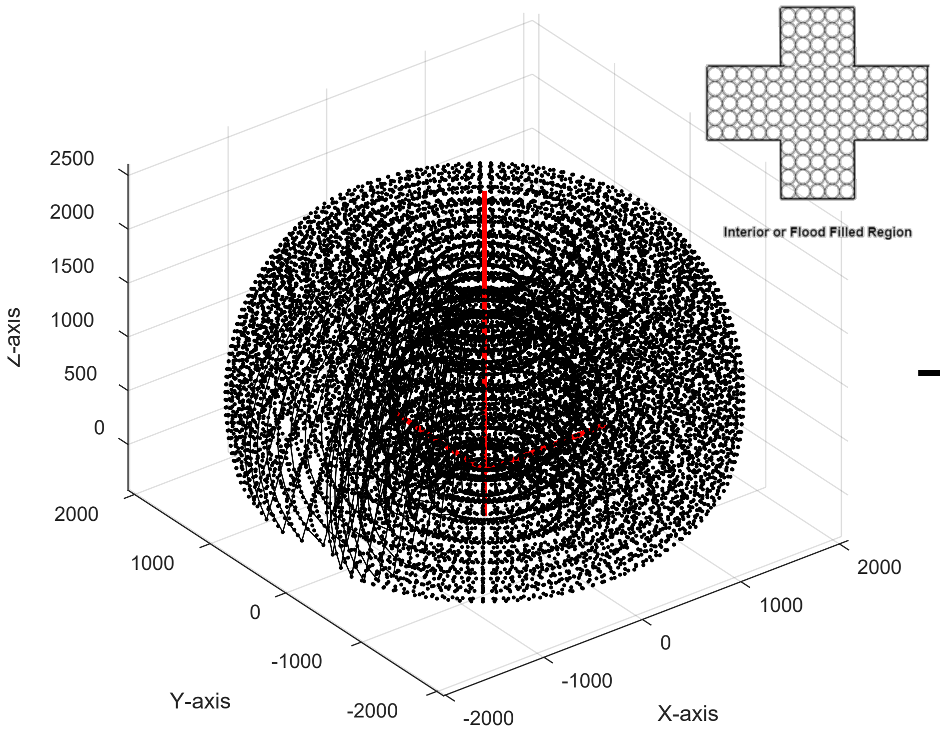
Coor	X	Y	Z
Max X	**2086.78**	0	710.24
Min X	**-2015.68**	540.1	710.24
Max Y	-33.45	**2086.51**	710.24
Min Y	-33.45	**-2086.51**	710.24
Max Z	-1065.17	285.41	**2119.98**
Min Z	-965.98	258.83	**-424.9**

$$Iterate\ [x, y, z] = [99, 28, 24]$$



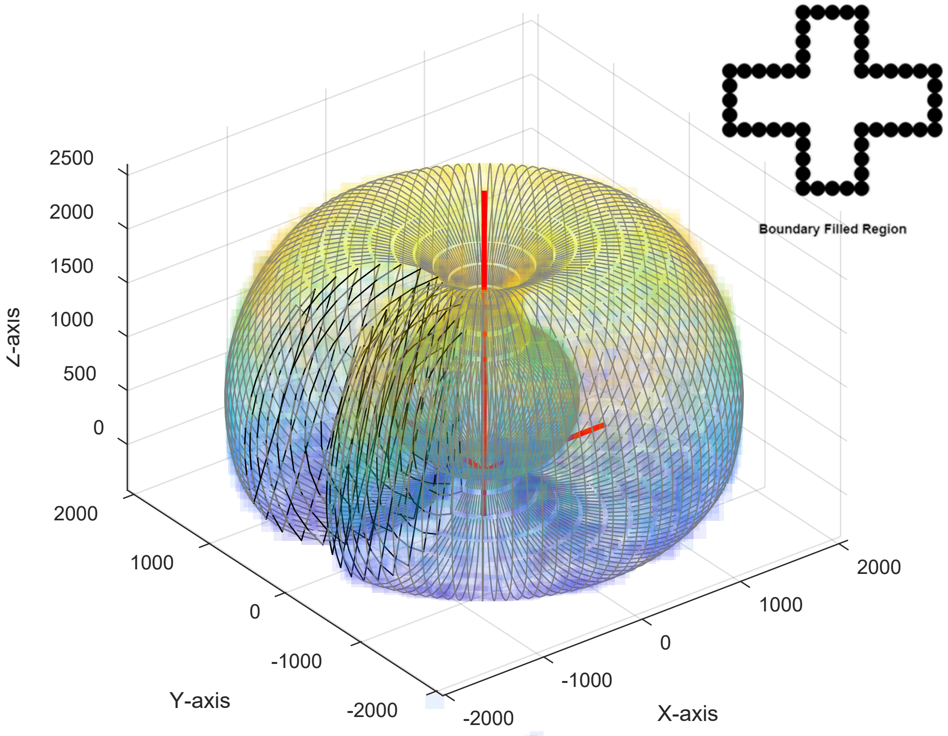
BOUNDARY FILLED ALGORITHM

4X Reduction



Raw Point Cloud Data

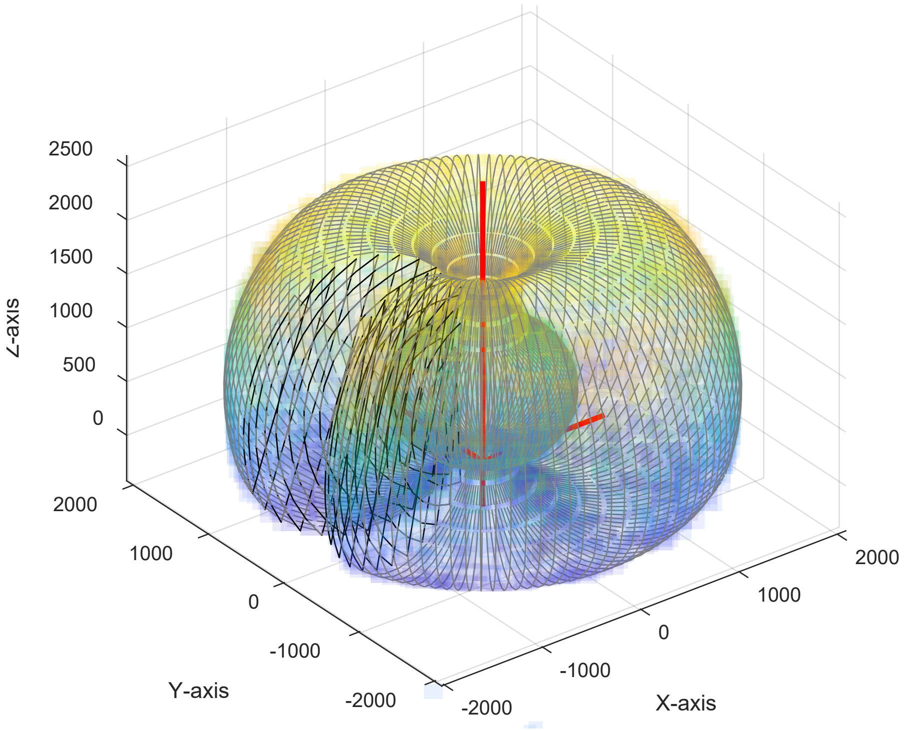
3 x 22176 Array

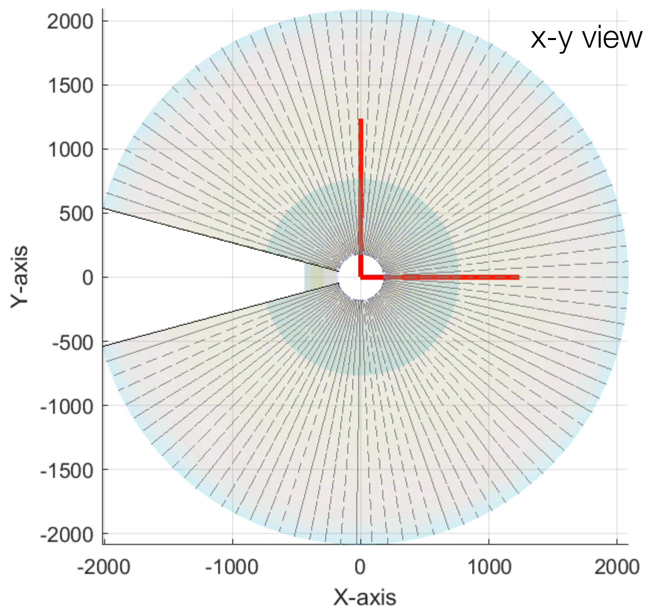


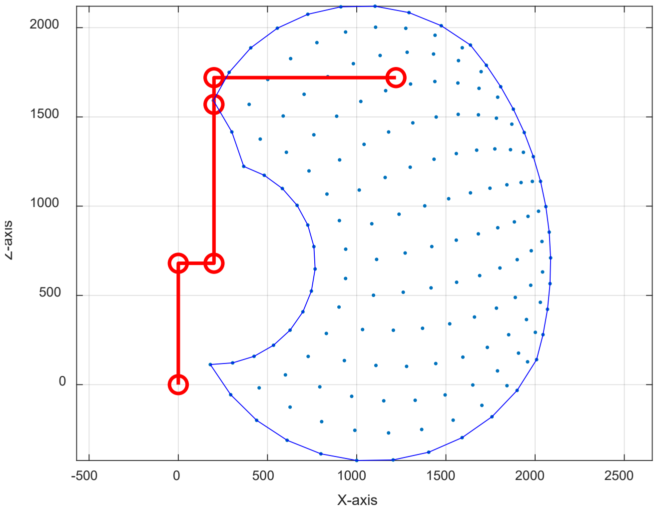
3D Boundary Fitted Data

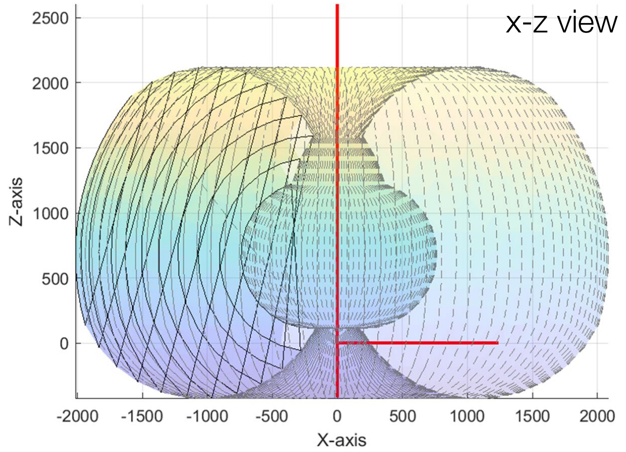
3 x 5089 Array

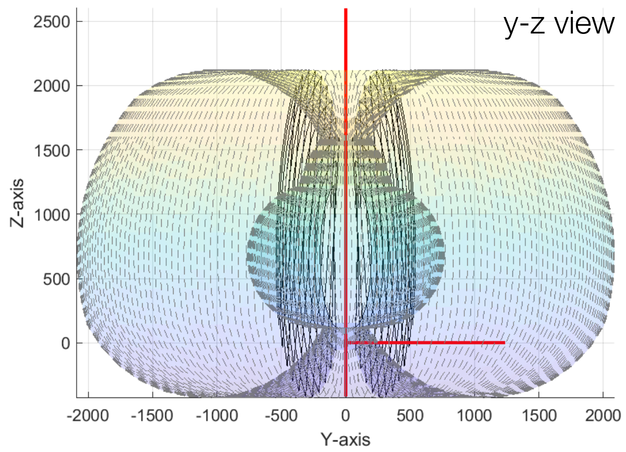
3D Workspace
Forward Kinematics







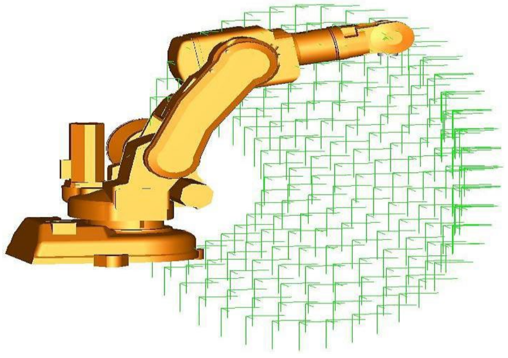




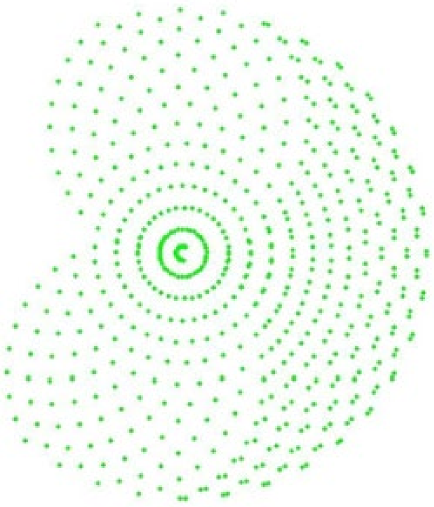
WORKSPACE REVIEW

A methodology to determine the functional workspace of a 6R robot using forward kinematics and geometrical methods

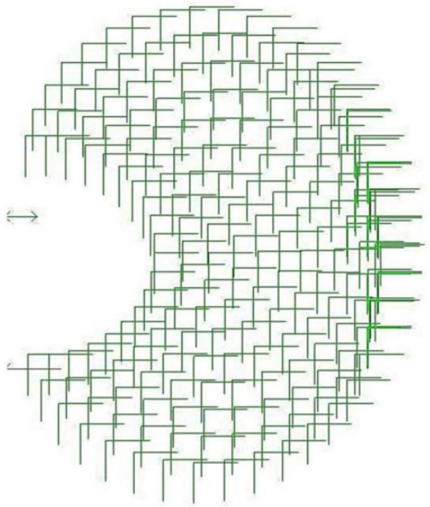
Arun Gowtham Gudla
University of Windsor



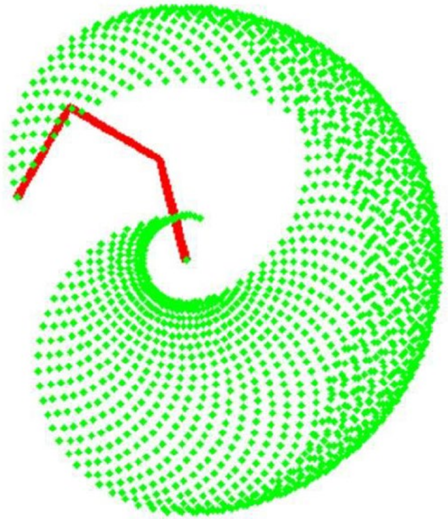
Manual Point generation Method



Analytical approach



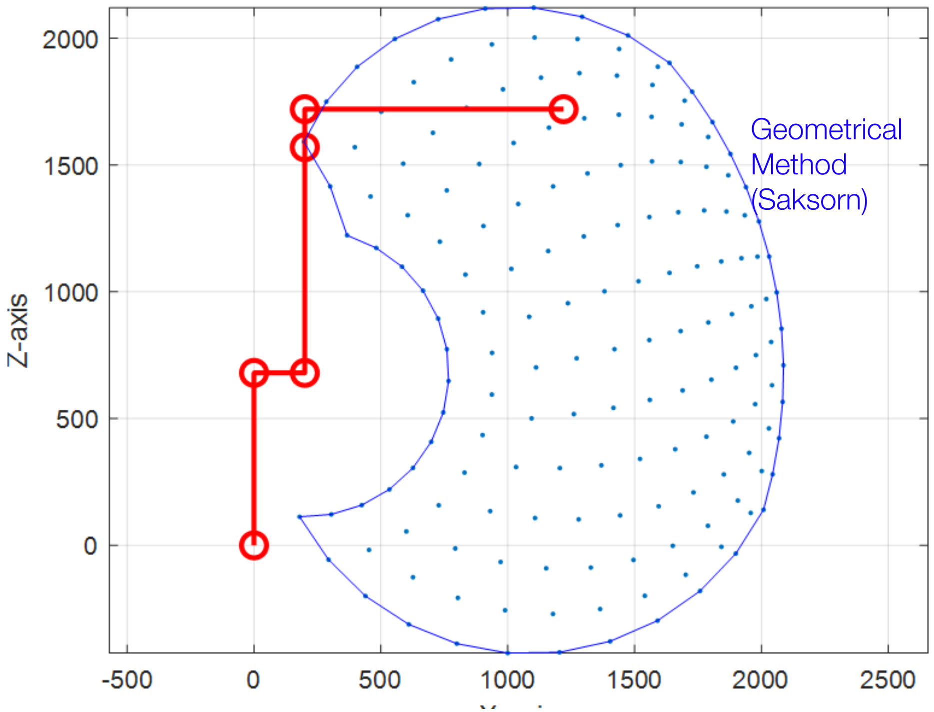
Empirical method

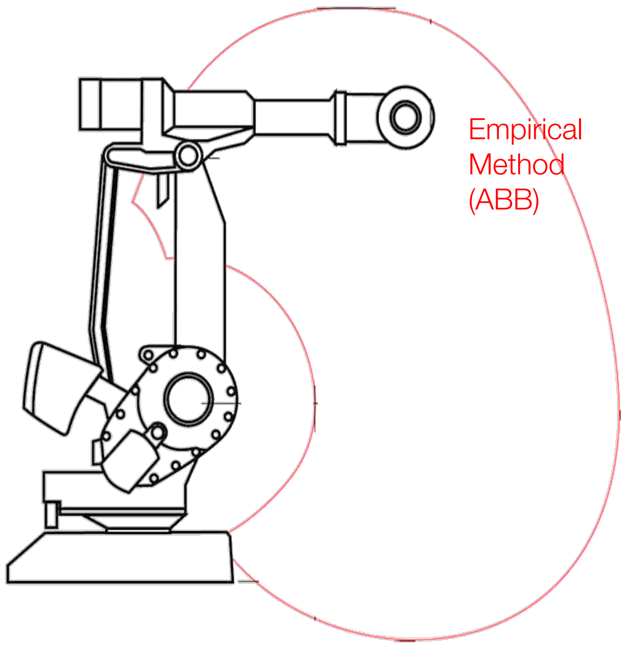


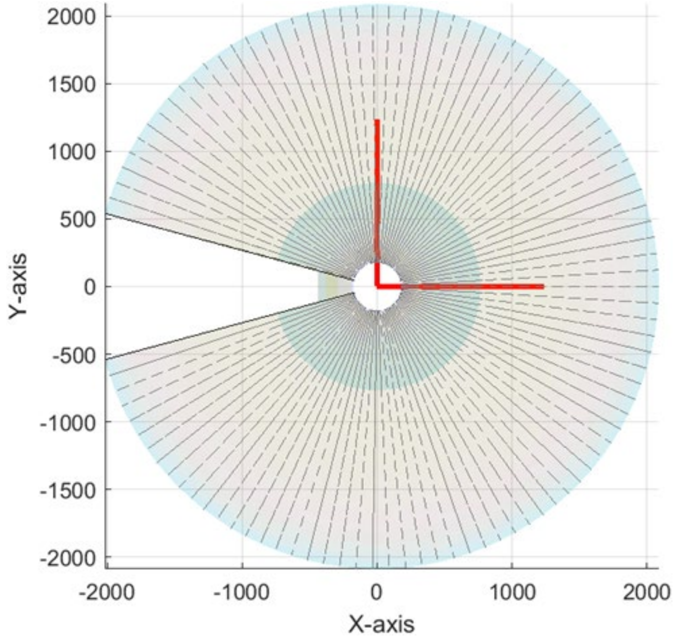
Geometrical Solution

UNWORKABLE SPACE

They are some difference







WHY?

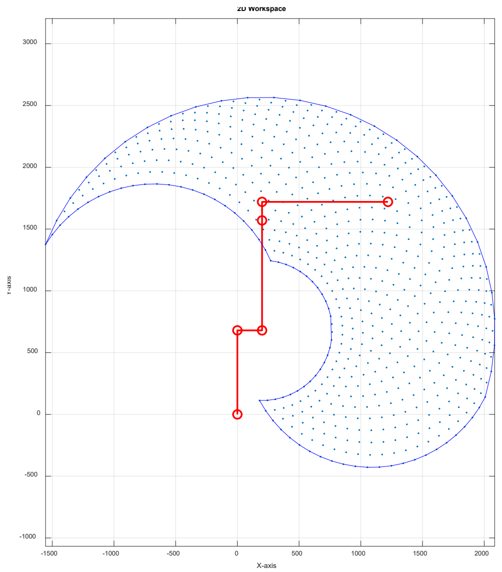
UNWORKABLE SPACE

Positions at wrist center (mm) and angle (degrees):

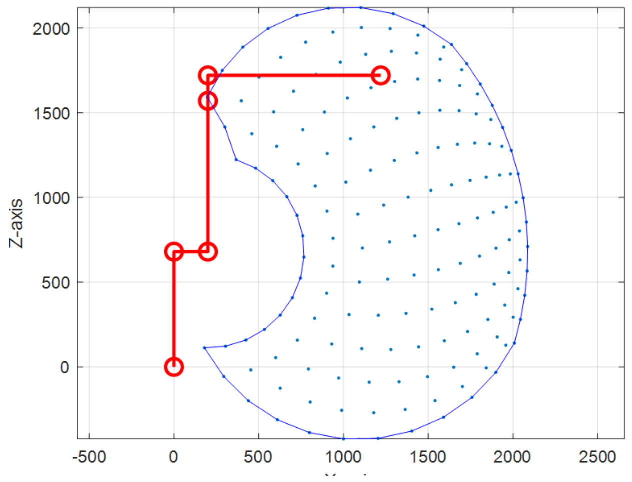
Position no (see figure above)	Position (mm) X	Position (mm) Z	Angle (degrees) Axis 2	Angle (degrees) Axis 3
0	1080	1720	0	0
1	887	2140	0	-30
2	708	836	0	65
3	1894	221	95	-60
4	570	-126	95	40
5	51	1554	-70	40
6	227	1210	-70	65

The θ_2 *can't be* -70 *and* $\theta_3 = -60$ *at the same time*

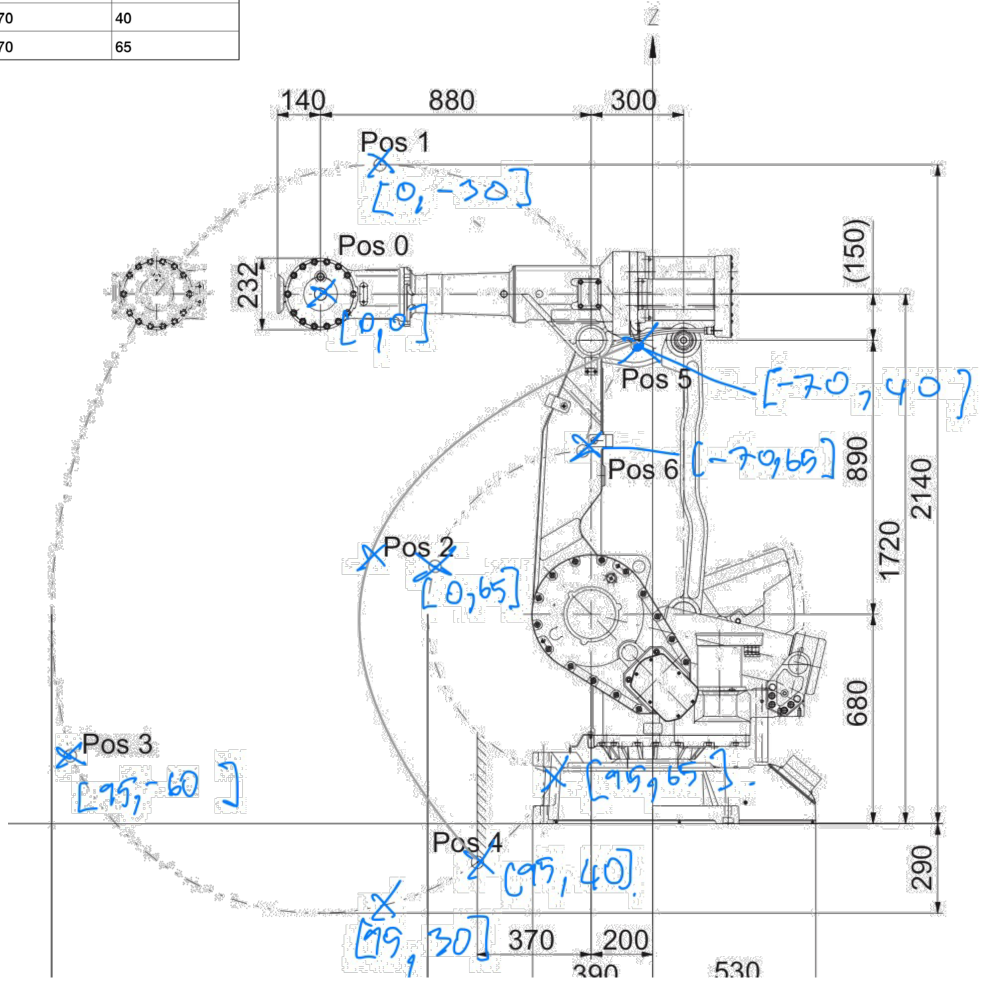
Solve by using **linear regression**



Old



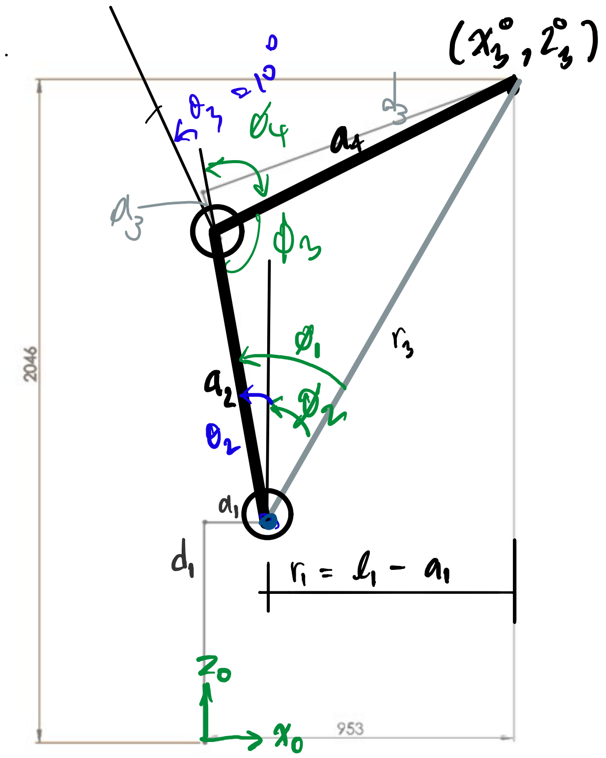
New



Actual

INVERSE KINEMATICS

Geometry Approach



Side View

From Cosine Rule

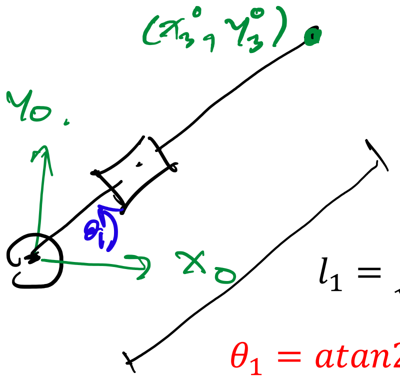
$$\phi_1 = \cos^{-1}\left(\frac{a_3^2 + d_3^2 - a_2^2 - r_3^2}{-2a_2 r_3}\right)$$

$$\phi_2 = atan2(r_1, r_2)$$

From Geometric Side View

$$\theta_2 = \phi_1 - \phi_2$$

$$\theta_2 = \cos^{-1}\left(\frac{a_3^2+d_3^2-a_2^2-r_3^2}{-2a_2 r_3}\right) - atan2(r_1, r_2)$$



$$l_1 = \sqrt{(x_3^0)^2 + (y_3^0)^2}$$

$$\theta_1 = atan2(y_3^0, x_3^0)$$

Top View

From Cosine Rule

$$\phi_3 = cos^{-1}\left(\frac{r_3^2 - r_4^2 - a_2^2}{-2r_4 a_2}\right)$$

From Geometric Side View

$$\phi_4 + \theta_3 = atan2(a_3, d_3)(1)$$

$$\phi_4 = 180^\circ - \phi_3 \ (2)$$

Sub (2) *in* (1)

$$\theta_3 = \phi_3 - 180^\circ + atan2(a_3, d_3)$$

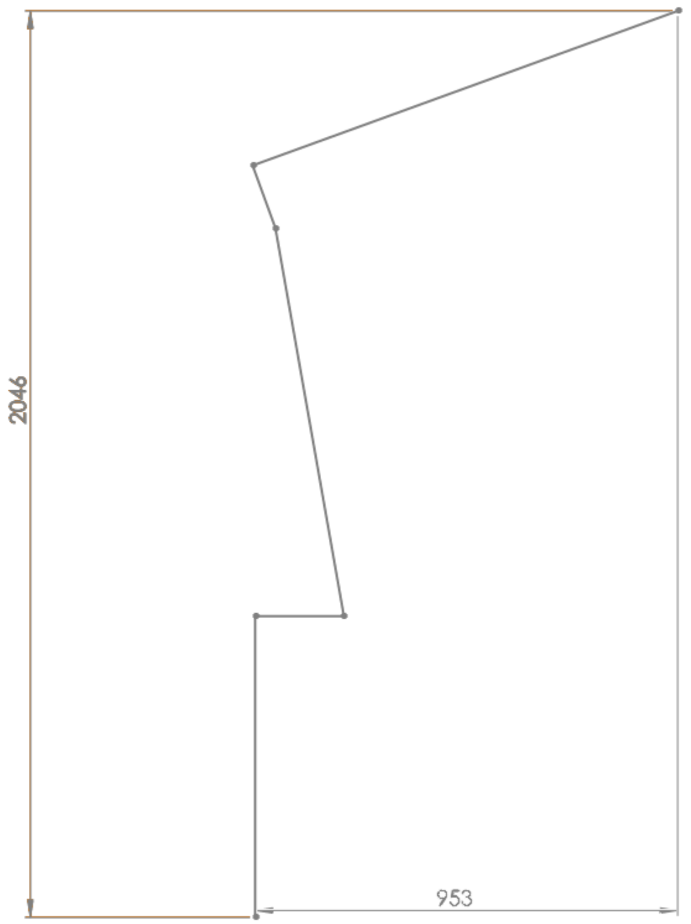
$$\theta_3 = cos^{-1}\left(\frac{r_3^2 - r_4^2 - a_2^2}{-2r_4 a_2}\right) - 180^0 + atan2(a_3, d_3)$$

CHECK INV 1

$theta_{in=}$ [10 10 10]
From Solidworks

X,y,z = [953, 168, 2046]

$theta_{out=}$ [9.9977 9.9958 10.0283]

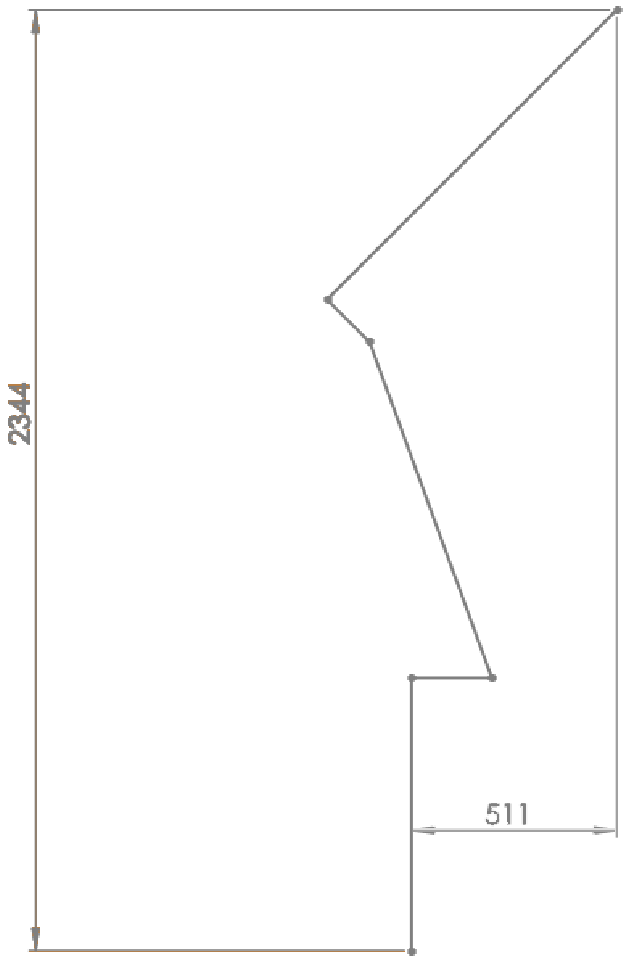


CHECK INV 2

$theta_{in=}$ [0 20 25]

X,y,z = [511, 0, 2344]

$theta_{out=}$ [0 19.9981 25.0459]



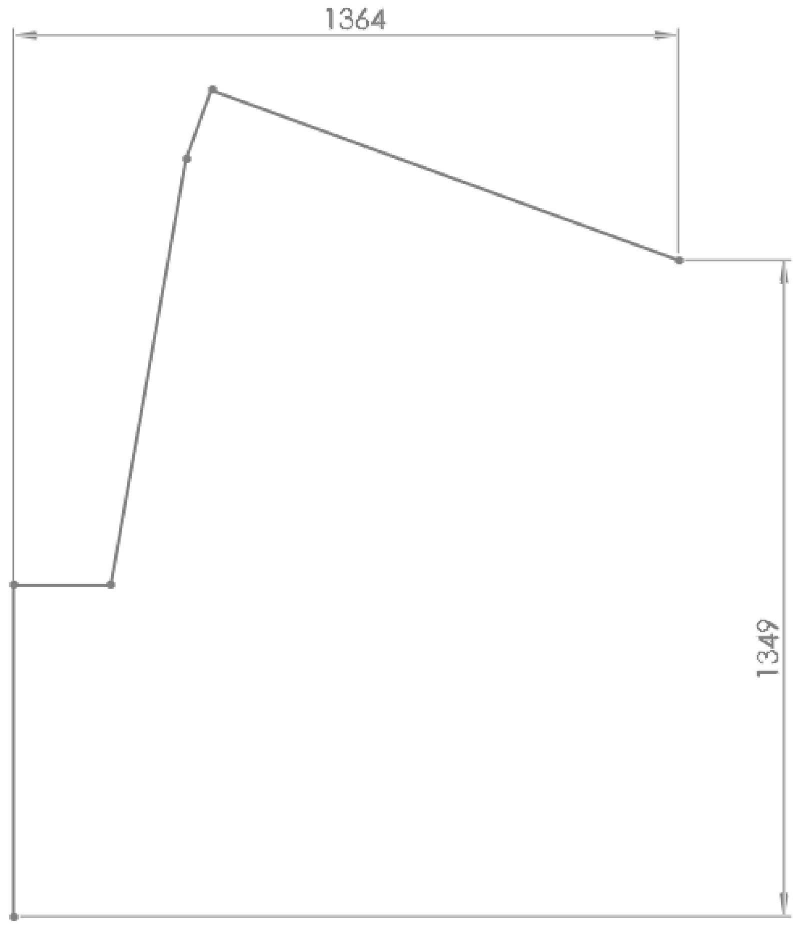
CHECK INV 3

$theta_{in=}$ [0 -10 -10]

X,y,z = [1364, 0, 1349]

$theta_{out=}$ [0 -9.9732 -10.0066]

X,y,z = [1364.336, 0, 1348.572]



JACOBIAN

$$\begin{bmatrix} v_n^0(t) \\ \omega_n^0(t) \end{bmatrix} = \begin{bmatrix} J_v(q(t)) \\ J_\omega(q(t)) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} J_{v_1}(q(t)) & J_{v_2}(q(t)) & J_{v_3}(q(t)) \\ J_{\omega_1}(q(t)) & J_{\omega_2}(q(t)) & J_{\omega_3}(q(t)) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} z_1 \times (P_3^0 - P_0^0) & z_2 \times (P_3^0 - P_1^0) & z_3 \times (P_3^0 - P_2^0) \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

Where as

$$z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad z_2 = z_3 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}; P_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad P_1 = \begin{bmatrix} a_1c_1 \\ a_1s_1 \\ d_1 \end{bmatrix}; \quad P_2 = \begin{bmatrix} a_1c_1 - a_2s_2c_1 \\ a_1s_1 - a_2s_2c_1 \\ d_1 + a_2c_2 \end{bmatrix}; P_3 = \begin{bmatrix} a_1c_1 + d_3s_1 + a_2c_1c_2 + a_3c_3c_2c_1 - a_3s_3s_2c_1 \\ a_1s_1 - d_3c_1 + a_2c_2s_1 + a_3c_3c_2s_1 - a_3s_3s_2s_1 \\ d_1 + a_2s_2 + a_3c_2s_3 + a_3c_3s_2 \end{bmatrix}$$

Thus

$$\begin{bmatrix} v_n^0(t) \\ \omega_n^0(t) \end{bmatrix} =$$

$$\begin{bmatrix} -(a_1s_1 - d_3c_1 + a_2c_2s_1 + a_3c_3c_2s_1 - a_3s_3s_2s_1) & -c_1(-d_3c_1 + a_2c_2s_1 + a_3c_3c_2s_1 - a_3s_3s_2s_1) & d_3s_1 + a_3c_3c_2c_1 - a_3s_3s_2c_1 \\ a_1c_1 + d_3s_1 + a_2c_1c_2 + a_3c_3c_2c_1 - a_3s_3s_2c_1 & -s_1(d_3s_1 + a_2c_1c_2 + a_3c_3c_2c_1 - a_3s_3s_2c_1) & -d_3c_1 + a_3c_3c_2s_1 - a_3s_3s_2s_1 \\ 0 & d_1 + a_2s_2 + a_3c_2s_3 + a_3c_3s_2 & a_3c_2s_3 + a_3c_3s_2 \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}_{6\times 3} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

Robot Application

(IRB4400 M94A)

Robot Capabilities
(IRB4400 M94A)

Enable simplicity

Increase productivity

Ensure consistent quality production

Enhance flexibility

Optimize performance

Help employees with their tasks

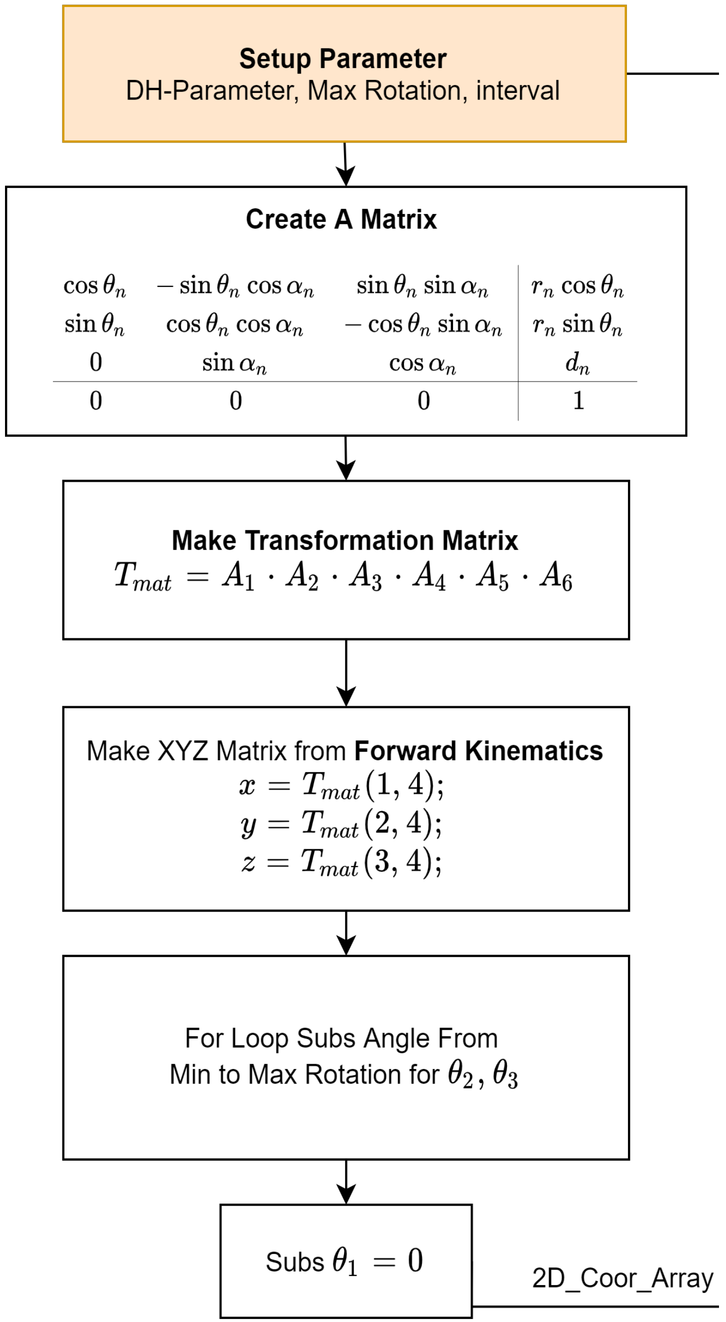
Handling operations, machine tending
- Metal casting
- Plastic. moulding
- Stamping, forging, bending
- Inspection, testing
- Packaging, picking, placing
- Palletizing
- Material handling

Welding
- Arc welding
- Spot welding
- Laser welding
- Soldering

Assembling, disassembling
Cleanroom, Dispensing, painting

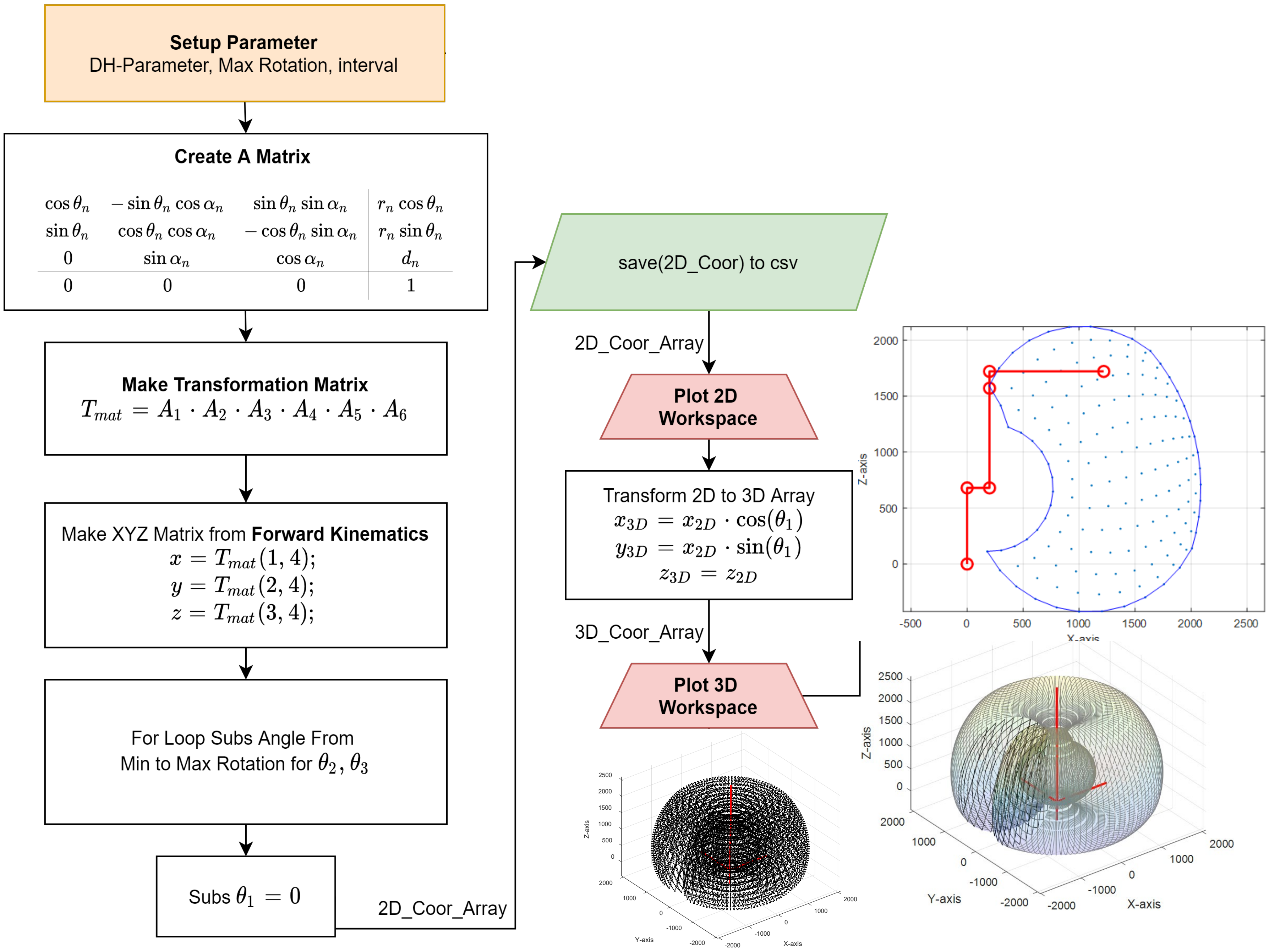
Processing
- **Laser cutting**
- **Water jet cutting**
- Mechanical cutting, grinding, deburring, milling, polishing

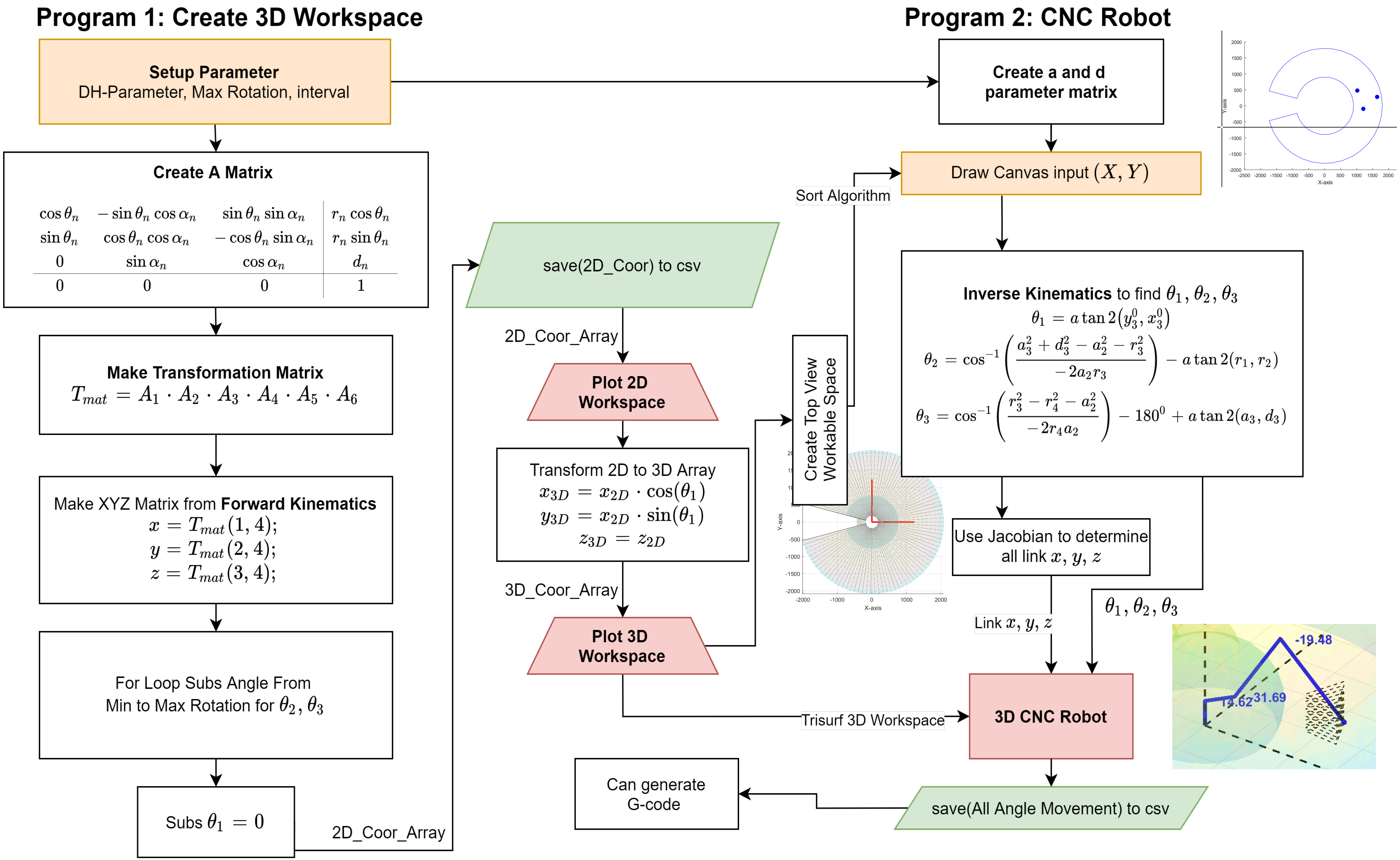
FLOW CHART



Setup Parameter
DH-Parameter, Max Rotation, interval
Create A Matrix
$$\left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$
Make Transformation Matrix
$T_{mat} = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$
Make XYZ Matrix from **Forward Kinematics**
$x = T_{mat}(1, 4);$
$y = T_{mat}(2, 4);$
$z = T_{mat}(3, 4);$
For Loop Subs Angle From
Min to Max Rotation for θ_2, θ_3
Subs $\theta_1 = 0$
2D_Coor_Array

FLOW CHART

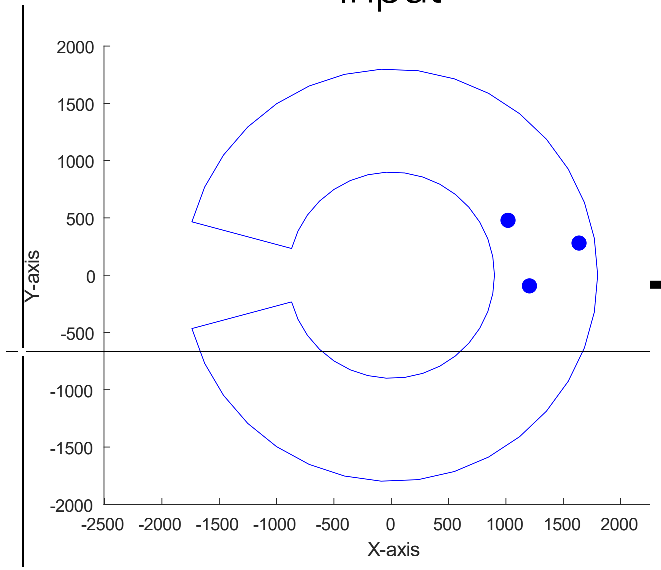

Setup Parameter
DH-Parameter, Max Rotation, interval
Create A Matrix
$$\left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$
Make Transformation Matrix
$T_{mat} = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$
Make XYZ Matrix from **Forward Kinematics**
$x = T_{mat}(1,4);$
$y = T_{mat}(2,4);$
$z = T_{mat}(3,4);$
For Loop Subs Angle From
Min to Max Rotation for θ_2, θ_3
Subs $\theta_1 = 0$
2D_Coor_Array
save(2D_Coor) to csv
2D_Coor_Array
Plot 2D Workspace
Transform 2D to 3D Array
$x_{3D} = x_{2D} \cdot \cos(\theta_1)$
$y_{3D} = x_{2D} \cdot \sin(\theta_1)$
$z_{3D} = z_{2D}$
3D_Coor_Array
Plot 3D Workspace
Z-axis
X-axis
Y-axis



Program 1: Create 3D Workspace
Program 2: CNC Robot
Setup Parameter
DH-Parameter, Max Rotation, interval
Create a and d parameter matrix
Create A Matrix
$\left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$
Make Transformation Matrix
$T_{mat} = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$
Make XYZ Matrix from **Forward Kinematics**
$x = T_{mat}(1, 4);$
$y = T_{mat}(2, 4);$
$z = T_{mat}(3, 4);$
For Loop Subs Angle From
Min to Max Rotation for θ_2, θ_3
Subs $\theta_1 = 0$
2D_Coor_Array
save(2D_Coor) to csv
2D_Coor_Array
Plot 2D Workspace
Transform 2D to 3D Array
$x_{3D} = x_{2D} \cdot \cos(\theta_1)$
$y_{3D} = x_{2D} \cdot \sin(\theta_1)$
$z_{3D} = z_{2D}$
3D_Coor_Array
Plot 3D Workspace
Create Top View Workable Space
Sort Algorithm
Draw Canvas input (X, Y)
Inverse Kinematics to find $\theta_1, \theta_2, \theta_3$
$\theta_1 = a\tan 2(y_3^0, x_3^0)$
$\theta_2 = \cos^{-1}\left(\frac{a_3^2 + d_3^2 - a_2^2 - r_3^2}{-2a_2 r_3}\right) - a\tan 2(r_1, r_2)$
$\theta_3 = \cos^{-1}\left(\frac{r_3^2 - r_4^2 - a_2^2}{-2r_4 a_2}\right) - 180^0 + a\tan 2(a_3, d_3)$
Use Jacobian to determine all link x, y, z
Link x, y, z
$\theta_1, \theta_2, \theta_3$
Trisurf 3D Workspace
3D CNC Robot
save(All Angle Movement) to csv
Can generate G-code
X-axis
Y-axis
-19.48
14.62
31.69

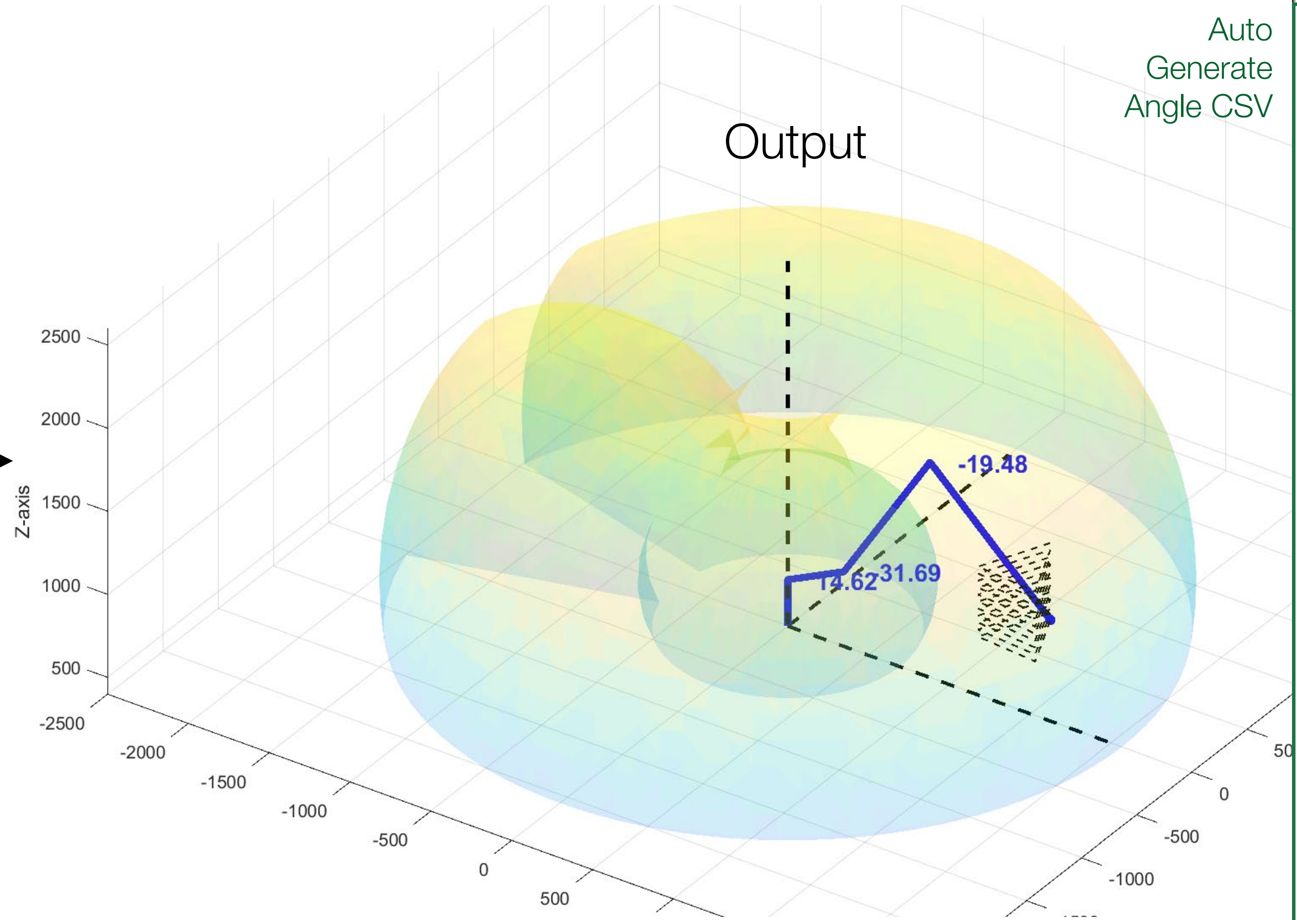
PROGRAM WORKFLOW

Input



Press Enter

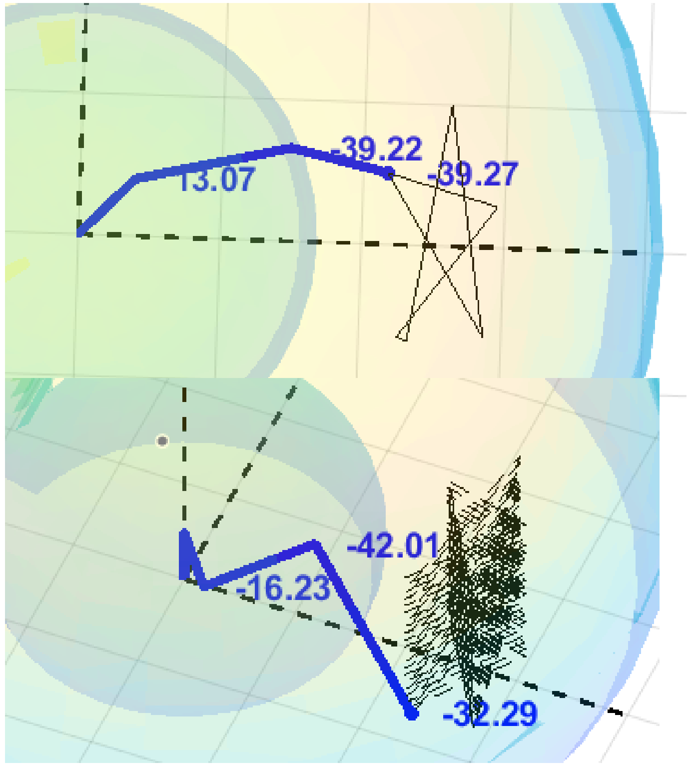
Output



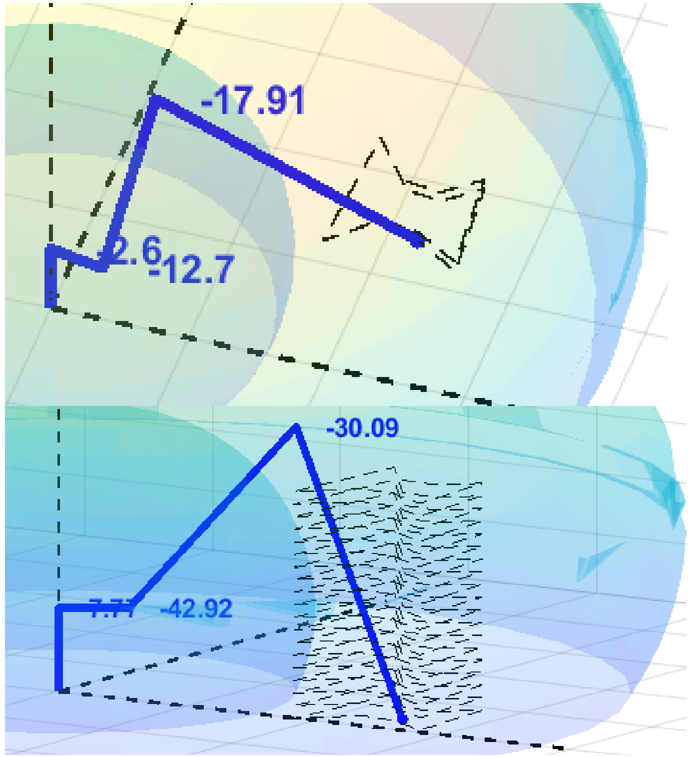
Auto Generate Angle CSV

495.5899	1.268855	106.6155
554.951	1.241202	141.3415
610.2779	1.208673	182.1895
660.9419	1.17164	228.6953
706.3671	1.130521	280.3303
746.0373	1.085786	336.5078
779.5015	1.037941	396.5892
806.3795	0.987532	459.8916
826.3658	0.93513	525.6957
839.2332	0.881332	593.2535
844.8354	0.826749	661.7972
843.1089	0.772001	730.5478
834.0733	0.71771	798.7239
817.8311	0.664494	865.5507
794.5671	0.612958	930.2686
764.5456	0.563686	992.1422
728.1078	0.51724	1050.468
685.6679	0.474146	1104.584
637.7082	0.434895	1153.874
584.7738	0.399933	1197.778
527.4663	0.369657	1235.797
466.437	0.344412	1267.5
402.3796	0.324483	1292.525
336.0221	0.310098	1310.589
268.1186	0.301421	1321.486
199.4409	0.298549	1325.093
130.7696	0.301516	1321.367
62.88507	0.310287	1310.352
581.1579	1.306496	59.34689
645.0648	1.271375	103.4507
703.9134	1.231036	154.1073
757.0348	1.185937	210.7408

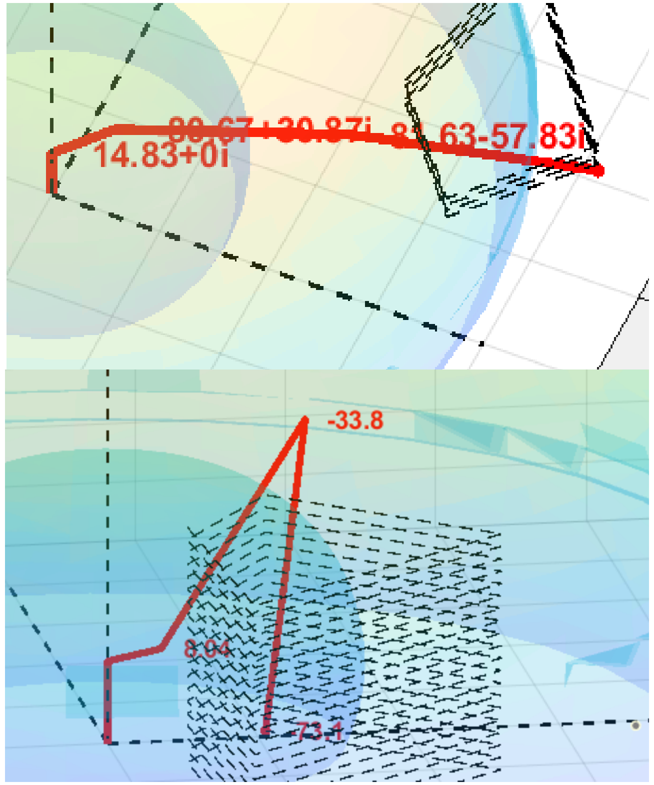
DEMO
HARRY CNC BOT



Can be Drawn
COMPLEX SHAPE



Can be Drawn
ANY SHAPE



Error When
Exceed Workspace

ENJOY

ROBOTICS!

Saksorn Ruangtanusak 61070505063

HARRY CNC BOT