时间序列 HW1

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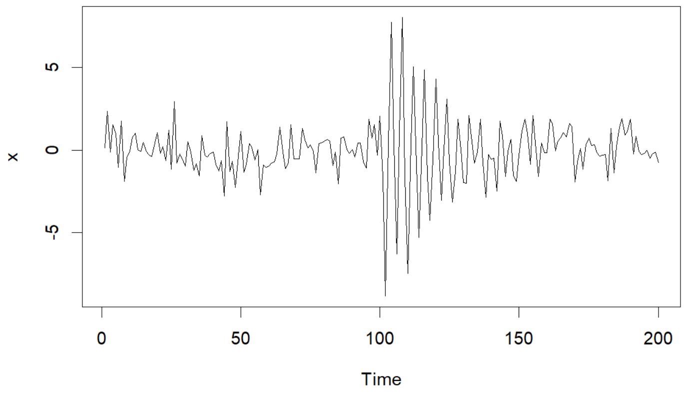
Pb 1.2

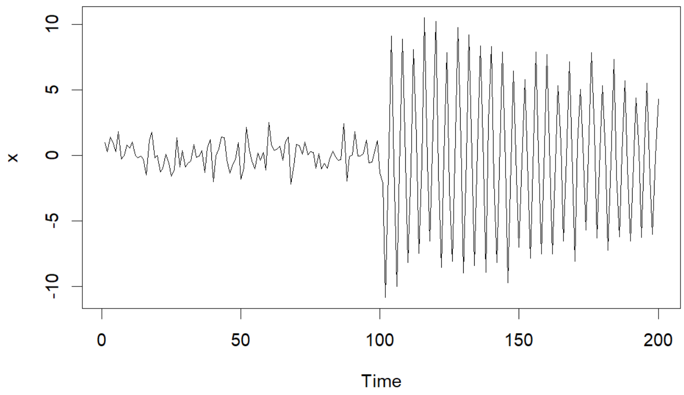
a

```
s = c(rep(0,100), 10*exp(-(1:100)/20)*cos(2*pi*1:100/4))
x = s + rnorm(200)
plot.ts(x)
```



b

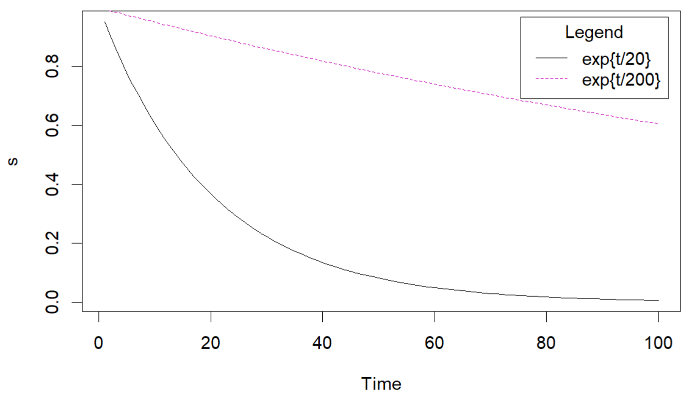
```
s = c(rep(0,100), 10*exp(-(1:100)/200)*cos(2*pi*1:100/4))
x = s + rnorm(200)
plot.ts(x)
```



c

t=0:100 时，(a)(b)均与地震序列的前半段接近；t=101:200 时,(a)与爆炸的后半段相近呈现快速衰减 而(b)与地震的后半段相近呈现不衰减或极缓慢地衰减

```
s =   exp(-(1:100)/20)
plot.ts(s,col=1)
s =   exp(-(1:100)/200)
lines(s,col=6,lty=2)
legend('topright', inset=.02, title="legend", c("exp{t/20}","exp{t/200}"),lty=c(1, 2),
col=c(1,6))
```

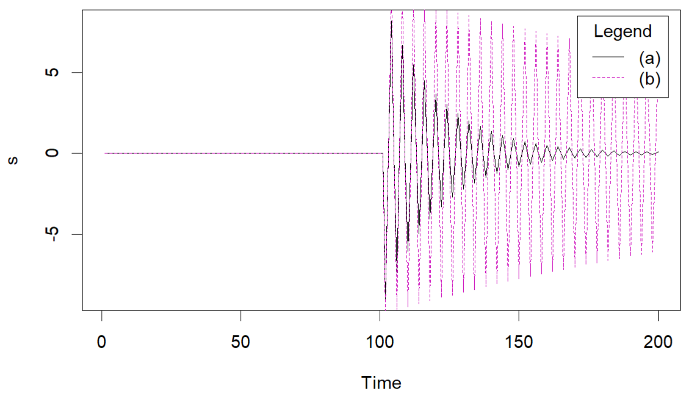


可以观察出，(a)比(b)下降快很多，另外(a)的下降速率由快变慢而(b)的下降速率则相对平稳

Pb 1.5

a

$$\mu_t = Ex_t = Es_t + Ew_t = s_t \qquad (w_t \sim WN \Rightarrow Ew_t = 0)$$



b

$$\gamma(s,t) = \mathrm{E}\left[\left(x_s - \mu_s\right)\left(x_t - \mu_t\right)\right] = \mathrm{E}[w_s w_t] = \{ \begin{matrix} 1, & s=t \\ 0, & o.w \end{matrix}$$

Pb 1.8

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a

$$\begin{cases} x_t - x_{t-1} & = & \delta + w_t, \\ \quad \vdots & & \\ x_1 - x_0 & = & \delta + w_1, \end{cases}$$

$$\Rightarrow x_t - x_0 = \delta \cdot t + \textstyle\sum_1^t w_i = x_t$$

b

$$\mu(t) = E(x_t) = E(\delta \cdot t + \sum_1^t w_i) = \delta \cdot t + \sum_1^t E(w_i) = \delta \cdot t$$

$$\gamma(s,t) = E[(x_s - \mu_s)(x_t - \mu_t)] = E[(\sum_1^s w_k)(\sum_1^t w_i)] = E[\sum_1^{t \wedge s} w_i^2 + \sum_{k,j} w_k w_j] = t \wedge s$$

c

平稳性要求$x_t - x_s \sim x_{t-s}$,从方差来看
$V(x_t - x_s) = t^2 + s^2 - 2Cov(x_t, x_s) = t^2 + s^2 - 2(t \wedge s), V(x_{t-s}) = (t-s)^2$
并不相等,所以不平稳

d

$$\begin{aligned} \rho_x(t-1,t) &= \frac{\gamma(t-1,t)}{\sqrt{\gamma(t-1,t-1)\gamma(t,t)}} \\ &= \frac{t-1}{\sqrt{(t-1)t}} \\ &= \sqrt{\frac{t-1}{t}} \longrightarrow 1 \end{aligned}$$

说明随着t的增大，序列中相邻变量的相关程度越来越大

e

$$
\begin{gathered}
y_t \doteq x_t - x_{t-1} = \delta + w_t \\
u_y(t) = E(y_t) = \delta \equiv Const \\
\gamma(y_s, y_t) = E[(y_s - \mu_y(s))(y_t - \mu_y(t))] = E[w_s w_t] = I(|t-s| = 0) \\
\therefore |t-s| \longmapsto \gamma(y_s, y_t)
\end{gathered}
$$

均值函数为常数且$\gamma(y_s, y_t)$只由|t-s|决定，因此是平稳的。

Pb 1.25

a

假设x_t是某一平稳分布序列，则：

$$
\begin{aligned}
& Var(\sum_1^n a_i x_i) \geq 0, \\
\Longrightarrow & \sum_i \sum_j a_i Cov(x_i, x_j) a_j \geq 0, \\
\because \ & \{x_t\}\text{为平稳分布序列} \\
\therefore \ & \gamma(i-j) = \gamma(x_i, x_j) = Cov(x_i, x_j), \\
\therefore \ & \sum_i \sum_j a_i \gamma(i-j) a_j \geq 0, \quad \forall a \\
& i.e.\ \gamma(h)\text{半正定}.
\end{aligned}
$$

b

不妨先取均值化，令$y_t = x_t - \overline{x}$，然后：

$$
\text{记} \ \ A = \begin{pmatrix}
0 & \cdots & 0 & y_1 & y_2 & \cdots & y_{N-1} & y_N \\
0 & \cdots & y_1 & y_2 & y_3 & \cdots & y_N & 0 \\
\vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
y_1 & \cdots & y_{N-1} & y_N & 0 & \cdots & 0 & 0
\end{pmatrix}
$$

$$
\begin{aligned}
& \because \ \ \hat{\gamma}(|i-j|) = \frac{1}{N} \sum_{k=1}^{n-|i-j|} y_{k+|i-j|} \cdot y_k \\
& \therefore \ \ \hat{\gamma}(|i-j|) = \frac{1}{N} (AA^T)_{i,j} \\
& \therefore \ \ \sum_i^N \sum_j^N a_i \hat{\gamma}(|i-j|) a_j = a^T \cdot \frac{1}{N}(AA^T) \cdot a = \frac{1}{N} \left\| A^T a \right\|_2^2 \geq 0 \quad \forall a \\
& i.e.\ \hat{\gamma}(h)\text{半正定}.
\end{aligned}
$$