时间序列HW08

> 18300290007 加兴华

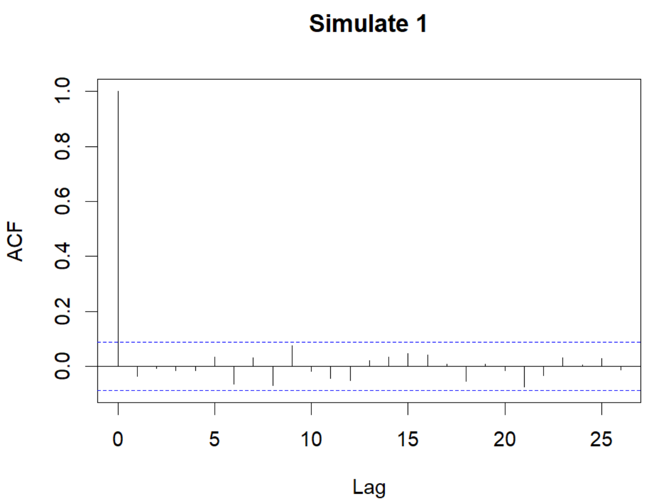
Pb 3.20

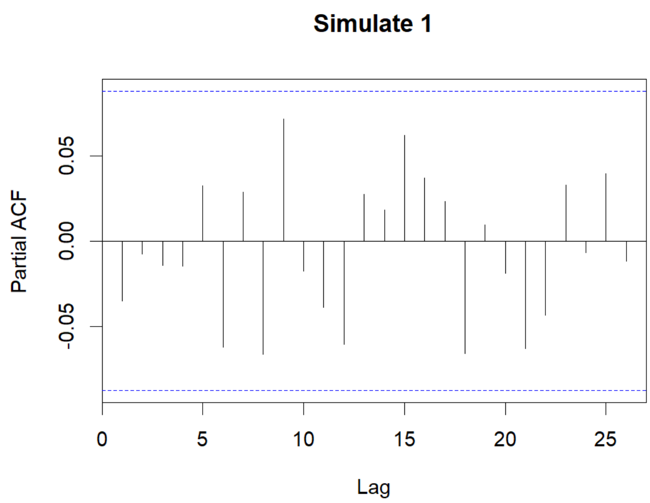
3.20 Repeat the following numerical exercise three times. Generate $n = 500$ observations from the ARMA model given by

$$x_t = .9x_{t-1} + w_t - .9w_{t-1},$$

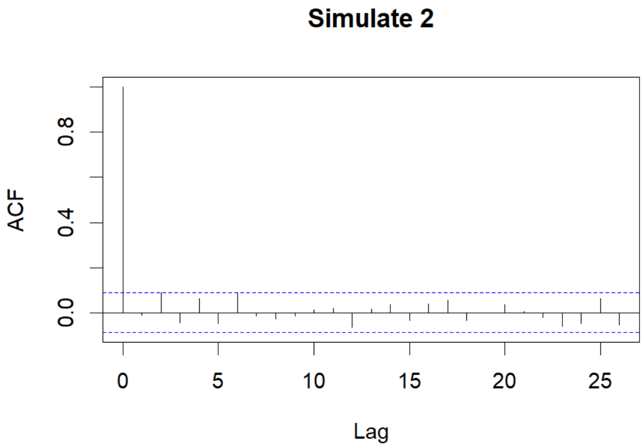
with $w_t \sim$ iid N(0, 1). Plot the simulated data, compute the sample ACF and PACF of the simulated data, and fit an ARMA(1, 1) model to the data. What happened and how do you explain the results?

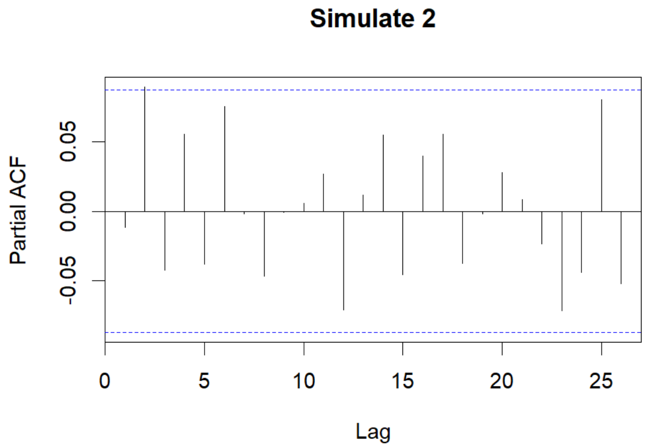
```r
set.seed(i)
n <- 500
x <- arima.sim(list(order=c(1,0,1), ar=c(0.9), ma=c(-0.9)), n=n)
par(mfrow=c(1,2))
acf(x,main='Simulate i')
pacf(x,main='Simulate i')
arima(x, order=c(1,0,1))
```



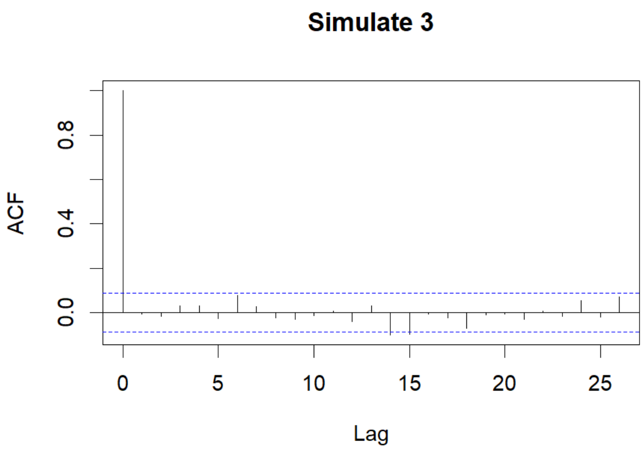


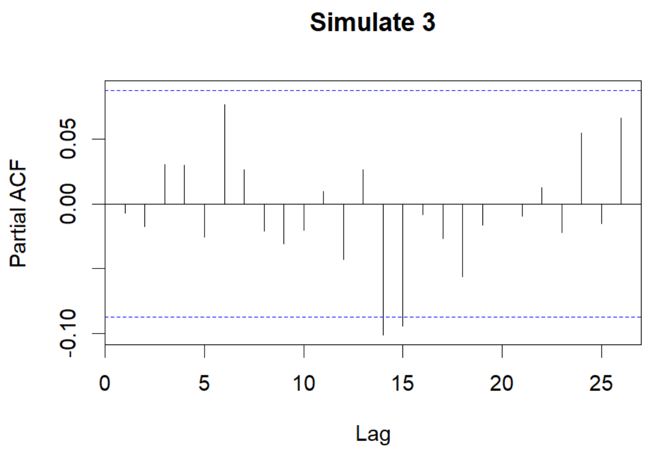
```
Coefficients:
           ar1      ma1    intercept
        0.5313  -0.5653     -0.0031
s.e.    0.9832   0.9642      0.0423
sigma^2 estimated as 1.04:  log likelihood = -719.39,  aic = 1446.78
```






```
Coefficients:
           ar1      ma1    intercept
        -0.851   0.7970      0.0910
s.e.     0.095   0.1067      0.0439
sigma^2 estimated as 1.024:  log likelihood = -715.48,  aic = 1438.95
```






```
Coefficients:
           ar1       ma1    intercept
        0.1669   -0.1759      0.0516
s.e.    1.4665    1.4670      0.0452

sigma^2 estimated as 1.043:  log likelihood = -719.99,  aic = 1447.97
```

观察三次生成，ACF与PACF的图像总体上一致，除了ACF(0)=1外其余均不显著，这是因为在该模型在约简后等价于$x_t = w_t$，从而$x_{i\neq t}\perp x_t$；

参数估计则三次三次生成差异显著，第一次生成真值甚至与估计域相差很远，猜想是如上所述，模型可以约简，参数冗余导致估计不稳定。针对猜想，我将参数θ 改成非-0.9重复进行试验，发现估计结果稳定，说明猜想正确。

Pb 3.21

3.21 Generate 10 realizations of length $n = 200$ each of an ARMA(1,1) process with $\phi = .9, \theta = .5$ and $\sigma^2 = 1$. Find the MLEs of the three parameters in each case and compare the estimators to the true values.

```
n <- 200
y=matrix(0,10,3)
for(i in 1:10){
set.seed(i)
x=arima.sim(list(order=c(1,0,1), ar=c(0.9), ma=c(0.5)), n=n)
model=arima(x, order=c(1,0,1))
y[i,]=cbind(model$coef[1],model$coef[2],model$sigma2)
}
colnames(y)<-cbind(’ar1’,’ma1’,’sigma2’)
rownames(y)<-1:10
show(y)
```

	ar1	ma1	sigma2
1	0.9062811	0.4574039	0.9683910
2	0.8544712	0.5011655	1.0326099
3	0.9024316	0.4957944	1.0447328
4	0.8063899	0.4157756	0.8622443
5	0.8730442	0.4522822	0.9542421
6	0.8193428	0.4990442	0.9372741
7	0.8952263	0.5166450	0.9374252
8	0.7858231	0.5566621	1.0152871
9	0.8419338	0.5260541	0.9066649
10	0.8525341	0.5284628	0.9053378

观察输出，大部分估计值都接近模型的真值(0.9,0.5,1)，但也有一些与真值距离较远，可能是由于生成的序列长度短导致，为了验证猜想，我将n改成2000后再次实验，结果如下：

	ar1	ma1	sigma2
1	0.8996847	0.4901138	1.0908708
2	0.9114919	0.4856297	1.0088062
3	0.8834436	0.5280234	1.0048732
4	0.8860683	0.4709238	0.9587053
5	0.8899475	0.5116074	1.0002551
6	0.8975001	0.5179718	0.9962989
7	0.9011873	0.5007041	0.9988119
8	0.8801503	0.4541335	1.0275313
9	0.9057963	0.4866031	0.9500186
10	0.8985915	0.5008334	1.0397976

可以看到，当序列长度为2000后，估计值能稳定在真值附近，说明之前的猜想正确。