金融计量学 HW1

23210980044 加兴华

Problem 1 (Exercise 1.6)

The prices and dividends of a stock are given in the table below

t	P_t	D_t
1	82	0.1
2	85	0.1
3	83	0.1
4	87	0.125

a. What is $R_3(2)$?
b. What is $r_4(3)$?

$$R_t(k) = \frac{P_t}{P_{t-k}} - 1,$$
$$r_t = log(1 + R_t(k)),$$

a

$$R_3(2) = \frac{P_3 + D_3}{P_1} - 1 = \frac{83 + 0.1}{82} - 1 \approx 0.01341$$

b

$$r_4(3) = log(\frac{P_4 + D_4}{P_1}) = log(\frac{87 + 0.125}{82}) \approx 0.0606$$

Problem 2 (Exercise 1.8)

Suppose that $X_1, X_2, \dots$ is a lognormal geometric random walk with parameters (μ, σ^2). More specifically, suppose that $X_k = X_0 \exp\{r_1 + \dots + r_k\}$, where X_0 is a fixed constant and $r_1, r_2, \dots$ are i.i.d. $N(\mu, \sigma^2)$.

a. Find $P(X_2 > 1.3X_0)$.
b. Use (A.4) in the textbook to find the density of X_1.
c. Find a formula for the 0.9 quantile of X_k for all k.
d. What is the expected value of X_k^2 for any k? (Find a formula giving the expected value as a function of k.)
e. Find the variance of X_k for any k.

a

$$P(X_2 > 1.3X_0) = P(e^{r_1+r_2} > 1.3), \quad r_1 + r_2 \sim N(2\mu, 2\sigma^2)$$

$$\therefore P(X_2 > 1.3X_0) = P(Z > \frac{log(1.3) - 2\mu}{\sqrt{2}\sigma}), \quad Z \sim N(0,1)$$

$$\therefore P(X_2 > 1.3X_0) = 1 - \Phi(\frac{log(1.3) - 2\mu}{\sqrt{2}\sigma})$$

b

$$X_1 = X_0 \cdot e^{r_1} \Rightarrow r_1 = log(\frac{X_1}{X_0}),$$

$$F_{X_1}(x) = P(X_1 \le x) = P(r_1 \le log\frac{x}{X_0}) = F_{r_1}(log\frac{x}{X_0}),$$

$$f_{X_1}(x) = f_{r_1}(log\frac{x}{X_0}) \cdot |\frac{\partial log\frac{x}{X_0}}{\partial x}| = \frac{1}{x} \cdot f_{r_1}(log\frac{x}{X_0}),$$

$$\therefore f_{X_1}(x) = \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{(log\frac{x}{X_0} - \mu)^2}{2\sigma^2}\}, \quad x > 0$$

c

$$X_k = X_0 \cdot exp\{r_1 + r_2 + \cdots + r_k\},$$

$$F_{X_k}(x) = P(X_k < x) = P(r_1 + r_2 + \cdots + r_k < log\frac{x}{X_0}), \quad r_1 + r_2 + \cdots + r_k \sim N(k\mu, k\sigma^2)$$

$$\therefore F_{X_k}(x) = P(Z < \frac{log\frac{x}{X_0} - k\mu}{\sqrt{k}\sigma}),$$

$$\therefore \frac{log\frac{F_{X_k}^{-1}(v)}{X_0} - k\mu}{\sqrt{k}\sigma} = \Phi^{-1}(v)$$

$$\therefore F_{X_k}^{-1}(0.9) = exp\{\Phi^{-1}(0.9)\sqrt{k}\sigma + k\mu\} \cdot X_0$$

d

$$X_k^2 = X_0^2 \cdot exp\{2r_1 + 2r_2 + \cdots + 2r_k\} \sim LN(2k\mu, 4k\sigma^2) \cdot X_0^2,$$

$$\therefore EX_k^2 = e^{2k\mu + 2k\sigma^2} \cdot X_0^2$$

e

$$\begin{aligned} Var(X_k) &= E(X_k^2) - (EX_k)^2 \\ &= X_0^2 \cdot [e^{2k\mu+2k\sigma^2} - (e^{k\mu+\frac{k\sigma^2}{2}})^2] \\ &= X_0^2 \cdot e^{2k\mu+k\sigma^2}(e^{k\sigma^2} - 1) \end{aligned}$$

Problem 3 (Exercise 8.1)

Kendall's tau rank correlation between X and Y is 0.55. Both X and Y are positive. What is Kendall's tau between X and $1/Y$? What is Kendall's tau between $1/X$ and $1/Y$?

$$\rho_\tau(X,Y) = 0.55 = P((X_1 - X_2)(Y_1 - Y_2) > 0) - P((X_1 - X_2)(Y_1 - Y_2) < 0),$$

$$\begin{aligned}
\rho_\tau(X, 1/Y) &= P((X_1 - X_2)(1/Y_1 - 1/Y_2) > 0) - P((X_1 - X_2)(1/Y_1 - 1/Y_2) < 0) \\
&= P((X_1 - X_2)(Y_1 - Y_2) < 0) - P((X_1 - X_2)(Y_1 - Y_2) > 0) \\
&= -0.55,
\end{aligned}$$

$$\begin{aligned}
\rho_\tau(1/X, 1/Y) &= P((1/X_1 - 1/X_2)(1/Y_1 - 1/Y_2) > 0) - P((1/X_1 - 1/X_2)(1/Y_1 - 1/Y_2) < 0) \\
&= P((X_1 - X_2)(Y_1 - Y_2) > 0) - P((X_1 - X_2)(Y_1 - Y_2) < 0) \\
&= \rho_\tau(X,Y) \\
&= 0.55
\end{aligned}$$

Problem 4 (Exercise 8.3)

Show that an Archimedean copula with generator function $\phi(u) = -\log(u)$ is equal to the independence copula C_0. Does the same hold when the natural logarithm is replaced by the common logarithm, i.e., $\phi(u) = -\log_{10}(u)$?

$$\phi(u) = -\log(u) \Rightarrow \phi^{-1}(v) = e^{-v},$$
$$c(u_1, u_2) = \phi^{-1}(\phi(u_1) + \phi(u2)) = e^{-(-log(u_1) - log(u_2))} = u_1 + u_2,$$
$$\therefore c(u_1, u_2) = c_\pi(u_1, u_2)$$

同理可证底数为10时的情况也有同样的结论。

Problem 5 (Exercise 16.2)

Suppose there are two risky assets, C and D. The tangency portfolio is 65% C and 35% D, and the expected return and standard deviation of the return on the tangency portfolio are 5% and 7%, respectively. Suppose also that the risk-free rate of return is 1.5%. If you want the standard deviation of your return to be 5%, what proportions of your capital should be in the risk-free asset, asset C and asset D?

$$R_T = 0.65R_C + 0.35R_D, \quad E_T = 0.05, \quad SR_T = 0.07, \quad \mu_f = 0.015.$$
$$R \doteq \omega R_T + (1-\omega)R_f,$$
$$\sigma_R = \omega SR_T = 0.05 \Rightarrow \omega = \frac{5}{7}$$
$$ER = \mu_f + \omega(E_T - \mu_f) = 0.015 - \omega(0.05 - 0.015)$$
$$\therefore R = \frac{5}{7}(0.65R_C + 0.35R_D) + \frac{2}{7}R_f \approx 0.46R_C + 0.25R_D + 0.29R_f$$

Problem 6

Use the stock return data of Apple, Amazon and Google from 2015/01/01 to 2016/12/31 to find the tangency portfolio weight and the minimum variance. Plot the efficient frontier.

```
library(xlsx);library(quadprog)

data <- read.xlsx('./hw1.xlsx',1);data
R <- 100*data[,2:4]; head(R)
mean_vect <- apply(R,2,mean); print(mean_vect)
cov_mat <- cov(R); print(cov_mat)
num_assets <- 3
```

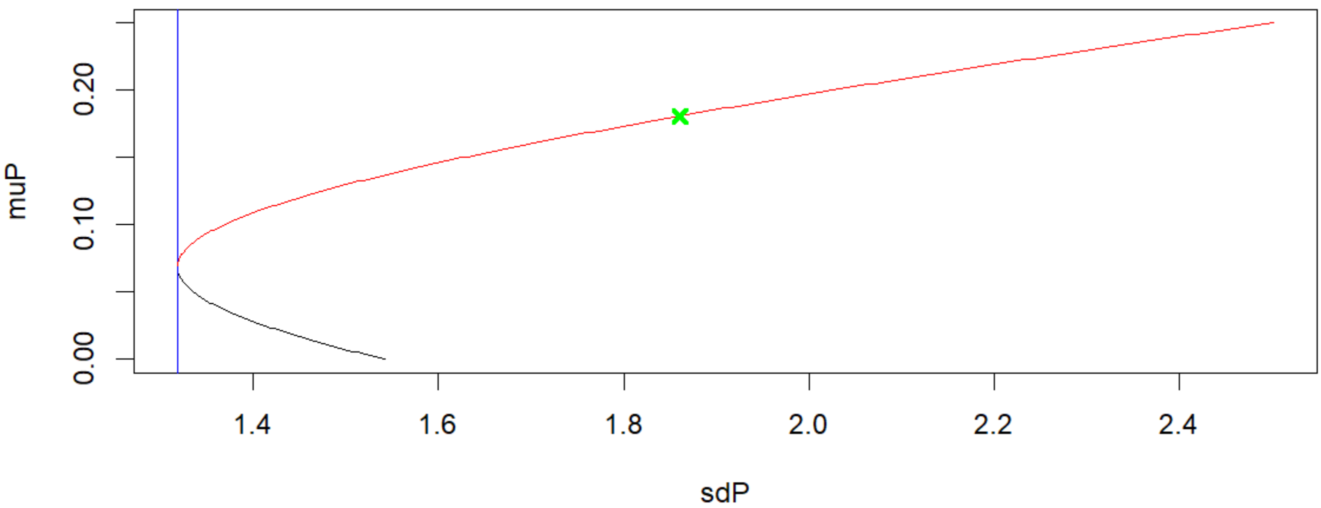
```
Amat <- rbind(rep(1,num_assets),diag(num_assets))
# 使用quadprog包求解每个设定μ下的sd和权重
muP <- seq(0,0.25,length=300)
sdP <- muP
Amat <- rbind(rep(1,3),mean_vect)
weights <- matrix(0,nrow=300,ncol=num_assets)
for(i in 1:length(muP)){
   bvec=c(1,muP[i])
   result=solve.QP(Dmat=2*cov_mat,dvec=rep(0,num_assets),Amat=t(Amat),bvec=bvec,meq=2)
   sdP[i]=sqrt(result$value)
   weights[i,]=result$solution
}
head(weights)
min_var_ind <- (sdP==min(sdP)); print(sdP[min_var_ind])   #最小方差组合
efficient_fronter_ind <- (muP>muP[min_var_ind])

sharpe <- muP[efficient_fronter_ind]/sdP[efficient_fronter_ind ]; # 计算μf=0时的夏普比率
tan_ind <- (sharpe==max(sharpe)); print(weights[tan_ind,])   # 切点组合

# 画图
plot(sdP,muP,type="l") # 抛物线
abline(v=sdP[min_var_ind],col="blue") # 最小方差垂线
lines(sdP[efficient_fronter_ind ],muP[efficient_fronter_ind],col='red') # 有效前沿
points(sdP[tan_ind ],muP[tan_ind], col = "green", pch=4, lwd=3) # 最优组合点
```

由于不含无风险资产，取μ_f=0.

代码显示最优组合权重(Apple, Amazon, Google)=[-0.05544401   0.83233067   0.22311333]，对应图中绿叉

最小方差组合标准差=1.318429， 对应图中蓝线

有效前沿如下图中红线所示


muP
0.00
0.10
0.20
sdP
1.4
1.6
1.8
2.0
2.2
2.4