Homework 3

DATA130013 Financial Econometrics

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Problem 1 (Example 1.32)

Show that the Ornstein-Uhlenbeck process is a Gaussian process with zero mean function and covariance function $\frac{\sigma^2}{2c}(e^{c(t+s)} - e^{c(t-s)})$ for $s < t$.

Problem 2 (Exercise 3.1)

Write an algorithm for Brownian motion with drift $q > 0$ such that

$$\tilde{B}_t = B_t + qt, \quad t \in [0, T],$$

where B is the standard Brownian motion. Moreover, simulate 10000 sample paths with $q = 0.1$ and sketch the paths in one plot.

Problem 3 (Example 3.8)

We want to estimate $Cauchy(0, 1)$ tail probability p with the following four equivalent forms

$$p = \int_2^\infty \frac{1}{\pi(1+x^2)}dx = \frac{1}{2}\int_{|x|>2} \frac{1}{\pi(1+x^2)}dx = \frac{1}{2} - \int_0^2 \frac{1}{\pi(1+x^2)}dx = \int_0^{1/2} \frac{x^{-2}}{\pi(1+x^{-2})}dx.$$

Construct four approximations based on the four representations by Monte Carlo method and monitor their variances. What can you conclude?

Problem 4 (Example 3.10)

Suppose $X \sim f$, the density of $t(\nu, \theta, \sigma^2)$ with location θ and scale σ. Consider $\mathbb{E}h_i(X)$ with h_i being

$$h_1(x) = \sqrt{\left|\frac{x}{1-x}\right|}, \quad h_2(x) = x^5 I(x \geq 2.1), \quad h_3(x) = \frac{x^5}{1+(x-3)^2} I(x \geq 0).$$

Compare the approximation results of Monte Carlo method, importance sample with instrumental distribution $Cauchy(0, 1)$ and importance sample with instrumental distribution $N\left(0, \frac{\nu}{\nu-2}\right)$.