Homework 4 for ECE 251A

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1. Problem from the text: 2.12, 4.2 (Minimum-phase system for part a), 4.7 (no need for partial autocorrelation).

2.12 Consider the system function of a third-order FIR system

$$H(z) = 12 + 28z^{-1} - 29z^{-2} - 60z^{-3}$$

(*a*) Determine the system functions of all other FIR systems whose magnitude responses are identical to that of $H(z)$.

(*b*) Which of these systems is a minimum-phase system and which one is a maximum-phase system?

(*c*) Let $h_k(n)$ denote the impulse response of the kth FIR system determined in part (*a*) and define the energy delay of the kth system by

$$\mathcal{E}_k(n) \triangleq \sum_{m=n}^{\infty} |h_k(m)|^2 \qquad 0 \le n \le 3$$

for all values of k. Show that

$$\mathcal{E}_{\min}(n) \le \mathcal{E}_k(n) \le \mathcal{E}_{\max}(n) \qquad 0 \le n \le 3$$

and

$$\mathcal{E}_{\min}(\infty) = \mathcal{E}_k(\infty) = \mathcal{E}_{\max}(\infty) = 0$$

where $\mathcal{E}_{\min}(n)$ and $\mathcal{E}_{\max}(n)$ are energy delays of the minimum-phase and maximum-phase systems, respectively.

4.2 Consider a zero-mean random sequence $x(n)$ with PSD

$$R_x(e^{j\omega}) = \frac{5 + 3\cos\omega}{17 + 8\cos\omega}$$

(*a*) Determine the innovations representation of the process $x(n)$.

(*b*) Find the autocorrelation sequence $r_x(l)$.

4.7 Use the Yule-Walker equations to determine the autocorrelation and partial autocorrelation coefficients of the following AR models, assuming that $w(n) \sim \text{WN}(0, 1)$.

(*a*) $x(n) = 0.5x(n-1) + w(n)$.

(*b*) $x(n) = 1.5x(n-1) - 0.6x(n-2) + w(n)$.

What is the variance σ_x^2 of the resulting process?

2. Computer Assignment: Repeat Prob. 2 from Homework 3, with the Welch method for data sizes of 128 and 256 samples. In each case, experiment (average over several trials) with an overlap

of 50% and 75%, segments of length 32, 64 and 128. Compare the performance for the rectangular and Hamming window.