April 4 (Monday).

Topics: 1.30 GE focused on Shadow Computation.

Example: Given

1. x_w-y_w-z_w world coordinate Right Hand System. $\vec{P_i} + \lambda(\vec{P_s} - \vec{P_i})$

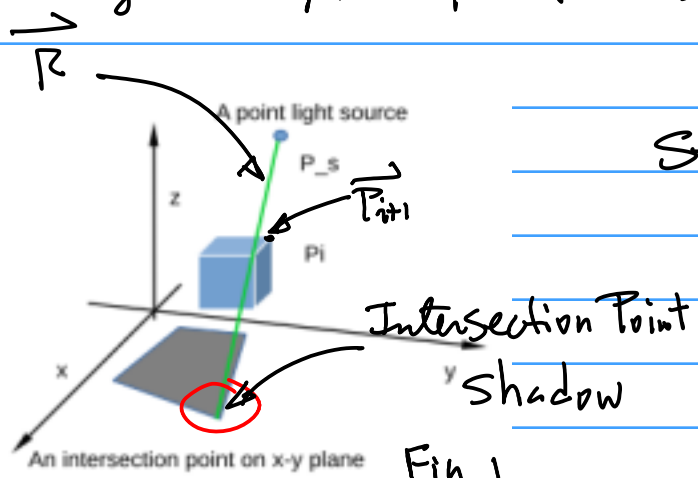


Fig. 1

2. A Given Cube $\{\vec{P_i}(x_i, y_i, z_i) \mid i = 1, 2, \cdots, N\}$

Counter Clockwise

3. A given Point light Source $\vec{P_s}(x_s, y_s, z_s)$

4. Generate Ray Equation / Ray Cast x_w-y_w plane, Produces Shadow if Blocked by the Cube.

References:

2021F-101b-notes-cmpe240-2021-12-1.pdf

Step 1. Generate A Ray Cast / Equation

$$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) \quad \ldots (1)$$

Starting from $\vec{P_s}$, Passing through $\vec{P_i}$

$$\vec{d}(x, y) = \vec{P_s} - \vec{P_i} \quad \ldots (1b)$$

Intersection Point: $\vec{P_i}$ Between Ray $\vec{R}(x, y)$, and x_w-y_w plane.

Step 2. To find the intersection Point on x_w-y_w plane

Define a plane equation.

a. Define A Normal vector

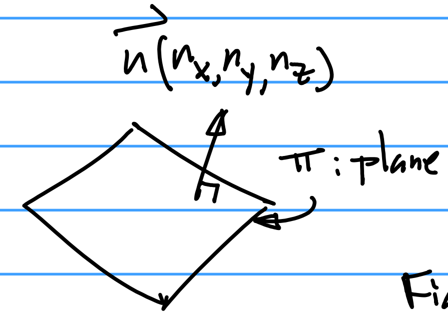


Fig. 2

use the normal vector to define the plane π.

b. Form a vector (on a plane) from $\vec{P_i}$ and $\vec{P_{i+1}}$, $(\vec{P_{i+1}} - \vec{P_i})$ a line

$$\vec{n} \cdot (\vec{P_{i+1}} - \vec{P_i}) = 0 \quad \ldots (2)$$

Take

$\vec{v}(v_x, v_y, v_z)$, $\vec{a}(a_x, a_y, a_z)$ to Replace $\vec{P_i}$, $\vec{P_{i+1}}$

Assume $\vec{a}(a_x, a_y, a_z)$ is a Known Vector; And $\vec{v}(v_x, v_y, v_z)$ is unknown, But an arbitrary Point on the plane π.

Hence, Eqn (2) becomes

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0 \quad \dots (2^*)$$

Now, find the intersection point defined By the Ray Eqn (1). In order to that, we will need to find λ.

Since, the intersection point $\vec{P_i}$ is the Common Point By the Ray and the plane π. we have

$$\begin{cases} \vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) & \dots (3a) \\ \vec{n} \cdot (\vec{v} - \vec{a}) = 0 & \dots (3b) \end{cases}$$

$\vec{n}(n_x, n_y, n_z)$, Normal Vector has to be Known,

$\vec{a}(a_x, a_y, a_z)$ is known on π.

Starting from the plane Eqn (3b).

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

where $\vec{v} = \vec{R}$, e.g.

$$\vec{n} \cdot (\vec{v} - \vec{a}) \Big|_{\vec{v} = \vec{R}} = 0 \quad \dots (4)$$

$$\vec{n} \cdot (\vec{R} - \vec{a}) \Big|_{\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i})} = 0$$

$$\vec{n} \cdot (\vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) - \vec{a}) = 0$$

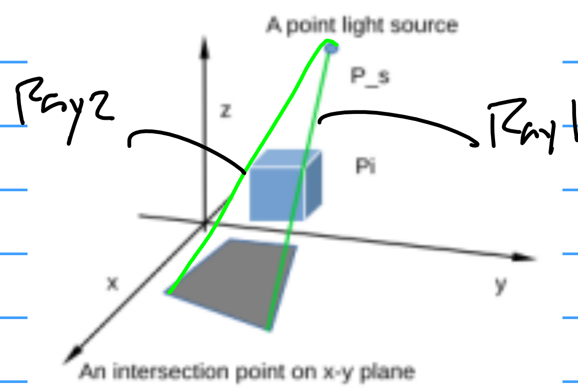
$$\vec{n} \cdot \vec{P_i} + \lambda \vec{n} \cdot (\vec{P_s} - \vec{P_i}) - \vec{n} \cdot \vec{a} = 0$$

$$\lambda \vec{n} \cdot (\vec{P_s} - \vec{P_i}) = \vec{n} \cdot \vec{a} - \vec{n} \cdot \vec{P_i}$$

$$\lambda = \frac{\vec{n} \cdot \vec{a} - \vec{n} \cdot \vec{P_i}}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})}$$

$$= \frac{\vec{n} \cdot (\vec{a} - \vec{P_i})}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})} \quad \dots (5)$$

Note λ is NOT the intersection Pt. it allows us to use Ray Eqn (1) to find the intersection.



use Eqn (5) to find more than one intersection

April 7th (Wed)

Example: Ref (see pp.1. github LectureNotes, 2021F-101b-~)
Given $\vec{P_s}(-20, 110, 200)$, A vertex of a given Cube $\vec{P_i}(100, 100, 110)$. Find the intersection pt to Draw Shadow.

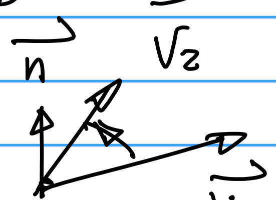
Sol First, Ray Equation, Eqn(1):

$$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) \quad \ldots (1)$$

then, plane Eqn for X_w-Y_w plane

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

Find $\vec{n}$,



$$\vec{V_1} \times \vec{V_2} = \vec{n} \quad \ldots (2)$$

Let $\vec{V_1} = x_w \vec{i}$, $\vec{V_2} = y_w \vec{j}$

$$\vec{n} = x_w \vec{i} \times y_w \vec{j}$$

$$= (0, 0, 1)$$

Then, 2nd find λ for the Ray Equation, from Eqn.

$$\lambda = \frac{\vec{n} \cdot (\vec{a} - \vec{P_i})}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})} \quad \ldots (5)$$

where $\vec{a} = (0, 0, 0)$, Hence,

$$\lambda = \frac{-\vec{n} \cdot \vec{P_i}}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})}$$

Now, hand Calculation, also for Coding

$$\lambda = \frac{-\vec{n} \cdot \vec{P_i}}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})}$$

$$= -\frac{n_x x_i + n_y y_i + n_z z_i}{n_x(x_s - x_i) + n_y(y_s - y_i) + n_z(z_s - z_i)}$$

From the given Condition $\vec{n}(0, 0, 1)$, Therefore

$$\lambda = \frac{-\cancel{n_z} \cdot z_i}{\cancel{n_z} \cdot (z_s - z_i)} = -\frac{z_i}{z_s - z_i}$$

$$= -\frac{110}{200 - 110} = -\frac{110}{90} = -11/9$$

Now, back to the Ray Equation

$$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i})$$

$$\vec{R} = (100,100,110) - \frac{11}{9}(-20-100, 110-100, 200-110)$$

$$= (100,100,110) - \frac{11}{9}(-120, 10, 90)$$

$$= \left(\frac{1100\times120}{9}, -\left(100-\frac{110}{9}\right), 110-110\right)$$

$$= \left(\frac{1100\times120}{9}, \frac{110}{9}-100, 0\right) = (246.7, 87.8, 0)$$

please finish this calculation.

Now, Coding part. Same Code on github.

Note:

a. Define Normal vector $\vec{n}$ for X_w-Y_w plane

2018F-116-11diffuse20181114.cpp

```
world.X[47] = 0; world.Y[47] = 0; world.Z[47] = 1; // normal vector for x-y plane
```

b. Note the "typedef struct" for Defining 3D points.

```
typedef struct {
    float X[UpperBD], Y[UpperBD], Z[UpperBD];
} pworld;
```

```
pworld  world;
pviewer viewer;
```

$$\lambda = \frac{\vec{n}\cdot(\vec{a}-\vec{P_i})}{\vec{n}\cdot(\vec{P_s}-\vec{P_i})} \quad \cdots \quad (5)$$

$$= -\frac{n_x x_i + n_y y_i + n_z z_i}{n_x(x_s-x_i)+n_y(y_s-y_i)+n_z(z_s-z_i)}$$

Now, λ Calculation. Eqn (5)

```
//----------lambda for intersection pt on xw-yw plane----------*
float temp = (world.X[47]*(world.X[46]-world.X[45]))
            +(world.Y[47]*(world.Y[46]-world.Y[45]))
            +(world.Z[47]*(world.Z[46]-world.Z[45]));
float lambda = temp / ((world.X[47]*(world.X[45]-world.X[7]))
                      +(world.Y[47]*(world.Y[45]-world.Y[7]))
                      +(world.Z[47]*(world.Z[45]-world.Z[7])));
float lambda_2 = temp / ((world.X[47]*(world.X[45]-world.X[6]))
                        +(world.Y[47]*(world.Y[45]-world.Y[6]))
                        +(world.Z[47]*(world.Z[45]-world.Z[6])));
```

Note, Substitute λ to Ray Equation to find the intersection point

```
//-----------ray equation to find intersection pts------------*
world.X[48] = world.X[45] + lambda*(world.X[45] - world.X[7]); // Intersection pt p7
world.Y[48] = world.Y[45] + lambda*(world.Y[45] - world.Y[7]); // Intersection pt p7
world.Z[48] = 0.0;

world.X[49] = world.X[45] + lambda_2*(world.X[45] - world.X[6]);  //intersection pt p6
world.Y[49] = world.Y[45] + lambda_2*(world.Y[45] - world.Y[6]);  //intersection pt p6
world.Z[49] = 0.0;
```

$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) \rightarrow$

$$x = x_i + \lambda(x_s - x_i)$$
$$y = y_i + \lambda(y_s - y_i)$$
$$z = z_i + \lambda(z_s - z_i)$$

Assignment in-Class Show & Tell. Implement Intersection Computation on LPC1769, "Show+Tell" Demo in Class. On April 11 (Monday),

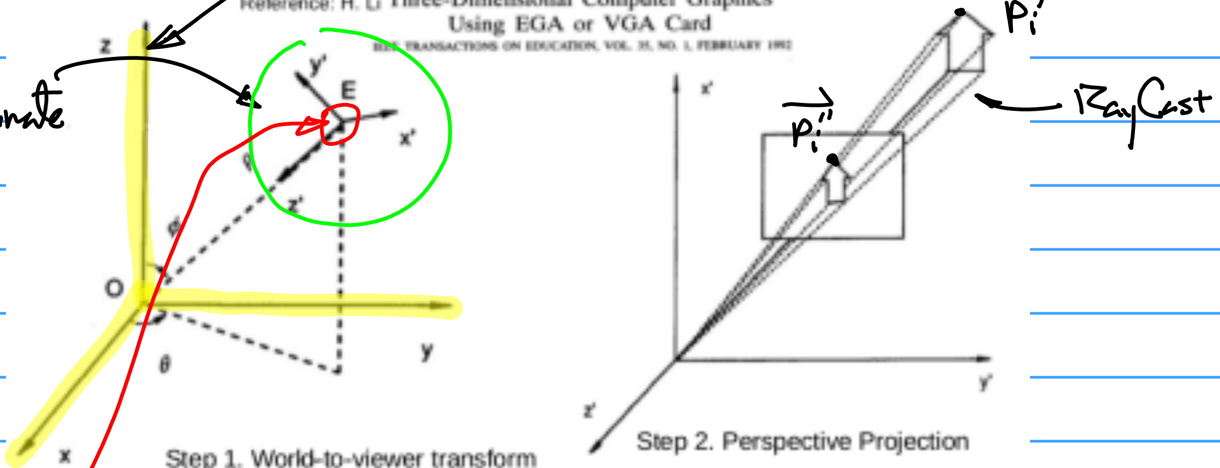
To Be Able to Display 3D Graphics on 2D Display Devices. Let's Define Transformation Pipeline. 1. Define World-Coordinate System, x_w-y_w-z_w; Right Hand System

3D Transformation Pipeline Technique

Reference: H. Li Three-Dimensional Computer Graphics Using EGA or VGA Card, IEEE TRANSACTIONS ON EDUCATION, VOL. 35, NO. 1, FEBRUARY 1992

2. Viewer Coordinate System x_v-y_v-z_v

Left-Hand System



Step 1. World-to-viewer transform

Step 2. Perspective Projection

3. Virtual Camera is located $\vec{E}$ (e_x, e_y, e_z)

$$\mathbf{T} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x_p = x_e\left(\frac{D}{z_e}\right)$$
$$y_p = y_e\left(\frac{D}{z_e}\right)$$

Harry Li, Ph.D.

Example: Display Shadows on 2D Display Device.

Step 1. World To Viewer Transform.

$$T = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \cdots (1)$$

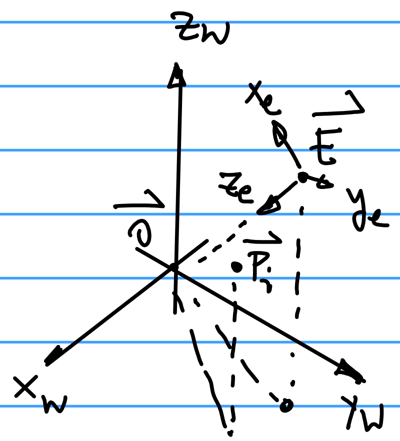


Fig. 1.

Everything is defined in the World Coordinate System X_w-Y_w-Z_w including a Virtual Camera.

$\vec{E}(x_e, y_e, z_e)$, X_e-Y_e-Z_e "Viewer" Coordinate System.

Given $\vec{P_i}(x_i, y_i, z_i)$ in X_w-Y_w-Z_w World Coordinate, Represent this point in X_e-Y_e-Z_e Coordinate System.

$$\vec{P_i'} \quad \begin{bmatrix} x_i' \\ y_i' \\ z_i' \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix} \overset{\vec{P_i}}{\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}} \quad \cdots (1\text{-}b)$$

Assume $\vec{E}(x_e, y_e, z_e) = (200, 200, 200)$

Physical meaning of Transformation Matrix T.

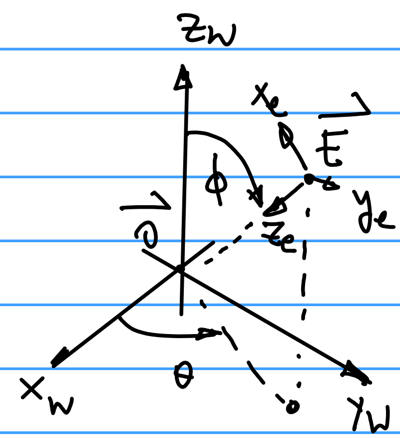


Fig. 2

θ: Angle from the dash line on X_w-Y_w plane w.r.t positive X_w-axis

ϕ: Angle Between Z_w & Z_e.

ρ (rho): $\rho = \sqrt{x_e^2 + y_e^2 + z_e} \quad \cdots (2)$

distance from $\vec{E}$ to the origin $\vec{O}$ of X_w-Y_w-Z_w.

Suppose $\vec{E}(200, 200, 200)$ is given, Find $\cos\theta$, $\sin\theta$, $\cos\phi$, $\sin\phi$ for T-matrix.

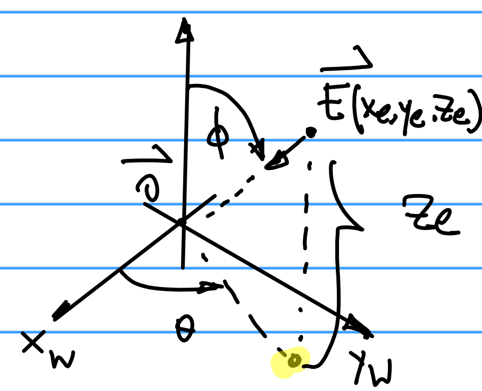
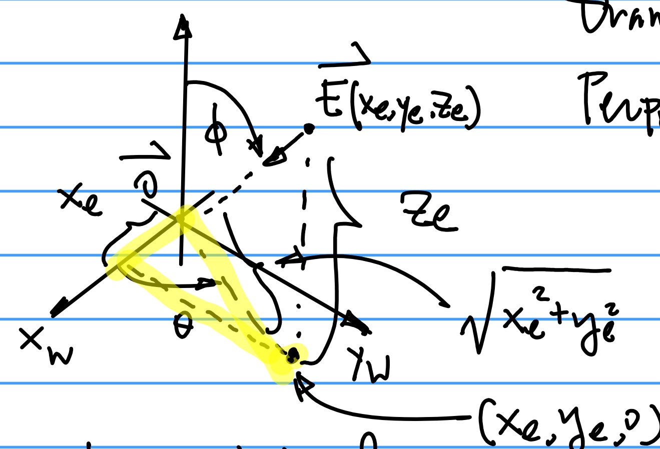


Fig. 3

Draw A Line Passing through $\vec{E}$ Perpendicular to X_w-Y_w $\Rightarrow$ Form an intersection point.

Draw A Line Passing through the intersection point on X_w-Y_w plane on the plane and Perpendicular to X_w-axis.

Fig.4

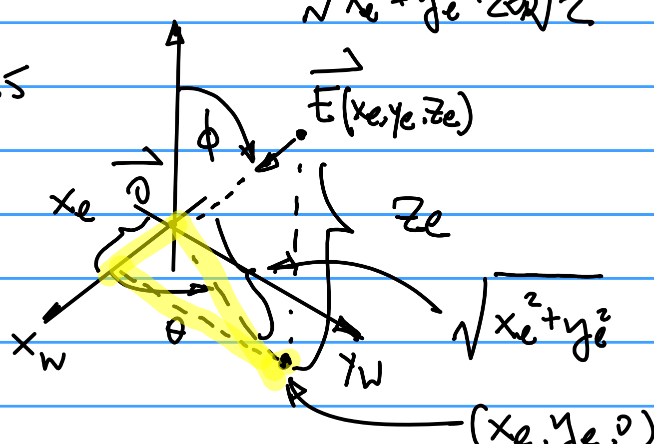


We can form a triangle on X_w-Y_w plane, as in Fig.4, hence

$$\cos\theta = \frac{x_e}{\sqrt{x_e^2+y_e^2}} = \frac{200}{200\sqrt{2}} = \frac{\sqrt{2}}{2}$$

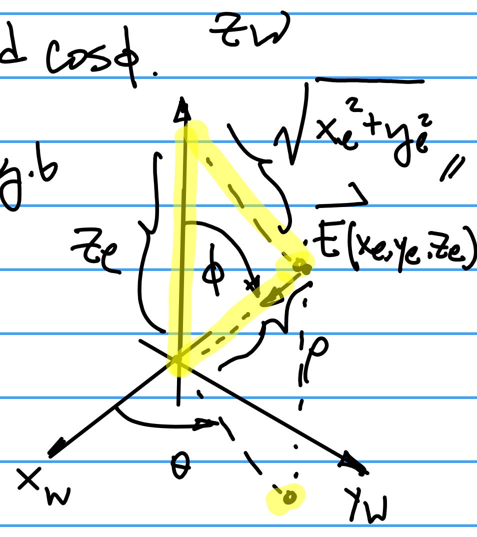
similarly, $\sin\theta = \dfrac{y_e}{\sqrt{x_e^2+y_e^2}} = \dfrac{200}{200\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Fig.5



Find $\cos\phi$.

Fig.6



Draw A Line Passing $\vec{E}(x_e, y_e, z_e)$ Perpendicular to Z_w-axis

$$\cos\phi = \frac{z_e}{\rho} = \frac{z_e}{\sqrt{x_e^2+y_e^2+z_e}}$$

$$= \frac{200}{200\sqrt{3}} = \sqrt{3}/3$$

$$\sin\phi = \frac{\sqrt{x_e^2+y_e^2}}{\sqrt{x_e^2+y_e^2+z_e}} = \frac{200\sqrt{2}}{200\sqrt{3}}$$

$$= \frac{\sqrt{2}\cdot\sqrt{3}}{3} = \frac{\sqrt{6}}{3}$$

Homework: Due April 18th (Monday)

1. Draw A world Coordinate System X_w-Y_w-Z_w axis, with X_w Red, Y_w Green, Z_w Blue

2 Draw A cube, size Length = 100, floats 10 unit Above $X_w - Y_w$ plane. in other word $\vec{P_i}(100, 100, 110)$;

3. Draw a point light Source $\vec{P_s}(-20, 110, 200)$, And Ray Cast to Connect $\vec{P_s}$ to $\vec{P_i}$; use Green Color.

4. Compute the Shadow point $\vec{P_i'}$, Draw the intersection Point to link $\vec{P_s} - \vec{P_i} - \vec{P_i'}$

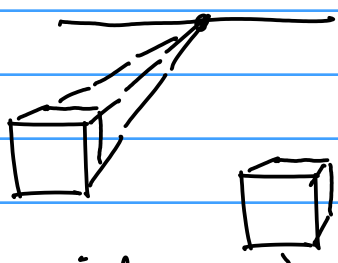
Note: You may want to Adjust the $\vec{P_s}$ Position, So this $\vec{P_i'}$ (Intersection Point) Can be Visible on Your Display

Step 2. Perspective Projection

$$x_p = x_e\left(\frac{D}{z_e}\right)$$
$$y_p = y_e\left(\frac{D}{z_e}\right) \quad \ldots (3)$$

Vanishing pt

Horizon



Eqn(3) project a point $\vec{P_i}(x_i, y_i, z_i)$ in $X_e - Y_e - Z_e$

Onto a 2D Display Device, like LCD.

April 13 (Wed)

Topics: 1° Perspective Projection

2° Diffuse Reflection

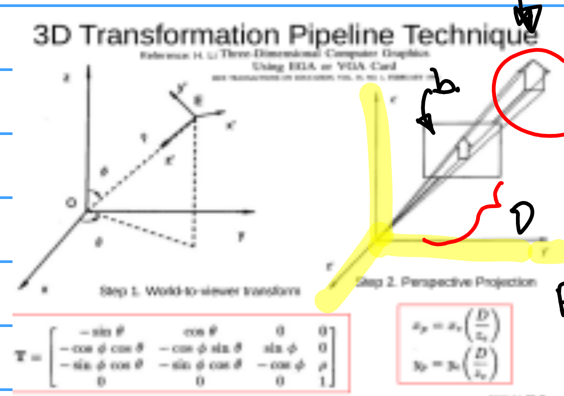


Fig.1

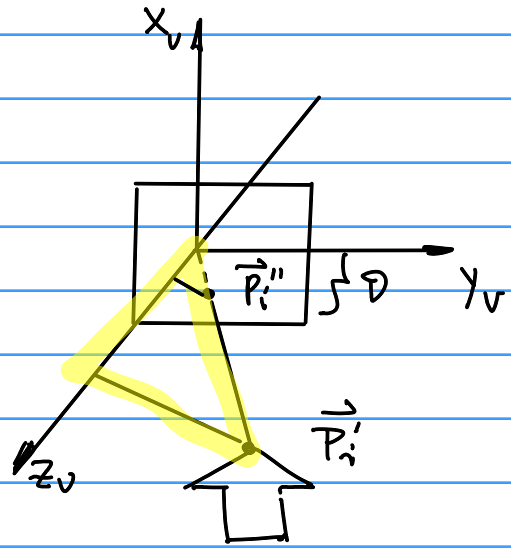
Note: $X_v - Y_v - Z_v$ is Left Hand System.

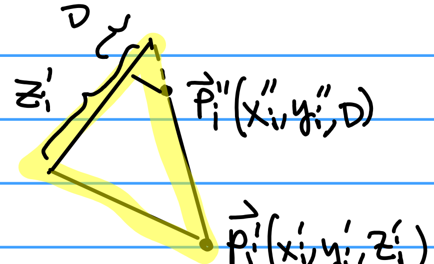
a. It is a 3D object

b. projection Plane, distance to the viewer Coordinate System $(0,0,0) \to (X_e, Y_e, Z_e)$ in $X_w Y_w Z_w$. D is the distance.

c. Origin of $X_v - Y_v - Z_v$. → Camera Location, Camera is modeled as "pin-Hole" model.

Projection is formulated By using Similar Triangles in Fig.1





$x_p = \frac{D}{Z_e} x_e$ from Eqn(8).

Or,

$x_i'' = \frac{D}{Z_i'} x_i'$

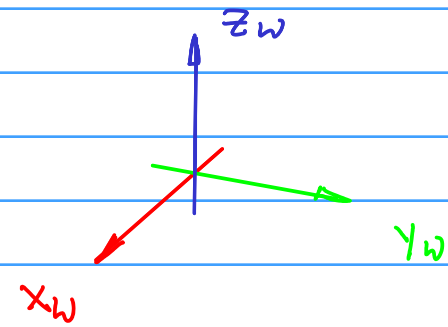
Similarly,

$y_p = \frac{D}{Z_e} y_e$ or

$y_i'' = \frac{D}{Z_e} y_i'$

Homework, Due A week from Today
April 20th

1. Draw a World Coordinate System, X_W : Red, Y_W : Green, Z_W: Blue, Design the size (X_W, Y_W, Z_W, 50 units)



2. Design By Defining Dimension of a Cube.
(Example: length = 100)
$\vec{P}_i(x_i, y_i, z_i) = (100, 100, 110)$
Elevate the cube By 10 units.

3. Draw the Cube on the LCD

4. Submission:
   a. Screen Capture of your XPRESSO Screen, which shows your Name (Folder Name) And your Program (Partial) 0.5 pt
   b. Take a photo of your display. With Entire Prototype System of your own 0.5 pt.

5. Submission to CANVAS.

Note: You will need transform from a virtual coordinate System to physical coordinate

Consider Diffuse Reflection.

2018S-23-lec7-DiffuseReflection-v6-2018-4-25.pdf

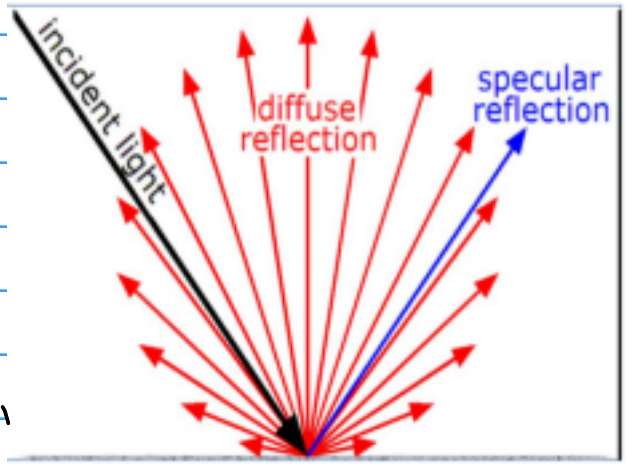


Fig. 1

https://en.wikipedia.org/wiki/Diffuse_reflectio

Definition: A Reflection from an object <u>Surface</u> <u>uniformly</u> in <u>all</u> directions.

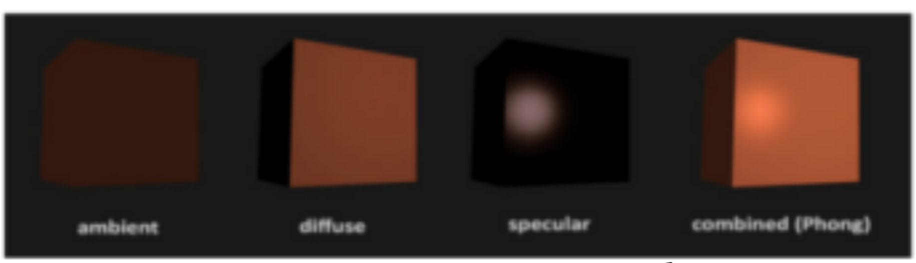


Fig2.

1. Define Reflectivity, A property of An Object surface.

$$\vec{K}_r = (k_r, k_g, k_b) = (r, g, b) \quad \cdots (1)$$

$$0 \le r, g, b \le 1$$

Note: for a Black object,

$r=0,\ g=0,\ b=0$

for a green leaf.

$r \approx 0;\ g \neq 0, 0 < g;\ b \approx 0$

2. Color Space. R, G, B

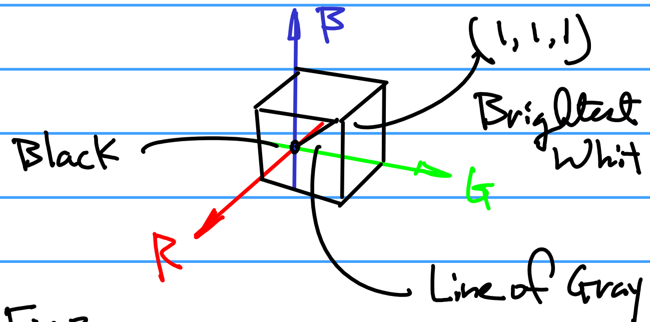
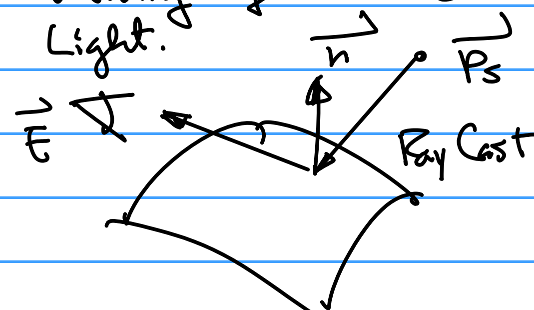


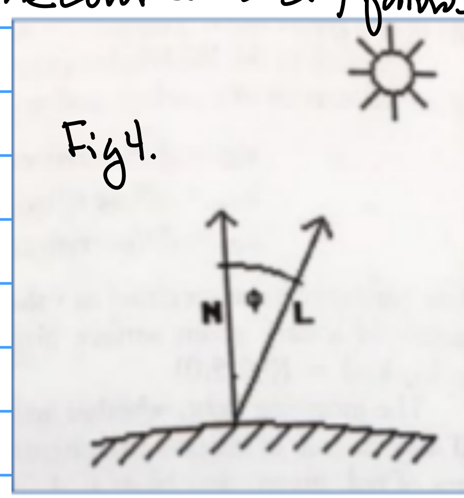
Fig3

3. Viewing Angle v.s. Incident Light.



a. Perceived color is independent of viewing Angle.

b. Normal vector $\vec{N}$ and incident Light $\vec{L}$ ($\vec{R}$ Ray Cast) form An Angle ϕ, the color Intensity follows



Cos φ function.

Color Intensity.

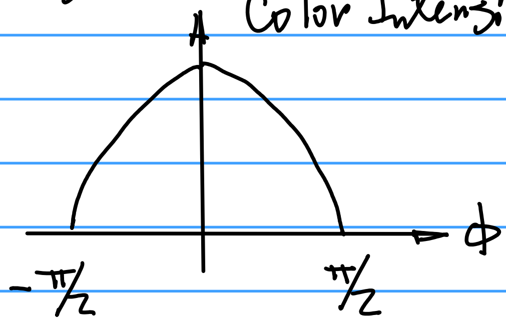


Fig 5.

$\phi = 0$, $\cos\phi = 1$, Strongest Reflection, highest Intensity.

$\phi = \pi/2$, $\cos\phi = 0$, No Reflection, hence, no color.

4. Distance.

April 18 (Monday)

Topics: Continue on Diffuse Reflection.

Light Intensity Satisfies the relationship Below.

$$\text{Intensity} \sim \frac{1}{\|\vec{r}\|^2} \quad \dots (1)$$

$\vec{r}$ Ray Equation.

$$\vec{r} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i})$$

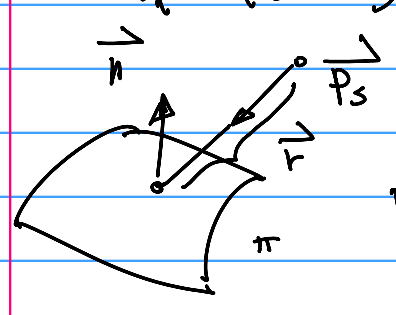


Fig. 1

Example: Given $\vec{P_s}(x_s, y_s, z_s)$ And A Cube.

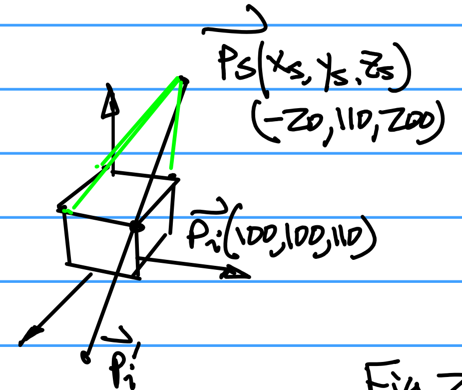


Fig. 2

$\vec{P_i}$ shadow Point.

Find Diffuse Reflection at $\vec{P_i}(x_i, y_i, z_i)$

Sol: Formulation.

$$\vec{I}_{diff}(x, y, z) = (I_{diff,r}(x,y),\ I_{diff,g}(x,y),\ I_{diff,B}(x,y))$$

$$\vec{K_d} = (K_r, K_g, K_b) \quad \dots (2)$$

$$0 \le K_r, K_g, K_b \le 1$$

Note: For Simplicity, we will focus on one type of Reflectivity for Now, "r" red

$$\boxed{I_r = K_{d_r} \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \frac{1}{\|\vec{r}\|^2}} \quad \dots (3)$$

Reflectivity for "r"

Cos φ. $\vec{n} \cdot \vec{r} = \|\vec{n}\| \|\vec{r}\| \cos\phi$

$$\|\vec{r}\|_2 = \sqrt{(x_s-x_i)^2+(y_s-y_i)^2+(z_s-z_i)^2} \quad \ldots(4)$$

$$\|\vec{r}\|_2^2 = (x_s-x_i)^2+(y_s-y_i)^2+(z_s-z_i)^2$$

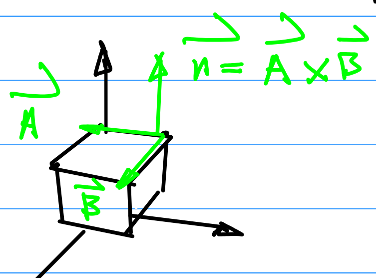
Suppos $\vec{P_s}(-20,110,200)$, $\vec{P_i}(100,100,110)$

Assum: $K_{dr}=0.8$

Find Normal vector for the Cube Surface.

$\vec{n}$ Defined By Vector Cross Product

$$\vec{n}=\vec{A}\times\vec{B}$$



$\vec{p_i}$ Fig 3

But the Surface of the Cube is in Parallel with X_w-Y_w plane.

$\therefore \vec{n}(0,0,1)$

Now, find

$$\cos\phi = \frac{\vec{n}\cdot\vec{r}}{\|\vec{n}\|\|\vec{r}\|}$$

Directional vector of $\vec{r}$

$$= \frac{(n_x,n_y,n_z)\cdot(x_i-x_s, y_i-y_s, z_i-z_s)}{\sqrt{n_x^2+n_y^2+n_z^2}\sqrt{(x_i-x_s)^2+(y_i-y_s)^2+(z_i-z_s)^2}}$$

Substitute the given Condition, So

$$\cos\phi = \frac{z_i-z_s}{\sqrt{(100+20)^2+10^2+90^2}} = \frac{110-200}{\sqrt{\Delta}}$$

Therefore $\cos\phi = \left|\frac{110-200}{\sqrt{\Delta}}\right|$

$$= \left\|-\frac{90}{\sqrt{\Delta}}\right\| = \frac{90}{\sqrt{\Delta}} \qquad \frac{1}{\sqrt{\Delta}} \ll 1.$$

And the distance from $\vec{P_s}$ to $\vec{P_i}$

$$\|\vec{r}\|^2 = (x_s-x_i)^2+(y_s-y_i)^2+(z_s-z_i)^2$$
$$= 120^2+10^2+90^2$$

Therefore, we have

$$I_{diff} = 0.8\times\frac{90}{\sqrt{\Delta}}\times\frac{1}{120^2+10^2+90^2}$$

Note: Very Small! Need Post Processing for Better Visualization.

$$= 0.8\times\frac{90}{\sqrt{120^2+10^2+90^2}}\times\frac{1}{120^2+10^2+90^2} = 2.12\times10^{-5}$$

Since the result is very small, we need to Perform Post Processing.

Note: Directional vector $\vec{P_i}-\vec{P_s}$ gives Negative Value, $\vec{P_s}-\vec{P_i}$ Positive → take Absolute Value.

Objective: To scale up the result in Eqn(3), to match the entire dynamic range of the display device.

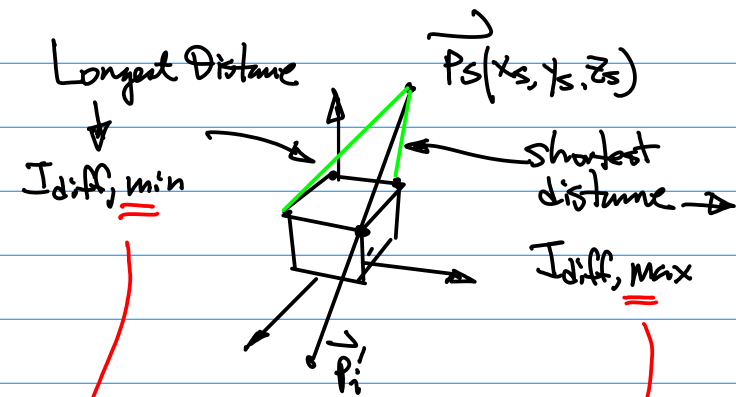
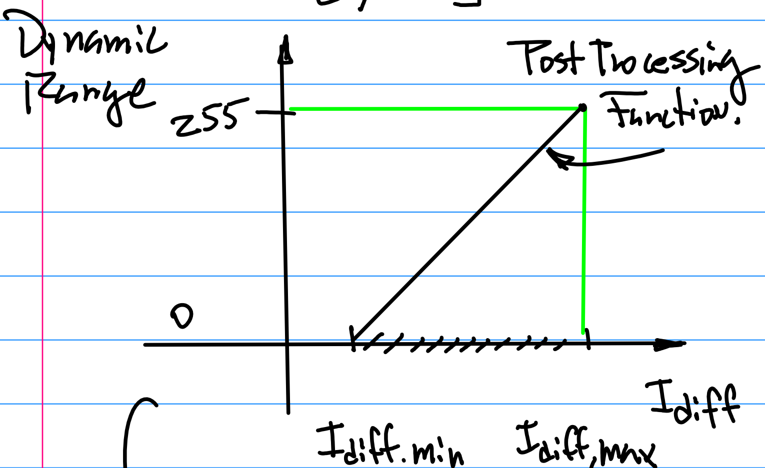


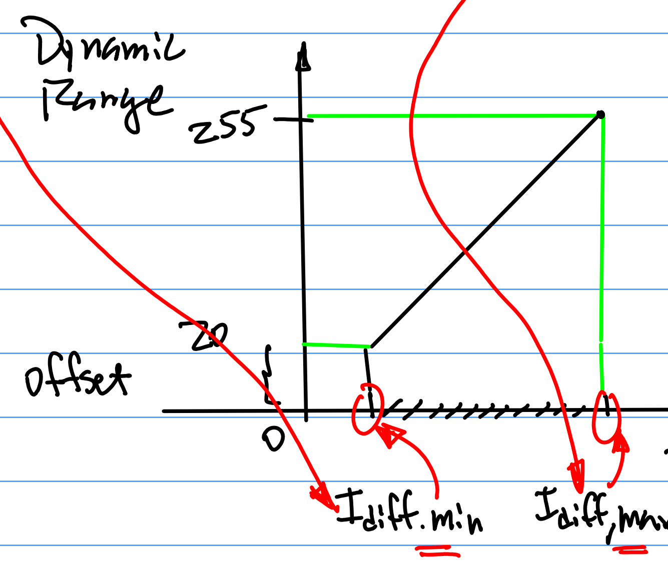
Fig.5

1. Dynamic Range of the Display Device → 8 bits for Each Primitive color.

$2^8 = 256$. Dynamic Range $[0, 255]$



By Eqn(3)   Fig(4-a)



Post Processing function

$$\frac{x - x_2}{y - y_2} = \frac{x_1 - x_2}{y_1 - y_2}$$

Add An offset to this function

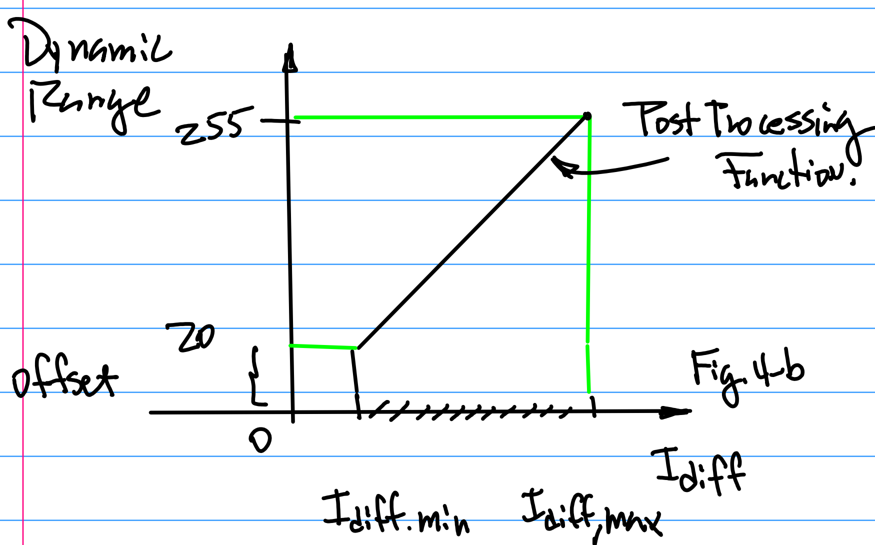


Fig.4-b