CmpE240 April 4, 22 Part II-1

April 4 (Monday).

Topics: 1:30 GE focused on Shadow Computation.

Example: Given

1. x_w-y_w-z_w world coordinate Right Hand System.

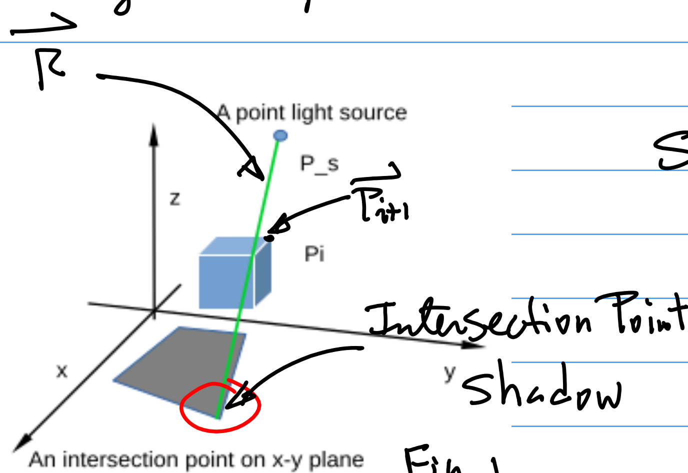


Fig. 1

2. A Given Cube $\{\vec{P_i}(x_i, y_i, z_i) \mid i=1,2,\cdots,N\}$ Counter Clockwise.

3. A given point light Source $\vec{P_s}(x_s, y_s, z_s)$

4. Generate Ray Equation/Ray Cast x_w-y_w plane, Produces Shadow if Blocked by the Cube.

References:

2021F-101b-notes-cmpe240-2021-12-1.pdf

Step1. Generate A Ray Cast/Equation

$$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) \quad \ldots (1)$$

Starting from $\vec{P_s}$, Passing through $\vec{P_i}$

$$\vec{d}(x, y) = \vec{P_s} - \vec{P_i} \quad \ldots (1b)$$

Intersection Point: $\vec{P_i}$

Between Ray $\vec{R}(x,y)$, and x_w-y_w plane.

Step 2. To find the intersection Point on x_w-y_w plane

Define a plane equation.

a. Define A Normal vector

$\vec{n}(n_x, n_y, n_z)$

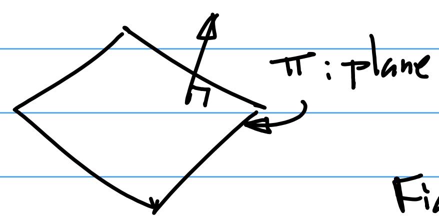


Fig. 2

use the normal vector to define the plane π.

b. Form a vector (on a plane) from $\vec{P_i}$ and $\vec{P_{i+1}}$, $(\vec{P_{i+1}} - \vec{P_i})$ a line

$$\vec{n} \cdot (\vec{P_{i+1}} - \vec{P_i}) = 0 \quad \ldots (2)$$

Take

$\vec{v}(v_x, v_y, v_z)$, $\vec{a}(a_x, a_y, a_z)$

to Replace $\vec{P_i}$, $\vec{P_{i+1}}$

Assume $\vec{a}(a_x, a_y, a_z)$ is a Known Vector; And $\vec{v}(v_x, v_y, v_z)$ is unknown, But an arbitrary Point on the plane π.

Hence, Eqn (2) becomes

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0 \quad \ldots (2*)$$

Now, find the intersection point defined By the Ray Eqn (1). In order to that, we will need to find λ.

Since, the intersection point $\vec{P_i}$ is the Common Point By the Ray and the plane π. we have

$$\begin{cases} \vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) & \ldots (3a) \\ \vec{n} \cdot (\vec{v} - \vec{a}) = 0 & \ldots (3b) \end{cases}$$

$\vec{n}(n_x, n_y, n_z)$, Normal Vector has to be Known,

$\vec{a}(a_x, a_y, a_z)$ is known on π.

Starting from the plane Eqn (3b).

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

where $\vec{v} = \vec{R}$, e.g.

$$\vec{n} \cdot (\vec{v} - \vec{a}) \Big|_{\vec{v} = \vec{R}} = 0 \quad \ldots (4)$$

$$\vec{n} \cdot (\vec{R} - \vec{a}) \Big|_{\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i})} = 0$$

$$\vec{n} \cdot (\vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) - \vec{a}) = 0$$

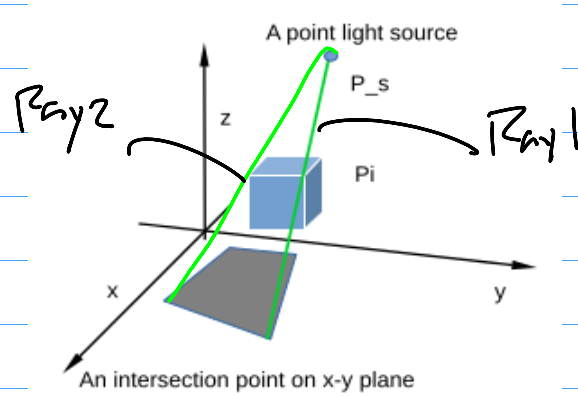
$$\vec{n} \cdot \vec{P_i} + \lambda \vec{n} \cdot (\vec{P_s} - \vec{P_i}) - \vec{n} \cdot \vec{a} = 0$$

$$\lambda \vec{n} \cdot (\vec{P_s} - \vec{P_i}) = \vec{n} \cdot \vec{a} - \vec{n} \cdot \vec{P_i}$$

$$\lambda = \frac{\vec{n} \cdot \vec{a} - \vec{n} \cdot \vec{P_i}}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})}$$

$$= \frac{\vec{n} \cdot (\vec{a} - \vec{P_i})}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})} \quad \ldots (5)$$

Note λ is <u>NOT</u> the intersection pt. it allows us to use Ray Eqn (1) to find the intersection.



use Eqn (5) to find more than one intersection

Example: