April 4 (Monday).

Topics: 1.30 GE focused on Shadow Computation.

Example: Given

1. x_w-y_w-z_w world coordinate Right Hand System. $\vec{P_i} + \lambda(\vec{P_s} - \vec{P_i})$ $\vec{R}$

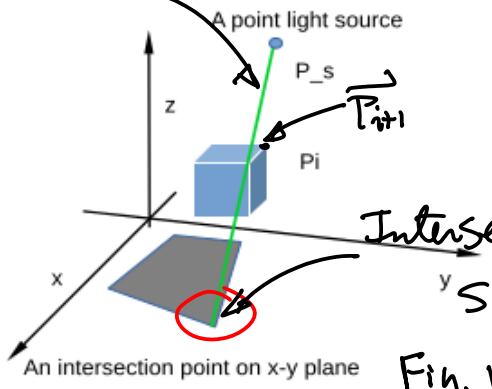


Fig. 1

2. A given Cube $\{\vec{P_i}(x_i, y_i, z_i) \mid i = 1, 2, \cdots, N\}$ Counter Clockwise

3. A given point light source $\vec{P_s}(x_s, y_s, z_s)$

4. Generate Ray Equation / Ray Cast x_w-y_w plane, Produces Shadow if Blocked by the Cube.

References:

2021F-101b-notes-cmpe240-2021-12-1.pdf

Step 1. Generate A Ray Cast / Equation

$$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) \quad \cdots (1)$$

Starting from $\vec{P_s}$, Passing through $\vec{P_i}$

$$\vec{d}(x, y) = \vec{P_s} - \vec{P_i} \quad \cdots (1b)$$

Intersection Point: $\vec{P_i}$ Between Ray $\vec{R}(x, y)$, and x_w-y_w plane.

Step 2. To find the intersection Point on x_w-y_w plane

Define a plane equation.

a. Define A Normal vector

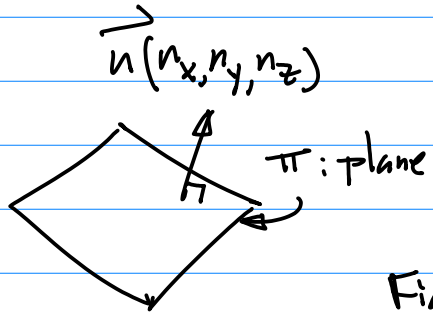


Fig. 2

use the normal vector to define the plane π.

b. Form a vector (on a plane) from $\vec{P_i}$ and $\vec{P_{i+1}}$, $(\vec{P_{i+1}} - \vec{P_i})$ a line

$$\vec{n} \cdot (\vec{P_{i+1}} - \vec{P_i}) = 0 \quad \cdots (2)$$

Take

$\vec{v}(v_x, v_y, v_z)$, $\vec{a}(a_x, a_y, a_z)$ to Replace $\vec{P_i}$, $\vec{P_{i+1}}$

Assume $\vec{a}(a_x, a_y, a_z)$ is a Known Vector; And $\vec{v}(v_x, v_y, v_z)$ is unknown, But an arbitrary Point on the plane π.

Hence, Eqn(2) becomes

$$\vec{n}\cdot(\vec{v}-\vec{a})=0 \quad \dots (2^*)$$

Now, find the intersection point defined By the Ray Eqn(1). In order to that, we will need to find λ.

Since the intersection point $\vec{P_i}$ is the common Point By the Ray and the plane π. we have

$$\begin{cases} \vec{R}=\vec{P_i}+\lambda(\vec{P_s}-\vec{P_i}) & \dots (3a) \\ \vec{n}\cdot(\vec{v}-\vec{a})=0 & \dots (3b) \end{cases}$$

$\vec{n}(n_x, n_y, n_z)$, Normal Vector has to be known,

$\vec{a}(a_x, a_y, a_z)$ is known on π.

Starting from the plane Eqn (3b).

$$\vec{n}\cdot(\vec{v}-\vec{a})=0$$

where $\vec{v}=\vec{R}$, e.g.

$$\vec{n}\cdot(\vec{v}-\vec{a})\Big|_{\vec{v}=\vec{R}}=0 \quad \dots (4)$$

$$\vec{n}\cdot(\vec{R}-\vec{a})\Big|_{\vec{R}=\vec{P_i}+\lambda(\vec{P_s}-\vec{P_i})}=0$$

$$\vec{n}\cdot\left(\vec{P_i}+\lambda(\vec{P_s}-\vec{P_i})-\vec{a}\right)=0$$

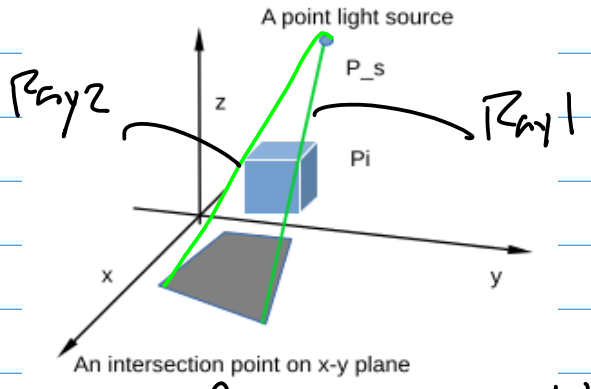
$$\vec{n}\cdot\vec{P_i}+\lambda\vec{n}\cdot(\vec{P_s}-\vec{P_i})-\vec{n}\cdot\vec{a}=0$$

$$\lambda\vec{n}\cdot(\vec{P_s}-\vec{P_i})=\vec{n}\cdot\vec{a}-\vec{n}\cdot\vec{P_i}$$

$$\lambda=\frac{\vec{n}\cdot\vec{a}-\vec{n}\cdot\vec{P_i}}{\vec{n}\cdot(\vec{P_s}-\vec{P_i})}$$

$$=\frac{\vec{n}\cdot(\vec{a}-\vec{P_i})}{\vec{n}\cdot(\vec{P_s}-\vec{P_i})} \quad \dots (5)$$

Note λ is NOT the intersection Pt. it allows us to use Ray Eqn(1) to find the intersection.



use Eqn(5) to find more than one intersection

April 7th (Wed)

Example: Ref (see pp. 1. github Lecture Notes, 2021F-101b-n)

Given $\vec{P_s}(-20, 110, 200)$, A vertex of a given Cube $\vec{P_i}(100, 100, 110)$. Find the intersection pt to Draw Shadow.

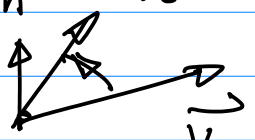
Sol First, Ray Equation, Eqn(1):

$$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) \quad \cdots (1)$$

then, plane Eqn for X_w-Y_w plane

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

Find $\vec{n}$,



$$\vec{V_1} \times \vec{V_2} = \vec{n} \quad \cdots (2)$$

Let $\vec{V_1} = x_w \vec{i}$, $\vec{V_2} = y_w \vec{j}$

$$\vec{n} = x_w \vec{i} \times y_w \vec{j}$$

$$= (0, 0, 1)$$

Then, 2nd find λ for the Ray Equation, from Eqn.

$$\lambda = \frac{\vec{n} \cdot (\vec{a} - \vec{P_i})}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})} \quad \cdots (5)$$

where $\vec{a} = (0, 0, 0)$, Hence,

$$\lambda = \frac{-\vec{n} \cdot \vec{P_i}}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})}$$

Now, hand Calculation, also for coding

$$\lambda = \frac{-\vec{n} \cdot \vec{P_i}}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})}$$

$$= -\frac{n_x x_i + n_y y_i + n_z z_i}{n_x(x_s - x_i) + n_y(y_s - y_i) + n_z(z_s - z_i)}$$

From the given condition $\vec{n}(0, 0, 1)$, Therefore

$$\lambda = \frac{-n_z z_i}{n_z \cdot (z_s - z_i)} = -\frac{z_i}{z_s - z_i}$$

$$= -\frac{110}{200 - 110} = -\frac{110}{90} = -11/9$$

Now, back to the Ray Equation

$$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i})$$

$\vec{R} = (100,100,110) - \frac{11}{9}(-20-100, 110-100, 200-110)$

$= (100,100,110) - \frac{11}{9}(-120, 10, 90)$

$= \left(\frac{1100 \times 120}{9}, -\left(100 - \frac{110}{9}\right), 110-110\right)$

$= \left(\frac{1100 \times 120}{9}, \frac{110}{9} - 100, 0\right) = (2467, 87.8, 0)$

please finish this calculation.

Now, Coding part. Same Code on github.

2018F-116-11diffuse20181114.cpp

Note:

a. Define Normal vector $\vec{n}$ for x_w-y_w plane

```
169    world.X[47] = 0; world.Y[47] = 0; world.Z[47] = 1; // normal vector for x-y plane
```

b. Make the "typedef struct" for Defining 3D points.

```
39  typedef struct {
40      float X[UpperBD], Y[UpperBD], Z[UpperBD];
41  } pworld;
```

```
72      pworld  world;
73      pviewer viewer;
```

Now, λ Calculation. Eqn (5)

$$\lambda = \frac{\vec{n} \cdot (\vec{a} - \vec{P}_i)}{\vec{n} \cdot (\vec{P}_s - \vec{P}_i)} \quad \cdots (5)$$

$$= - \frac{n_x x_i + n_y y_i + n_z z_i}{n_x(x_s - x_i) + n_y(y_s - y_i) + n_z(z_s - z_i)}$$

```
171     //----------lambda for Intersection pt on xw-yw plane----------*
172     float temp = (world.X[47]*(world.X[46]-world.X[45]))
173                 +(world.Y[47]*(world.Y[46]-world.Y[45]))
174                 +(world.Z[47]*(world.Z[46]-world.Z[45]));
175     float lambda = temp / ((world.X[47]*(world.X[45]-world.X[7]))
176                           +(world.Y[47]*(world.Y[45]-world.Y[7]))
177                           +(world.Z[47]*(world.Z[45]-world.Z[7])));
178     float lambda_2 = temp / ((world.X[47]*(world.X[45]-world.X[6]))
179                             +(world.Y[47]*(world.Y[45]-world.Y[6]))
180                             +(world.Z[47]*(world.Z[45]-world.Z[6])));
```

Note, Substitute λ to Ray Equation to find the intersection point

```
182     //----------ray equation to find intersection pts-----------*
183     world.X[48] = world.X[45] + lambda*(world.X[45] - world.X[7]); // Intersection pt p7
184     world.Y[48] = world.Y[45] + lambda*(world.Y[45] - world.Y[7]); // Intersection pt p7
185     world.Z[48] = 0.0;
186
187     world.X[49] = world.X[45] + lambda_2*(world.X[45] - world.X[6]);  //intersection pt p6
188     world.Y[49] = world.Y[45] + lambda_2*(world.Y[45] - world.Y[6]);  //intersection pt p6
189     world.Z[49] = 0.0;
```

$\vec{R} = \vec{P_i} + \lambda(\vec{P_s} - \vec{P_i}) \rightarrow$

$x = x_i + \lambda(x_s - x_i)$
$y = y_i + \lambda(y_s - y_i)$
$z = z_i + \lambda(z_s - z_i)$

Assignment in-Class show & Tell. Implement Intersection Computation on LPC 17b9, "Show+Tell" Demo in Class. On April 11 (Monday),

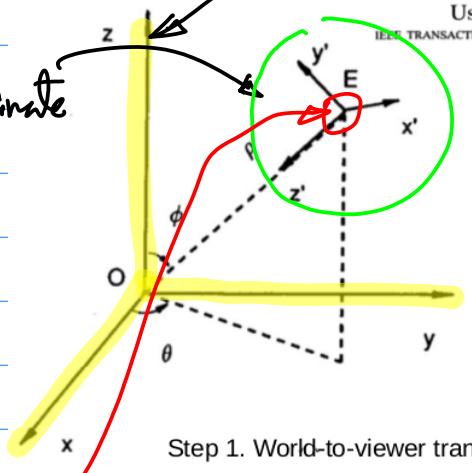
To Be Able to Display 3D Graphics on 2D Display Devices. Let's Define Transformation Pipeline. 1. Define World-Coordinate System: x_w-y_w-z_w Right Hand System

3D Transformation Pipeline Technique

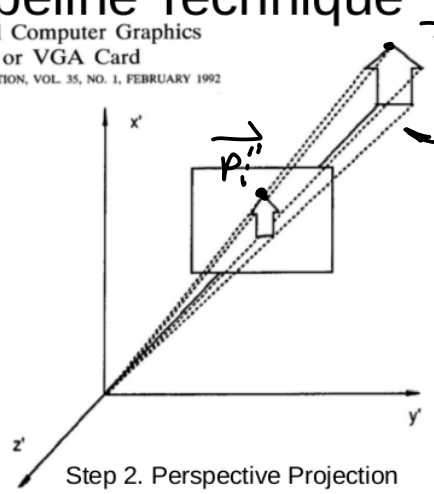
Reference: H. Li Three-Dimensional Computer Graphics Using EGA or VGA Card
IEEE TRANSACTIONS ON EDUCATION, VOL. 35, NO. 1, FEBRUARY 1992

2. Viewer Coordinate System x_v-y_v-z_v Left-Hand System

3. Virtual Camera is located $\vec{E}$ (e_x, e_y, e_z)



Step 1. World-to-viewer transform



Step 2. Perspective Projection

$$\mathbf{T} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x_p = x_e\left(\frac{D}{z_e}\right)$$
$$y_p = y_e\left(\frac{D}{z_e}\right)$$

Harry Li, Ph.D.

Example: Display Shadows on 2D Display Device.

Step1. World To Viewer Transform.

$$T = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \cdots (1)$$

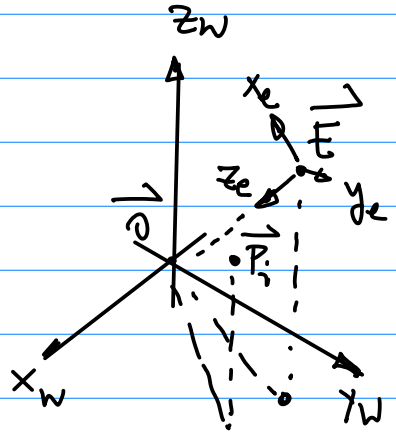


Fig.1.

Everything is defined in the World Coordinate System x_w-y_w-z_w including a Virtual Camera.

$\vec{E}(x_e, y_e, z_e)$, x_e-y_e-z_e "Viewer" Coordinate System.

Given $\vec{P_i}(x_i, y_i, z_i)$ in x_w-y_w-z_w World Coordinate, Represent this point in x_e-y_e-z_e Coordinate System.

$\vec{P_i}$

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} \quad \cdots (1\text{-}b)$$

Assume $\vec{E}(x_e, y_e, z_e) = (200, 200, 200)$

Physical meaning of Transformation Matrix T.

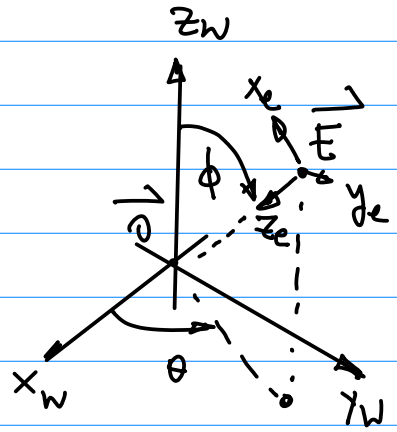


Fig.2

θ: Angle from the dash line on x_w-y_w plane w.r.t positive x_w-axis

ϕ: Angle Between z_w & z_e.

ρ (rho): $\rho = \sqrt{x_e^2 + y_e^2 + z_e} \quad \cdots (2)$

distance from $\vec{E}$ to the origin $\vec{O}$ of x_w-y_w-z_w.

Suppose $\vec{E}(200, 200, 200)$ is given, Find $\cos\theta$, $\sin\theta$, $\cos\phi$, $\sin\phi$ for T-matrix.

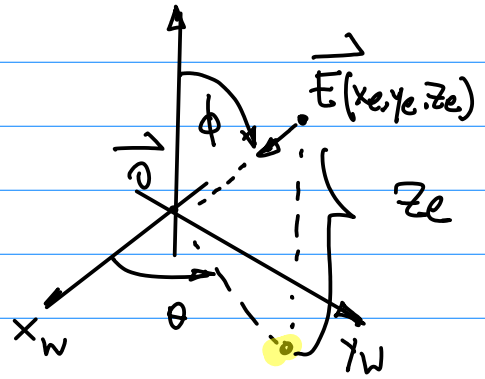
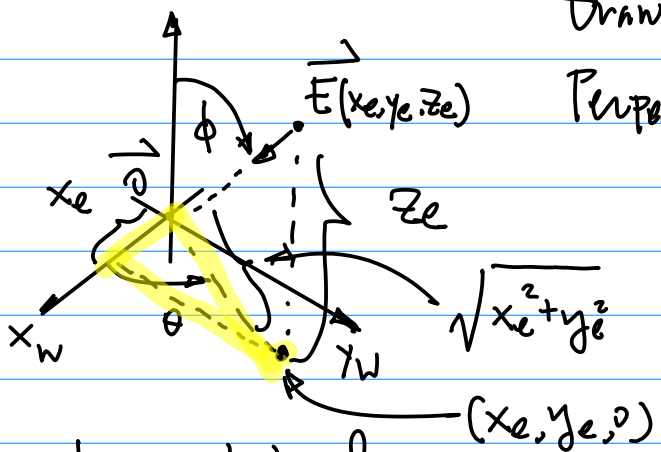


Fig.3

Draw A Line Passing through $\vec{E}$ Perpendicular to X_w-Y_w → Form an intersection point.

Draw A Line Passing through the intersection point on X_w-Y_w plane on the plane and Perpendicular to X_w-axis.

Fig.4

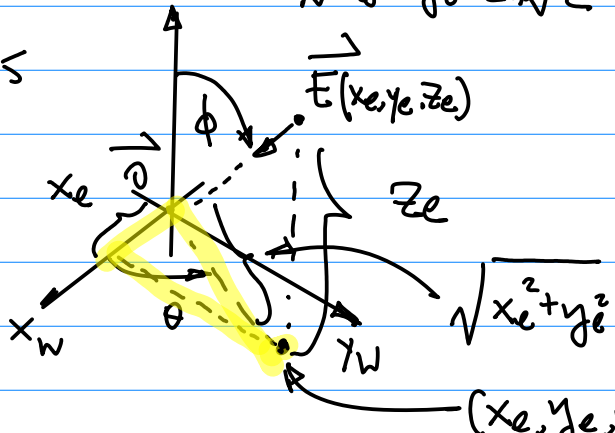


We can form a trimgle on X_w-Y_w plane, as in Fig.4, hence

$$\cos\theta = \frac{x_e}{\sqrt{x_e^2+y_e^2}} = \frac{200}{200\sqrt{2}} = \frac{\sqrt{2}}{2}$$

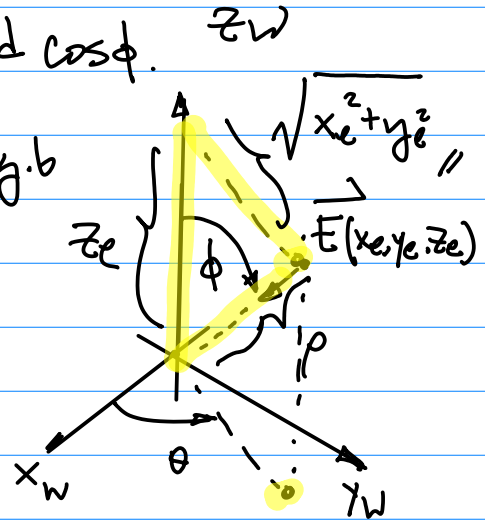
similarly, $\sin\theta = \frac{y_e}{\sqrt{x_e^2+y_e^2}} = \frac{200}{200\sqrt{2}} = \frac{\sqrt{2}}{2}$

Fig.5



Find $\cos\phi$.

Fig.6



Draw A Line Passing $\vec{E}(x_e,y_e,z_e)$ Perpendicular to z_w-axis

$$\cos\phi = \frac{z_e}{\rho} = \frac{z_e}{\sqrt{x_e^2+y_e^2+z_e}}$$

$$= \frac{200}{200\sqrt{3}} = \sqrt{3}/3$$

$$\sin\phi = \frac{\sqrt{x_e^2+y_e^2}}{\sqrt{x_e^2+y_e^2+z_e}} = \frac{200\sqrt{2}}{200\sqrt{3}}$$

$$= \frac{\sqrt{2}\cdot\sqrt{3}}{3} = \frac{\sqrt{6}}{3}$$

Homework: Due April 18th (Monday)

1. Draw A world Coordinate System X_w-Y_w-Z_w axis, with X_w Red, Y_w Green, Z_w Blue

2 Draw A cube, size Length = 100, floats 10 unit Above X_w-Y_w plane. in other word $\overrightarrow{P_i}(100,100,110)$;

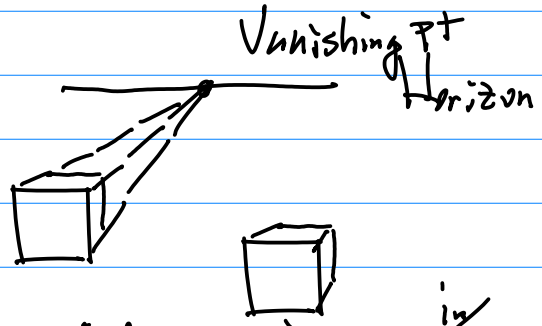
3. Draw a point light Source $\overrightarrow{P_s}(-20,110,200)$. And RayCast to connect $\overrightarrow{P_s}$ to $\overrightarrow{P_i}$; use Green Color.

4. Compute the Shadow point $\overrightarrow{P_i'}$, Draw the intersection Point to link $\overrightarrow{P_s} - \overrightarrow{P_i} - \overrightarrow{P_i'}$

Note: You may want to Adjust the $\overrightarrow{P_s}$ position, So this $\overrightarrow{P_i'}$ (Intersection Point) can be visible on your Display

Step2. Perspective Projection

$$x_p = x_e\left(\frac{D}{z_e}\right)$$
$$y_p = y_e\left(\frac{D}{z_e}\right) \quad \dots (3)$$



Eqn(3) project a point $\overrightarrow{P_i}(x_i, y_i, z_i)$ in $X_e Y_e Z_e$

a

Onto 2D Display Device, like LCD.

April 13 (Wed)

Topics: 1° Perspective Projection

2° Diffuse Reflection

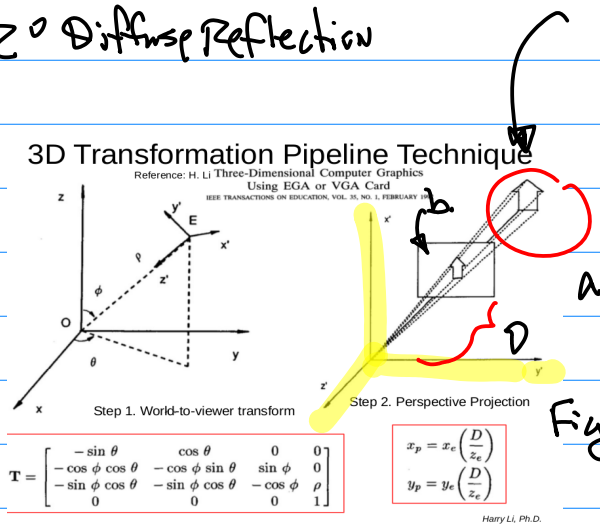


Fig1.

$$\mathbf{T} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x_p = x_e\left(\frac{D}{z_e}\right)$$
$$y_p = y_e\left(\frac{D}{z_e}\right)$$

Note: X_v-Y_v-Z_v is Left Hand System.

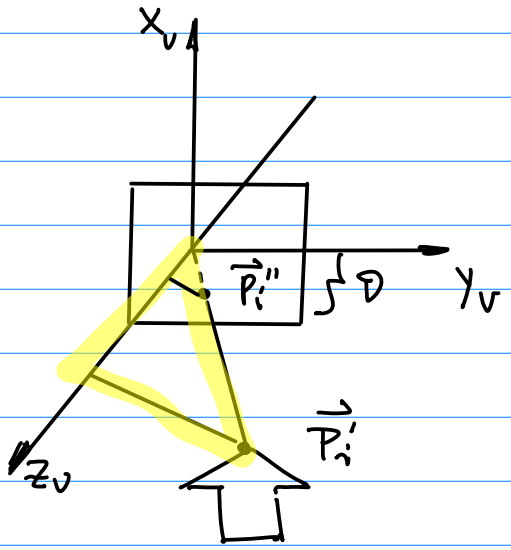
a. It is a 3D object

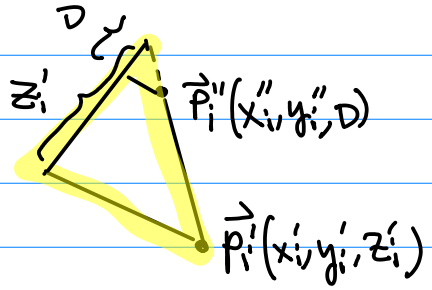
b. projection Plane, distance to the viewer Coordinate System (0,0,0) → (X_e, Y_e, Z_e) in $X_w Y_w Z_w$

D is the distance.

c. Origin of X_v-Y_v-Z_v. → Camera Location, Camera is modeled as "pin-Hole" model.

Projection is formulated By using Similar Triangles in Fig.1





$$x_p = \frac{D}{z_e} x_e \quad \text{from Eq(s).}$$

Or,

$$x_i'' = \frac{D}{z_i'} x_i'$$

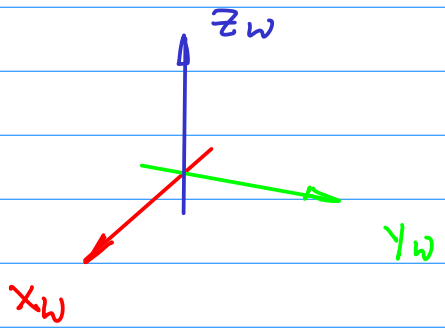
Similarly,

$$y_p = \frac{D}{z_e} y_e \quad \text{or}$$

$$y_i'' = \frac{D}{z_e} y_i'$$

Homework, Due A week from Today
April 20th

1. Draw a World Coordinate System, x_w: Red, y_w: Green, z_w: Blue, Design the size (x_w, y_w, z_w, 50 units)



2. Design By Defining Dimension of a Cube.
(Example: length = 100)
$\vec{P_i}(x_i, y_i, z_i) = (100, 100, 110)$
Elevate the cube By 10 units.

3. Draw the Cube on the LCD

4. Submission:
   a. Screen Capture of your Xpresso Screen, which Shows your Name (Folder Name) And your Program (Partial) — 0.5 pt
   b. Take a photo of your display. With Entire Prototype System of your own — 0.5 pt.

5. Submission to CANVAS.

Note: You will need transform from a virtual coordinate System to physical coordinate

Consider Diffuse Reflection.

2018S-23-lec7-DiffuseReflection-v6-2018-4-25.pdf

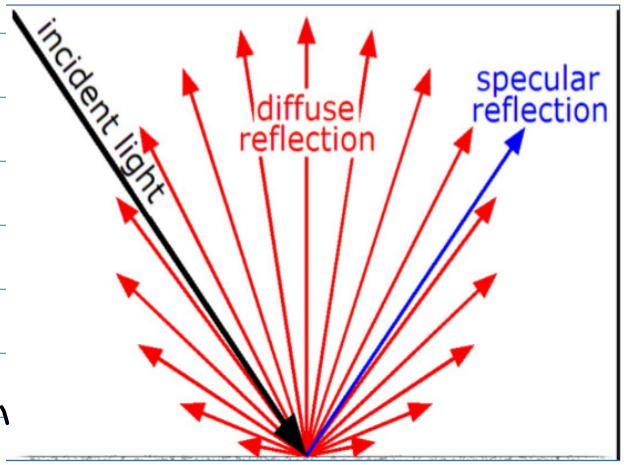
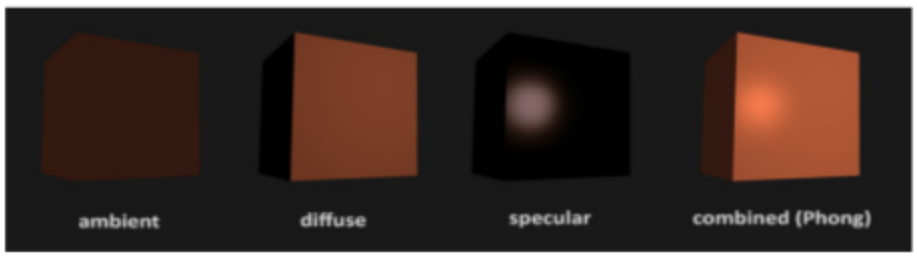


Fig. 1

https://en.wikipedia.org/wiki/Diffuse_reflectio

Definition: A Reflection from an object surface uniformly in all directions.



1. Define Reflectivity, A property of An object surface.

$\vec{K_r} = (K_r, K_g, K_b) = (r, g, b)$ ...(1)

$0 \le r, g, b \le 1$

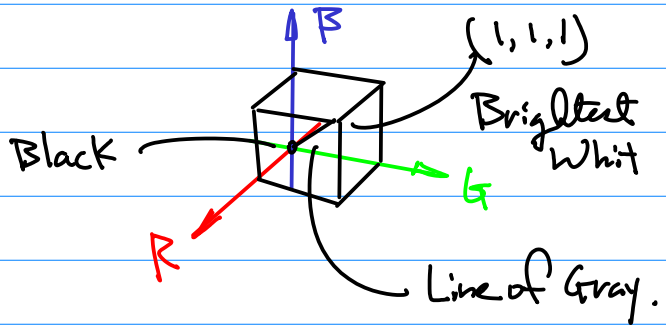
Note: for a Black object,

$r=0, g=0, b=0$

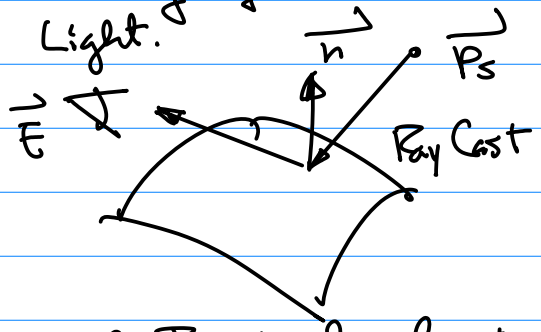
for a green leaf.

$r \cong 0;\ g \neq 0, 0 < g;\ b \cong 0$

2. Color Space. R, G, B

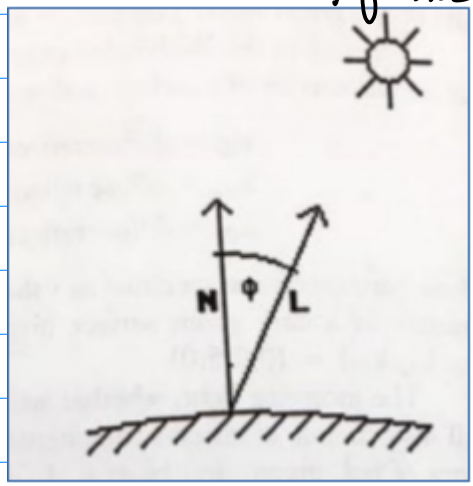


3. Viewing Angle v.s. Incident Light.

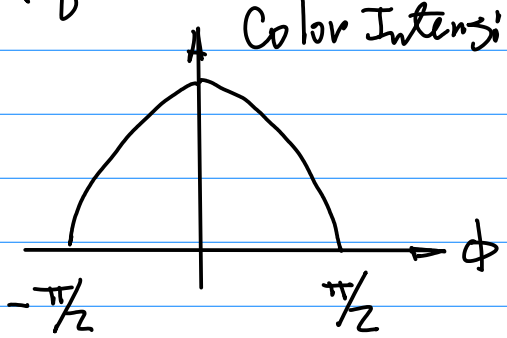


a. Perceived color is independent of viewing Angle.

b. Normal vector $\vec{N}$ and incident Light $\vec{L}$ ($\vec{R}$ Ray Cast) form An Angle ϕ, the color Intensity follows



Cosϕ function.



ϕ=0, Cosϕ=1, Strongest Reflection.
highest Intensity.

ϕ=π/2, Cosϕ=0, No Reflection,
theme, no color.

4. Distance.