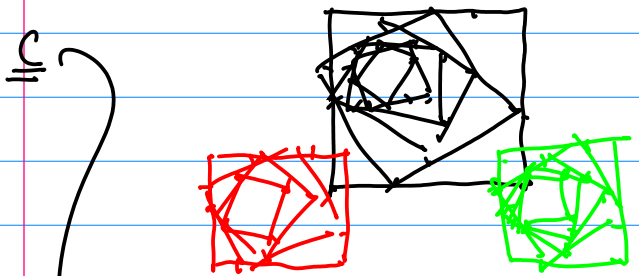


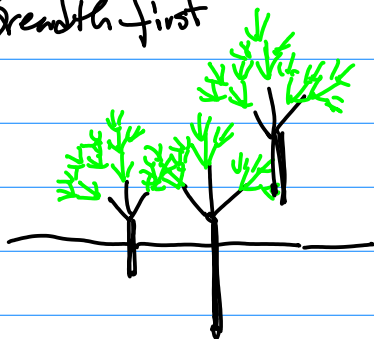
- a A set of Squares Rotation with one same color
- b Change location/color/size



Keep the patterns, please don't erase them.

Part II Once Part I is Done Switch to Part I.

a Breadth first

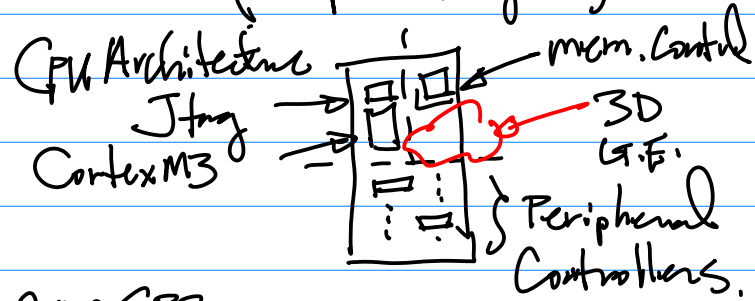


b  $L \geq 7$  Create A patch of Forest  $x_w$    
 Changing location/size

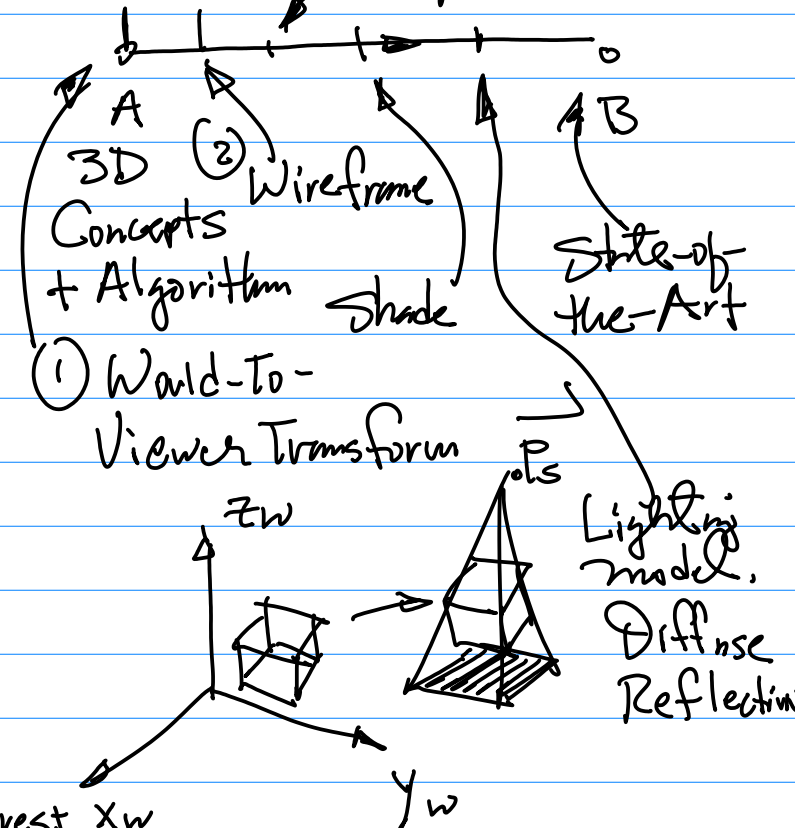
Note: Please Don't Erase the Drawing Till All the trees are constructed.

Note: Report Writing.

3D G.E. (Graphics Engine) <sup>22</sup>



GPP SPRs, SPI SPRs. Solid obj. HL, HS. Removal



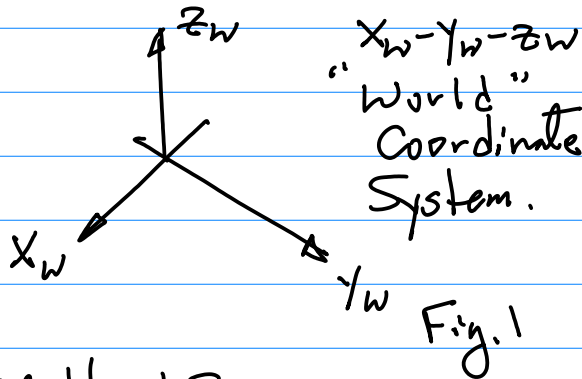
March 15 (Monday)

Note: 1<sup>st</sup> Midterm A week from this Wednesday (March 24th)

2<sup>nd</sup> Smartphone (Cam Capability) A piece of Paper to the Class. Test the Environment

Ref: github: 2018F-114-3D Graphics

1°



Right-Hand System,

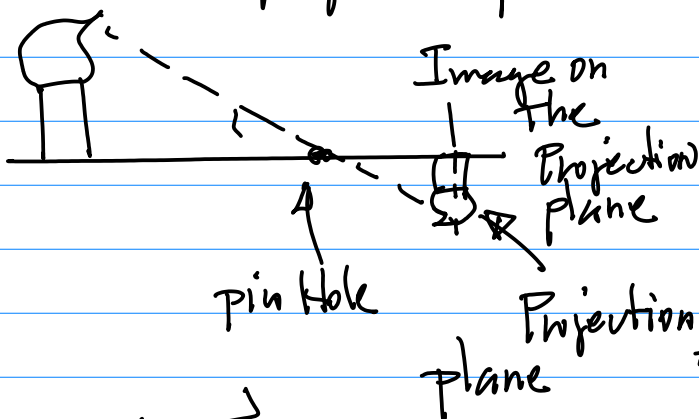
2° Virtual Camera

$\underline{a}$  Virtual Enclosure

$\underline{b}$  Optic lens (Pin Hole)

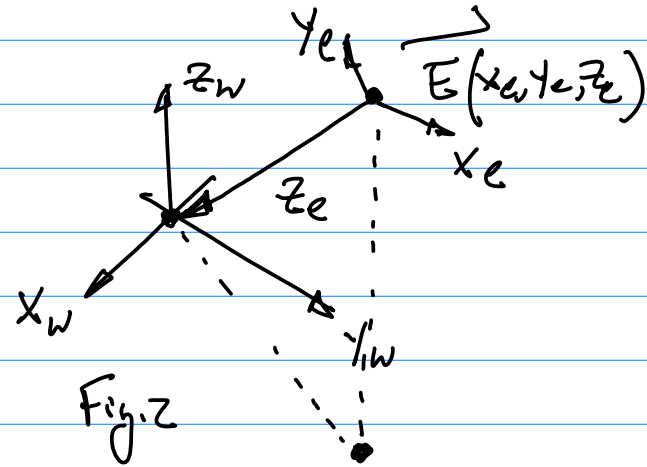
$\phi_d \approx \phi$  Very Small diameter

$\underline{c}$  Projection plane, the plane to form projected Image when light passing through the lens and reaches the projection plane.



3. Denote  $\vec{E}(x_e, y_e, z_e)$  as

Virtual Camera Location



Projection of  $\vec{E}(x_e, y_e, z_e)$  on to  $X_w - Y_w$  plane in the  $X_w - Y_w - Z_w$  World Coordinate System

4. Viewer Coordinate System

Viewer ~ Virtual Camera

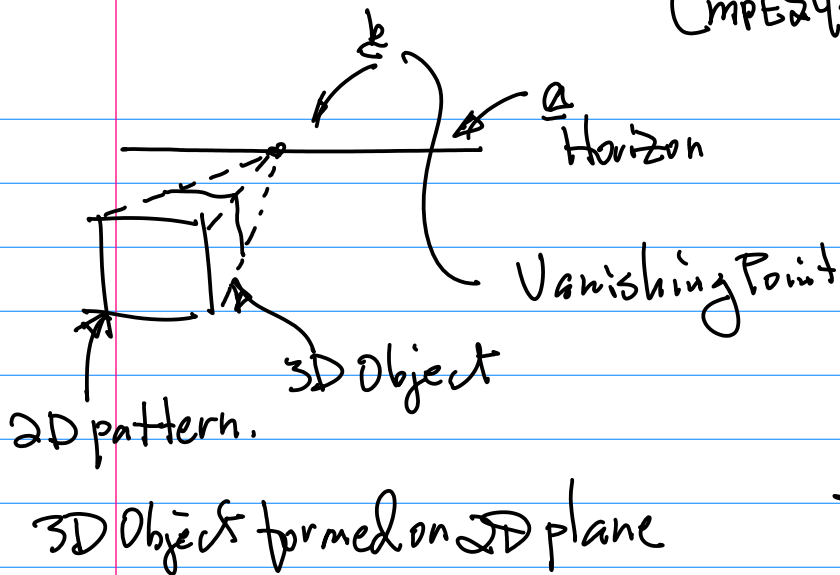
$X_e - Y_e - Z_e$  Viewer Coordinate System. "Eye"

$\underline{a}$   $\vec{E}(x_e, y_e, z_e)$  AS the origin.

$\underline{b}$  Left-Hand System.

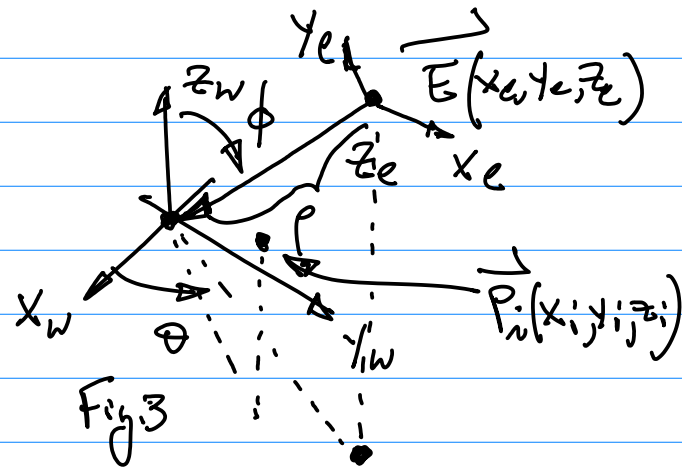
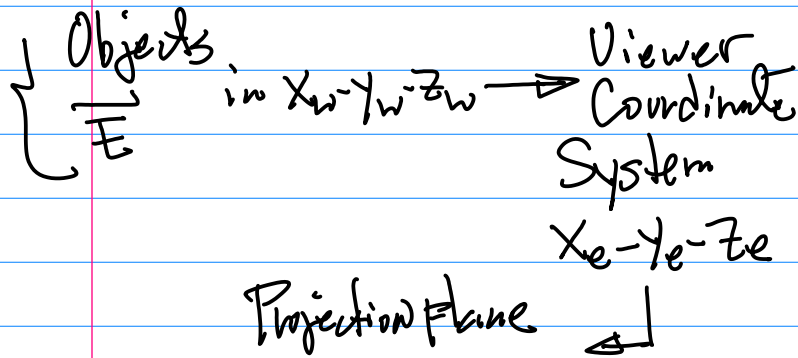
$\underline{c}$   $Z_e$ -axis points to the origin.

5. Perspective Projection



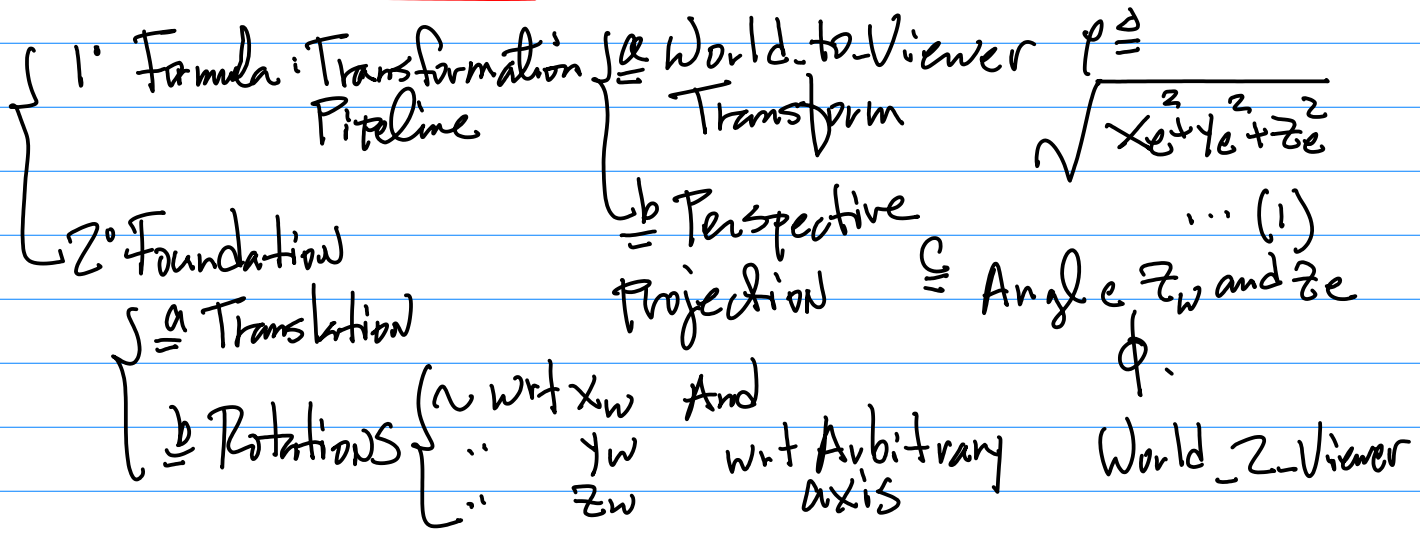
7. Transformation Pipeline  
To Display 3D object(s) on to  
2D LCD Display Screen,  
W2V (World-Viewer)  
Transform  
Perspective Projection.

6. Perspective Projection  $\rightarrow$  @ the  
Virtual Camera in  $X_w-Y_w-Z_w$ ,  
in  $X_e-Y_e-Z_e$ .



3 parameters:  
 $\theta$  theta  $\theta$  Angle  
 $\phi$  Vector  $\vec{E}$  to  $\vec{D}$   
"roh", Distance

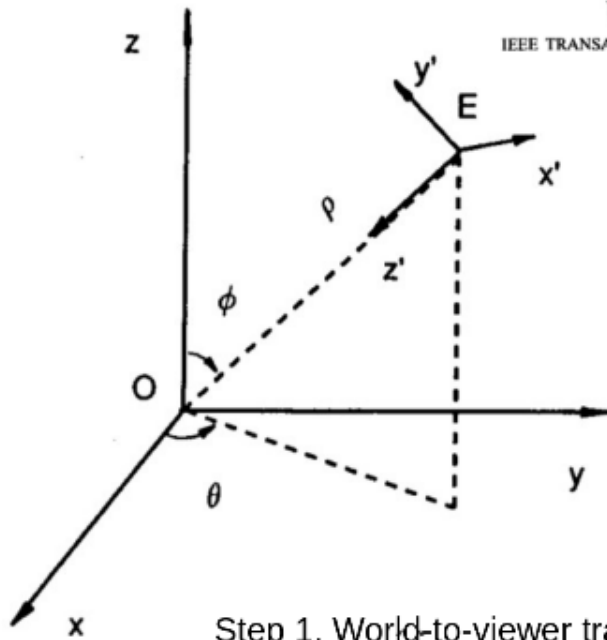
## Mathematical Formulation



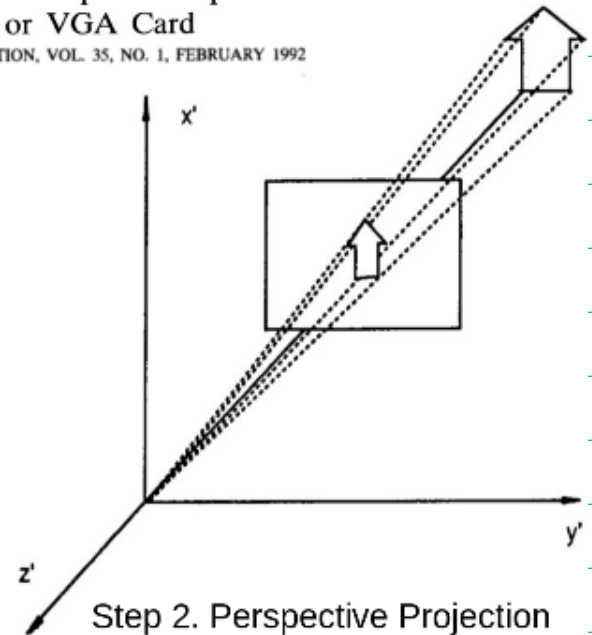
# 3D Transformation Pipeline Technique

Reference: H. Li Three-Dimensional Computer Graphics  
Using EGA or VGA Card

IEEE TRANSACTIONS ON EDUCATION, VOL. 35, NO. 1, FEBRUARY 1992



Step 1. World-to-viewer transform



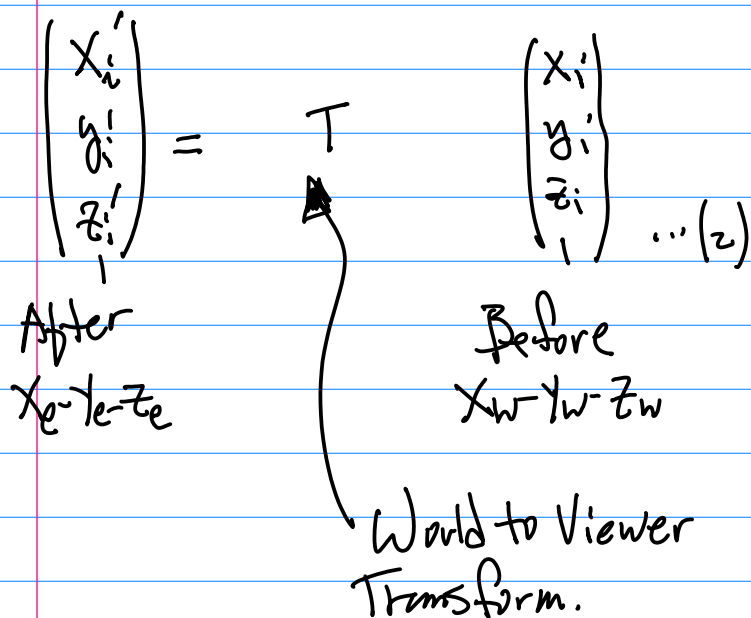
Step 2. Perspective Projection

$$\mathbf{T} = \begin{bmatrix} -\sin \theta & \cos \theta & 0 & 0 \\ -\cos \phi \cos \theta & -\cos \phi \sin \theta & \sin \phi & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & -\cos \phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x_p = x_e \left( \frac{D}{z_e} \right)$$

$$y_p = y_e \left( \frac{D}{z_e} \right)$$

Transform, Map  $P_i$  from the World-coordinate to Viewer Coordinate.



March 17 (Wed)  
Topics: 1° Hardware Architecture  
2° Software SPRs  
Init & Config.

Example: 2D & 3D G.E.  
SPI I/F LCD Display to work with LPC1769.

GPIO (GPP) SPI  
A set of b n'

System Configuration  
\* Configuration of the Peripheral Cont.

Per. Cont.  
PWR  
CLK  
multiplexing

$$\begin{pmatrix} \sin\theta & \cos\theta & 0 & 0 \\ \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ Part I. } \theta$$

Composite Rotation Matrix

$$\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ \cos\phi & \cos\phi & \sin\phi & 0 \\ \sin\phi & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ Part II } \phi$$

RTOS  
Peripheral Controller

GPP  
SPI

SPI's SPR.

1. Naming Convention

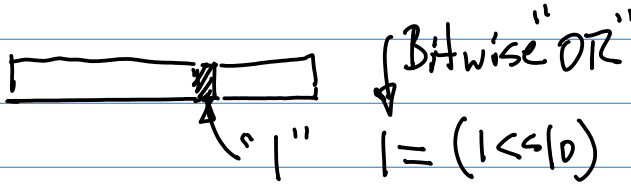
LPC\_SC → PCOMP

2. Power Up the Selected Peripheral Controller By Setting the Corresponding.

$$\begin{pmatrix} - & - & - & P \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ Part III } \text{roh}$$

# CompE240

26



Tech Spec.

1° 8 bit Transfer

$$CR\phi[3:0] = 0111 = 0X7$$

2° SPI

$$CR\phi[5:4] = 00 \text{ (SPI)}$$

3° Clock  $f_{SPI}$

$$a \text{ } CR\phi[15:8]$$

8 bits  $255 = 2^8$

$$f = \frac{PCLK}{SPI (SCR+1) CPSDVSR}$$

... (1)

[0, 255]

much 2 and.

Today's Topics:

1° Midterm Review

2° SPRs, CR $\phi$ , CR1 for SPI I/F

Ref:

1° CPU Datasheet.

PP431-433.

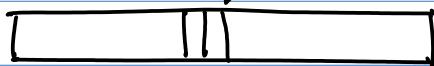
CR $\phi$

2° Sample code

SPI init (Drawa Line)

$l = \sim (3 \leq 20)$   
 "AND" "1" Negation, "00"

Clear 2 Bits



Set 2 bits  
 as "01"

$l = \sim ((3 \leq 18) | (3 \leq 16) | (3 \leq 14));$   
 "AND" Negn. "11" Total 6 bits < 18  
 Clear 6 bits

$l = ((2 \leq 18) | (2 \leq 16) | (2 \leq 14) | 1)$   
 "OR" Set "10"

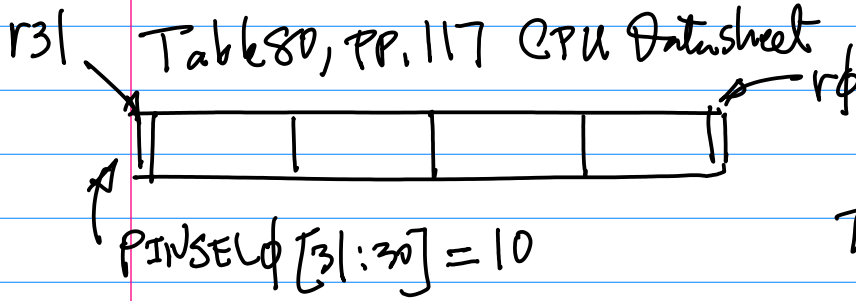
Note:  $\left\{ \begin{array}{l} CR\phi \\ CR1 \end{array} \right\}$  LPC\_SSP1  $\rightarrow$  CR $\phi$   
 Control Register

Table 571



# CMPE240

Example: SSP.C Source Code  
Walk-Through. 151-208  
Line 162-165 PINSEL0



$$f_{SPI} = \frac{1 \times 10^6}{(7+1) * DVR} \quad 27$$

To find  $f_{SPI}$ .

$$2^8 = 256, \text{ SCR}[0, 255]$$

TP. 433. CPSDVSR & [2, 254]

Midterm Review.

1° Video On, Mandatory.

a Submission to CANVAS  
15 min. File Uploading  
No Late Submissions  
After the Deadline

Paper will be disqualified

IF CANVAS Disrupted,  
then E-mail Submission  
= file in "Zip"

Table 81. PINSEL1

= 0x2

10 From CPU Datasheet

PINSEL1[1:0], PINSEL1[3:2],

PINSEL1[5:4] → SPI

SSEL0, MOSI0, MISO0

Line 173

CR = 0x0707 → Tech

(Spec. = file in "Zip")

.... 10000 0111 10000 0111

16 Upper  
Bits all  
zeros.

CR0[15:8] SCR

= Clock

CR[5:4] = 00 SPI

From  
Datasheet  
PP 431

8 bit Transfer

FirstName + 4 Digits + CMPE240  
SID mid.zip

2° 3 Questions ±

Hardware { CPU Block Diagram  
Memory map  
SPRs.

Ckt, SCH Design

$$f_{SPI} = \frac{PCLK}{(SCR+1) * DVR}$$

CR0[15:8] = 0x7

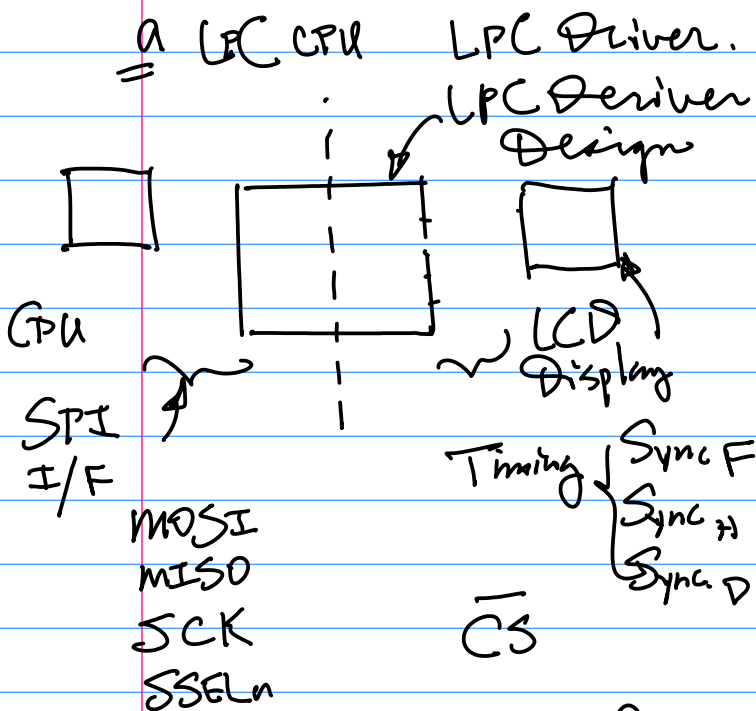
CR1[2] = 0  
for "Master"

Software: Coding



SPR Coding  $\rightarrow$  Binary Pattern for Init & Config Debugging Purpose

Algorithm: 2D Vector Graphics. G.E.  $\rightarrow$  Tech Spec.



a  $\rightarrow$  Virtual v.s. physical Display Transform.

$$\vec{P} = \vec{P}_i + \lambda (\vec{P}_{i+1} - \vec{P}_i)$$

Screen Saver.  $\rightarrow$  i. Rotation! No w/o Rotation matrix  $\rightarrow$  2D Transforms

Composition of 2D Transform.

$\begin{cases} R_{3 \times 3} \\ T_{3 \times 3} \end{cases} \rightarrow$  Tree.

Preprocess  $\rightarrow R_{3 \times 3}$  Post  $\sim$

Formula: One Page Formula Sheet; is allowed. However, No Example, or Verbal Explanation is Not Allowed. Submission of the formula Page is required with your mid-term paper. No multiple choice question.

SCH: Requires All the pins needed in the design to have Label; wire: "Arrow" to indicate direction.

Block Diagram: wire(s), Label(s) direction (Arrow)

CPU Datasheet will be provided

C Code program will be provided for Answering questions, or for Redesign.

Calculator is allowed;



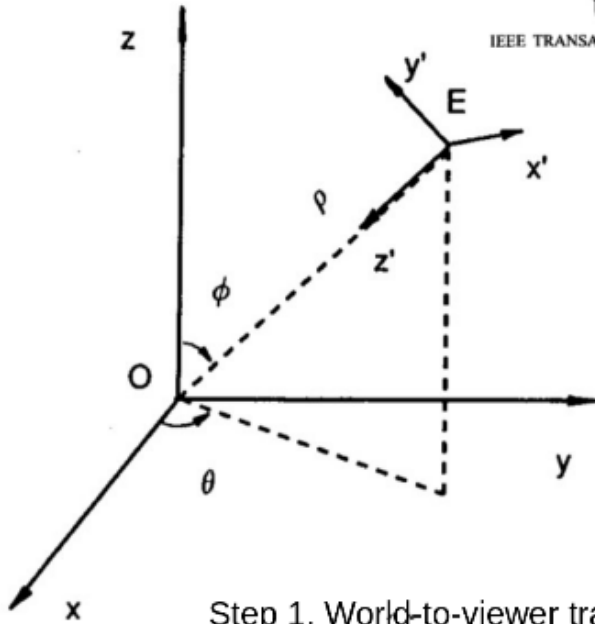
April 5 (Monday)

1. Midterm key on github, "Key" To search.

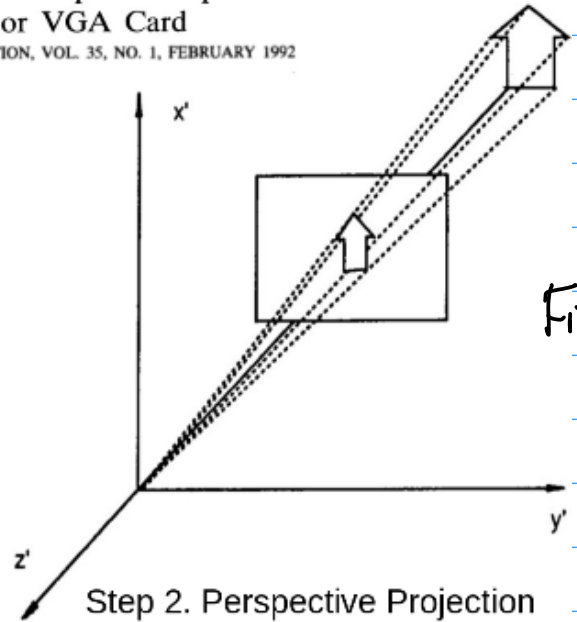
# 3D Transformation Pipeline Technique

Reference: H. Li Three-Dimensional Computer Graphics  
Using EGA or VGA Card

IEEE TRANSACTIONS ON EDUCATION, VOL. 35, NO. 1, FEBRUARY 1992



Step 1. World-to-viewer transform



Step 2. Perspective Projection

Fig1.

$$T = \begin{bmatrix} -\sin \theta & \cos \theta & 0 & 0 \\ -\cos \phi \cos \theta & -\cos \phi \sin \theta & \sin \phi & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & -\cos \phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

... (1)

$$x_p = x_e \left( \frac{D}{z_e} \right)$$

$$y_p = y_e \left( \frac{D}{z_e} \right)$$

... (2)

Harry Li, Ph.D. mem.

Today's Topics: 3D G.E.

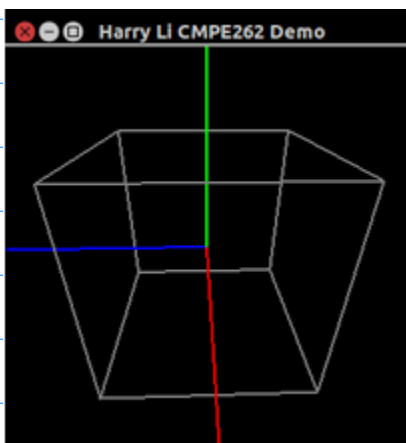


Fig2a

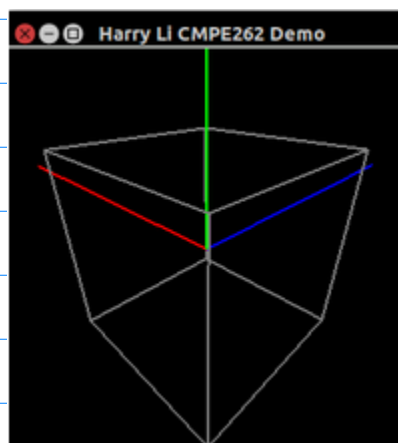
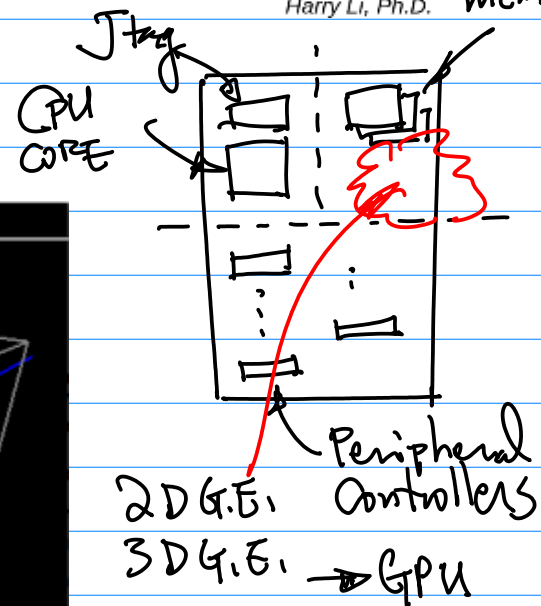


Fig2b



from github. Key "111a," "111b," ...

Note: 2D G.E. { Vector Graphics  
Transformation  
Primitive Graphics } { a D.D.A.  
b line Transistor/Gate  
c Arc/circle etc. Level.

Example: 3D Wireframe Model

First,  $X_w - Y_w - Z_w$  World Coordinate System.

a Right System, r-g-b  
for  $X_w, Y_w, Z_w$  axis

b Transformation Pipeline

1st World-Z Viewer

$$T_{4 \times 4}: (X_w, Y_w, Z_w) \rightarrow (X_v, Y_v, Z_v) \dots (3)$$

2nd Perspective Projection

$$P: (X_v, Y_v, Z_v) \rightarrow (X_p, Y_p) \dots (3b)$$

Second. Design of Dataset, e.g.,

4 Vertices for  $X_w - Y_w - Z_w$  Axis

$\vec{P}(x, y, z)$  3D pt. in  $X_w - Y_w - Z_w$

Step 1.  $\vec{P}(x, y, z) \in \mathbb{R}^3$

for the origin  $\vec{P}_0(x_0, y_0, z_0) = (0, 0, 0) \dots (4)$

$$\vec{P}_x(x_x, y_x, z_x) = (100, 0, 0), \dots (4-1)$$

$$\vec{P}_y(x_y, y_y, z_y) = (0, 100, 0), \text{ and } \dots (4-2)$$

$$\vec{P}_z(x_z, y_z, z_z) = (0, 0, 100) \dots (4-3)$$

#define  $X_w$ -axis ~~~~~

#define  $Z_w$ -axis ~~~~~

Now, Implementation (Drawing  
r-g-b axis)  $\swarrow$  ATul 12/11

Homework: Draw r-g-b axis  
on your LPC Display.

Bring your Program Board  
to the Next Class.

Step 2. World-Z-Viewer

$$T_{4 \times 4}: (X_w, Y_w, Z_w) \in \mathbb{R}^3 \rightarrow$$

$$(X_v, Y_v, Z_v) \in \mathbb{R}^3$$

$$\begin{pmatrix} X_v \\ Y_v \\ Z_v \\ 1 \end{pmatrix} = T_{4 \times 4} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \dots (5)$$

"After"

"Before"

Now, find the  $X_w$ -axis  
in Viewer Coordinate

## Step 3. Perspective Projection

$$P: (x_e, y_e, z_e) \in \mathbb{R}^3 \rightarrow (x_i'', y_i'') \in \mathbb{R}^2$$

Coordinate ON your  
LPC1764 Display  
Device.

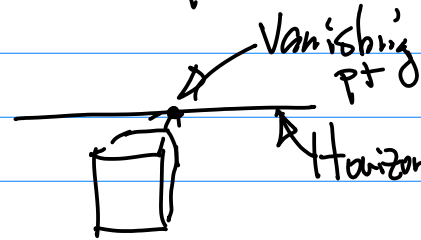
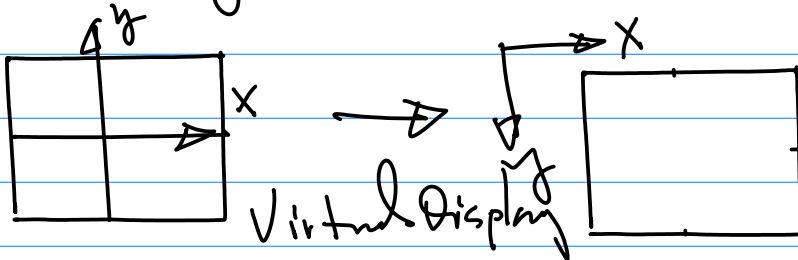
Note:

$(x_i'', y_i'')$  is defined on 2D  
Virtual Display Coordinate  
Need to perform Virtual to  
Physical Transformation in order  
to plot your Result.

Back projection Plane: ~  
the only light that can  
reach the projection plane  
is the light passing  
through the "pin hole".  
(plane is at the Back)  
projection

Frontal projection plane: ~  
move the back projection  
plane to the outside of  
the virtual camera.

$D$ : Distance from the projection  
plane to the "pin hole".



$$\begin{aligned} X_p &= \frac{D}{z_v} X_r \quad (a1) \quad (X_r, y_r, z_v) \in \mathbb{R}^3 \\ y_p &= \frac{D}{z_v} y_r \quad (a2) \end{aligned}$$

Virtual Cam focal length

In Homework,  
 $D \approx 20$

$(X_p, y_p)$   
On your 2D  
Display

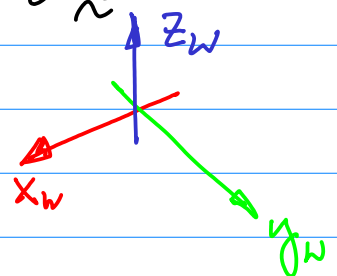
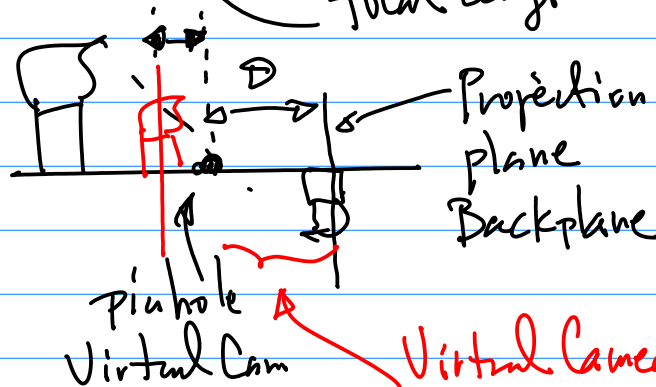


Fig1.

Virtual Camera

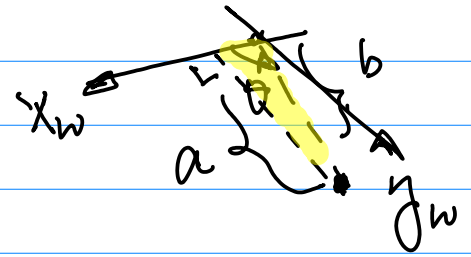
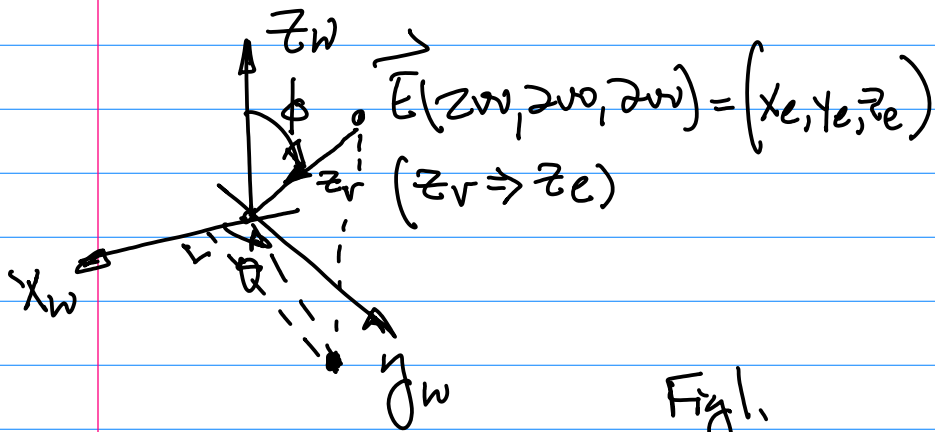


Fig 1.

$$\sin \theta = \frac{a}{b}$$

$$a = y_e = 2w$$

$$b = \sqrt{x_e^2 + y_e^2} = 2w\sqrt{2}$$

$$\therefore \sin \theta = \frac{2w}{2w\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{x_e}{b} = \frac{2w}{2w\sqrt{2}} = \frac{\sqrt{2}}{2}$$

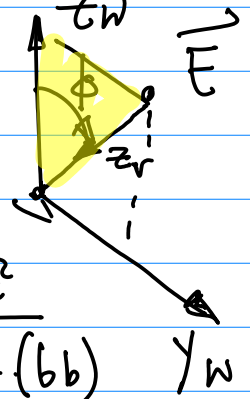
for  $\phi$  (phi)

$$\cos \phi = \frac{z_e}{\rho} \quad \dots (5b)$$

$$\rho = \sqrt{x_e^2 + y_e^2 + z_e^2}$$

"rho" ... (6)

$$\sin \phi = \frac{\sqrt{x_e^2 + y_e^2}}{\rho} \quad \dots (6b)$$



a Vector Passing Through  $E$ , Perpendicular to  $x_w$ - $y_w$  plane, So form an intersection pt.

b Vector Passing through the intersection pt on  $x_w$ - $y_w$ , perpendicular to  $x_w$  axis.

Note: Angle  $\theta$  (Theta) on  $x_w$ - $y_w$  plane  
 { wrt positive  $x_w$ -axis And  
 [Counter Clockwise Direction]

Note: Angle  $\phi$  (phi) on  $z_w$ - $z_w$  plane.

Now, find each entry on  $4 \times 4$  matrix,

So World-2 Viewer Transform Can be performed.

for Angle  $\theta$  (Theta)

$\sin \theta$  and  $\cos \theta$

April 7 (Wed)

Note:  $x_w$ - $y_w$ - $z_w$  Left Hand System

$$\cos \phi = \frac{z_e}{\rho} = \frac{2w}{\sqrt{2w^2 + 2w^2 + 2w^2}} = \frac{2w}{2w\sqrt{3}} = \frac{\sqrt{3}}{3}$$

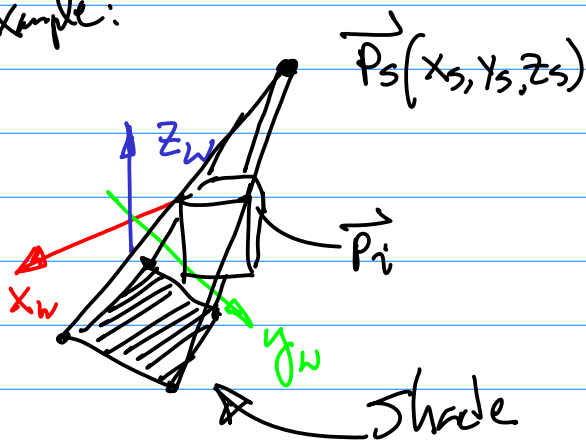
CMPE240

Similarly

$$\sin \phi = \sqrt{x_c^2 + y_c^2} / \rho = \frac{2\sqrt{2}}{2\sqrt{3}} = \sqrt{2/3}$$

Consider "Shade" Calculation.

Example:



1.  $x_w - y_w - z_w$ .

April 12 (Monday)

Note: 1<sup>st</sup> Homework Submission (E-mail)

$x_w - y_w - z_w$  Drawing.

Source code — Photo

(Exported Project) By Wednesday.

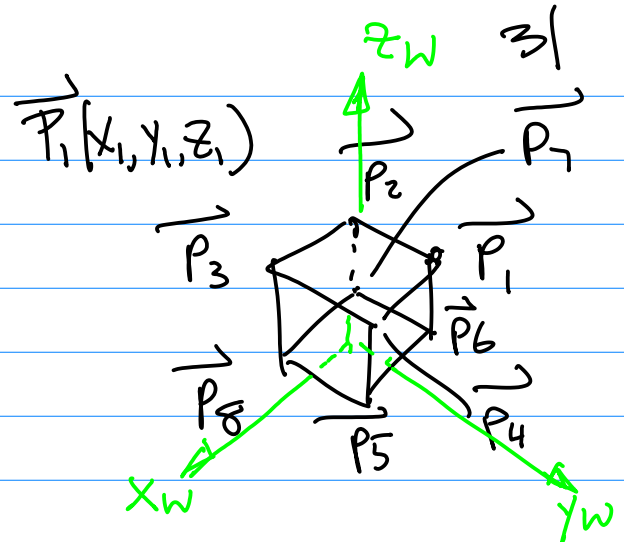
2<sup>nd</sup> Bring your Prototype

Board to Each Session for Inspection, Show & Tell.

Discussion on 3D Shade Computation

Conditions for this discussion

1. Define  $\{\vec{P}_i(x_i, y_i, z_i) | i=1, 2, \dots, 8\}$



One side of the cube overlapped with  $z_w$ -axis size of 100.

$\vec{P}_1(0, 100, 110)$

$\vec{P}_2(0, 0, 110), \vec{P}_3(100, 0, 110)$

$\vec{P}_4(100, 100, 110)$

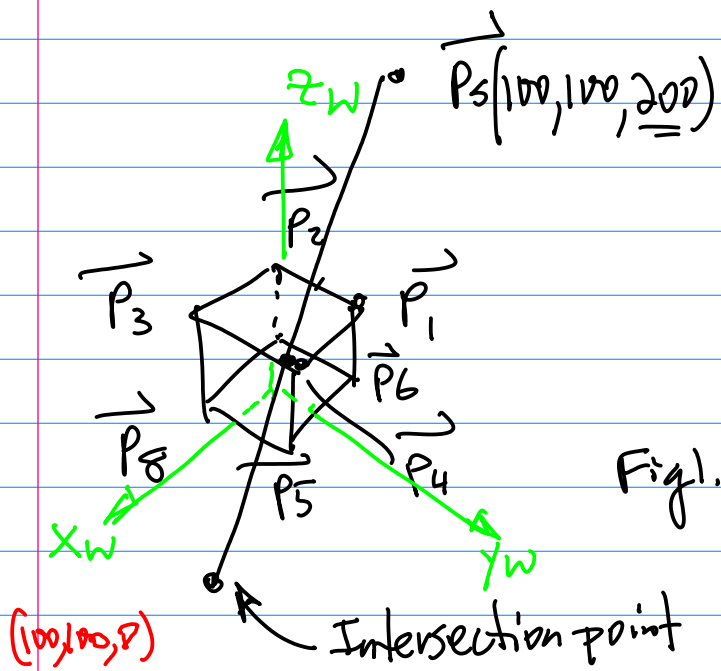
Note:  $P_1, P_2, \dots$ , are arranged Counter Clockwise

$\vec{P}_5(100, 100, 10), \vec{P}_6(0, 100, 10)$

$\vec{P}_7(0, 0, 10), \vec{P}_8(100, 0, 10)$

2. Point Light Source

$\vec{P}_s(100, 100, 200)$



3. Ray Equation, Connecting  $\vec{P}_3$  to  $\vec{P}_4$ ,

$$\vec{R} = \vec{P}_3 + \lambda(\vec{P}_4 - \vec{P}_3) \dots (1)$$

$$\vec{P}_4 = \vec{P}_4$$

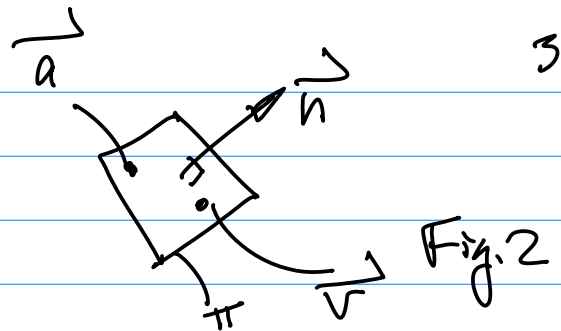
OR,

$$\vec{P} = \vec{P}_3 + \lambda(\vec{P}_4 - \vec{P}_3) \dots (1b)$$

Find intersection point on  $x_w - y_w$  plane.

4. plane Equation

4a Normal vector  $\vec{n}(n_x, n_y, n_z)$  perpendicular to the given plane,  $x_w - y_w$  plane



4b. plane  $\pi$ .

An Known, Arbitrary pt on  $\pi$  denoted

$$\vec{a}(a_x, a_y, a_z)$$

An arbitrary point

$$\vec{v}(v_x, v_y, v_z) \text{ on } \pi$$

$$\vec{n} \cdot (\vec{a} - \vec{v}) = 0 \dots (2)$$

OR

$$\vec{n} \cdot (\underbrace{\vec{v} - \vec{a}}_{\text{line segment}}) = 0 \dots (2-a)$$

normal vector

5. Intersection pt.

$$\vec{R} = \vec{P}_3 + \lambda(\vec{P}_4 - \vec{P}_3) \dots (3a)$$

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0 \dots (3b)$$

From (3b),

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

$$\vec{v} = \vec{r}, \vec{v} = \vec{p}_s + \lambda(\vec{p}_i - \vec{p}_s)$$

Sub.  $\vec{v} = \vec{r}$  Above into (3b)

$$\vec{n} \cdot (\vec{v} - \vec{a}) \Big|_{\vec{v} = \vec{r}} = 0$$

$$\vec{n} \cdot (\vec{r} - \vec{a}) \Big|_{\vec{r} = \vec{p}_s + \lambda(\vec{p}_i - \vec{p}_s)} = 0$$

$$\vec{n} \cdot (\vec{p}_s + \lambda(\vec{p}_i - \vec{p}_s) - \vec{a}) = 0$$

$$\vec{n} \cdot \vec{p}_s + \lambda \vec{n} \cdot (\vec{p}_i - \vec{p}_s) - \vec{n} \cdot \vec{a} = 0$$

$$\lambda \vec{n} \cdot (\vec{p}_i - \vec{p}_s) = \vec{n} \cdot \vec{a} - \vec{n} \cdot \vec{p}_s$$

$$\lambda = \frac{\vec{n} \cdot (\vec{a} - \vec{p}_s)}{\vec{n} \cdot (\vec{p}_i - \vec{p}_s)} \quad \dots (4)$$

from Eqn (3a), Ray Eqn.

With  $\lambda$  we can find the intersection Point.

In C/C++ Coding.

$$\lambda = \frac{\vec{n} \cdot (\vec{a} - \vec{p}_s)}{\vec{n} \cdot (\vec{p}_i - \vec{p}_s)} = \frac{(n_x, n_y, n_z) \cdot (a_x - x_s, a_y - y_s, a_z - z_s)}{(n_x, n_y, n_z) \cdot (x_i - x_s, y_i - y_s, z_i - z_s)}$$

$$= \frac{n_x(a_x - x_s) + n_y(a_y - y_s) + n_z(a_z - z_s)}{n_x(x_i - x_s) + n_y(y_i - y_s) + n_z(z_i - z_s)} \quad \dots (4a)$$

Example: Compute the shade by finding intersection pt formed by  $\vec{p}_s$  and  $\vec{p}_u$ .

Sol: From Eqn (3a), we have

$$\vec{r} = \vec{p}_s + \lambda(\vec{p}_u - \vec{p}_s) = (x_s, y_s, z_s) + \lambda(x_u - x_s, y_u - y_s, z_u - z_s)$$

from Eqn (4), where

$$\vec{n}(n_x, n_y, n_z) = (0, 0, 1)$$

$$\vec{a}(a_x, a_y, a_z) = (0, 0, 0)$$

Hence,

$$\lambda = \frac{0 \cdot (a_x - x_s) + 0 \cdot (a_y - y_s) + 1 \cdot (a_z - z_s)}{0 \cdot (x_u - x_s) + 0 \cdot (y_u - y_s) + 1 \cdot (z_u - z_s)}$$

$$= \frac{a_z - z_s}{z_u - z_s} = \frac{0 - 200}{110 - 200} = \frac{200}{110}$$

$$= 20/11$$

Therefore, Sub  $\lambda$  Back to the Ray Eqn.



$$\begin{aligned}
 \vec{R} &= \vec{P}_s + \lambda (\vec{P}_4 - \vec{P}_s) \mid \lambda = 20/q \\
 &= (100, 100, 200) + \frac{20}{q} (x_4 - x_s, y_4 - y_s, z_4 - z_s) \\
 &= (100, 100, 200) + \frac{20}{q} (100 - 100, 100 - 100, 110 - 200) \\
 &= (100, 100, 200) + \frac{20}{q} (0, 0, -90) \\
 &= (100, 100, 200) + (0, 0, -\frac{20}{q} \times 90) \\
 &= (100, 100, 200) + (0, 0, -200) = (100, 100, 0)
 \end{aligned}$$

Note: 1. Finish Last Homework,  
then Expand to a Cube.  
(Display it).

2. Compute/Implement this  
Algorithm, to calculate (Hand)

Each of Every 4 pts of top  
Surface of the Cube.

Note: Homework Submission Extended to 18th  
Sunday, 11:59pm.

a e-mail; to Subject:

First Name + SID (4 Digits) + CMPE240 + HW2  
April 14 (Th).

See the figure Next page

```

float Xe = 200.0f;
float Ye = 200.0f;
float Ze = 200.0f;
float Rho = sqrt(pow(Xe,2) + pow(Ye,2) + pow(Ze,2));
float D_focal = 20.0f;

```

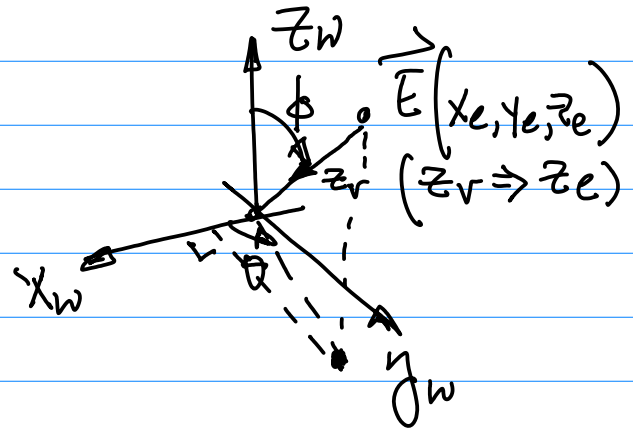
```
//define the x-y-z world coordinate
world.X[0] = 0.0; world.Y[0] = 0.0; world.Z[0] = 0.0; // origin
world.X[1] = 50.0; world.Y[1] = 0.0; world.Z[1] = 0.0; // x-axis
world.X[2] = 0.0; world.Y[2] = 50.0; world.Z[2] = 0.0; // y-axis
world.X[3] = 0.0; world.Y[3] = 0.0; world.Z[3] = 50.0; // z-axis
```

for  $X_w, Y_w, Z_w$  World-Coordinate System

```
typedef struct {
    float X[UpperBD];
    float Y[UpperBD];
    float Z[UpperBD];
} pworld;
```

for  $X_v, Y_v, Z_v$  Viewer (Virtual Camera)

```
typedef struct {
    float X[UpperBD];
    float Y[UpperBD];
    float Z[UpperBD];
} pviewer;
```



for Perspective Projection

```
typedef struct {
    float X[UpperBD];
    float Y[UpperBD];
} pperspective;
```

Declaration of Each Coordinate System

```
pworld world;
pviewer viewer;
pperspective perspective;
```

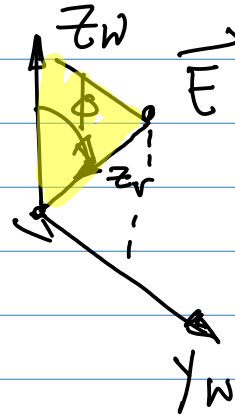
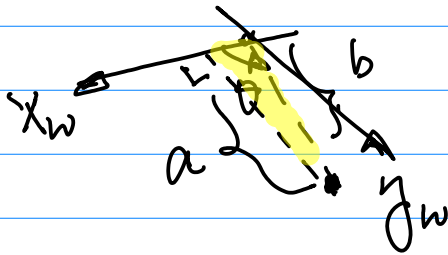
Initialization, By Defining 4 vectors

(pts)  $\vec{P}_1, \vec{P}_2, \vec{P}_3, \vec{P}_4$  for  $X_w, Y_w, Z_w$  axis

//define the x-y-z world coordinate

```
world.X[0] = 0.0; world.Y[0] = 0.0; world.Z[0] = 0.0; // origin
world.X[1] = 50.0; world.Y[1] = 0.0; world.Z[1] = 0.0; // x-axis
world.X[2] = 0.0; world.Y[2] = 50.0; world.Z[2] = 0.0; // y-axis
world.X[3] = 0.0; world.Y[3] = 0.0; world.Z[3] = 50.0; // z-axis
```

From PP.31. Fig. Below



We define  $\sin\theta$ ,  $\cos\theta$  for  $T_{4 \times 4}$

World-To-Viewer Transform.

```
float sPheta = Ye / sqrt(pow(Xe,2) + pow(Ye,2));
float cPheta = Xe / sqrt(pow(Xe,2) + pow(Ye,2));
float sPhi = sqrt(pow(Xe,2) + pow(Ye,2)) / Rho;
float cPhi = Ze / Rho;
```

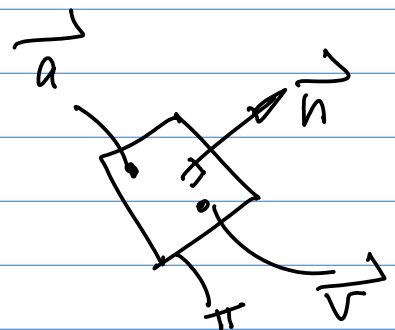
Define plane Equation By

a Normal Vector  $\vec{n} = (n_x, n_y, n_z) = (0, 0, 1)$

b Arbitrary pt  $\vec{a} = (0, 0, 0)$

Then a point light Source  $P_s(x_s, y_s, z_s)$

From PP.32



```
world.X[45] = -200.0; world.Y[45] = 50.0; world.Z[45] = 200.0; // Ps (point source)
world.X[46] = 0; world.Y[46] = 0; world.Z[46] = 0; // arbitrary vector A on x-y plane
world.X[47] = 0; world.Y[47] = 0; world.Z[47] = 1; // normal vector for x-y
```

Now, the Implement for Computing World-to-Viewer By  $T_{4 \times 4}$

```

for(int i = 0; i <= UpperBD; i++)
{
    viewer.X[i] = -sPheta * world.X[i] + cPheta * world.Y[i];
    viewer.Y[i] = -cPheta * cPhi * world.X[i]
    - cPhi * sPheta * world.Y[i]
    + sPhi * world.Z[i];
    viewer.Z[i] = -sPhi * cPheta * world.X[i]
    - sPhi * cPheta * world.Y[i]
    - cPheta * world.Z[i] + Rho;
}

```

from TP, 29, T444

$$T = \begin{bmatrix} -\sin \theta & \cos \theta & 0 & 0 \\ -\cos \phi \cos \theta & -\cos \phi \sin \theta & \sin \phi & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & -\cos \phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} X_v \\ Y_v \\ Z_v \\ 1 \end{pmatrix} = T_{4 \times 4} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \quad \begin{array}{l} \text{World-to-} \\ \text{Viewer} \\ \text{Transform} \end{array}$$

"After"      "Before"

Now, Perspective Projection from TP, 29

```

perspective.X[i] = D_focal * viewer.X[i] / viewer.Z[i];
perspective.Y[i] = D_focal * viewer.Y[i] / viewer.Z[i];

```

$$\begin{aligned} x_p &= x_e \left( \frac{D}{z_e} \right) \\ y_p &= y_e \left( \frac{D}{z_e} \right) \end{aligned}$$

For Intersection Computation  
 find  $x$  in  $X_w - Y_w - Z_w$ ,  
 find intersection pt(s)  
 in  $X_w - Y_w - Z_w$

Then, Computer Virtual Display (2D)  
 to Physical Display ! Then plot the points !  
 Then Done !

Then Transform  
 intersection pipeline

# Diffuse Reflection.

## Background/Basic Concepts

1. Objective: To generate realistic Looking 3D Graphics

### Lighting Models

After Trans. Pipeline

$$\vec{I}(x,y) = (r(x,y), g(x,y), b(x,y))$$

Graphics (Vector) Graphics  
Primitive Colors ... (1)

r, g, b : 8 bit Resolution.

r, g, b  $\in [0, 255]$ , for 15 bits, or 16 bit  
r, g, b. is Common.

## 2. Color Space

April 19 (Monday)

Diffuse Reflection

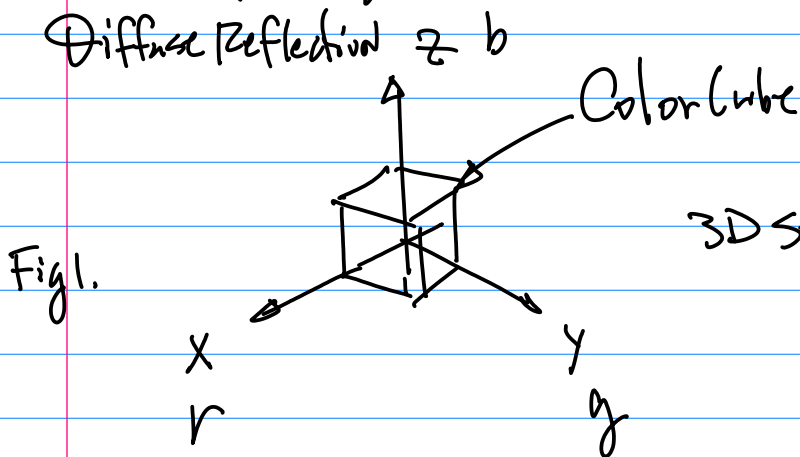
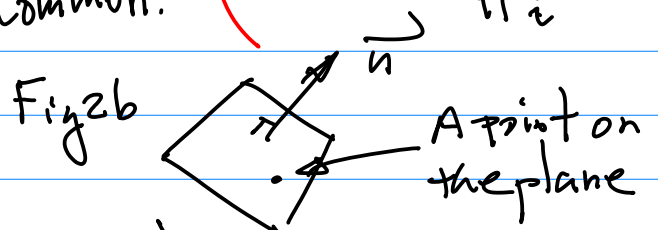
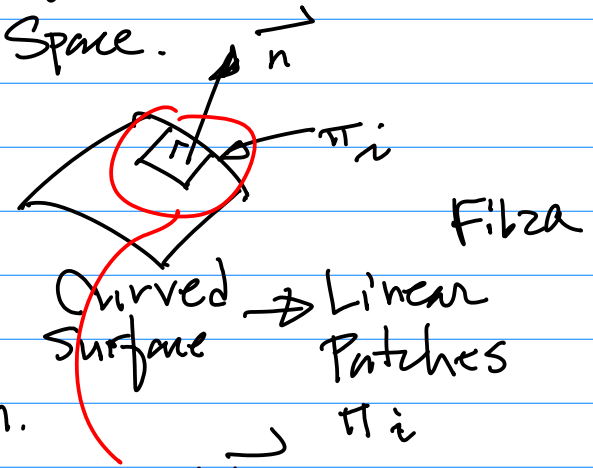


Fig 1.

Note: in r-g-b Color Space,  
(255, 0, 0) : Brightest Red  
(0, 255, 0) : Brightest Green, etc.

line connecting (0, 0, 0) to (1, 1, 1) : gray, (0, 0, 0) Black, (1, 1, 1) Brightest White

3. Color Contribution to Each pixel on a Surface of a given Object in 3D Space.

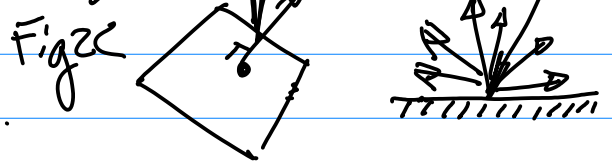


$$\vec{I}(x,y,z) = \vec{I}_1(x,y,z) + \vec{I}_2(x,y,z) + \vec{I}_3(x,y,z) + \dots (2)$$

3D Space in  $x_w - y_w - z_w$

$\vec{I}(x,y,z)$  : Diffuse Reflection

Diffuse Reflection (Ref. on github) :



Diffuse Reflection:  $\sim$  that reflects incoming light uniformly in all different directions.

Example: Diffuse Reflection model.

Step 1.  $\vec{I}(x, y, z)$  Color on a pt. of a surface in 3D space.

1. Reflectivity for all 3 different light models ( $I_r, I_g, I_b$ )

$$\vec{I}(x, y, z) = (\vec{I}_r(x, y, z), \vec{I}_g(x, y, z), \vec{I}_b(x, y, z))$$

Define  $\vec{R} = (r_r, r_g, r_b) \dots [3]$

Simplify the discussion by focusing on one primitive color

$$0 \leq r_r, r_g, r_b \leq 1$$

$r_r = r_g = r_b = 0$  Black

$r_r = r_g = r_b = 1$  white

$r_r = 0, r_g = 0.75, r_b = 0$  : Green

$$\vec{I}_r(x, y, z)$$

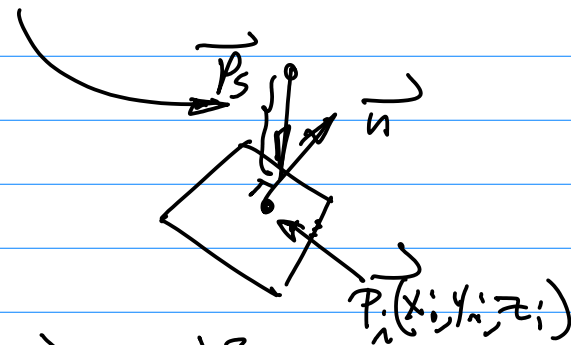
Energy of Photons from the Source reaching the pt. is inverse proportional to the squared distance

Physical Characteristics of the given surface

Now, Specular Reflection

$I_z$ : Reflection produces high light, it is a function of  $E$  position.

Fig 3

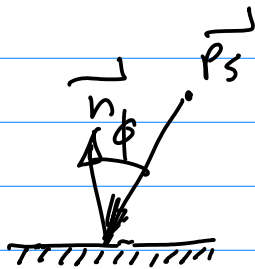


$$\|\vec{P}_s - \vec{P}_i\|^2 = \|\vec{r}\|^2 \dots (4)$$

$I_b$ : Ambient light, from indirect light source, can be simulated by adding constant values to  $r, g, b$ .

Ray Equation  $\vec{r}$

$$\vec{I}_r(x, y, z) \sim \frac{1}{\|\vec{r}\|^2} \dots (5)$$



$\phi$ . ph: Angle  
of the incoming  
light

$\phi = 0$ , Strongest incoming light  $\rightarrow$  "1" Normalize it  
 $\phi = \pi/2$  photons miss the pt. on the  
 surface  $\rightarrow$  Normalized it.  
 "0"

Cos  $\phi$  Rule. Dot Product

$$\vec{n} \cdot (-\vec{P}_s) = \|\vec{n}\| \|\vec{-P}_s\| \cos \phi \quad \dots (5)$$

$$\cos \phi = \frac{\vec{n} \cdot (-\vec{P}_s)}{\|\vec{n}\| \|\vec{-P}_s\|} = -\frac{\vec{n} \cdot \vec{P}_s}{\|\vec{n}\| \|\vec{P}_s\|} \quad \text{From Eqn (5), (a), (10)}$$

$\dots (b)$

define  $\vec{-P}_s$  as Ray Vector

$$I_r(x, y, z) = K_r \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \frac{1}{\|\vec{r}\|^2} \quad \dots (11)$$

\*  $\vec{r} \triangleq \vec{P}_i - \vec{P}_s \quad \dots (7)$

Hence

$$\cos \phi = \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \quad \dots (8)$$

where  
 $\|\vec{r}\|^2 = \|\vec{P}_i - \vec{P}_s\|^2$

$$\vec{I}_r(x, y, z) \sim \cos \phi \left( = \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \right) = \frac{(x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2}{\|\vec{r}\|^2}$$

$\vec{I}_r(x, y, z) \sim K_r$  Reflectivity for  $r$  Preparation for  
 $\dots (9)$   
 $\dots (10)$