

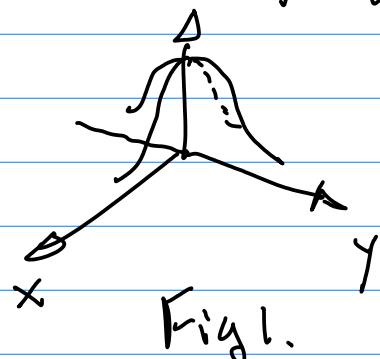
CmPE242

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \dots (1)$$

March 10 (Wed) $\frac{(x^2+y^2)}{26^2}$

$$G(x,y) = \frac{1}{2\pi 6} e^{-\frac{(x^2+y^2)}{26^2}} \dots (2)$$

Note $M_x = M_y = 0$, $G_x = G_y = 0$



Note: $\text{LoG}(x)$ is NOT ²²

Exactly the Computation
for derivatives, But we
use it, for its Low Pass
feature, and 2nd order
derivatives.

Sol.

(1) "Mapping" to a Kernel
Build a kernel with
"Odd" Number of grids,
elements

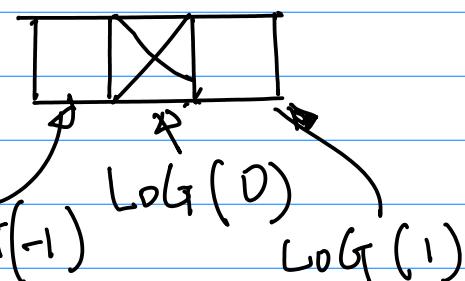
From Eqn(4), Ppt from the github
2018S-15-LecB-V3 ...

Let $y=0$, to have one indep.
Variable x .

$$\nabla^2 G(x,y) \Big|_{y=0} = \nabla^2 G(x,0) \dots (3)$$

$$-\frac{1}{2\pi 6^3} e^{-\frac{x^2}{26^2}} + \frac{x^2}{2\pi 6^5} e^{-\frac{x^2}{26^2}}$$

$K \times 1$
No. of elements One Row
for $K=3$



$\text{Log}(x)$, or $\nabla^2 G(x,0)$

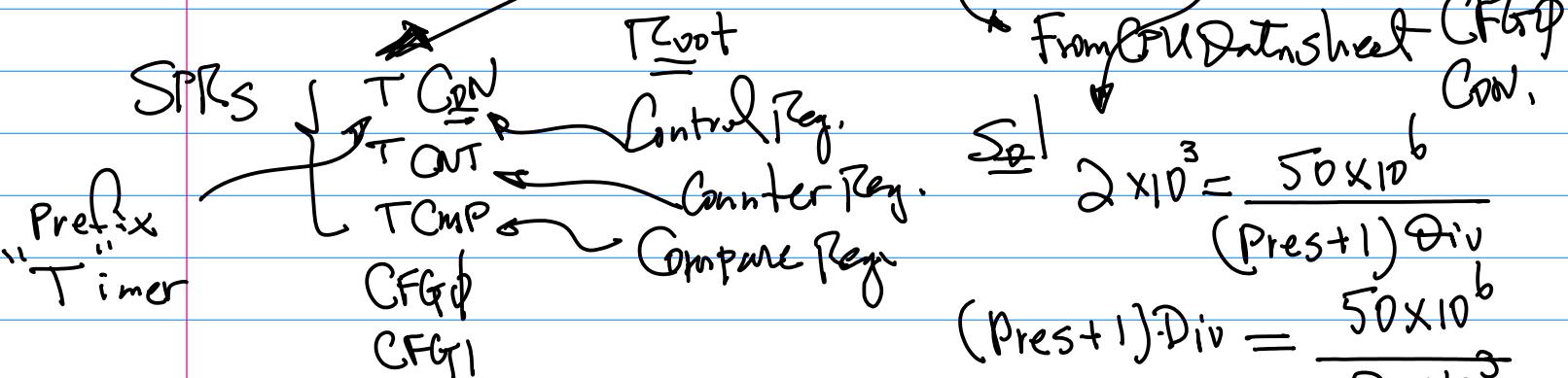
Example: ① Use $\text{LoG}(x)$ to Build
a convolutional Kernel (z) to
Compute Derivatives of the Error

\cong Identify the
Center Reference
 \Leftarrow from $\text{LoG}(x)$ (or
 $\nabla^2 G(x,0)$). map it
to the kernel

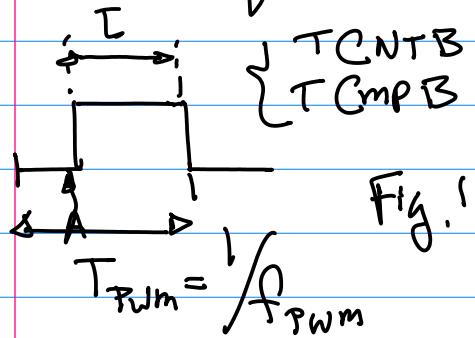
Solve for f_{Pwm} . . .
 Driver Implementation. } from
 D.C. }
 2018S-10-0 ~
 PWM Driver

Add Duty Cycle Function to
 Device Driver.

Theoretical Aspect
 Implementation C Code.



Pwm Output Square Waveform



$$f_{Pwm} = \frac{CLK_p}{(\text{Prescaler}+1)(\text{divider})} \quad \dots (3)$$

PP1118, Q11 Datasheet

Prescaler : 8 bit, [0, 255]

divider ; 1, 2, 4, 8, 16

Note CFG / Con are responsible for Setting Prescaler / Divider value.

$$f_{Pwm} = \frac{50 \times 10^6}{(Pres+1) \cdot Div} \quad \dots (4)$$

If we need $f_{Pwm} = 2 \times 10^3$
 Find SPR, Set SPR. to Realize this frequency.

From PLD Datasheet CFG & CON.

$$2 \times 10^3 = \frac{50 \times 10^6}{(Pres+1) \cdot Div}$$

$$(Pres+1) \cdot Div = \frac{50 \times 10^6}{2 \times 10^3}$$

$$(Pres+1) \cdot Div = 25 \times 10^3$$

Let Div = 16,

Solve for Pres.

$$Pres+1 = \frac{25 \times 10^3}{16}$$

$$Pres = \frac{25 \times 10^3 - 1}{16} \approx 255$$

Iteration,

Change PCLK to 10 MHz,
 then, we have

$$\text{Pres} = \frac{10 \times 10^6 - 16 \times 7 \times 10^3}{16 \times 2 \times 10^3}$$

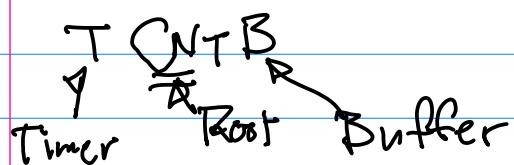
if it is still
too big

therefore, then low the CLK_p

try $\text{CLK}_p \approx 2 \times 10^6$. please verify it!

Arm11 Datasheet
 $\left\{ \begin{array}{l} T_{\text{INTB}} \xrightarrow{\text{---}} "N" \text{ counts} \\ T_{\text{CmpB}} \\ f_{\text{PWM}} \end{array} \right.$

Note: SPR Responsible for f_{PWM}



$$f_{\text{PWM}} = 1 \times 10^3 \text{ given.}$$

f Master Clock
Peripheral

N Counts for CNT SPR.

$$f = f_{\text{PWM}} \cdot N$$

Given Target Unknown to be Calculated

Note: T_{CmpB}

for Duty Cycle

2nd Counts Value for "Cmp"

Derived from Duty Cycle.

March 17 (Wed)

f_{PWM} By Setting SPR's
Duty Cycle value

Define one period ;
the CNT

Duty Cycle $\rightarrow \%$ \rightarrow Counts
Percentage \downarrow
Cmp

GPFCON P.P. 522

$$\text{GPFCON}[29:28] = 10 \rightarrow \text{Pwm}$$

$$\text{GPFCON}[31:30] = 10 \rightarrow \text{Pwm}$$

define S3C64xx - 1

GPFCON

0x7...

Comparison Register

x.f

$g = n(0x31 \ll 28)$

"AND" \rightarrow "00", "11"

$\rightarrow (0x21 \ll 28)$

"DR" "ID" Unsigned

Set 2 Bits

$$\text{GPFCON}[29:28] = 10$$

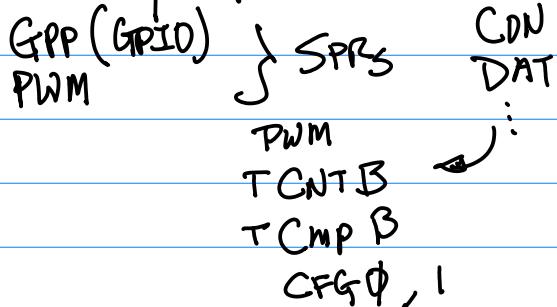
March 22nd (Mon)

Review.

1^o 3+ Questions.

a Basic Concepts
 CPU Architecture
 Block Diagram.

Memory Map, Peripheral Controllers

Architecture \rightarrow Mem \rightarrow SPIC

Code
 User Space
 programming

KConf
 Script
 Define Compilation + Build Process.

Programming Requirements, No Programs

However!
 Writing
 Code
 Debug/Change the
 existing code is Needed;

b Design-Related Question(s)

SCH. Design, CIC for PWM

Pin(s), f_{PWM}
 motor Drive
 = Pin(s), Label(s) Stepper motor I/F

GPP I/O Testing ("Hello, the world")

Input Testing CIC

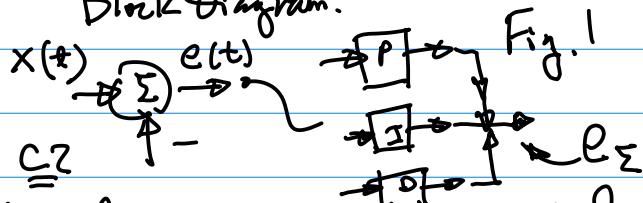
Output Testing CIC

Resistance Value
 Calculation \leq Theoretical Aspects

C1. PID Controller Design

Basic Concepts

Block Diagram.

Kernels F.D. $2 \times 1 \rightarrow$ CentralB.D. $2 \times 1 \frac{1}{2} (F.D + B.D)$

With Noise Reduction 3x1

Low Pass Filter: G(x) Gaussian.

∇^2 : 2nd Order Derivation as in
 Computer Vision

$$\nabla^2: \text{Laplacian } \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \rightarrow \frac{\partial^2}{\partial x^2}$$

LG(x)

Note: One page formula sheet is
 Allowed, However No Verbal
 Description And/or Examples Allowed.

Note: Calculation IS Allowed.

Close Book, Close Notes

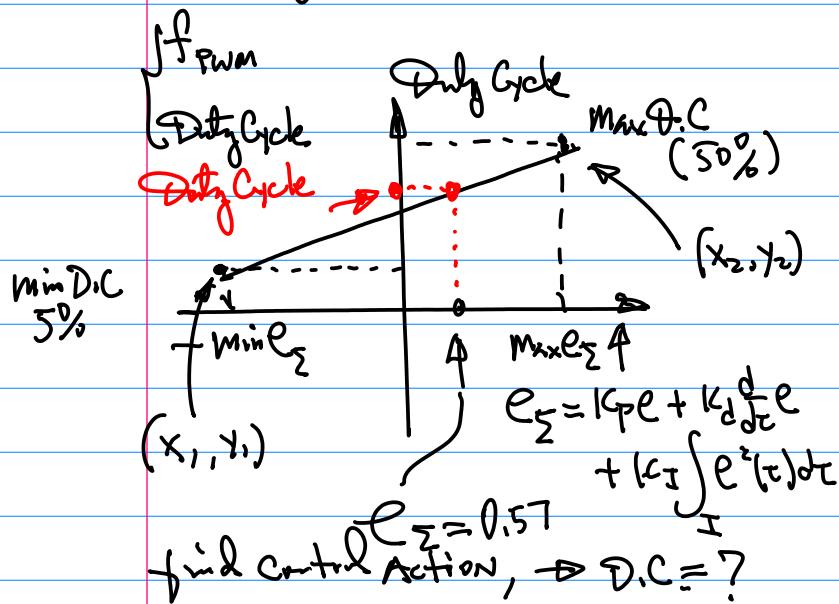
Datasheets if needed will be
 provided;

Convolution with Kernel(s)

Table of E(t), find $\int e(t)E(t)$ Convolution

$$\int K_I e^2(t) dt = \sum_{i=0}^I K_I e_i^2(t)$$

Mapping to Control function PWM



To perform init & Config:

1° Binary Pattern for SPR.

Ready/modify user Application Programs / Kernel Space Device Driver Program.

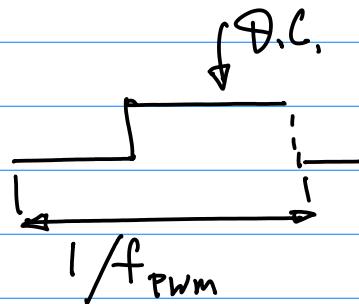
C Code for this purpose.

Note: PWM Waveform, e.g., Duty Cycle Calculation.

$$\frac{N_{\text{for CNT}}}{N_{\text{for Cmp}}} \frac{f_{\text{PLCK}}}{f_{\text{PWM}}} = \frac{N_B}{T_B}$$

$$f_{\text{PWM}} = \frac{\text{PCLK}}{(Prest+1) \times DUR}$$

$$\%(\text{D.C.}) \times N = N' \rightarrow \text{Cmp}$$



Implementation: Reference Platform
ARM11

Architecture
CPU Block Diagram → Memory map.
→ 32 (4 GB)
8 Equal Banks

* SPRs.
(Peripheral Controllers)
GPP → GPX C DN
GPX DAT
T CNT B
T Cmp B

S PWM
= GPX
GPP

Design Spec. → SPRs → CPU
(pins) Datasheet
on Target Board
CON r3 | r4

- Pre-requist
 - 1° O.S. Source Distribution
 - 2° Tool Chain Distro.
 - 3° "Cross Comp" Datasheet.
- Tool Chain Installed
- Running make menuconfig

242

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Continued \rightarrow KConf (at \drivers
 ↓ ~ \Char)

Script. Add your
DeviceDriver

make menuconfig

involve your Change,
Compile & Build
(Module Only for
Simplicity Purpose)

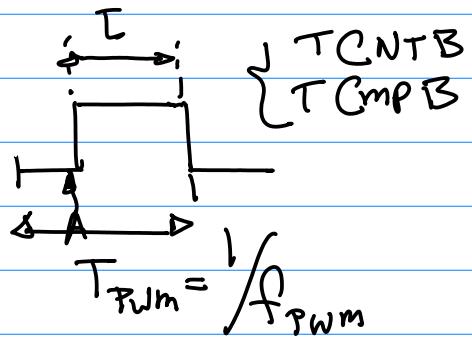
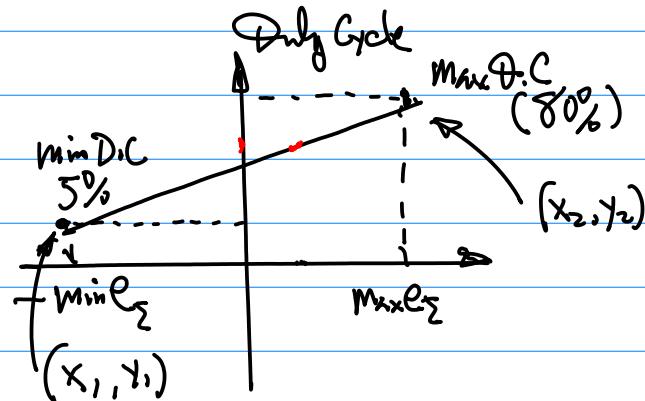
Object "KO".

Copy "USB" $\xrightarrow{\text{Upload}}$
by "CP" Copy
Command to

your target

"insmod" mytest.ko (To make
it as a part of Kernel Image)

Run your user application
program (By Calling the module)



April 5th (Monday)

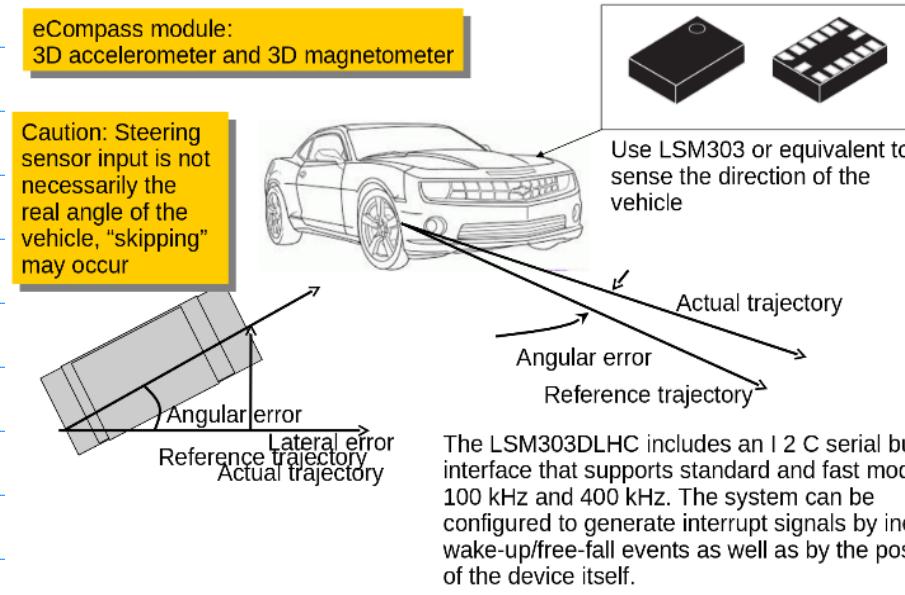
1. Midterm Graded, the key was posted online, github, search under folder 2014S, "Key"

2. 2nd half of this course. I₂T (Industrial)
I₂T (Inertial)

Sensors I/F
 Digital Sensors — I₂C I/F.
 Analog Sensors — ADC

[CMPE242-Embedded-Systems- / 2018S-16-AngularSensing-i2c-LSM303- final HL 2017-3-13.pdf](#)

Sensors for Driving Direction and Turning Angle



3D Accelerometer and 3D Magnetometer LMS303

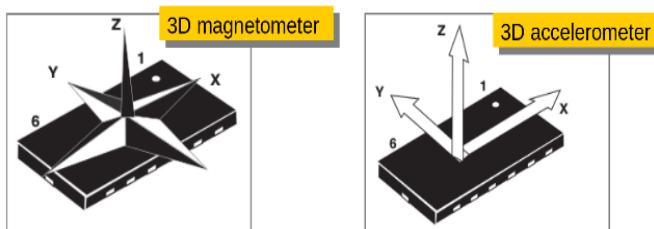


Table 9

Pin name	Pin description
SCL	I ₂ C serial clock (SCL)
SDA	I ₂ C serial data (SDA)

I₂C Interface

- (1) The transaction started through a START (ST) signal, defined as a high-to-low on the data line while the SCL line is held high.
- (2) After ST, the next byte contains the slave address (the first 7 bit), bit 8 for if the master is receiving or transmitting data.

• F.F.T to find (Characterize)
 Analog Sensor Data
 (Nyquist Theorem)
 Validation of Sensor
 Data,
 OpAmps to Build
 Processing Circ.
 "SPICE"
 Simulation.

Example: LSM303

Note: Next Project

use LSM303.

LSM303

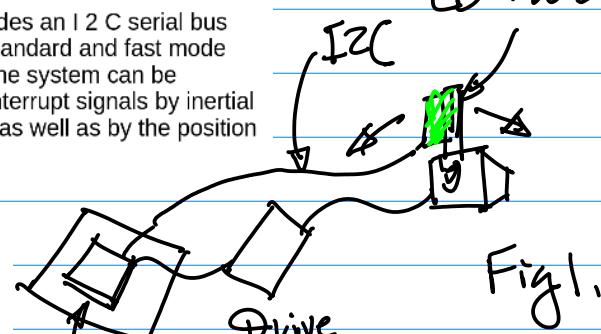


Fig. 1.

Configuration I
 for PID controller
 Homework: Implementation

I₂C LSM303 Sensor
 I/F. Due April 16(Fri)

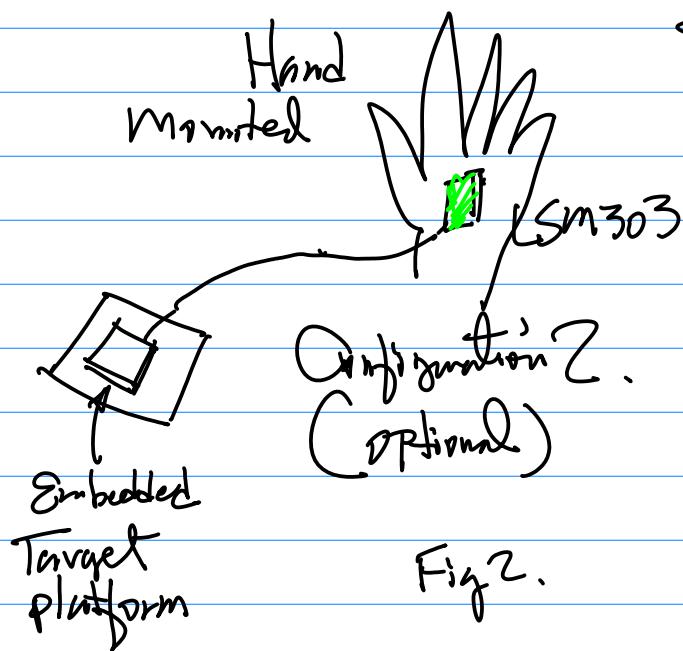


Fig 2.

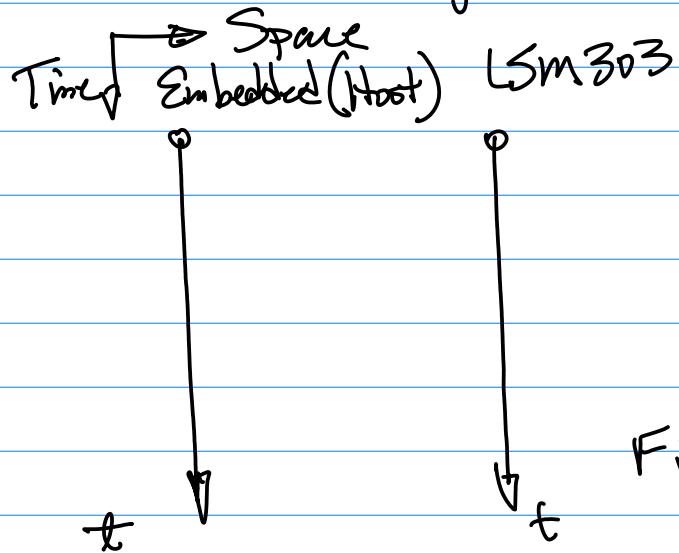
3. a Space-Time Diagram

Fig 3

Submission On CANVAS
Objective 1 To be Able to Read
Sensor Input,
② To be Able Config the
Sensor.

1. Note: Lsm303 for ST-micro
Sensor Supports { Acceleration
X-y-z axis
Magnetometer
Temperature }

2. I₂C
{ SDA (Serial Data) Bidirectional
SCL (Serial Clock) Data; }

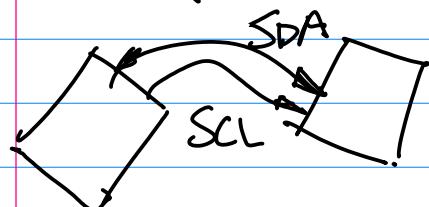
I₂C Device
(Lsm303)

Fig 4.

b To Describe "Hand-Shaking"
Three Small Steps.

Step 1. → Step 2 → Step 3
Host Slave "Ack" Data
Command. Transmission
to the Target will start
via Address
for Init & Config

* Be sure read Datasheet to
map the Steps of the I/F
to Space-Time Diagram.

3. Datasheet Table 9 & 11.

TP2P.

Notation:

A Frame

- 1° St → DSP
"Start" "Stop"

2^o The Notations in Table II

SAD, SAD+w, SNB, DATA,

SAK, etc. pp.20

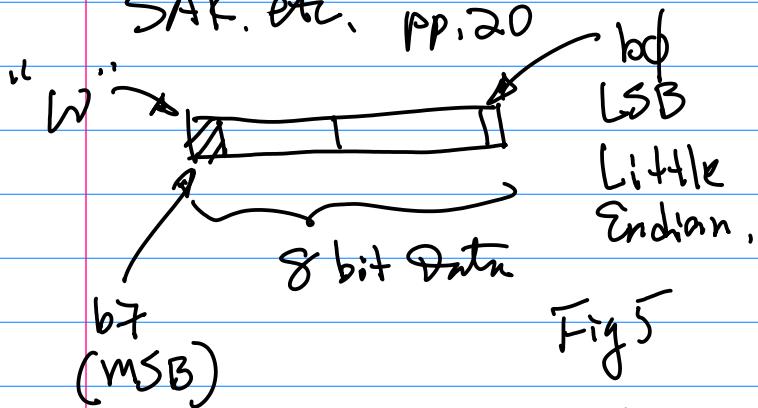


Fig 5

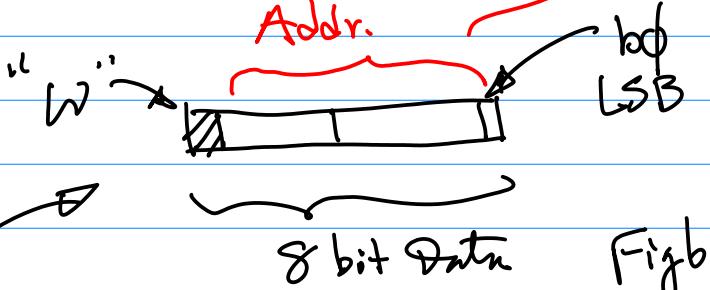
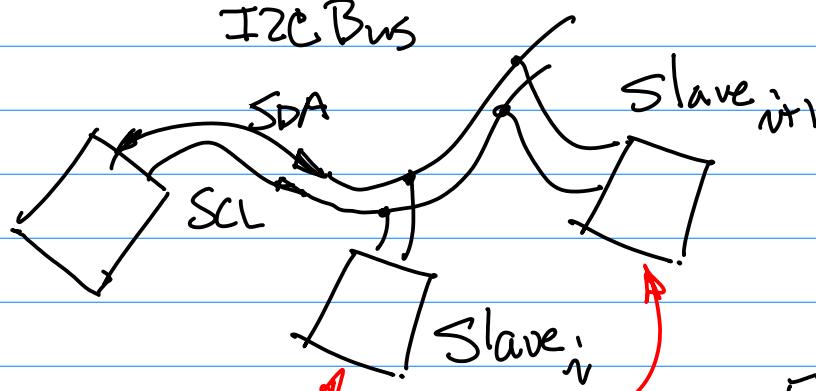
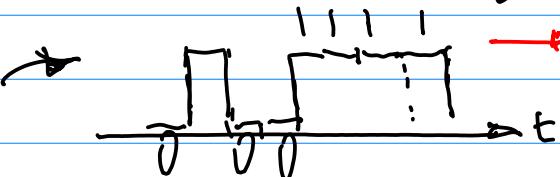
3^o from pp.20 (Datasheet)I₂C Bus

Fig b

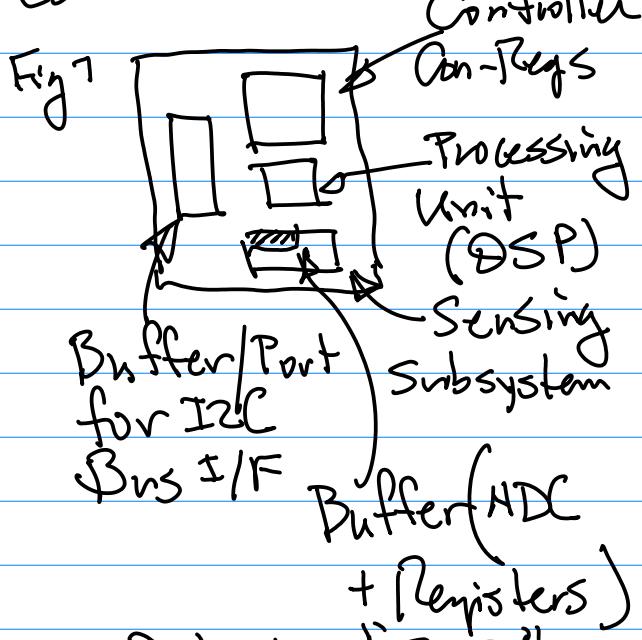
4^o 1st Address (7 bits), 2nd Address lower SUB.

All from the host (Target platform)

0xf2
1111 : 0010

Consider A Slave device

LSM303



2nd Address "SNB" is for Identifying the target inside the Slave Device.

5. 127 Devices Possible (Theoretically) on I₂C Bus, In Reality this has to be checked By "FAN-IN" or FAN-DUT

128 Internal Addresses

→ Special Purpose Registers.

6. Most Significant Bit is transmitted first

Example: From Datasheet (Lsm303)
TP.19 Table 11, 12, 13

Homework (1pt) Due A week
from Today, April 14, Due
On CANVAS

1° Build I2C Bus Interface
with your target platform
as a host, Lsm303 Slave.

To be able:

\cong hardware Implementation.

(e.g. mount Lsm303
on the Stepper motor,
or mount it to your
hand)

\cong Read Acceleration Data

X, Y, Z, displaying on
your terminal.

\cong Read Magnetometer

Data and display it on
the terminal

Note: Sample code is posted

"as is" basis.

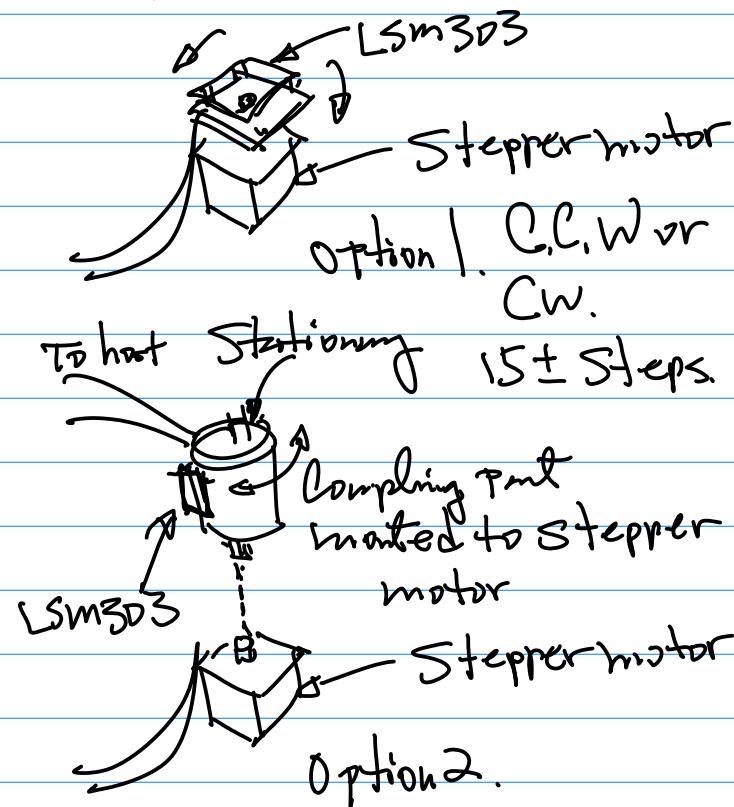
Report: 20-2021S-10-Lsm

2° Submission

\cong Source Code, \cong Readme.txt

\cong photo(s) of your
Implementation

3x photos, 1 for the entire
System (with Laptop); 2nd for
the Host Side, Expansion
Connector is the focus;
3 for Lsm303



\cong 5 seconds Video Clip(s)

720P or 1080P (1920x1280)
Compressed, MP4G4, avi ?

file Naming: first Name 4 Digits -
242.zip

Note: Table 11 & 12 (PP19)

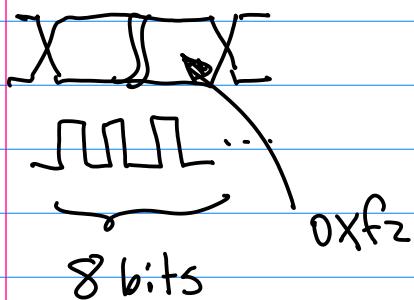
CMRF/EE2442

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One Byte Writing \rightarrow Multiple Bytes Writing Tech Spec:

Host
Table (Logical Behavior)

Timing (Waveform)



Now, the Address for the Sensor(s) and Addresses for Registers
 ↴ Control Register — Init & Config
 ↴ Data Register

SAD[5:1] Address + SAD[0] for w/r
 $\begin{cases} =1 \text{ for w} \\ =0 \text{ for r} \end{cases}$

Note: Use info from Table 14

\rightarrow fill in SAD+w, SAD+r
 in tables 11~13.

Note: Section 5.1.3 Magnetometer

Example: Table 8. Control Register A

for Magnet



CTRL_REG1-A [7:0] 8 bits.

1's Complement
 By Negation
 $"0" \leftrightarrow "1"$
 $"1" \rightarrow "0"$

Tech Spec \rightarrow Binning Pattern

i. 400 Hz Data Rate

ii. X-Y axis.

iii. Sensor Active (No LowPower)

CTRL_REG1-A [3] = 0

CTRL_REG1-A [2] = 0

CTRL_REG1-A [1] = 1

CTRL_REG1-A [0] = 1

\uparrow
 0×3
 Home,

CTRL_REG1-A [7:0] =
 0×13 ✓

Section 7.1.8 Status Registers

Section 7.1.9 ~ 7.1.11 Data Registers.

2's Complement form

CTRL_REG1-A [7:4] = 0x7

(for 400 Hz) + 1

April 12 (Monday)

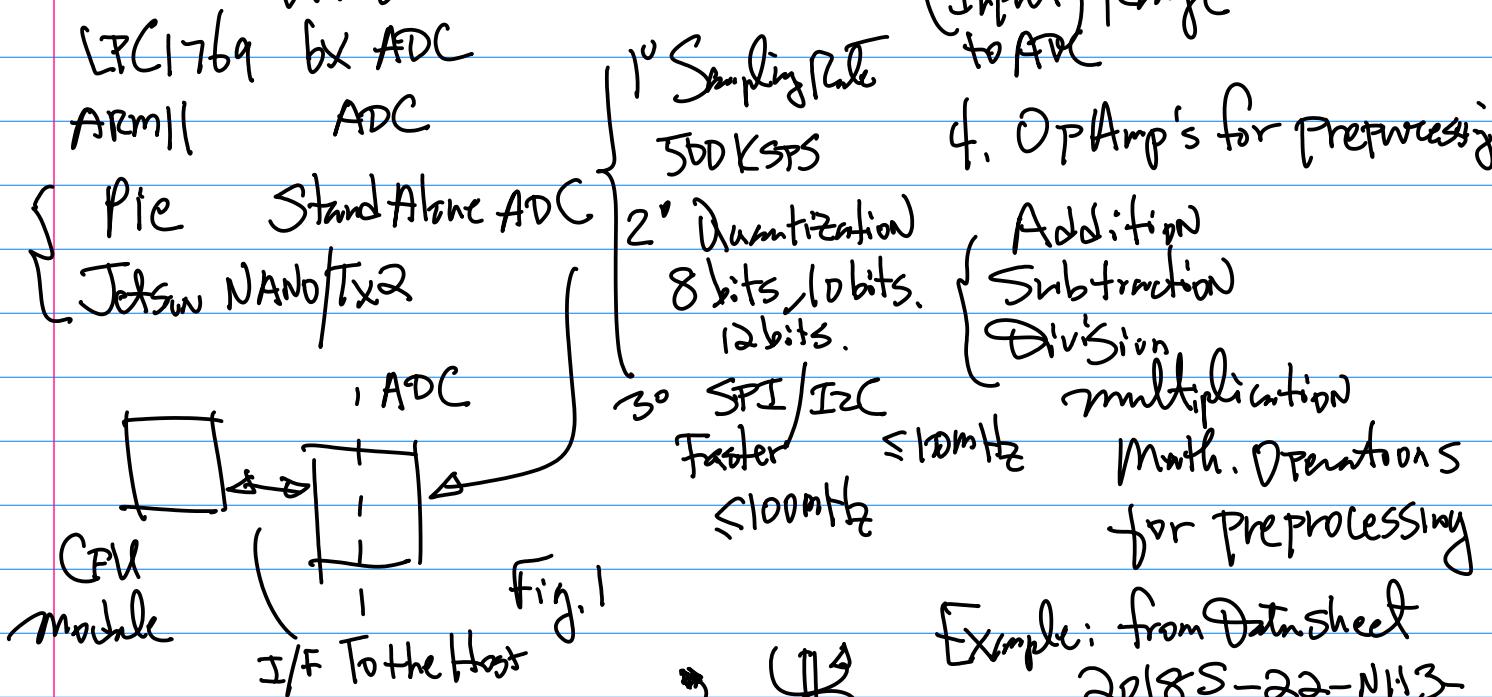
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Homework Extension to April 19
(Monday)

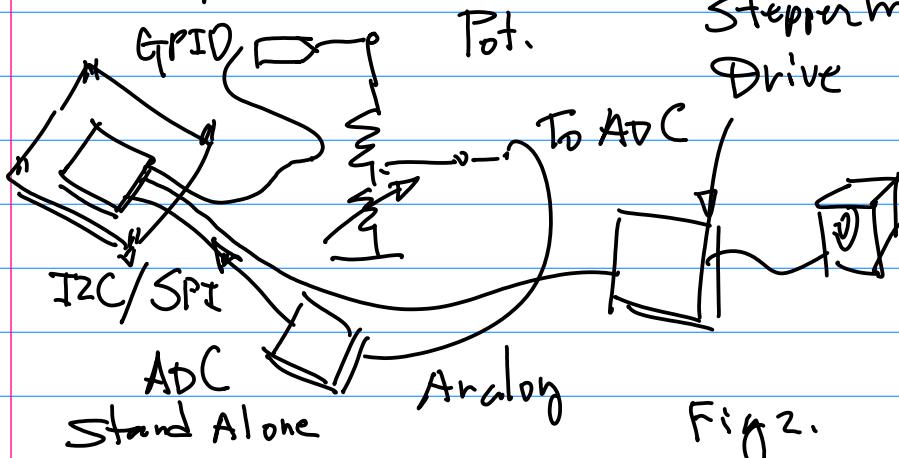
Industrial Analog Sensor I/F Design

Example: NHI3 Analog Sensor (ISN
Selective Electrode) Interface.

1. ADC (Analog to Digital Conversion
Unit)



2. Prototype to Build



3. Analog Interface Design.

Start with Characteristic
Curve

Linearization

OptAmp Preprocessing CKT

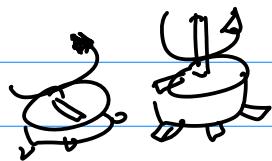
Optimized dynamic
(Input) Range
to ADC

4. OP Amp's for Preprocessing

Addition
Subtraction
Division
Multiplication
Math. Operations
for Preprocessing

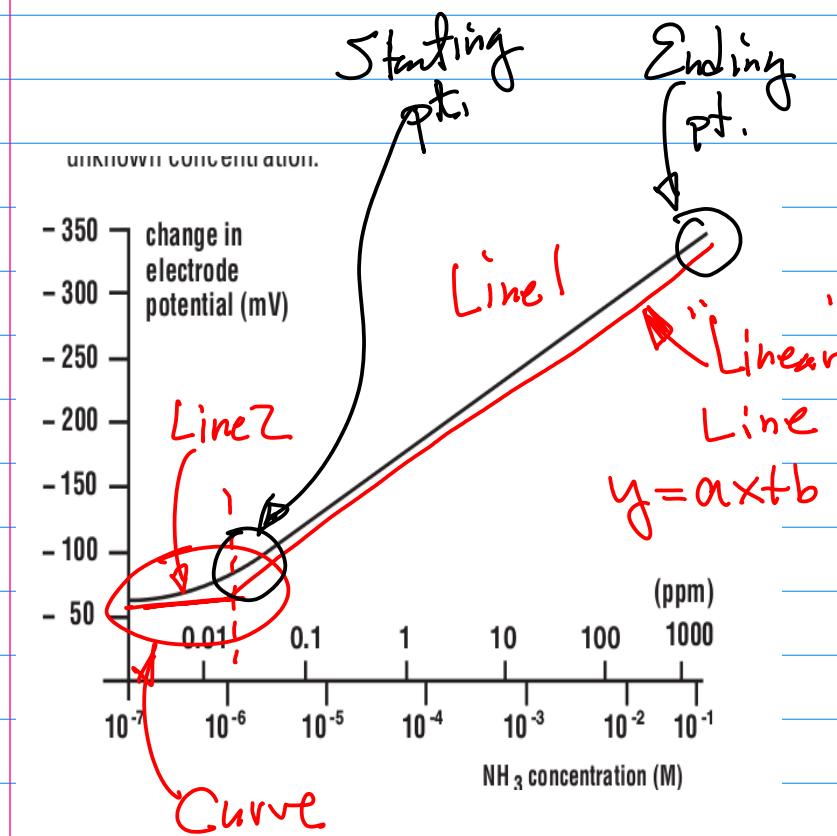
Example: from Data Sheet
2D/8S-22-NH3

TP13, Fig. 14.



Stepper motor
Drive

Step1. Linearization
Characteristic
Curve



Step 2. Map the Dynamic Range of Sensor to the Dynamic Range of ADC Input.

2.1 "Shifting" OR Offset to move the Characteristic Curve to the origin.

Linearization: Line Equation(s) to replace Non-Linear Curve.

Simplification: Just Keep Line 1.

Note: Linearization By Identifying Starting and Ending Points.

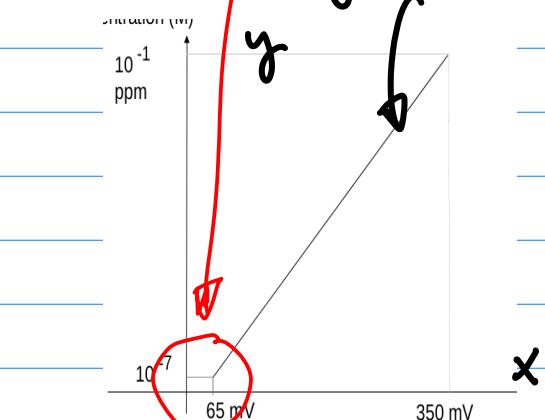
$$(V_0, C_0)$$

$$(V_N, C_N)$$

Voltage vs. Concentration

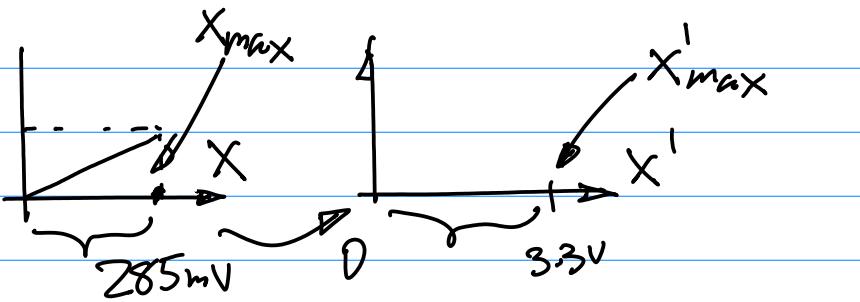
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \dots (1)$$

$$(x_1, y_1) \Rightarrow (V_0, C_0), (x_2, y_2) \Rightarrow (V_N, C_N)$$



x now is changed
By "Shifting"
 $a x \rightarrow a(x - \Delta x)$

2.2 Dynamic Range fits to ADC Dynamic Range



$$\frac{X'^{\max}}{X^{\max}} = \frac{3.3V}{0.285V} \text{ Let } A \dots (2)$$

Gain
("Scaling Factor")

$$X \cdot A = X' \dots (3)$$

Review { Inverting Configuration OpAmps
OpAmp Non-Inverting Configuration

April 14 (Th).

OpAmps for Pre-processing Design

1. OpAmp { a Very High Input Impedance
b Very Big Open-loop Gain
c Very Small Output Resistance

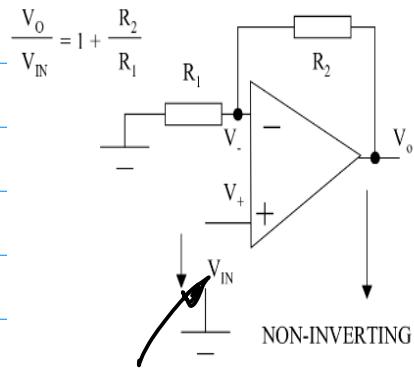
2. Using OpAmps As Basic

Building Blocks (B^3) for

Arithmetic Computation { Add/Sub
Multiplication

Integration/Derivatives Division

3. Configurations { Non-Inverting
Inverting ~



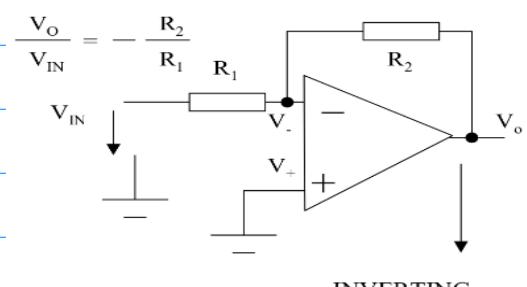
a Input: Positive Polarity;

b feedback Ckt

$V_{out} \rightarrow V_{in}$ (V_-)
Via R_f

c Draw the Ckt.

$$A = \frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_1} \dots (1)$$



a Input: $V_- \neq 0$

b feedback Ckt

$V_o \rightarrow V_{in}$ (V_-) $\propto -R_f$

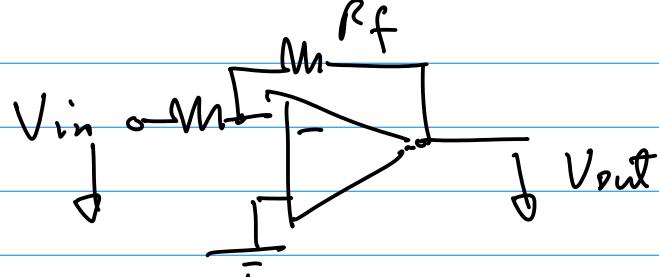
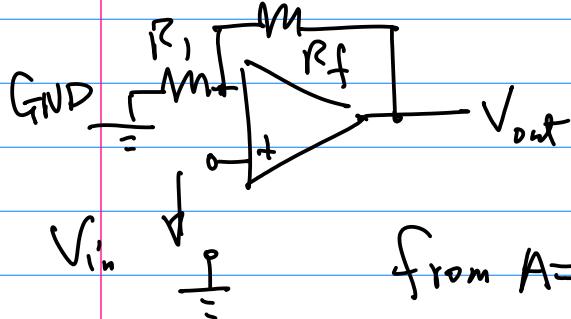
$$A = -\frac{R_f}{R_1} \dots (2)$$

4. Use OpAmp Circuits for Math. Operations.

Addition. $X_1 + X_2$

More than One Approach is possible, But Let's use Inverting Configuration

Non-Inverting Configuration



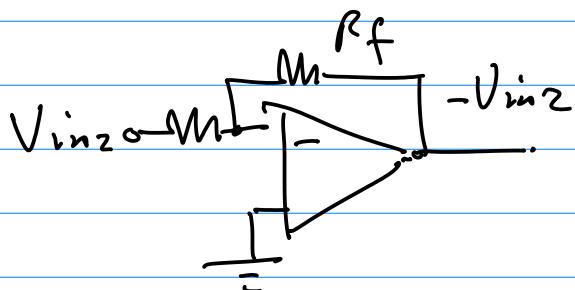
$$\text{from } A = 1 + \frac{R_f}{R_1} \quad \frac{V_{out}}{V_{in}} = A = -\frac{R_f}{R_1}$$

$$\text{Or, } A = \frac{V_{out}}{V_{in}}, \quad V_{out} = A \cdot V_{in}$$

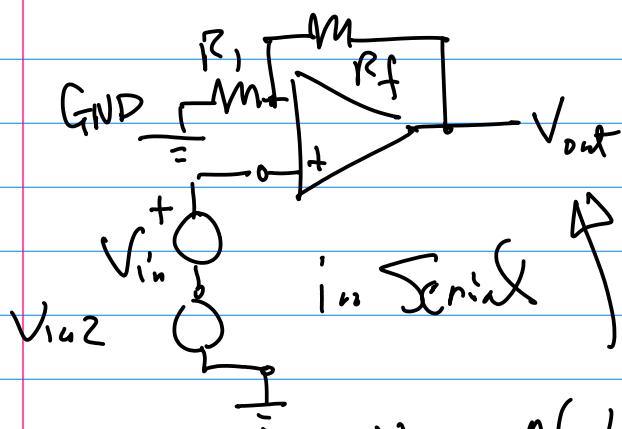
... (3)

$$\frac{V_{out}}{V_{in}} = A = -\frac{R_f}{R_1}$$

$V_{in1} + V_{in2}$ Addition



Let input circuit as follows



$$V_{out} = A(V_{in1} + V_{in2})$$

make $V_{out} = -V_{in2}$, Let
 $R_f = R_1 = 1 k\Omega$
 (741 OpAmp, 384 Quad-Pak)

Note: Not too Big Current,
 Power Consumption is going to be a problem.

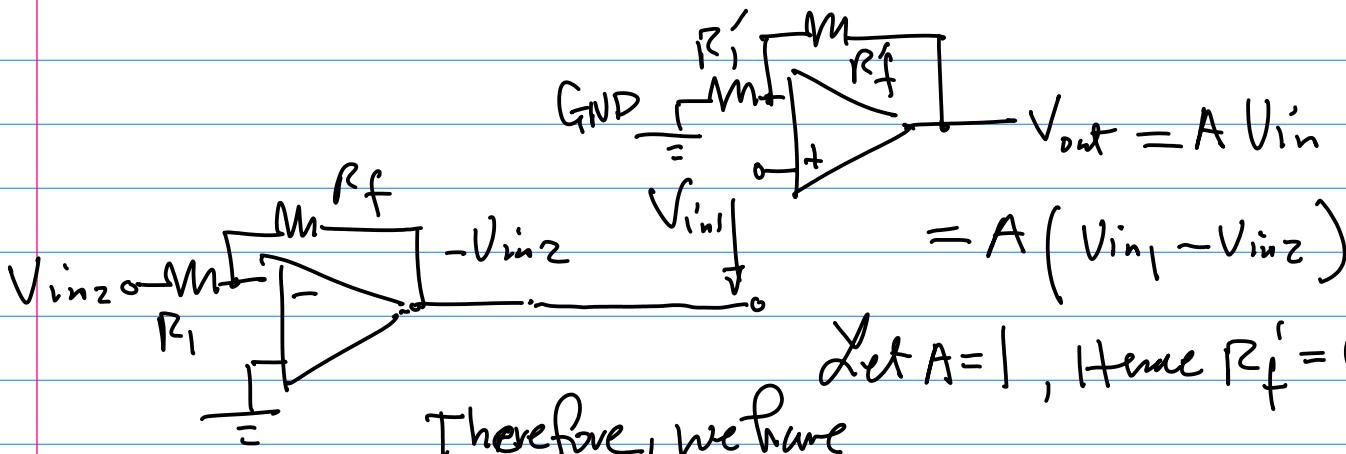
Let $A = 1, \rightarrow R_f = 0 \Omega$

Not too small, Noise will distort the Signal

Subtraction $X_1 - X_2 (V_{in1} - V_{in2})$

Then, Combine it with Add CKT

So, we have $V_{in_1}, -V_{in_2}$



$$\text{Let } A=1, \text{ Hence } R_f' = 0\Omega$$

Therefore, we have

Subtraction

$$V_{out} = V_{in_1} - V_{in_2}$$

$\underline{\text{Multiplication}}: y = ax$

use Non-Inverting Configuration $a > 1$

Configuration

$$\frac{V_{out}}{V_{in}} = A \quad A = 1 + \frac{R_f}{R_1}$$

$$V_{out} = A \cdot V_{in} = \left(1 + \frac{R_f}{R_1}\right) V_{in}$$

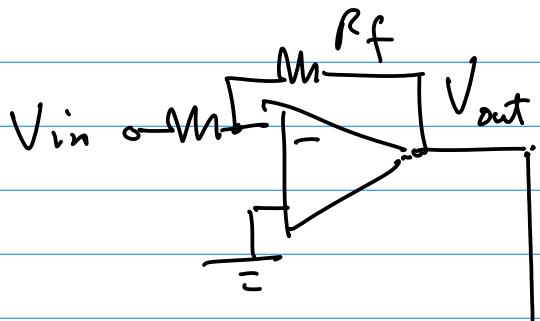
Note: For Linear System, the multiplication is done by multiplying a gain, But Not Another X (or V_{in}).

$\underline{\Delta \text{ Division}}$ (is a multi-
plication!)

for the multiplier A less than 1.

Two Stage Inverting Configuration, 1st Stage does the division But with negative Sign, 2nd Stage with gain = 1, But change to positive by 2nd negative.

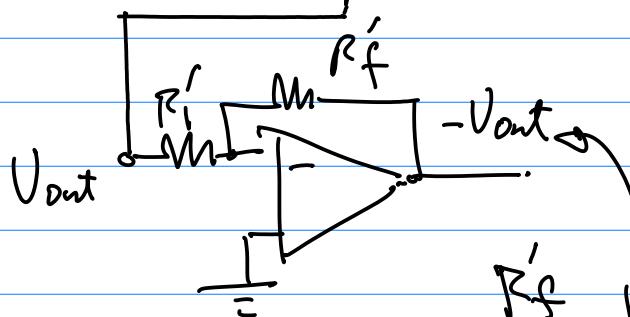
Example: 2 stage Inverting



$V_{out} = -\frac{R_f}{R_1} V_{in}$, where $\frac{R_f}{R_1}$ is a fractional number for division, for example

$$0.32, \quad \frac{R_f}{R_1} = 0.32 \text{ if } R_1 = 1K$$

$$R_f = 320\Omega$$



to make gain = -1

$$\frac{R_f'}{R_1'} = 1, \text{ Let } R_1' = 1K\Omega$$

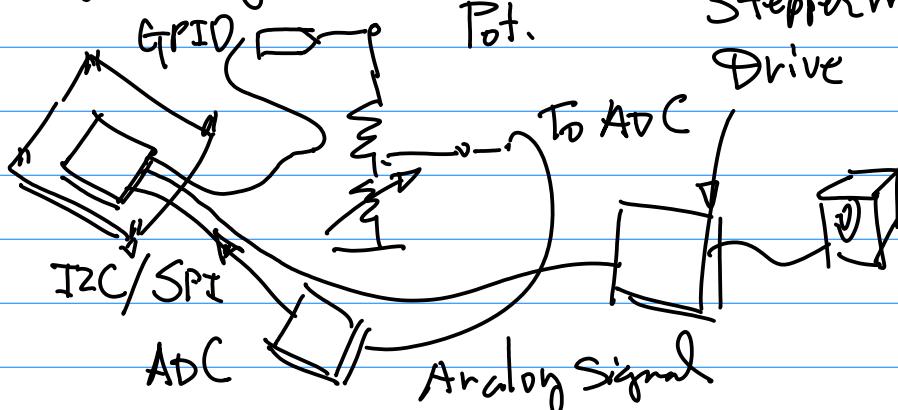
then solve for

$$R_f' = R_1' = 1K\Omega.$$

Note: Analog Sensors
have to meet the Output
Current Requirement, e.g.
 4 mA or 20 mA

April 19 (mon).

1) Preparation for the Coming
Project. Fig 2. PP. 37.

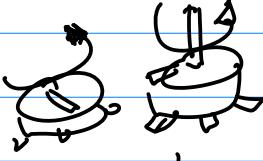


Project Due May 9 (Sunday)
11:59 P.M.

April 19 (mon)

Design On Preprocessing CKT.

Example: See Fig. on github

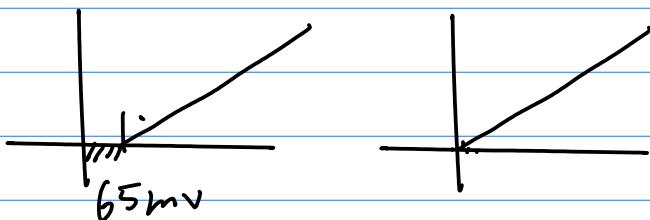


Characteristic Curve.
Shifting \rightarrow Enlargement
of the Dynamic Range.

a Shifting, add/sub.

b Dynamic Gain
Range mult.

Shifting (Subtraction).



Inverting Configuration

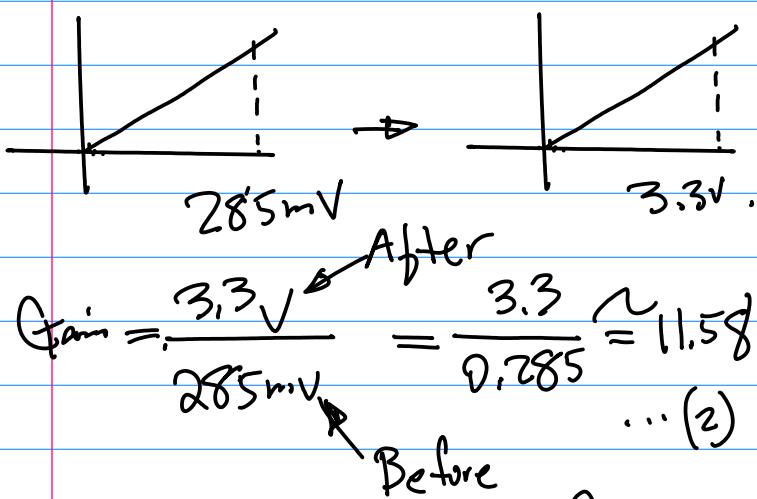
$$\frac{V_D}{V_{in}} = -\frac{R_f}{R_1}$$

Output from the Reference
(External Voltage Source)

Desired Shifting

$$\frac{65 \times 10^{-3}}{5.0} = \frac{R_f}{R_1}, \text{ Let } R_f = 1 \text{ k}\Omega \quad \dots (1)$$

Now, Dynamic Range Design



Since Gain is positive, we have
Non-inverting configuration

$$A = \left(1 + \frac{R_f}{R_1}\right), \text{ Substitute (2) into this Eqn.}$$

$$11.58 = 1 + \frac{R_f}{R_1}$$

Let $R_1 = 1 \text{ k}\Omega$, find / solve
for R_f . Hence $R_f = 10.58 \text{ k}\Omega$

Then, Integrate OPAMPS Together
to form Preprocessing CLK.

Note:

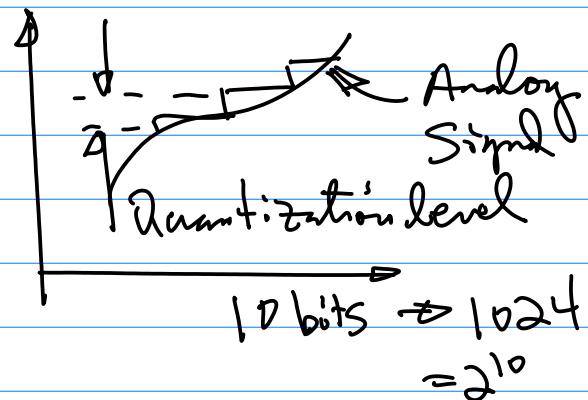
1° Analyze Sensor Characteristic
Curve, Define Arithmetic/
Math Operations, for
Shifting and Magnification
(of the gain, to cover 3.3V
Dynamic Range)

2° To be Able to use
Inverting And/Or Non-inverting
Configuration to Realize the
design Requirements, e.g.
Shifting, and magnification

Now, Let's Consider the ADC
Design. Scope:

System Level for ADC
Data Validation

DFT (Discrete Fourier Transform) \rightarrow Power Spectrum
 ↓
 Nyquist Sampling Theorem.

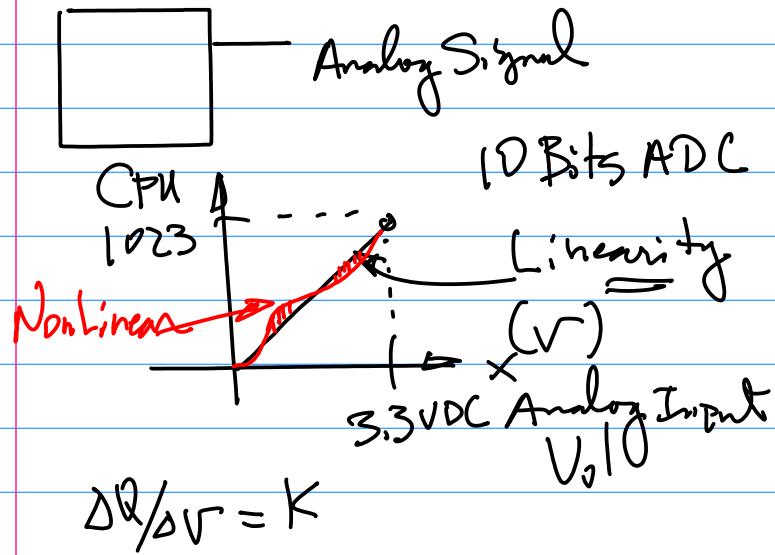


April 21 (Wed)

Data Validation: To make sure
 ADC Digitized (\rightarrow CPU)

Δt : Sampling interval

(Q)



$$\frac{dQ}{dV} = K$$

→ Nyquist Theorem

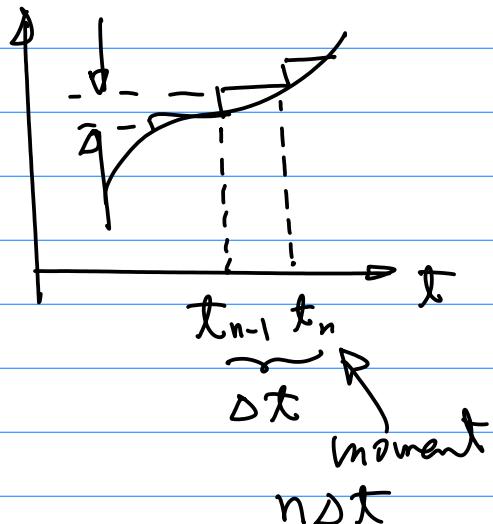
$$f_{\text{Sampling}} \geq 2 f_{\max} \dots (1)$$

Sampling frequency has to be greater than or equal to twice the highest frequency of the signal itself.

Note:

- Denote Digital Signal as $x(n)$
 or $x(h)$

Digitized, limited
 Quantization level
 Index: integer $n \neq$ Fourier Transform



2. Introduce Discrete Fourier Transform

Basic
 Background: Building
 Blocks to
 Formulation. Characterize
 or to Build
 a given
 Signal.

$$C_n = \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f_c \tau} d\tau \quad \dots (1)$$

$$f(x) = f(x_0) + \frac{f'(x)}{1!}(x-x_0) + \frac{f''(x)}{2!}(x-x_0)^2 + \dots + R_n(x) \dots (2)$$

Basic Building Blocks

$$f(x) \approx \sum_{n=-\infty}^{+\infty} C_n (\cos 2\pi n f t + \phi) \quad \dots (3)$$

3. Notation

$\mathcal{X}(m)$ Discrete Fourier Transform
of a Signal $x(n)$

m : Index i.e Frequency Domain

Hence, D.F.T. (Discrete Fourier Transform)

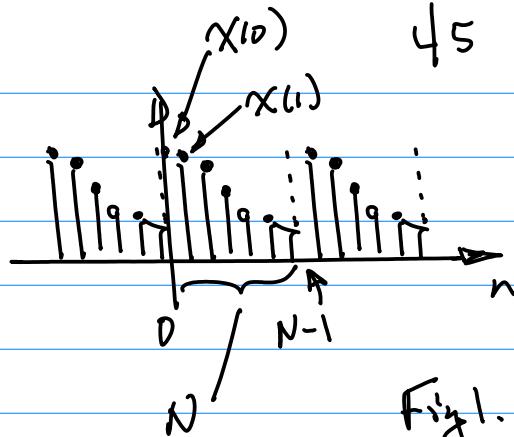
$$\mathcal{X}(m) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{mn}{N}} \quad \dots (4)$$

Physical meaning:

$x(n) = x(n+KN)$ Periodic function.

Period: N , where $K=0, 1, 2, \dots$
(Natural Number)

$\frac{1}{N}$ Scaling factor, N No. of Total
pts for One Period.



$n=0, n=1, n=2, \dots$

$x(0), x(1), x(2), \dots$

$$e^{-j2\pi \frac{mn}{N}}$$

for Imaginary Axis

m : Frequency Index

$$\mathcal{X}(m) \Big|_{m=0} = \mathcal{X}(0)$$

D.C.
Component

$$\mathcal{X}(1), \mathcal{X}(2), \dots, \mathcal{X}(N-1)$$

Higher Freq.

Cmp.

$$\mathcal{X}(m) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{mn}{N}}$$

$$= x(0)e^{-j2\pi \frac{m \cdot 0}{N}} + x(1)e^{-j2\pi \frac{m \cdot 1}{N}} +$$

$$\dots + x(N-1)e^{-j2\pi \frac{m(N-1)}{N}}$$

from Right Hand Side of

Eqs.(4).

(For Simplicity, $\frac{1}{N}$ Removed)

ComptEE 242

for $m=0$, $\frac{-j2\pi 0 \cdot 0}{N} + \frac{-j2\pi \frac{0 \cdot 1}{N}}{N}$

$$X(0) = X(0)e^{-j2\pi \frac{0 \cdot 0}{N}} + X(1)e^{-j2\pi \frac{0 \cdot 1}{N}} + \dots + X(N-1)e^{-j2\pi \frac{0 \cdot (N-1)}{N}}$$

D.C.

$$= X(0) \cdot 1 + X(1) \cdot 1 + \dots + X(N-1) \cdot 1$$

$$= \underbrace{X(0) + X(1) + \dots + X(N-1)}$$

Divided by $\frac{1}{N}$, Average \rightarrow D.C.

for $m=1$.

$$X(1) = X(0)e^{-j2\pi \frac{1 \cdot 0}{N}} + X(1)e^{-j2\pi \frac{1 \cdot 1}{N}} + \dots + X(N-1)e^{-j2\pi \frac{1 \cdot (N-1)}{N}}$$

:

for $m=2, 3, \dots$

$$X(N-1) = X(0)e^{-j2\pi \frac{(N-1) \cdot 0}{N}} + X(1)e^{-j2\pi \frac{(N-1) \cdot 1}{N}} + \dots + X(N-1)e^{-j2\pi \frac{(N-1) \cdot (N-1)}{N}}$$

D.F.T.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix}$$

$$= \frac{1}{N}$$

$$\begin{bmatrix} (X(0)) \\ X(1) \\ X(2) \\ \vdots \\ (X(N-1)) \end{bmatrix}$$

$$\cdots (5)$$

April 26 (Mon) 4/6
 Note:
 - May 21 (Fri)
 12:15 - 14:30

Example: Eqn(s)

Left Hand: $X(m)$
 D.F.T. $\xrightarrow{\quad m \quad}$ Freq. Index

$$m=0, 1, 2, \dots, N-1$$

N pts. One Period

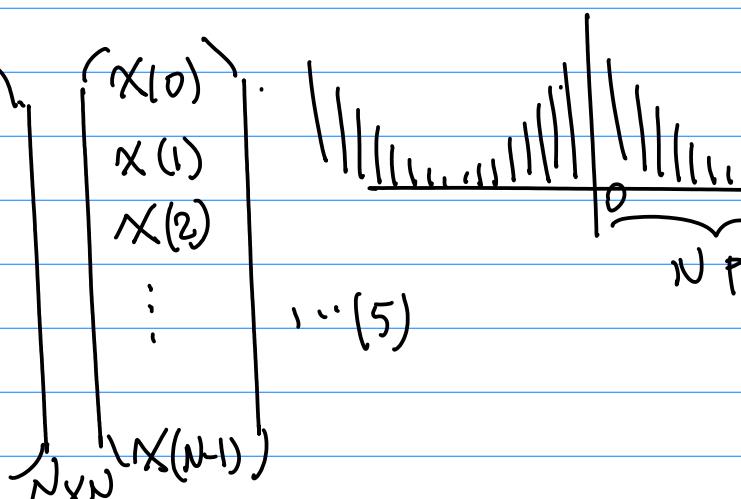
$$X(m) = X(m + KN) \dots (1)$$

Periodic Function,

Period = N magnitude



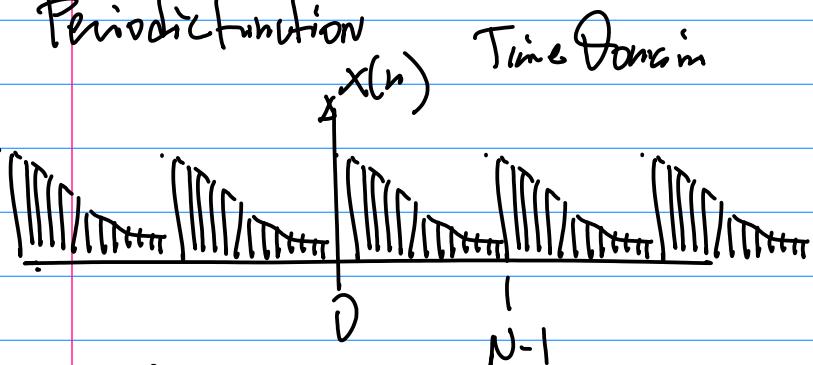
Fig 1



2. Right Hand Input
Digital Signal (Digit)
from ADC) $x(n)$
Signal Time Index

$$x(n) = x(n + pN) \dots (2)$$

Periodic Function Time Domain



$$\begin{aligned} x(n) &= x(n + pN) \\ \mathcal{X}(n) &= \mathcal{X}(n + kN) \end{aligned}$$

3. Denote Eqn (5) as follows.

$$\begin{bmatrix} \mathcal{X}(0) \\ \mathcal{X}(1) \\ \mathcal{X}(2) \\ \vdots \\ \mathcal{X}(N-1) \end{bmatrix} = \frac{1}{N} E_{N \times N} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}$$

Note: (Optional) The process

of Digitizing a function in One

domain, either in Time Domain,

or in Frequency Domain, will

lead its counterpart of the

function become periodic function,

$$x(t) \xrightarrow{\text{F.T.}} \mathcal{X}(f)$$

in time-Domain

$$x(n)$$

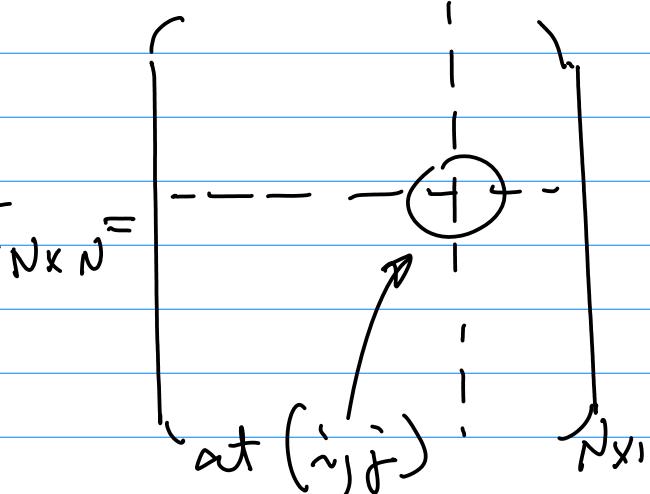
Digitized in
Time

in frequency
Domain

$$\mathcal{X}(n)$$

Digitized (Conceptually)

Freq.



$$e^{-j \frac{2\pi}{N} i \cdot j} \dots (3)$$

Example: Find Entry of the
 $E_{N \times N}$ matrix at
(2, 3) location, Suppose
 $N = 4$.

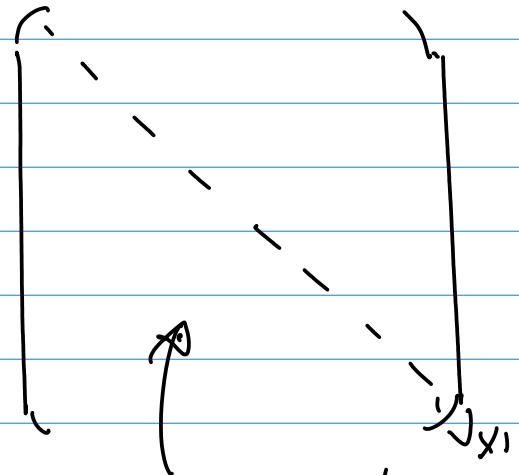
From Eqn(3)

$$e^{-j2\pi \frac{m \cdot n}{N}} \Big|_{\begin{array}{l} i=2 \\ j=3 \end{array}} = ?$$

Final Equation

$$e^{-j2\pi \frac{m \cdot n}{N}} = \cos\left(2\pi \frac{m \cdot n}{N}\right) - j \sin\left(2\pi \frac{m \cdot n}{N}\right)$$

... (4)

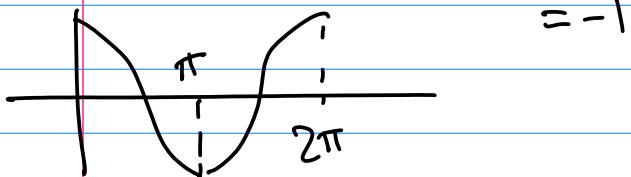
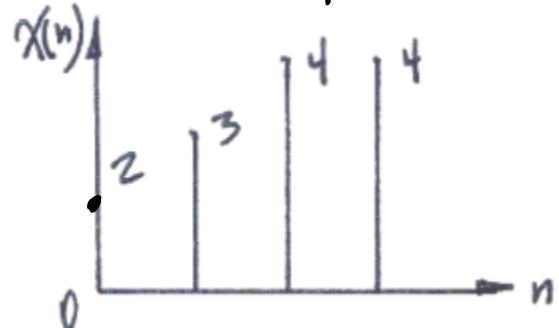
Symmetric Along
main Diagonal.

Based Eqn(4), q given Condition

$$i=2, j=3,$$

$$e^{-j2\pi \frac{2 \cdot 3}{4}} = \cos\left(2\pi \frac{2 \cdot 3}{4}\right) - j \sin\left(2\pi \frac{2 \cdot 3}{4}\right)$$

$$= \cos(3\pi) - j \sin(3\pi) = -1 - j \cdot 0$$

Example: Given $x(n)$ below,
with $N=4$ 

Find/Compute its D.F.T

 $\underline{\underline{SD}}$ From Eqn(5),

Note, This way, we can

Evaluate Each of Every Entry
of the Matrix.4. Entry for $E_{N \times N}$ is

$$e^{-j2\pi \frac{m \cdot n}{N}} = e^{-j2\pi \frac{n \cdot m}{N}}$$

$$\begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{pmatrix} = \frac{1}{N} E_{N \times N} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{pmatrix}$$

Ques/Ex2/2

4a

$$N=4; X(0)=2, X(1)=3,$$

$$X(2)=4, X(3)=1;$$

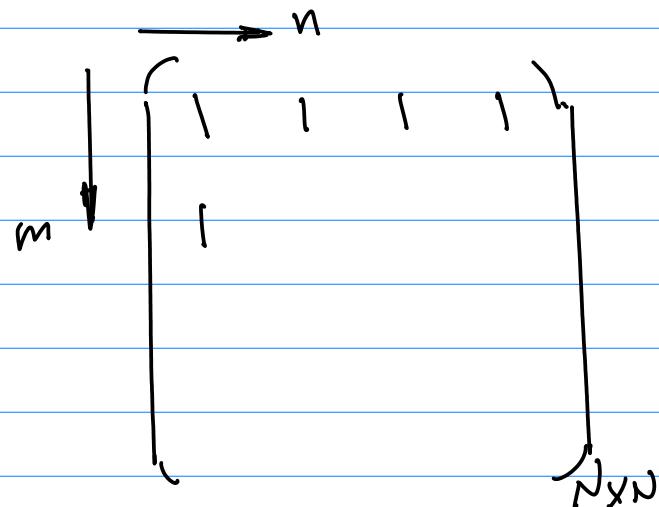
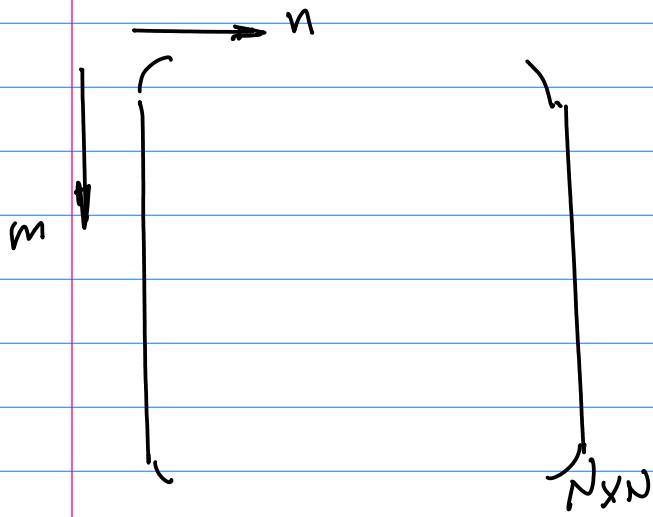
for 1st Row, 1st column Location

$$m=0, n=0.$$

Then use Eqn(4*) we evaluate from Eqn(4**)

Each Entry.

$$e^{-j2\pi \frac{mn}{N}} = e^{-j2\pi \frac{0 \cdot 0}{4}} = e^{-j0} = 1$$



Visualize the following property as if you are standing at 1st Row & 1st Column position,

As you move top down, m index is increasing by 1 per row while n-index is unchanged

and, As you move from left

to Right, n index is increasing by 1 per column

while your m-Index is a constant.

For 1st Row, 2nd Col.

$$e^{-j2\pi \frac{m \cdot n}{N}} = e^{-j2\pi \frac{0 \cdot 1}{4}} = 1$$

For 1st Row, 3rd Col.

$$e^{-j2\pi \frac{m \cdot n}{N}} = e^{-j2\pi \frac{0 \cdot 2}{4}} = 1$$

Now, for the 2nd Row, m=1

2nd Row, 1st col. n=0

$$e^{-j2\pi \frac{m \cdot n}{N}} = e^{-j2\pi \frac{1 \cdot 0}{4}} = 1$$

2nd Row, 2nd col, n=1

$$e^{-j2\pi \frac{m \cdot n}{N}} = e^{-j2\pi \frac{1 \cdot 1}{4}} = e^{-j\pi/2} = -j$$

$$\begin{aligned} \text{For } i=1, j=2 \rightarrow \text{Symmetric Entry.} \\ \text{For } i=2, j=1 \\ e^{-j2\pi \frac{i+j}{N}} &= e^{j2\pi \frac{1+2}{N}} \\ &= e^{-j2\pi \frac{2+1}{N}} \quad (\text{see github Handout}) \end{aligned}$$

Now, Using Symmetric Property, Together with Euler Equation (4*)
PP. 48!
we can build $E_{N \times N}$, for
 $N=4$.

Note: $N=2^x \dots (1)$
for F.F.T (Fast Fourier Transform), the Above Condition has to hold good.

If Data Points from your ADC is not satisfied by Eqn(1), Then, pad "0"s to make it good.

Example (Continued from PP. 49)

Find D.F.T. (See Handout Online)

Now, Define Power Spectrum.

$$P(m) \stackrel{def}{=} \sqrt{\text{Re}[\mathcal{X}(m)]^2 + \text{Im}^2[\mathcal{X}(m)]}$$

↑ ↑ ... (2)

Index Real Part of Imaginary
freq. the D.F.T. Part of the
 Q.F.T.

Physical meaning: Energy Distribution of a given Signal in Frequency Domain

Note: 1. D.C. Comp. $\mathcal{X}(0)$, $m=0$

$$\text{Re}[\mathcal{X}(1)] = -\frac{1}{4}, \text{Im}[\mathcal{X}(1)] = \frac{1}{4}$$

$$\mathcal{X}(m) = \text{Re}[\mathcal{X}(m)] + j \text{Im}[\mathcal{X}(m)]$$

↑
D.F.T.

from the Handout

$$\text{Re}[\mathcal{X}(3)] = -\frac{1}{4}; \text{Im}[\mathcal{X}(3)] = -\frac{1}{4}$$

2. Computation of $P(m)$ from $\mathcal{X}(m)$

$$P(m) \Big|_{m=0} = \sqrt{\text{Re}[\mathcal{X}(0)]^2 + \text{Im}^2[\mathcal{X}(0)]}$$

$$= \sqrt{3.25^2 + 0^2} = 3.25$$

$$P(1) = \frac{1}{4} \sqrt{(-2)^2 + 1^2} = \frac{1}{4} \sqrt{5}$$

Note:

$$P(m) = P(m+KN) \dots (3)$$

Common Power Spectrum is
A Banded Band Signal Power
Spectrum.

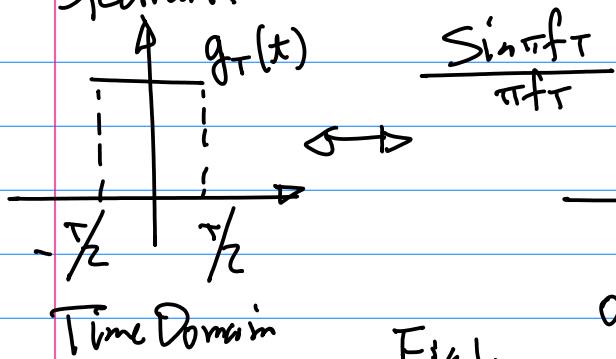
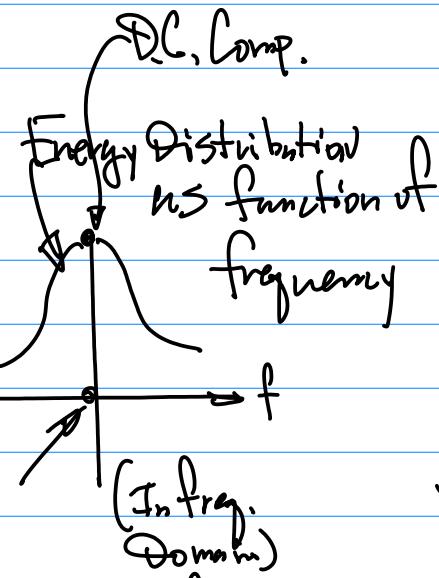


Fig 1.

Note:

$$P(m) = P(-m) \dots (4)$$

Even function.



Proof, Verification of Eqn (3)

$$P(m) = \sqrt{Re[\mathcal{X}(m)] + Im^2[\mathcal{X}(m)]}$$

$$\mathcal{X}(m) = \mathcal{X}(m+KN) \quad]$$

$$P(m+KN) = \sqrt{Re^2[\mathcal{X}(m+KN)] + Im^2[\mathcal{X}(m+KN)]}$$

$$Im^2[\mathcal{X}(m+KN)]$$

$$= \sqrt{Re^2[\mathcal{X}(m)] + Im^2[\mathcal{X}(m)]}$$

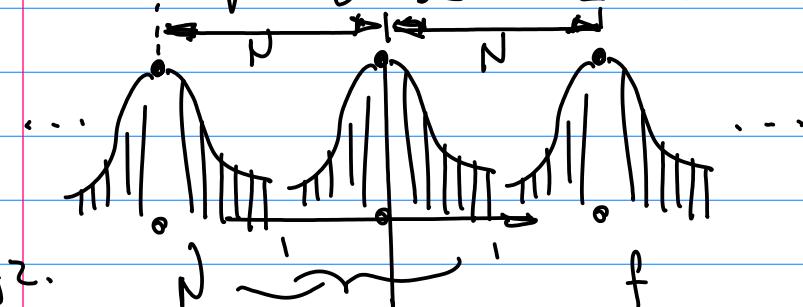


Fig 2.

Symmetric.
"Mirror," Equal
Even Function

