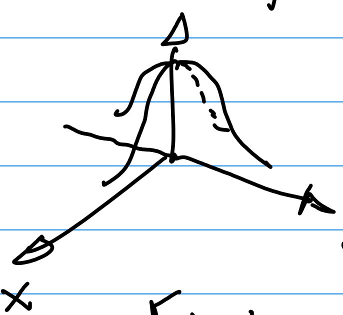
CmpE242

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \ldots (1)$$

March 10 (Wed)

$$G(x,y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \quad \ldots (2)$$

Note $\mu_x = \mu_y = 0$, $\sigma_x = \sigma_y = \sigma$



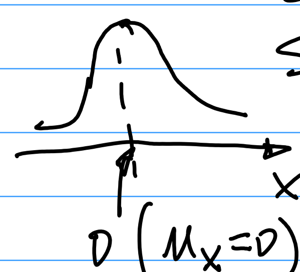


Fig 1.

From Eqn (4), ppt from the github 2018S-15-Lec6-V3 ...

Let $y=0$, to have one indep. variable x.

$$\nabla^2 G(x,y)\big|_{y=0} = \nabla^2 G(x,0) \quad \ldots (3)$$

$$= -\frac{1}{\sqrt{2\pi}\,\sigma^3} e^{-\frac{x^2}{2\sigma^2}} + \frac{x^2}{\sqrt{2\pi}\,\sigma^5} e^{-\frac{x^2}{2\sigma^2}}$$

LoG(x), or $\nabla^2 G(x,0)$

Example: (1) Use LoG(x) to Build a Convolutional Kernel (z) to Compute Derivatives of the Error

22

Note: LoG(x) is NOT Exactly the Computation for derivatives, But we use it, for its Low pass feature, and 2nd order derivative.

Sol.

(1) "Mapping" to a Kernel

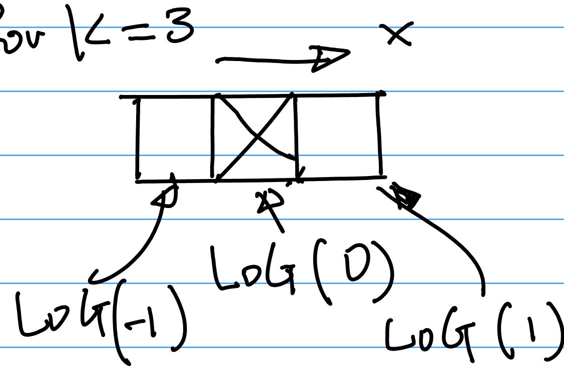
Build a kernel with "odd" Number of grids, elements

$K \times 1$

No. of elements

One Row

for $K=3$



a. Identify the Center Reference

b. from LoG(x) (or $\nabla^2 G(x,0)$), map it to the Kernel

Solve for y..

Driver Implementation. $\begin{cases} f_{PWM} \\ D.C. \end{cases}$

2018S-10-0 ~
PWM Driver

Add Duty Cycle Function to Device Driver.

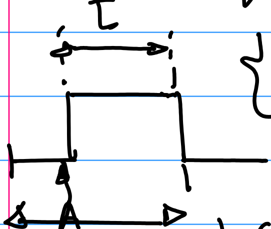
$\begin{cases} \text{Theoretical Aspect} \\ \text{Implementation C Code.} \end{cases}$

SPRs $\begin{cases} \text{TCON} \leftarrow \text{Control Reg.} \\ \text{TCNT} \leftarrow \text{Counter Reg.} \\ \text{TCMP} \leftarrow \text{Compare Reg.} \end{cases}$ Root

CFG0
CFG1

Prefix "Timer"

PWM Output Square Waveform



$\begin{cases} \text{TCNTB} \\ \text{TCmpB} \end{cases}$

Fig. 1

$T_{PWM} = 1/f_{PWM}$

$$f_{PWM} = \frac{CLK_P}{(\text{Prescaler}+1)(\text{divider})} \quad \ldots (3)$$

PP 1118, CPU Datasheet

Prescaler : 8 bit, [0, 255]

Divider; 1, 2, 4, 8, 16

Note CFG/CON are responsible for Setting Prescaler/Divider value.

$$f_{PWM} = \frac{50\times10^6}{(\text{Pres}+1)\cdot\text{Div.}} \quad \ldots (*)$$

If we need $f_{PWM} = 2\times10^3$

Find SPR, Set SPR. to Realize this frequency.

From CPU Datasheet CFG0 CON.

Sol

$$2\times10^3 = \frac{50\times10^6}{(\text{Pres}+1)\text{Div}}$$

$$(\text{Pres}+1)\text{Div} = \frac{50\times10^6}{2\times10^3}$$

$$(\text{Pres}+1)\text{Div.} = 25\times10^3$$

Let Div=16,
Solve for Pres.

$$\text{Pres}+1 = \frac{25\times10^3}{16}$$

$$\text{Pres} = \frac{25\times10^3 - 16}{16} \simeq > 255$$

Iteration,

Change PCLK to 10MHz, then, we have

CMPE/EE242

$$Pres = \frac{10\times10^6 - 16\times2\times10^3}{16\times2\times10^3}$$ if it is still too big

therefore, then Low the CLK_p

try $CLK_p \simeq 2\times10^6$. please verify it!

Note: SPR Responsible for f_{PWM}

T CNT B

Timer, Root, Buffer

$f_{PWM} = 1\times10^3$ given.

f ~~master~~ Clock peripherial

N Counts for CNT SPR.

$$f = f_{PWM} \cdot N \quad \ldots (4)$$

Given, Target, Unknown to be Calculated

Note: T Cmp B

Comparison Register for Duty Cycle

2nd Counts Value for "Cmp" Derived from Duty Cycle.

March 17 (Wed) f_{PWM} By Setting SPR's value; Duty Cycle

Arm11, ~~Datasheet~~

T CNT B ⟵ "N" Counts ⇕ f_{PWM}

T Cmp B

2018S-10-

$$f_{PCLK}/f_{PWM} = N \quad \ldots (1)$$

Define ~~the~~ one period; the CNT

Duty Cycle → % → Counts → Cmp

Percentage

GPFCON PP. 522

GPFCON[29:28] = 10 → PWM

GPFCON[31:30] = 10 → PWM

#define S3C64xx_GPFCON

0x7...

*.h

&= ~(0x3U<<28)

"AND" "Neg" "11" "00"

|= (0x2U<<28)

"OR" "10" unsigned

Set 2 Bits GPFCON[29:28] = 10

March 22nd (mon)
Review.

1° 3± Questions.

a. Basic Concepts CPU Architecture, Block Diagram.

Memory Map. Peripheral Controllers

GPP (GPIO) } SPRs CON
PWM DAT

PWM
TCNTB
TCMPB
CFG0, 1

Architecture → Mem → SPR
Code

{ User Space Programming
{ Kernel Space Device Driver

Kconf Script ←

Define Compilation + Build Process.

Programming Requirements, No Program Code Writing

However! Debug/Change the existing code is Needed;

b. Design Related Question(s)

SCH. Design, CKT for PWM

Pin(s), f_{PWM} — GPIO

motor Drive
Pin(s), Label(s) Stepper motor I/F

GPP I/O Testing ("Hello, the World")

{ Input Testing CKT
{ Output Testing CKT

Resistance Value Calculation

c. Theoretical Aspects

C1. PID Controller Design
Basic Concepts
Block Diagram.

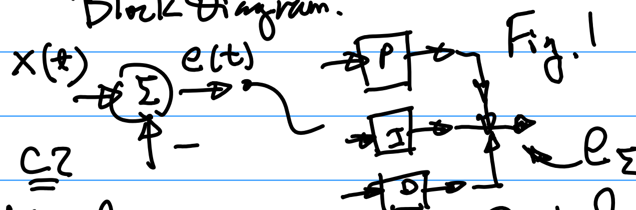


Fig. 1

C2
Kernels F.D. 2x1 → Central D.
B.D. 2x1 $\frac{1}{2}$(F.D+B.D)

With Noise Reduction 3x1
Low Pass Filter : G(x) Gaussian.
+ 2nd Order Derivation as in Computer Vision

∇^2 : Laplacian $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ → 1D $\frac{\partial^2}{\partial x}$

LoG(x)

Note: One page formula sheet is Allowed, However No verbal Description And/Or Examples Allowed.

Note: Calculator Is Allowed.

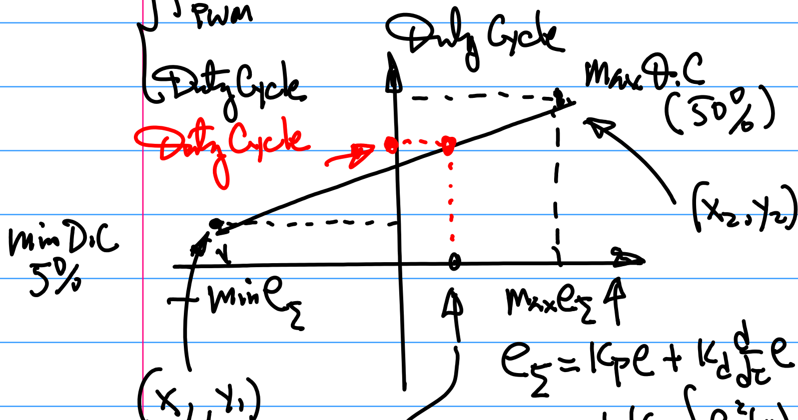
Close Book, Close Notes
Datasheets if needed will be provided;

Convolution with Kernel(s)
Table of E(t), find $\frac{d}{dt}E(t)$ Convolution

$\int K_I e^2(\tau)d\tau \simeq \sum_{i=0}^{I} K_I e^2(\tau)$

Mapping to Control function PWM

$\begin{cases} f_{PWM} \\ \text{Duty Cycle} \end{cases}$



$e_\Sigma = K_p e + K_d \frac{d}{dt} e + K_I \int_I e^2(\tau)d\tau$

$e_\Sigma = 0.57$

find Control Action, → D.C = ?

Implementation: Reference Platform ARM11

Architecture
CPU Block Diagram → memory map.

2^{32} (4 GB)

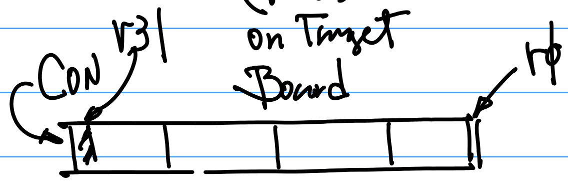
8 Equal Bank
↓ 3 Addr Bits
$a_{31} a_{30} a_{29}$

* SPRs. ← (Peripheral Controllers)

$\begin{cases} PWM \\ GPP \end{cases}$

GPx CON
GPx DAT
T CNT B
T Cmp B

Design Spec. → SPRs → CPU Datasheet
(Pins) on Target Board



To perform init & Config:
1° Binary Pattern for SPR.

Read/modify user Application Programs | Kernel Space Device Driver Program.

C Code for this Purpose.

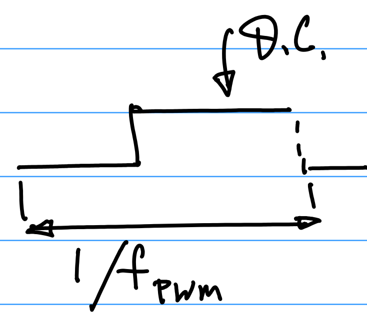
Note: PWM Waveform, e.g. Duty Cycle Calculation.

N. for CNT
N' for Cmp
T B

$f_{PCLK} / f_{PWM} = N$

$f_{PWM} = \frac{PCLK}{(Prest 1) * DVR}$

$\%(D.C) * N = N' \rightarrow Cmp$



Pre-requisit $\begin{cases} 1° \text{ O.S Source Distribution} \\ 2° \text{ Tool Chain Distro. "Cross Comp"} \\ 3° \text{ Datasheet.} \end{cases}$

↓

Tool Chain Installed Running

make menuconfig

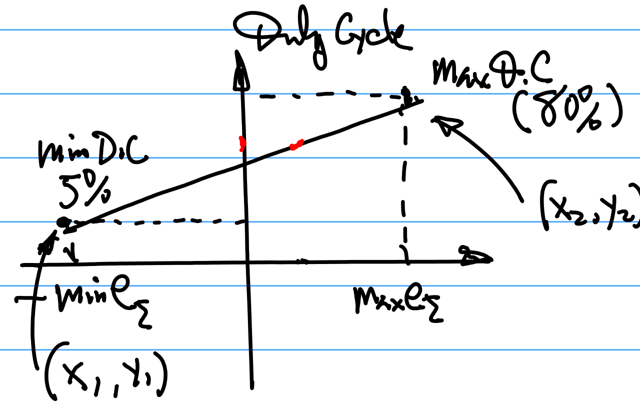
Continued → Kconf (at \drivers ~\char)

↓

Script. Add your Device Driver

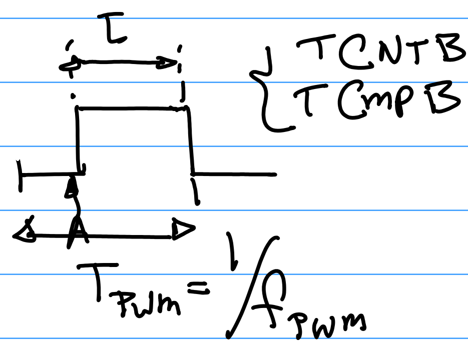
↓

make menuconfig



↓

invoke your change, Compile & Build (Module Only for Simplicity Purpose)



↓

Object "KO".

↓

Copy "usB" upload by "CP" Copy Command to your target

↓

"insmod" mytest.Ko (To make it as a part of Kernel Image)

↓

run your user application program (By calling the module)

https://github.com/hualili/CMPE242-Embedded-Systems-/blob/master/2018S-16-AngularSensing-i2c-L8

CMPE/EE242

April 5th (Monday)

1. Midterm Graded, the key was posted online, github, search under folder 2019S, "key"

2. 2nd half of this course. IIoT (Industrial IoT)

Sensors I/F { Digital Sensors — I2C I/F.
Analog Sensors — ADC

FFT to find/characterize Analog Sensor Data (Nyquist Theorem) Validation of Sensor Data. 32

OpAmps to Build Processing CKT. "SPICE" Simulation.

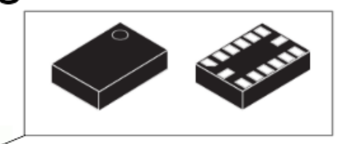
Example: LSM303

Note: Next Project use LSM303.

CMPE242-Embedded-Systems- / 2018S-16-AngularSensing-i2c-LSM303- final HL 2017-3-13.pdf

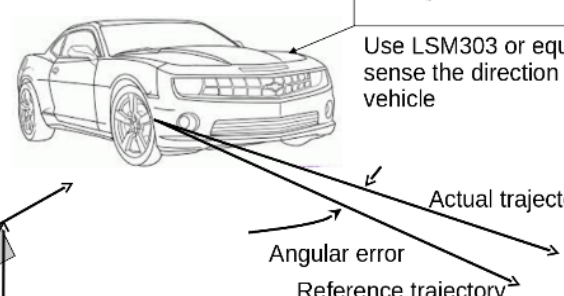
Sensors for Driving Direction and Turning Angle

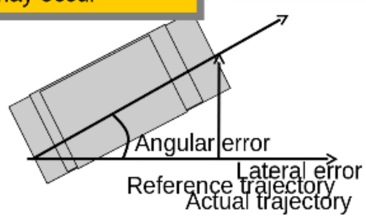
eCompass module:
3D accelerometer and 3D magnetometer



Use LSM303 or equivalent to sense the direction of the vehicle

Caution: Steering sensor input is not necessarily the real angle of the vehicle, "skipping" may occur





The LSM303DLHC includes an I 2 C serial bus interface that supports standard and fast mode 100 kHz and 400 kHz. The system can be configured to generate interrupt signals by inertial wake-up/free-fall events as well as by the position of the device itself.

Iarry Li, Ph.D. April 2015

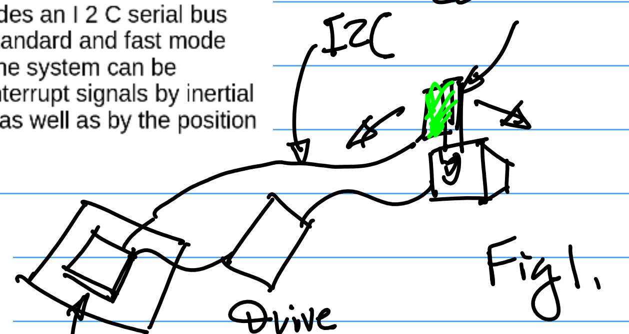
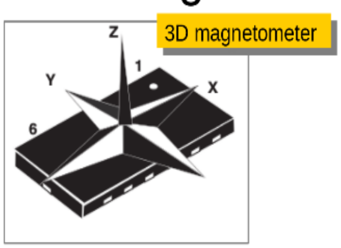


Fig 1. Configuration I for PID controller

3D Accelerometer and 3D Magnetomete LMS303



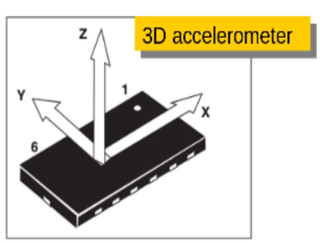


Table 9

Pin name	Pin description
SCL	I^2C serial clock (SCL)
SDA	I^2C serial data (SDA)

I2C Interface

(1) The transaction started through a START (ST) signal, defined as a high-to-low on the data line while the SCL line is held high.
(2) After ST, the next byte contains the slave address (the first 7 bit), bit 8 for if the master is receiving or transmitting data.

Homework: Implementation I2C LSM303 Sensor I/F. Due April 16 (Fri)

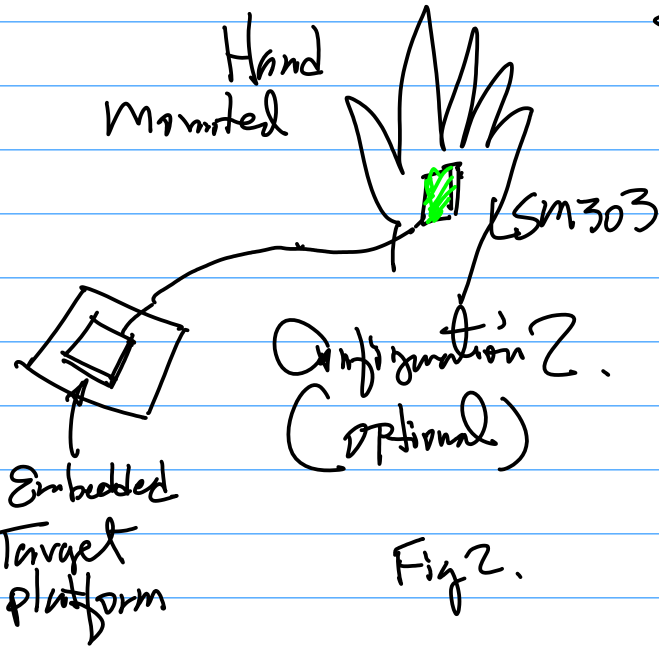


Fig 2.

3. a. Space-Time Diagram

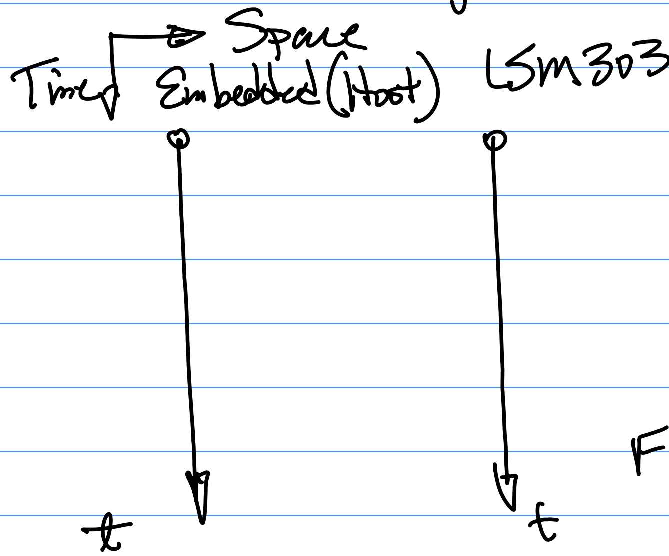


Fig 3

Submission On CANVAS

Objective ① To be Able to Read Sensor Input,

② To be Able Config the Sensor.

b. To Describe "Hand-Shaking"

Three Small Steps.

Step 1. → Step 2 → Step 3

Host Command. for the Target via Address for Init & Config

Slave "Ack"

Data Transmission will start

1. Note: LSM303 for ST-micro

Sensor Supports { Acceleration X-y-z axis, Magnetometer, Temperature

* Be sure read Datasheet to map the Steps of the I/F to Space-Time Diagram.

2. I2C

{ SDA (Serial Data) Bidirectional Data;
SCL (Serial Clock)

3. Datasheet Table 9 & 11. Pp. 20.

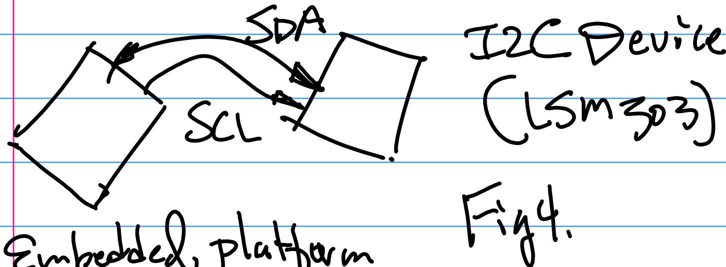


Fig 4.

Notation:

1° ST → SP

"Start" "Stop"

A Frame

2. The Notations in Table 11
SAD, SAD+W, SUB, DATA, SAK, etc. pp. 20

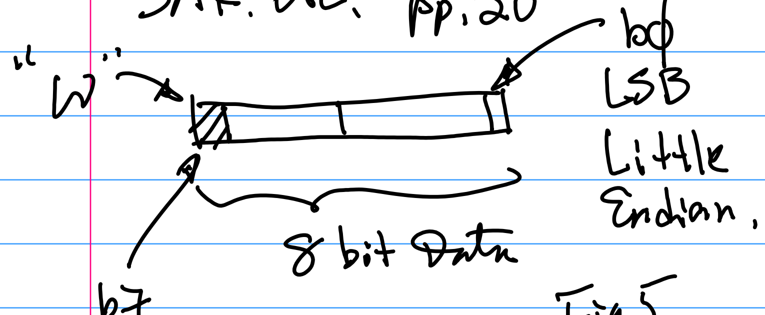
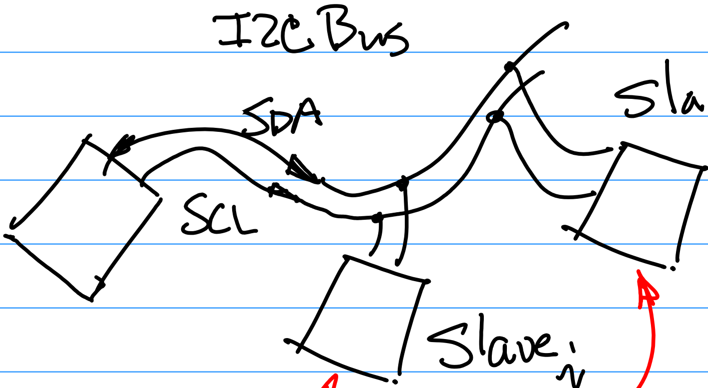


Fig 5

3. from pp. 20 (Datasheet)



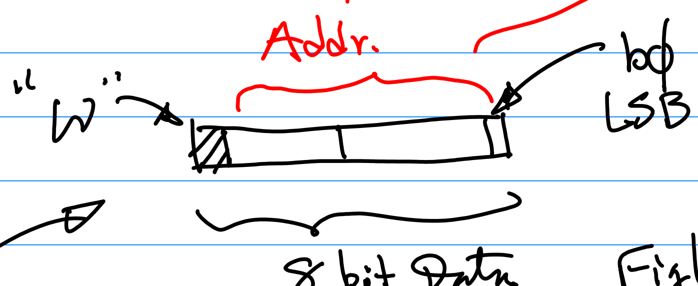
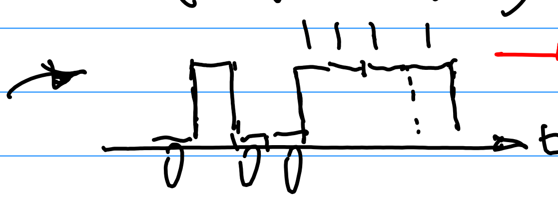


Fig 6

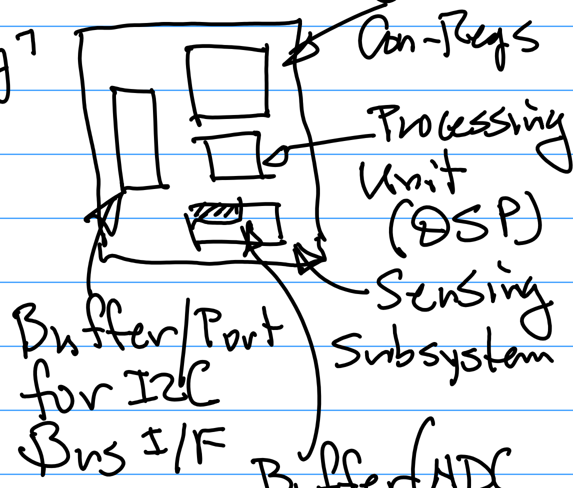
4. 1st Address (7 bits, lower), 2nd Address SUB.
All from the host (Target platform)

0xf2
1111 : 0010



Consider A slave device LSM303

Fig 7



2nd Address "SUB" is for Identifying the target inside the Slave Device.

5. 127 Devices Possible (Theoretically) on I2C Bus, In Reality this has to be Checked By "FAN-IN" "FAN-OUT"

128 Internal Addresses → Special Purpose Registers.

6. Most Significant Bit is transmitted first

Example: From Datasheet (LSM303)
PP.19 Table 11, 12, 13

Homework (1pt) Due A week from Today, April 14, Due On CANVAS

1° Build I2C Bus Interface with your target platform as a host, LSM303 Slave.
To be able:

a Hardware Implementation. (e.g. mount LSM303 on the Stepper motor, OR mount it to your hand)

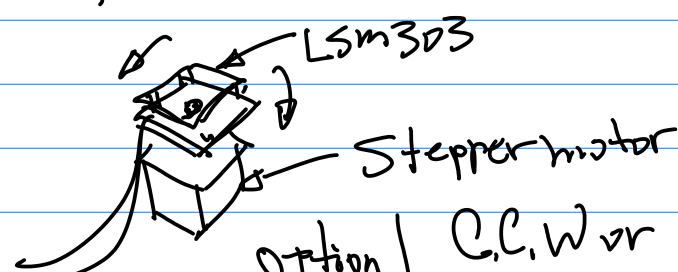
b Read Acceleration Data X, Y, Z, Display it on your terminal.

c Read Magnetometer Data and display it on the terminal

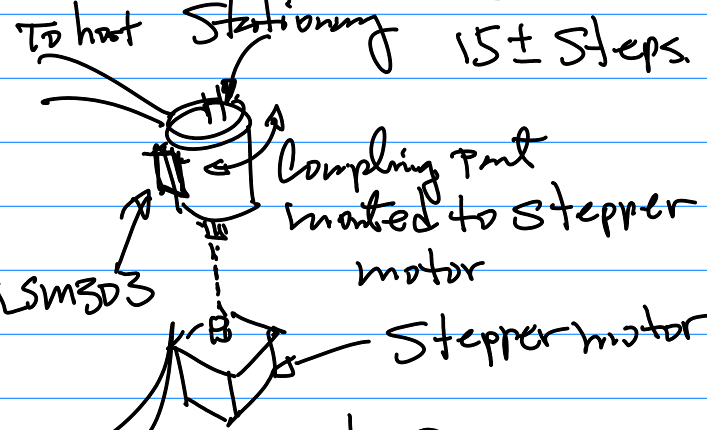
Note: Sample code is posted "as is" basis.
Repo: 20-2021S-10-lsm

2° Submission
a Source Code, b Readme.txt

c photo(s) of your Implementation
3x photos, 1 for the entire System (with Laptop); 2nd for the Host Side, Expansion Connector is the focus; 3rd for LSM303



Option 1. C.C.W or C.W. 15± Steps.



Option 2.

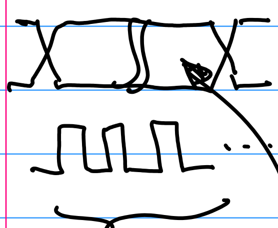
d 5 seconds Video Clip(s)
720P OR 1080P (1920x1280)
Compressed, mpeg4, .avi ?

file Naming: firstName_4Digits_242.zip

Note: Table 11 & 12 (PP19)

One Byte Writing → Multiple Bytes Writing
Host   Host

Table (Logical Behavior)
↓
Timing (Waveform)



0Xf2

8 bits

Now, the Address for the Sensor(s)
and Addresses for Registers
Control Register — Init & Config
Data Register

SAD[7:1] Address + SAD[0] for W/R
=1 for W
=0 for R

Note: Use info from Table 14
to fill in SAD+W, SAD+R
in tables 11~13.

Note: Section 5.1.3 Magnetometer

Example: Table 18. Control Register A
for Magnet   CTRL_REG1_A [7:0] 8 bits.

CTRL_REG1_A[7:4] = 0X7
(for 400Hz)

TechSpec → Binary Pattern

TechSpec:
i. 400Hz Data Rate
ii. X-Y axis.
iii. Sensor Active (No LowPower)

CTRL_REG1_A [3] = 0
CTRL_REG1_A [2] = 0
CTRL_REG1_A [1] = 1
CTRL_REG1_A [0] = 1

→ 0X3

Hence,

CTRL_REG1_A [7:0] =
0X73 ✓

Section 7.1.8 Status Registers

Section 7.1.9 ~ 7.1.11
Data Registers.
2's Complement form
↓
1's Complement By Negation
"0" → "1"
"1" → "0"
+1

April 12 (Monday)

Homework Extension to April 19 (Monday)

Industrial Analog Sensor I/F Design

Example: NH_3 Analog Sensor (Ion Selective Electrode) Interface.

1° ADC (Analog to Digital Conversion Unit)

LPC1769 6x ADC

ARM11 ADC

Pie Stand Alone ADC

Jetson NANO/Tx2

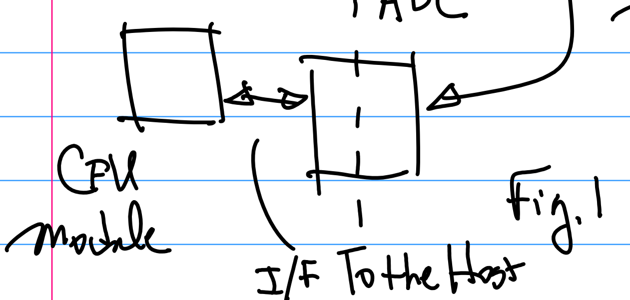


Fig. 1

1° Sampling Rate
500 KSPS

2° Quantization
8 bits, 10 bits, 12 bits.

3° SPI/I2C
Faster ≤100MHz
≤10MHz

Stepper motor Drive

2. Prototype to Build

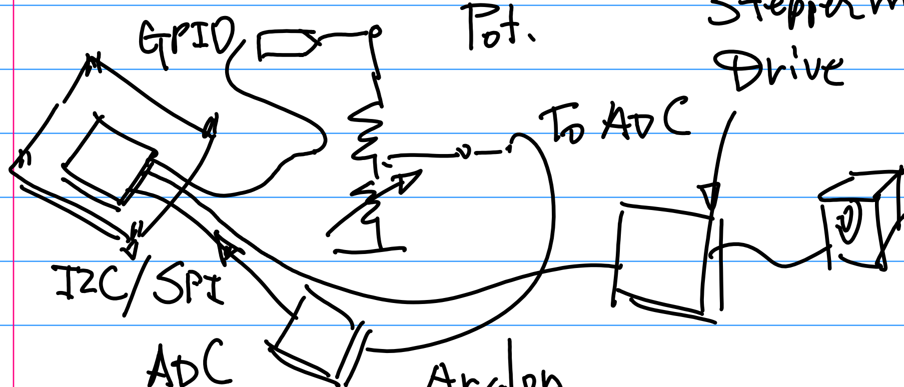


Fig 2.

3. Analog Interface Design.
Start with Characteristic Curve
↓
Linearization
↓
OpAmp Preprocessing CKT
↓
Optimized dynamic (Input) Range to ADC

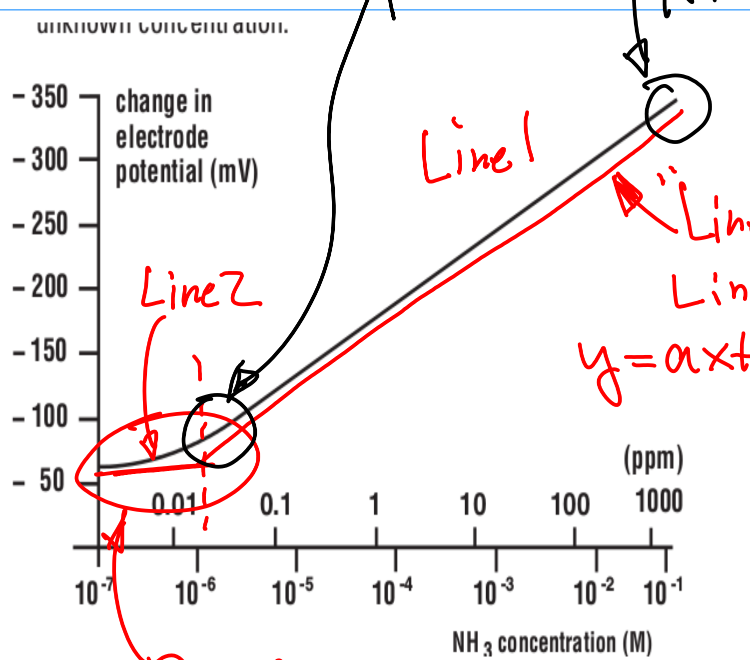
4. OpAmp's for Preprocessing
Addition
Subtraction
Division
multiplication
Math. Operations for Preprocessing

Example: from Datasheet 2018S-22-NH3-
TP13, Fig. 14.

Step 1. Linearization
Characteristic Curve

Starting pt.

Ending pt.



Non-Linear. $y = ax^2 + bx + C$ 2nd order

Step 2. Map the Dynamic Range of Sensor Output to the Dynamic Range of ADC Input.

2.1 "Shifting" OR Offset to move the Characteristic Curve to the origin.

Linearization: Line Equation(s) to Replace Non-Linear Curve.

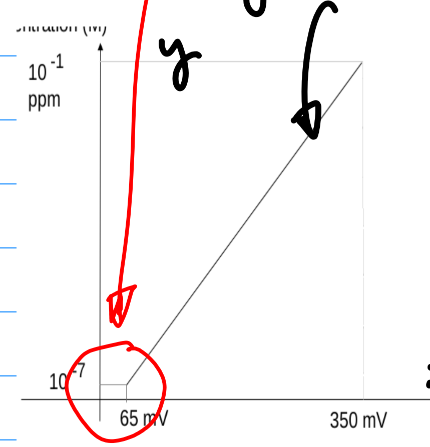
Simplification: Just Keep Line 1.

Note: Linearization By Identifying Starting and Ending Points.

(V_0, C_0) (V_N, C_N)

Voltage v.s. Concentration

$y = ax + b$



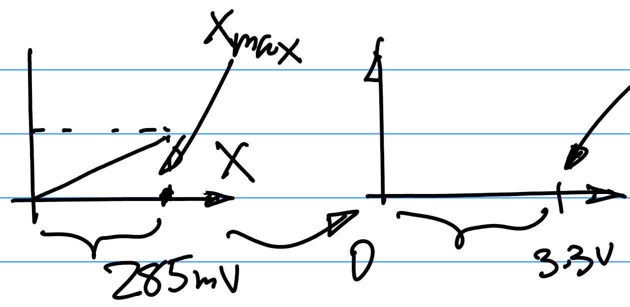
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \quad \dots (1)$$

$(x_1, y_1) \to (V_0, C_0)$, $(x_2, y_2) \to (V_N, C_N)$

x_{New} is changed By "Shifting"

$ax \to a(x - \Delta x)$

2.2 Dynamic Range fits to ADC Dynamic Range



$$\frac{X'_{max}}{X_{max}} = \frac{3.3V}{0.285V} \overset{Let}{=} A \quad \cdots (2)$$

Gain

("Scaling Factor")

$$X \cdot A = X' \quad \cdots (3)$$

Review OpAmp { Inverting Configuration OpAmps
Non-Inverting Configuration

April 14 (Th).

OpAmps for Pre-processing Design

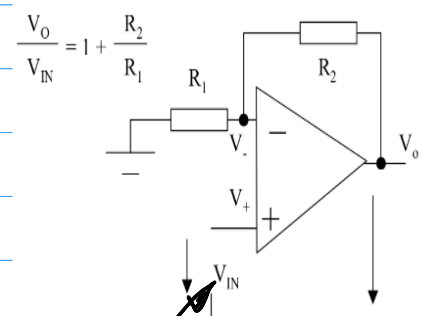
1. OpAmp { a Very High Input Impedence
b Very Big Open-Loop Gain
c Very Small output Resistance

2. Using OpAmps As Basic Building Blocks (B^3) for

Arithmetic Computation { Add/Sub
Multiplication/Division

Integral/Derivatives

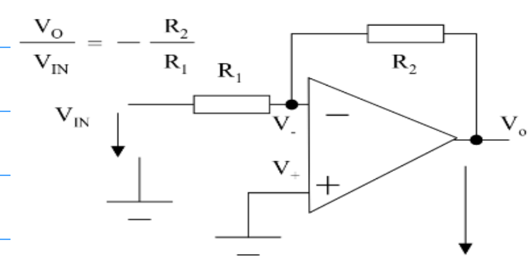
3. Configurations { Non-Inverting ~
Inverting ~



NON-INVERTING

a Input: Positive Polarity;
b feedback CKT $V_{out} \rightarrow$ Input (V_- pin) Via R_f
c Draw the CKT.

$$A = V_{out}/V_{in} = 1 + R_f/R_1 \quad \cdots (1)$$



INVERTING

a Input: V_- pin
b feedback CKT $V_o \rightarrow V_-$ (in) Via R_f

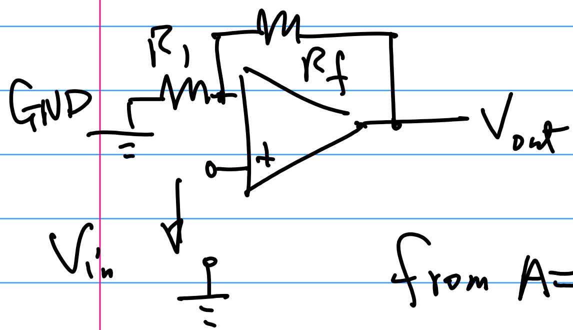
c $A = -\frac{R_f}{R_1} \quad \cdots (2)$

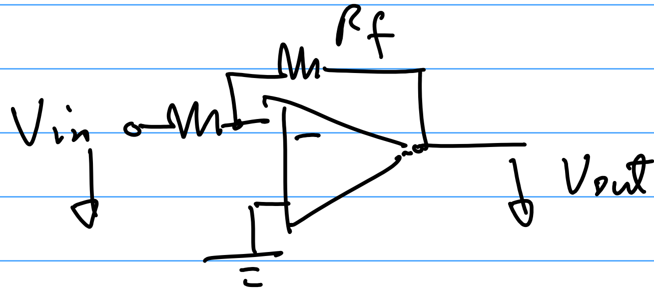
4. Use OpAmp CKT(s) for Math. Operations.

More than One Approach is possible, But Let's use Inverting Configuration

a Addition. $X_1 + X_2$

Non-Inverting Configuration

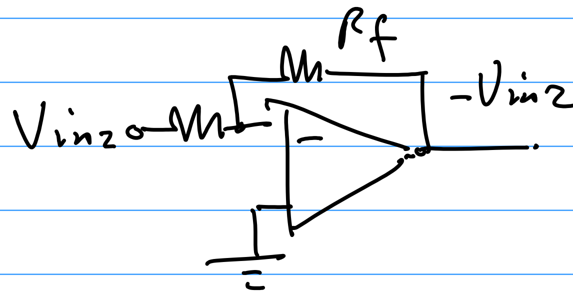




from $A = 1 + \frac{R_f}{R_1}$

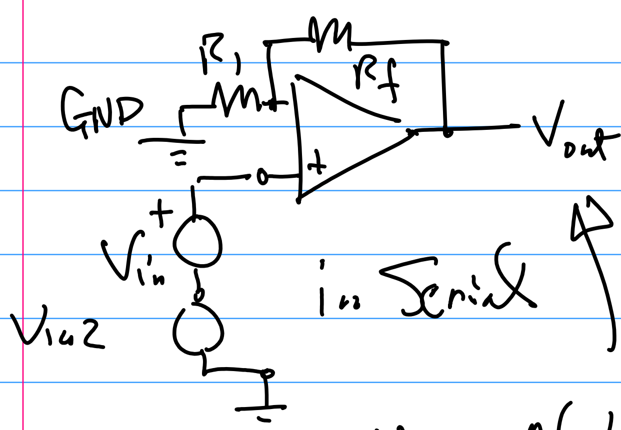
$\frac{V_{out}}{V_{in}} = A = -\frac{R_f}{R_1}$

OR, $A = V_{out}/V_{in}$, $V_{out} = A \cdot V_{in}$ ...(3)



$V_{in1} + V_{in2}$ Addition

Let input Circuit as follows



make $V_{out} = -V_{in2}$, Let $R_f = R_1 = 1K\Omega$

(741 opAmp, 384 Quad-Pack)

Note: Not too Big Current, PWR Consumption is going to be a problem.

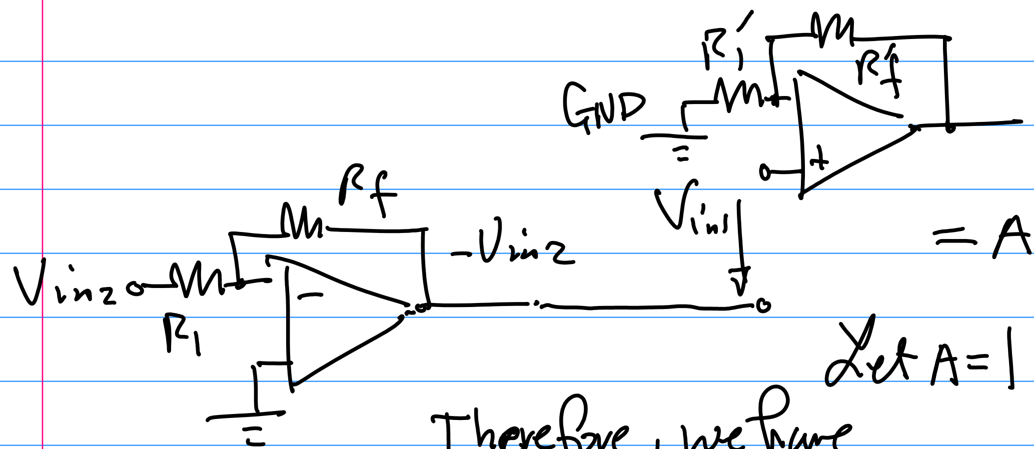
Not too small, Noise will distore the Signal

$V_{out} = A(V_{in1} + V_{in2})$

Let $A = 1$, $\Rightarrow R_f = 0\Omega$

b Subtraction $X_1 - X_2$ $(V_{in1} - V_{in2})$

Then, Combine it with Add CKT
So, we have $V_{in_1} - V_{in_2}$



$= A(V_{in_1} - V_{in_2})$

Let $A = 1$, Hence $R_f' = 0\Omega$

Therefore, we have

Subtraction
$V_{out} = V_{in_1} - V_{in_2}$

c multiplication. $y = ax$, $a > 1$

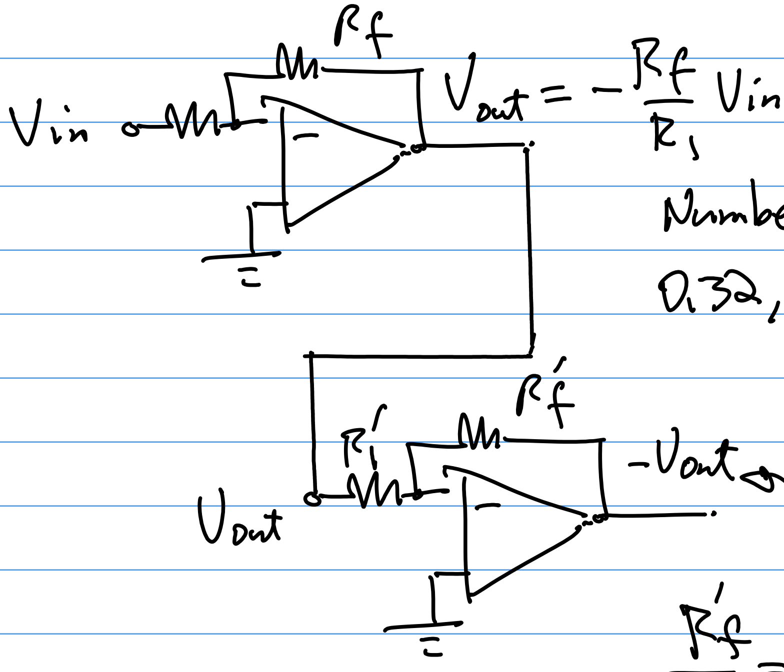
use Non-Inverting Configuration

$$\frac{V_{out}}{V_{in}} = A \Bigg|_{A = 1 + \frac{R_f}{R_1}}$$

$$V_{out} = A \cdot V_{in} = \left(1 + \frac{R_f}{R_1}\right) V_{in}$$

Note: For Linear System, the multiplication is done by multiplying a gain, But Not Another X (or V_{in}).

d Division (is a multiplication!) for the multiplier A less than 1.
Two Stage Inverting Configuration, 1st Stage does the division But with negative Sign, 2nd stage with gain = 1, But Change to positive by 2nd negative.
Example: 2 stage Inverting



$V_{out} = -\frac{R_f}{R_1} V_{in}$, where $\frac{R_f}{R_1}$ is a fractional Number for division, for example 0.32, $R_f / R_1 = 0.32$, if $R_1 = 1K$

$R_f = 320\Omega$

to make gain = -1

$\frac{R_f'}{R_1'} = 1$, Let $R_1' = 1K\Omega$, then Solve for

$R_f' = R_1' = 1\,K\Omega$.

//

Note: Analog Sensors have to meet the Output Current Requirement, e.g. 4~20mA //

Project Due May 9 (Sunday) 11:59 P.m.

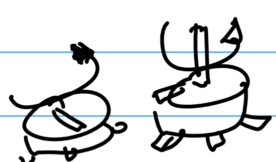
April 19 (Mon)

Design ON Preprocessing CKT.

Example: See Fig. on github

Characteristic Curve.

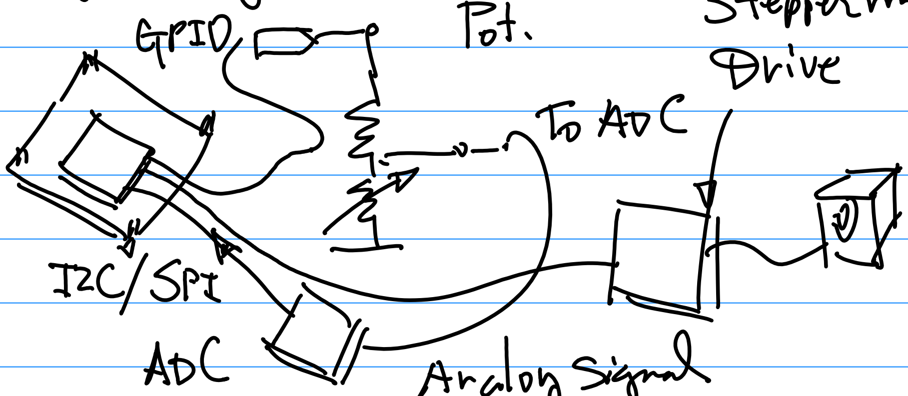
Shifting → Enlargement of the Dynamic Range.



Stepper motor Drive

April 19 (Mon).

1° Preparation for the Coming Project. Fig 2, PP.37.



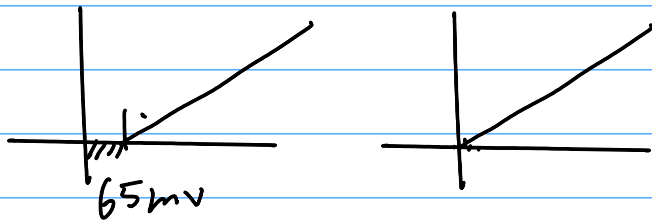
a Shifting. Add/Sub.

b Dynamic Range: Gain mult.

CmpE/EE242

4/3

Shifting (Subtraction).



Inverting Configuration

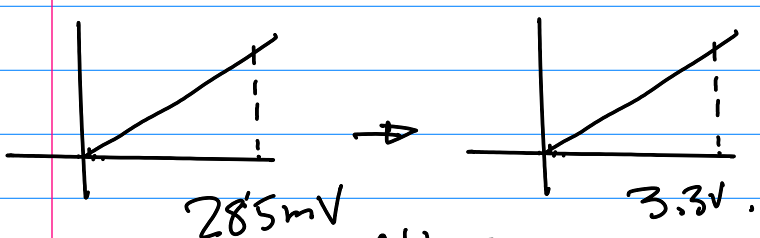
$$\frac{V_o}{V_{in}} = -\frac{R_f}{R_1}$$

Desired Shifting

Output from the Reference (External Voltage Source)

$$\frac{65\times10^{-3}}{5.0} = \frac{R_f}{R_1}, \quad \text{Let } R_f = 1K\Omega \quad \cdots (1)$$

Now, Dynamic Range Design



$$\text{Gain} = \frac{3.3V}{285mV} = \frac{3.3}{0.285} \approx 11.58 \quad \cdots (2)$$

After

Before

Since Gain is positive, we have Non-inverting configuration

$$A = \left(1 + \frac{R_f}{R_1}\right)$$, substitute (2) into this Eqn.

$$11.58 = 1 + \frac{R_f}{R_1}$$

Let $R_1 = 1K\Omega$, find/solve for R_f. Hence $R_f = 10.58\ K\Omega$

Then, Integrate OPAmps Together to form Preprocessing CKT.

Note:

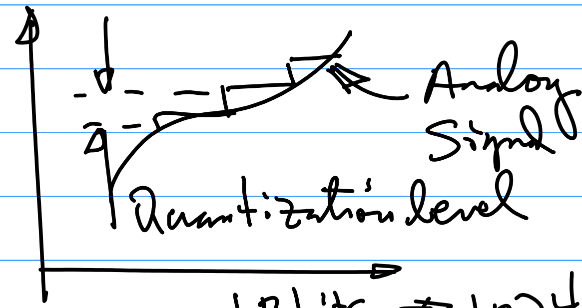
1° Analyze Sensor Characteristic Curve, Define Arithematic/Math Operations, for Shifting and magnification (of the Gain, to Cover 3.3V Dynamic Range)

2° To be Able to use Inverting And/OR Non-inverting Configuration to Realize the design Requirements, e.g. Shifting, and magnification

Now, Let's Consider the ADC Design. Scope:

- System Level for ADC
- Data Validation

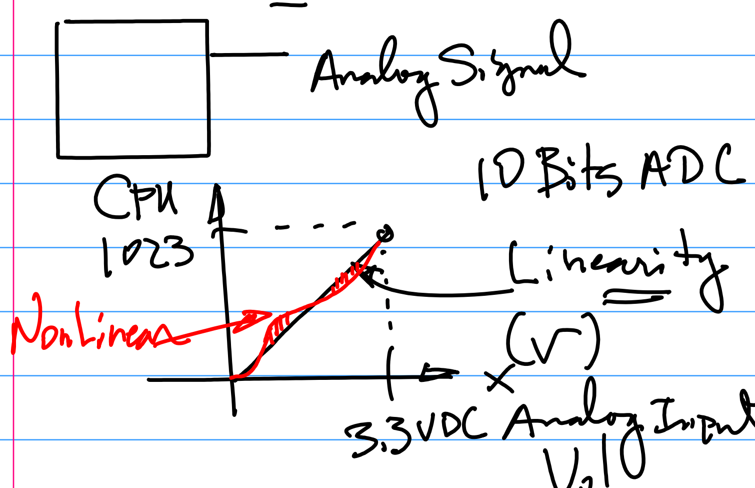
D.F.T. (Discrete Fourier Transform) → Power Spectrum
↓
Nyquist Sampling Theorem.



10 bits → 1024 $= 2^{10}$

April 21 (Wed)

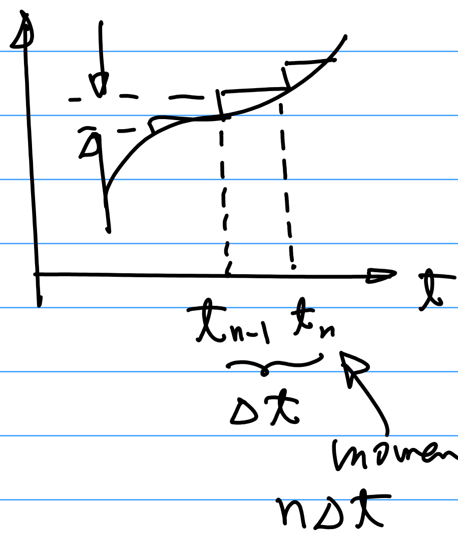
Data Validation: To make sure ADC Output (to CPU)



(Q)

$\Delta Q / \Delta V = K$

Δt : Sampling interval



→ Nyquist Theorem

$$f_{sampling} \geq 2 f_{max} \quad \cdots (1)$$

Sampling frequency has to be greater than OR Equal to Twice the highest frequency of the Signal itself.

Note:

1. Denote Digital signal as $x(n)$ or $x(k)$

Digitized, limited Quantization level

Index integer $n \Delta t$

2. Introduce Discrete Fourier Transform

a. Background : Basic Building Blocks to Characterize or to Build a given Signal.

b. Formulation.

↓

Fourier Transform

$$C_n = \int_T x(\tau) e^{-j2\pi f\tau} d\tau \quad \ldots(1)$$

$$f(x) = f(x_0) + \frac{f'(x)}{1!}(x-x_0) + \frac{f''(x)}{2!}(x-x_0)^2 + \cdots + R_n(x) \quad \ldots(2)$$

Basic Building Blocks

$$f(x) \cong \sum_{n=-\infty}^{+\infty} C_n(\cos 2\pi n f t + \phi) \quad \ldots(3)$$

3. Notation

$X(m)$ Discrete Fourier Transform of a Signal $x(n)$

m: Index in Frequency Domain

Hence, D.F.T. (Discrete Fourier Transform)

$$X(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{mn}{N}} \quad \ldots(4)$$

Physical meaning:

$x(n) = x(n+kN)$ periodic function.

Period: N, where $k = 0, 1, 2, \ldots$ (Natural Number)

$\frac{1}{N}$ Scaling factor, N No. of Total pts for One Period.

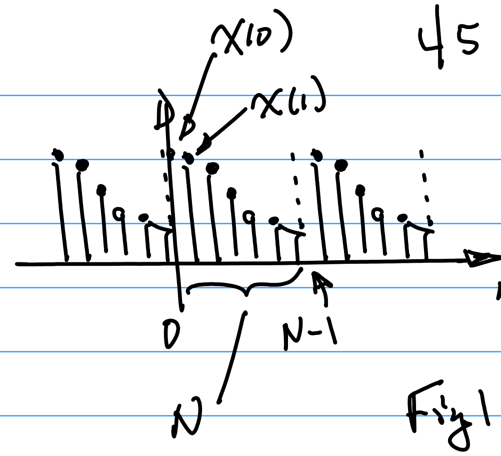


Fig 1.

$n=0,\ n=1,\ n=2,\ \ldots$

$x(0),\ x(1),\ x(2),\ \ldots$

$e^{-j2\pi \frac{mn}{N}}$ for Imaginary Axis

m: Frequency Index

$X(m)\big|_{m=0} = X(0)$ D.C. Component

$X(1), X(2), \ldots X(N-1)$ Higher Freq. Comp.

$$X(m) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{mn}{N}} = x(0)e^{-j2\pi \frac{m\cdot 0}{N}} + x(1)e^{-j2\pi \frac{m\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi \frac{m(N-1)}{N}}$$

from Right Hand side of Eq.(4).

(For Simplicity, $\frac{1}{N}$ Removed)

CmpE/EE 242

April 26 (Mon) 46

Note: Final Exam
- Mazal (Fri)
1215 - 1430

for $m=0$,

$$\underset{\uparrow}{X}(0) = x(0)e^{-j2\pi\frac{0\cdot 0}{N}} + x(1)e^{-j2\pi\frac{0\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{0(N-1)}{N}}$$

D.C.

$$= x(0)\cdot 1 + x(1)\cdot 1 + \cdots + x(N-1)\cdot 1$$

$$= \underbrace{x(0) + x(1) + \cdots + x(N-1)}$$

Divided by $1/N$, Average $\rightarrow$ D.C.

Example: Eqn (5)

Left Hand: $X(m)$

D.F.T. $\quad$ m Freq. Index

$$m = \underbrace{0, 1, 2, \cdots, N-1}_{N \text{ pts. One Period}}$$

for $m=1$.

$$X(1) = x(0)e^{-j2\pi\frac{1\cdot 0}{N}} + x(1)e^{-j2\pi\frac{1\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{1(N-1)}{N}}$$

$$X(m) = X(m+kN) \quad \cdots (1)$$

Periodic Function,
Period = N

⋮

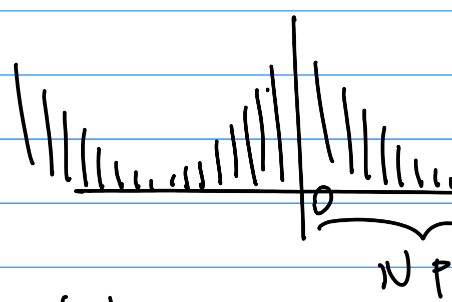
for $m = 2, 3, \cdots$

$$X(N-1) = x(0)e^{-j2\pi\frac{(N-1)\cdot 0}{N}} + x(1)e^{-j2\pi\frac{(N-1)\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{(N-1)(N-1)}{N}}$$

Fig 1

D.F.T.

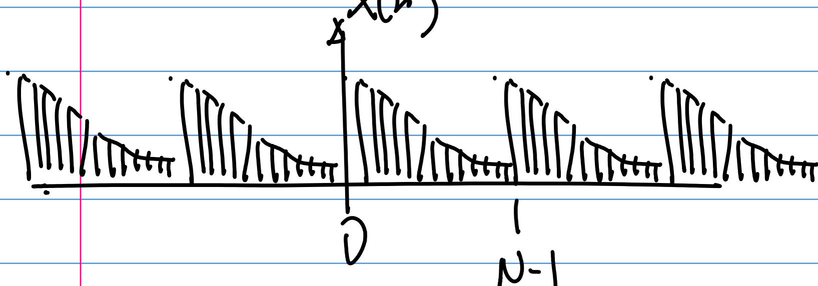
$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix} = \frac{1}{N} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}_{N\times N} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix} \quad \cdots (5)$$



2. Right Hand input Digital Signal (Output from ADC) $x(n)$
Signal ↑ ↖ Time Index

$x(n) = x(n+PN) \quad \cdots (2)$

Periodic Function   Time Domain



Note: (Optional) The process of Digitizing a function in One domain, either in Time Domain, or in Frequency Domain, will lead its Counterpart of the function become periodic function,

$x(t) \xleftrightarrow{F.T.} X(f)$

in time-Domain   in frequency Domain

$x(n)$ Digitized in Time $X(m)$ Digitized (Conceptually) Freq.

$x(n) = x(n+pN)$

$X(m) = X(m+kN)$

3. Denote Eqn (5) as follows.

$$\begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{pmatrix} = \frac{1}{N} E_{N\times N} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{pmatrix}_{N\times 1}$$

$$E_{N\times N} = \begin{pmatrix} & & \vdots & \\ \cdots & \cdots & \bigcirc & \cdots \\ & & \vdots & \end{pmatrix}$$

at (i, j)

$$e^{-j2\pi \frac{i \cdot j}{N}} \quad \cdots (3)$$

Example: Find Entry of the $E_{N\times N}$ matrix at $(2,3)$ location, Suppose $N=4$.

From Eqn (3)

$$e^{-j2\pi\frac{i\cdot j}{N}}\Big|_{\substack{i=2\\ j=3}} = ?$$

Euler Equation

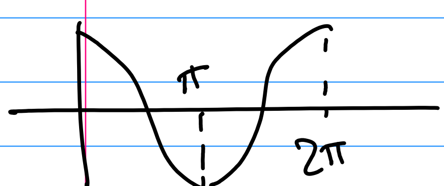
$$e^{-j2\pi\frac{mn}{N}} = \cos\left(2\pi\frac{mn}{N}\right) - j\sin\left(2\pi\frac{mn}{N}\right) \quad \cdots (4)$$

Based Eqn (4), & given Condition $i=2$, $j=3$,

$$e^{-j2\pi\frac{2\cdot 3}{4}} = \cos\left(2\pi\frac{2\cdot 3}{4}\right) - j\sin\left(2\pi\frac{2\cdot 3}{4}\right)$$

$$= \cos(3\pi) - j\sin(3\pi) = -1 - j\cdot 0$$

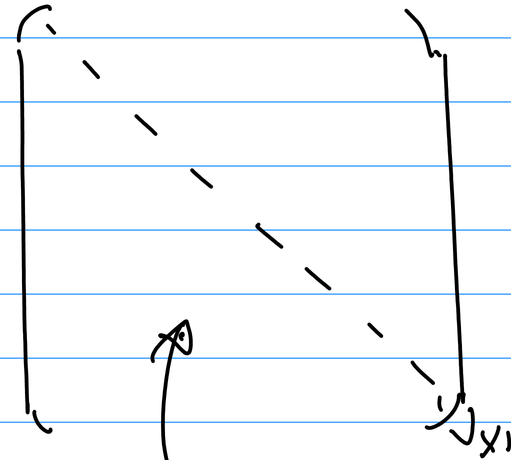
$$= -1$$



Note, This way, we can Evaluate Each of Every Entry of the matrix.

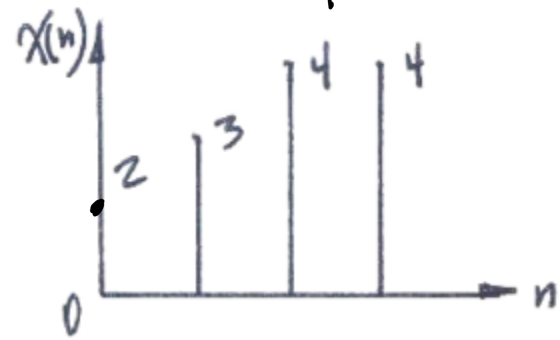
4. Entry for $E_{N\times N}$ is

$$e^{-j2\pi\frac{m\cdot n}{N}} = e^{-j2\pi\frac{n\cdot m}{N}}$$



Symmetric Along main Diagonal.

Example: Given $x(n)$ below, with $N=4$



Find/Compute its D.F.T

Sol From Eqn (5),

$$\begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{pmatrix} = \frac{1}{N} E_{N\times N} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{pmatrix}$$

$N=4$; $x(0)=2$, $x(1)=3$, $x(2)=4$, $x(3)=4$;

Then use Eqn (4*), we evaluate Each Entry.

$$\begin{array}{c c} & \xrightarrow{\quad n \quad} \\ m \downarrow & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}_{N \times N} \end{array}$$

Visualize the following property as if you are standing at 1st Row & 1st Column Position,

As you move top down, m index is increasing by 1 per row while n-index is unchanged

and, As you move from Left to Right, n index is increasing by 1 per colum while your m-Index is a Constant.

For 1st Row, 1st column Location $m=0, n=0$.

From Eqn (4*)

$$e^{-j2\pi\frac{mn}{N}} = e^{-j2\pi\frac{0\cdot 0}{N}} = e^{-j0} = 1$$

$$\begin{array}{c c} & \xrightarrow{\quad n \quad} \\ m \downarrow & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & & & \\ & & & \\ & & & \end{bmatrix}_{N \times N} \end{array}$$

For 1st Row, 2nd col. $m=0$ $n=1$

$$e^{-j2\pi\frac{m\cdot n}{N}} = e^{-j2\pi\frac{0\cdot 1}{N}} = 1$$

For 1st Row, 3rd col.

$$e^{-j2\pi\frac{m\cdot n}{N}} = e^{-j2\pi\frac{0\cdot 2}{N}} = 1$$

Now, for the 2nd Row, $m=1$

2nd Row, 1st col. $n=0$

$$e^{-j2\pi\frac{m\cdot n}{N}} = e^{-j2\pi\frac{1\cdot 0}{N}} = 1$$

2nd Row, 2nd col, $n=1$

$$e^{-j2\pi\frac{m\cdot n}{N}} = e^{-j2\pi\frac{1\cdot 1}{4}} = e^{-j\pi/2} = -j$$