

April 3rd (Monday)

Road map for the 2nd half of the Semester:

ITDT (Industrial IoT):

ADC Data Validation → ① F.I.T.
(Fast Fourier Transform)
Analog Sensor interface Design.

② Power Spectrum Technique.
③ Hardware Architecture Aspects

Ion Selective Electrode Sensors → Many Applications in different Industry Sector.

the Semester.

Example: for Analog ISE Sensor interface Design.

Ref:



ammonia/ammonium ele

Cat No. S-05722-16 model 9512BNWP

Ammonia (NH₃)
Ammonium (NH₄⁺)

1. For both drinking water and was
2. EPA-approved for ISE analysis
3. The Orion ammonia electrode is with a chemical-resistant transluce
4. The easy-to-fill electrode comes to avoid overfilling and to monitor ti
5. Membrane replacement options loose membranes or preassembled membrane for the convenience of i

20 replacement membranes, preas body with membrane, 60-mL of filli cable with BNC connector.

620

Fig.1

Google Definition:

Principle of ion-selective electrode (I.S.E.) An ideal I.S.E. consists of a thin membrane across which only the intended ion can be transported. The transport of ions from a high conc. to a low one through a selective binding with some sites within the membrane creates a potential difference.

Example of A Battery

Homework Extension Next Monday with Demo.

Project: Due the 2nd of the Semester.

Implementation (1/3) 33% to
PID.
I₂C Sensor
PWM motor Control
Pre-processing C.F.T.
ADC

Research Part: (1/3)
① P.R.T Presentation ON State-of-the-Art Technology in the Embedded world.

② Report (Guideline)

③ Proposal (one page), Submit to the CANVAS for Approval.

— By Wednesday / Monday Next
(1/3) Demo & Presentation: By the end of

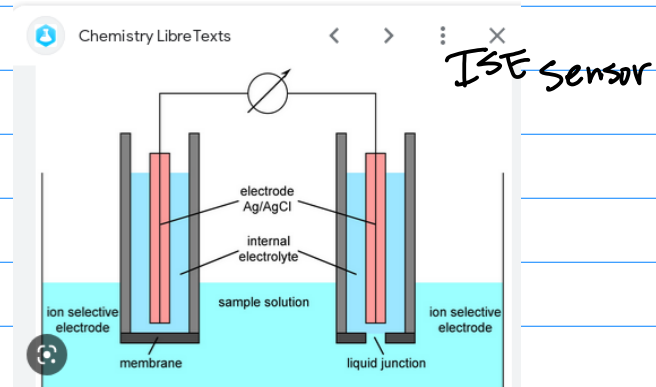


Fig.2

Working Principle of Battery - Electrical E...

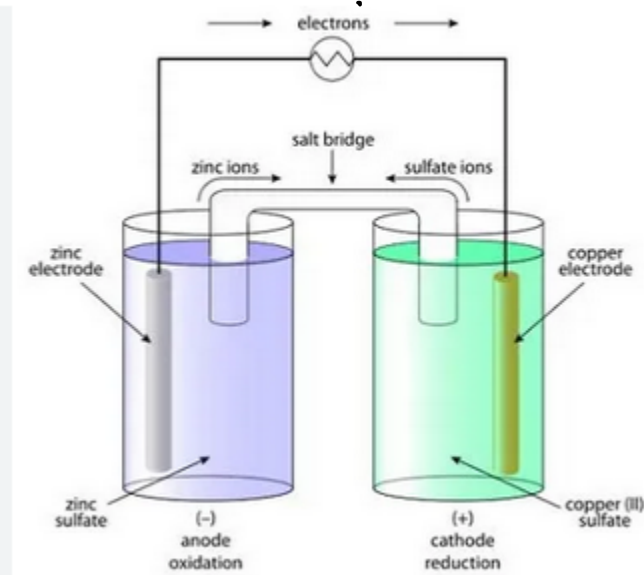


Fig. 3

Observation: Use Battery As An Example to Demonstrate Ion Selective Electrode Sensor. See $\text{NH}_3/\text{NH}_4^+$ sensor in Fig. 1.

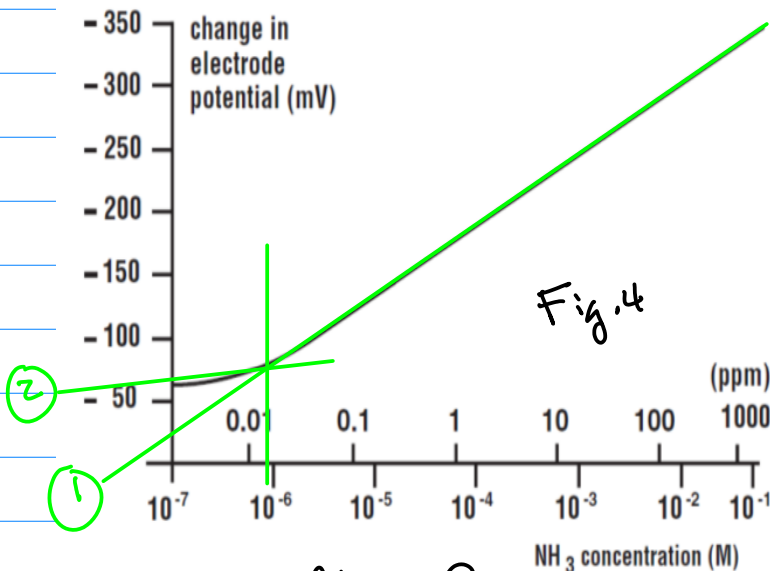
CharacteristicTypical NH_3 Calibration Curve

Fig. 4

Note 1. We like to have the Linear Characteristics from the Calibration Curve. Such as $[1, 10]$, $[10, 100]$, $[100, 1000]$, etc.

Visit

Note 2: For the Non Linear Part, Let's perform Linearization — By using piece-wise Linear Lines.

Piece-wise Line 1.
Piece-wise Line 2.

Next step is to formulate each line by using Linear Equation.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1} \quad \dots (1)$$

Solve for $y = bx + c$ (see the previous Notes).

With Simplification By Removing Very Low Concentration Part, we have

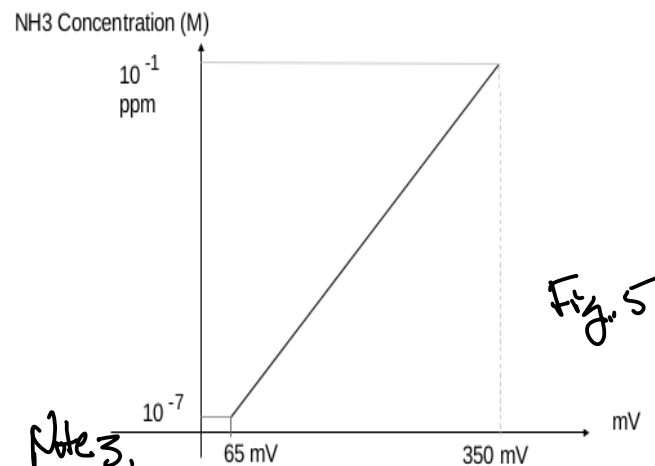


Fig. 5

Note 3.

Then, Change the Cal-Curve to the Characteristic Curve e.g. Horizontal axis is voltage for the Design of interface.

Industrial Analog Sensor Interface Protocol:

- 1° Output has to be defined by Current.
- 2° The Range $4 \sim 20 \text{ mA}$.

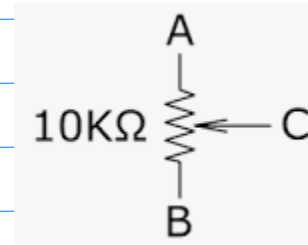


Fig. 1



Control.com

<https://control.com> > Technical Articles

Why is 4-20 mA Current Used for Industrial Analog Sensors?

Note 4. Fit to the Dynamic Range of your target platform.

External ADC Needed

CMOS Range ADC [0, 3.3]

TTL " " [0, 5.0]

April 5th (Wed)

Note 1° New updated Due Date for the Motor Control, please use PWM from your target platform as the input to the Motor Controller.

2° ADC Unit for the project Selection Guide.

1° Interface protocol I2C; $\sim 4 \text{ Mbps}$

2° $0.5 \sim 2 \text{ MSPS}$
(Million Samples per Second)

3° $10 \sim 12 \text{ bits per Sample}$.

Example: P.O.T.

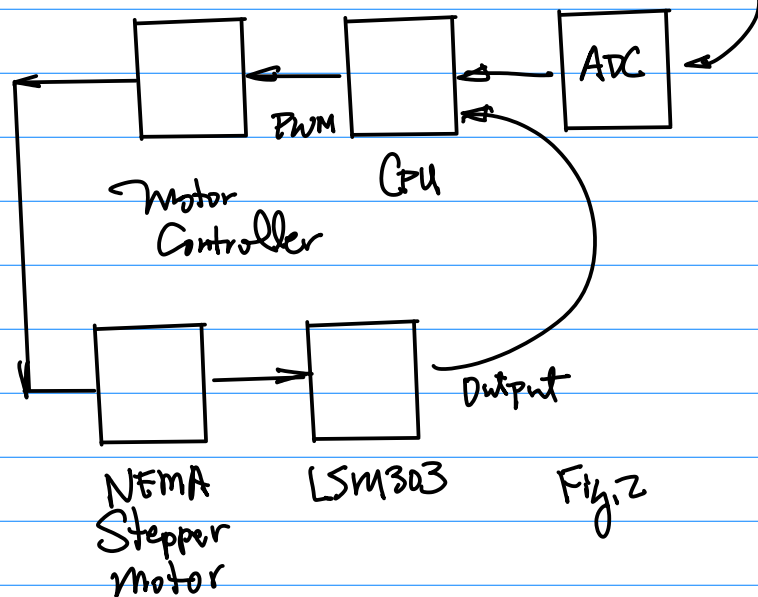
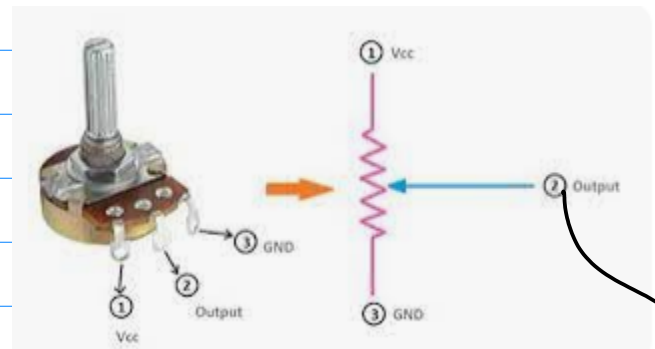


Fig. 2

<https://www.adafruit.com/product/1083>

ADS1015 12-Bit ADC - 4 Channel with Programmable Gain Amplifier - STEMMA QT / Qwiic

Product ID: 1083

\$9.95

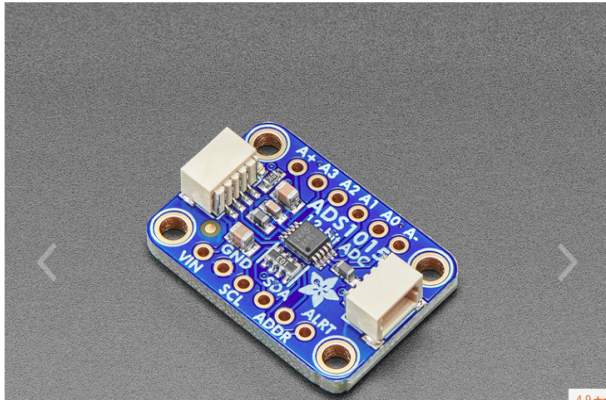


Fig.3



www.ti.com

SBAS473C –MAY 2009–REVISED OCTOBER 2009

ADS1013
ADS1014
ADS1015

Note: Input Voltage Range

ELECTRICAL CHARACTERISTICS

All specifications at -40°C to $+125^{\circ}\text{C}$, $V_{DD} = 3.3\text{V}$, and Full-Scale (FS) = $\pm 2.048\text{V}$, unless otherwise noted. Typical values are at $+25^{\circ}\text{C}$.

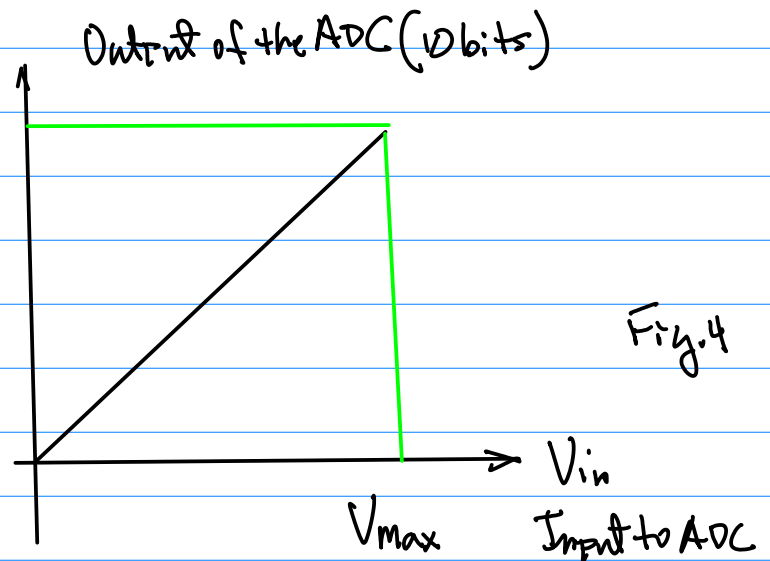
PARAMETER	TEST CONDITIONS	ADS1013, ADS1014, ADS1015			UNIT
		MIN	TYP	MAX	
ANALOG INPUT					
Full-scale input voltage ⁽¹⁾	$V_{IN} = (AIN_P) - (AIN_N)$		$\pm 4.096/\text{PGA}$		V
Analog input voltage	AIN_P or AIN_N to GND	GND		V_{DD}	V
Differential input impedance			See Table 2		
Common-mode input impedance	$FS = \pm 6.144V^{(1)}$		10		MΩ
	$FS = \pm 4.096V^{(1)}, \pm 2.048V$		6		MΩ
	$FS = \pm 1.024V$		3		MΩ
	$FS = \pm 0.512V, \pm 0.256V$		100		MΩ

Example: Design Objective .

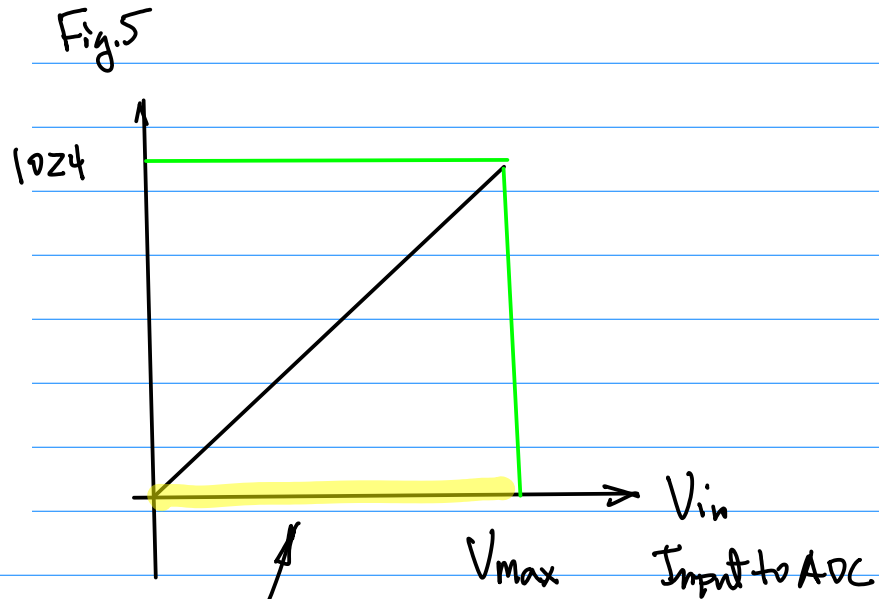
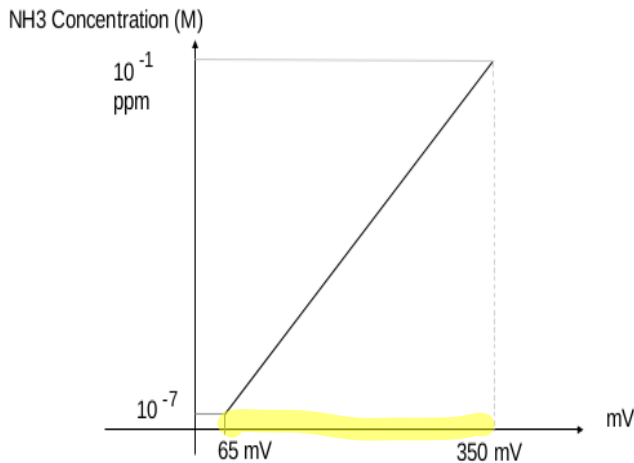
1^o Selection of An ADC . 1024

Construct the Characteristic Curve of the ADC

$V_{\text{max}} = 3.3\text{V}$ for CMOS



2^o Design Objective: To Design
A pre-process unit to make the
Analog Sensor Output match to
the ADC input dynamic Range.



Step 1. 65 mV \rightarrow 0
350 mV \rightarrow V_{max}

In general,

$$V_{S,min} \rightarrow V_{ADC,min} \quad \dots (1)$$

$$V_{S,max} \rightarrow V_{ADC,max} \quad \dots (2)$$

As info (mon).

Example: Continuation of the
Preprocessing Design.

Note 1. Check 1015 ADC Dynamic
Range for the Input.

[0, 3.3V]. Verification is needed

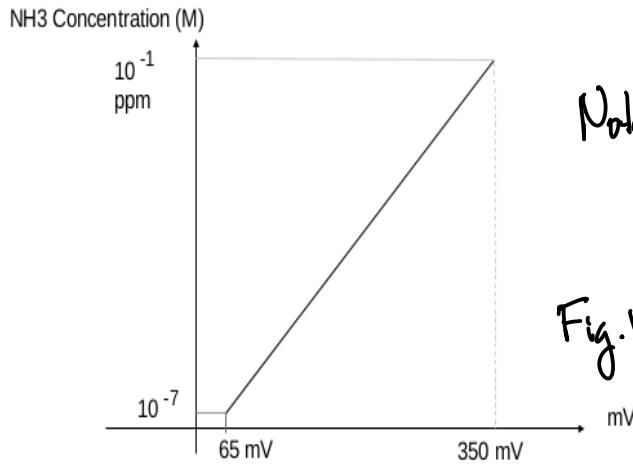
Theoretical Analysis:

Step 1. Provide "offset" to shift

The Sensor dynamic Range,
Subtraction Can be utilized for
this purpose. e.g.

$$V_{sen} - V_{offset} \quad \dots (1)$$

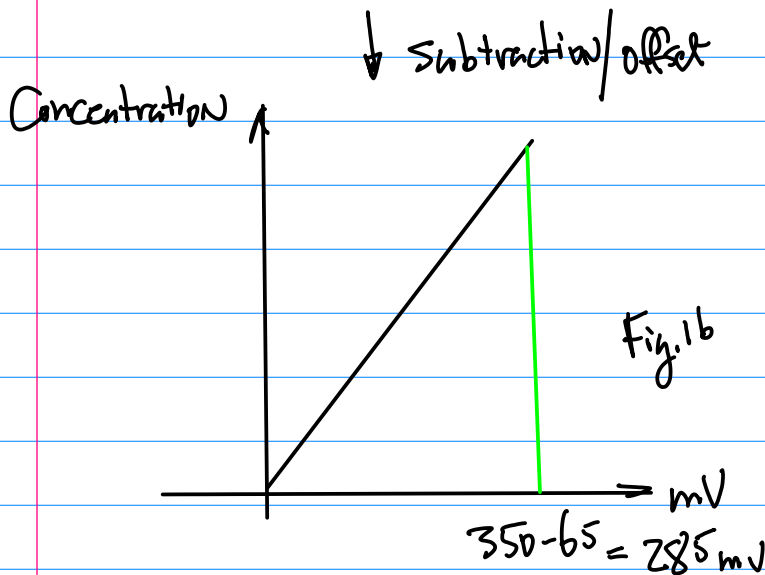
which will lead to the Result Below.



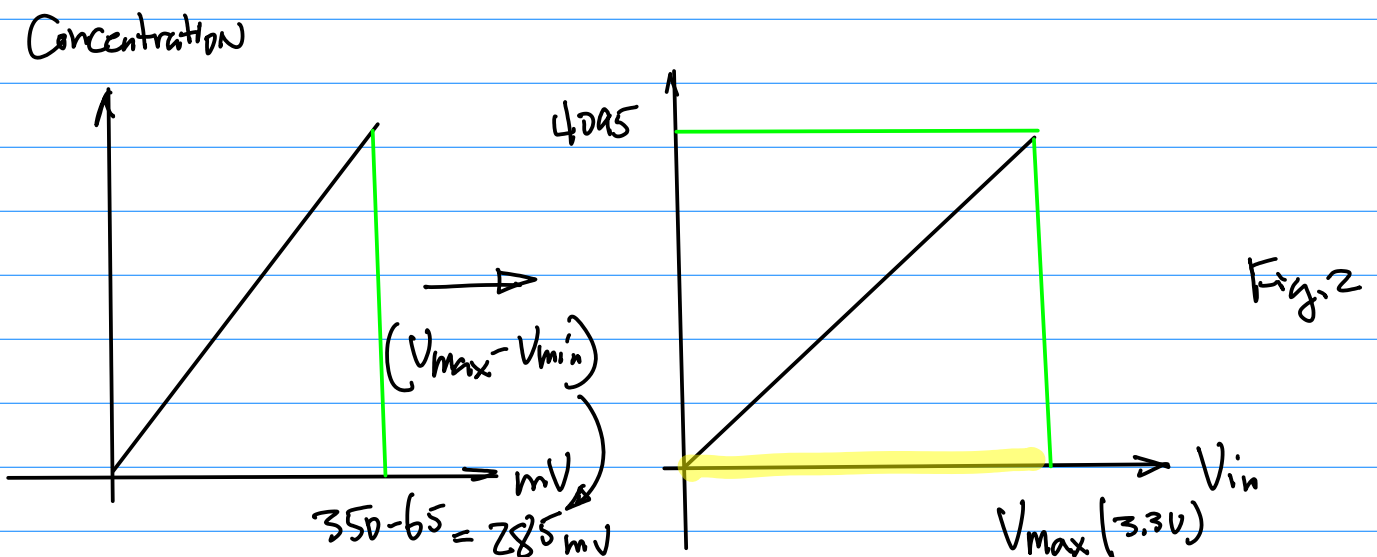
Note 1. For more generalized Case, Let $V_{min} = 65 \text{ mV}$, $V_{max} = 350 \text{ mV}$.
So, the offset $= -V_{min}$.

then, the Upper Bound after offset is

$$V_{max} - V_{min}$$



Step 2. To magnify the sensor output range to match the entire Dynamic Range of the ADC.



Find the Gain for the Magnification

$$A = \frac{V_{\text{Output Range}}}{V_{\text{Input Range}}} = \frac{3.3}{285 \times 10^{-3}} \approx 11.58$$

Where 3.3 VDC is from 1015 ADC for Example.

Example: Hardware Design for the pre-processing.
Ref.

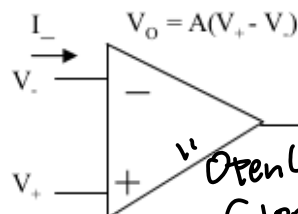
1-0Lecture 10 OpAmp Circuits.pdf

Note 1: Using OpAmp for Pre-processing
Not for the Buffering.

OpAmp Device As a Buffering Stage

Both Analog and Digital Circuit

Note 2: Background



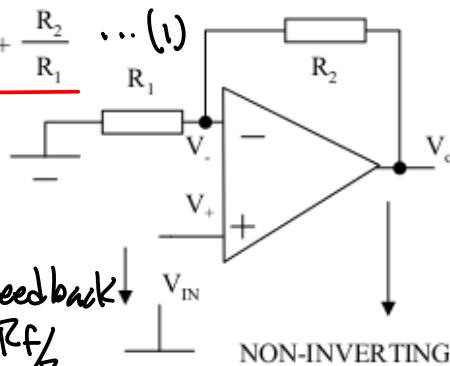
(1) To protect the previous stage's output signal, which is the input to the next stage, while sampling/connecting the signal to its next stage logic circuit. (2) Unit gain non-inverting OpAmp configuration is an excellent choice.

Ideal OpAmp Properties: (1) very large gain, $A \gg M$; (2) draws very little current, $I \sim 0$, e.g., very high impedance; (3) $V_O = A(V_+ - V_-)$ is finite range, which leads to $V_+ = V_-$.

for Example
100 MΩ or
higher.

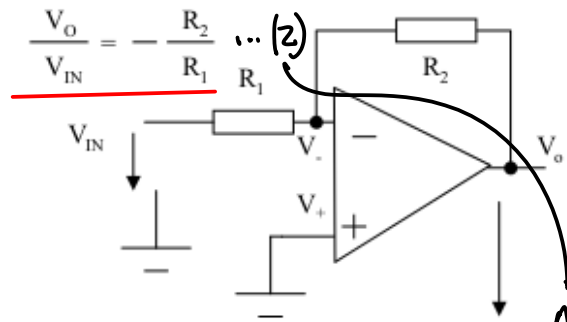
so R_2 for feedback
 $A = 1 + \frac{R_2}{R_1}$

Harry Li, PH.D. SJSU



NON-INVERTING

Fig. 3



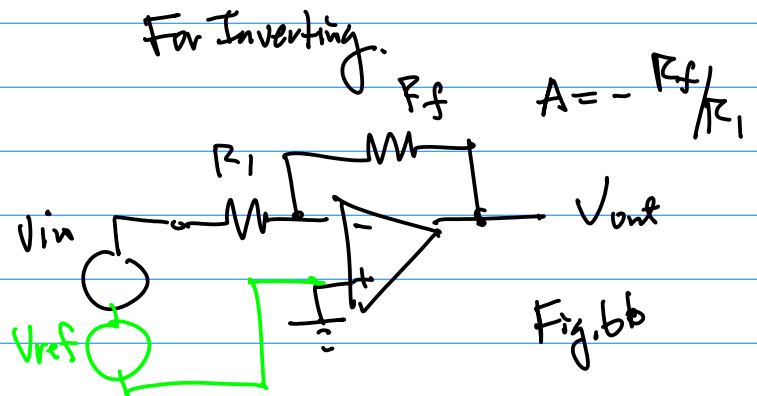
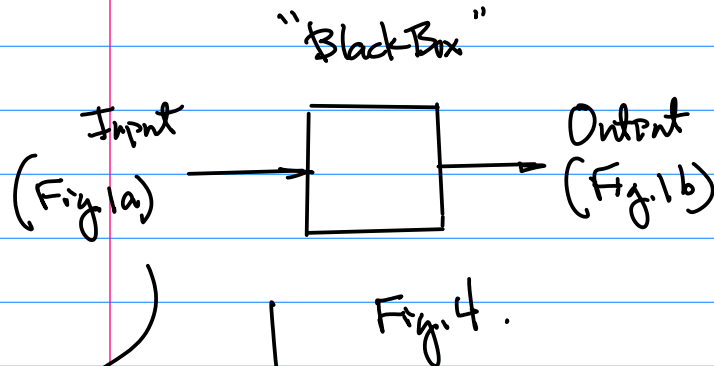
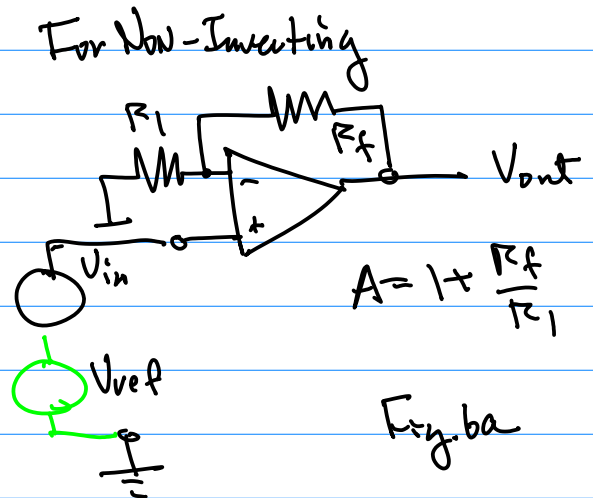
INVERTING

$$A = -\frac{R_2}{R_1}$$

$1 \times 10^{-9} \text{ A}$ or
Smaller
 V_1
 V_2
 $I \ll 8$

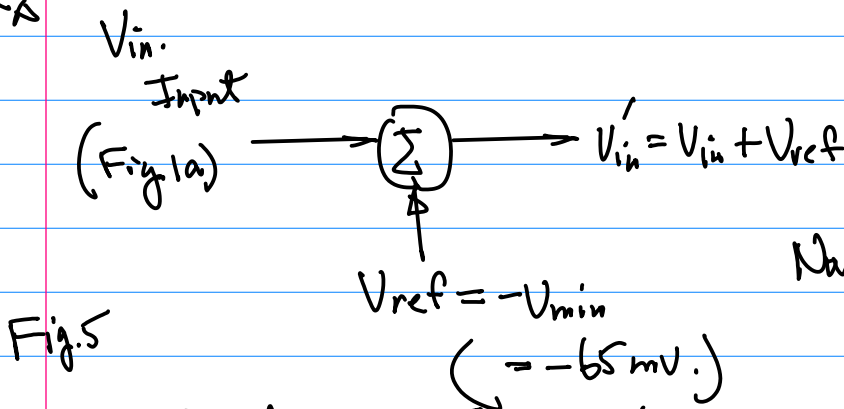
Now, consider the design Implementation for 2-Step Process.

Tools $\left\{ \begin{array}{l} \text{Non-Inverting} \sim / \text{Inverting} \sim \\ \text{Linear System: SuperImpose} \\ \text{One Signal (offset)} \\ \text{on to the other} \\ \text{Signal (Input).} \end{array} \right.$



Linear System: "SuperImpose
Signal for
Offset"

Note: Selection of Non- ~ v.s.
Inverting Configuration Depends
on your design Need. This
Design is an illustration of
SuperImposing an "offset", e.g.
 V_{ref} .



In the Circuit Design. Just Connect 2
inputs together.

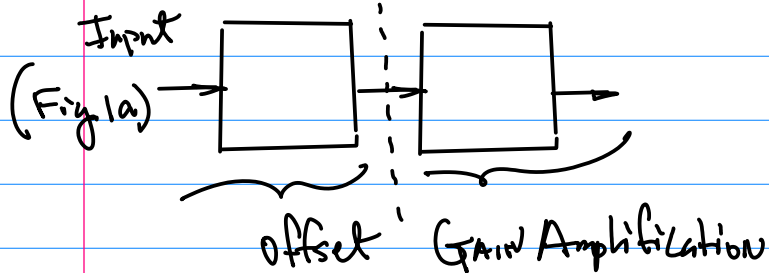
Now, to Magnify the Signal to match
ADC's Dynamic Range.
Choose Non-Inverting Configuration
(see Fig. 2)

Hence

$$A = 1 + \frac{R_f}{R_1} = 11.58$$

Choose $R_1 = 1 \text{ k}\Omega$
 Solve for $R_f \approx 10.58 \text{ k}\Omega$?
 please verify it!

Stage 1 Stage 2
 Box 1 Box 2

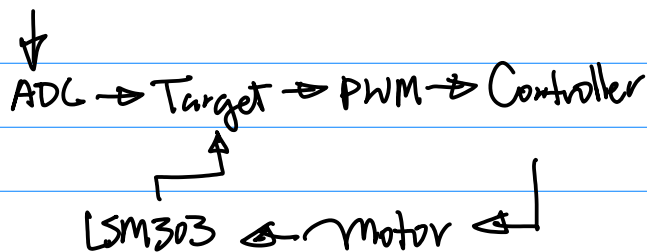


April 12 (Wed).

Note 1. The Last Project Preparation.
 (Requires the Semester End Presentation).

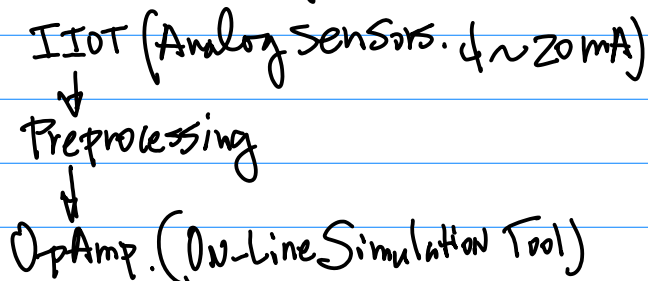
Note 2. Implementation of ADC Unit.

P.O.T. 47K or 470K or Similar.



Note 3. ADC Data Validation
 FFT. Power Spectrum.

Note 4. Road Map.



Analog Devices
<https://www.analog.com/ltspice-simulator>

LTspice Information Center

LTspice® is a powerful, fast, and free SPICE simulator software, schematic capture and waveform viewer with enhancements and models for improving the ...

Free for Download,
 Originated from "Linear"
 A Silicon Valley Company.

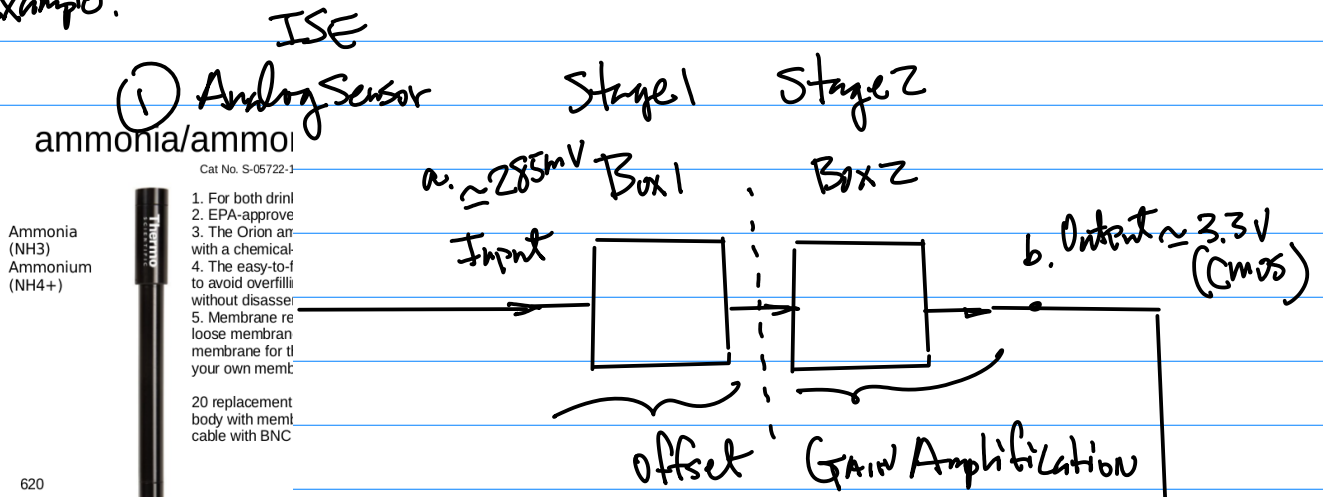
EasyEDA
<https://easyeda.com>

EasyEDA - Online PCB design & circuit simulator

EasyEDA is a free and easy to use circuit design, circuit simulator and in your web browser.

Requirements: To Be Able to Run
 SPICE Simulator.

Example:



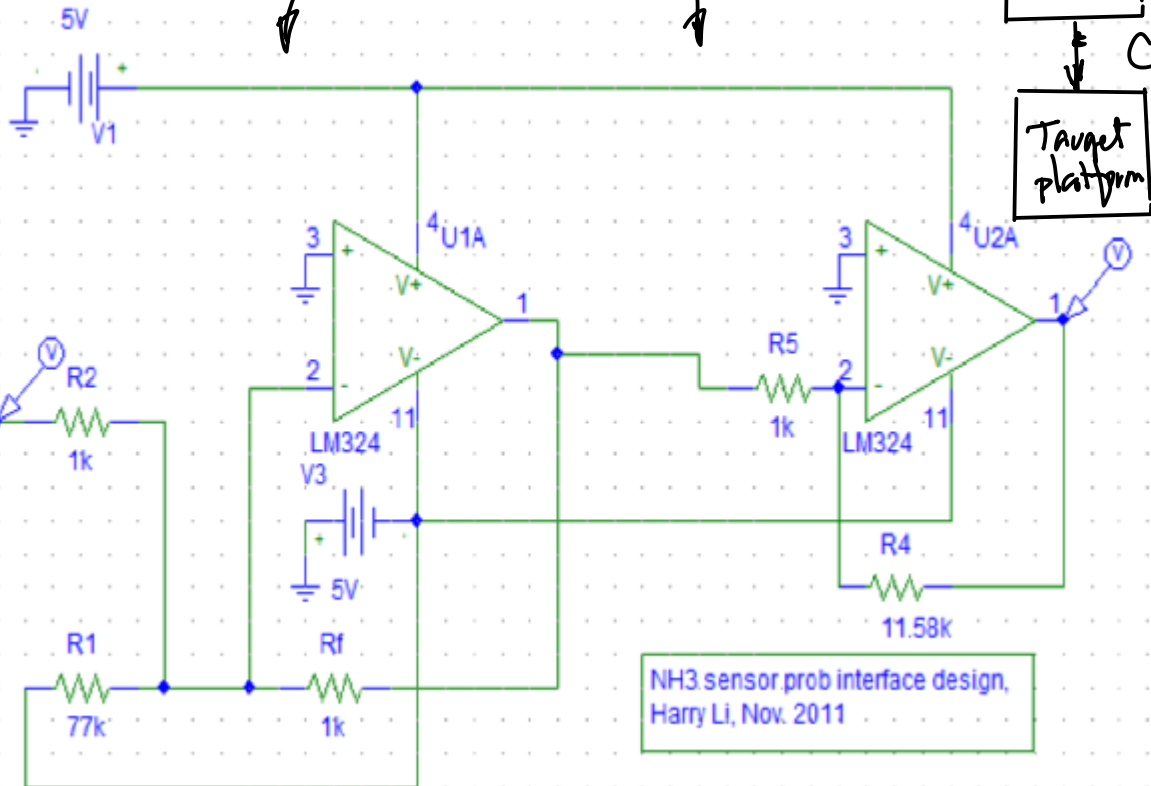
Note: 1° Analog

Sensor—
Signal
Source
Sine
Wave
Signal

2° for min.

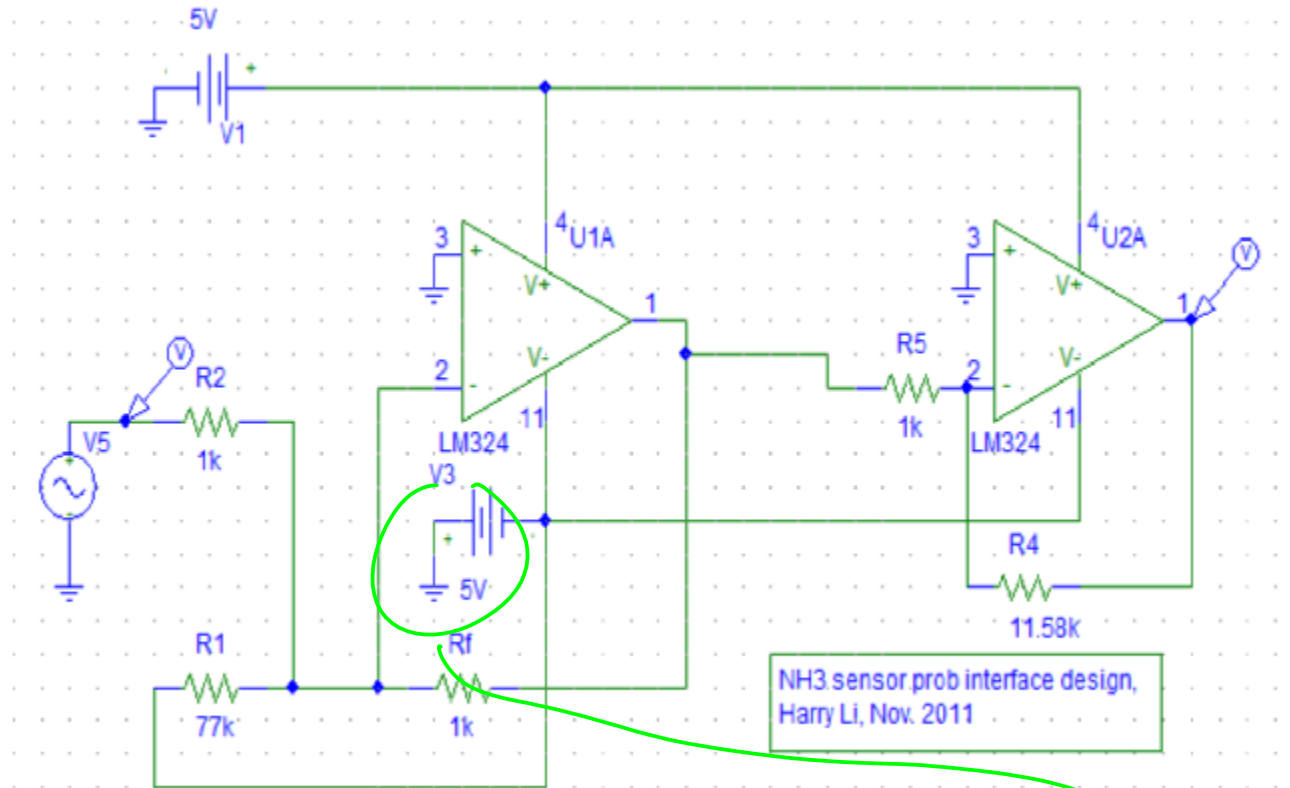
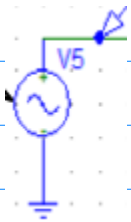
Input from the
Sensor. ($\approx 65\text{mV}$)
We want to get
ADC Output = 0.

for Max Input from the Sensor,
We want to get Output: (3.3V_{DC}) 4095
ADC



12-bit
ADC
Output
 ≈ 4095

Note: 3. Simulation of the Sensor Output as the input to the pre-processing circuit.



April 17 (Monday).

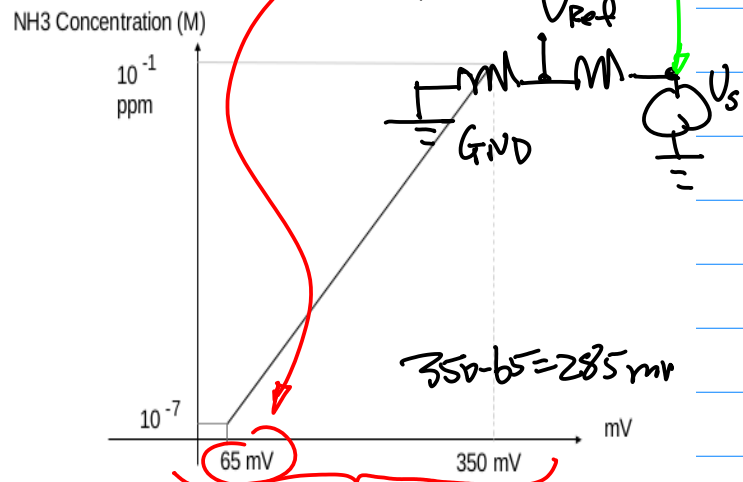
1^o Project (Integration of Homework + ADC). Due May 7 (Sunday) plus Research Part / Presentation.

2^o Bonus Points (50%) for BLDC motor Control; 3-phase (u, v, w) motor Control.

3^o SPICE Simulation for

Pre-processing Circuit Simulation.

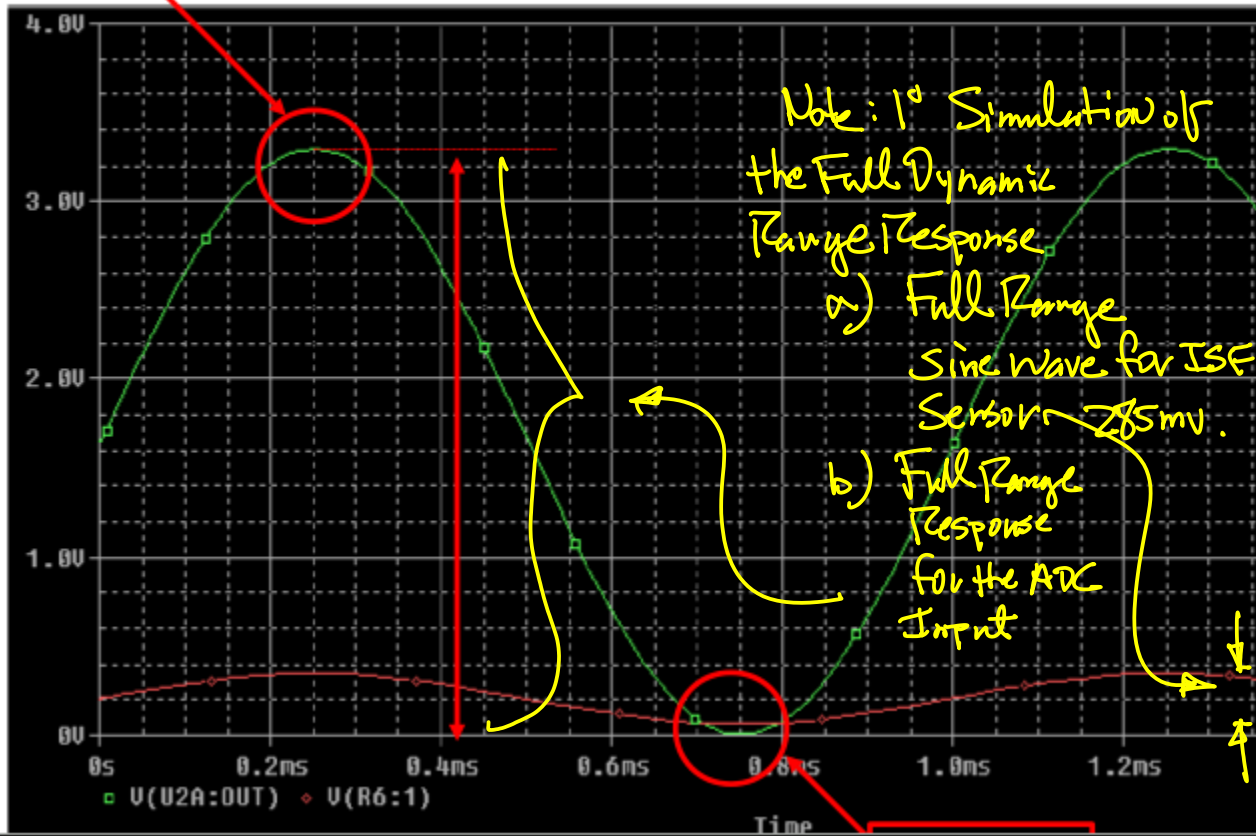
Example: Continuation. Provide "offset" By Voltage Divider V_{ref}



2018S-20-NH3- 4Design2018-1-16.pdf

Simulation Result

The output:
3.3V

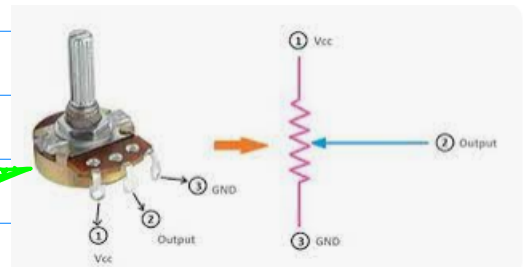


ADC Data Validation

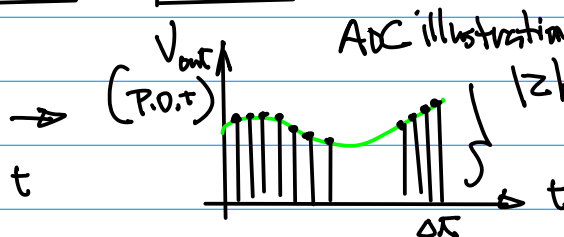
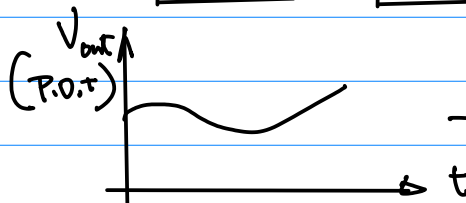
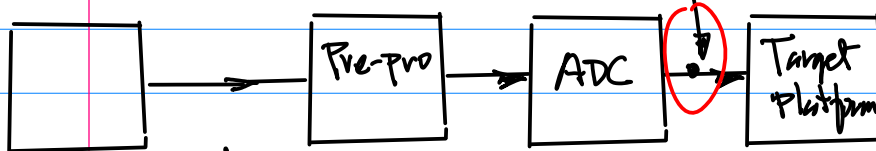
Tool: Fourier Spectrum Analysis.

Data Validation

P.O.T. Human Operator



See PP.48 For Details.



ADC illustration

12bit ADC.

ADC Output

$\{x(n) | n=0,1,2,\dots\}$

$[0, 4095]$

Background/Formulation.

To Validate $\{x(n)\}$, or $\{x(n) | n=0, 1, 2, \dots\}$
 $x(n)$.

D.F.T (Discrete Fourier Transform) is defined as follows.

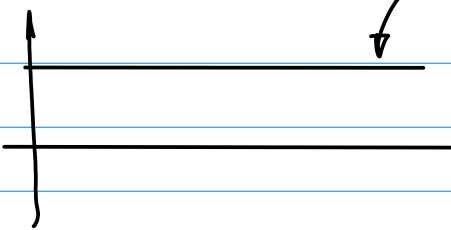
$$X(m) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{mn}{N}} \quad \dots (1)$$

Time Index

Physical meaning: $X(m)$, Discrete Fourier Transform.

m : Frequency Index

$m=0$, DC index; $X(0)$ DC Component.



$m=1$, $X(1)$ Fundamental Frequency Component.

N : One Period. Total No. of Points Per a Period. Such as
 $N=1024, 2048, 4096$, etc.

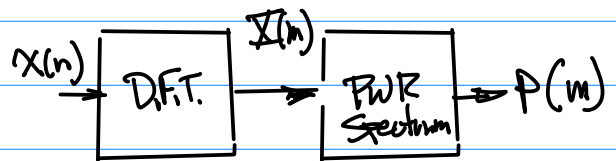
$N=2^x$ for FFT Only.
(Fast Fourier Transform)

$$e^{-j2\pi \frac{mn}{N}} = \cos 2\pi \frac{mn}{N} - j \sin 2\pi \frac{mn}{N} \quad \dots (2)$$

Euler Formula

$$e^{j\phi} = \cos \phi + j \sin \phi \quad \dots (3)$$

Power Spectrum of $X(m)$.



Let's Define the Power Spectrum as:

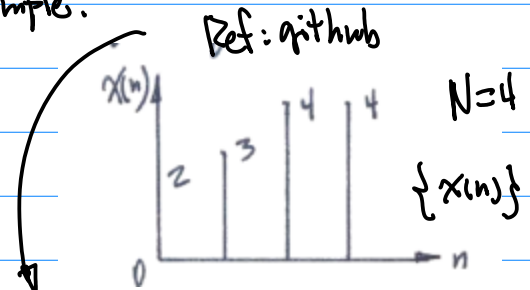
$$P(m) = \sqrt{\text{Re}^2[X(m)] + \text{Im}^2[X(m)]} \quad \dots (4)$$

Where

$$\text{Re}[X(m)] = \text{Re} \left[\frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{mn}{N}} \right]$$

$$\text{Im}[X(m)] = \text{Im} \left[\frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{mn}{N}} \right]$$

Example:



CMPE242-Embedded-Systems- / 2018S-26-1D-DFTv2.pdf

$$x(0) = 2, x(1) = 3, x(2) = 4, x(3) = 4.$$

Find $X(m)$ D.F.T.

$$X(m) = \frac{1}{4} \sum_{n=0}^3 x(n) e^{j2\pi \frac{mn}{4}} \quad \dots (5)$$

Next. From Eqn (5)

For $m=0$,

$$X(0) = \frac{1}{4} \sum_{n=0}^3 x(n) e^{-j2\pi \frac{0n}{4}} = \frac{1}{4} \sum_{n=0}^3 x(n)$$

$$= \frac{1}{4} (x(0) + x(1) + x(2) + x(3)) \quad \dots (ba)$$

For $m=1$

$$X(1) = \frac{1}{4} \sum_{n=0}^3 x(n) e^{-j2\pi \frac{1 \cdot n}{4}}$$

$$= \frac{1}{4} \left(x(0) \cdot 1 + x(1) e^{j2\pi \frac{1}{4}} + x(2) e^{-j2\pi \frac{2}{4}} + x(3) e^{j2\pi \frac{3}{4}} \right) \quad \dots (bb)$$

April 9 (Wed)

Final Exam Schedule: 18th (Thur)

Group I Classes

Group I classes are those classes which meet M, W, F, MTW, MWR, MTWF, MWRF, MTWRF, MW, WF, MWF, MF, TW, WR, MT, WS.

Regular Class Start Times	Final Examination Days	Final Examination Time
7:00 through 8:25 AM	Friday, May 19	7:15-9:30 AM
8:30 through 9:25 AM	Tuesday, May 23	7:15-9:30 AM
9:30 through 10:25 AM	Thursday, May 18	7:15-9:30 AM
10:30 through 11:25 AM	Monday, May 22	9:45 AM-12:00 PM
11:30 AM through 12:25 PM	Wednesday, May 17	9:45 AM-12:00 PM
12:30 through 1:25 PM	Friday, May 19	12:15-2:30 PM
1:30 through 2:25 PM	Tuesday, May 23	12:15-2:30 PM
2:30 through 3:25 PM	Thursday, May 18	12:15-2:30 PM
3:30 through 4:25 PM*	Monday, May 22	2:45-5:00 PM
4:30* through 5:25 PM*	Wednesday, May 17	2:45-5:00 PM

Note: Final Exam is in the
Same format as the midterm.
Be Sure to Bring your Prototype
System.

Note 2:

Project ON CANVAS. Part I & Part II.
25pts.

Part I: Integration of PID Control
with ADC

Part II: Research. PPT.

Note 3: Presentation Date

8th (Monday) ~ 10th (Wed) Final (18th, Th)

(1) Presentation.

(2) In-Class
Demo of the
Project!

12:15-2:30 pm.
Last Day of Class
15th. (Monday)
Review.

Example: Continuation of D.F.T. Example.

For $m=2$, from Eqn (1) pp 58

$$X(m) = \frac{1}{4} \sum_{n=0}^3 x(n) e^{-j2\pi \frac{2 \cdot n}{4}}$$

$$= \frac{1}{4} \left[x(0) e^{j0} + x(1) e^{j2\pi \frac{1 \cdot 2}{4}} + x(2) e^{-j2\pi \frac{2 \cdot 2}{4}} + x(3) e^{j2\pi \frac{3 \cdot 2}{4}} \right] \quad \dots (bc)$$

for $m=3$,

$$X(m) = \frac{1}{4} \sum_{n=0}^3 x(n) e^{-j2\pi \frac{3 \cdot n}{4}}$$

$$= \frac{1}{4} \left[x(0) e^{j0} + x(1) e^{j2\pi \frac{3 \cdot 1}{4}} + x(2) e^{-j2\pi \frac{3 \cdot 2}{4}} + x(3) e^{j2\pi \frac{3 \cdot 3}{4}} \right] \quad \dots (bd)$$

Hence, we can form a Col. Vector
By Arranging $X(0)$,
 $X(1)$, \dots , $X(3)$ as follows.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \frac{1}{4} E_{4 \times 4} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} \quad \dots (1)$$

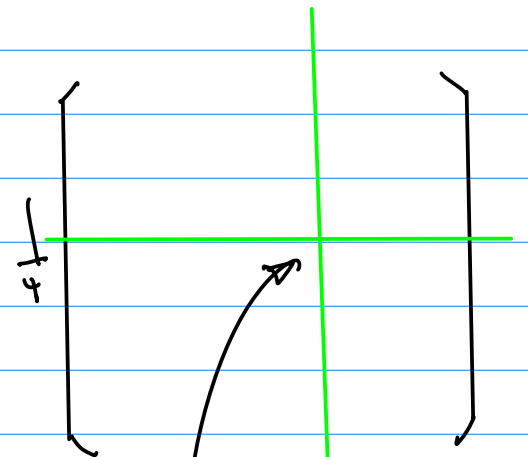
Where

$$E_{4 \times 4} = \begin{bmatrix} W_{00} & W_{01} & W_{02} & W_{03} \\ W_{10} & W_{11} & W_{12} & W_{13} \\ W_{20} & W_{21} & W_{22} & W_{23} \\ W_{30} & W_{31} & W_{32} & W_{33} \end{bmatrix}_{4 \times 4} \quad \dots (2)$$

Note: W_{ij} ... (3)
Index for Row. Index for Col.

From Eqn (1) & (2), we have

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}_m = \frac{1}{4} \begin{bmatrix} W_{00} & W_{01} & W_{02} & W_{03} \\ W_{10} & W_{11} & W_{12} & W_{13} \\ W_{20} & W_{21} & W_{22} & W_{23} \\ W_{30} & W_{31} & W_{32} & W_{33} \end{bmatrix}_{4 \times 4} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}_n \quad \dots (3)$$



W_{ij} , i for Row, j for Col.

Where $W_{ij} = e^{-j2\pi \frac{i \cdot j}{N}} \dots (4)$

From Euler Equation

$$e^{-j2\pi \frac{i \cdot j}{N}} = \cos 2\pi \frac{i \cdot j}{N} - j \sin 2\pi \frac{i \cdot j}{N} \quad \dots (5)$$

Find the entry of E matrix
at 1st Row, 1st Col. Location.

Since, we have Eqn (5).

$$e^{-j2\pi \frac{i \cdot j}{N}} = e^{-j2\pi \frac{i \cdot j}{4}}$$

$$= e^{-j2\pi \frac{i \cdot j}{4}} \Big|_{\substack{i=0 \\ j=0}} = 1$$

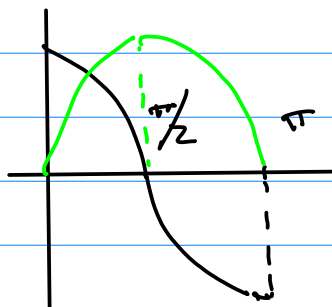
for the 3rd Row, 2nd Col.

$$i=2, j=1$$

From

$$e^{-j2\pi \frac{i \cdot j}{N}} \Big|_{\substack{i=2, j=1 \\ N=4}} = e^{-j2\pi \frac{2}{4}}$$

$$e^{-j2\pi \frac{2}{4}} = e^{-j\pi} = \cos \pi - j \sin \pi = -1 \quad P(m) = \sqrt{R_e^2[X(m)] + I_m^2[X(m)]}$$



$$P(0) = \sqrt{R_e^2[X(0)] + I_m^2[X(0)]} = \sqrt{3.25^2 + 0^2} = 3.25$$

Now, Consider the Last Row, Last Col. $i = N-1 = 3, j = N-1 = 3$.

From

$$e^{-j2\pi \frac{3 \cdot 3}{4}} = e^{-j \frac{9\pi}{2}} = e^{-j \frac{8\pi}{2} - j \frac{\pi}{2}}$$

$$= e^{-j4\pi - j \frac{\pi}{2}} = 1 \cdot e^{-j \frac{\pi}{2}}$$

$$= \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} = 0 - j \cdot 1 = -j$$

$$\cos \frac{\pi}{2} - j \sin \frac{\pi}{2} = 1 \checkmark$$

$$X(0) = \frac{1}{4}(2+3+4+4) = 3.25$$

$$X(1) = \frac{1}{4}(2-3j-4+4j) = \frac{1}{4}(-2+j)$$

$$X(2) = \frac{1}{4}(2-3+4-4) = \frac{1}{4}(-1)$$

$$X(3) = \frac{1}{4}(2+3j-4-4j) = \frac{1}{4}(-2-j)$$

for $m=1$,

$$P(1) = \sqrt{R_e^2[X(1)] + I_m^2[X(1)]}$$

$$= \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2} \text{ finish the evaluation}$$

April 24 (Monday).

Example: FFT.C

From the Handout, Ref from the github.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

So,

Step.

...(3*)

Step 1. Find $E_{N \times N}$. 2. Find D.F.T.

Using $E_{N \times N}$ And $x(n)$.

Step 3. Find $P(m)$ for $m=0, 1, \dots, N-1$

CMPE212

Spring 2023

Note: You can Compile/Build for Your Laptop platform. No Need for 62
ARM-Linux-gcc

```
1 /*****
2 * Program is for CMPE class use, see Dr. Harry Li's lecture notes for details *
3 * Reference: Digital Signal Processing, by A.V. Oppenheim;
4 * fft.c for calculating 4 points input, but you can easily expand this to 2^x inputs;
5 * : x0.1; Date: Sept. 2009;
6 * Note: cross compiled for arm-linux-gcc, be sure to modify make file
7 to link math lib when compiling, by adding -lm
8 This code then was tested on ARM11 board. Feb 2015.
9 *****/
```

Cross Compiler.

2⁰ Link Math Library

~/Desktop/SJSU/befor2018/EE264/EE264Ubuntu/OpenGL/CT/lec22FFTIFFT\$./fft

```
harry@harry-laptop: ~/Desktop/SJSU/befor2018/EE264
harry@harry-laptop:~/Desktop/SJSU/befor2018/EE264/
IFFT$ ls
fft fft_cpp.cpp fft_ifft_cpp.cpp ifft_cpp.cpp
harry@harry-laptop:~/Desktop/SJSU/befor2018/EE264/
IFFT$ ./fft
*****Before*****
X[1]:real == 2.000000 imaginary == 0.000000
X[2]:real == 3.000000 imaginary == 0.000000
X[3]:real == 4.000000 imaginary == 0.000000
X[4]:real == 4.000000 imaginary == 0.000000

*****After*****
X[1]:real == 13.000000 imaginary == 0.000000
X[2]:real == -2.000000 imaginary == 1.000000
X[3]:real == -1.000000 imaginary == 0.000000
X[4]:real == -2.000000 imaginary == -1.000000
It took me 145 clicks (0.000000 seconds).
harry@harry-laptop:~/Desktop/SJSU/befor2018/EE264/
IFFT$
```

```
harry@harry-laptop: ~/Desktop/SJSU/befor2018/EE264/
//*****
//This program is converted from a FORTRAN program
9 *
//book by A.V. Oppenheim; *
//Status: Tested; *
//gcc fft.cpp -o fft *
//Additional information: See Dr. Hua Harry Li's ha
//fft.cpp for calculating 4 pts input, but can easil
*
//*****
```

Note: To Compile -lm on x86 platform.

Note: $P(m) = P(m + KN) \dots (1)$

Periodical Function.

Period = N

$P(m) = P(-m) \dots (2)$

Even function

