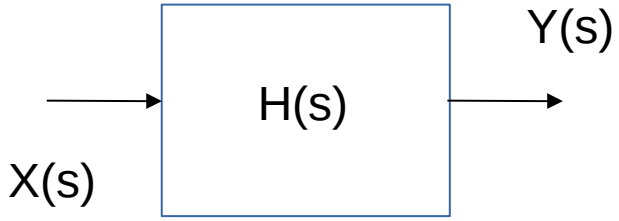
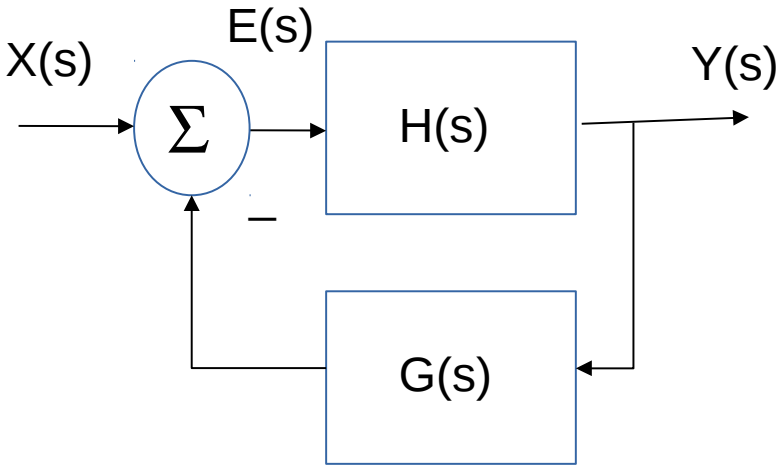
Close Loop Transfer Function



Start with open loop, where plant H(s) can be a stepper motor for example, so we have

$$Y(s) = H(s)\, X(s) \quad \ldots \quad (1)$$

Now we can add a sensor to form a feedback loop as follows



So, sensor G(s) forms a feedback loop as shown in Figure 2.

$$E(s) = X(s) - G(s)\, Y(s) \quad \ldots (2)$$

$$Y(s) = H(s)\, E(s) \quad \ldots (3)$$

From (2) we have

$$X(s) = E(s) + G(s)\, Y(s) \quad \ldots (4)$$

Substitute (3), we have

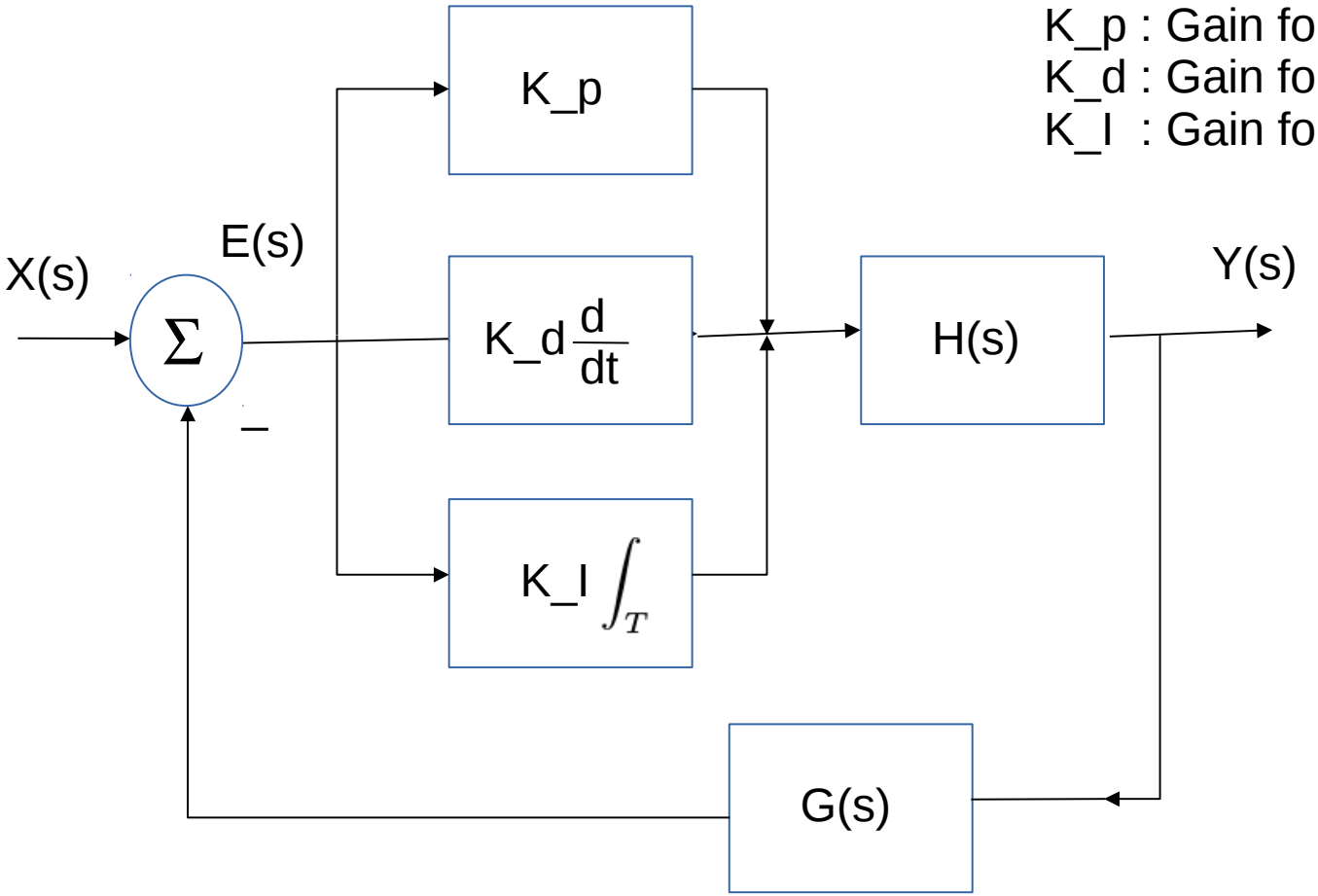
$$X(s) = E(s) + G(s)\, H(s)\, E(s)$$

Hence, the transfer function is

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)\, H(s)} \quad \ldots (5)$$

PID Control

K_p : Gain for proportional control
K_d : Gain for derivative control
K_I : Gain for integral control

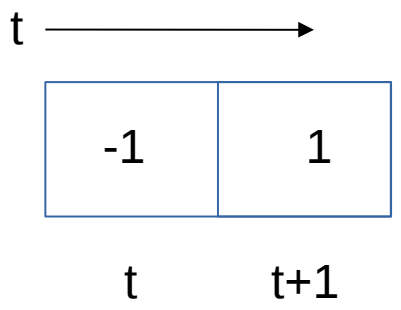


Central Difference And Its Kernel

Forward difference:

$$FD = \frac{de(t)}{dt} \approx e(t+1) - e(t)$$

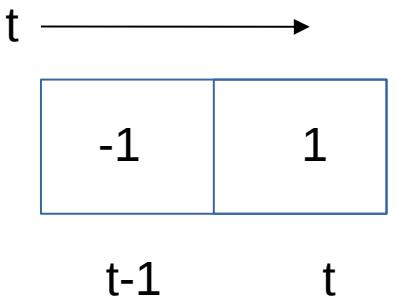
Map to a kernel



Backward difference:

$$BD = \frac{de(t)}{dt} \approx e(t) - e(t-1)$$

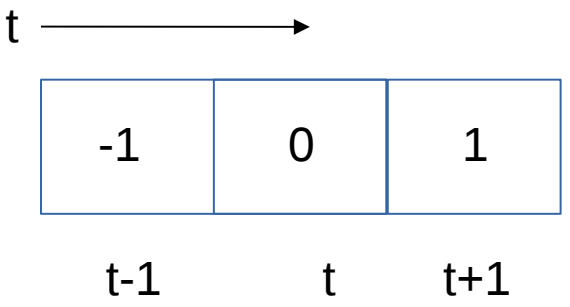
Map to a kernel



Central differnece:

$$CD = \frac{1}{2}(FD + BD) = \frac{1}{2}[e(t+1) - e(t-1)]$$

Map to a kernel



LoG Kernel And Convolution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \quad \ldots(1)$$

Then taking the 2nd order derivative of equation (1), as shown below (note I have done 2D example on LoG, in the case of 1D, LoG, just let 2nd independent variable y = 0.

$$\frac{\partial^2}{\partial x^2} G(x,y) = -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{x^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \ldots (4)$$

Based on the result from equation (4) (let y = 0 for 1D case), so we can construct K by K kernel (K as an odd number) as illustrated below:

Example: Illustration of K by K kernel

Evaluate each coefficient of the K by K kernel, by setting the x = 0 at the center of the kernel, and then you can find coefficient for both x = +1 and x = -1 (symmetric, they are equal, and so on.)

LoG Kernel And Convolution

Derive LoG Kernel

Example: LoG stands for Laplace of Gaussian
First, Laplace operator is given as:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{...(1)}$$

First, 1D Gaussian function

$$G_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u)^2}{2\sigma^2}}$$

Then, 2D Gaussian function,

$$G_\sigma(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u_x)^2+(y-u_y)^2}{2\sigma^2}} \quad \text{...(5)}$$

Assume $u_x = u_y = 0$

$$\frac{\partial}{\partial x} G(x,y) = -\frac{x}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \text{... (3)}$$

Hence

$$\frac{\partial^2}{\partial x^2} G(x,y) = -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{x^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \text{... (4)}$$

$$\frac{\partial^2}{\partial y^2} G(x,y) = -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{y^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \text{... (5)}$$

$$\nabla^2 G(x,y) = \frac{x^2+y^2-2\sigma^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

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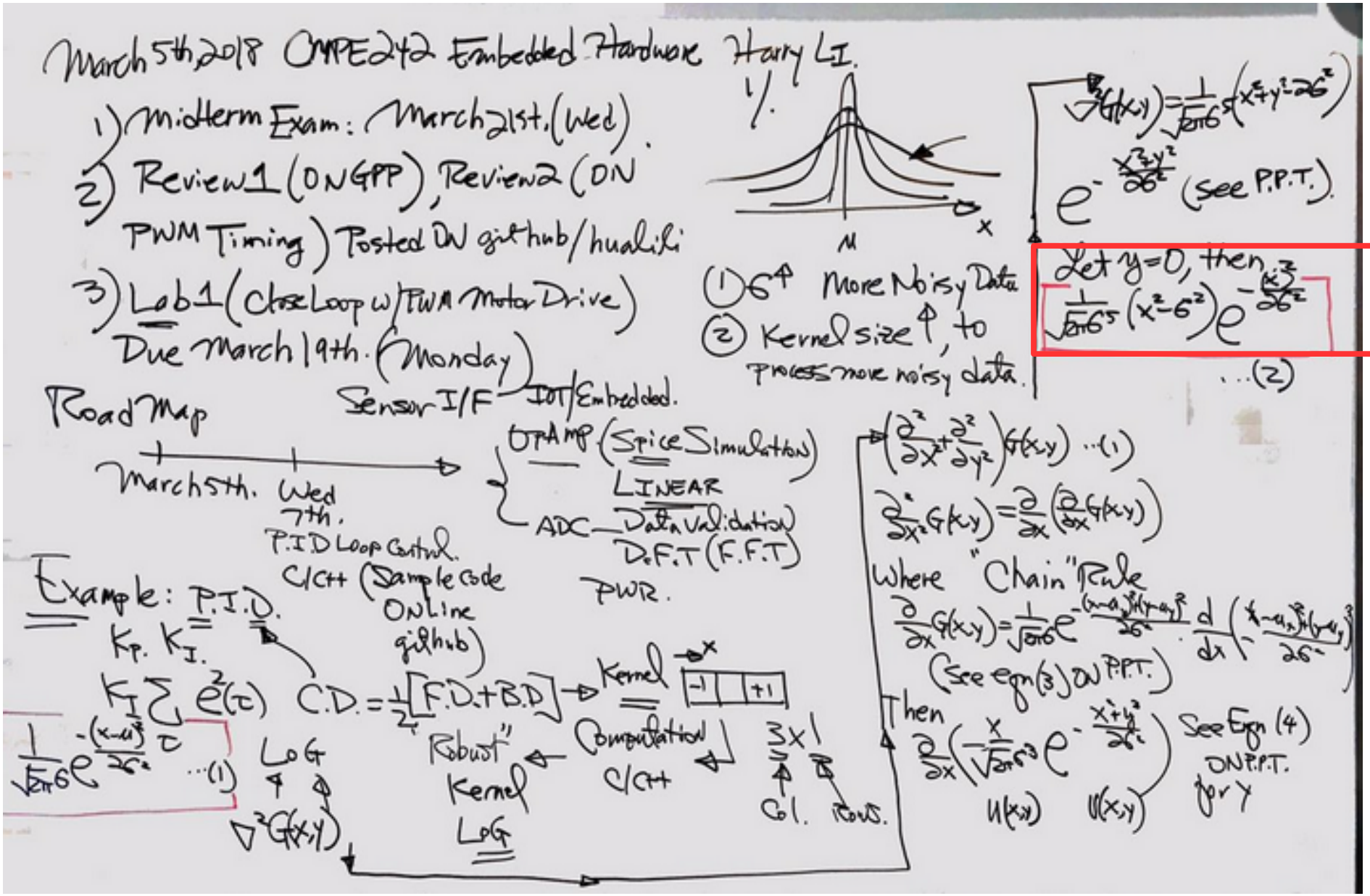
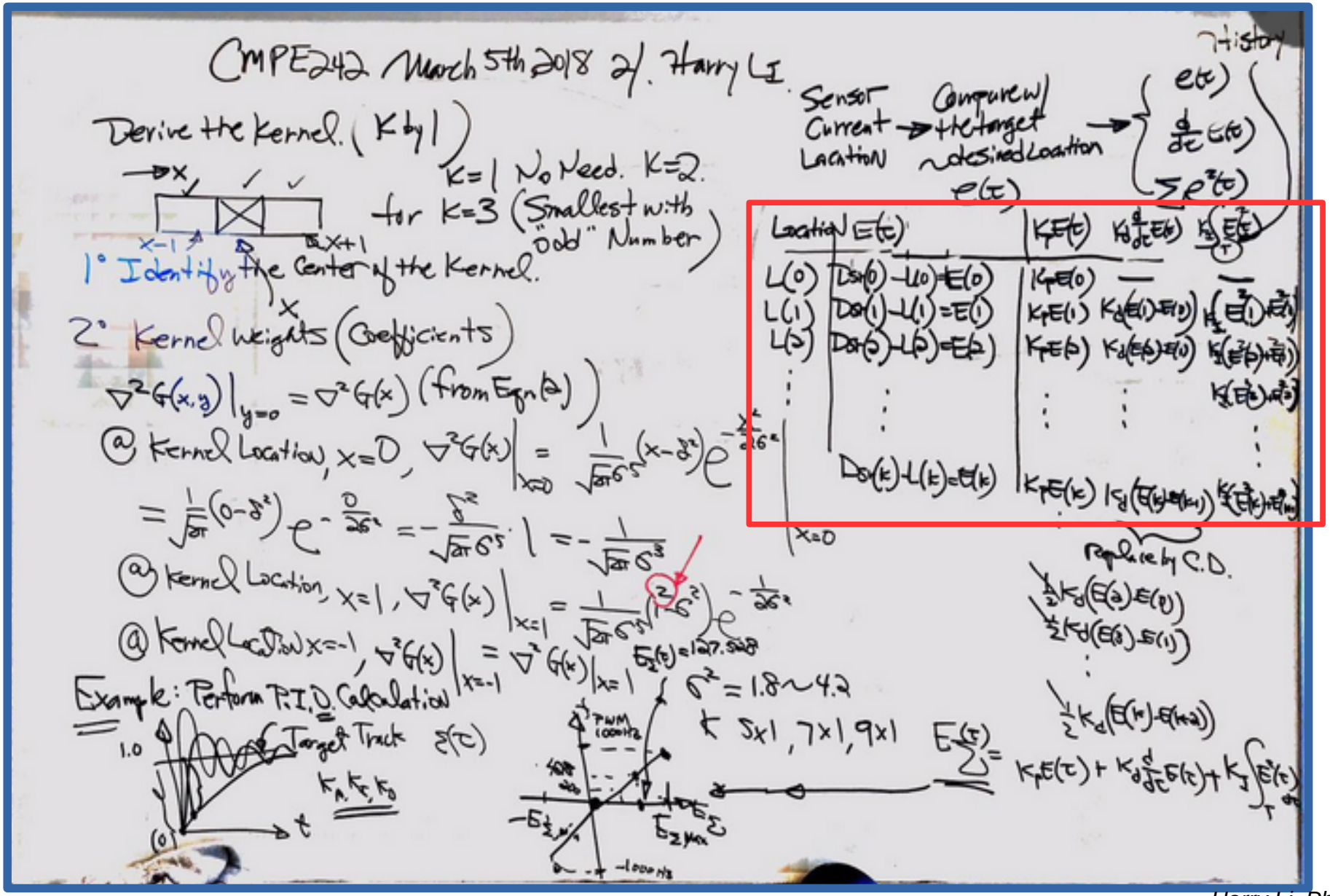
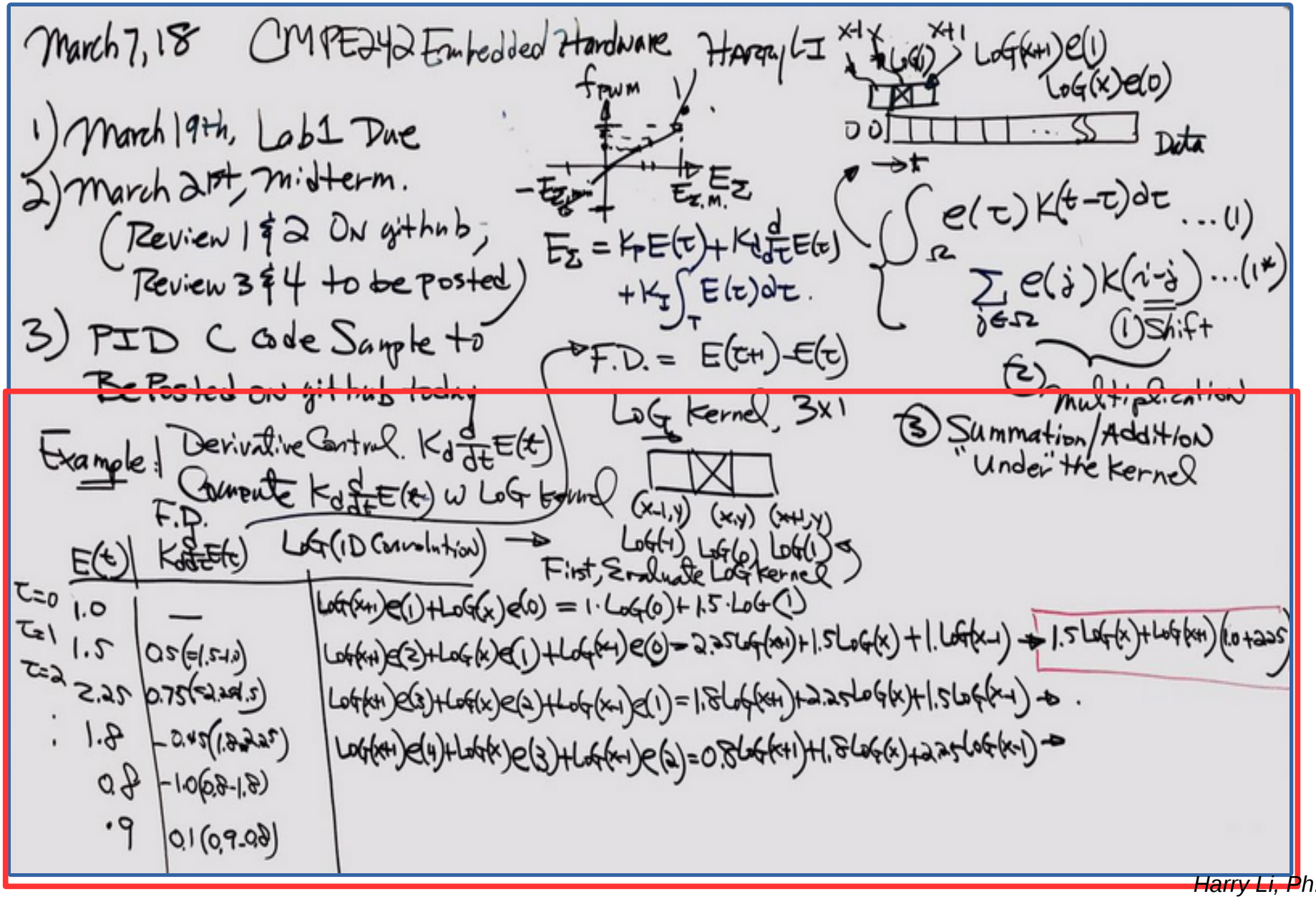
1D LoG Kernel



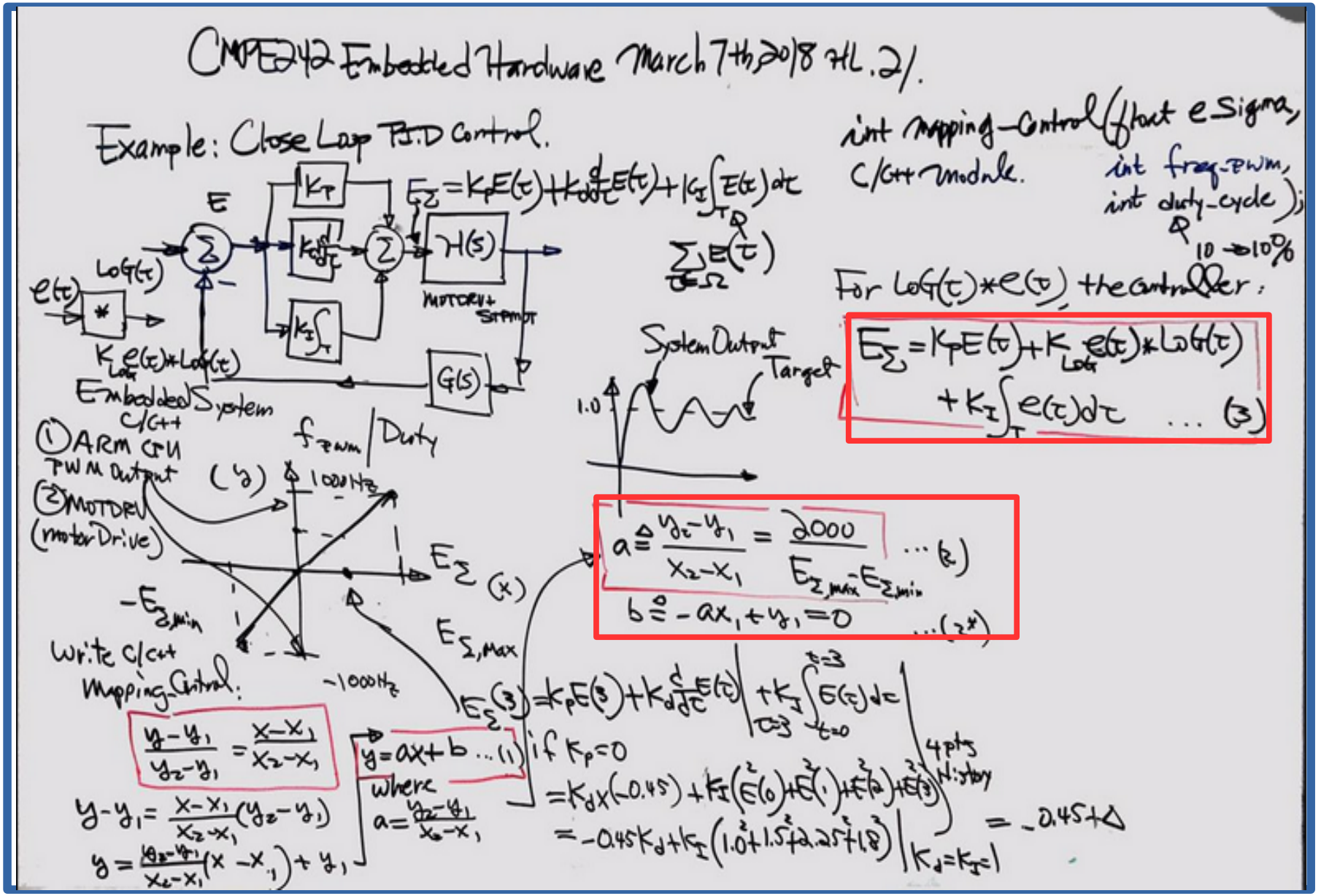
Calculate PID of Error



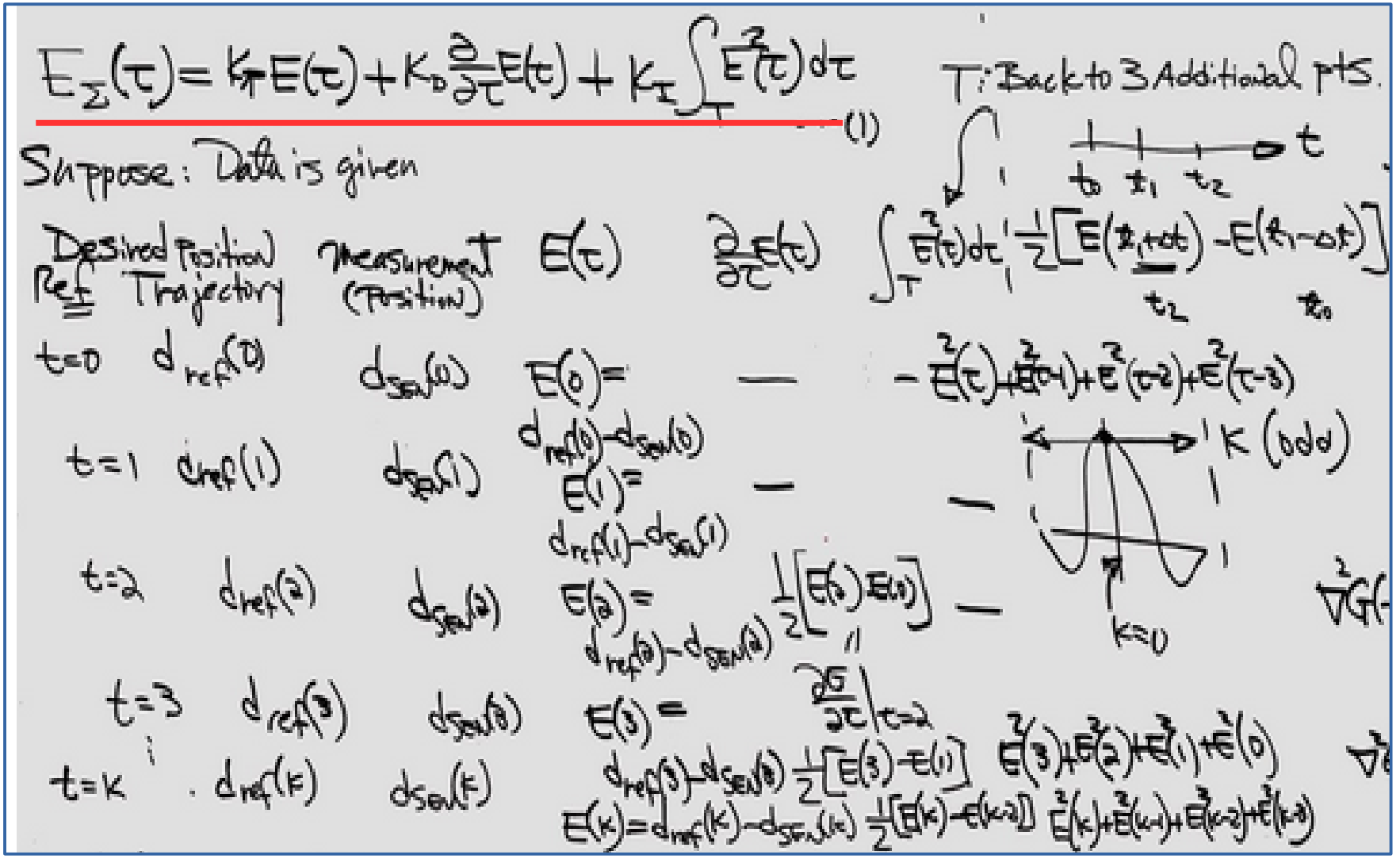
Convolution With LoG



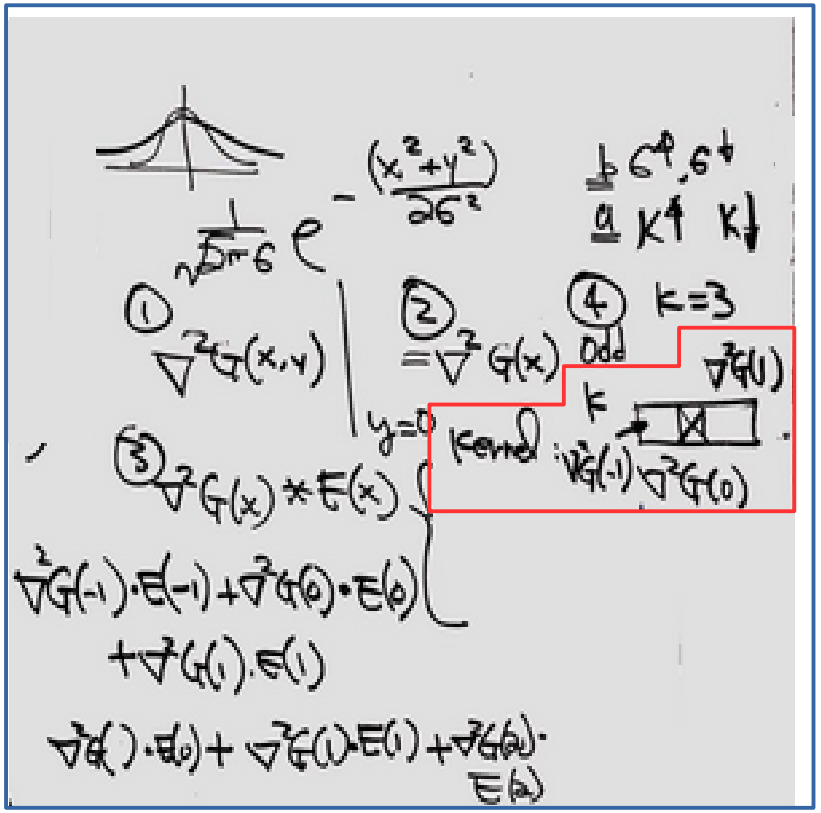
E-Sigma Mapping To PWM Control

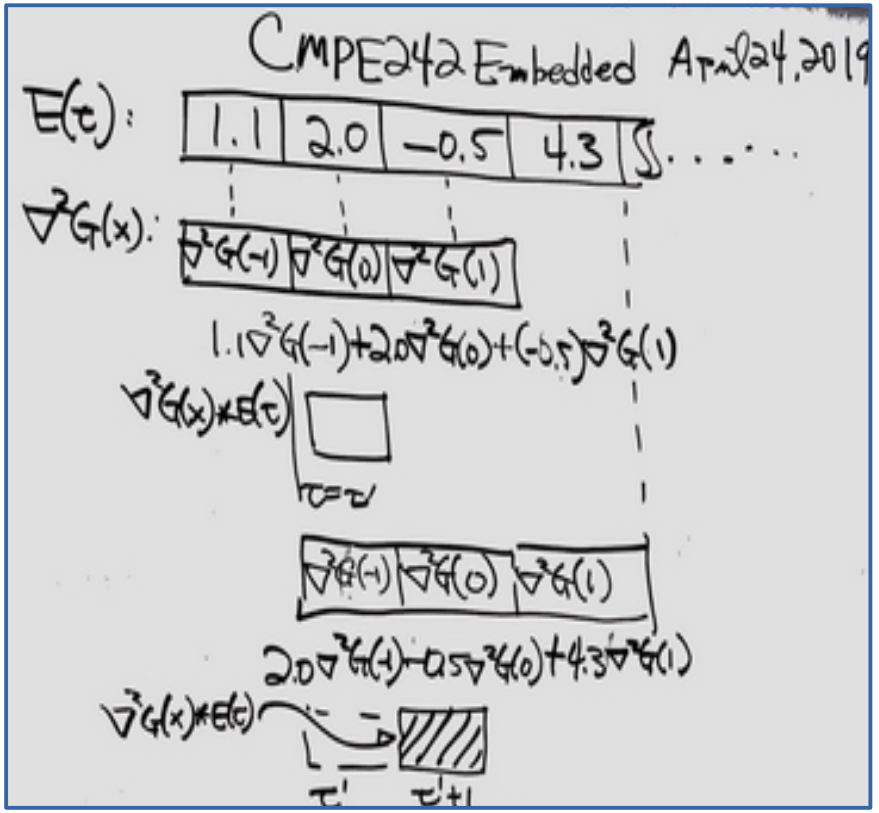


April 2019 PID Computation (1)



April 2019 PID Computation (2)





Homework: write C/C++ code to PID of the error given in the above example with K_p = 1, K_d = 2, and K_i = 4.