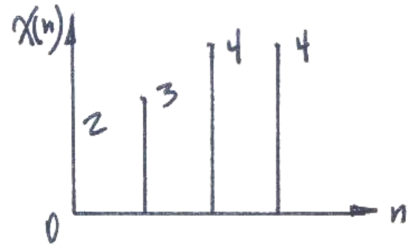
EE264 Discrete Fourier Transform
HL, 2009.1.23

1) Given A Discrete Signal $x(n)$ as follows, find its D.F.T.



Sol

By Definition 1D D.F.T. is

$$X(m)=\frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi\frac{mn}{N}} \quad \cdots(1)$$

Where $m=0,1,2,\cdots,N-1$

And the inverse D.F.T. is

$$x(n)=\sum_{m=0}^{N-1} X(m)e^{j2\pi\frac{mn}{N}} \quad \cdots(2)$$

From (1),

$$X(m)=\frac{1}{N}[x(0)W_N^0+x(1)W_N^m+\cdots x(N-1)W_N^{(N-1)m}]$$

Where $W_N^{mn}=e^{-j2\pi\frac{mn}{N}}$

hence, for $m=0$,

$$X(0)=\frac{1}{N}[x(0)W_N^0+x(1)W_N^0+\cdots+x(N-1)W_N^0]$$

$$X(1)=\frac{1}{N}[x(0)W_N^0+x(1)W_N^1+\cdots+x(N-1)W_N^{N-1}$$

$$X(2)=\frac{1}{N}[x(0)W_N^0+x(1)W_N^2+\cdots+x(N-1)W_N^{2(N-1)}]$$

etc.

Write the above N equations in matrix form, we have

$$\begin{bmatrix} X(0)\\ X(1)\\ \vdots\\ X(N-1)\end{bmatrix}=\frac{1}{N}\begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0\\ W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1}\\ W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)}\\ \vdots & & & & \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)}\end{bmatrix}\begin{bmatrix} x(0)\\ x(1)\\ \vdots\\ x(N-1)\end{bmatrix} \quad \cdots(3)$$

Now, for $N=4$,

$W_N^0=1$, $W_N^1=e^{-j2\pi\frac{1}{4}}=e^{-j\pi/2}=\cos\pi/2-j\sin\pi/2=0-j=-j$

$W_N^2=e^{-j2\pi\frac{2}{4}}=e^{-j\pi}=\cos\pi-j\sin\pi=-1$

$W_N^3=e^{-j2\pi\frac{3}{4}}=e^{-j\frac{3\pi}{2}}=\cos\frac{3\pi}{2}-j\sin\frac{3\pi}{2}=j$

Hence, (3) becomes

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\end{bmatrix}=\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}\begin{bmatrix} 2\\ 3\\ 4\\ 4\end{bmatrix} \quad \cdots(3^*)$$

So,

$X(0)=\frac{1}{4}(2+3+4+4)=3.25$

$X(1)=\frac{1}{4}(2-3j-4+4j)=\frac{1}{4}(-2+j)$

$X(2)=\frac{1}{4}(2-3+4-4)=\frac{1}{4}(-1)$

$X(3)=\frac{1}{4}(2+3j-4-4j)=\frac{1}{4}(-2-j)$

2) Based on question 1, find its Power spectrum.

Power Spectrum Definition

$$P(m) \triangleq \mathrm{Sqrt}\left[\mathrm{Re}^2(X(m)) + \mathrm{Im}^2(X(m))\right]$$

From the Computation of D.F.T. we have

$$P(m)\Big|_{m=0} \triangleq 3.25$$

$$P(1) = \frac{1}{4}\sqrt{2^2+1^2} = \sqrt{5}/4$$

$$P(2) = \frac{1}{4}\sqrt{(-1)^2+0^2} = 1/4$$

$$P(3) = \frac{1}{4}\sqrt{(-2)^2+(-1)^2} = \sqrt{5}/4.$$

(END)