Binary Images (Part 1) (Basics)

HL .1.10

1) Given a digital image $f(x,y)$ find Binary Image Based on threshold $T = 135$.

Sol

Based on $T=135$ and

$f(x,y)$ (x down, y across)

70	70	80	159
75	85	140	121
90	210	101	90
211	209	115	85

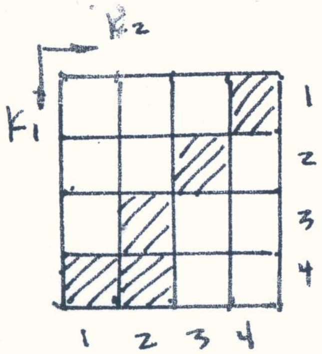
$$B(x,y) = \begin{cases} 255 & \text{if } f(x,y) \geq T \\ 0 & \text{o/w} \end{cases} \quad \ldots(1)$$

we have

(K_1 down, K_2 across)

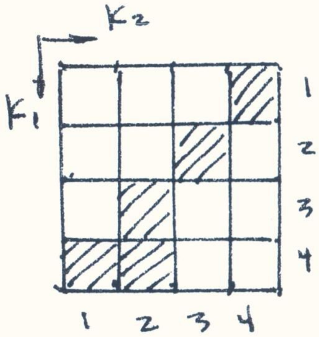
0	0	0	255
0	0	255	0
0	255	0	0
255	255	0	0

For the matter of Simplicity, we can further make normalization of the image intensity. the Brightest level is 1, and the darkest level is 0. So we have:



2) Given a $B(x,y)$, find (1) its size, (2) $\bar{X}$, (3) $\bar{Y}$

Sol



(1) Find its size

$$A = \sum_{K_1=1}^{4} \sum_{K_2=1}^{4} B(K_1,K_2)$$

$$= \sum_{K_1=1}^{4} B(K_1,1) + \sum_{K_1=1}^{4} B(K_1,2) + \sum_{K_1=1}^{4} B(K_1,3) + \sum_{K_1=1}^{4} B(K_1,4)$$

$$= (0+0+0+1) + (0+0+1+1) + (0+1+0+0) + (1+0+0+0)$$

$$= 5.$$

(2) Find $\bar{X}$.

$$\bar{X} \triangleq \frac{\iint_{\Omega} x B(x,y)\,dx\,dy}{\iint_{\Omega} B(x,y)\,dx\,dy}$$

So, in discret form,

$$\bar{X} = \frac{\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} K_1 B(K_1,K_2)}{\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} B(K_1,K_2)}$$

Since $\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} K_1 B(K_1,K_2) = \sum_{K_2=1}^{4} 1\cdot B(1,K_2) + \sum_{K_2=1}^{4} 2\cdot B(2,K_2) + \sum_{K_2=1}^{4} 3\cdot B(3,K_2) + \sum_{K_2=1}^{4} 4\cdot B(4,K_2)$

$$= 1\cdot B(1,1) + 1\cdot B(1,2) + 1\cdot B(1,3) + 1\cdot B(1,4)$$
$$+ 2\cdot B(2,1) + 2\cdot B(2,2) + 2\cdot B(2,3) + 2\cdot B(2,4)$$
$$+ 3\cdot B(3,1) + 3\cdot B(3,2) + 3\cdot B(3,3) + 3\cdot B(3,4)$$
$$+ 4\cdot B(4,1) + 4\cdot B(4,2) + 4\cdot B(4,3) + 4\cdot B(4,4)$$

$= 0+0+0+1\cdot B(1,4)$
$+0+0+2\cdot B(2\cdot 3)+0$
$+0+3\cdot B(3,2)+0+0$
$+4\cdot B(4,1)+4\cdot B(4,2)+0+0$
$= 1\cdot B(1,4)+2\cdot B(2,3)+3\cdot B(3,2)$
$+4\cdot B(4,1)+4\cdot B(4,2)$
$= 1\cdot 1+2\cdot 1+3\cdot 1+4\cdot 1+4\cdot 1$
$= 1+2+3+4+4=14$

AND,
$$\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} B(K_1,K_2) = A \text{ (size)}$$

From (1) we have already calculated
$A=5$.

SO,
$$\bar{X} = \frac{14}{5} = 2.8 \simeq 3.$$

Now,
$$\bar{Y} = \frac{\iint_\Omega y B(x,y)\,dx\,dy}{\iint_\Omega B(x,y)\,dx\,dy}$$

Or,
$$\bar{Y} = \frac{\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} K_2 B(K_1,K_2)}{\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} B(K_1,K_2)}$$

Where $\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} B(K_1,K_2)=A$,

e.g. $A=5$,

and
$$\sum_{K_1=1}^{4}\sum_{K_2=1}^{4} K_2 B(K_1,K_2)$$
$$= \sum_{K_1=1}^{4} 1\cdot B(K_1,1)+\sum_{K_1=1}^{4} 2\cdot B(K_1,2)+\sum_{K_1=1}^{4} 3\cdot B(K_1,3)+\sum_{K_1=1}^{4} 4\cdot B(K_1,4)$$
$= 1\cdot B(1,1)+1\cdot B(2,1)+1\cdot B(3,1)+1\cdot B(4,1)$
$+2B(1,2)+2\cdot B(2,2)+2\cdot B(3,2)+2\cdot B(4,2)$
$+3\cdot B(1,3)+3\cdot B(2,3)+3\cdot B(3,3)+3\cdot B(4,3)$
$+4\cdot B(1,4)+4\cdot B(2,4)+4\cdot B(3,4)+4\cdot B(4,4)$

$=0+0+0+1\cdot B(4,1)$
$+0+0+2\cdot B(3,2)+2B(4,2)$
$+0+3\cdot B(2,3)+0+0$
$+4\cdot B(1,4)+0+0+0$
$= 1\cdot B(4,1)+2\cdot B(3,2)+2\cdot B(4,2)$
$+3\cdot B(2,3)+4\cdot B(1,4)$
$= 1\cdot 1+2\cdot 1+2\cdot 1+3\cdot 1+4\cdot 1=12$

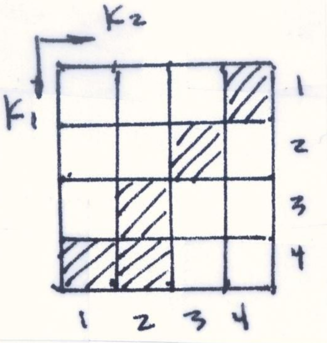
Hence,
$$\bar{Y} = \frac{12}{5} = 2.4 \simeq 2_{\shortparallel}$$

Binary Image Processing

Part II. HL

1) Given A Binary Image Below, find its perimeter P.

Sol By Definition, A pixel b(x,y) of P has at least one of its Neighbors belongs to background.



Hence

$$P = \int_{\Omega} f(x,y)\,dl \quad \cdots (1)$$

Or,

$$P = \sum_{K_1=1}^{N} \sum_{K_2=1}^{M} B'(x,y) \quad \cdots (2)$$

Where

$B'(x,y)$: A collection of Pixels whose Neighboring pixel(s) belong to the background.

Therefore,

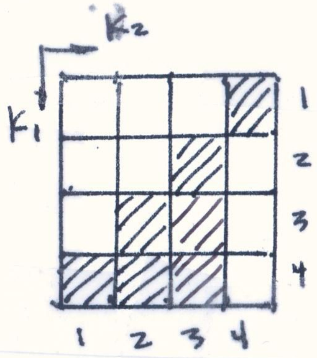
$$P = \sum_{K_1=1}^{4} \sum_{K_2=1}^{4} B'(x,y)$$

$$= B'(1,4) + B'(2,3) + B'(3,2) + B'(4,1) + B'(4,2)$$

$$= 5$$

Note: The above calculation is based on 4-connected Neighbors

2) Find Perimeter of the following Binary Image.



Sol

Since $P = \int_{\Omega} f(x,y)\,dl$

Or,

$$P = \sum_{K_1=1}^{N} \sum_{K_2=1}^{M} B'(x,y)$$

we have

$$P = \sum_{K_1=1}^{4} \sum_{K_2=1}^{4} B'(x,y)$$

$$= B'(1,4) + B'(2,3) + B'(3,2) + B'(3,3) + B'(4,1) + B'(4,3)$$

$$= 6$$

Note: $B'(4,2)$ does not have 4-connected Neighbors belonging To the Background.

(END)