CmpE258
Spring 2023 (Part III) 1/

April 20 (Thursday).

Note 1. Final Exam.

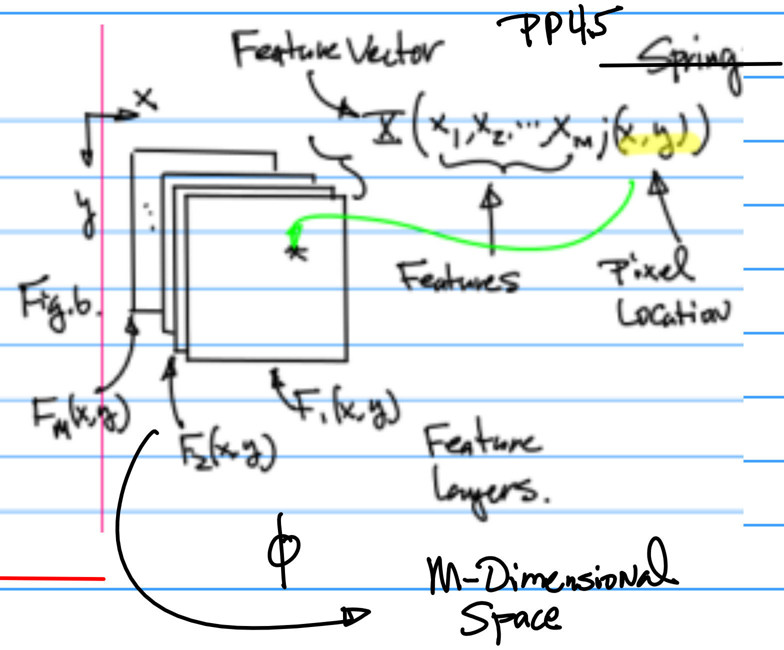
Cluster Analysis. "Mapping"

## Group II Classes

Group II classes are those classes which meet TR, T, R, TWR, MTR, TRF, MTI
MTWR, TWRF, RF, TF, TRS.

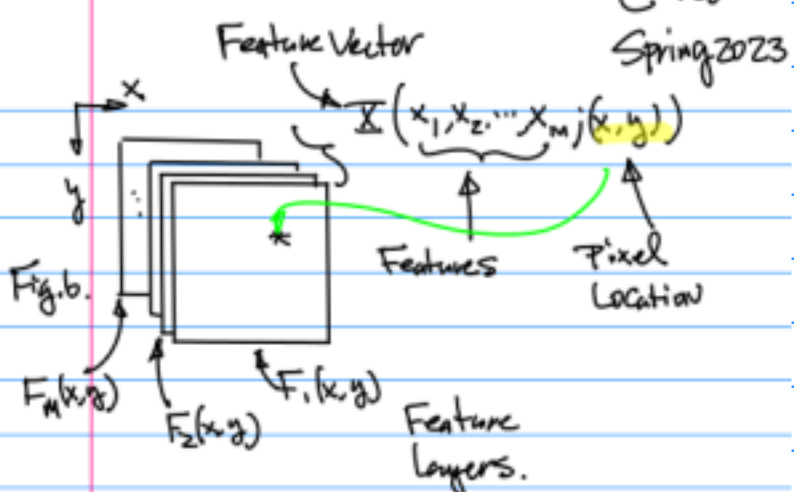
| Regular Class Start Times | Final Examination Days | Final Examinatior |
|---|---|---|
| 7:00 through 8:25 AM | Monday, May 22 | 7:15-9:30 AM |
| 8:30 through 9:25 AM | Wednesday, May 17 | 7:15-9:30 AM |
| 9:30 through 10:25 AM | Friday, May 19 | 9:45 AM-12:00 PM |
| 10:30 through 11:25 AM | Tuesday, May 23 | 9:45 AM-12:00 PM |
| 11:30 AM through 12:25 PM | Thursday, May 18 | 9:45 AM-12:00 PM |
| 12:30 through 1:25 PM | Monday, May 22 | 12:15-2:30 PM |
| 1:30 through 2:25 PM | Wednesday, May 17 | 12:15-2:30 PM |
| 2:30 through 3:25 PM | Friday, May 19 | 2:45-5:00 PM |
| 3:30 through 4:25 PM* | Tuesday, May 23 | 2:45-5:00 PM |
| 4:30* through 5:25 PM* | Thursday, May 18 | 2:45-5:00 PM |

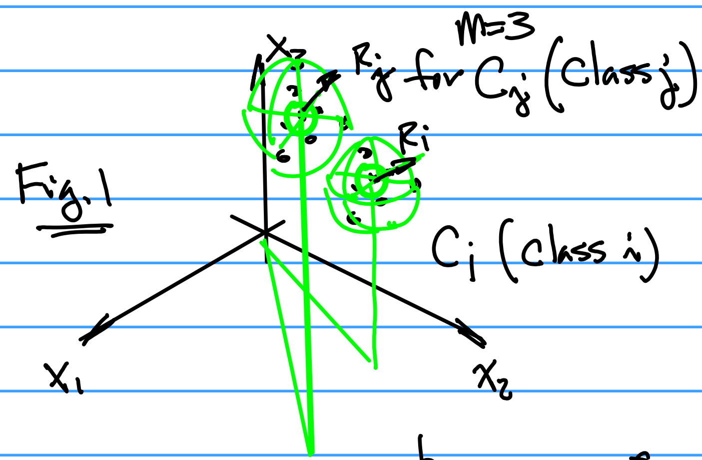
PP45 ~~Spring~~

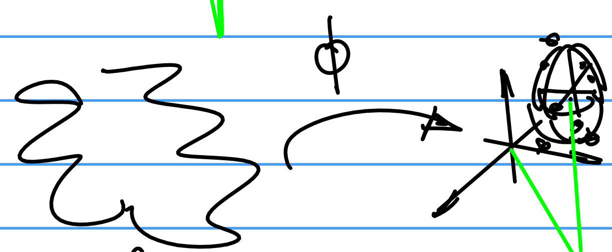



From Notes Part II, pp.45

CmpE258
Spring 2023



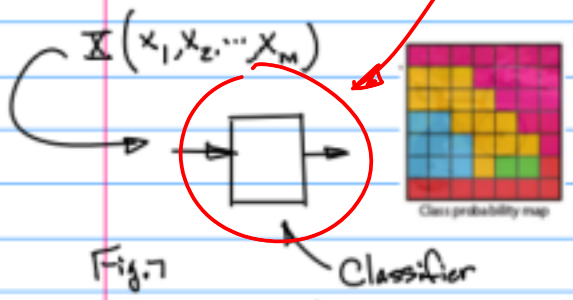




$\underline{X}(x_1, x_2, \cdots, x_N)$

$\underline{X}(x_1, x_2, \cdots x_M; (x, y))$

$\downarrow$




Cluster Analysis Technique.

$I(x,y)$ pixel(s) $\rightarrow$ Feature $\underline{X}(x_1, x_2, \cdots, x_N)$ $\xrightarrow{\phi}$ Classification $C_i$

Prob($C_i$)

CMPE258
Spring2023

(III) 2/

Note: $\sum_{i=1}^{N} \text{Prob}(C_i) = 1$ (Previously M)

Number of Classes.
from "Heuristics" Expert Knowledge.

Consider K-mean Cluster Algorithm.

github.

2022S-114c-Kmean-handCalculation...

2022S-114c-KmeanCluster-v3-2022-... PPT

Example: K-mean Cluster Algorithm.

First, Notation.

Note 1. Vectors

Given a set of observations ($\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_n$),

e.g. $\underline{X}_1 = (x_{11}, x_{12}, \cdots, x_{1N})$ ... (1)

first observation

For $\underbrace{\underline{X}_i}_{\text{Observation } i}$ $\quad x_{ij}$

Component $j$ for the Observation $i$

Note: if for d-dimensional Vector, then Eqn(1) has its $N = d$

K-mean

partition the $n$ observations into $k$ ($\leq n$) sets $\mathbf{S} = \{S_1, S_2, …, S_k\}$ ... (2)

$\{\underline{X}_i \mid i = 1, 2, \cdots, N\}$

K-Classes.

$S_i$ for Class $i$

Note 1. Note 2. Squared Distance

$$\underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} \sum_{\mathbf{x} \in S_i} \|\mathbf{x} - \boldsymbol{\mu}_i\|^2 = \underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} |S_i| \operatorname{Var} S_i \quad \ldots (3)$$

a. "Argmin" minimization

b. "S" Domain, Scope of the minimization

Example for $\|\vec{x} - \vec{\mu}_i\|^2$,

if $\vec{x} = (x_1, x_2)$, $\vec{\mu}_i = (\mu_{i1}, \mu_{i2})$

then

$$\|\vec{x} - \vec{\mu}_i\|^2 = \left(\sqrt{(x_1 - \mu_{i1})^2 + (x_2 - \mu_{i2})^2}\right)^2$$

$$= (x_1 - \mu_{i1})^2 + (x_2 - \mu_{i2})^2 \quad \ldots (4)$$

Note 3. "$\sum_{x \in S_i}$" Notation $\sum_{i=1}^{M} x_i$

Summation for each & every $x$ as Long as it is from the set $S_i$

Note 4: from Eqn(3), we have

$S_i$: Collection of Vectors $\underline{X}$ belonging to Class $i$

Note 5: $\sum_{i=1}^{k}$ → to Cover the Collection of all Classes.

Note 6: $\mu_i$ ① $\vec{\mu}_i = (\mu_{i1}, \mu_{i2}, \cdots, \mu_{id})$

② Cluster for the Class $i$

CmpE258

Spring2023

3

April 25 (Tue)

Note 1. Quick update on Project Progress Report. (Next Lecture)

Example: Continuation of K-mean Cluster Algorithm.

$$S_i^{(t)} = \{x_p : \|x_p - m_i^{(t)}\|^2 \le \|x_p - m_j^{(t)}\|^2 \ \forall j, 1 \le j \le k\} \quad \ldots (1)$$

Ref

20225-114c-Kmean-handCalculation1-c

Note 1: A Set of Feature Vectors

$S_i^{(t)}$ ← Captured at Step t

$i$ ← Class id: ith Class

$S_i^{(t)} = \{ \vec{X}_p \quad \}$

index, $p = 1, 2, \ldots$ just like Notation $i$, $j$, or $k$

$S_i^{(t)} = \{ \vec{X}_p : \quad \}$

Condition

$\|\vec{X}_p - \vec{m}_i^{(t)}\|^2$

Distance (Squared) at time (t) to the Cluster of class $i$

$\|\vec{X}_p - \vec{m}_j^{(t)}\|^2 \quad \ldots \quad \ldots$ to the Cluster class $j$

" $\forall j$ " for Any $j$, such as

$1 \le j \le K$ Covers all the different classes.

Hand Calculation Example

Given the following feature vectors use K-mean Algorithm to find the clusters.

$X_1 = \begin{bmatrix}0\\0\end{bmatrix}$ $X_2 = \begin{bmatrix}1\\0\end{bmatrix}$ $X_3 = \begin{bmatrix}0\\1\end{bmatrix}$ $X_4 = \begin{bmatrix}1\\1\end{bmatrix}$

$X_5 = \begin{bmatrix}2\\1\end{bmatrix}$ $X_6 = \begin{bmatrix}1\\2\end{bmatrix}$ $X_7 = \begin{bmatrix}2\\2\end{bmatrix}$ $X_8 = \begin{bmatrix}3\\2\end{bmatrix}$

$X_9 = \begin{bmatrix}6\\6\end{bmatrix}$ $X_{10} = \begin{bmatrix}7\\6\end{bmatrix}$ $X_{11} = \begin{bmatrix}8\\6\end{bmatrix}$ $X_{12} = \begin{bmatrix}6\\7\end{bmatrix}$

$X_{13} = \begin{bmatrix}7\\7\end{bmatrix}$ $X_{14} = \begin{bmatrix}8\\7\end{bmatrix}$ $X_{15} = \begin{bmatrix}9\\7\end{bmatrix}$ $X_{16} = \begin{bmatrix}7\\8\end{bmatrix}$

$X_{17} = \begin{bmatrix}8\\8\end{bmatrix}$ $X_{18} = \begin{bmatrix}9\\8\end{bmatrix}$ $X_{19} = \begin{bmatrix}8\\9\end{bmatrix}$ $X_{20} = \begin{bmatrix}9\\9\end{bmatrix}$

Sol: Step 1. Define K=2 per Heuristics. Expert Knowledge

Note: "0" Initial step.

Let cluster

$\vec{m}_1^0 = \vec{X}_1 = \begin{pmatrix}0\\0\end{pmatrix} \ldots (1)$

$\vec{m}_2^0 = \vec{X}_2 = \begin{pmatrix}1\\0\end{pmatrix} \ldots (2)$

Initial Arbitrary Values

"1" Class 1

And Arbitrarily assign Feature Vectors into 2 Classes.

Step 2. Use Eqn (1) To Compute the distance

$\|\vec{X}_p - \vec{m}_i^{(t)}\|^2$

and $\|\vec{X}_p - \vec{m}_j^{(t)}\|^2$

To Evaluate the Grouping of $\vec{X}_p$ to the Class $i$ per Eqn (1).

If Eqn(1) holds good, then $\vec{X}_p$ Stays in the Class $i$.
O/w Re-assign $\vec{X}_p$ to the Class $j$.

Step 3. Update the Cluster
(when New Grouping is formed)

$$m_1^{(t)}\Big|_{t=1} = \frac{1}{N}\sum_{\vec{X}_p \in S_1} \vec{X}_p$$

$$m_i^{(t+1)} = \frac{1}{|S_i^{(t)}|}\sum_{x_j \in S_i^{(t)}} x_j \quad \dots (2)$$

$\vec{X}_j$

Total Number of Feature Vectors in the Class $i$

Step 4. Carry out the Computation with the New Cluster.
(To Decide if the grouping is final or to Continue updating Cluster Values)

Note: "Stop" if No Regrouping
O/w. Continue By Repeating the process, e.g. update Cluster Values, the Evaluate the grouping.

Step 5. Perform the Computation as Described in Step 4. which Leads to

$C_1$ (Class 1) : $S_1 = \{\vec{X}_1, \vec{X}_2, \cdots, \vec{X}_8\}$

$S_2 = \{\vec{X}_9, \vec{X}_{10}, \cdots, \vec{X}_{20}\}$

then, Update the Cluster $\vec{m}_1^{(t)}, \vec{m}_2^{(t)}$.

↓

Check. No New Grouping

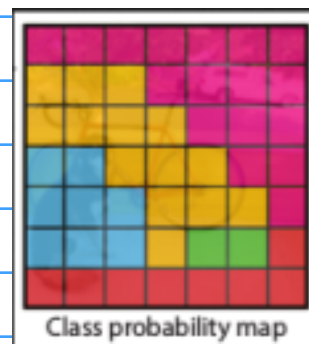
↓

$\vec{m}_1^{(t+1)}, \vec{m}_2^{(t+1)}$ are the Same.

∴ Stop. (Converged).

Discussion On Probability Distribution Map.

(a)



Class probability map

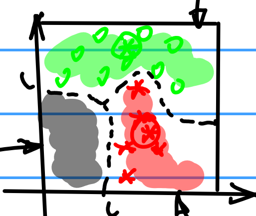
Feature Vector Map.

(b)

Boundary for the Classification

Regular K-mean Cluster Algorithm

(c)



$C_3$

$C_1$

$C_2$

Fig. 1

$$Prob(C_1) \stackrel{?}{=} \frac{\text{Area of Black Pixels}}{\text{Area of } \Omega \text{ (Image Plane)}} \quad \dots (3a)$$

where Area of Black Pixels
can be computed.

Area of $\Omega$(Image Plane) = Resolution
of the image plane.
For Example, 448×448

Similarly, find

$$Prob(C_2) = \frac{\text{Area of Red Pixels}}{\text{Area of } \Omega(\text{Image Plane})} \quad \ldots (3b)$$

$$Prob(C_3) = \frac{\text{Area of green Pixels}}{\text{Area of } \Omega(\text{Image Plane})} \quad \ldots (3c)$$

then

$$\sum_{i=1}^{N} \Big|_{N=3} Prob(C_i) = 1 \quad \ldots (4)$$

Since

$$\sum_{i=1}^{3} Prob(C_i) = Prob(C_1) + Prob(C_2) + Prob(C_3) \quad \ldots (4\text{-}b)$$

$$= \frac{\text{Area of Black Pixels}}{\text{Area of } \Omega(\text{Image Plane})} + \frac{\text{Area of Red Pixels}}{\text{Area of } \Omega(\text{Image Plane})} + \frac{\text{Area of green Pixels}}{\text{Area of } \Omega(\text{Image Plane})}$$

$$= \frac{\text{Area of } \Omega(\text{Image Plane})}{\text{Area of } \Omega(\text{Image Plane})} = 1.$$

//

(The following Notes were added After the Class After Recovering from the Laptop Computer Shut down. For Additional Lecture Notes Check the Class Zoom Recording).

2022S-114c-Kmean-prob-map-hl-2023-4-26.pdf

Step1. Feature Vectors

## Probability Distribution Map and Kmean Cluster Technique


$$\text{Prob}(C1) = \frac{8}{16} = \frac{1}{2}$$

$$\text{Prob}(C2) = \frac{8}{16} = \frac{1}{2}$$

From Probability Distribution Map.

Prob(Ci) = Prob(Ci/Obj) Prob(Obj) ... (1)
Prob(Ci/Obj) = Prob(Ci) / Prob(Obj) ... (2)

Application Example: Given ROI_i(x,y) with feature vector X, find it Prob(Ci/Obj) = ? by using equation (2)

Harry Li, Ph.D., SJSU

Detected

$X_1=\begin{bmatrix}0\\0\end{bmatrix}$ $X_2=\begin{bmatrix}1\\0\end{bmatrix}$ $X_3=\begin{bmatrix}0\\1\end{bmatrix}$ $X_4=\begin{bmatrix}1\\1\end{bmatrix}$

$X_5=\begin{bmatrix}2\\1\end{bmatrix}$ $X_6=\begin{bmatrix}1\\2\end{bmatrix}$ $X_7=\begin{bmatrix}2\\2\end{bmatrix}$ $X_8=\begin{bmatrix}3\\2\end{bmatrix}$

$X_9=\begin{bmatrix}6\\6\end{bmatrix}$ $X_{10}=\begin{bmatrix}7\\6\end{bmatrix}$ $X_{11}=\begin{bmatrix}8\\6\end{bmatrix}$ $X_{12}=\begin{bmatrix}6\\7\end{bmatrix}$

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Objective: To find the Probability of an object Belonging to Class i.

Requirements: 1° Hand Calculation of K-mean Cluster Algorithm;

2° Use K-mean Algorithm to Perform Image Segmentation, then Find Probability Distribution Map.

CMPE258

Spring2023

Note 1: Loss function Composition : $f_{Loss} = \sum_{i=1}^{k} \lambda_i f_{Loss,i}$ ...(1)

where $\sum_{i=1}^{k} \lambda_i = 1$.

No. of Bounding Boxes on a grid S

Entire Image

$$\lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{obj}} \left[ (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 \right]$$ Location

$$+ \lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{obj}} \left[ \left(\sqrt{w_i} - \sqrt{\hat{w}_i}\right)^2 + \left(\sqrt{h_i} - \sqrt{\hat{h}_i}\right)^2 \right]$$ Shape

$$+ \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{obj}} \left( C_i - \hat{C}_i \right)^2$$ ← Confidence

$\lambda'$

$$+ \lambda_{\text{noobj}} \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{noobj}} \left( C_i - \hat{C}_i \right)^2$$

$$+ \sum_{i=0}^{S^2} \mathbb{1}_{i}^{\text{obj}} \sum_{c \in \text{classes}} \left( p_i(c) - \hat{p}_i(c) \right)^2$$ Probability Distribution

$\lambda''$

Note 2:

Unit function

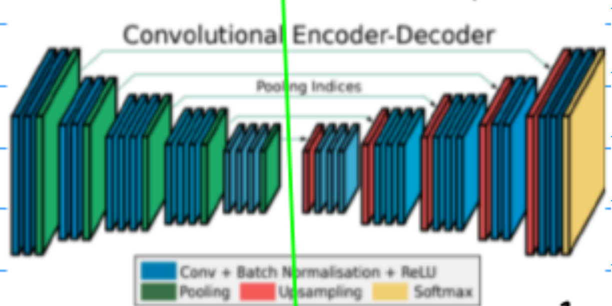
$\mathbb{I}_{ij}^{obj} = \begin{cases} 1 & \text{if Object exists} \\ 0 & \text{o/w} \end{cases}$ ...(2)

Location 50%, Shape 20%, ... etc.

Semantic Segmentation

2022F-109-semantic-seg-part1-HL-2022- ← Ref: PPT.

pp52 Fig.3




$\underbrace{\qquad\qquad}$ Deep Convolutional Neural Network:

Encoder:

{ Feature Extraction — Convolutions
{ Classification — Feedforward NN

$\underbrace{\qquad\qquad}$ Decoder:

Target Pixel by pixel

Recognition of Object

May 2nd (Tue)

Example: Super Sampling. A Technique to project Lower Resolution Feature Map / Image to A Higher Resolution Feature / Image.

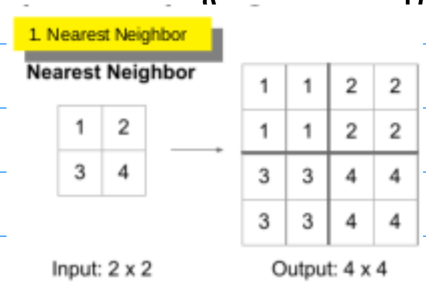
Ref: PPT. pp3.

2022F-109-semantic-seg-part1-HL-2022-11-10.pdf

Notes & Examples. PP53

2022F-101-cmpe258-note-part2-2022-12-6.pdf

Techniques 1° Nearest Neighbor Technique



# Bi-Linear Interpolation Technique

Step 1. Project "Anchor" Points

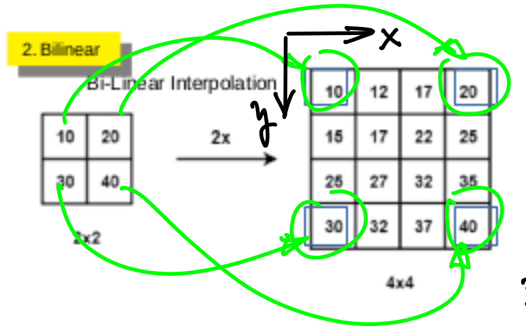



Fig. 1

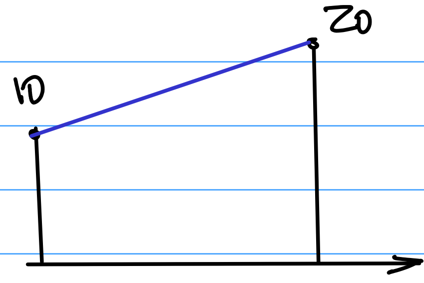



Fig. 2-c

Step 2. Bi-Linear Interpolation with 4 Anchor Points to fill up the rest of the feature points.

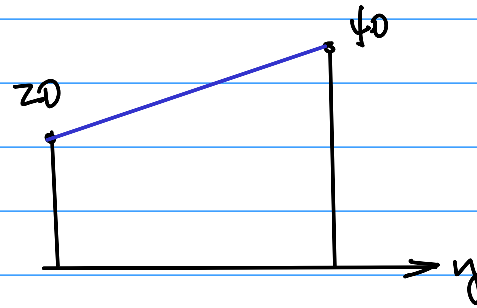



Fig. 2-d

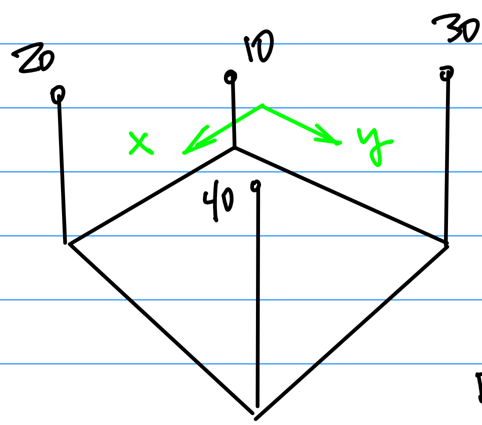



Fig. 2-a

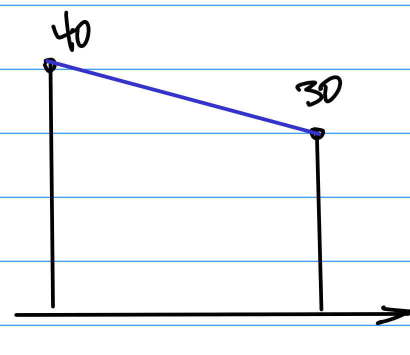
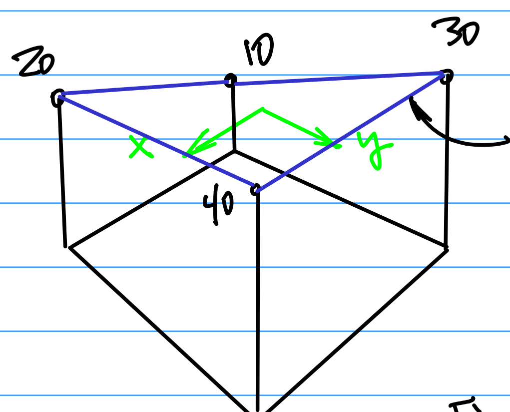



Fig. 2-e

"Right Hand" System




Straight Line

↓

Linear

↓

$y = ax + b$ ... (1)

Fig. 2-b

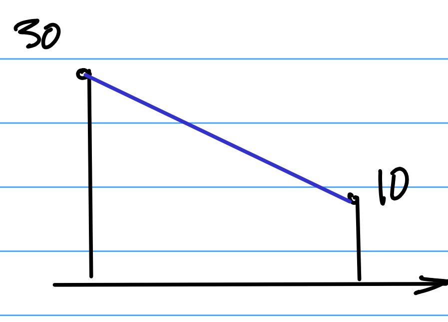



Fig. 2-f

From Fig. 2-C ~ Fig. 2-e,

The Interpolation is performed Linear W.r.t. x Variable; that is the 1st Linear Interpolation;

From Fig 2d ~ 2f,

The Interpolation is performed Linear W.r.t. y Variable; that is the 2nd Linear Interpolation;

Hen[ce] ... we ... ion.

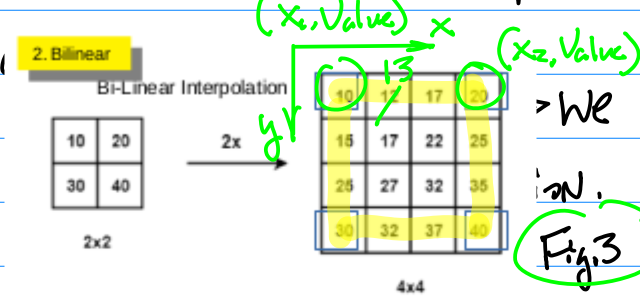



Fig.3

PP53

$$y = \underbrace{\frac{y_2 - y_1}{x_2 - x_1}}_{a} x \underbrace{- \frac{y_2 - y_1}{x_2 - x_1} x_1 + y_1}_{b} \quad \cdots (3)$$

$$a = \frac{y_2 - y_1}{x_2 - x_1} \quad \cdots (3\text{-}b)$$

$$b = -\frac{y_2 - y_1}{x_2 - x_1} x_1 + y_1 \quad \cdots (3\text{-}c)$$

Note: $y = ax + b$, whose $a$ & $b$ are derived in Eqn (3), (3-b) (3-c).

Example: Hand Calculation, PP55

From the given Condition

Coordinate Feature Value

$(x_1, y_1) = (0, 10)$ See Fig. 5-b.

$(x_2, y_2) = (3, 20)$

A

Hence $a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 10}{3 - 0} = 10/3$

and

$$b = -\frac{y_2 - y_1}{x_2 - x_1} x_1 + y_1$$

$$= -\frac{20 - 10}{3 - 0} \cdot 0 + 10 = 10$$

Therefore, from Eqn (3), we have

$$y = ax + b = \frac{10}{3} \cdot x + 10 \Big|_{x=1} = \frac{10}{3} + 10$$

$$\cong 3.3 + 10 = 13.$$

Please Carry out the Calculation for the Next Feature Value Using the Same Linear interpolation w.r.t. x.

Background: Given $(x_1, y_1)$, $(x_2, y_2)$ and $x_3$, Find $y_3 = ?$

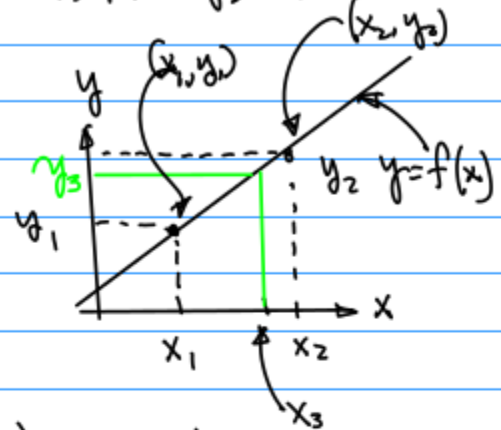



Fig.7

$y = f(x)$, $\underline{y = ax + b}$ $\quad \cdots (1)$

Which is a linear function, (since x is Not in 2nd, 3rd, or higher order).

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1} \quad \cdots (2)$$

Solve for a and b in the Above equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Now, Consider the 2nd Linear Interpolation w.r.t y.

From Ref. pp 55

B. $(x_1, y_1) = (0, 10)$

$(x_2, y_2) = (\cancel{0}\ 3, 30)$

Feature Value (Function) from the feature map.

Independent Variable → "y"

Along the y-axis of the feature map.

Then use Eqn (3-b) & (3-c) to find slope a and offset b; as follows

$$a = \frac{30-10}{3-0} = 20/3$$

From Ref. pp.55

$y_2'$ $y_1'$

$$b = -\frac{y_2 - y_1}{x_2 - x_1} x_1 + y_1$$

$$= -\frac{30-10}{3-0} \cdot y_1' + 10 \Big|_{y_1' = 0}$$

$$= 10$$

Therefore, we have

pp55

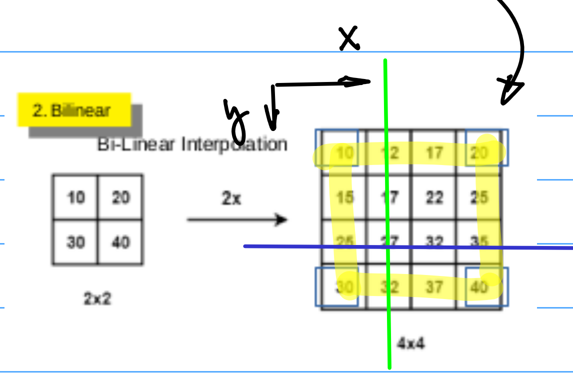
$$y = ax + b$$

$$= 20/3 \cdot x + 10 = 20/3 \cdot y' + 10$$

$$\simeq 17$$

see the illustration below.

Carry out the Calculations, we Can have all Boundary points fixed as illustrated below



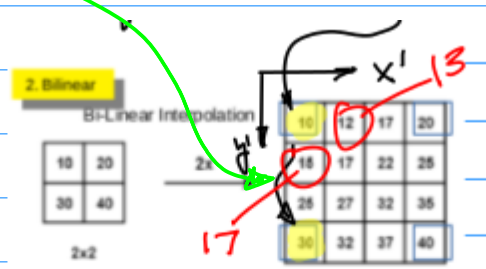

then, for the interior

May 4 (Th). Example: Bi-Interpolation for Up-Sampling/Super Sampling.

Continuation of finding the interior feature point. (Green Line, Blue Line in the Figure Above)

Use "Green" Line, interpolation w.r.t y see "B" on this page;

OR, equivalently, use "Blue", the interpolation w.r.t x. See A on pp.9.



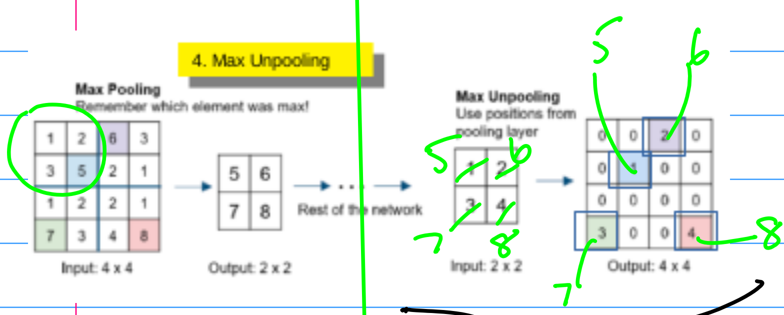



Fig. 1.

1° Locations (Registered During Max-Pooling).
2° Values go to the Registered Location.

Note: for Up-Sampling.

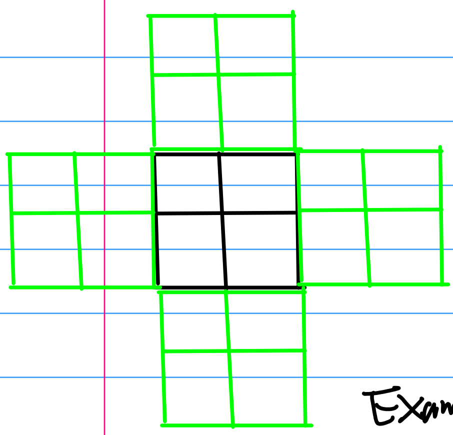


Fig. 2

2022F-109-semantic-seg-part1-HL-2022-11-10.pdf

Example: Ref. pp. 5

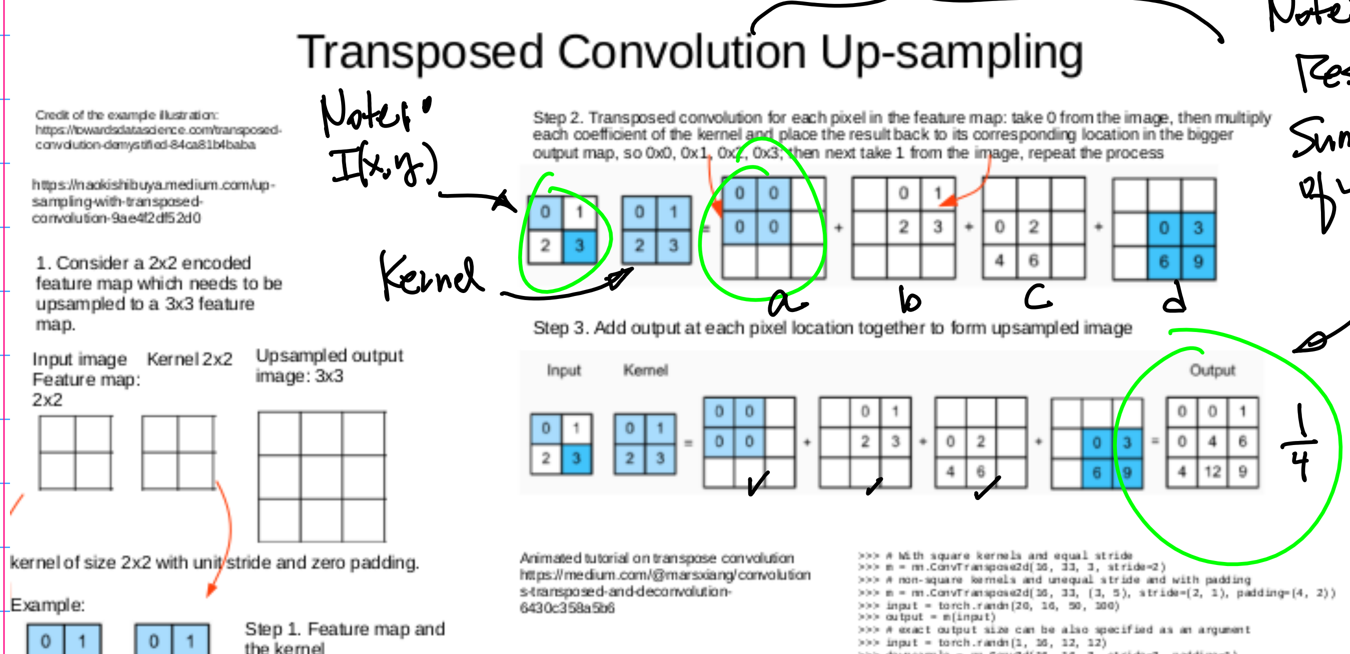
Note 2: 4 Convolved Results

Fig. 3

Note 3: Result Summation of 4 ~

Note 1: I(x, y)

Kernel




$\frac{1}{4}$

CMPE258
Spring 2023



Step 1. Take a coefficient at the top left corner from the Kernel, which is 0; then multiply it for each & every pixel of the image, then,

$0*0=0$, $0*1=0$

$I(x,y)\big|_{x=0,y=0}$ $\quad$ $I(x,y)\big|_{x=1,y=0}$

$0*2=0$, $0\times 3=0$

$I(x,y)\big|_{x=0,y=1}$ $\quad$ $I(x,y)\big|_{x=1,y=1}$

To place the Results back to the processed feature plane with matching to their locations.

Step 2. Take the 2nd Coef. from the Kernel, which is equal to 1. Repeat the Computation with this coefficient, so

$1*0=0$, $1*1=1$

$I(x,y)\big|_{x=0,y=0}$ $\quad$ $I(x,y)\big|_{x=1,y=0}$

$1*2=2$, $1*3$

$I(x,y)\big|_{x=0,y=1}$ $\quad$ $I(x,y)\big|_{x=1,y=1}$

place the Results to their Corresponding Locations. see Fig.3.b.

Step 3. Then take Coefficient = 2, Repeat the Computation.

$2*0=0$, $2*1=2$
$2*2=4$, $2*3=6$.

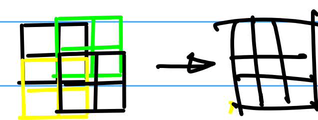
So, we have c in Fig.3

Step 4. Similarly, we have the Result d in Fig 3.

Step 5. Combine the 4 Computation Results together to form one feature map.

Note: Scaling factor $\frac{1}{4}$ is applied to keep the final Result in the same dynamic Range

up-sampled by



Transposed Convolution.

Note: 1° Team Project Presentation Starting on Tue (May the 9th). Please check the Name List, identify your presentation schedule.

First 25 people on the List → if your Coordinator on the List → presentation on the 9th.

CmpE258
Spring 2023



10 min. Time slot for Each team; 7 min for PPT, then Demo; 1~2 min. for Q&A.

Final Exam:

1° 3~4 Questions. 2 hr. 15 mins.

| | | |
|---|---|---|
| 4:30* through 5:25 PM* | Thursday, May 18 | 2:45-5:00 PM |

Thur. May 18, 2:45-5:00 pm.

Same format as the midterm Exam;

2° Video Cam ON All the time, Video Recording is ON:

3° please make sure Laptop/Desktop Computer is Ready; Homework Code Projects Code Ready. You will Need to Run the Code;

85%~90% Newer Material Since the midterm;

4° One page formula is allowed, No NoteBook/Notes Allowed;

5° Time Limit → Submission ON CANVAS only, No Exceptions.

6° Scopes of the Questions:

Architecture(s); Math Formulation and Analysis; Hand Calculation, Examples in the lectures; Coding in Homeworks & Projects.