

COMP1163 August 20 (Fri)

Organizational Meeting

1) Harry Li Email:

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(650) 400-1116 Text

Office: M,W, 3:40-4:40pm.

Zoom ID + Passcode

is the same as

what you have today.

Lecture Zoom Link sent to
the class today.

Note: Homework, Projects
Announcements will be made
in Class, posted online as
github

CANVAS, Submission of homework
Projects will be
on CANVAS.

Text Books + References (optional)

a. Unity Tutorial, 3D Graphics

Game Dev. Engine

b. Other Optional Text Books —

Reference Only.

Programming Languages + Software IDE

1. Unity, Student or Personal

Edition. → Karting Game

2. Python for Graphics
Video, Version 3.6 or
higher.

Anaconda; Tool for
Python Programming →

3. C/C++ for 2D & 3D
Graphics, Videos.

4. C# for Interface to
Unity IDE.

→ 5. OpenGL Homework:
Installation of OpenGL,
In 2 weeks Sept. 2nd (Th)

→ 6. OpenGL Installation
of OpenGL. Homework:
Installation, and have it
ready by next week

Aug. 26 (Th) Before
4:00 pm..

→ 7. O.S. Ubuntu 18.04

Installation of Unity
By Aug. 26 (Th),
Before 4:00 pm..

2/

Grading Policy:

30% Projects/Homework etc.
30% midterm (ONE)
40% Final (Comprehensive)

"GAME"-Like Environment

{ Robotics
Self-Driving.

Conduct of the Class

- 1° Lecture
- 2° Show+Tell
- 3° Form A team, 2-3 person Team.

All homework, Coding have to be individual, however teamwork is encouraged, and be required

Projects, Homework: Assigned Projects.
(3 projects)

plus A-Semester-long project (Team)

a 2-3 person team;

b Proposal of A-Semester-long Project;

c Progress Report & Presentation During Class Show+Tell

d Final Presentation (PPT. Demo)

3 projects.

Project to Build 3D Animated Graphics.

Virtual Camera + Video

3D

Graphics + Video

a Scene View Window

b Hierarchy Window

3D

Graphics + Video

August.26 (Th)

Topics 1° Software Development Tool

2° Vertex Graphics
2D Vertex Graphics.

Reference Link: [github/valdih](https://github.com/valdih)

Software Tool: First, Unity Up

By Friday

OpenGL Installation on your Machine

Example: Running Unity ,
"Karting" GAME

Start the Unity .

Step1. On the right hand UI. Interactive Tutorial Panel (Window)

Select/Go through 2 Tutorial

First.Tutorial — play the Karting GAME

Step2. UI Editor

a Scene View Window

b Hierarchy Window

Comp1b3

3

Hierarchy: "Everything" defined in this Window,

$\stackrel{a}{=} \text{PAN}$;

$\stackrel{b}{=} \text{Zoom In/Out}$, $\stackrel{c}{=} \text{Orbit Movement}$
(Virtual Camera)

Use this platform to modify the "Karting" Game. Removal of Some/all

3D Objects

Re-Building 3D Scene. System
(3D World Coordinate)

2D Vector Definition of a Line

Segment

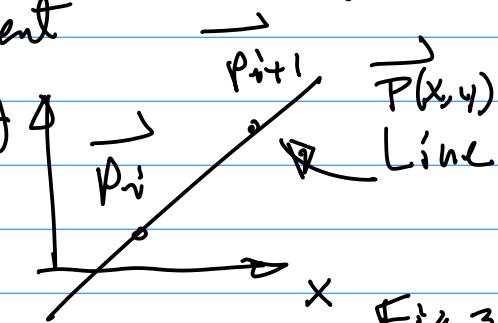


Fig.3

x-y Coordinate System

"Virtual Display" Coordinate System

Introduction to 2D Vector Graphics.

Dimensional
Description

Vertices (Vertex)

To Define Graphics

Pattern(s)

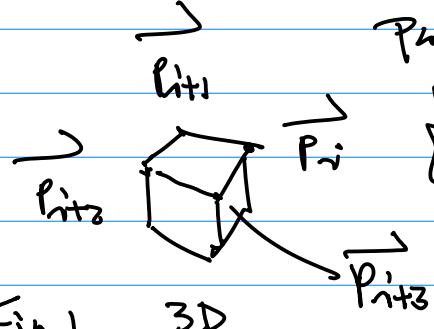


Fig.1. 3D

Primitive Graphics

2 pts to uniquely define a line

\vec{P}_i, \vec{P}_{i+1}

Notation

\vec{P}_i Short Hand Notation

$P_i(x_i, y_i), x_i-, y_i-$ comp.

$P_i(x_i, y_i) = (x_i, y_i)$ for

Coding in C/C++, Python, ...

Vector \rightarrow Vertex \rightarrow Point



To Define A line

① Direction of the Line

$$\vec{d} = \vec{P}_{i+1} - \vec{P}_i \quad \dots (1)$$

\uparrow

\uparrow

Ending Pt. Starting Pt

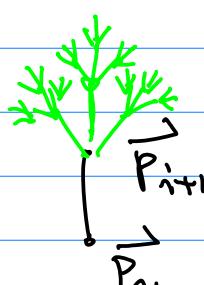


Fig.2

2D Vector
Graphics

Eqn(1), can be written as follows

$$\vec{d}(x_d, y_d) = \vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \vec{P}_i(x_i, y_i) \\ \dots (1-a)$$

For Coding purpose,

$$x_d = x_{i+1} - x_i \quad (1-b)$$

$$y_d = y_{i+1} - y_i \quad (1-c)$$

Write C code for the directional vector in Eqn(1-b), (1-c)

Question: How to find the Ending pt from Eqn(2a) ?

if $\lambda = 1$

$$\begin{aligned} \vec{P}(x, y) &= \vec{P}_i(x_i, y_i) + 1 \cdot (\vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \\ &\quad \vec{P}_i(x_i, y_i)) \\ &= \vec{P}_i(x_i, y_i) + \vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \\ &\quad \vec{P}_i(x_i, y_i) \\ &= \vec{P}_{i+1}(x_{i+1}, y_{i+1}) \text{ Ending pt.} \end{aligned}$$

$$\begin{aligned} x_d[i] &= x[i+1] - x[i] ; // \text{for } x\text{-Comp of the directional vector} \\ y_d[i] &= y[i+1] - y[i] ; // \text{for } y\text{-Comp. of the directional Vector.} \\ &\dots (1-d), (1-e) \end{aligned}$$

(2) Need A pt to make an Unique Line

$$\vec{P}(x, y) = \vec{P}_i(x_i, y_i) + \lambda \vec{d}(x, y) \quad \dots (2)$$

Where λ is scalar

Physical meaning: $\vec{P}(x, y)$ Any pt. on the Line

$\vec{P}_i(x_i, y_i)$ A given pt (Known) on this Line

$\vec{d}(x, y)$, A directional vector of the Line

Let $\lambda = 0$, $\vec{P}(x, y) = \vec{P}_i(x_i, y_i)$ starting pt.

From Eqn(2), $\vec{P}(x, y) = \vec{P}_i(x_i, y_i) + \lambda \left(\vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \vec{P}_i(x_i, y_i) \right) \dots (2a)$

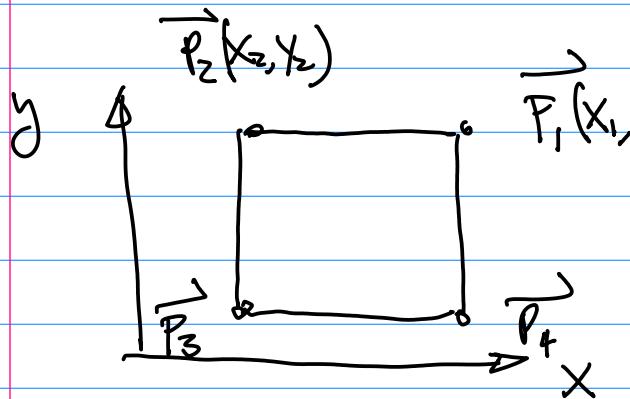
ScreenSaver a collection of 2D

Rotating Patterns. (Squares)

Example: Using Eqn(2a) to Create
2D Rotating Squares as a Screen
Saver.

Step 1. Define 2 vectors (\vec{P}_i s)

$\vec{P}_i(x_i, y_i)$, and $\vec{P}_{i+1}(x_{i+1}, y_{i+1})$



$$\vec{P}_1(x_1, y_1) = (60, 60), \vec{P}_2(x_2, y_2) = (10, 60)$$

And to Define A Line in Parallel with \vec{P}_1 & \vec{P}_2 $(1-d), \frac{1}{2}(1-e)$

$$\vec{P}_3(x_3, y_3) = (10, 10), \vec{P}_4(x_4, y_4) = (60, 10)$$

Connect \vec{P}_2 to \vec{P}_3 , Similarly \vec{P}_1 to \vec{P}_4

Therefore, we have formed A Square

Line Equation for Line (Top Line)

$$\vec{P}(x, y) = \vec{P}_1(x_1, y_1) + \lambda (\vec{P}_2(x_2, y_2) - \vec{P}_1(x_1, y_1)) \dots (3a)$$

Line for $\vec{P}_2(x_2, y_2)$ and $\vec{P}_3(x_3, y_3)$

$$\vec{P}(x, y) = \vec{P}_2(x_2, y_2) + \lambda_2 (\vec{P}_3(x_3, y_3) - \vec{P}_2(x_2, y_2)) \dots (3b)$$

And for the other 2 Lines

$$\vec{P}(x, y) = \vec{P}_3(x_3, y_3) + \lambda_3 (\vec{P}_4(x_4, y_4) - \vec{P}_3(x_3, y_3)) \dots (3c)$$

And

$$\vec{P}(x, y) = \vec{P}_4(x_4, y_4) + \lambda_4 (\vec{P}_1(x_1, y_1) - \vec{P}_4(x_4, y_4)) \dots (3d)$$

These 4 equations define
the Boundary of the Square.

From Coding Aspect:

From Sample/Example

Eqn(3a) becomes

$$\begin{cases} x = x_1 + \lambda (x_2 - x_1) \\ y = y_1 + \lambda (y_2 - y_1) \end{cases} \dots (4a) \quad \dots (4b)$$

Define A buffer for x,

And a buffer for y.

Each x_1, y_1, x_2, y_2 are also

Therefore C/C++ Coding Implementation
for (h-a), (h-b) can be
done accordingly.

Homework: Install OpenGL on your
Machine By Next Lecture,
So we will use it for
Rotating Sphere implementation.



Sept 2nd (Th)

Topics: 1° 2D Screen Saver
Implementation;

Ref: [github/kanalilj/OpenGL/](https://github.com/kanalilj/OpenGL/)

Homework: (To Be Submitted in
1 week) Submission 4:00 pm.

Sept. 9th (Th) (1 pt)

Visit homework Assignment
on OpenGL, Source code .CPP
has been posted.

Example: OpenGL CPP code

1° Create A program header
template, Start your
Program with this
Unified template

- a. Program Name
- b. Coded by
- c. Date , d. Version
- e. Status (Debugging, Release).
- f. Compilation and Build; g.

Ref. (URL)

```
b. j glBegin( );  
j glEnd( );  
glClear();  
GL_POLYGON Keyword.
```

Vertex (pt)

Homework: GL_LINES

modify the Sample code
to draw a line with

$$\begin{aligned} P_1(x_1, y_1) &= P_1(x_1, y_1) = (50, 50) \\ P_{1t}(x_{1t}, y_{1t}) &= P_2(x_2, y_2) = (60, 100) \end{aligned}$$

Compile your program, run it.
 E-mail your Screen Capture,
 or 5 seconds Video Clips.

Submission

in e-mail,

Before Sept 9

4:10pm.

(No point)

7.

Create Rotating Squares for
 a Screen Saver.

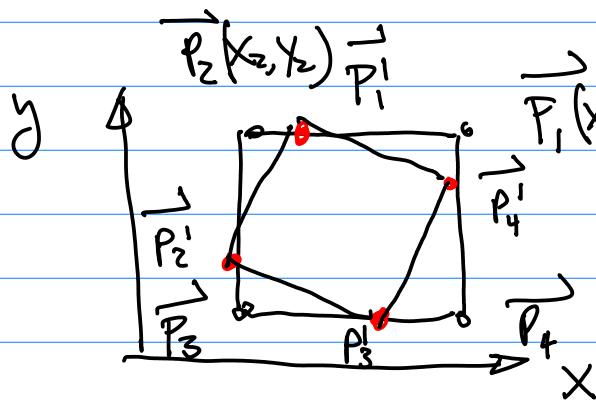


Fig 1. Note: $\vec{P_1}, \vec{P_2}, \dots, \vec{P_4}$ are
 defined in A Counter Clock wise
 direction (Later in 3D Graphics
 we will do Hidden Line/Surface
 Removal)

Repeat the Same Process, however
 with New set of Points, $\vec{P_1}', \vec{P_2}',$
 $\vec{P_3}', \vec{P_4}'$

Continue this process, we have:

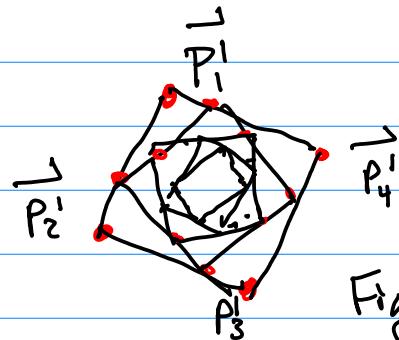


Fig. 2

To generalize this process,
 we introduce a superscript
 j as follows,

From Eqn(za)

$$\vec{P}(x, y) = \vec{P}_1(x_1, y_1) + \lambda (\vec{P}_2(x_2, y_2) - \vec{P}_1(x_1, y_1))$$

Let $\lambda = 0.8$ To S

$$\vec{P}(x, y) = \vec{P}_1(x_1, y_1) + \lambda (\vec{P}_2(x_2, y_2) - \vec{P}_1(x_1, y_1)) \quad \dots (za)$$

If $\lambda = 0.8$, \vec{P} is 80% pt on the line
 formed by \vec{P}_1 & \vec{P}_2 ;

$$\vec{P}(x, y) = \vec{P}_i(x_i, y_i) + \lambda (\vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \vec{P}_i(x_i, y_i)) \dots (2a)$$

becomes

$$\vec{P}_i^{j+1}(x_i^{j+1}, y_i^{j+1}) = \vec{P}_i^j(x_i^j, y_i^j) + \lambda (\vec{P}_{i+1}^j(x_{i+1}^j, y_{i+1}^j) - \vec{P}_i^j(x_i^j, y_i^j)) \dots (1)$$

The Above Equation can be written in Explicit form (x - Comp, y - Comp)

for x -Comp.

$$x_i^{j+1} = x_i^j + \lambda (x_{i+1}^j - x_i^j) \dots (2a)$$

$$y_i^{j+1} = y_i^j + \lambda (y_{i+1}^j - y_i^j) \dots (2b)$$

C/C++ Code

$$x_buf[i][j+1] = x[i][j] - \text{lambda} * (x[i+1][j] - x[i][j]);$$

$$y_buf[i][j+1] = y[i][j] - \text{lambda} * (y[i+1][j] - y[i][j]);$$

Sample Code Example: [github/hualili/OpenCV.../1-line.cpp](https://github.com/hualili/OpenCV.../1-line.cpp).

https://github.com/hualili/opencv/blob/master/ComputerGraphics_AR/F2018/1_line.c

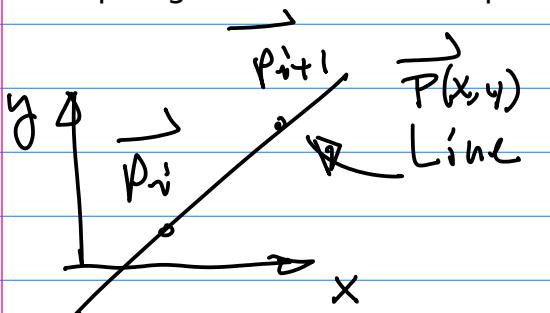


Fig.4

```

1  ****
2  * Program: line.c  Coded by: Harry Li
3  * Version: x1.0;  status: tested;
4  * Compile and build:
5  * gcc main.cpp -o main.o -lGL -lGLU -lglut
6  * Date: Jun 5, 2014
7  * Purpose: Graphics Demo.
8  ****
9  #include<GL/glut.h>
10 #include<stdio.h>
```

① Header

② Libraries

11 void mydisplay()
12 {
13 float p1x=1.0f, p1y=1.0f; //the window coordinates (-1.0, 1.0)
14 float p2x=-1.0f, p2y=-1.0f;

(3) $\vec{P}_1(x_1, y_1), \vec{P}_2(x_2, y_2), \vec{P}_1(x_1, y_1) = (1, 1), \vec{P}_2 = (-1, -1)$

```
15 glClear(GL_COLOR_BUFFER_BIT);
16 glLoadIdentity();
```

(4)

Note: House Keeping for 2D Graphics

(5) {
glBegin();

GL_LINES

```
17 glBegin(GL_LINES);
18 glVertex2f(p1x, p1y);
19 glVertex2f(p2x, p2y);
20 glEnd();
```

$$\text{glVertex2f}(x, y) \rightarrow \overrightarrow{P_i} (x_i, y_i) = (x_i, y_i)$$

Note: In your homework, please
2D Sample code,

Example: Creating a Tree By 2D
vector graphics.

Sept. 9 (Th)

1. Show-And-Tell
2. Today's Topics: 2D Vector
Graphics for Screen Saver
Application, Squares (Rotating
Pattern), Trees

Show+Tell Ken, Bill.

Patrick, Anh

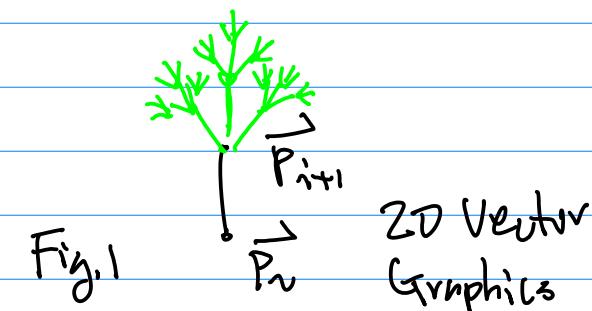
Homework: Implementation of
Rotating Squares Based on
Eqn (a)-(2b), or pp 8.

The A week from Today.

Show+Tell

Optional Homework: Write
a script CS (C#) to
control your Design.

~ 2 weeks



- Note:
- 1° The levels of Iteration should be at least 7 or higher;
 - 2° Random Function Generator to Allow Each tree to be placed Random Locations.
 $\text{at } \text{rand}();$
 - 3° Python Version OpenGL for the implementation is encouraged.

Description of the Algorithm:

1. You Design the length of A Tree Trunk By 2D Vectors. P_i, P_{i+1} ,
 P_i : Starting pt; P_{i+1} Ending pt.

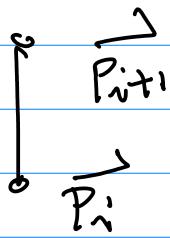


Fig. 2

$$\vec{P}_i(0, -10) \quad \vec{P}_{i+1}(0, 0)$$

--- (x)

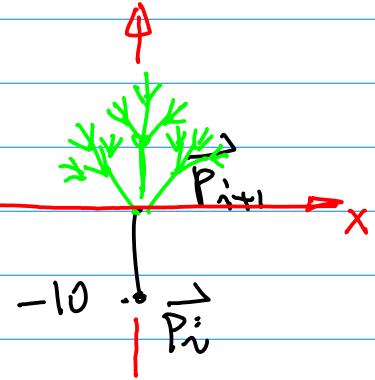


Fig. 4

2. Generate Another (Next Level)

Tree Branch \rightarrow Main Branch

(Same Direction) as its previous level;

a. Same Direction
b. Reduction By 20%

From Eqn.(1a), (1b),
with the given condition (x),
we have

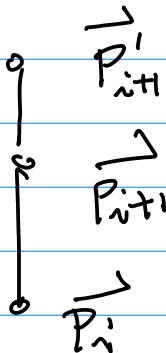


Fig. 3

$$\begin{aligned}\vec{P}'_{i+1} &= \vec{P}_{i+1} + \lambda(\vec{P}_{i+1} - \vec{P}_i) \\ &= (0, 0) + \lambda((0, 0) - (0, -10)) \\ &= (0, 0) + \lambda(0, 10) \dots (2)\end{aligned}$$

$$\vec{P}'_{i+1}(x'_{i+1}, y'_{i+1}) = \vec{P}_{i+1} + \lambda(\vec{P}_{i+1} - \vec{P}_i) \dots (1)$$

Starting Pt.

Direction
Vector of the
previous level

To find New Ending pt \vec{P}'_{i+1}
Let $\lambda = 0.8$, Substitute λ
into the above equation

$$\vec{P}'_{i+1} = (0, 0) + 0.8(0, 10)$$

$$\begin{aligned}&= (0 + 0.8 \times 0, 0 + 0.8 \times 10) \\ &= (0, 8)\end{aligned}$$

$$\begin{cases} x'_{i+1} = x_{i+1} + \lambda(x_{i+1} - x_i) \dots (1a) \\ y'_{i+1} = y_{i+1} + \lambda(y_{i+1} - y_i) \dots (1b) \end{cases}$$

3. Create the Other 2 Side Branches (Left Branch, Right Branch)

Define a pt $\vec{P}_i(x_i, y_i)$
Rotate this pt. $\vec{P}'_i(x'_i, y'_i)$
By α

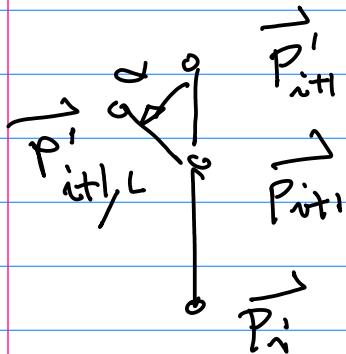


Fig 5.

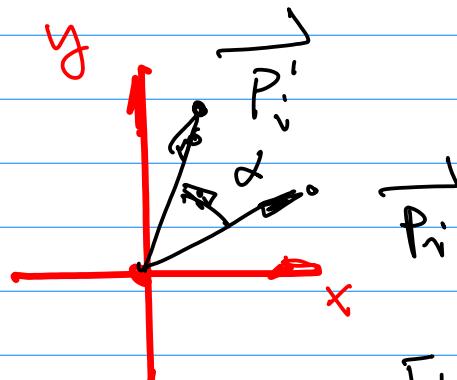


Fig b.

Left Branch: Rotating the main branch (at the same level) counter clockwise by α (Angle)

Denote the New Branch as $\vec{P}'_{i+1,L}$

Note: Need a Newer Math. Formulation for this Rotation.

$$R_{3 \times 3} = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \dots (3)$$

$$\begin{aligned} \vec{P}_i(x_i, y_i) &= (x_i, y_i), \\ \vec{P}_{i+1}(x_{i+1}, y_{i+1}) &= (x_{i+1}, y_{i+1}) \\ \vec{P}_i(x_i, y_i) &= (x_i, y_i)^T = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \end{aligned}$$

↓ Col. Vector

$$\vec{P}_{i+1}^T = \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix}$$

For Simplicity,

$$\vec{P}_i(x_i, y_i) = \begin{pmatrix} x_i \\ y_i \end{pmatrix},$$

$$\vec{P}_{i+1}(x_{i+1}, y_{i+1}) = \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix}$$

CMPE163

$A \cdot \vec{P}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$, Before the Rotation,

$\vec{P}_i \cdot P_i' = \begin{pmatrix} x_i' \\ y_i' \end{pmatrix}$, After the Rotation

?

Now, for the Branch
to the Right,
¹²

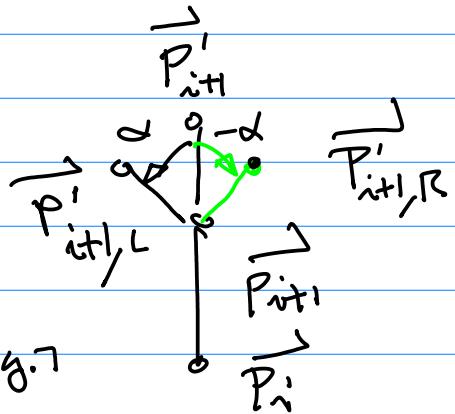


Fig. 7

After

$$\begin{pmatrix} x'_i \\ y'_i \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \quad \dots (3b)$$

Before

$$\begin{pmatrix} x'_{i+1,R} \\ y'_{i+1,R} \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} \quad \dots (5)$$

Therefore,

$$-\sin(-\alpha) = \sin\alpha$$

Rotation x)

Suppose $\alpha = 30^\circ$

$$\begin{cases} x'_i = x_i \cos\alpha - y_i \sin\alpha \dots (4a) \\ y'_i = x_i \sin\alpha + y_i \cos\alpha \dots (4b) \end{cases}$$

Note: 1° Positive α : α in Counter
Clockwise direction;

Negated α : Clockwise

2° The Rotation in (3b)

defines the Rotation

w.r.t the origin of the x-y Coordinate System.

$$\begin{cases} x'_{i+1,R} = \cos\alpha \cdot x'_{i+1} + \sin\alpha \cdot y'_{i+1} \\ y'_{i+1,R} = -\sin\alpha \cdot x'_{i+1} + \cos\alpha \cdot y'_{i+1} \end{cases} \dots (6a)$$

$$\begin{cases} x'_{i+1,R} = -\sin\alpha \cdot x'_{i+1} + \cos\alpha \cdot y'_{i+1} \\ y'_{i+1,R} = -\cos\alpha \cdot x'_{i+1} - \sin\alpha \cdot y'_{i+1} \end{cases} \dots (6b)$$

Team 1 { Christiana 1702,
Anh 3807,
Charles 042b
Ken 3381

Team 2 { Christoffer 7230
Patric 8001
Ammol 9654

Team 3. | Yong 423b
Zhonglin 6764

Semester Long - Project Proposal.

A Paragraph, 50 words, proposal.

Tech. Elements:

1. 3D Graphics
2. Interactive
3. Presentation mode — Script Features
4. Integrate Real/Live Video from
File(s) And/or from a Camera

Guidelines for Topics Selection

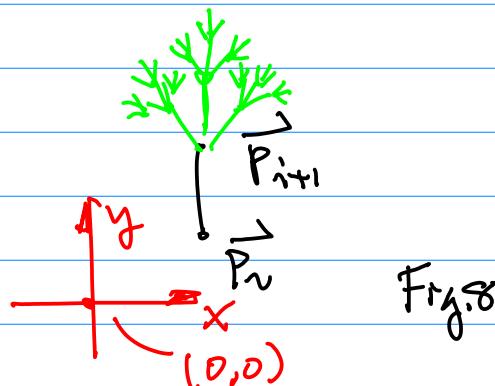
Enhance/Improve Productivity

Promote/Present Better Communication

By Sept. 23rd Submission to Email:
= zhua.li@sjtu.edu

Show + Tell

Note: For more general cases,



Sept 16 (Th)

Topics: 1° 2D Transforms

To Project Implementation
Trees/Forest

Note: Homework (Show+Tell)

Homework (Un-Official)

Rotating Squares Open GL
Reference: On git
github.com/wallli/OpenGL-Computer-~F2018/~2021F-b-...-Homework

(2 pts.)

Sept. and

Homework: Implementation of
Rotating Squares Based on
Euler (2D), on PP8.

Example: On Window.

Note:

1. Always start your
program with a header;

a. Program Name:

Coded by:

Date:

Version:

Status:

Note: (Environment, OS
Compilation & Build)

2. Implementation.

Vector Formulation w/o
Rotation Matrix.

Note: $\sin x$ or $\cos x$

Taylor Expansion (Taylor Series)

$$f(x) = f(x_0) + \frac{f'(x)}{1!}(x-x_0) + \frac{f''(x)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x)}{n!}(x-x_0)^n$$

Look-up table.



3. Change Polygons to Lines

2D vertex is better for this implementation;

4. $\lambda = 0.8$ to begin with, then when finish the entire program execution, modify λ , $\lambda = 0.95$, observe the difference patterns.

+ try $x=0.05$

5. Add Delay.

Frame Rate (Refresh Rate for Display).

30 FPS (Frames per Second)

$$f = 30 \text{ Hz}$$

$$T = \frac{1}{f} = 33.3 \text{ ms}$$

6. Keep one color at time for a set of all rotating squares

7. Create multiple sets of rotating squares, by defining "Anchor point" for each set. Change 2D vertex (x_i, y_i)

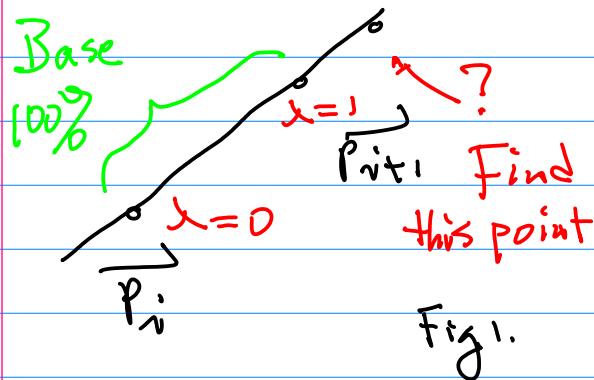
+ to

2D vertex $(x_i + x_0, y_i + y_0)$
where my anchor point is (x_0, y_0)

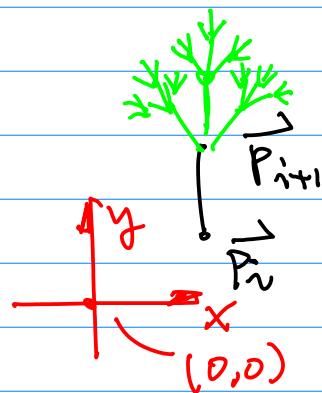
Later, you can randomize the anchor point.

8. Modify the Line width

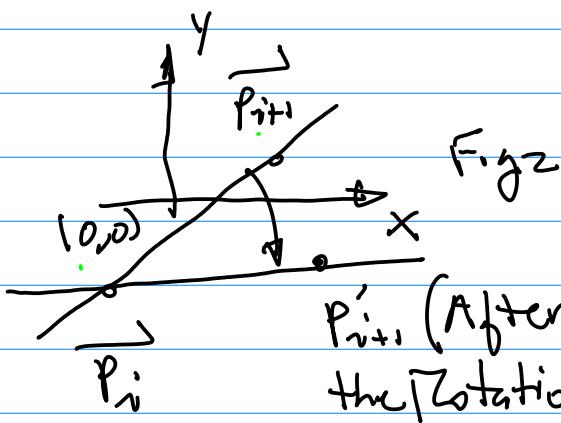
Note for the Homework, 1.3



Now, Consider Question 2



To Rotate the Tree Branches, we want to make sure the rotation is performed w.r.t the origin.



Translate \vec{P}_{i+1} to the origin,
then, Rotation.

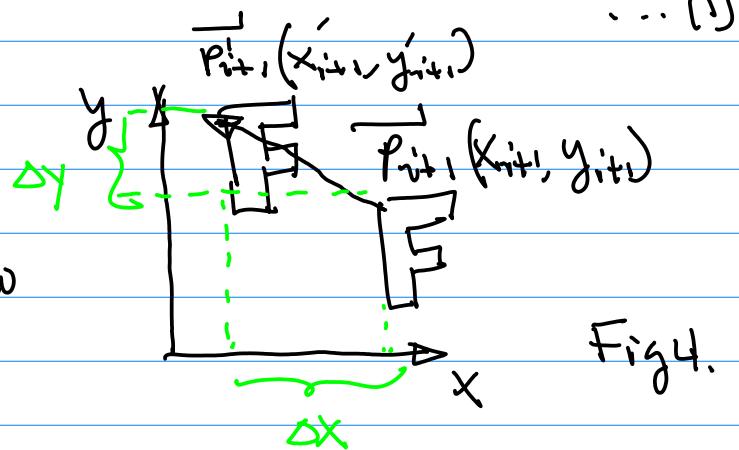
Pre-processing : to move \vec{P}_i to
the origin $(0,0)$

$$x'_{i+1} = x_{i+1} + \Delta x \quad (\text{After}) \quad (\text{Before})$$

then, Rotation;

Post-Processing.

Sept. 23rd: Composition.
Topic: Trees & 2D Transformation
Homework (Due A week)



Composition of 2D Transforms

$$y'_{i+1} = y_{i+1} + \Delta y \dots (1b)$$

$$\begin{pmatrix} x_{i+1}' \\ y_{i+1}' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta X \\ 0 & 1 & \Delta Y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \dots (z)$$

Before)

After

Post Processing to undo the
Pre-processing.

$$T^{-1} = \begin{pmatrix} 1 & 0 & -\Delta X \\ 0 & 1 & -\Delta Y \\ 0 & 0 & 1 \end{pmatrix} \dots (4)$$

$$\Delta X = -10$$

$$\Delta Y = -10$$



Substitute the given condition,
we have

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 1 \end{pmatrix}$$

verified.

$$T^{-1} = \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix}$$

Then, do this for $\vec{P}_2(10, 20)$, find \vec{P}'_2
then, Rotation

$$R = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\alpha = 30^\circ$$

Integrate this into the
Composition matrices, we
have

$$T^{-1} R T = \quad \text{1st applied to the input vector}$$

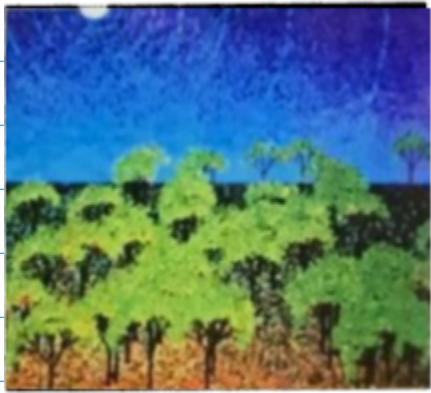
$$\begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 1 \end{pmatrix}$$

... (3)

$$\begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix}$$

... (5)

Then Post Processing,



$$\begin{pmatrix} \cos\alpha & -\sin\alpha & \Delta x \cos\alpha - \Delta y \sin\alpha \\ \sin\alpha & \cos\alpha & \Delta x \sin\alpha + \Delta y \cos\alpha \\ 0 & 0 & 1 \end{pmatrix}$$

Step 2.

$$T^{-1} RT =$$

$$T^{-1} \begin{pmatrix} \cos\alpha & -\sin\alpha & \Delta x \cos\alpha - \Delta y \sin\alpha \\ \sin\alpha & \cos\alpha & \Delta x \sin\alpha + \Delta y \cos\alpha \\ 0 & 0 & 1 \end{pmatrix}$$

Sept. 30 (Thu)

Today's Topics

1° Unity. 2° 3D Computer

Graphics.

Note:

1. New Homework Due A week from

Today

Continue the discussion of Eqn(5).

Step 1. To Take care of $R T$

$$= \begin{pmatrix} 1 & 0 & -\Delta x \\ 0 & 1 & -\Delta y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & \Delta x \cos\alpha - \Delta y \sin\alpha \\ \sin\alpha & \cos\alpha & \Delta x \sin\alpha + \Delta y \cos\alpha \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha & -\sin\alpha & \Delta x \cos\alpha - \Delta y \sin\alpha - \Delta x \\ \sin\alpha & \cos\alpha & \Delta x \cos\alpha + \Delta y \sin\alpha - \Delta y \\ 0 & 0 & 1 \end{pmatrix}$$

Hence,

... (b)

$$\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} \cos\alpha \cdot 1 & -\sin\alpha \cdot 1 & \Delta x \cos\alpha - \Delta y \sin\alpha \\ \sin\alpha \cdot 1 & \cos\alpha \cdot 1 & \Delta x \sin\alpha + \Delta y \cos\alpha \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x'_i \\ y'_i \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & \Delta x \cos\alpha - \Delta y \sin\alpha - \Delta x \\ \sin\alpha & \cos\alpha & \Delta x \cos\alpha + \Delta y \sin\alpha - \Delta y \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

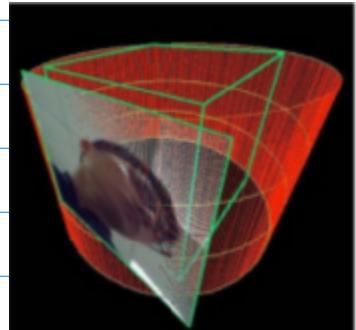
Therefore,

$$\begin{pmatrix} x'_i \\ y'_i \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha - \sin\alpha \cos\delta & \cos\alpha - \delta \sin\alpha & -\delta x_i \\ \sin\alpha \cos\delta & \cos\alpha + \delta \sin\alpha & -\delta y_i \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

↓
After Before

C/C++ Coding, we need

e. Decoration
Texture Mapping



$$\begin{cases} x'_i = \cos\alpha \cdot x_i - \sin\alpha \cdot y_i + \delta x \cos\delta - \delta y \sin\alpha - \delta x \\ y'_i = \sin\alpha \cdot x_i + \cos\alpha \cdot y_i + \delta x \sin\alpha + \delta y \cos\alpha - \delta y \end{cases}$$

$$\begin{cases} x'_i = \cos\alpha \cdot x_i - \sin\alpha \cdot y_i + \delta x \cos\delta - \delta y \sin\alpha - \delta x \dots (7a) \\ y'_i = \sin\alpha \cdot x_i + \cos\alpha \cdot y_i + \delta x \sin\alpha + \delta y \cos\alpha - \delta y \dots (7b) \end{cases}$$

Three Dimensional Graphics. f. Lighting models.

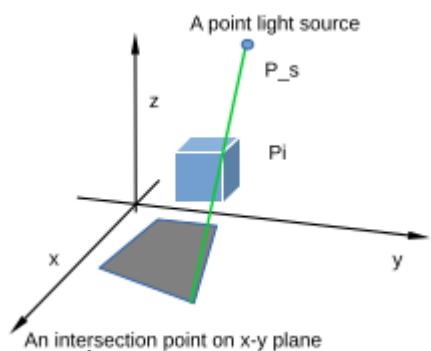
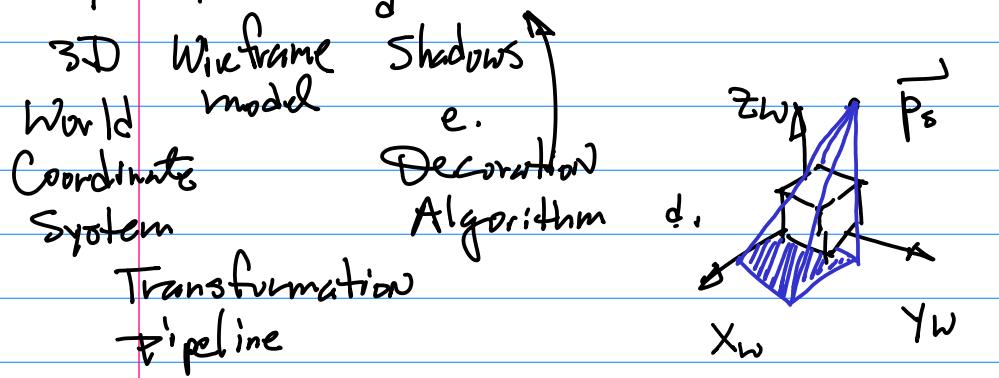
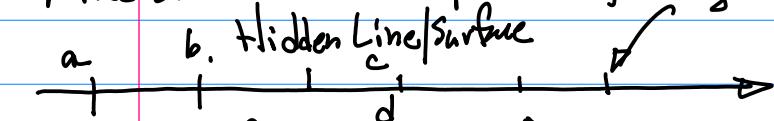
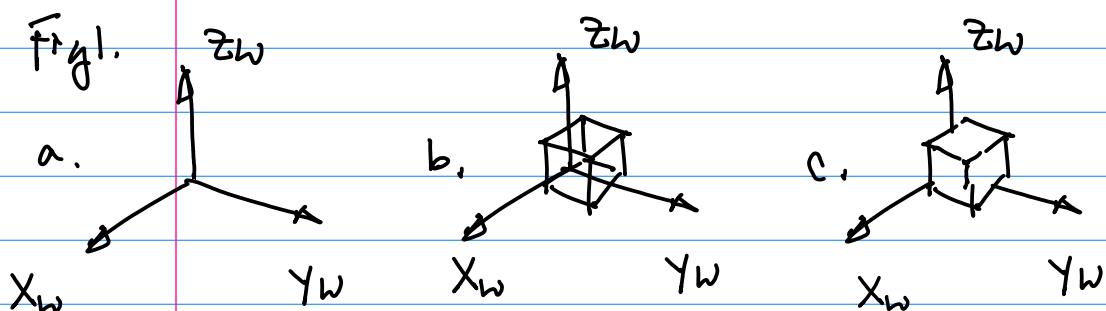
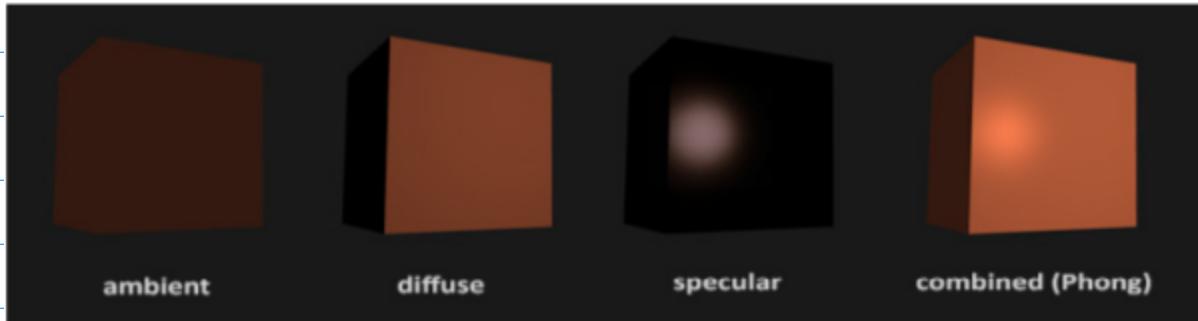


Fig. 1.



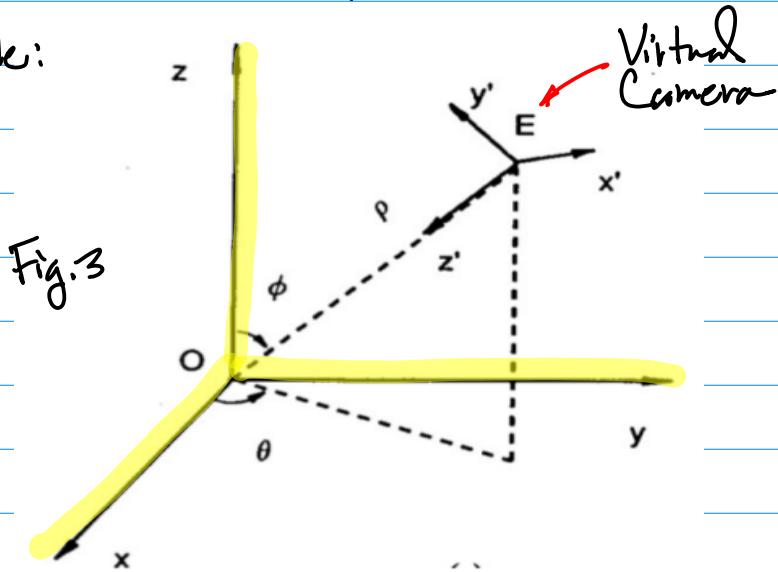
from H. Li's Book
manuscript.

f. Lighting models



World Coordinate System.

Example:



3D Transformation Pipelines

Step 1. Transform of a 3D Object from World Coordinate System to A Viewer Coordinate System.

Step 2. Project the Object from the Viewer Transform to 2D Display Screen
(Perspective Projection)

1° World Coordinate System (Yellow Highlight)

Subscript "w" —> World, $x_w - y_w - z_w$

2° Right-Hand System.

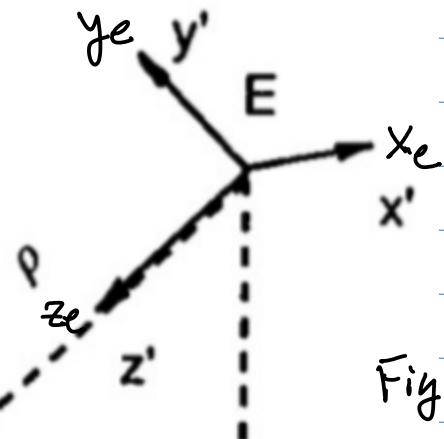
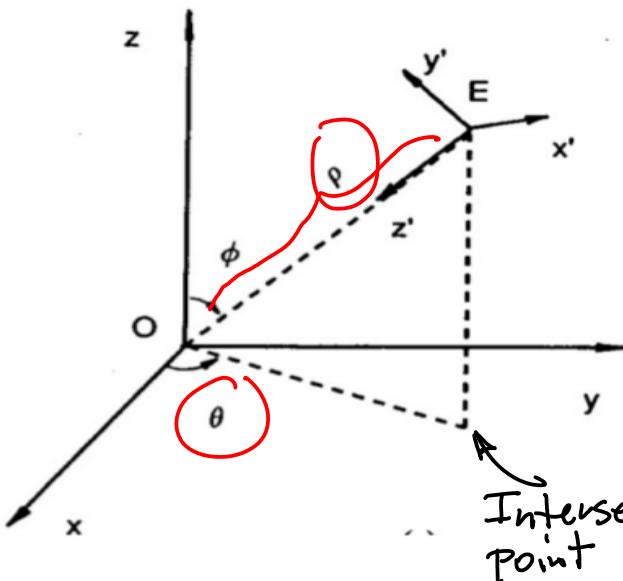
3° Define Everything in a World Coordinate System, including a Virtual Camera located at $E(x_e, y_e, z_e)$ 4° Viewer Coordinate System, a left hand system, $x_e - y_e - z_e'$, as in Fig 4.

Fig. 4

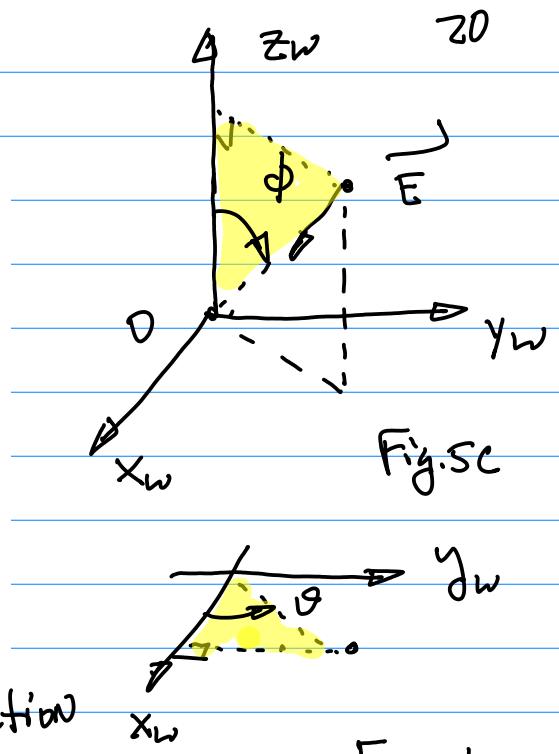
CmpE163

Example:

Figs



Intersection Point



Figs b

$$T = \begin{bmatrix} -\sin \theta & \cos \theta & 0 & 0 \\ -\cos \phi \cos \theta & -\cos \phi \sin \theta & \sin \phi & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & -\cos \phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

World-to-Viewer Transform

θ (Theta): Angle on x_w-y_w plane

... (1)

Formed by 2 vectors, 1st Vector

x_w -axis,

Project \vec{E} onto x_w-y_w plane,

Link the Intersection pt on the

x_w-y_w plane to the Origin $(0,0,0)$

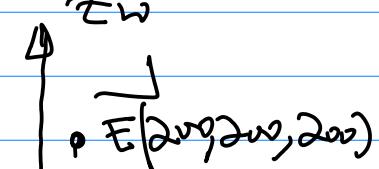
to form the 2nd vector.

Given the $\vec{E}(200, 200,$

$200)$, find World

To Viewer Transform

By Eqn(1).



ϕ (phi): Angle Between z_w and $\vec{E}O$ vector

ρ (rho): Distance, from \vec{E} to the
Origin of the World Coordinate
System.

Sol. From Eqn(1)

We need to find $\theta, \cos\theta, \sin\theta$; $\phi, \sin\phi, \cos\phi$; ρ .
 2. find New point $P_i' (x'_i, y'_i, z'_i)$ After the Transformation.

Oct. 7 (Th)

Tutorials: 1^o 3D Transformation Pipeline

2^o Homework 5 Key's
Presentation of Homework 5.

Homework 5 (1 pt) One week
Submission online (CANVAS)
(3D x_w - y_w - z_w Axis Display)

Example:

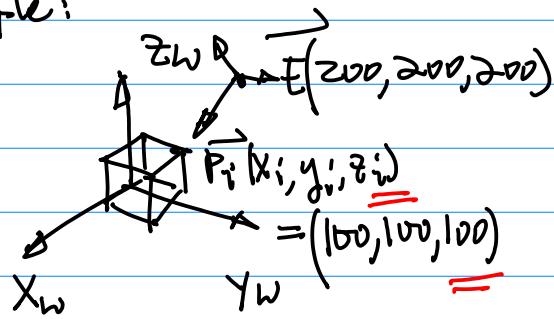


Fig.1.

$\vec{P}_i (x_i, y_i, z_i)$ Before the Transform
(in the world)

$\vec{P}_i' (x'_i, y'_i, z'_i)$ After the Transform.

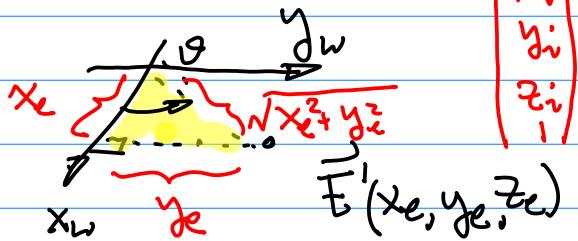
Find: 1. World-To-Viewer

Transformation matrix;

$$\begin{bmatrix} x'_i \\ y'_i \\ z'_i \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & -\cos\phi & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

First, find $\sin\theta$

Fig.2



$E'(x_e, y_e, z_e)$ is formed as an intersection point between a perpendicular line passing through $E(x_e, y_e, z_e)$ and x_w - y_w plane

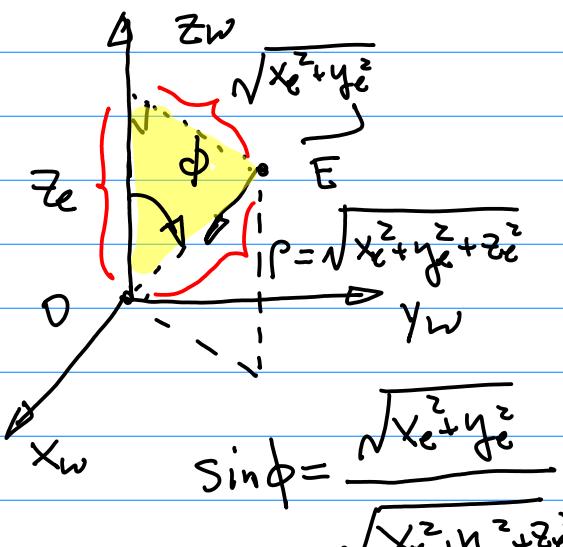
$$\text{Hence, } \sin\theta = \frac{y_e}{\sqrt{x_e^2 + y_e^2}} = \frac{200}{200\sqrt{2}} = \sqrt{2}/2$$

$$\cos\theta = \frac{x_e}{\sqrt{x_e^2 + y_e^2}} = \frac{200}{200\sqrt{2}} = \sqrt{2}/2$$

Similarly, find

$\sin\phi$

Draw a line passing E' perpendicular to z_w Axis to form a Triangle



$$= \frac{2\sqrt{2}\sqrt{2}}{2\sqrt{2}\sqrt{3}} = \sqrt{2}/3 = \sqrt{6}/3$$

$$\cos \phi = \frac{-z_e}{\sqrt{x_e^2 + y_e^2 + z_e^2}} = \frac{-700}{200\sqrt{3}}$$

$$= \sqrt{3}/3$$

$$\begin{bmatrix} -\sin \theta & \cos \theta & 0 & 0 \\ -\cos \phi \cos \theta & -\cos \phi \sin \theta & \sin \phi & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & -\cos \phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{6}}{2} \cdot \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{2} \cdot \frac{\sqrt{2}}{2} & \sqrt{6}/3 & 0 \\ -\frac{\sqrt{6}}{2} \cdot \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{2} \cdot \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{3} & 200\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \sqrt{6}/3 & 0 \\ -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 200\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, the point \vec{P}_i after the transform is \vec{P}'_i , given below

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \sqrt{6}/3 & 0 \\ -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 200\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

From Fig 1
Part 1

$$\text{where } \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 1 \end{bmatrix}$$

$$x'_i = -\frac{\sqrt{2}}{2} \cdot \frac{100}{200} + \frac{\sqrt{2}}{2} \cdot \frac{100}{200} = 0$$

$$y'_i = -\frac{\sqrt{6}}{6} \cdot \frac{100}{200} - \frac{\sqrt{6}}{6} \cdot \frac{100}{200} + \frac{\sqrt{6}}{3} \cdot \frac{100}{200}$$

$$= -\frac{\sqrt{6}}{3} \cdot \frac{100}{200} + \frac{\sqrt{6}}{3} \cdot \frac{100}{200} = 0$$

$$z'_i = -\frac{\sqrt{3}}{3} \cdot \frac{100}{200} - \frac{\sqrt{3}}{3} \cdot \frac{100}{200} - \frac{\sqrt{3}}{3} \cdot \frac{100}{200} + 200\sqrt{3}$$

$$= -\frac{\sqrt{3}}{3} \cdot \frac{100}{200} + 200\sqrt{3} = 100\sqrt{3}$$

Check physical meaning!

C/C++ Implementation. From Eqn(1)

$$\left. \begin{aligned} x'_i &= -\sin \theta \cdot x_i + \cos \theta \cdot y_i \\ y'_i &= -\cos \phi \cos \theta \cdot x_i - \cos \phi \sin \theta \cdot y_i + \sin \phi \cdot z_i \\ z'_i &= -\sin \phi \cos \theta \cdot x_i - \sin \phi \sin \theta \cdot y_i \\ &\quad - \cos \phi \cdot z_i + \rho \end{aligned} \right\} \dots (2)$$

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2nd Step of Transformation Pipeline:

Perspective Projection

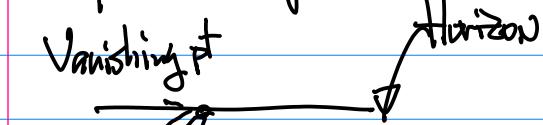


Fig 3a
Depth Perception

onal Computer Graphics
GA or VGA Card
EDUCATION, VOL. 35, NO. 1, FEBRUARY 1992

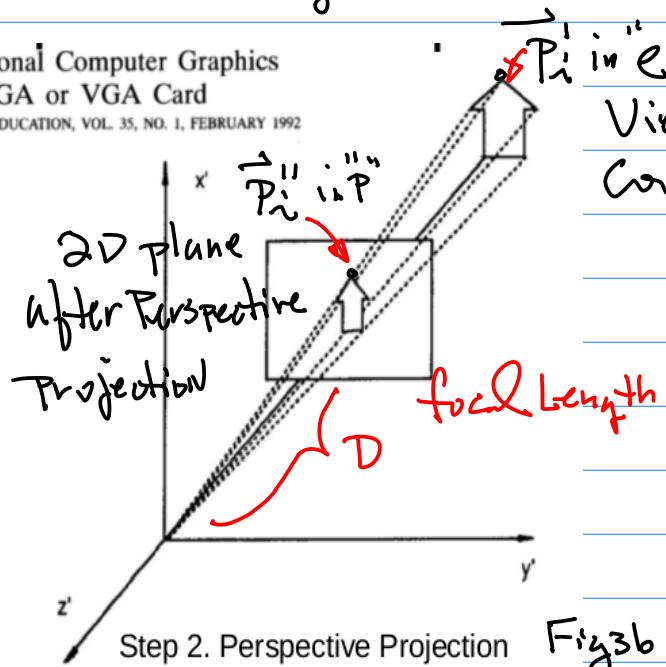


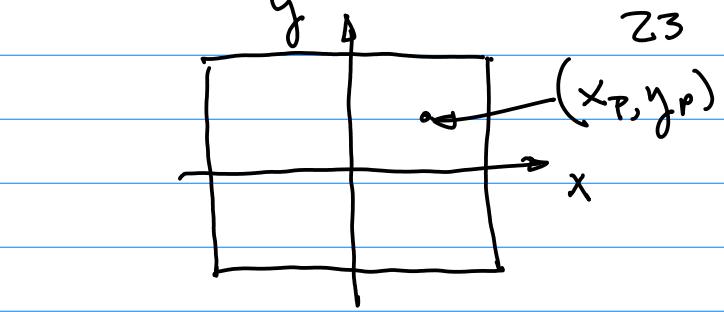
Fig 3b
Step 2. Perspective Projection

$$x_p = x_e \left(\frac{D}{z_e} \right)$$

$$y_p = y_e \left(\frac{D}{z_e} \right)$$

... (3)

where Subscript 'e' for Viewer Coordinate System, 'P' for Perspective Projection



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Example: Given a point \vec{P}_i' (after World to Viewer Transform), Compute Perspective projection

Coordinates S_D

(1) Assume $D=20$ from Eqn (3).

$$x_i'' = x_i' \frac{D}{z_i'} \Big|_{D=20} = \frac{20}{z_i'} \cdot x_i'$$

$$= \frac{20}{100\sqrt{3}} x_i' \Big|_{x_i'=0} = 0$$

$$y_i'' = y_i' \frac{D}{z_i'} = \frac{20}{z_i'} y_i'$$

$$= \frac{20}{100\sqrt{3}} \cdot 0 = 0$$

C/C++ Implementation:

$$\begin{cases} x_i'' = x_i' D / z_i' \\ y_i'' = y_i' D / z_i' \end{cases}$$

Oct.14 (Th)

Note: 1^o Homework Submission

Migrate All Submissions to CANVAS.

(1) Put a list of all the homeworks publish it on CANVAS;

(2) Submit All your homework in One zip. to CANVAS

One week from Today.

Oct 21st.

Midterm Exam: In 2 weeks

Oct. 28th.

Today's Topics :

1^o Examples of (World-To-Viewer, Perspective Projection) Transformation Pipelines.2^o Shadows

Example: Step1 for Homework3.

3. Implement world coordinate system and draw it on the screen.

Submission requirements:

- (1) source code;
- (2) compiled binary;
- (3) 5 second video clips;
- (4) put all in a zip file with the following naming convention:
FirstName-LastName-nameOfProject-4digitsID-YY-MM-DD.zip
for example:
Harry-Li-3Dworld-1116-2021-9-30.zip

Design Data set, e.g. a collection of Vectors, $\vec{P}_0, \vec{P}_1, \dots, \vec{P}_i$ to define the

World Coordinate System

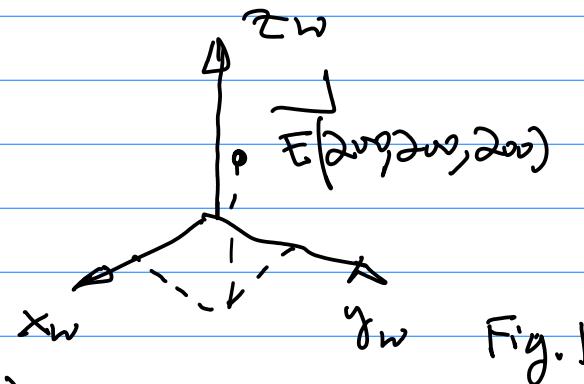


Fig. 1

 $P_i(x_i, y_i)$ to cover all axis

For 2 pts for the x-axis

$$\begin{aligned} \text{Origin } \vec{P}_0 & (x_0, y_0, z_0) = (0, 0, 0) \\ \vec{P}_1 & (x_1, y_1, z_1) = (50, 0, 0) \end{aligned}$$

C/C++ for x-axis: Group \vec{P}_0, \vec{P}_1 togetherfor y_w -axis, 2 pts,

$$\vec{P}_0 (x_0, y_0, z_0) = (0, 0, 0)$$

and

$$\vec{P}_2 (x_2, y_2, z_2) = (0, 50, 0)$$

for z_w -axis, 2 pts

$$\vec{P}_0 (x_0, y_0, z_0) = (0, 0, 0)$$

and

$$\vec{P}_3 (x_3, y_3, z_3) = (0, 0, 50)$$

Step 2. Design/Define a Virtual Camera Location

$$\vec{E}(x_e, y_e, z_e) = (200, 200, 200)$$

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Step 3. First step of the transformation pipeline,
e.g. World To Viewer
Transform From Eqn(1) pp22.

$$T_{w2v} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} & 0 \\ -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 2\sqrt{6}\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x'_i \\ y'_i \\ z'_i \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi & 0 \\ -\sin\phi\cos\theta & -\sin\phi\cos\theta & -\cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} \dots (1)$$

Now, for all the vectors,
 $\vec{P}_0, \vec{P}_1, \dots, \vec{P}_3$

for \vec{P}_0 , we have

Find $\sin\theta, \cos\theta, \sin\phi, \cos\phi, \rho$
from Eqn(1), we can derive 3
equations as follows

$$x'_i = -\sin\theta x_i + \cos\theta y_i$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} & 0 \\ -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 2\sqrt{6}\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\left\{ \begin{array}{l} y'_i = -\cos\phi\cos\theta x_i - \cos\phi\sin\theta y_i + \sin\phi z_i \\ z'_i = -\sin\phi\cos\theta x_i - \sin\phi\sin\theta y_i \\ \quad - \cos\phi z_i + \rho \end{array} \right. \dots (2)$$

$$\therefore \vec{P}_0 = (0, 0, 0)$$

for \vec{P}_1 ,

$$\rho = \sqrt{x_e^2 + y_e^2 + z_e^2}$$

$$\rightarrow \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} & 0 \\ -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 2\sqrt{6}\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 0 \\ 0 \\ 1 \end{bmatrix} =$$

To write the above equation as

in C Code,

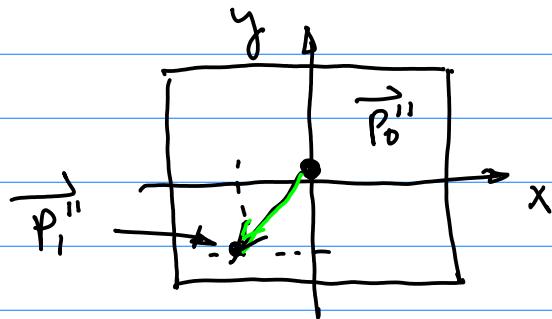
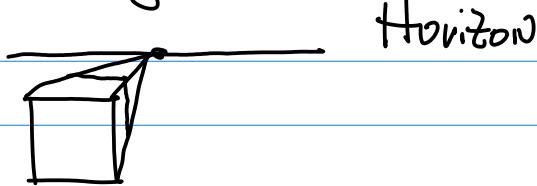
$$x_prim[i] = -\sin(\theta) * x[i] + \cos(\theta) * y[i];$$

$$\begin{bmatrix} -\frac{\sqrt{2}}{2} \cdot 50 \\ -\frac{\sqrt{6}}{6} \cdot 50 \\ -\frac{\sqrt{3}}{3} \cdot 50 + 2\sqrt{6}\sqrt{3} \end{bmatrix} = \begin{bmatrix} -25\sqrt{2} \\ -\frac{25\sqrt{6}}{3} \\ -\frac{\sqrt{3}}{3} \cdot 50 + 2\sqrt{6}\sqrt{3} \end{bmatrix} \dots (3)$$

You can compute for \vec{P}_3 (y_w -axis)
 \vec{P}_3 (z_w -axis)

Background on Perspective Projection

Vanishing Point



Next Step: Perspective Projection

Step 4: Perspective Projection

$$\left\{ \begin{array}{l} x_i'' = \left(\frac{D}{Z_i'} \right) x_i' \quad \dots (4a) \end{array} \right.$$

$$\left\{ \begin{array}{l} y_i'' = \left(\frac{D}{Z_i'} \right) y_i' \quad \dots (4b) \end{array} \right.$$

Since $\theta = 20^\circ$.For $i=0$, \vec{P}_0

$$\left\{ \begin{array}{l} x_0'' = \frac{Z_0}{Z_0'} \cdot x_0' = 0 \quad \dots (5a) \end{array} \right.$$

$$\left\{ \begin{array}{l} y_0'' = \frac{Z_0}{Z_0'} \cdot y_0' = 0 \quad \dots (5b) \end{array} \right.$$

Therefore $\vec{P}_0'' = (0, 0)$ For $i=1$, (x -axis)

$$x_1'' = \frac{D}{Z_1'} \cdot x_1'$$

$$= \frac{Z_0}{(\sqrt{3} \cdot 50 + 20\sqrt{3})} (-25\sqrt{2}) \quad \dots (ba)$$

Similarly, for \vec{P}_1, \vec{P}_2

plot x_w -axis's on 2D Display Screen. Plot the result in Green.

Sample code for Transformation
Pixelizing
 $\vec{E}(x_e, y_e, z_e)$

```
28 float Xe = 100.0f;
29 float Ye = 100.0f;
30 float Ze = 100.0f;
```

https://github.com/hualili/opencv/blob/master/ComputerGraphics_AR/F2018/10-world-20190925.cpp

$$P = \sqrt{x_e^2 + y_e^2 + z_e^2}$$

```
32 float Rho = sqrt(pow(Xe, 2) + pow(Ye, 2) + pow(Ze, 2));
33 float D_focal = 20.0f;
```

$$y_1'' = \frac{D}{Z_1'} \cdot y_1' = \frac{Z_0}{\sqrt{\frac{2}{3} \cdot 50 + 20\sqrt{3}}} \cdot (-25\sqrt{2})$$

Define $\vec{P}_i(x_i, y_i, z_i)$ struct in $x_w y_w z_w$ (World)

```
35 typedef struct {
36     float X[UpperBD];
37     float Y[UpperBD];
38     float Z[UpperBD];
39 } pworld;
```

```

41  typedef struct {
42      float X[UpperBD];
43      float Y[UpperBD];
44      float Z[UpperBD];
45  } pviewer;

```

X_v-Y_v-Z_v
ViewerCoordinate

```

47  typedef struct{
48      float X[UpperBD];
49      float Y[UpperBD];
50  } pperspective;

```

*PointStruct
for Perspective Projection
2D points*

Defining X_w-Y_w-Z_w World Coordinate Below,

```

74      //define the x-y-z world coordinate
75      world.X[0] = 0.0;    world.Y[0] = 0.0;    world.Z[0] = 0.0;    // origin
76      world.X[1] = 50.0;   world.Y[1] = 0.0;    world.Z[1] = 0.0;    // x-axis
77      world.X[2] = 0.0;    world.Y[2] = 50.0;   world.Z[2] = 0.0;    // y-axis
78      world.X[3] = 0.0;    world.Y[3] = 0.0;    world.Z[3] = 50.0;   // z-axis

```

World To Viewer Transform : (3 Equation, one for Each x'_i, y'_i, z'_i)

```

184      for(int i = 0; i <= UpperBD; i++)
185      {
186          viewer.X[i] = -sPheta * world.X[i] + cPheta * world.Y[i];
187          viewer.Y[i] = -cPheta * cPhi * world.X[i]
188          - cPhi * sPheta * world.Y[i]
189          + sPhi * world.Z[i];
190          viewer.Z[i] = -sPhi * cPheta * world.X[i]
191          - sPhi * cPheta * world.Y[i]
192          -cPheta * world.Z[i] + Rho;
193      }

```

$$\begin{cases} x'_i = -\sin\theta x_i + \cos\theta \cdot y_i \\ y'_i = -\cos\phi \cos\theta x_i - \cos\phi \sin\theta y_i + \sin\phi z_i \\ z'_i = -\sin\phi \cos\theta x_i - \sin\phi \sin\theta y_i \\ \quad - \cos\phi z_i + p \end{cases}$$

For Perspective Projection, we have

```

195     for(int i = 0; i <= UpperBD; i++)
196     {
197         perspective.X[i] = D_focal * viewer.X[i] / viewer.Z[i] ;
198         perspective.Y[i] = D_focal * viewer.Y[i] / viewer.Z[i] ;

```

$$\begin{cases} x''_i = \left(\frac{D}{z'_i} \right) x'_i \quad \dots (4a) \\ y''_i = \left(\frac{D}{z'_i} \right) y'_i \quad \dots (4b) \end{cases}$$

Out.21(Th)

Note: 1. Midterm Exam After

Review (All work, 12/14 A

Session), Nov 4th

2. Homework Review.

https://github.com/hualili/opencv/blob/master/ComputerGraphics_AR/F2018/2021F-7-homework-review1-cmpe163-2021-10-16.pdf

Both on GitHub and Submission to
CANVAS.

For Homework 1, Create a new folder with
or Name
Hw-1Review

Homework 2a, Screen Capture.

2.a Homework (no submission needed) Installation of OpenGL.
Auguster, 26, 2021, pp. 6, lecture notes.

Homework: Install OpenGL

Note: Writing OpenGL program to implement the techniques covered in the lecture is required.

① Submit Screen Capture to Show "GL_Lines"

2b. Homework (no submission needed) Installation of OpenGL. assigned date and/or due date:
Auguster, 26, 2021, pp. 6, lecture notes.

has been utilized in your program;

② The program is able to handle Data Points

Beyond Small Range, "Scaling" Capability is Implemented.

Homework: GL_LINES

Modify the Sample code

Note: No Rotation matrix allowed, stay with $\vec{P}(x,y) = \vec{P}_i + t(\vec{P}_{int} - \vec{P}_i)$

3. Homework (submission required). Rotating squares, assigned date and/or due date: Sept 9, 2021, pp. 9, lecture notes, 2021F-2-lecture-note-2021-10-14.pdf (the time stamp on this lecture note will be updated for each new lecture, so look for the document ID: 2021F-2-lecture-note ...)

3. Homework (submission required). Rotating squares, assigned date and/or due date: Sept 9, 2021, pp. 9, lecture notes, 2021F-2-lecture-note-2021-10-14.pdf (the time stamp on this lecture note will be updated for each new lecture, so look for the document ID: 2021F-2-lecture-note ...)

Homework: Implementation of Rotating Squares Based on Equation (2b), on pp 8.

2nd Part
Animation of Generating Rotating Squares.

3. Homework (submission required). Rotating squares, assigned date and/or due date: Sept 9, 2021, pp. 9, lecture notes, 2021F-2-lecture-note-2021-10-14.pdf (the time stamp on this lecture note will be updated for each new lecture, so look for the document ID: 2021F-2-lecture-note ...)

Homework: Implement Rotating Squares I Equation (2b), on

ChptE1b3

Note: Make submission if your team, if your team has not had discussion, just submit "Work in progress". 3D

4. Semester long project proposal (50 words, submission is open till 3D graphics is covered in lecture, e.g., can be submitted before Thanksgiving), Team 1 to Team 3, assigned date and/or due date: Sept 9, 2021, pp. 13, see lecture notes.

5. Homework (submission required). Rotating squares, assigned date and/or due date: Sept 16, 2021, pp. 13, lecture notes, 2021F-2-lecture-note-2021-10-14.pdf (the time stamp on this lecture note will be updated for each new lecture, so look for the document ID: 2021F-2-lecture-note ...)

Note: this assignment carries 2 points, this assignment is the continuation of assignment 3 on Sept 9, it is given again after class demo found some students implementation has some issues. You can submit rotating squares again here if there was some issues in your previous implementation.

Expectation/Requirements: To generate Screen Saver like the one shown in the lecture



6. Homework (submission required). Composition of 2D transformations, assigned date and/or due date: Sept 23, 2021, pp. 15, lecture notes, 2021F-2-lecture-note-2021-10-14.pdf (the time stamp on this lecture note will be updated for each new lecture, so look for the document ID: 2021F-2-lecture-note ...)

Requirements: To Compute/Implement C/C++ for

$$T \rightarrow RT, T = \begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix}, R = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

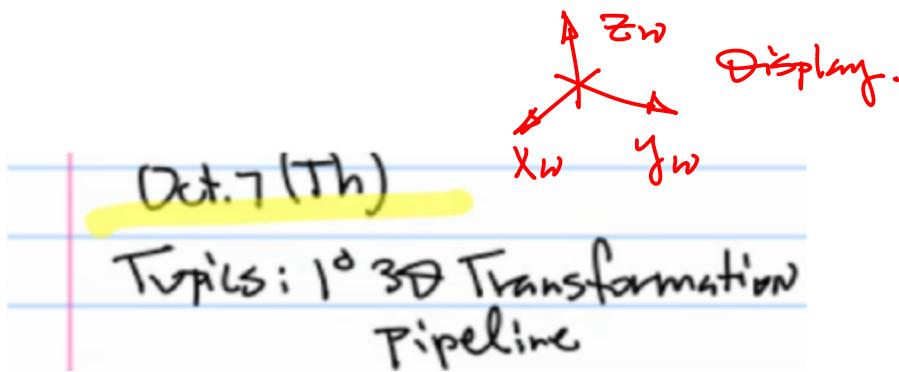
Sept. 23rd

Composition:

$$T^{-1} = \begin{pmatrix} 1 & 0 & -dx \\ 0 & 1 & -dy \\ 0 & 0 & 1 \end{pmatrix}$$

In program: 3 more Lines of code } for-loop
 { X-Comp.
 Y-Comp.

7. Homework (submission required, 1 pt). Plot x_w - y_w - z_w world coordinate, assigned date and/or due date: Oct 7, 2021, pp. 21, lecture notes, 2021F-2-lecture-note-2021-10-14.pdf (the time stamp on this lecture note will be updated for each new lecture, so look for the document ID: 2021F-2-lecture-note ...)



Note: Submission is Today Oct 21 (Thu), No e-mails.
Submission to SJSM CANVAS.

8. Homework (submission required). Review and collect all homework and put them into one zip file, assigned date and/or due date: Oct 44, 2021, pp. 15, lecture notes, 2021F-2-lecture-note-2021-10-14.pdf (the time stamp on this lecture note will be updated for each new lecture, so look for the document ID: 2021F-2-lecture-note ...)

Consider 3D Graphics for Shadow Computation.

Example: Objectives: To produce a shadow based on user defined single point light source.

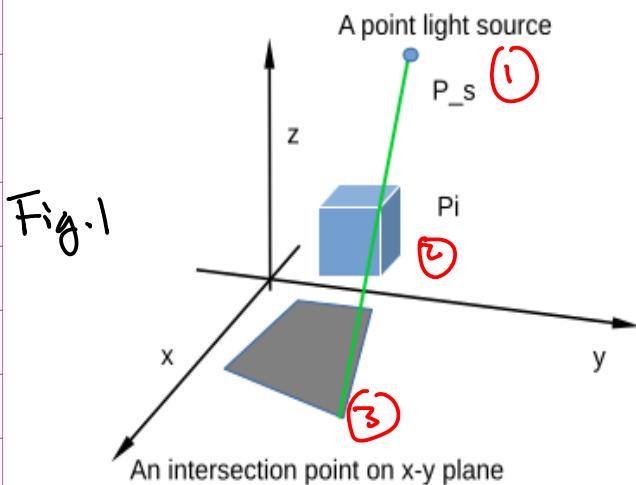


Fig. 1

A point

(1) Light Source

$$\vec{P}_s(x_s, y_s, z_s)$$

Defined in x_w - y_w - z_w coordinate system.

(2) An user defined cube

$$\vec{P}_1(x_1, y_1, z_1), \vec{P}_2(x_2, y_2, z_2)$$

$$\dots, \vec{P}_8(x_8, y_8, z_8)$$

Only interested in the top surface, just 4 pts to consider

$$P_1(x_1, y_1, z_1) = (0, 0, 10), P_2(x_2, y_2, z_2) = (100, 0, 10),$$

$$P_3(x_3, y_3, z_3) = (100, 100, 10), P_4(x_4, y_4, z_4) = (0, 100, 10),$$

$$P_5(x_5, y_5, z_5) = (0, 0, 110), P_6(x_6, y_6, z_6) = (100, 0, 110),$$

$$P_7(x_7, y_7, z_7) = (100, 100, 110), P_8(x_8, y_8, z_8) = (0, 100, 110). \quad (11.1)$$

③ Intersection pts (4 of them)
which form the shadow of the
Cube.

④ Ray Equation formed by Linking
the pt. light Source $\vec{P}_s(x_s, y_s, z_s)$
to one of the Vertices of the top
surface. $\vec{P}_i(x_i, y_i, z_i)$

$$r(x, y, z) = P_i(x_i, y_i, z_i) + \lambda(P_s(x_s, y_s, z_s) - P_i(x_i, y_i, z_i)). \quad (11.2)$$

a. use $\vec{P}_i(x_i, y_i, z_i)$ from
the Cube in this equation;

b. Direction Vector.

$$\vec{P}_s(x_s, y_s, z_s) - \vec{P}_i(x_i, y_i, z_i)$$

DR

$$\vec{P}_i(x_i, y_i, z_i) - \vec{P}_s(x_s, y_s, z_s)$$

Choose $\vec{P}_s - \vec{P}_i$ as a direction

vector.

④ $\vec{r}(x, y, z) = \vec{P}_i(x_i, y_i, z_i) +$
 $\lambda(\vec{P}_s(x_s, y_s, z_s) - \vec{P}_i(x_i, y_i, z_i))$
... (1)

$$x = x_i + \lambda(x_s - x_i) \dots (1a)$$

$$y = y_i + \lambda(y_s - y_i) \dots (1b)$$

$$z = z_i + \lambda(z_s - z_i) \dots (1c)$$

Let define $\vec{P}_s(x_s, y_s, z_s)$

$$= (x_s, y_s, z_s) = (50, 100, 40)$$

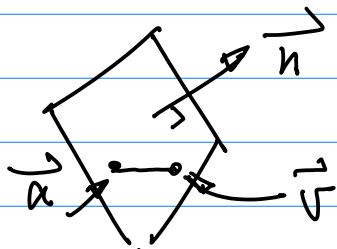
$$\vec{P}_i(x_i, y_i, z_i) = (x_i, y_i, z_i)$$

$$= (100, 100, 10)$$

Question: What λ value do we need to find the intersection point on x_w-y_w plane?

(Intersection pt of the Line equation, e.g. ray equation with x_w-y_w plane.)

Now, let's formulate x_w-y_w plane with a plane equation.



$n\hat{\ } (x_n, y_n, z_n)$ Normal vector of the plane, it defines the

orientation of the plane.

A Known point $\vec{a}(x_a, y_a, z_a)$ on the plane.

An Arbitrary unknown point

$$\vec{v}(x_v, y_v, z_v) = (x_v, y_v, z_v)$$

$\vec{a} - \vec{v}$ form a vector

$$\vec{n} \cdot (\vec{a} - \vec{v}) = 0 \dots (2) \text{ OR}$$

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

Find Intersection point (Shadow point) formed by the Ray Equation and plane Equation

$$\begin{cases} \vec{r}(x, y, z) = \vec{P}_i(x_i, y_i, z_i) + \lambda(\vec{P}_s(x_s, y_s, z_s) - \vec{P}_i(x_i, y_i, z_i)) \\ \vec{n} \cdot (\vec{r} - \vec{a}) = 0 \end{cases} \dots (3a)$$

... (3b)

(Ref: H.Li, Book, Chapter II) pp.140
(n-progress)

$$\begin{cases} r(x, y, z) = P_7(x_7, y_7, z_7) + \lambda(P_s(x_s, y_s, z_s) - P_7(x_7, y_7, z_7)) \\ n(v - a) = 0. \end{cases}$$

Question: what is $r(x, y, z)$ in Eqn(3)? "Ray", from the Ray Equation. Eqn(1).

Question: what is \vec{n} in Eqn(3b).

Normal vector.

$$\vec{n}(x_n, y_n, z_n) = (0, 0, 1)$$

$\vec{a}(0,0,0)$. On the X_w-Y_w plane

Question: What is the relationship

Between \vec{v} in Eqn(3b) and \vec{r} in Eqn(3a)? $\vec{v} = \vec{r}$

from Eqn(3b)

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

$$\vec{v} = \vec{r}$$

therefore

$$\vec{n} \cdot (\vec{v} - \vec{a}) = 0$$

$$\vec{v} = \vec{p}_i + \lambda(\vec{p}_s - \vec{p}_i)$$

$$n(v-a)|_{v=P_7(x_7,y_7,z_7)+\lambda(P_s(x_s,y_s,z_s)-P_7(x_7,y_7,z_7))}=0$$

$$\vec{n} \cdot (\vec{p}_i + \lambda(\vec{p}_s - \vec{p}_i) - \vec{a}) = 0$$

$$\vec{a} = (0,0,0)$$

$$\vec{n} \cdot (\vec{p}_i + \lambda(\vec{p}_s - \vec{p}_i)) = 0 \quad \dots (4)$$

Oct. 28 (Th).

Topics: 1^o Review for the mid-term

2^o 3D Shadow Computation

Midterm Exam Review.

1. 2-3 Questions, 1 hr

15 min. (Extra) To prepare

Paper Submission. (After 15 min. CANVAS will be closed)
No paper will be accepted.

2. Video UV is mandatory requirement. No Black Screen allowed.

If network/Zoom issue, then text me immediately
(650) 400-1116.

Question on Basic Concepts.

2D Vector Graphics, 3D Vector Graphics.

$$\vec{f}(x, y) = \vec{p}_i(x_i, y_i) + \lambda(\vec{p}_{i+1}(x_{i+1}, y_{i+1}) - \vec{p}_i(x_i, y_i))$$

for Starting pt? for Ending

Point? Any Arbitrary points Between the starting point and the end point?

2D Rotating patterns, $\lambda = ?$

2D Transformations, T_{3x3} Translation

Rotation R_{3x3} , Composition of 2D Transformations.

Rotation wrt Any Arbitrary point

$$\bar{R}^{-1} R T$$

\bar{R} Preprocessing

Post Processing.

Transformation Pipeline :

World-to-Viewer Transform
Perspective Projection

T_{WV} . (θ, ϕ, ρ Parameters)
 $\cos\theta, \sin\theta$ Virtual CAM
Triangle on Location
 $x_w - y_w - z_w$ plane.
 $\sin\phi, \cos\phi$ z_w and $\vec{E}(x_e, y_e, z_e)$ form

2nd Triangle.

$$\rho = \sqrt{x_e^2 + y_e^2 + z_e^2}$$

Perspective Projection. Convert 3D Data in $x_e - y_e - z_e$ into 2D Data (Display Screen).

D: focal length from the lens to the projection plane; \rightarrow Interpretation of projection result.

D' , projected Graphics

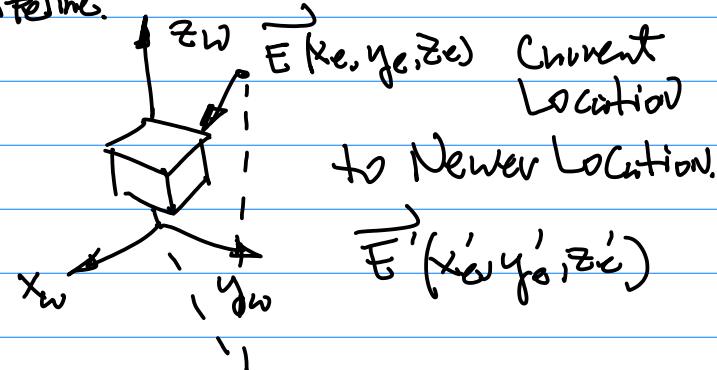
z'_v ("'" indicates this z_v

is defined in the Viewer Coordinate System). Z'_v , Graphics

Design Related Questions.

2D Graphics Design Related ~
3D Graphics Design Related ~

Typical 3D Design is tied to Transformation Pipeline.



$$\vec{E}(x_e, y_e, z_e) = (200, 200, 200) \rightarrow$$

$$\vec{E}'(x'_e, y'_e, z'_e) = (200, -100, -50)$$

Find T_{WV} (World-to-Viewer) and perspective projection

Draw the projected cube on your paper.

3D Shadow: Ray Equation.

$$\vec{r}(x, y, z) = \vec{P}_i(x_i, y_i, z_i) + \lambda (\vec{P}_s(x_s, y_s, z_s) - \vec{P}_i(x_i, y_i, z_i))$$

point light source $\vec{P}_s(x_s, y_s, z_s)$

\vec{n} Normal vector

$$(n_x, n_y, n_z), \text{ or } (x_n, y_n, z_n)$$



$$\text{Plane Eqn: } \vec{n} \cdot (\vec{r} - \vec{a}) = 0$$

$(\vec{r} - \vec{a})$: A vector, e.g. a

Line Segment on plane π

\vec{a} An Arbitrary known

vector.

\vec{r} An arbitrary unknown vector on the plane.

Unity 3D Graphics.

Version Number?

How to Create a Project?

How to Save a Project?

How to Create 3D

World Coordinate System?

Placing x_w, y_w, z_w axis

How to Virtual Camera Location?

How to place a Cube in

$x_w-y_w-z_w$?

Total 3D points, 30%.

Example: Continuation of previous discussion, so

$$\vec{n} \cdot (\vec{p}_i + \lambda (\vec{p}_s - \vec{p}_i)) = 0$$

$$\vec{n} \cdot \vec{p}_i + \lambda \vec{n} \cdot (\vec{p}_s - \vec{p}_i) = 0$$

$$\lambda \vec{n} \cdot (\vec{p}_s - \vec{p}_i) = -\vec{n} \cdot \vec{p}_i$$

$\underbrace{\lambda}_{\text{Scalar}}$

$$\frac{\lambda \vec{n} \cdot (\vec{p}_s - \vec{p}_i)}{\vec{n} \cdot (\vec{p}_s - \vec{p}_i)} = \frac{-\vec{n} \cdot \vec{p}_i}{\vec{n} \cdot (\vec{p}_s - \vec{p}_i)}$$

$$\lambda = -\frac{\vec{n} \cdot \vec{p}_i}{\vec{n} \cdot (\vec{p}_s - \vec{p}_i)} \quad \dots (5)$$

which defines the intersection point.

In C/C++, we have

$$\lambda = -\frac{\vec{n} \cdot \vec{p}_i}{\vec{n} \cdot (\vec{p}_s - \vec{p}_i)} = -\frac{(n_x, n_y, n_z) \cdot (x_i, y_i, z_i)}{(n_x, n_y, n_z) \cdot (x_s - x_i, y_s - y_i, z_s - z_i)}$$

$$= -\frac{n_x \cdot x_i + n_y \cdot y_i + n_z \cdot z_i}{n_x \cdot (x_s - x_i) + n_y \cdot (y_s - y_i) + n_z \cdot (z_s - z_i)} \quad \dots (5b)$$

Note: λ is not an intersection point. it is a parameter in the ray equation which gives intersection.

$$\vec{r} = \vec{p}_i + \lambda (\vec{p}_s - \vec{p}_i) \quad \left| \begin{array}{l} \lambda = -\frac{\vec{n} \cdot \vec{p}_i}{\vec{n} \cdot (\vec{p}_s - \vec{p}_i)} \end{array} \right.$$

which gives the intersection point.

To Do After the Midterm:

Each individual will present his/her Unity part of the Homework in Class.