CmpE258 Spring 2023 (Part III)

1/

April 20 (Thursday).

Cluster Analysis. "Mapping"

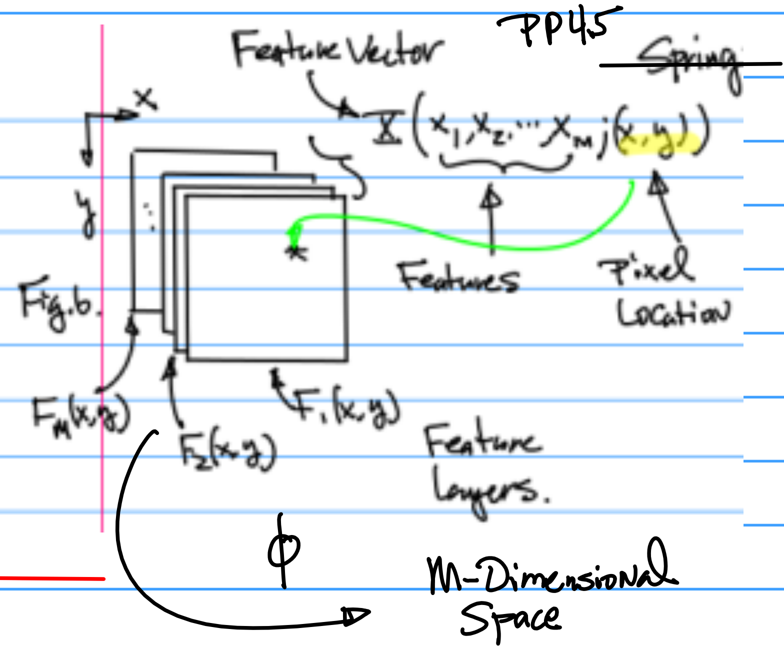
Note 1. Final Exam.

Group II Classes

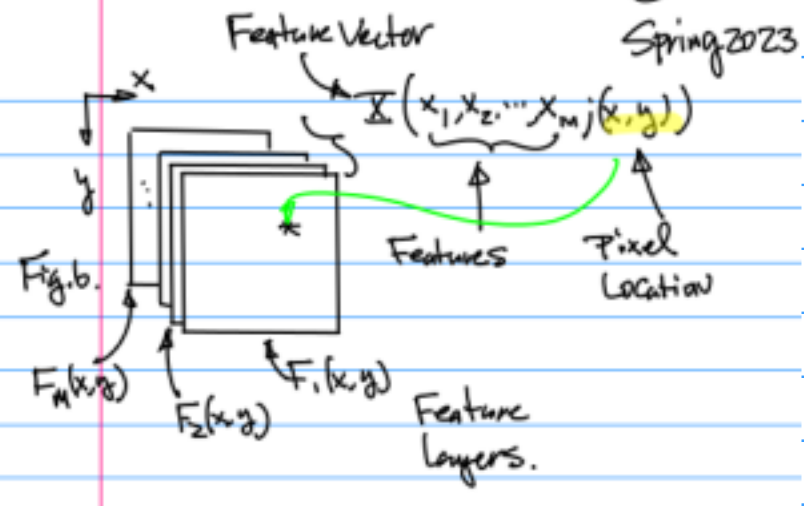
Group II classes are those classes which meet TR, T, R, TWR, MTR, TRF, MTI MTWR, TWRF, RF, TF, TRS.

Regular Class Start Times	Final Examination Days	Final Examination
7:00 through 8:25 AM	Monday, May 22	7:15-9:30 AM
8:30 through 9:25 AM	Wednesday, May 17	7:15-9:30 AM
9:30 through 10:25 AM	Friday, May 19	9:45 AM-12:00 PM
10:30 through 11:25 AM	Tuesday, May 23	9:45 AM-12:00 PM
11:30 AM through 12:25 PM	Thursday, May 18	9:45 AM-12:00 PM
12:30 through 1:25 PM	Monday, May 22	12:15-2:30 PM
1:30 through 2:25 PM	Wednesday, May 17	12:15-2:30 PM
2:30 through 3:25 PM	Friday, May 19	2:45-5:00 PM
3:30 through 4:25 PM*	Tuesday, May 23	2:45-5:00 PM
4:30* through 5:25 PM*	Thursday, May 18	2:45-5:00 PM

PP45 ~~Spring~~



From Notes Part II, PP.45



M=3

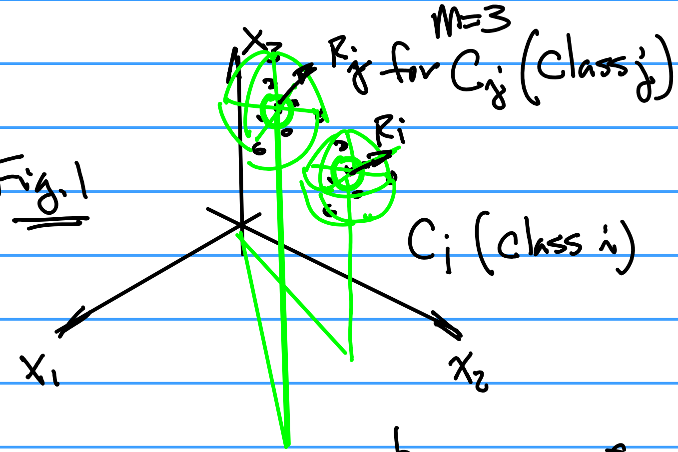
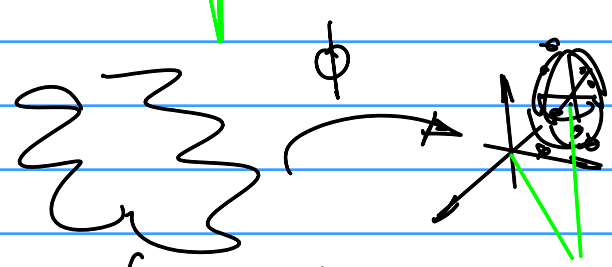


Fig.1



$\underline{X}(x_1, x_2, \cdots x_N)$

$\underline{X}(x_1, x_2, \cdots x_M; (x, y))$

$\downarrow$

Cluster Analysis Technique.

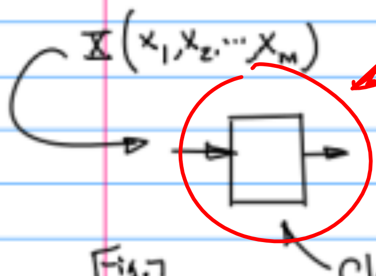


Fig.7

$I(x, y) \rightarrow$ Feature $\xrightarrow{\phi}$ Classification

pixel(s) $\quad \underline{X}(x_1, x_2, \cdots x_N) \quad C_i$

Prob(C_i)

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Note: $\sum_{i=1}^{N} \text{Prob}(C_i) = 1$ (Previously M)

N: Number of Classes.
from "Heuristics" Expert Knowledge.

Consider K-mean Cluster Algorithm.

github.

2022S-114c-Kmean-handCalculation...

2022S-114c-KmeanCluster-v3-2022-... ← PPT

Example: K-mean Cluster Algorithm.

First, Notation. Note 1. Vectors

Given a set of observations $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$,

e.g. $\underline{X}_1 = (x_{11}, x_{12}, \cdots, x_{1N})$ ...(1)
first observation

For $\underbrace{\underline{X}_i}_{\text{Observation } i}$ $\quad x_{ij}$

Component j for the Observation i

Note: if for d-dimensional Vector, then Eqn(1) has its $N=d$

K-mean: partition the n observations into k ($\leq n$) sets $\mathbf{S} = \{S_1, S_2, \ldots, S_k\}$ ...(2)

$\{\underline{X}_i \mid i = 1, 2, \cdots, N\}$

K-Classes. $\quad S_i$ for Class i

Note 1. Note 2. Squared Distance

$$\underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} \sum_{\mathbf{x} \in S_i} \|\mathbf{x} - \boldsymbol{\mu}_i\|^2 = \underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} |S_i| \operatorname{Var} S_i \quad \ldots(3)$$

a. "Argmin" minimization
b. "S" Domain, Scope of the minimization

Example for $\|\vec{x} - \vec{\mu}_i\|^2$,
if $\vec{x} = (x_1, x_2)$, $\vec{\mu}_i = (\mu_{i1}, \mu_{i2})$
then

$$\|\vec{x} - \vec{\mu}_i\|^2 = \left(\sqrt{(x_1 - \mu_{i1})^2 + (x_2 - \mu_{i2})^2}\right)^2$$

$$= (x_1 - \mu_{i1})^2 + (x_2 - \mu_{i2})^2 \quad \ldots(4)$$

Note 3. "$\sum_{x \in S_i}$" Notation $\sum_{i=1}^{M} x_i$

Summation for each & every x as Long as it is from the set S_i

Note 4: from Eqn (3), we have
S_i : Collection of Vectors $\underline{X}$ belonging to Class i

Note 5: $\sum_{i=1}^{k}$ → to Cover the Collection of all Classes.

Note 6: $\vec{\mu}_i$ ① $\vec{\mu}_i = (\mu_{i1}, \mu_{i2}, \cdots, \mu_{id})$
② Cluster for the Class i

April 25 (Tue)

Note 1. Quick update on Project Progress Report. (Next Lecture)

Example: Continuation of K-mean Cluster Algorithm.

$$S_i^{(t)} = \{x_p : \|x_p - m_i^{(t)}\|^2 \le \|x_p - m_j^{(t)}\|^2 \ \forall j, 1 \le j \le k\} \quad \dots (1)$$

Ref

2022S-114c-Kmean-handCalculation1-(

Note 1: A Set of Feature Vectors

$S_i^{(t)}$ ← Captured at Step t

i ← Class id: ith Class

$S_i^{(t)} = \{\vec{X}_p \quad \}$

index, $p = 1, 2, \dots$

just like Notation i, j, or k

$S_i^{(t)} = \{\vec{X}_p : \quad \}$

Condition

$\|\vec{X}_p - \vec{m}_i^{(t)}\|^2$

Distance (Squared) at time (t) to the Cluster of class i

$\|\vec{X}_p - \vec{m}_j^{(t)}\|^2$

... ... to the Cluster class j

"$\forall j$" for Any j, such as

$1 \le j \le k$ Covers all the different classes.

Hand Calculation Example

Given the following feature vectors use K-mean Algorithm to find the clusters.

$X_1 = \begin{bmatrix}0\\0\end{bmatrix}$ $X_2 = \begin{bmatrix}1\\0\end{bmatrix}$ $X_3 = \begin{bmatrix}0\\1\end{bmatrix}$ $X_4 = \begin{bmatrix}1\\1\end{bmatrix}$

$X_5 = \begin{bmatrix}2\\1\end{bmatrix}$ $X_6 = \begin{bmatrix}1\\2\end{bmatrix}$ $X_7 = \begin{bmatrix}2\\2\end{bmatrix}$ $X_8 = \begin{bmatrix}3\\2\end{bmatrix}$

$X_9 = \begin{bmatrix}6\\6\end{bmatrix}$ $X_{10} = \begin{bmatrix}7\\6\end{bmatrix}$ $X_{11} = \begin{bmatrix}8\\6\end{bmatrix}$ $X_{12} = \begin{bmatrix}6\\7\end{bmatrix}$

$X_{13} = \begin{bmatrix}7\\7\end{bmatrix}$ $X_{14} = \begin{bmatrix}8\\7\end{bmatrix}$ $X_{15} = \begin{bmatrix}9\\7\end{bmatrix}$ $X_{16} = \begin{bmatrix}7\\8\end{bmatrix}$

$X_{17} = \begin{bmatrix}8\\8\end{bmatrix}$ $X_{18} = \begin{bmatrix}9\\8\end{bmatrix}$ $X_{19} = \begin{bmatrix}8\\9\end{bmatrix}$ $X_{20} = \begin{bmatrix}9\\9\end{bmatrix}$

Sol: Step 1. Define $K=2$ per Heuristics. Expert Knowledge

Note: "0" Initial step.

Let cluster

$\vec{m}_1^0 = \vec{X}_1 = \begin{pmatrix}0\\0\end{pmatrix} \quad \dots (1)$

$\vec{m}_2^0 = \vec{X}_2 = \begin{pmatrix}1\\0\end{pmatrix} \quad \dots (2)$

Initial Arbitrary Values

"1" Class 1

And Arbitrarily assign Feature Vectors into 2 Classes.

Step 2. Use Eqn (1) To Compute the distance

$\|\vec{X}_p - \vec{m}_i^{(t)}\|^2$

and $\|\vec{X}_p - \vec{m}_j^{(t)}\|^2$

To Evaluate the Grouping of $\vec{X}_p$ to the Class i per Eqn (1).

If Eqn(1) holds good, then $\vec{X}_p$ Stays in the Class i.
O/w Re-assign $\vec{X}_p$ to the Class j.

Step 3. Update the Cluster (when New Grouping is formed)

$$m_1^{(t)}\Big|_{t=1} = \frac{1}{N}\sum_{\vec{X}_p \in S_1} \vec{X}_p$$

$$m_i^{(t+1)} = \frac{1}{|S_i^{(t)}|}\sum_{x_j \in S_i^{(t)}} x_j \quad \ldots (2)$$

$\vec{X}_j$

Total Number of Feature Vectors in the Class i

Step 4. Carry out the Computation with the New Cluster.
(To Decide if the grouping is final oR to Continue updating Cluster Values)

Note: "Stop" if No Regrouping
O/w. Continue By Repeating the process, e.g. update Cluster Values, the Evaluate the grouping.

Step 5. Perform the Computation as Described in Step 4. which Leads to

C_1 (Class 1): $S_1 = \{\vec{X}_1, \vec{X}_2, \cdots, \vec{X}_8\}$

$S_2 = \{\vec{X}_9, \vec{X}_{10}, \cdots, \vec{X}_{20}\}$

then, Update the Cluster $\vec{m}_1^{(t)}, \vec{m}_2^{(t)}$.

↓

Check. No New Grouping

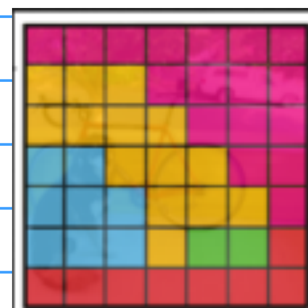
↓

$\vec{m}_1^{(t+1)}, \vec{m}_2^{(t+1)}$ are the Same.

$\therefore$ Stop. (Converged).

Discussion On Probability Distribution Map.

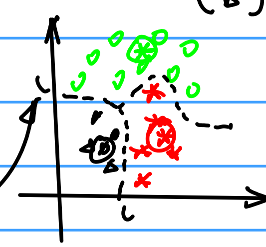
(a)



Class probability map

Feature Vector Map.

(b)



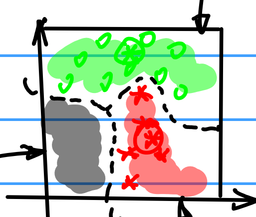
Boundary for the Classification

Regular K-mean Cluster Algorithm

(c)

C_3

C_1



C_2

Fig. 1

$$\text{Prob}(C_1) \stackrel{?}{=} \frac{\text{Area of Black Pixels}}{\text{Area of } \Omega \text{ (Image Plane)}} \quad \ldots (3a)$$

Where Area of Black Pixels
Can be computed.

Area of Ω(Image Plane) = Resolution of the image plane.
For Example, 448 x 448

Similarly, find

$$Prob(C_2) = \frac{\text{Area of Red Pixels}}{\text{Area of } \Omega(\text{Image Plane})} \quad \ldots (3b)$$

$$Prob(C_3) = \frac{\text{Area of green Pixels}}{\text{Area of } \Omega(\text{Image Plane})} \quad \ldots (3c)$$

then

$$\left.\sum_{i=1}^{N}\right|_{N=3} Prob(C_i) = 1 \quad \ldots (4)$$

Since

$$\sum_{i=1}^{3} Prob(C_i) = Prob(C_1) + Prob(C_2) + Prob(C_3) \quad \ldots (4\text{-}b)$$

$$= \frac{\text{Area of Black Pixels}}{\text{Area of } \Omega(\text{Image Plane})} + \frac{\text{Area of Red Pixels}}{\text{Area of } \Omega(\text{Image Plane})} + \frac{\text{Area of green Pixels}}{\text{Area of } \Omega(\text{Image Plane})}$$

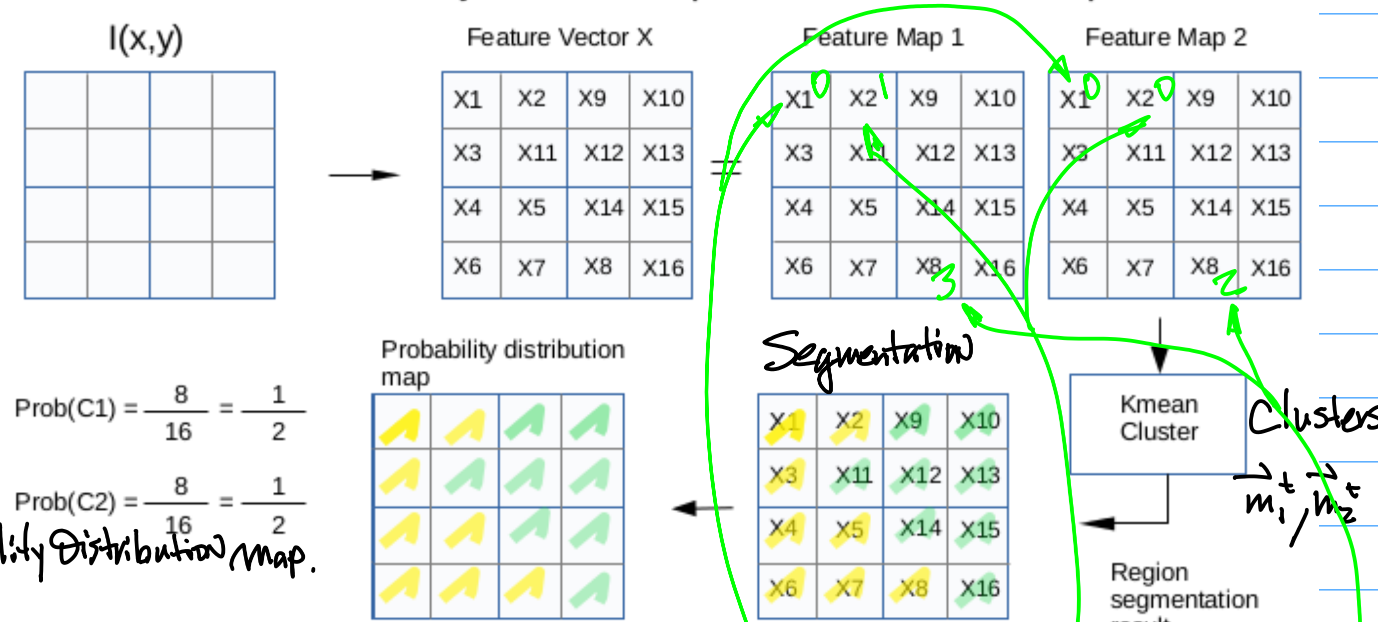
$$= \frac{\text{Area of } \Omega(\text{Image Plane})}{\text{Area of } \Omega(\text{Image Plane})} = 1. \; //$$

(The following Notes were added After the Class After Recovering from the Laptop Computer Shut down. For Additional Lecture Notes Check the Class Zoom Recording).

2022S-114c-Kmean-prob-map-hl-2023-4-26.pdf

Step 1. Feature Vectors

Probability Distribution Map and Kmean Cluster Technique



$$Prob(C1) = \frac{8}{16} = \frac{1}{2}$$

$$Prob(C2) = \frac{8}{16} = \frac{1}{2}$$

From Probability Distribution Map.

Prob(Ci) = Prob(Ci/Obj) Prob(Obj) … (1)

Prob(Ci/Obj) = Prob(Ci) / Prob(Obj) … (2)

Application Example: Given ROI_i(x,y) with feature vector X, find it Prob(Ci/Obj) = ? by using equation (2)

Harry Li, Ph.D., SJSU

$X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $X_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $X_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $X_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$X_5 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $X_6 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $X_7 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $X_8 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

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$X_{13} = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$ $X_{14} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$ $X_{15} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$ $X_{16} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

$X_{17} = \begin{bmatrix} 8 \\ 8 \end{bmatrix}$ $X_{18} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$ $X_{19} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$ $X_{20} = \begin{bmatrix} 9 \\ 9 \end{bmatrix}$

Objective: To find the Probability of an Detected object Belonging to Class i.

Requirements: 1° Hand Calculation of K-mean Cluster Algorithm;

2° Use K-mean Algorithm to Perform Image Segmentation, then Find Probability Distribution Map.

CMPE258

Spring2023

Note1: Loss function Composition: $f_{Loss} = \sum_{i=1}^{K} \lambda_i f_{Loss,i}$ ...(1)

where $\sum_{i=1}^{K} \lambda_i = 1$.

No. of Bounding Boxes on a grid S

Entire Image

$$\lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{obj}} \left[(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 \right]$$ Location

$$+ \lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{obj}} \left[\left(\sqrt{w_i} - \sqrt{\hat{w}_i} \right)^2 + \left(\sqrt{h_i} - \sqrt{\hat{h}_i} \right)^2 \right]$$ Shape

λ'

$$+ \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{obj}} \left(C_i - \hat{C}_i \right)^2$$ Confidence

$$+ \lambda_{\text{noobj}} \sum_{i=0}^{S^2} \sum_{j=0}^{B} \mathbb{1}_{ij}^{\text{noobj}} \left(C_i - \hat{C}_i \right)^2$$

λ''

$$+ \sum_{i=0}^{S^2} \mathbb{1}_{i}^{\text{obj}} \sum_{c \in \text{classes}} (p_i(c) - \hat{p}_i(c))^2$$ Probability Distribution

Note2:

Unit function

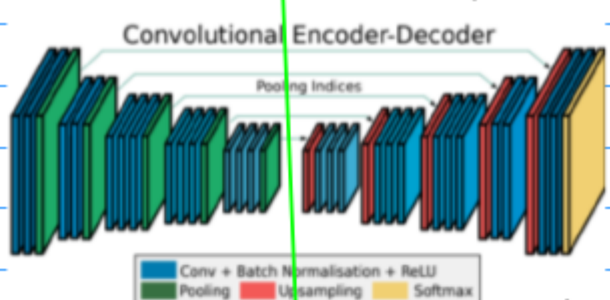
$$\mathbb{I}_{ij}^{obj} = \begin{cases} 1 & \text{if Object exists} \\ 0 & \text{o/w} \end{cases} \quad ...(2)$$

Location 50%, Shape 20%, ... etc.

Semantic Segmentation

2022F-109-semantic-seg-part1-HL-2022- Ref: PPT.

PP52 Fig.3



Deep Convolutional Neural Network:

Decoder:

Encoder:

Feature Extraction — Convolutions

Classification — Feedforward NN

Target Pixel by pixel

Recognition of Object